A $(1+\varepsilon)$-approximation for 1-center in $\mathbb{R}^d$

Sang-Sub Kim *Barbara Schwarzwald*

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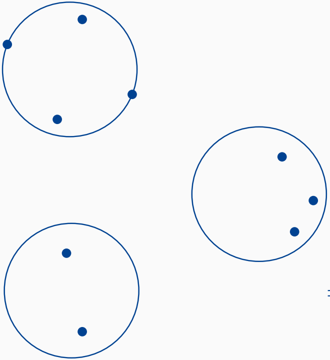
University of Bonn, Germany
schwarzwald@uni-bonn.de

The k-center problem

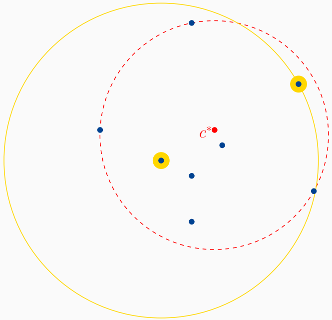


- Classic clustering problem
- Set of point P in $\mathbb{R}^d$
- Find k balls of minimum radius r^* covering all points of P

$\Leftrightarrow$ Find set of centers C minimizing $\max_{p \in P} \min_{c \in C} d(p, c)$

	general k	$k = 1$
bounded d	NP-hard APX-hard	$O(n)$
general d	NP-hard APX-hard	**no algorithm poly in d**

Approximating the 1-center problem

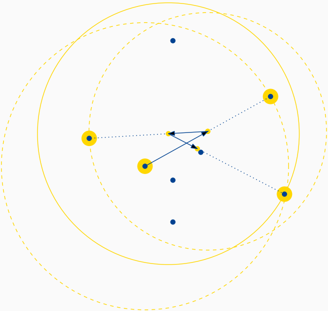


- Set of points P in $\mathbb{R}^d$
- Find a ball of minimum radius r^* covering all points of P

$\Leftrightarrow$ Find center c^* minimizing $\max_{p \in P} d(p, c^*)$

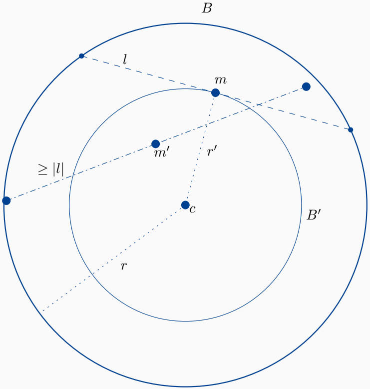
- p with $d(p, c^*) < \varepsilon r^*$ is a $(1 + \varepsilon)$-approximation
- Any point $p \in P$ is a 2-approximation
- Idea: Use farthest point to improve
- Bădoiu et.al., Kumar et.al.(2007): Build core-set of size $O(\frac{1}{\varepsilon})$ in time $O(\frac{nd}{\varepsilon} + \frac{1}{\varepsilon^{4.5}} \log \frac{1}{\varepsilon})$
- Simpler gradient descent?

Naive Gradient-Descent



- Begin with arbitrary point $p \in P$
- Find farthest point
- Move center straight towards it
  - $\frac{1}{2}$, $\frac{1}{3}$, $\ldots$, $\frac{1}{i+1}$ in step i
  - Arithmetic mean of all points so far
- Repeat $\lceil \frac{1}{\varepsilon^2} \rceil$ steps
- $(1+\varepsilon)$ approximation in time $O(\frac{nd}{\varepsilon^2})$

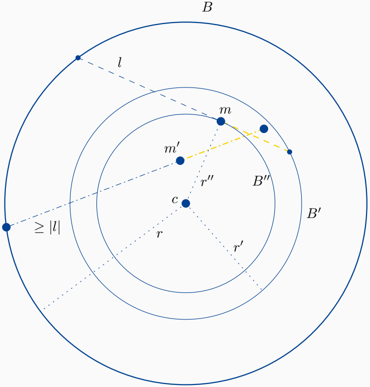
Structural Property Lemma (First step)



- Given a ball $B = B(c, r)$
- Place a line segment l in B as far away from c as possible
- This defines a ball $B' = B(c, r')$ tangential to l in its midpoint m
- $\frac{r'}{r}$ only depends on $\frac{|l|}{r}$
- Any line segment fully in B with length $\geq |l|$ has its midpoint in B'

- Can assume first two points have distance $> (1 + \varepsilon)r^*$
- Midpoint lies in ball B' with $\frac{r'}{r}$ defined by $\frac{|l|}{r} = \frac{1+\varepsilon}{1}$

Structural Property Lemma(Every further step)



- Given two balls $B = B(c, r)$ and $B' = (c, r')$ around the same center
- Place a line segment l in it as far away from c as possible with one endpoint in B'
- This defines $B'' = B(c, r'')$ tangential to l at some point m
- $\frac{r''}{r}$ depends on $\frac{|l|}{r}$ and $\frac{r'}{r}$
- Any line segment fully in B with length $\geq |l|$ and one endpoint in B' has the point m' at the same relative position as m inside B''

Need to know r^*? No, the relative position of m also depends only on $\frac{|l|}{r}$ and $\frac{r'}{r}$

Result

	1-**center**	k-**center**
gradient descent algorithm	$O(\frac{nd}{\varepsilon})$ using $\lfloor \frac{2}{\varepsilon} \rfloor$ points	$O(nd\,k^{\frac{k}{\varepsilon}})$ guessing points
coreset algorithm	$O(\frac{nd}{\varepsilon} + \frac{1}{\varepsilon^{4.5}} \log \frac{1}{\varepsilon})$ using $O(\frac{1}{\varepsilon})$ points	$O(nd\,k^{\frac{k}{\varepsilon}})$ guessing points

Thank you!