Scheduling drones to cover outdoor events

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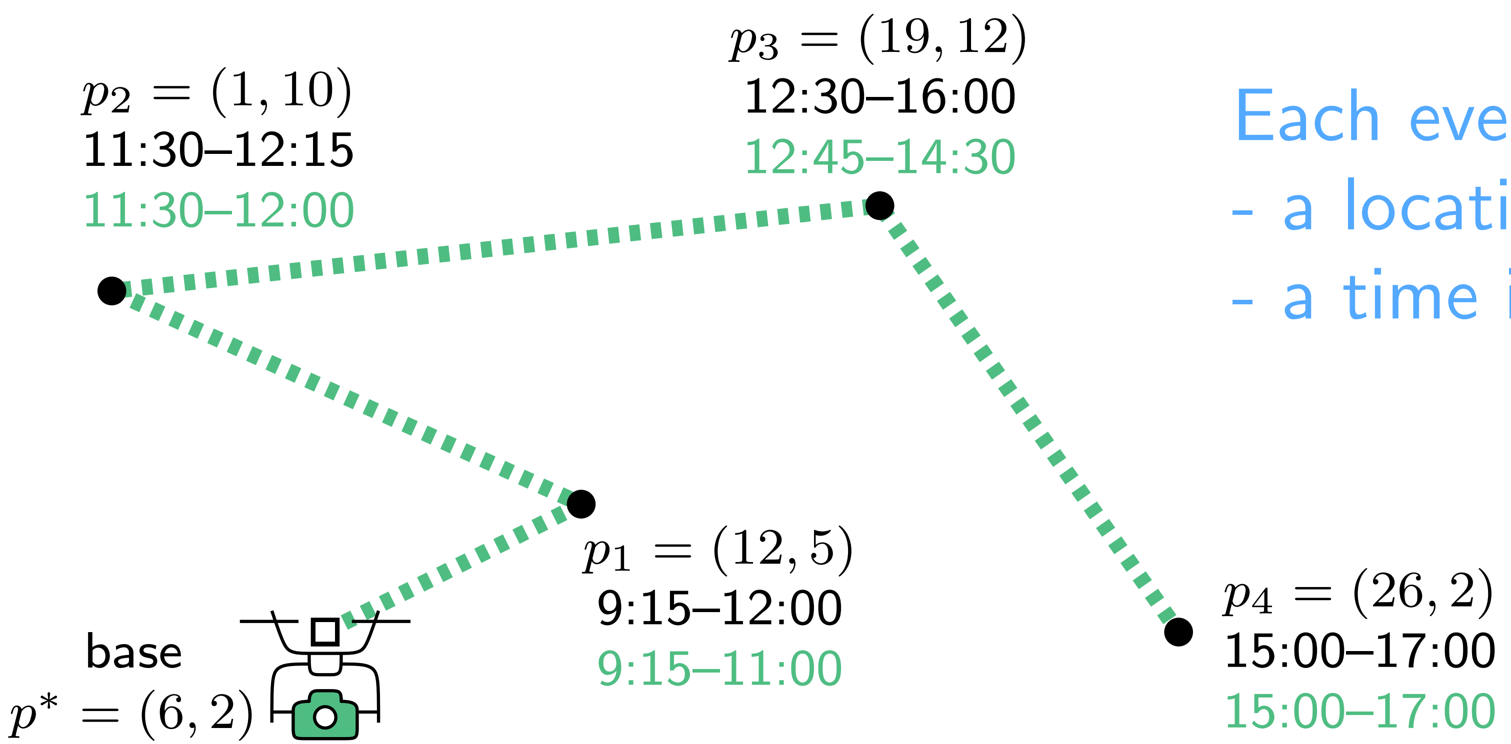
One drone (unlimited battery)

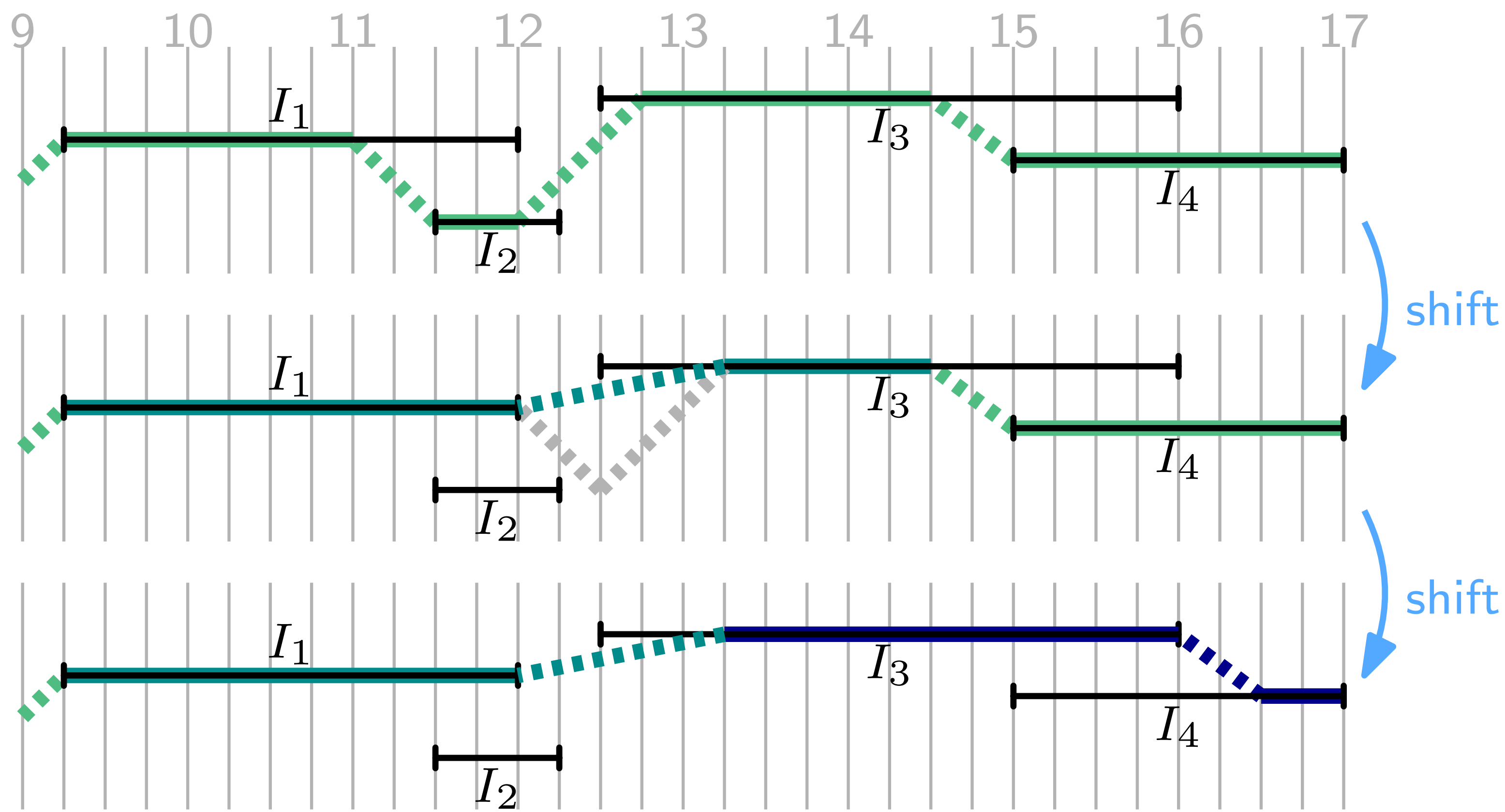


Each event i has:
- a location p_i and
- a time interval I_i.

Goal: Film as much (time) as possible.

Lemma: There is an optimal plan in which the drone does not leave an event before it has ended.

One drone (unlimited battery)



Goal: Film as much (time) as possible.

Lemma: There is an optimal plan in which the drone does not leave an event before it has ended.

One drone (unlimited battery)

We construct a directed (acyclic) graph $G = (V, E)$.

- V: base p^* and the points p_i.
- E: (p_i, p_j) iff a drone leaving p_i at the end of I_i can arrive to p_j at a time $t \in I_j := [a, b]$; weight $= b - t$. Every (p^*, p_i) is an edge with weight $|I_i|$.

We can compute E efficiently in $O(n^{5/3} + |E|)$ time: (x, y) at time $t \Rightarrow (x, y, t) \in \mathbb{R}^3 \Rightarrow (x, y, t, x^2, y^2, z^2) \in \mathbb{R}^6$ using halfspace reporting queries in $\mathbb{R}^6$ we determine E.

Our problem translates to finding a directed path in G from p^* of max weight: topo. sort + dynamic programming.

$$\boxed{\text{Optimal flight plan in } O(n^{5/3} + |E|) \text{ time.}}$$

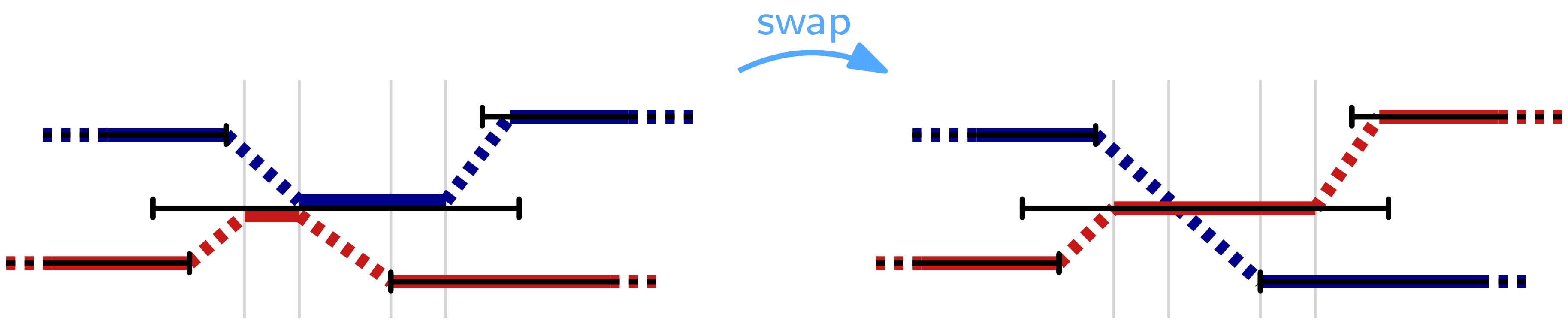
k drones (unlimited battery)

Lemma: There is an optimal plan in which:

a) no drone leaves an event before it has ended and

b) no two drones film at the same point at the same time.

We introduce a second operation:



- Do shifts until every drone leaves an event either at the end or when another drone arrives.
- Do swaps until a) is satisfied.

k drones (unlimited battery)

Lemma: There is an optimal plan in which:

a) no drone leaves an event before it has ended and
b) no two drones film at the same point at the same time.

We construct the same DAG $G = (V, E)$ as before.

Our problem translates to finding a set of k disjoint paths in G starting at p^* of max weight.

NP-complete for general graphs, but polynomial for DAGs.

Optimal flight plan in $O(n^2(\log n + k) + n|E|)$ time.

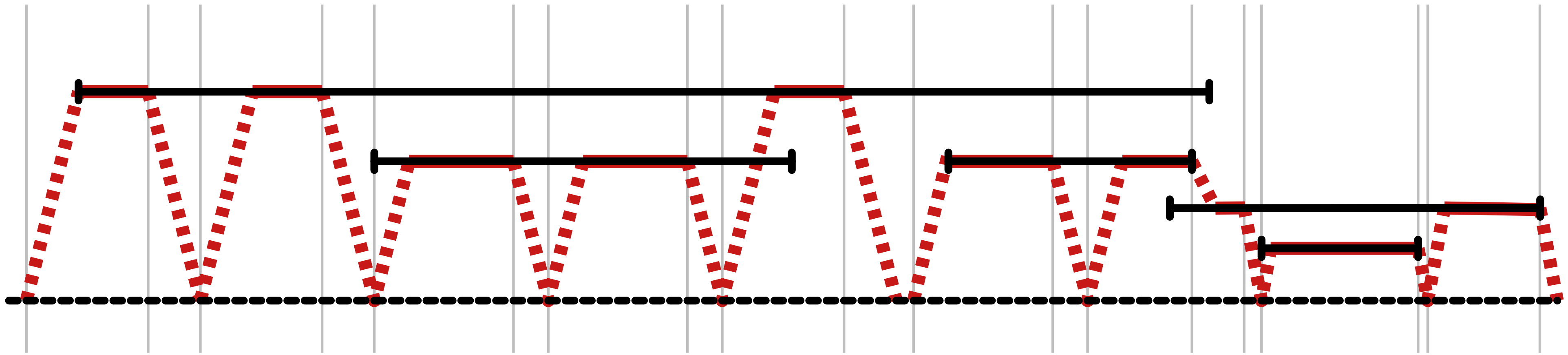
Min. $\#$ drones to cover it all in $O(n^{5/3} + \sqrt{n}|E'|)$ time.

One drone with limited battery

The set of theoretically relevant event-times for an optimal solution can be discretized.

Moreover, an optimal solution can be encoded using a linear number of driving instructions.

Applying dynamic programming we can compute an optimal sequence of instructions in polynomial time.



Conclusions

We studied the problem of optimally scheduling drones to film n events happening at certain time intervals in different places.

- One drone with no battery constraints: $O(n^{5/3} + |E|)$ algorithm, where $|E| = O(n^2)$.
- k drones with no battery constraints: $O(n^2(\log n + k) + n|E|)$ algorithm, where $|E| = O(n^2)$.
- Polynomial algorithm for one drone with limited battery.
- NEW! k drones with limited battery: NP-hard.

Thank you!