Hypergraph Representation via Axis-Aligned Point-Subspace Cover

Oksana Firman Joachim Spoerhase

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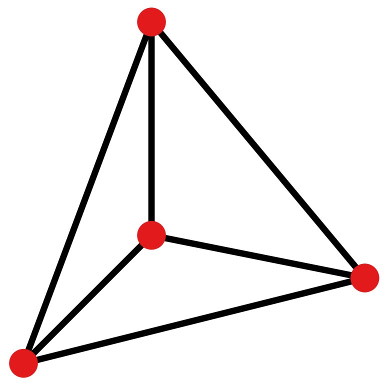
Geometric Representation of (Hyper)graphs



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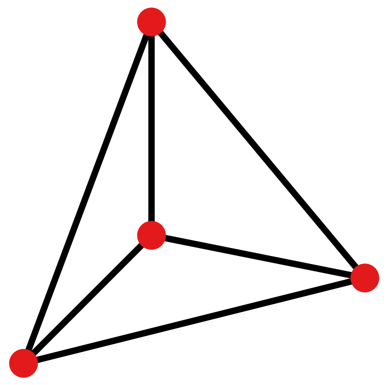
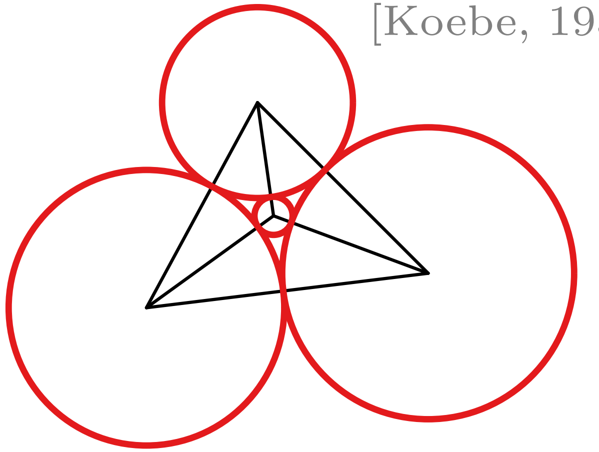


[Koebe, 1936]



contact representation

by discs

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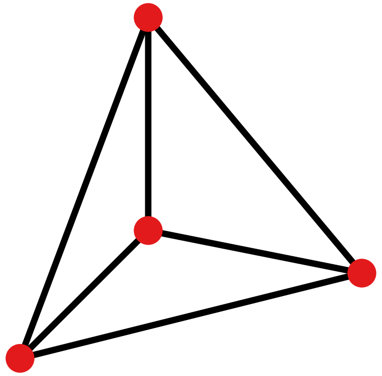

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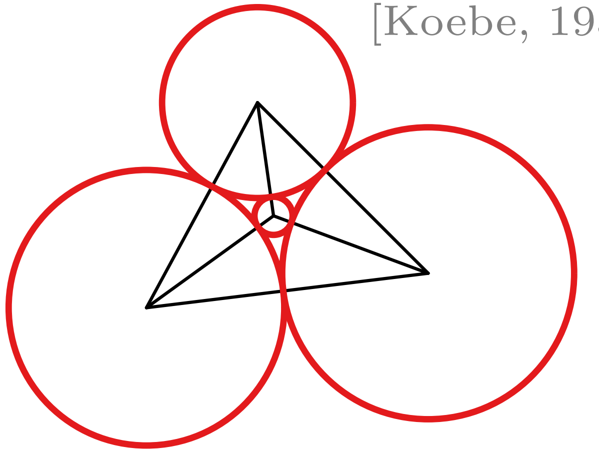

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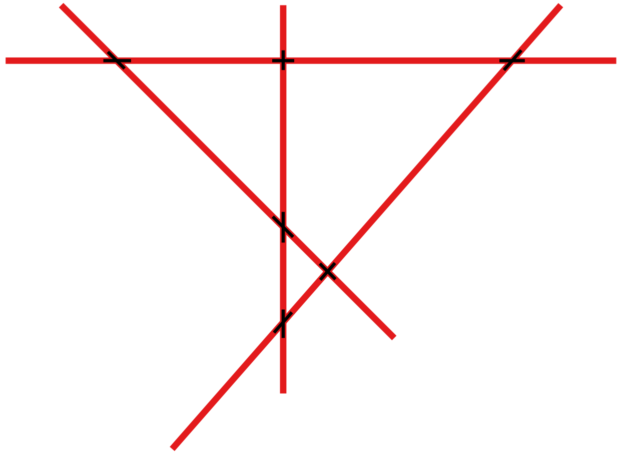
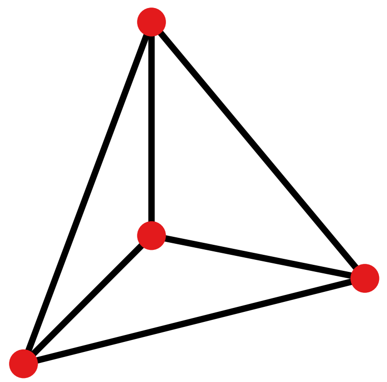
[Chalopin, Gonçalves, *STOC'09*]

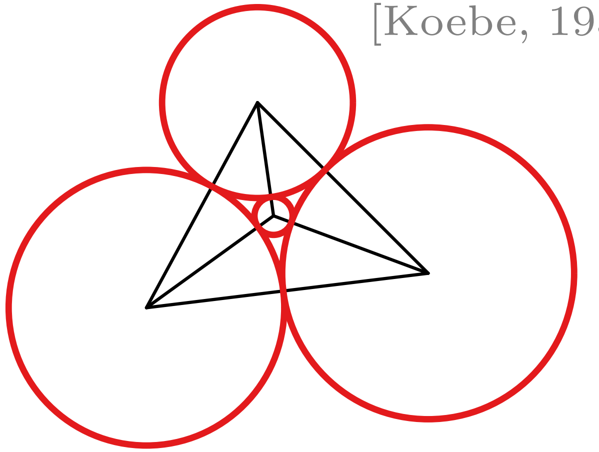


intersection representation

by segments

Geometric Representation of (Hyper)graphs





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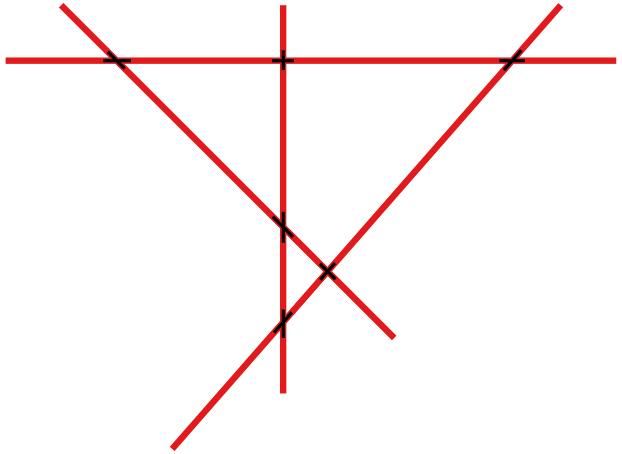

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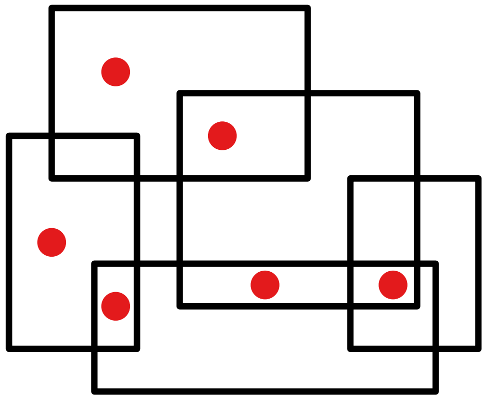
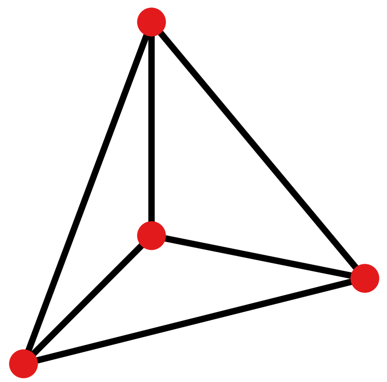


covering representation

points by rectangles

Geometric Representation of (Hyper)graphs



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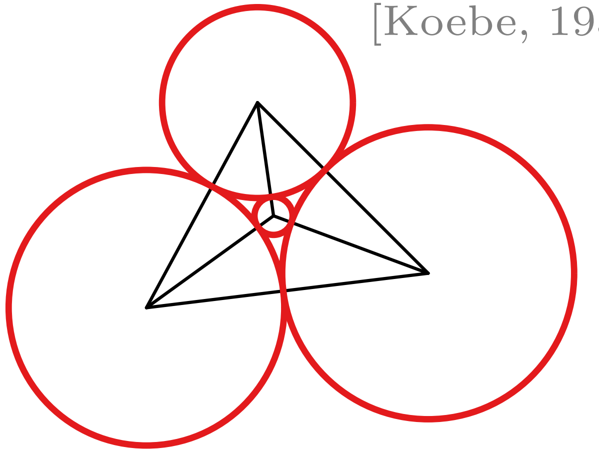

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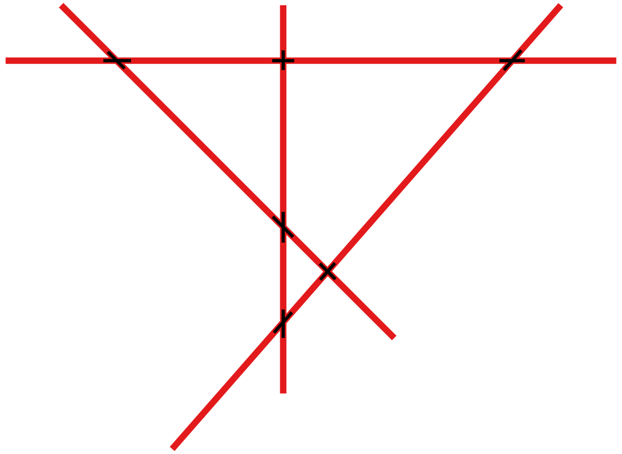

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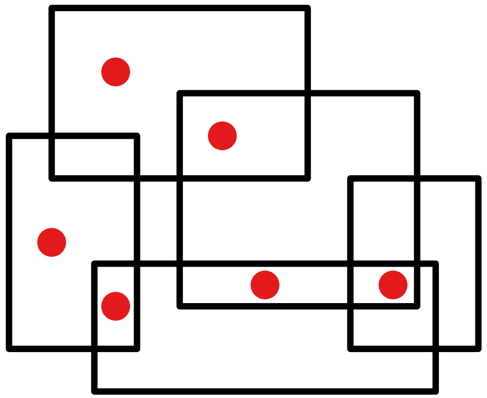

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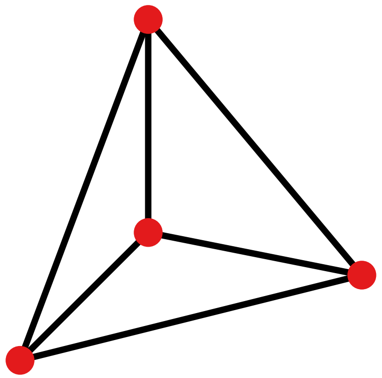

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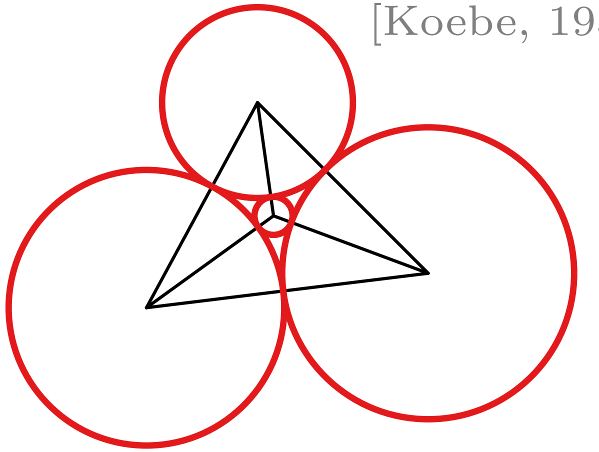

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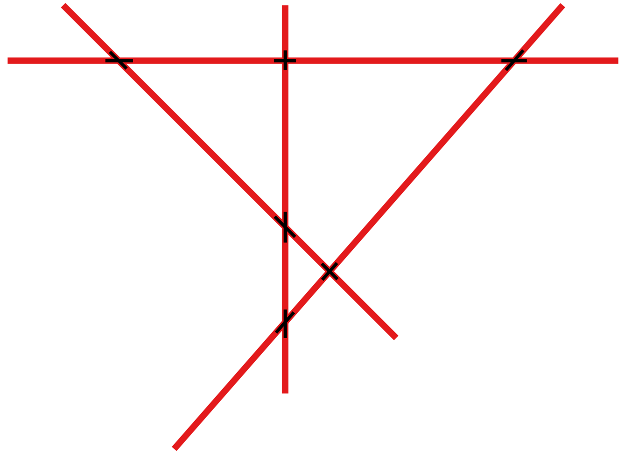

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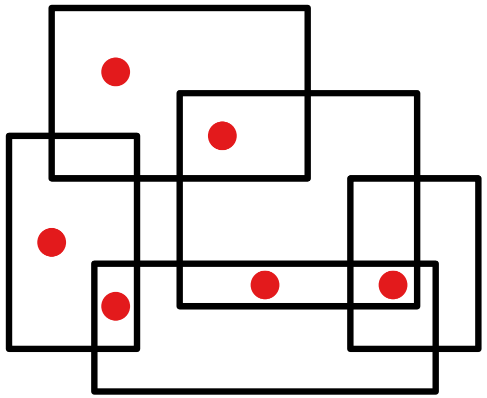

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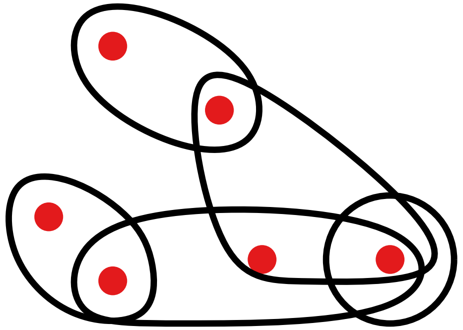
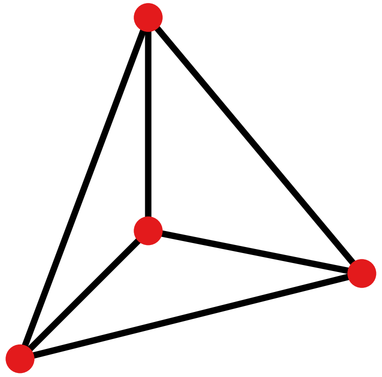
points by rectangles



points → vertices

covering objects → hyperedges

Geometric Representation of (Hyper)graphs



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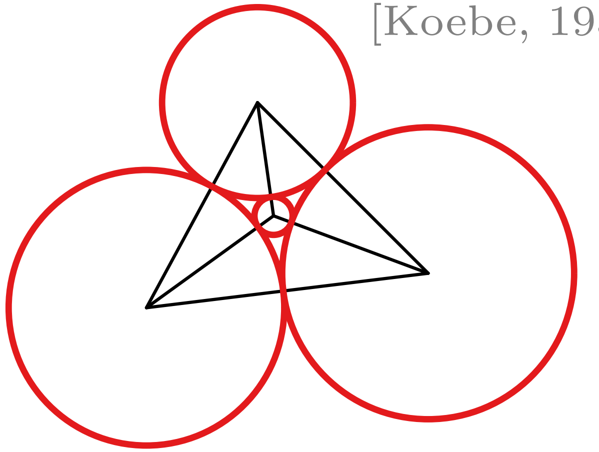

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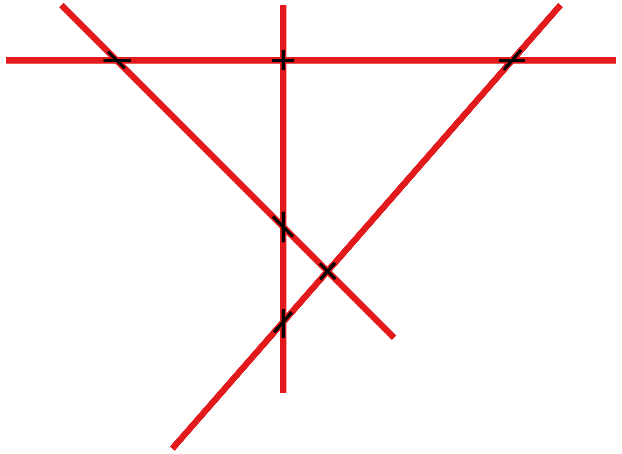

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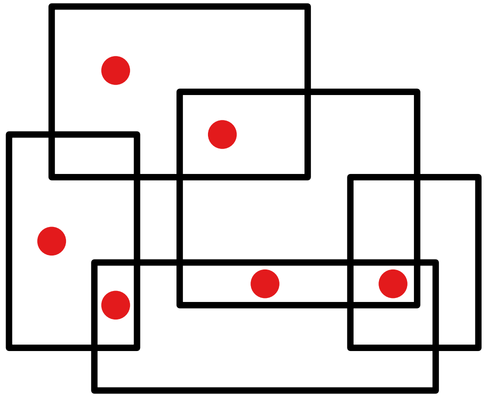
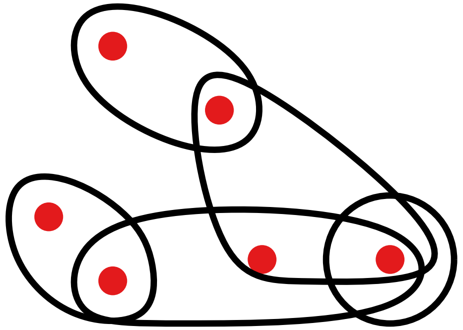
by segments



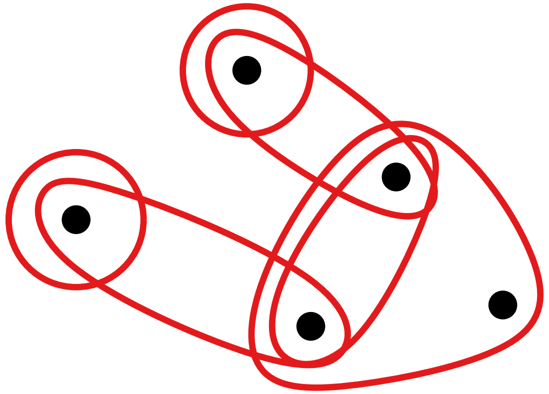
covering representation

points by rectangles



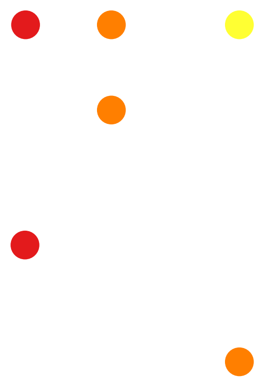
points ↔ vertices

covering objects ↔ hyperedges



Point Line Cover

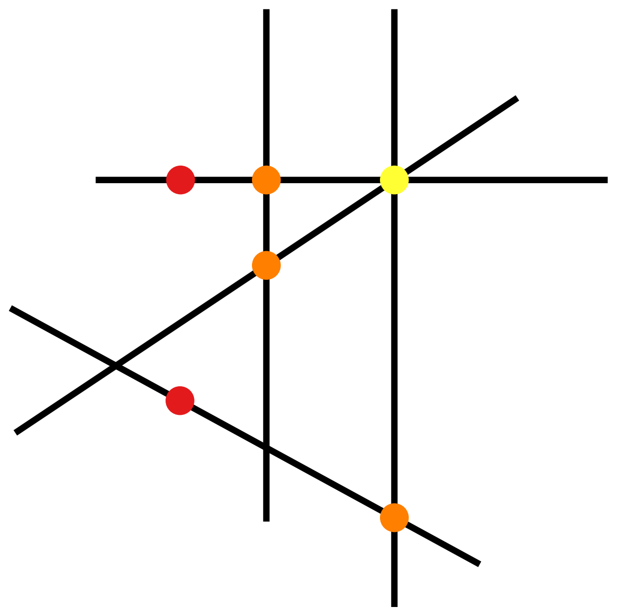
set of points P in 2D



Point Line Cover

set of points P in 2D

set of lines

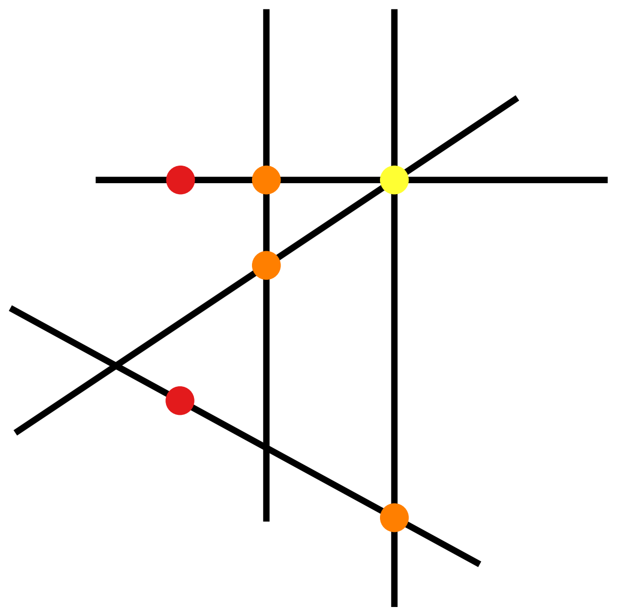


Point Line Cover

set of points P in 2D

set of lines

Hypergraph representation



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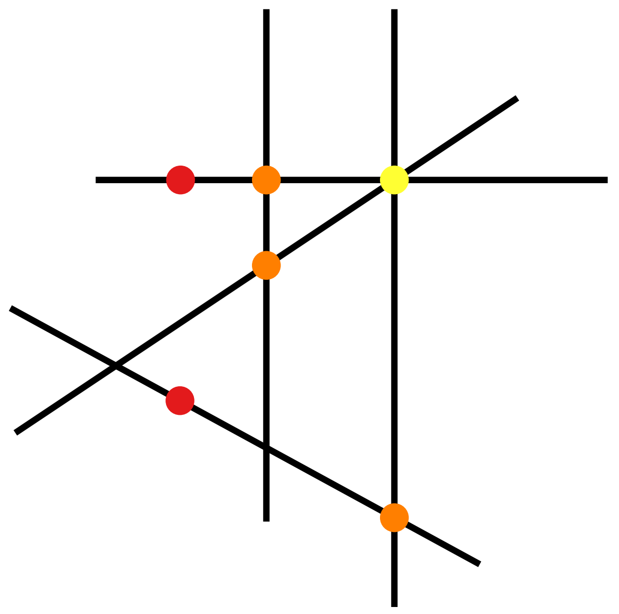
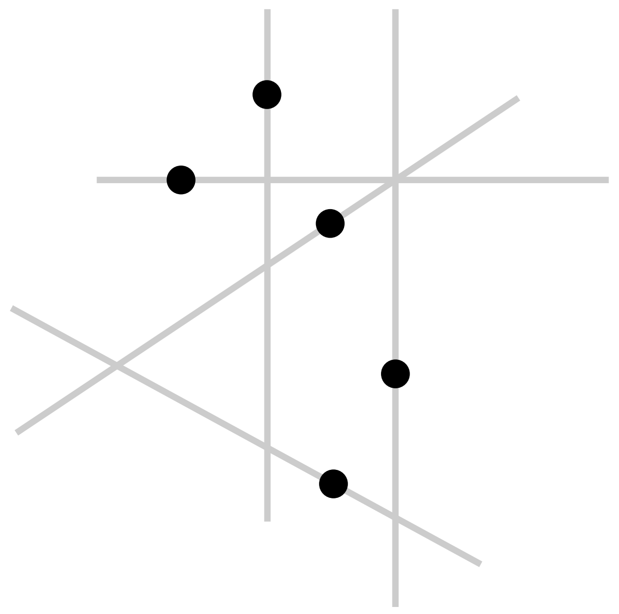


vertices

set of lines

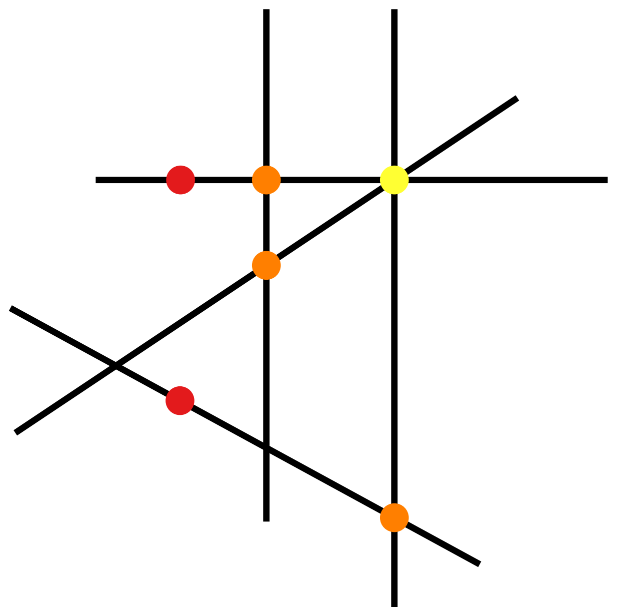
Hypergraph representation



Point Line Cover

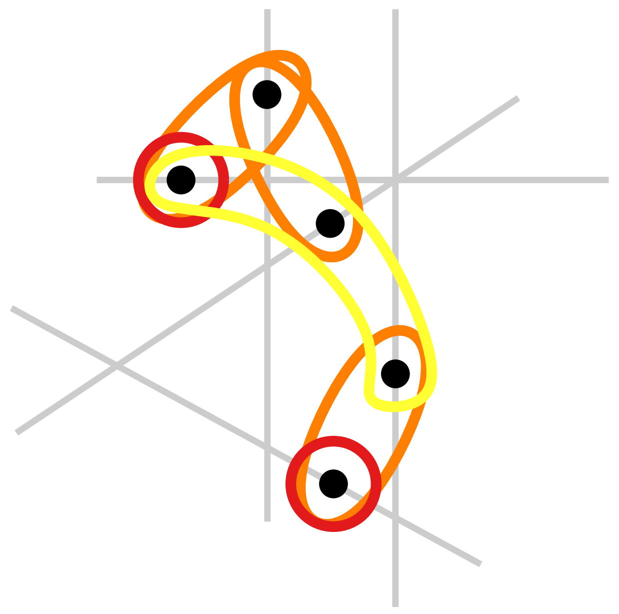
hyperedges

set of points P in 2D



set of lines

Hypergraph representation

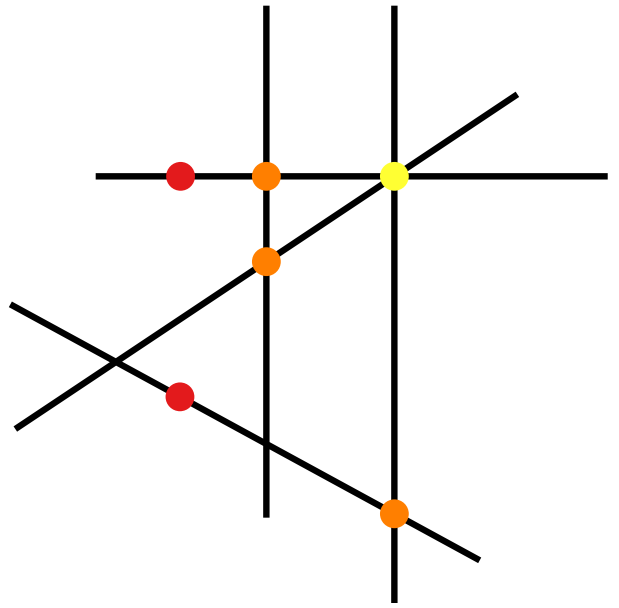


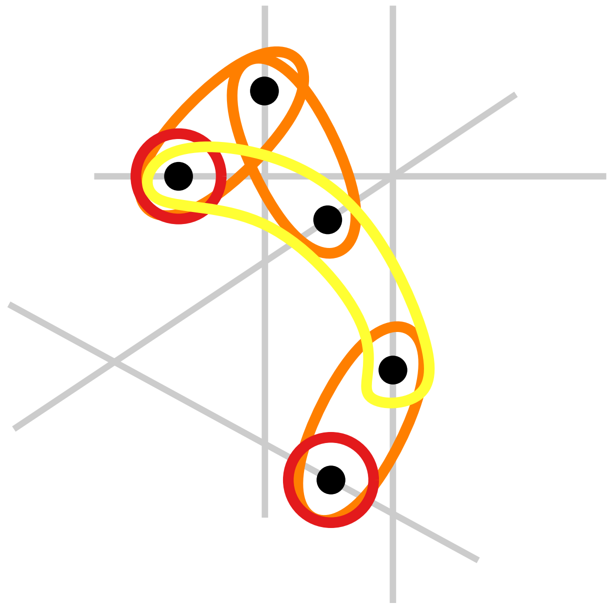
Point Line Cover – Motivation

set of points P in 2D

set of lines

Hypergraph representation





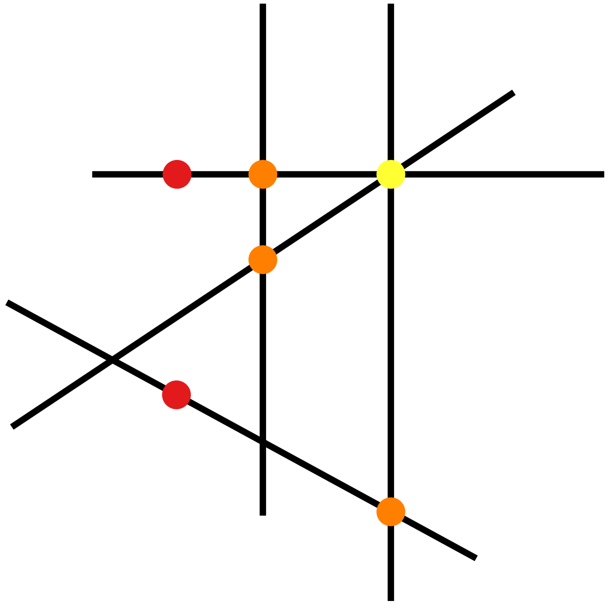
point line cover instances $\subset$ general hypergraphs

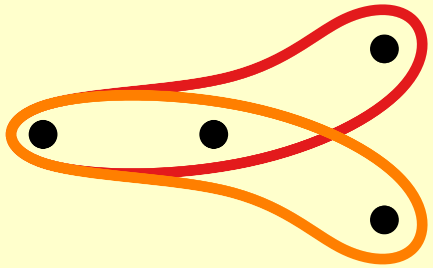
Point Line Cover – Motivation

set of points P in 2D

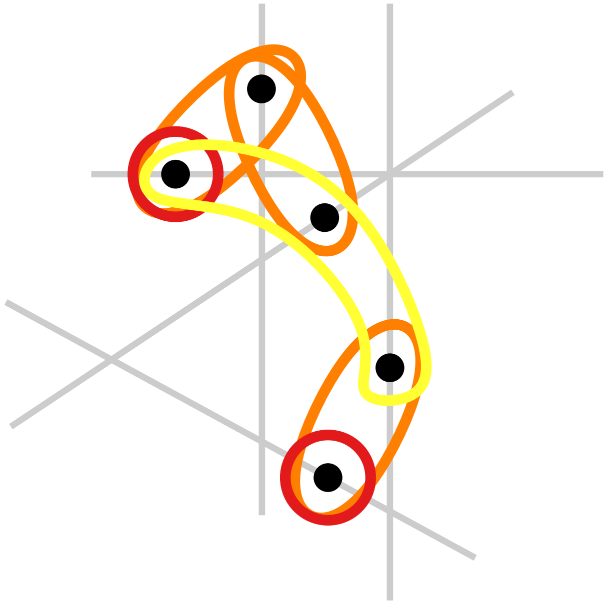
set of lines

Hypergraph representation





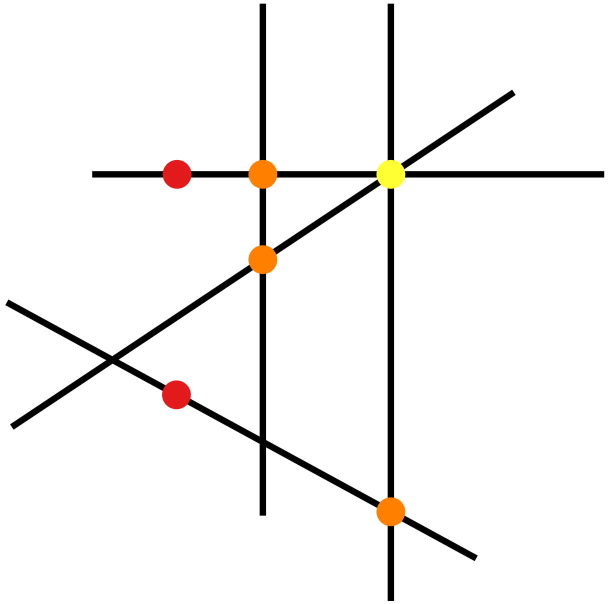
intersection property
does not hold



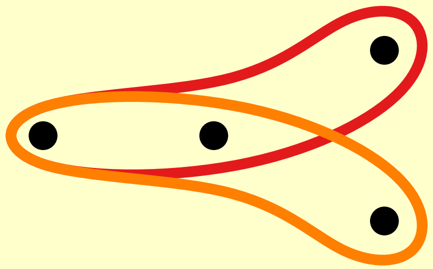
point line cover instances $\subset$ general hypergraphs

Point Line Cover – Motivation

set of points P in 2D



point line cover instances

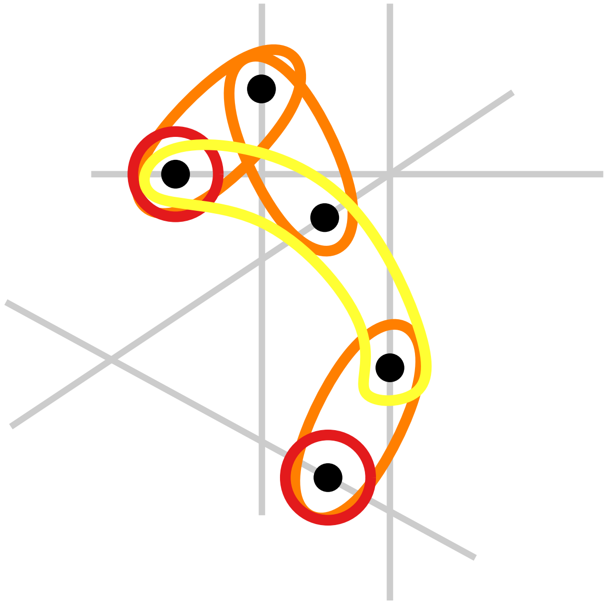


intersection property does not hold

$\subset$

set of lines

Hypergraph representation



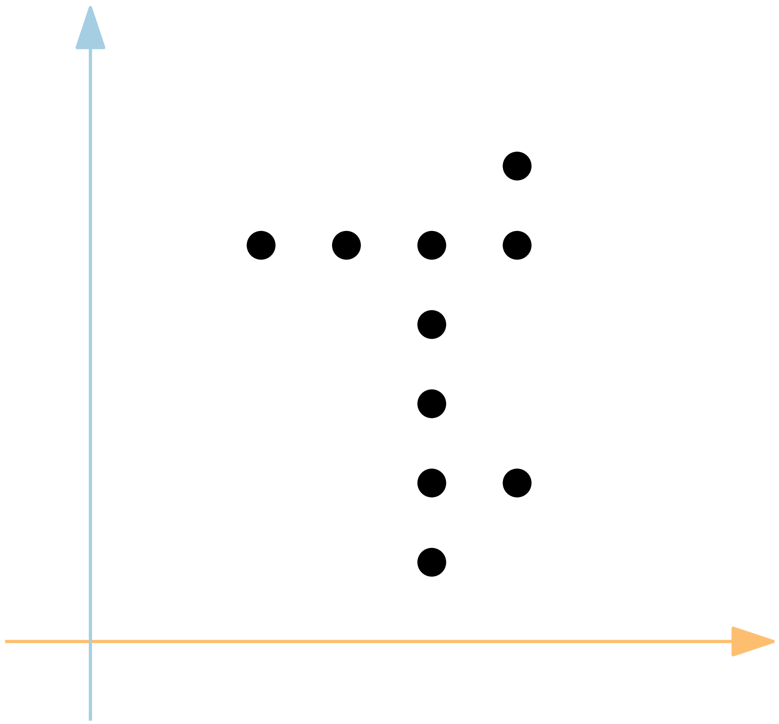
general hypergraphs

Is there a simple combinatorial characterization?

Open Q.

[Kumar, Ramesh, *ICALP* 2000]

Axis-Aligned Point Line Cover in 2D



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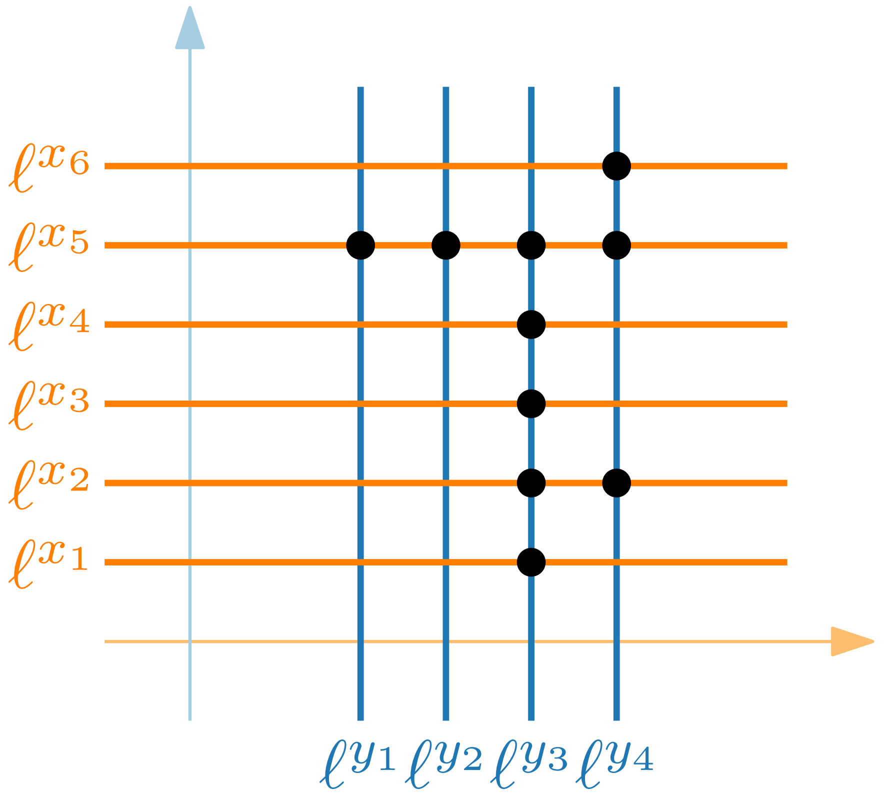

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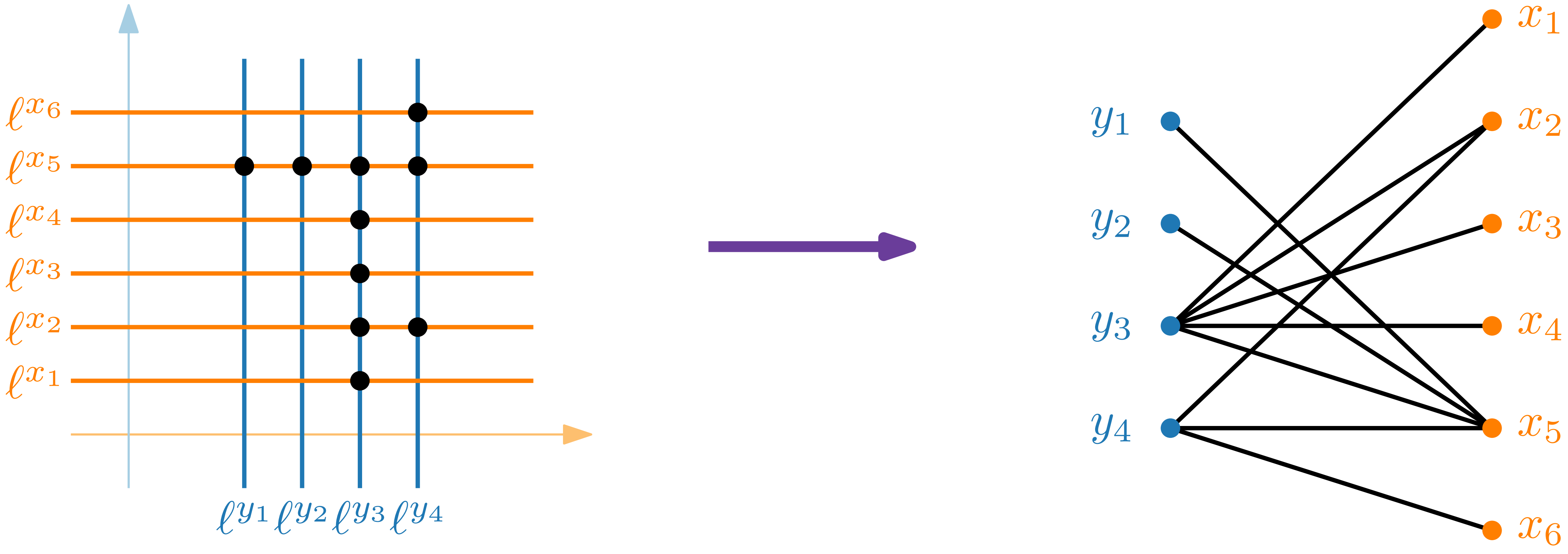

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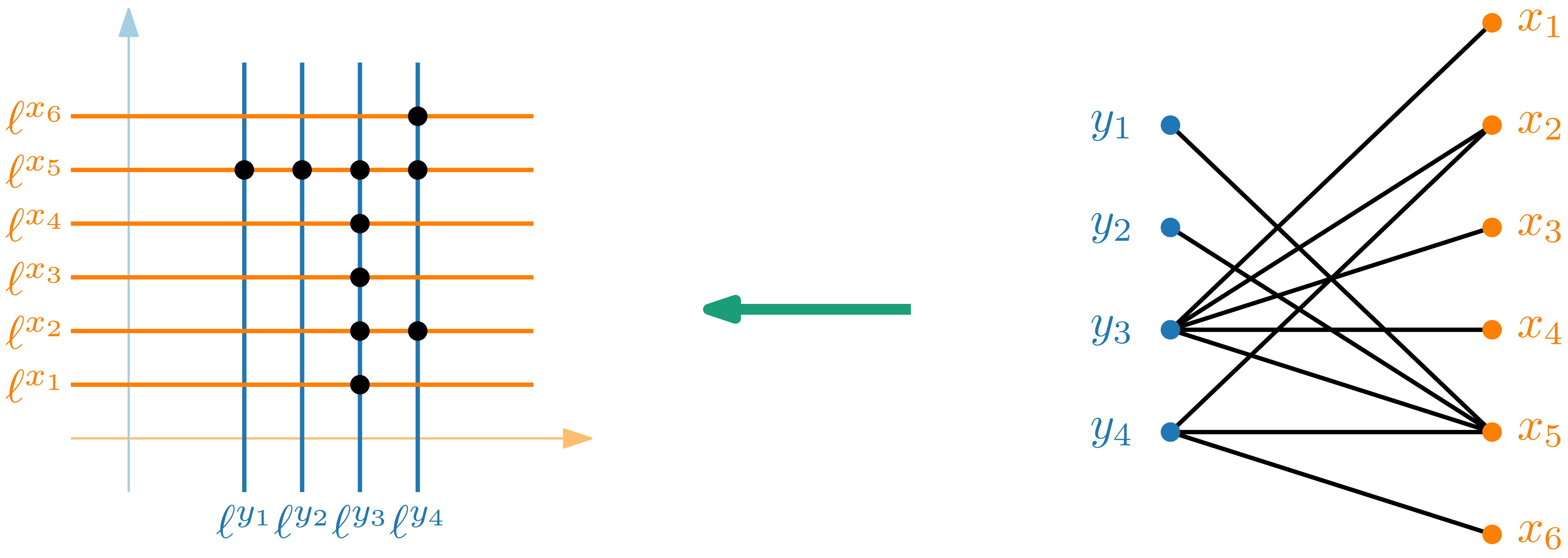

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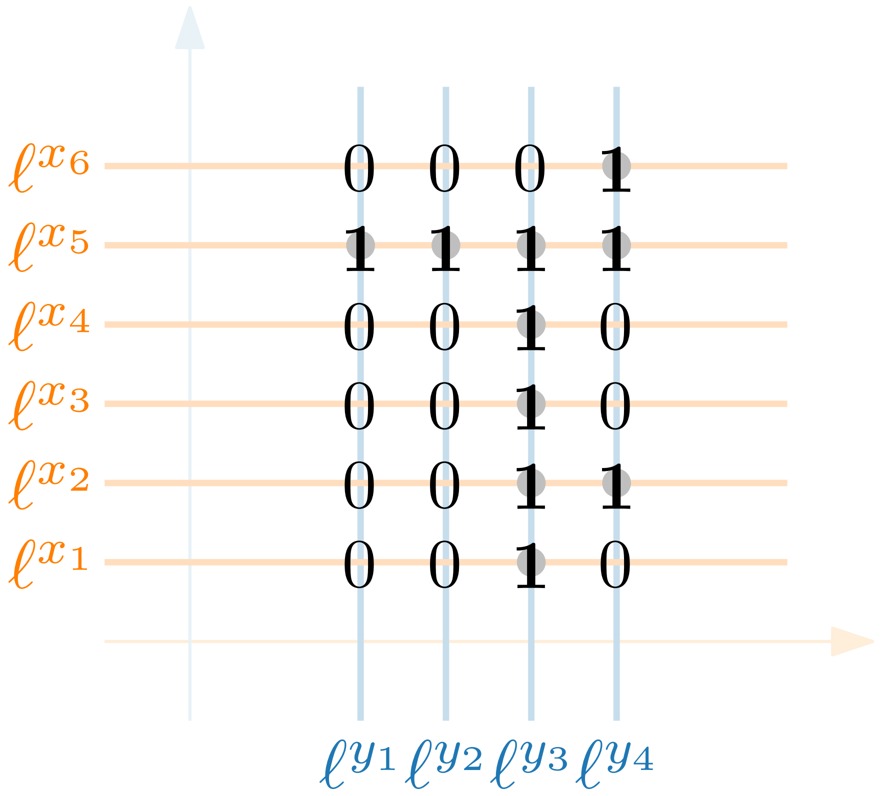
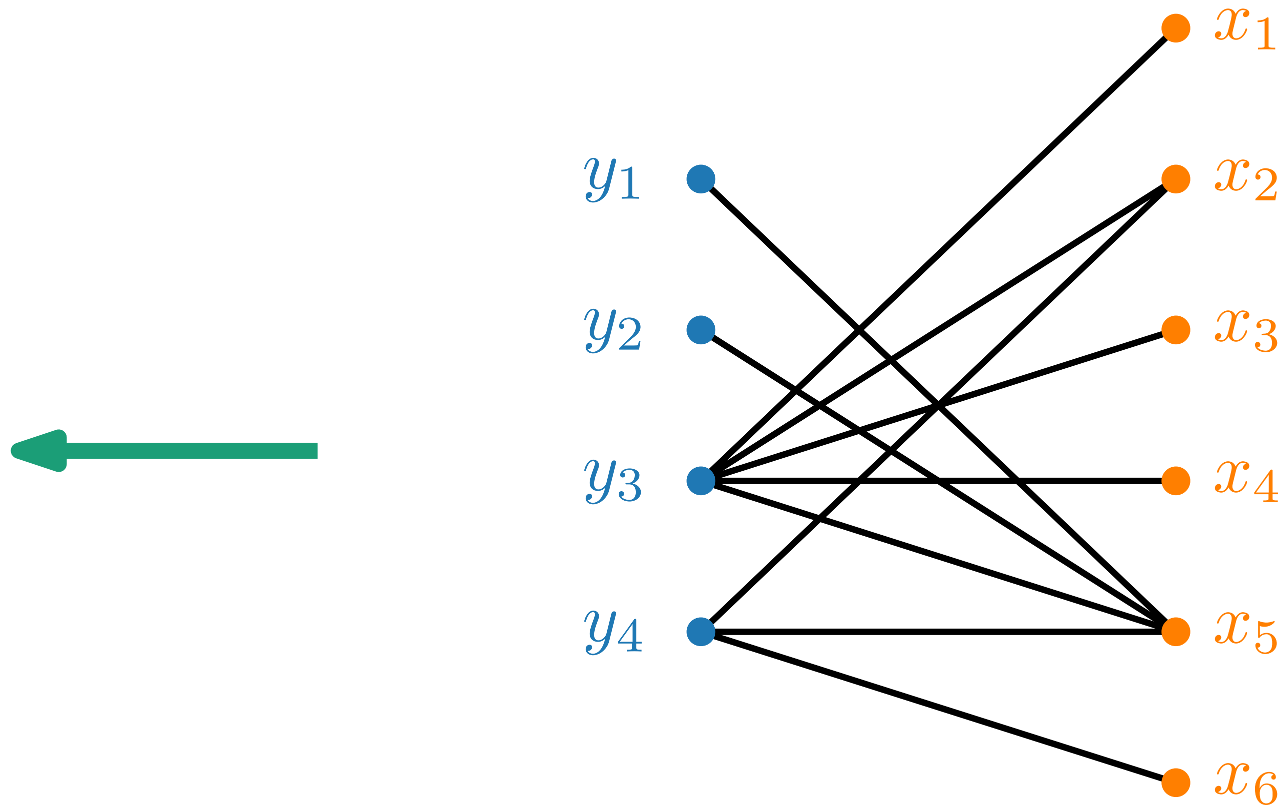
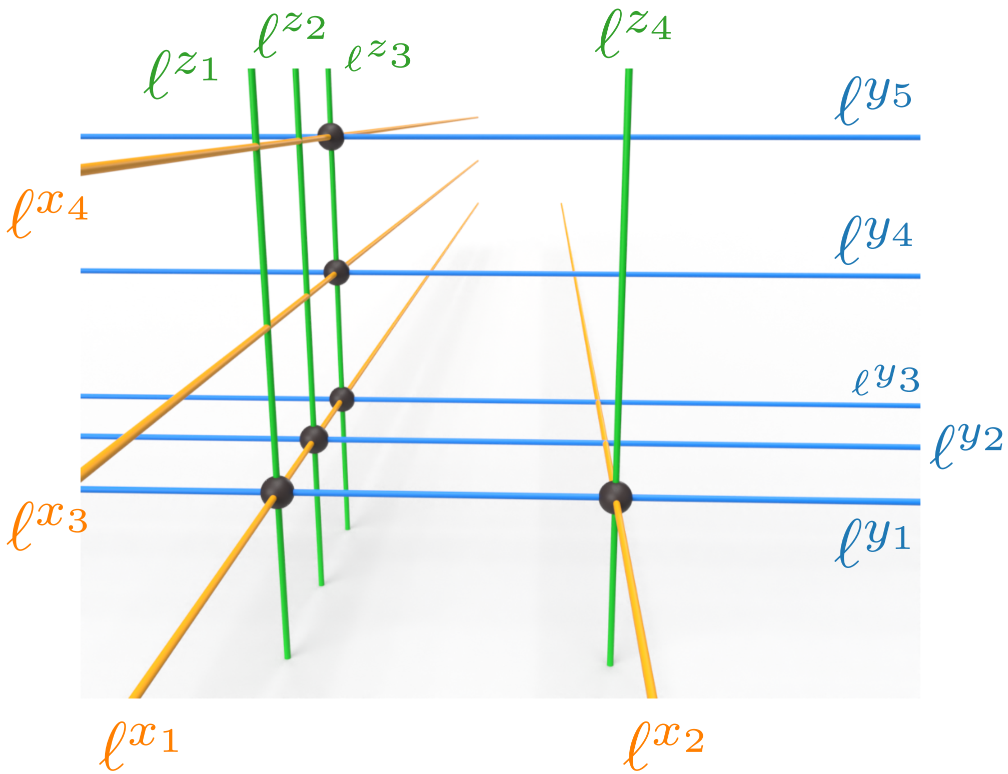




Axis-Aligned Point Line Cover in 3D



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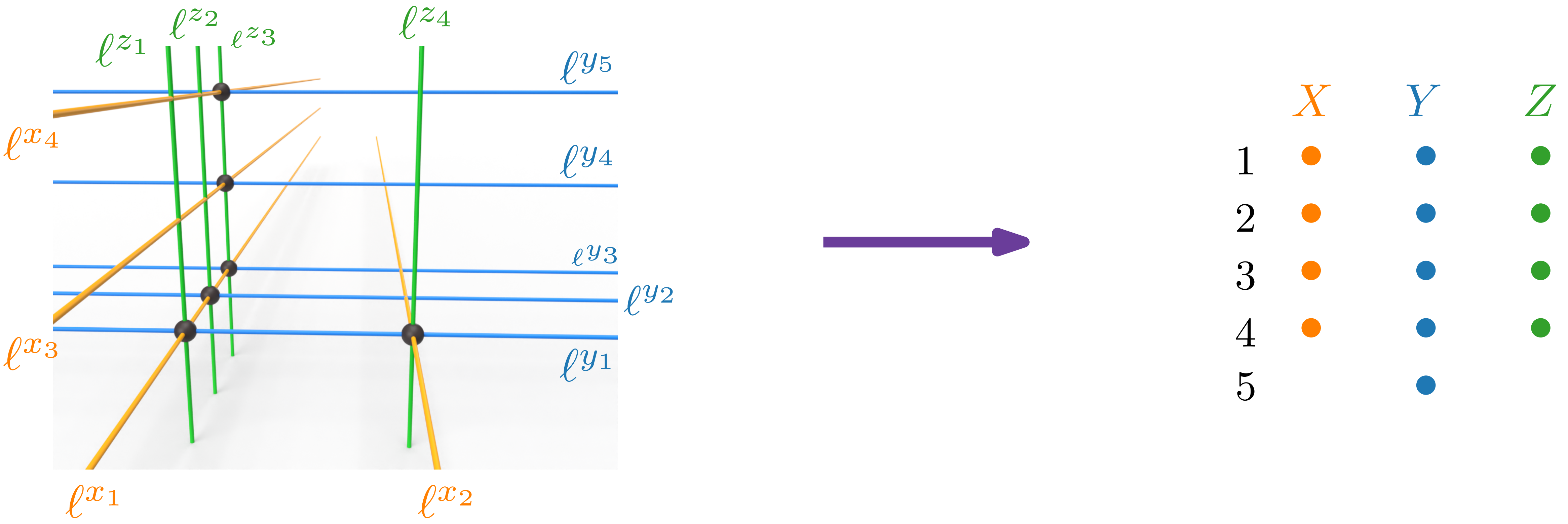

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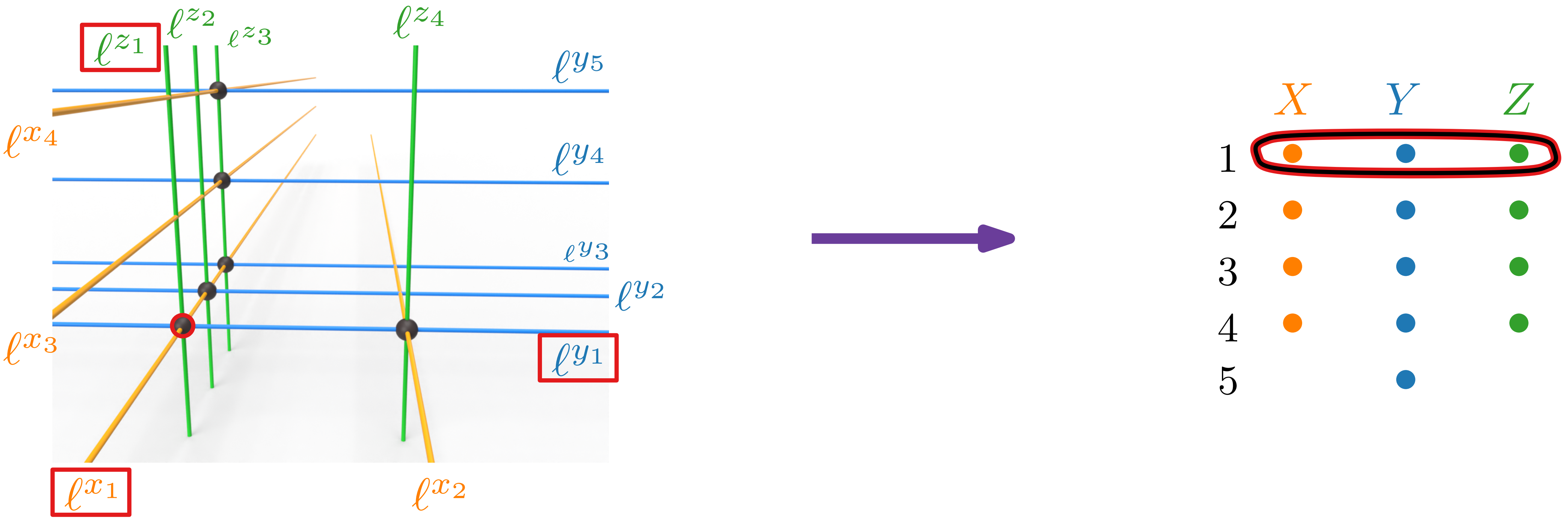

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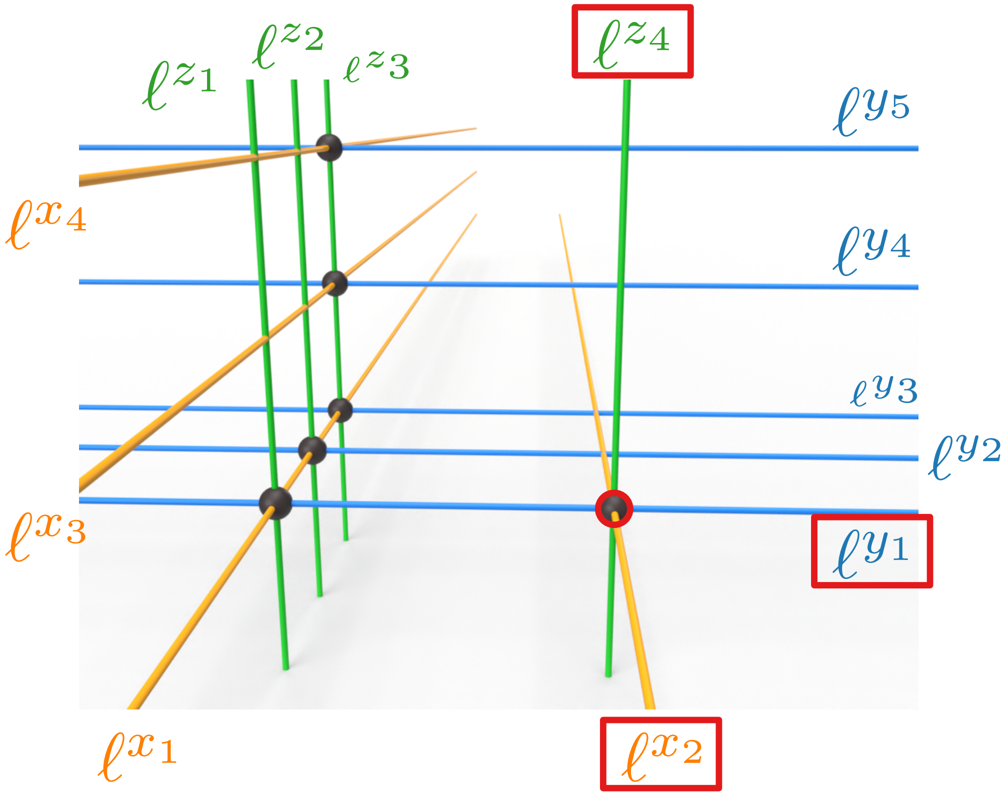

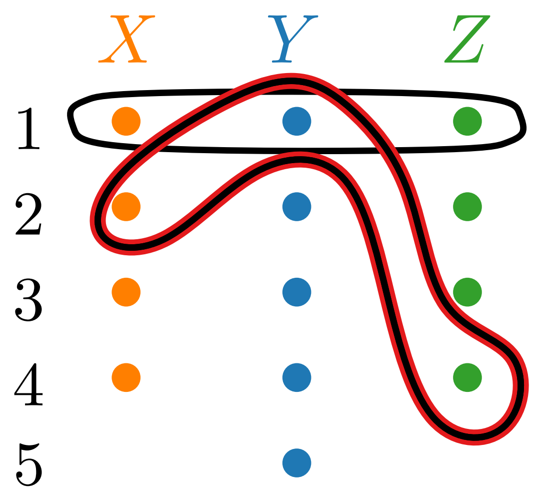

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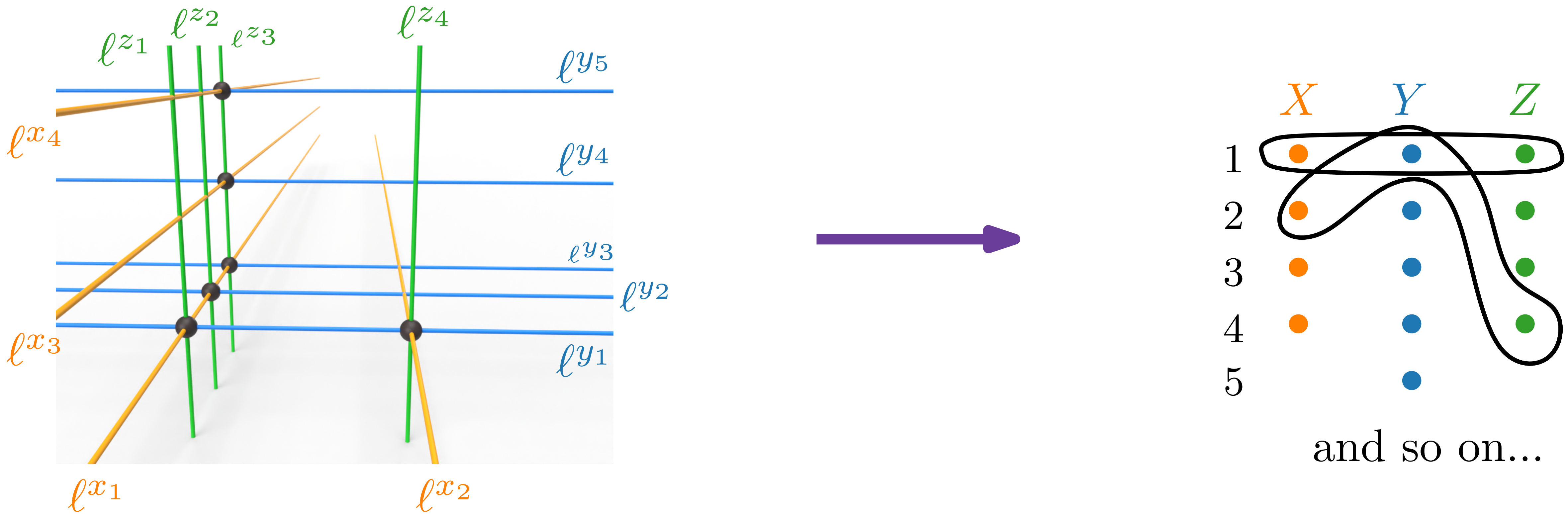

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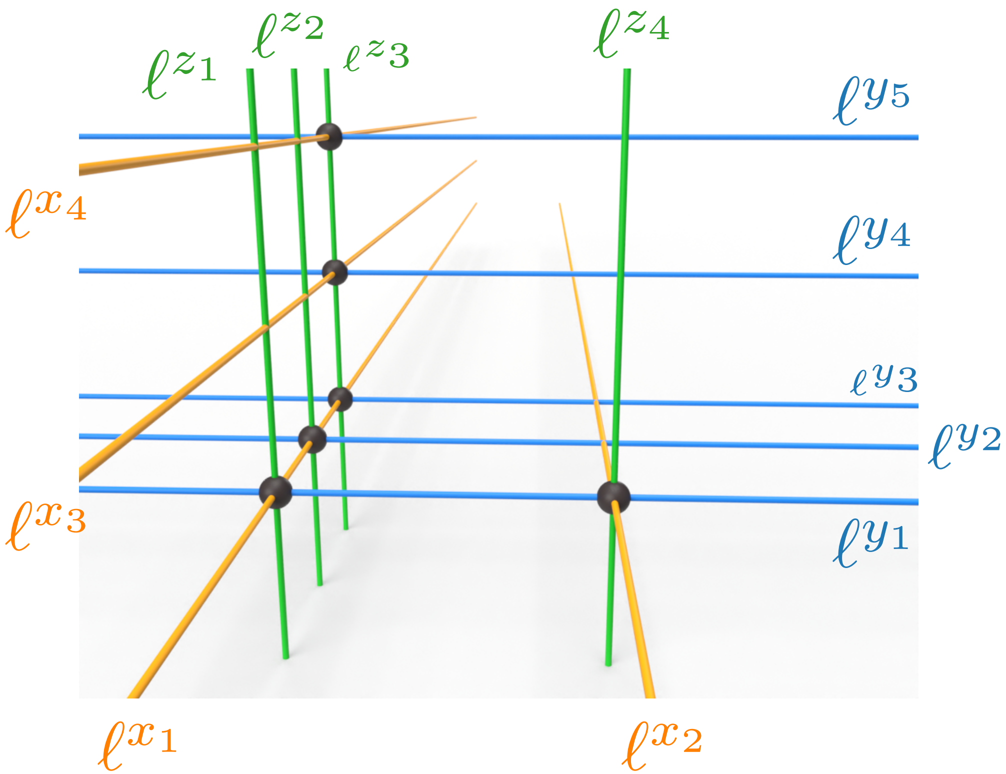
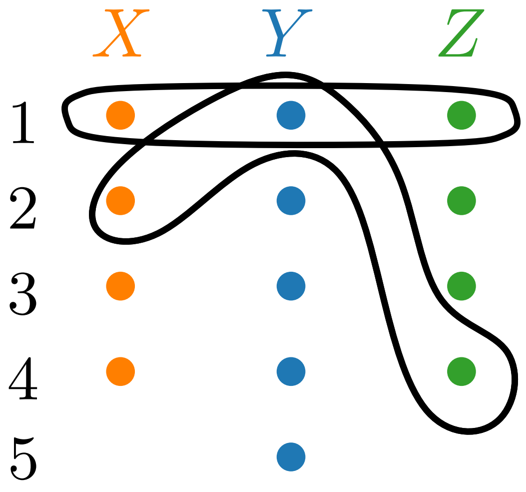




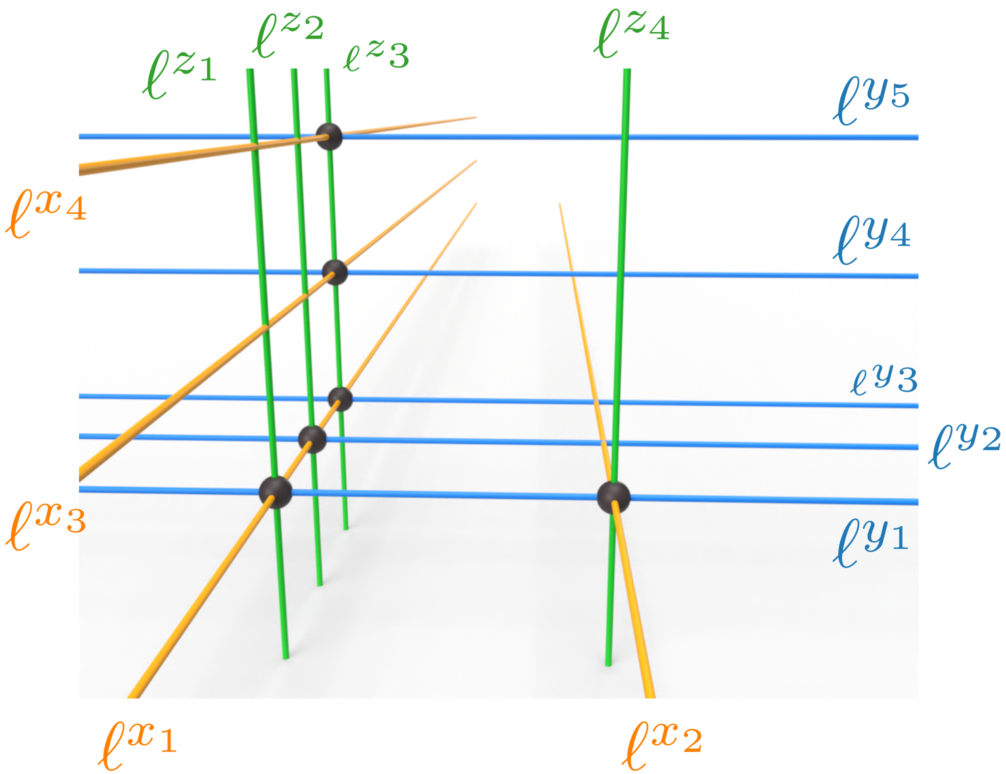
and so on...

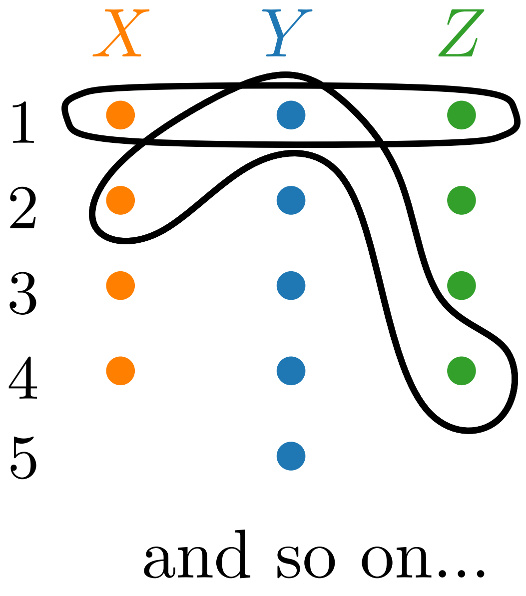
(every hyperedge has exactly 3 vertices, one from each group)

3-uniform

3-partite

Axis-Aligned Point Line Cover in 3D



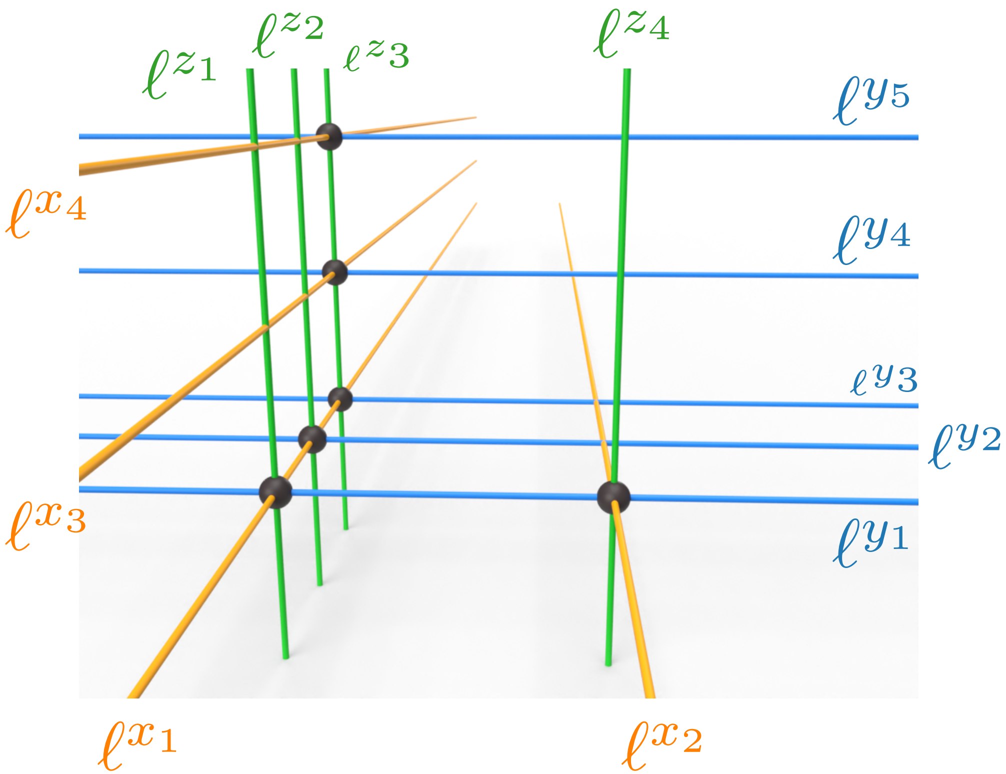


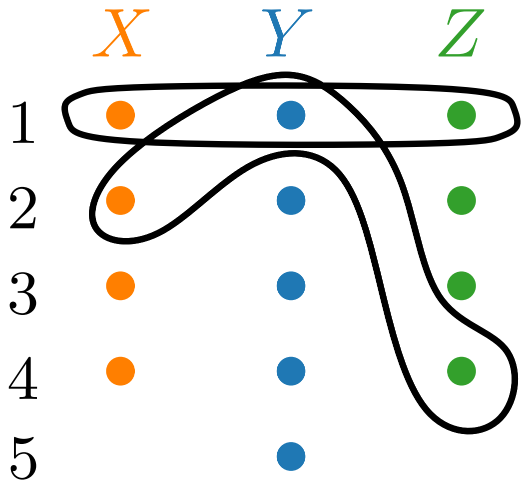
(every hyperedge has exactly 3 vertices, one from each group)

3-uniform — easy to check

3-partite — NP-hard

Axis-Aligned Point Line Cover in 3D





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3-uniform — easy to check

3-partite — NP-hard

3-hypergraph

Representable Hypergraphs

axis-aligned point line cover instance

↓

k-hypergraph

k-partite and k-uniform

Representable Hypergraphs

axis-aligned point line cover instance

↓ ↑ ?

k-hypergraph

k-partite and k-uniform

Representable Hypergraphs

axis-aligned point line cover instance

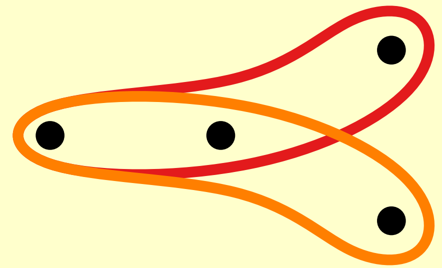
↓ ↑ ?

k-hypergraph

k-partite and k-uniform

No

exception: 2D



intersection property does not hold

Representable Hypergraphs

axis-aligned point line cover instance

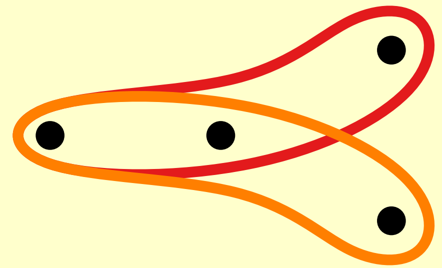
↓ ↑ ?

k-hypergraph

k-partite and k-uniform

No

exception: 2D



intersection property
does not hold

Which k-hypergraphs can be *represented*
via axis-aligned point line cover instances?

Paths

Notation.

$[k] = \{1, \ldots, k\}$ for

A hypergraph $G = (V, E)$
$V = V_1 \cup \ldots \cup V_k$

Paths

Def.

Let $s, t \in V$. An *s-t path* is a sequence of vertices $s = v_1, \ldots, v_r = t$ such that $\forall i \in [r-1]$ v_i and v_{i+1} belong to the same edge.

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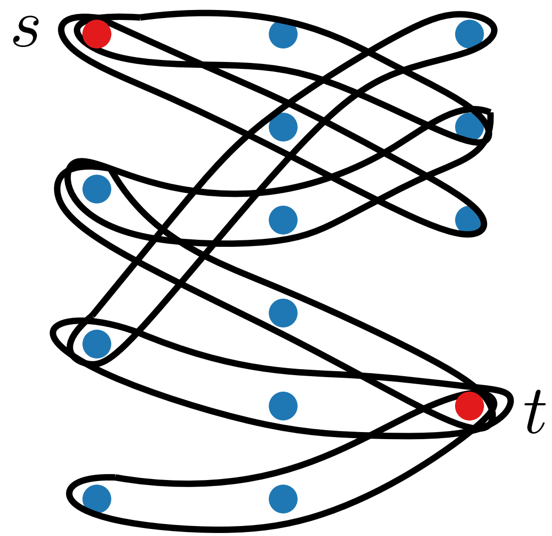

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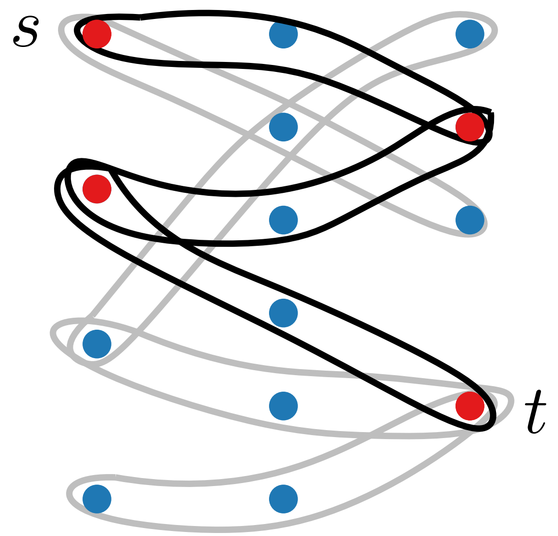

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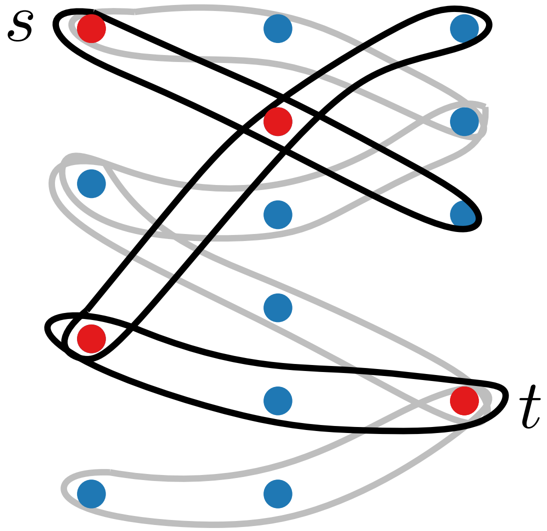
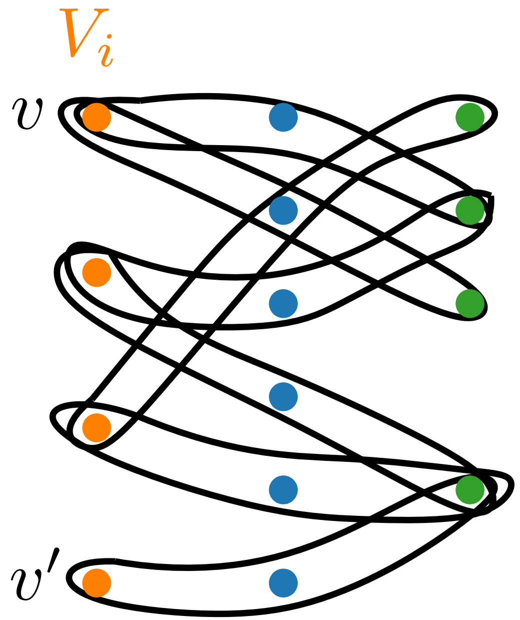


Separability – Key Property

Def. *Vertex separability*

For a given k-hypergraph G two distinct vertices v and v' from the same group V_i where $i \in [k]$ are *separable* if there exists $j \in [k]$ with $j \neq i$ such that every v-v' path contains a vertex in V_j.



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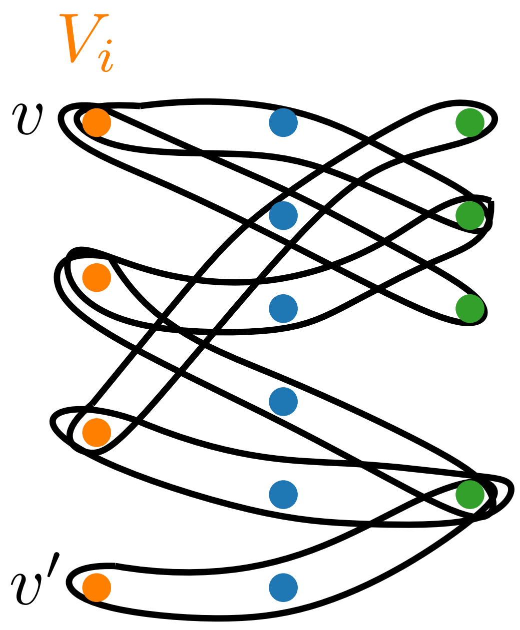

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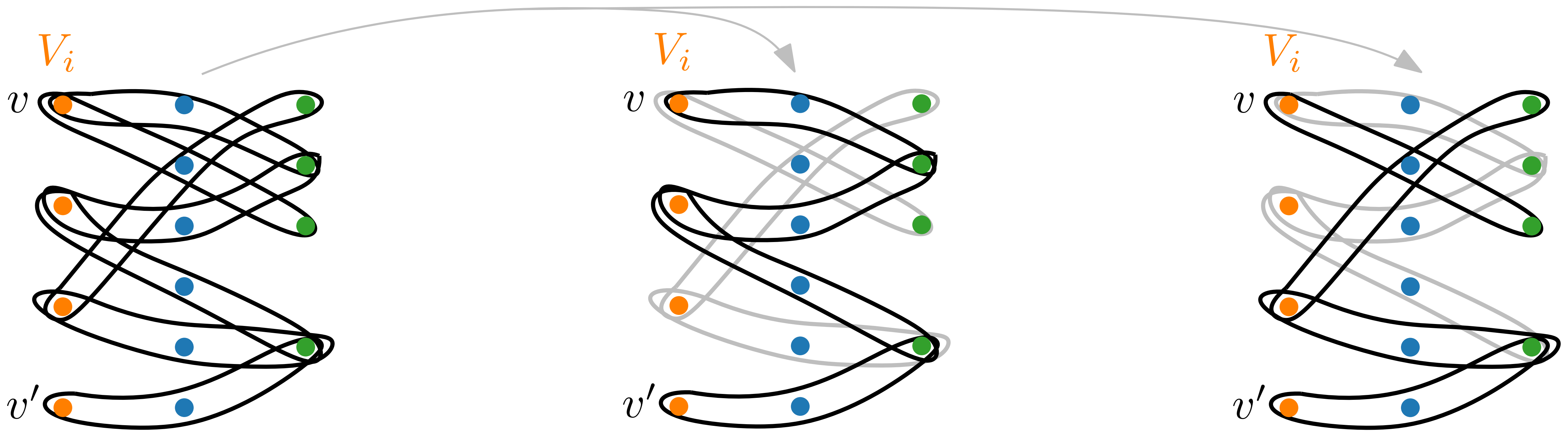

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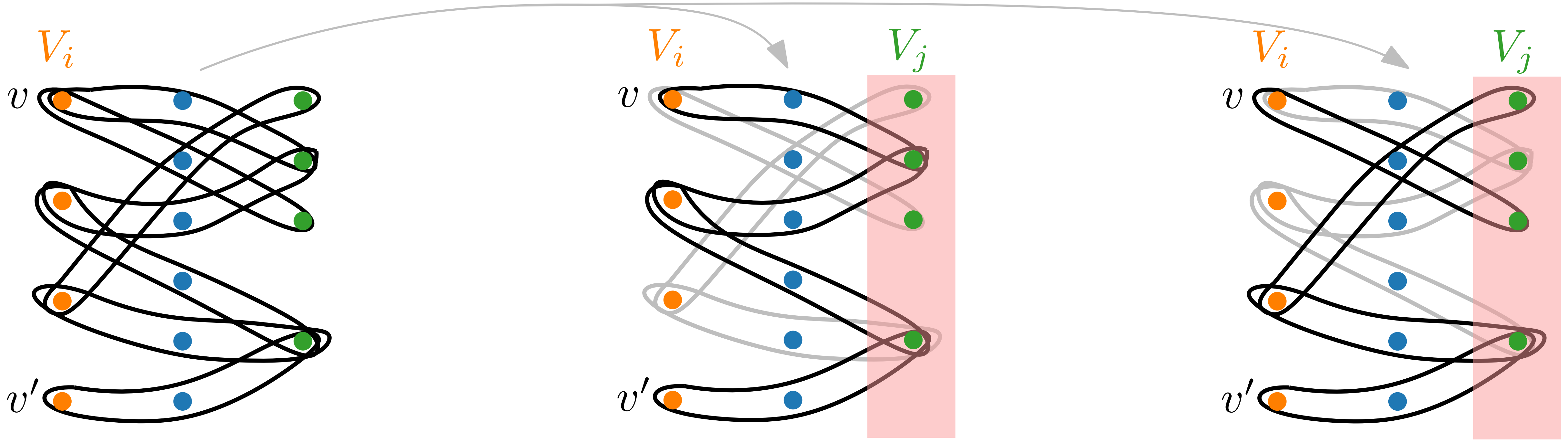

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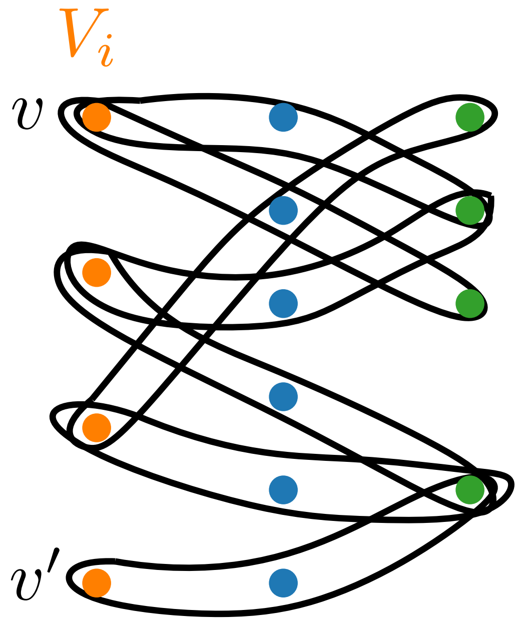

A k-hypergraph is called *vertex separable* if every two vertices from the same group are separable.

Main Result

Theorem

A k-hypergraph G is representable if and only if it is vertex separable.

Main Result – Construction

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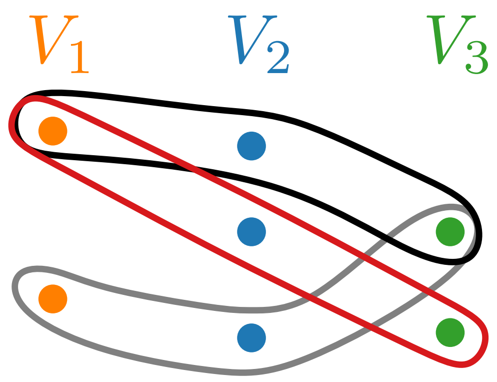
hyperedge e → point p^e
vertex v_i → line ℓ^{v_i}



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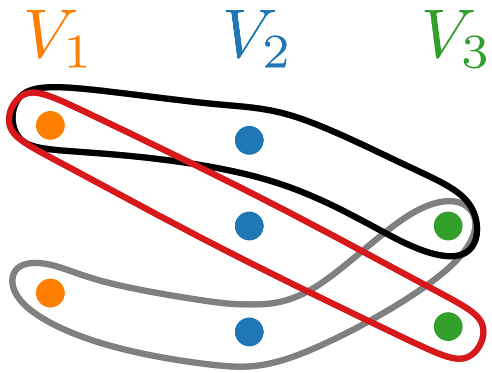

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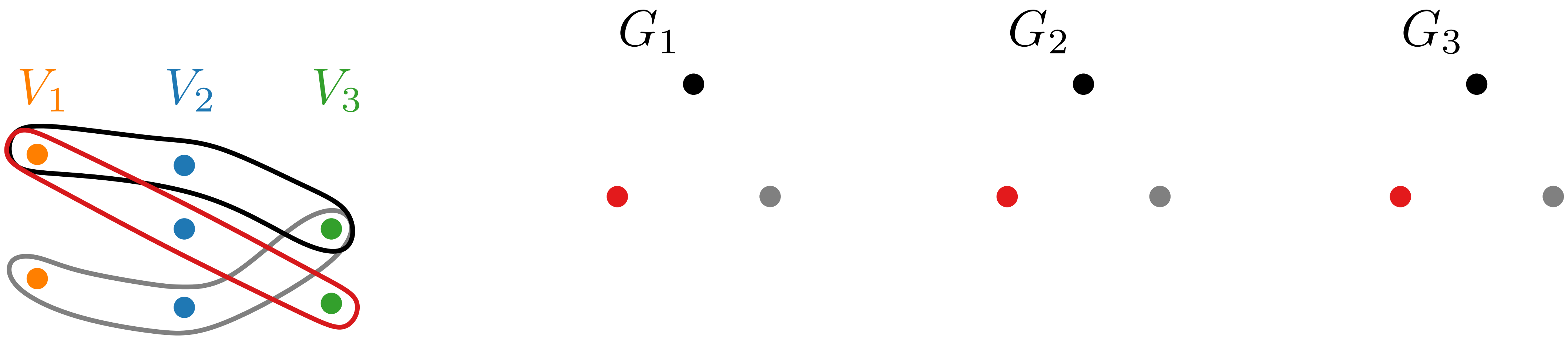

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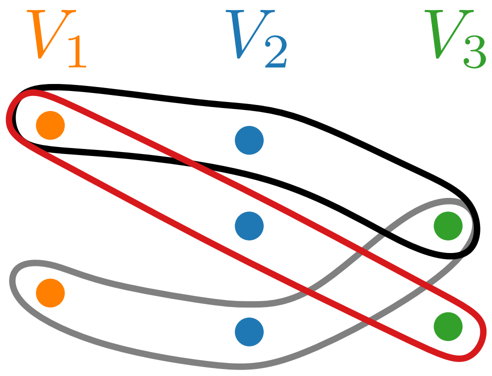

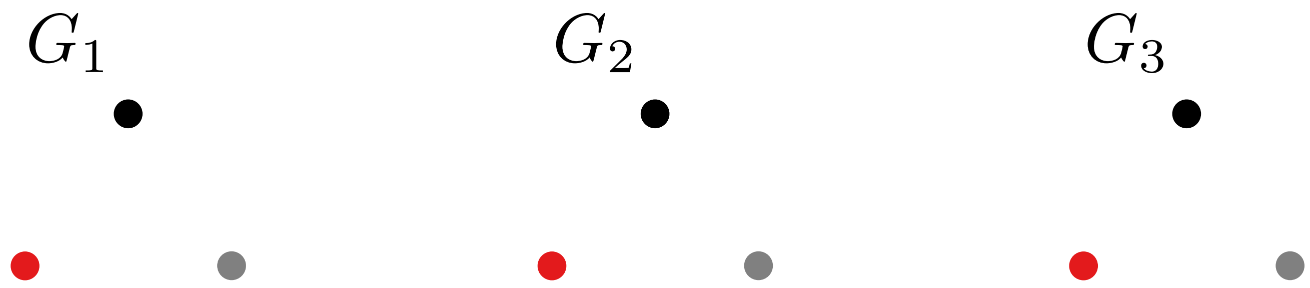

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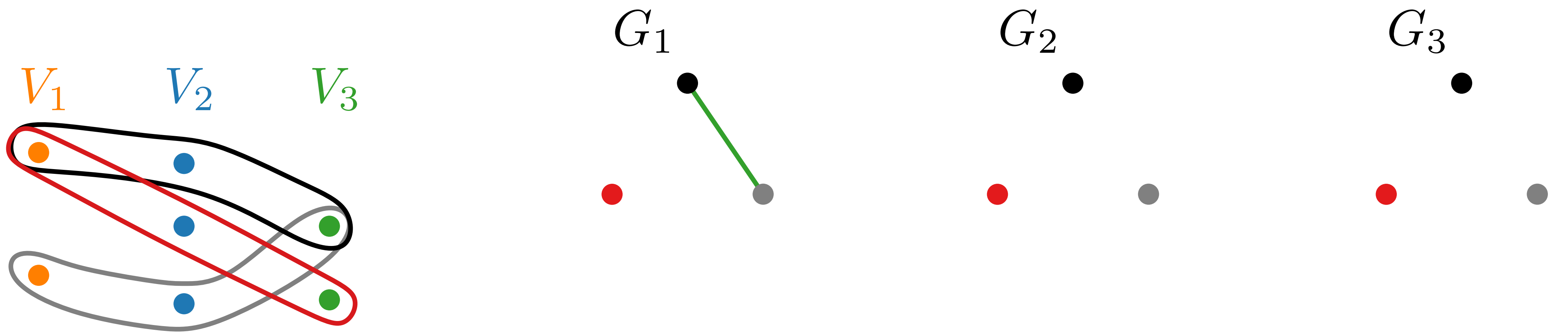

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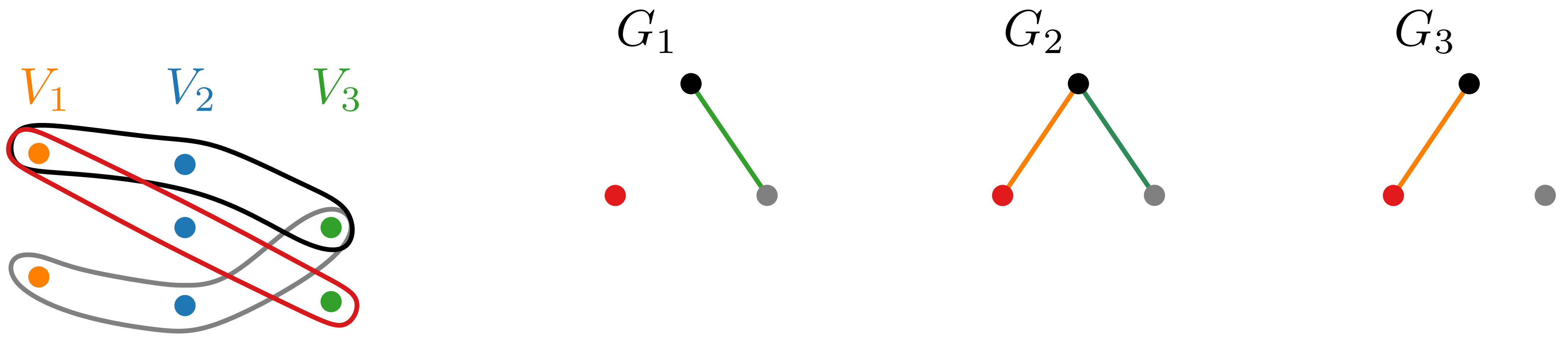

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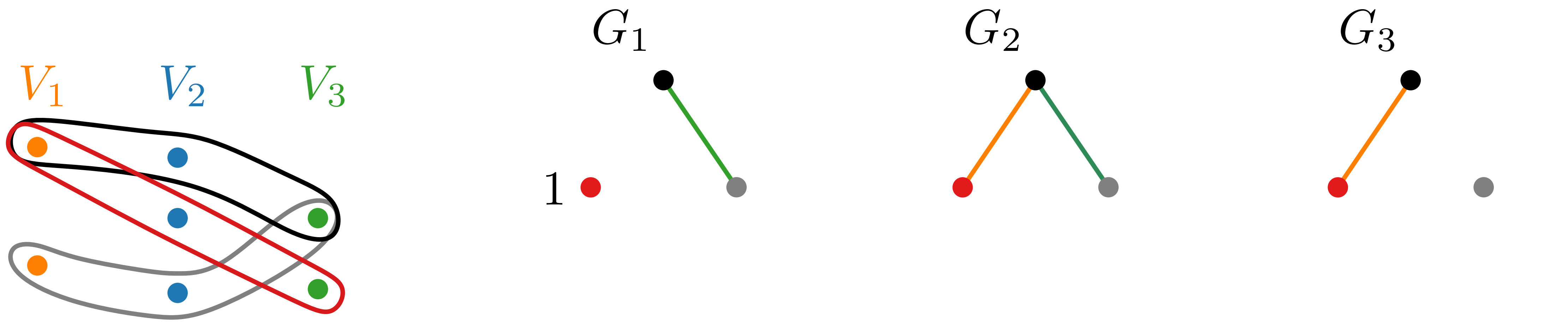

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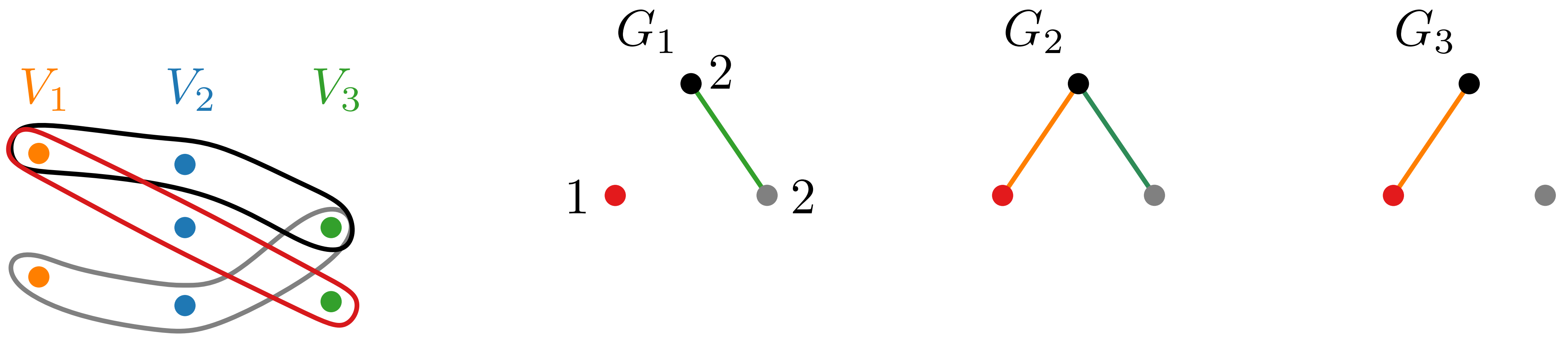

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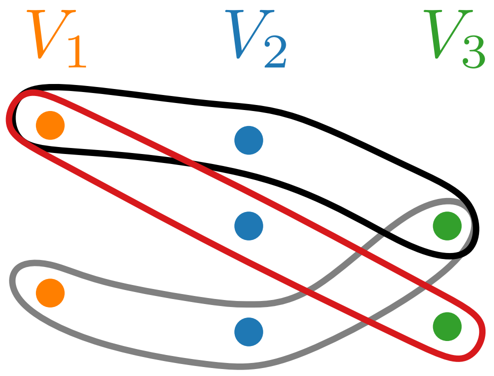

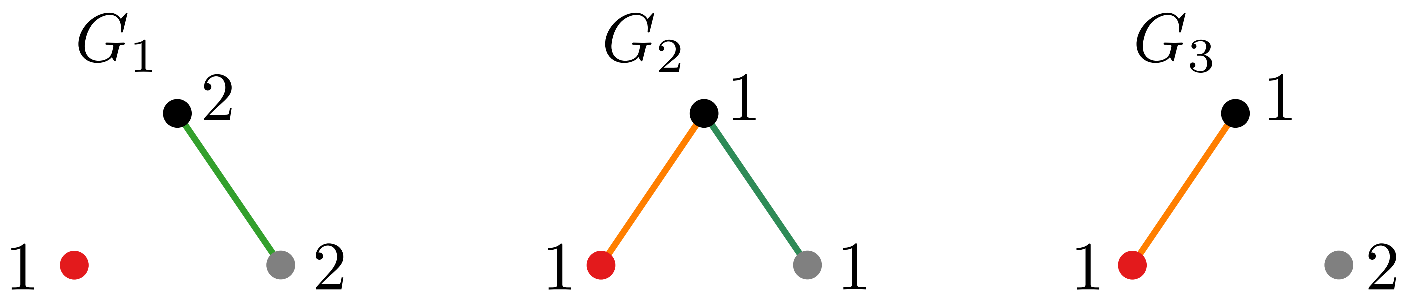

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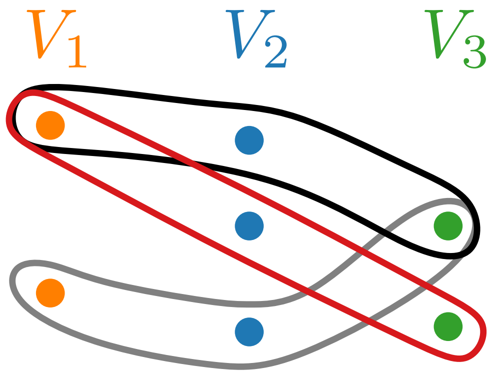

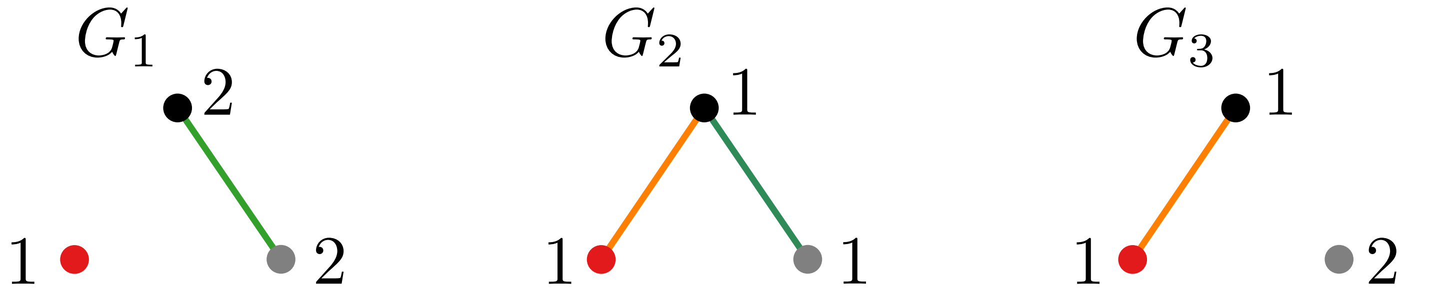

$p^{e_1} = (1, 1, 1)$ $p^{e_2} = (2, 1, 1)$ $p^{e_3} = (2, 1, 2)$

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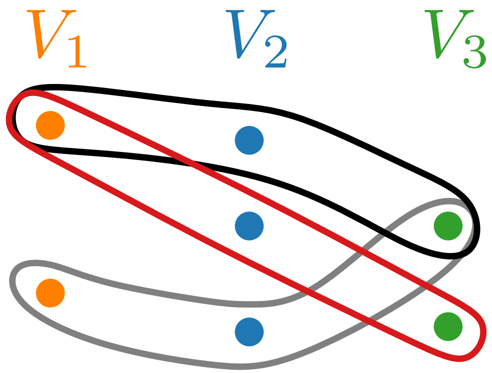

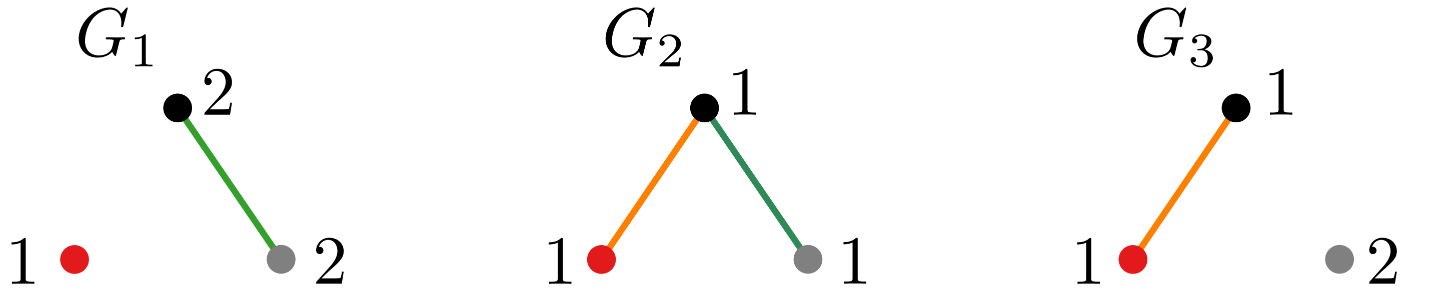

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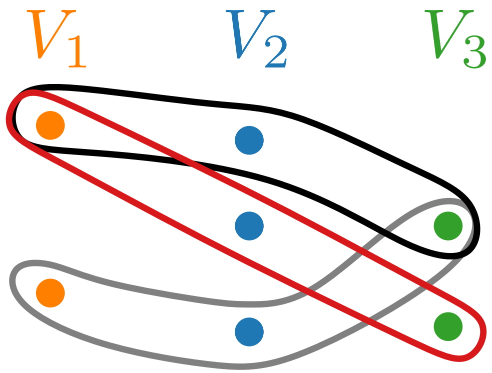

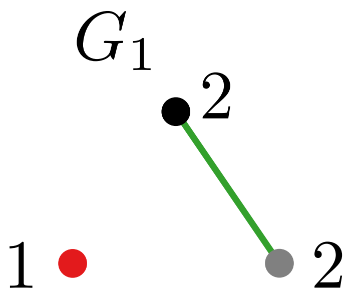

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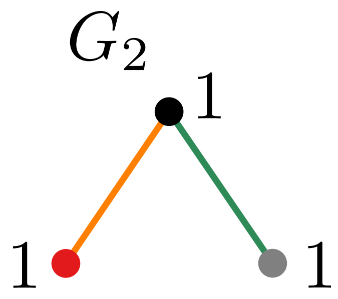

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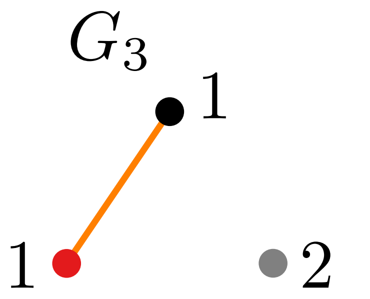

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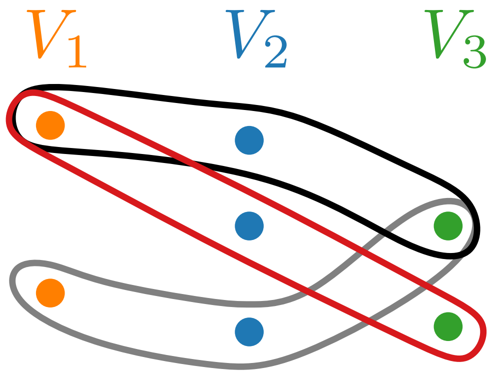

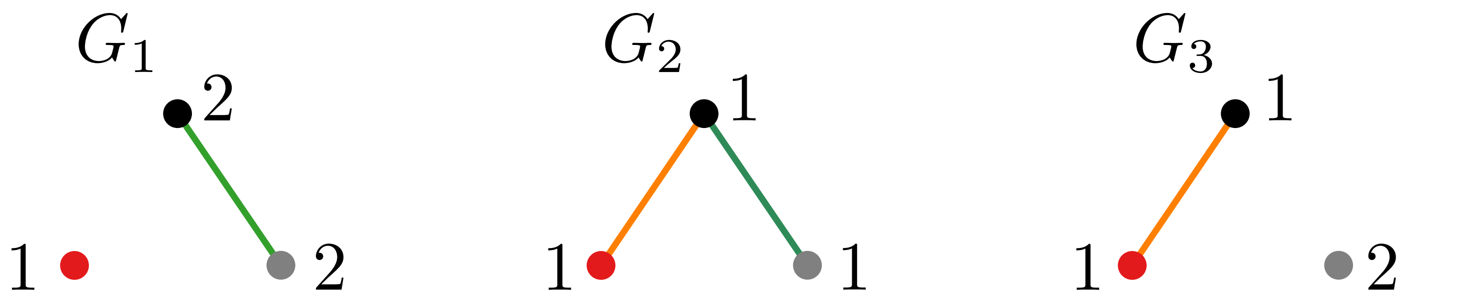

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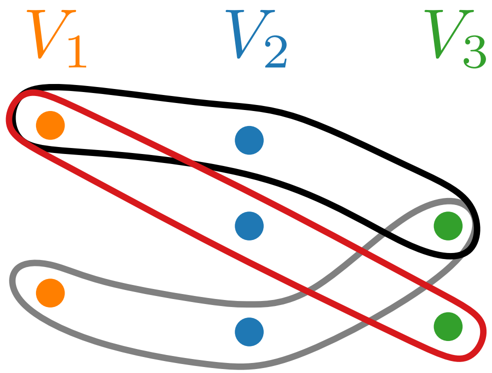

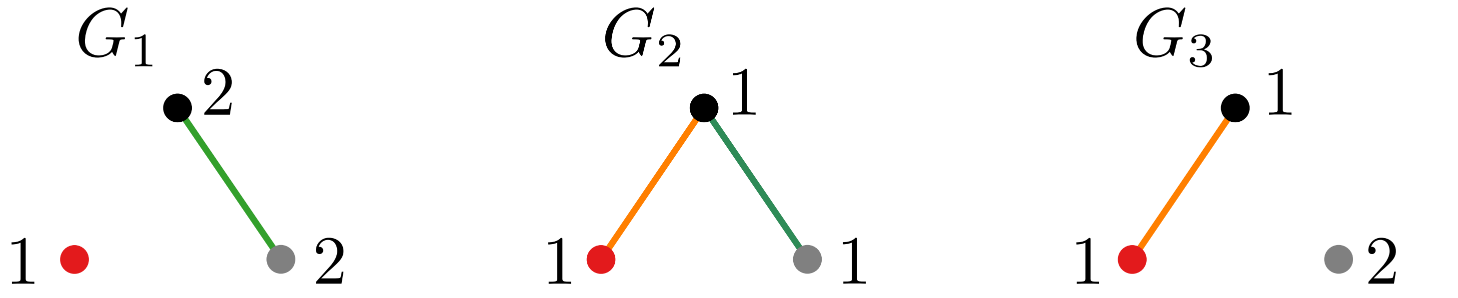

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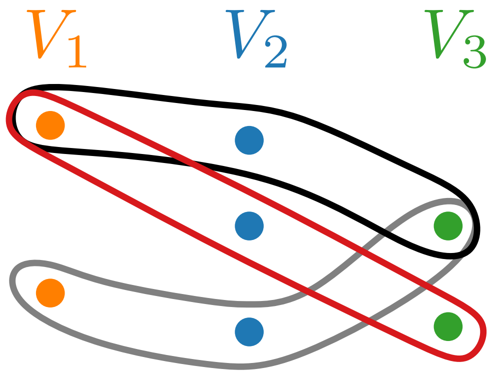

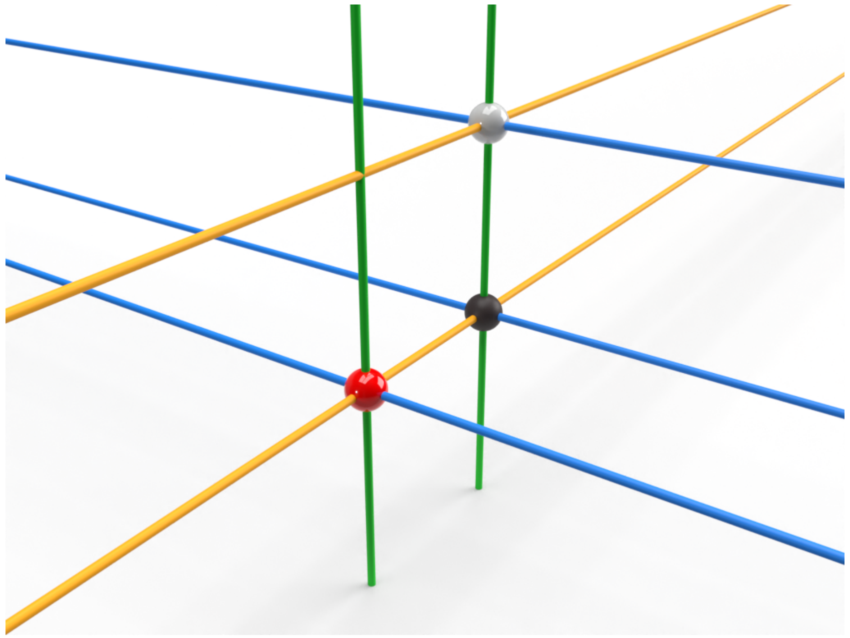

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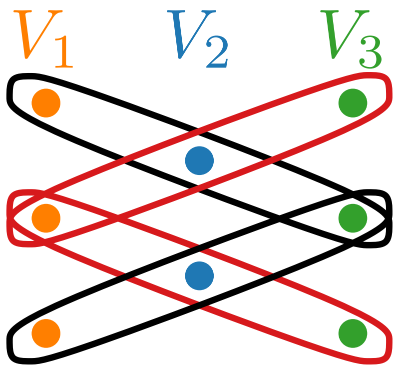
Proof – Part 2

vertex separable $\Leftarrow$ there is a representation

Assume that G is not vertex separable but it has a point line cover representation.



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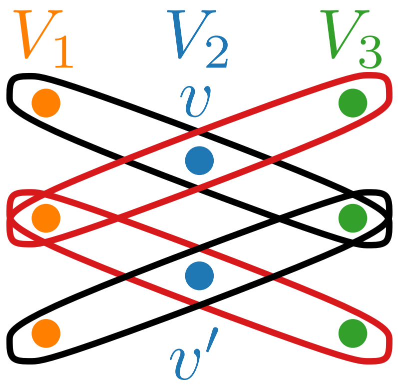

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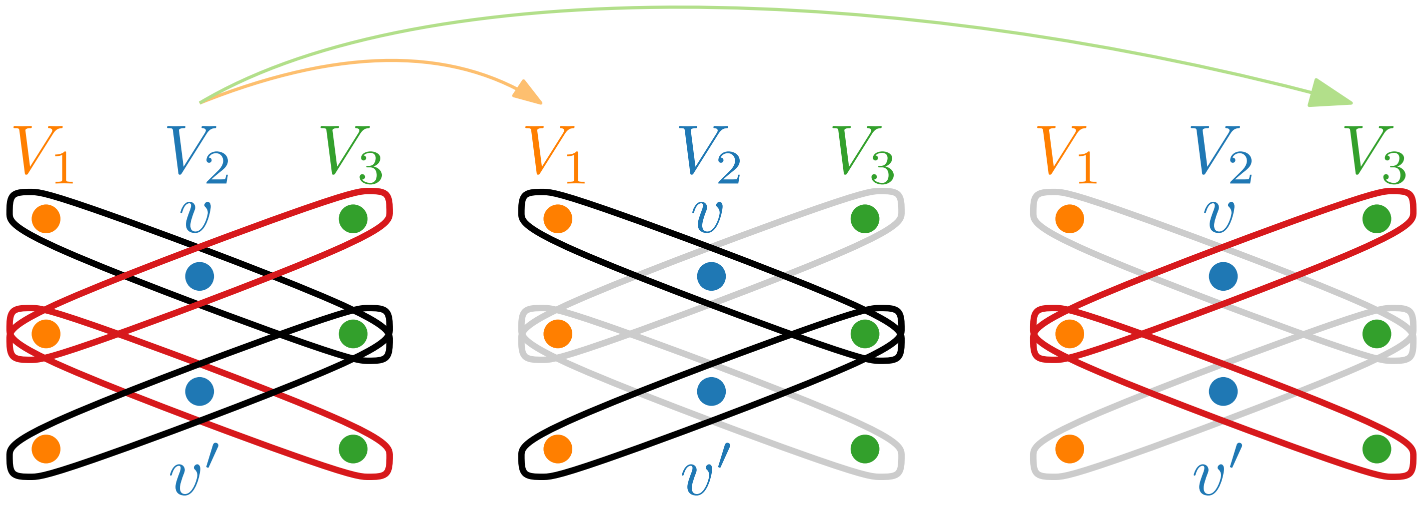

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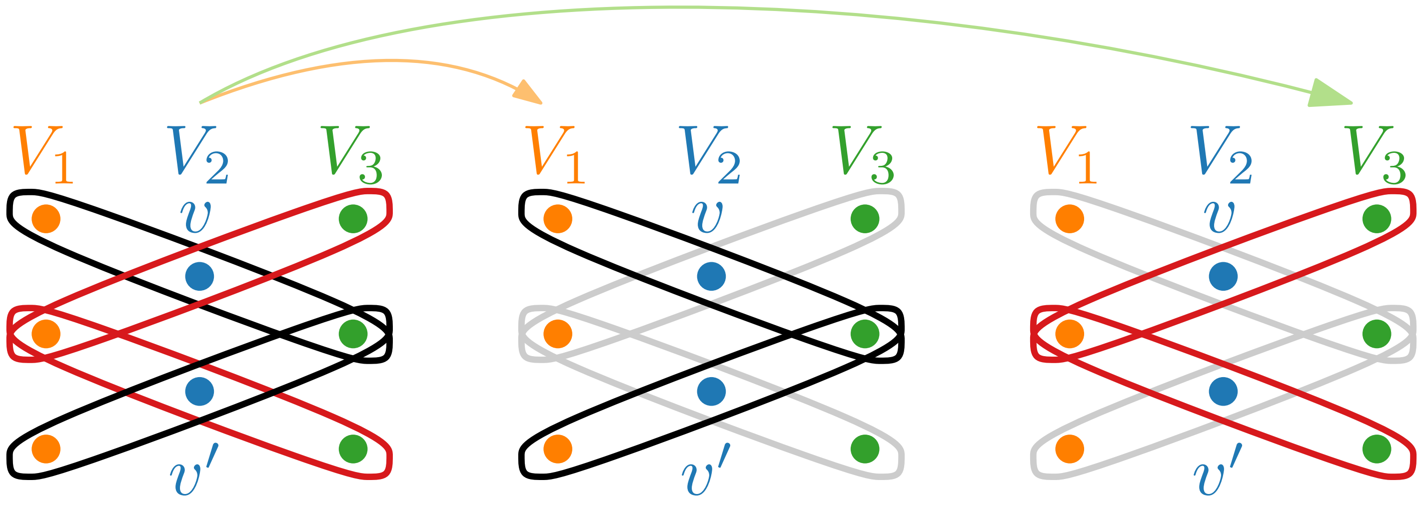

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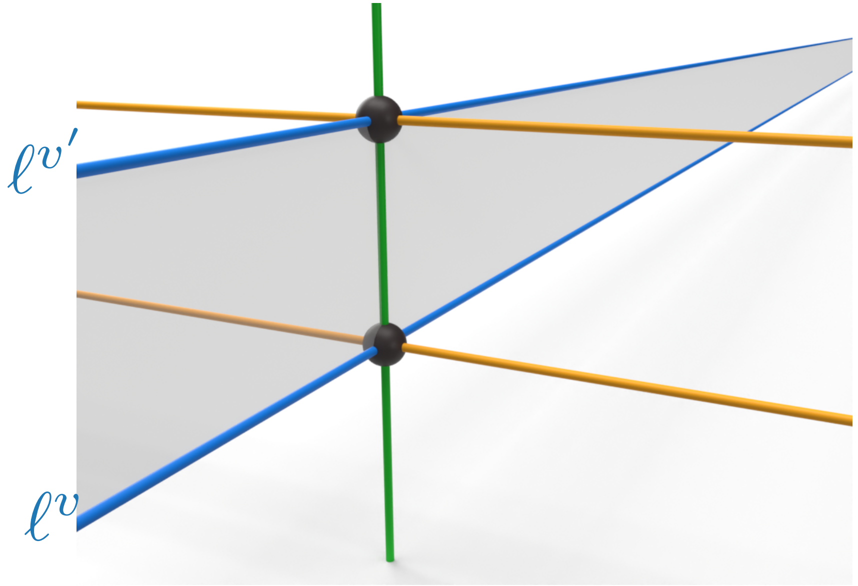

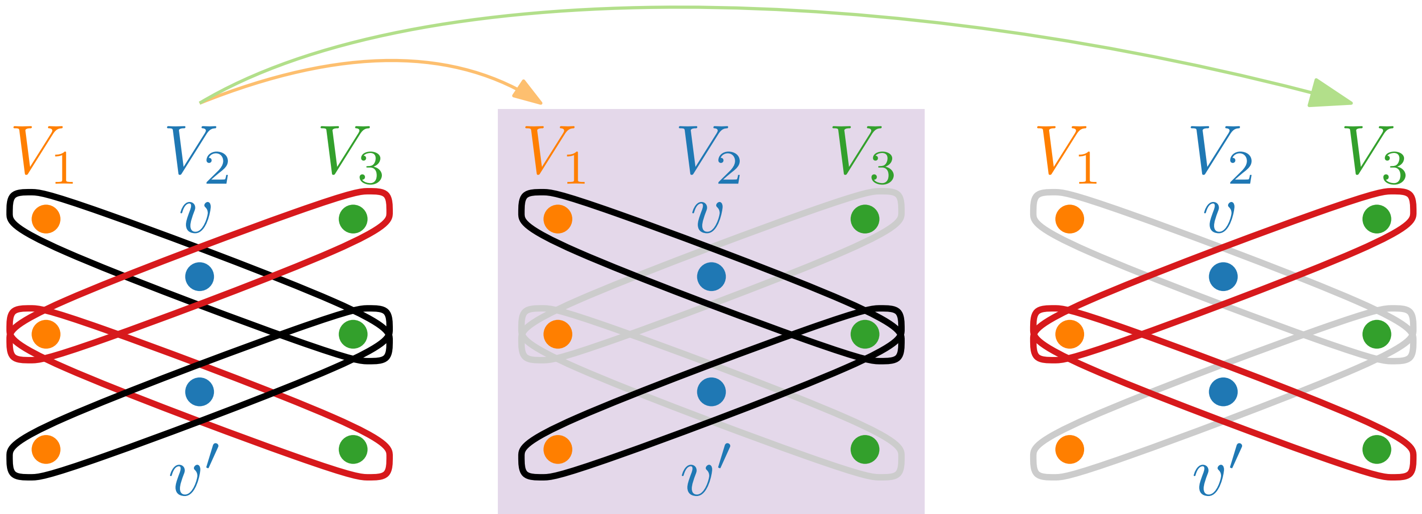

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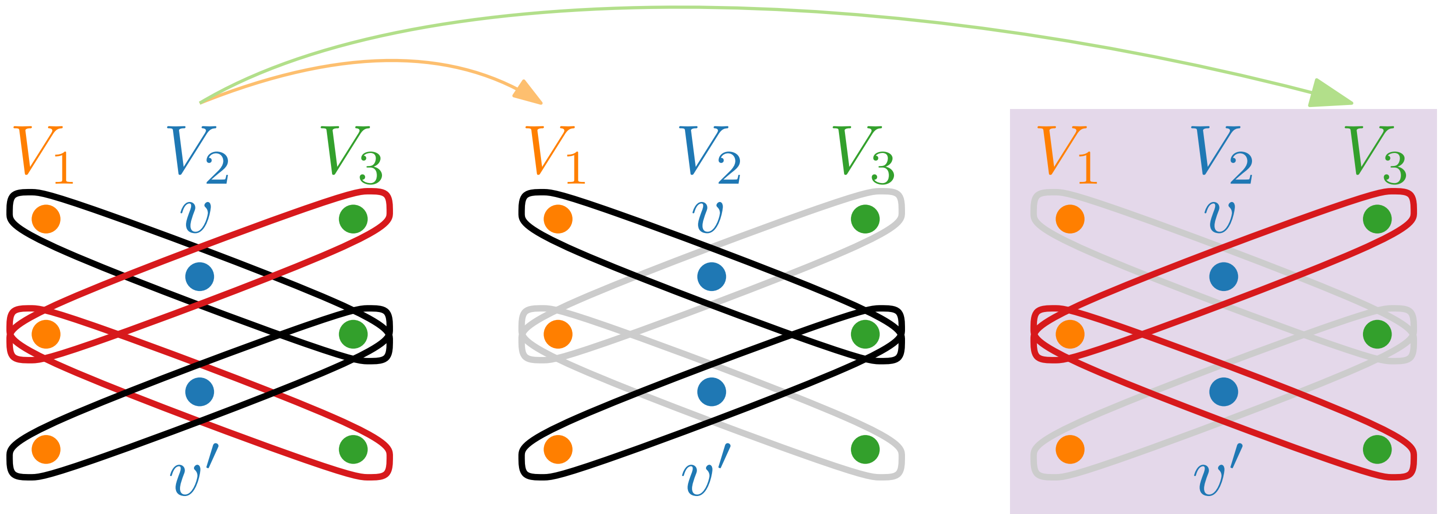

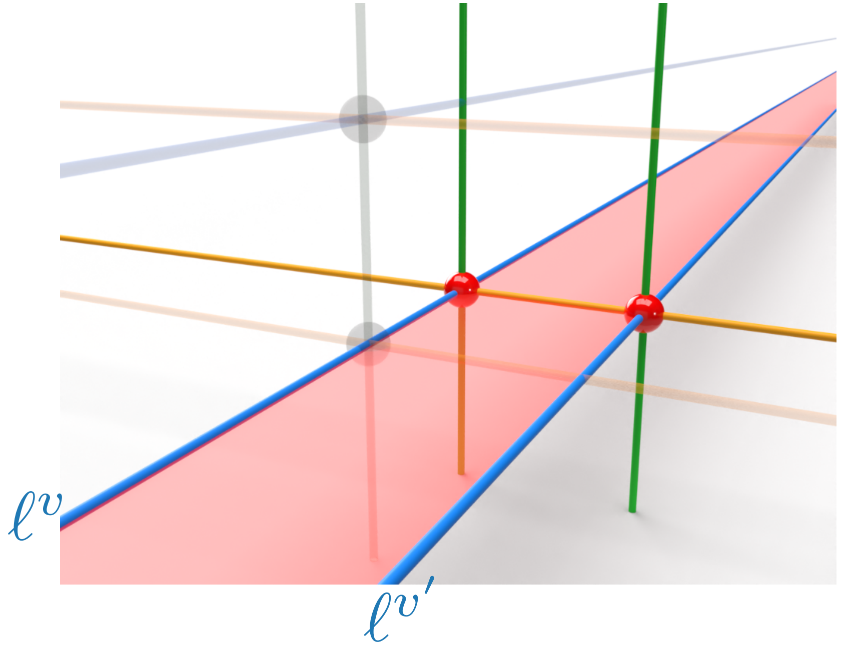

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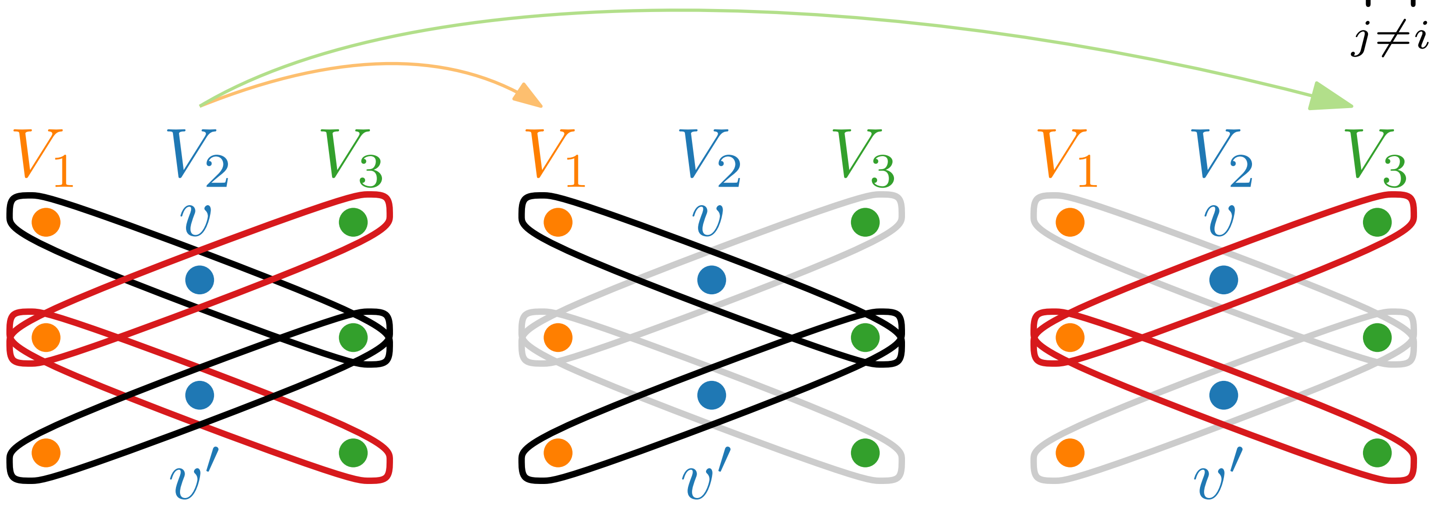
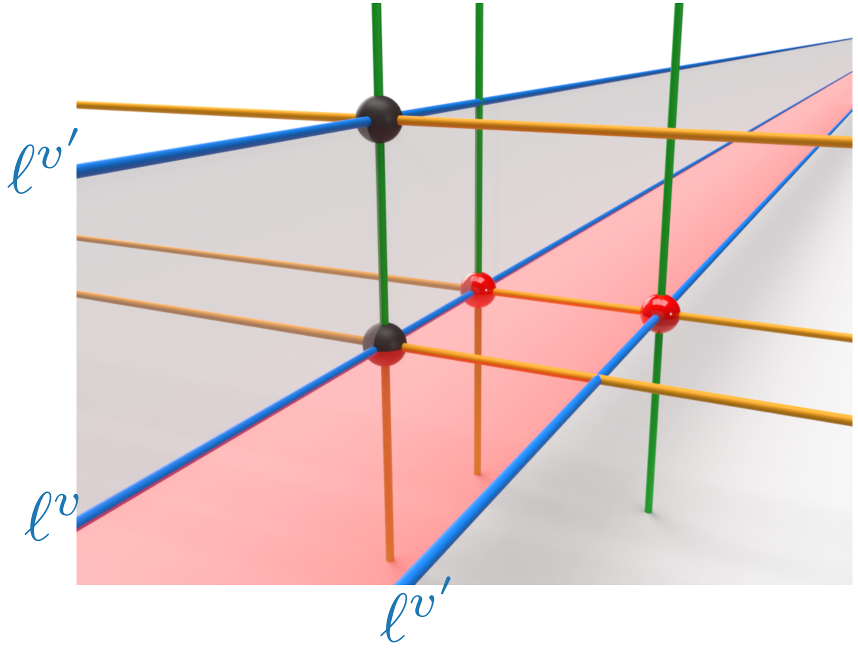
- all lines ℓ^{v_t} with $t \in [r]$ that represent the vertices $v_1, \ldots, v_r$ lie on the same hyperplane H_j perpendicular to the x_j-axis;
- the lines ℓ^v and $\ell^{v'}$ lie in the intersection $\bigcap_{j \neq i} H_j$.





Proof – Part 2

vertex separable $\Leftarrow$ there is a representation

Assume that G is not vertex separable but it has a point line cover representation.

- it contains at least two distinct vertices v and v' from the same group V_i that are not separable;
- for each group V_j with $j \neq i$, there exists a v-v' path $v = v_1, \ldots, v_r = v'$ such that $v_t \notin V_j$ for each $t \in [r]$;
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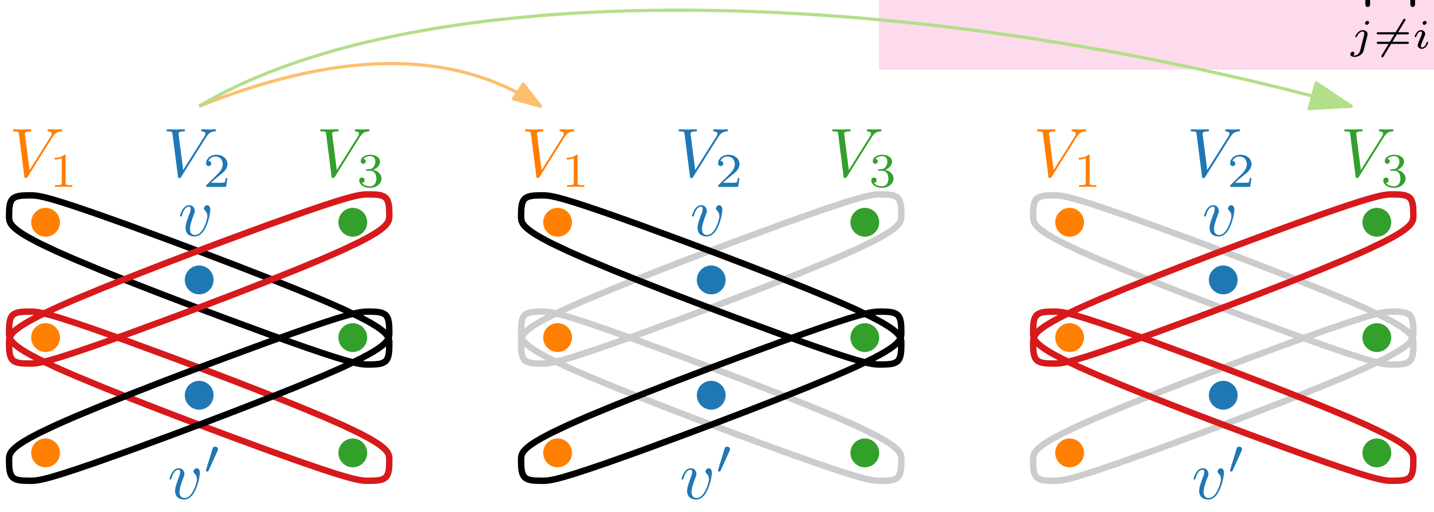

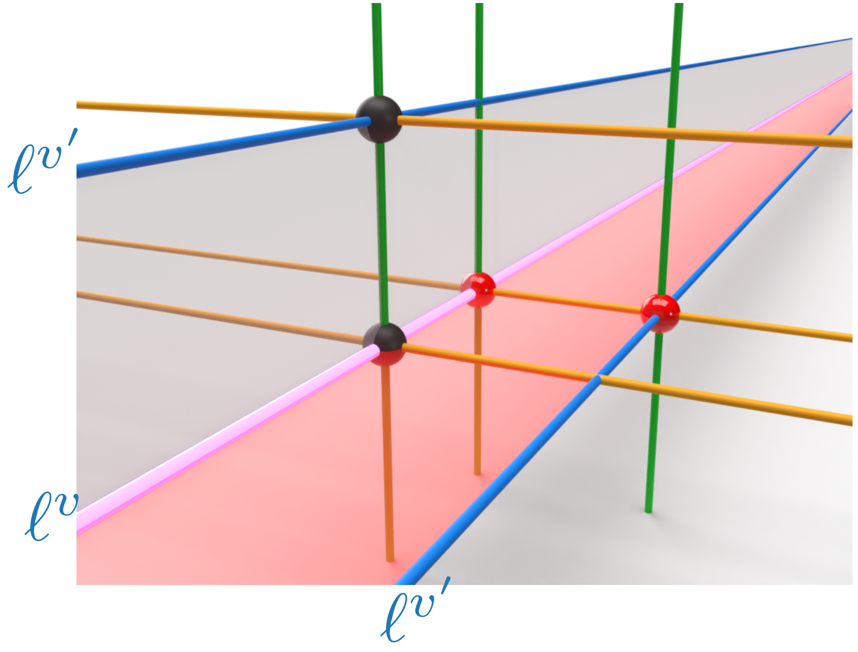

□

Further Results & Open Questions

- Generalization to ℓ-dimensional subspace, $\ell < d$

Further Results & Open Questions

- Generalization to ℓ-dimensional subspace, $\ell < d$

Space	d-dimensional
Covering objects	lines
Representable hypergraphs	vertex separable

Further Results & Open Questions

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Space	d-dimensional
Covering objects	lines
Representable hypergraphs	vertex separable
	polynomial recognition algorithm

Further Results & Open Questions

- Generalization to ℓ-dimensional subspace, $\ell < d$

Space	d-dimensional	
Covering objects	lines	$(d-1)$-dimensional subspaces
Representable hypergraphs	vertex separable	all
	polynomial recognition algorithm	

Further Results & Open Questions

- Generalization to ℓ-dimensional subspace, $\ell < d$

Space	d-dimensional	
Covering objects	lines	$(d-1)$-dimensional subspaces
Representable hypergraphs	vertex separable	all
	polynomial recognition algorithm	similar to representation of bipartite graphs in 2D

Further Results & Open Questions

- Generalization to ℓ-dimensional subspace, $\ell < d$

Space	d-dimensional		
Covering objects	lines	ℓ-dimension subspaces $2 \leq \ell \leq (d-2)$	$(d-1)$-dimensional subspaces
Representable hypergraphs	vertex separable	generalized vertex separable	all
	polynomial recognition algorithm		

Further Results & Open Questions

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Further Results & Open Questions

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What about non-constant d?

Further Results & Open Questions

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Representable hypergraphs	vertex separable	generalized vertex separable	all
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What about non-constant d?

- Design *improved* algorithms for vertex separable hypergraphs (e.g vertex cover, matching) parameterized by ℓ and d

Further Results & Open Questions

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What about non-constant d?

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Thank you!

Stand with Ukraine