

One-Bend Drawings of Outerplanar Graphs Inside Simple Polygons

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John Cabot
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Universität
Würzburg

**Andre
Löffler**
Universität
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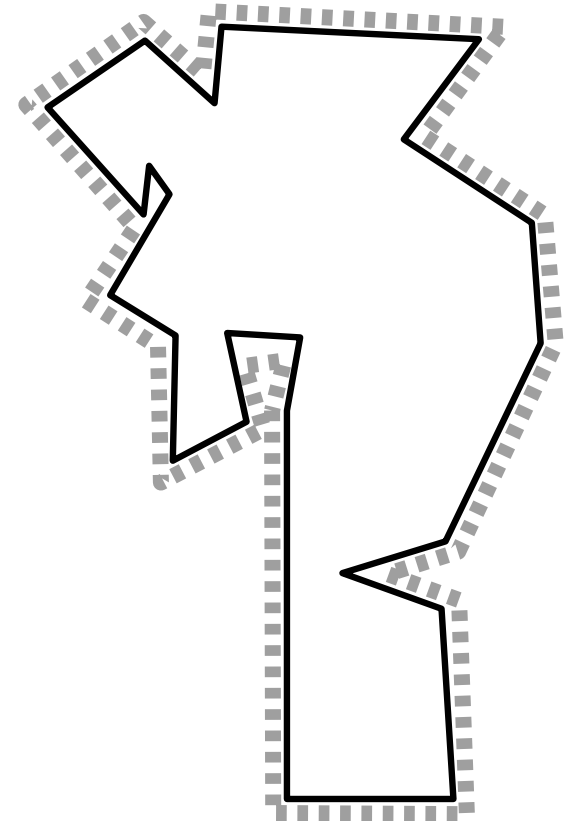
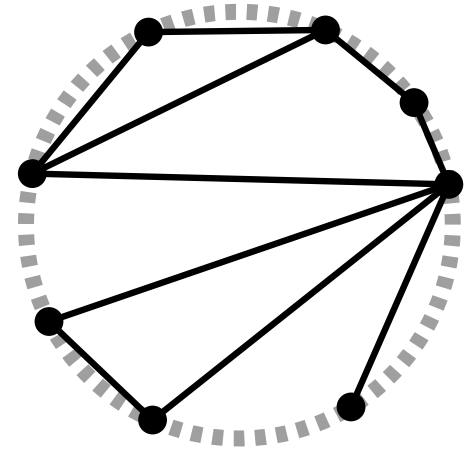
**Lena
Schlipf**
Universität
Tübingen

**Antonios
Symvonis**
National
Technical
University
Athens

Setting & Problem

Input:

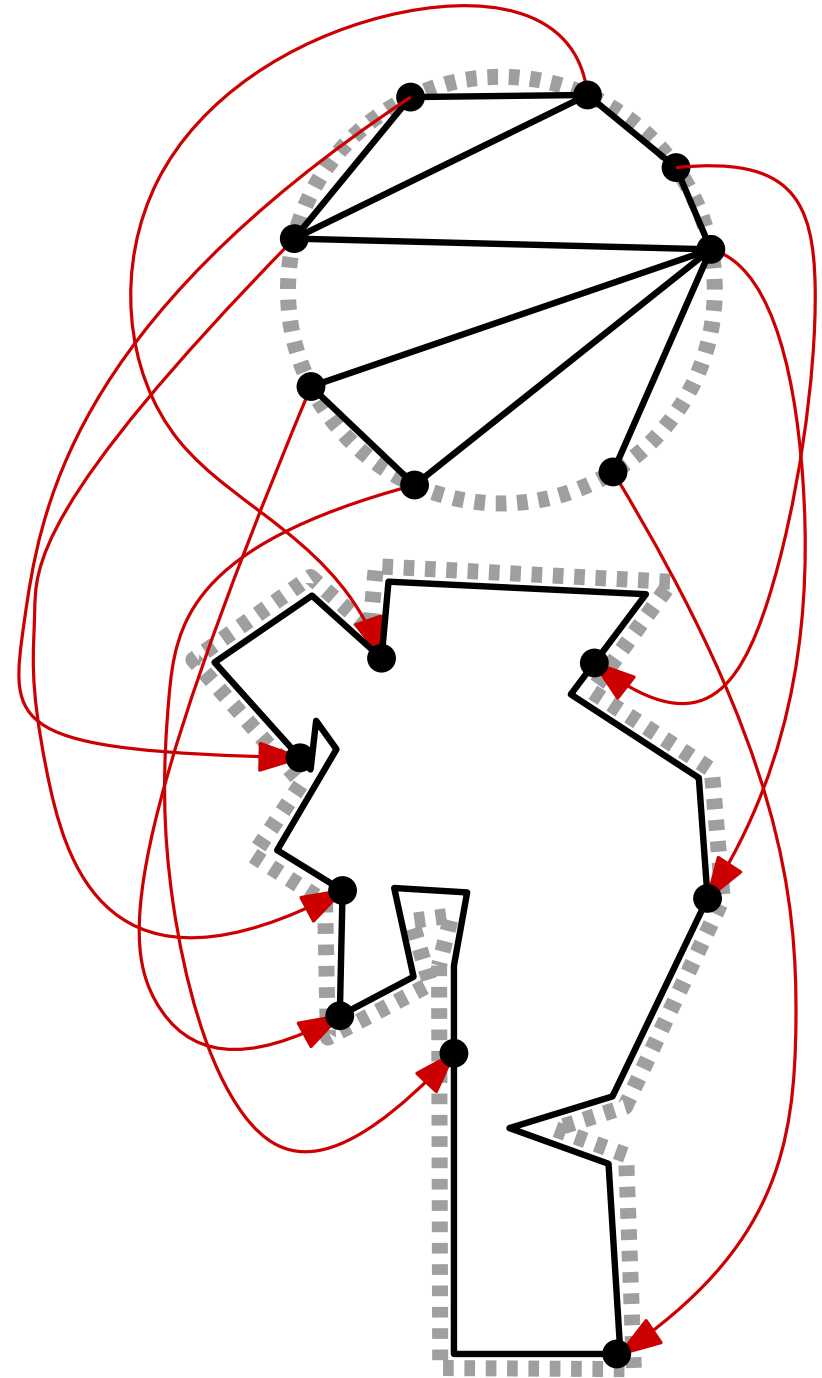
- Outerplanar Graph $G = (V, E)$
- Simple Polygon P



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Input:

- Outerplanar Graph $G = (V, E)$
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- Mapping of V to boundary of P



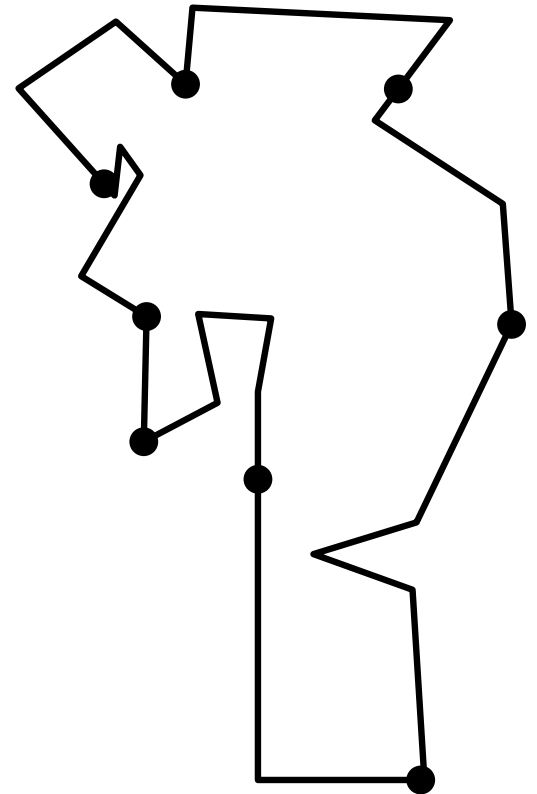
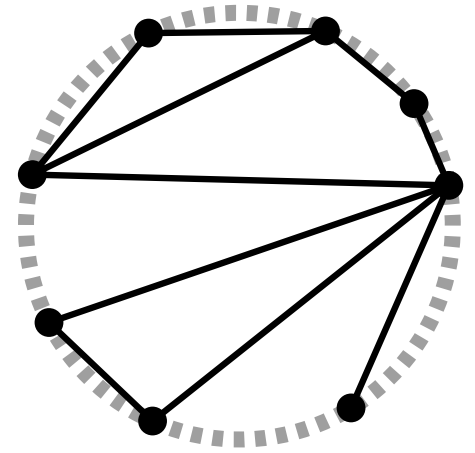
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- Bend points for edges s.t.



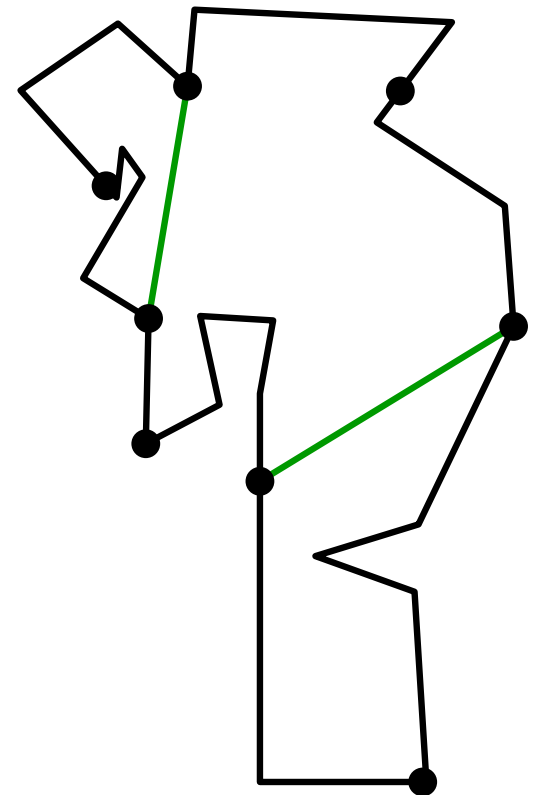
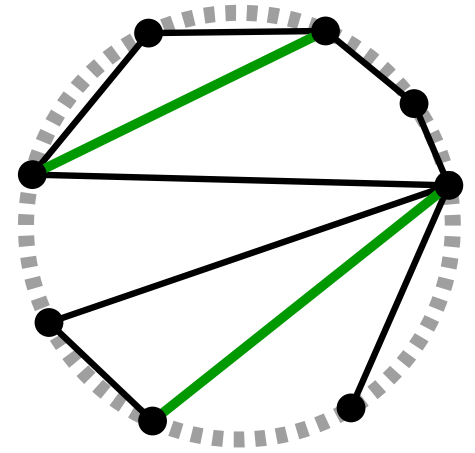
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Output:

- Bend points for edges s.t. all edges inside P



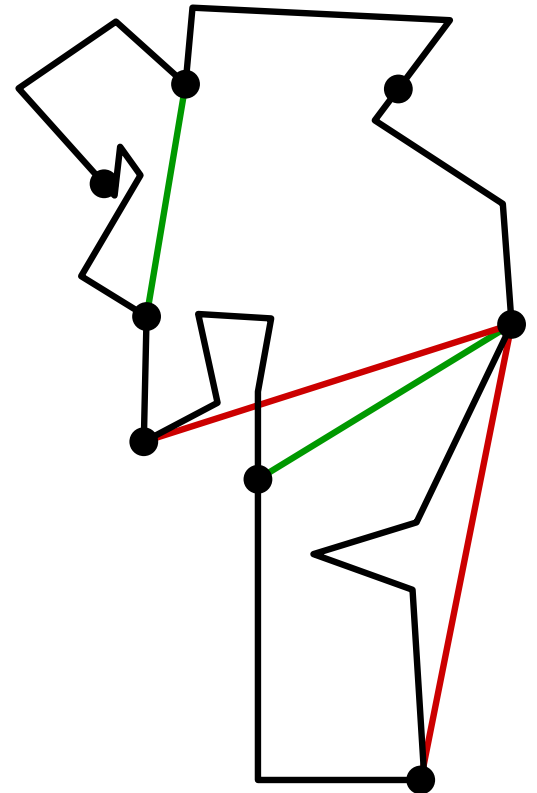
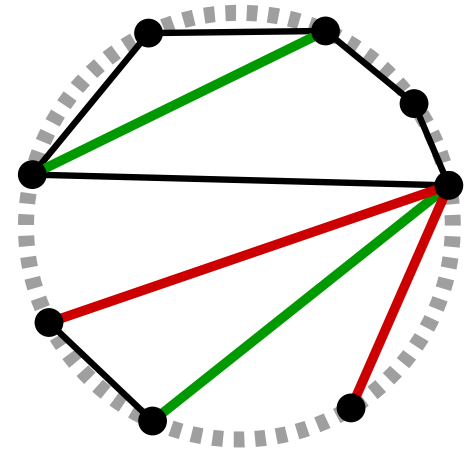
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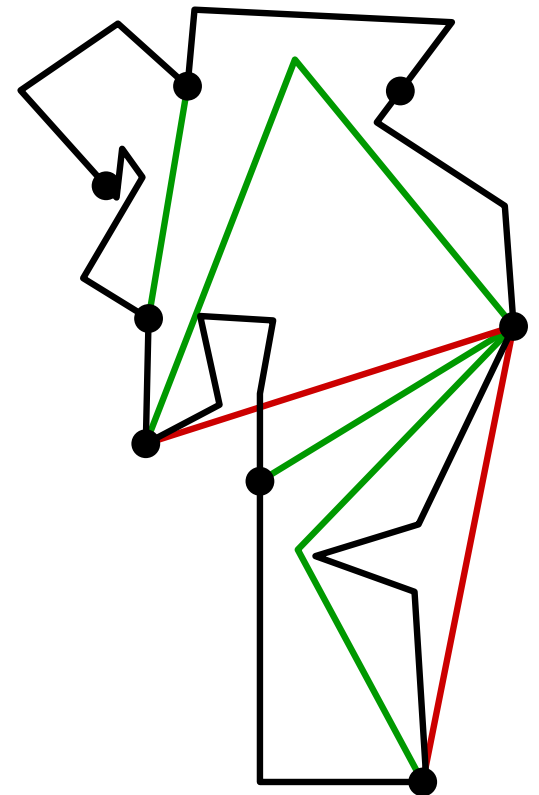
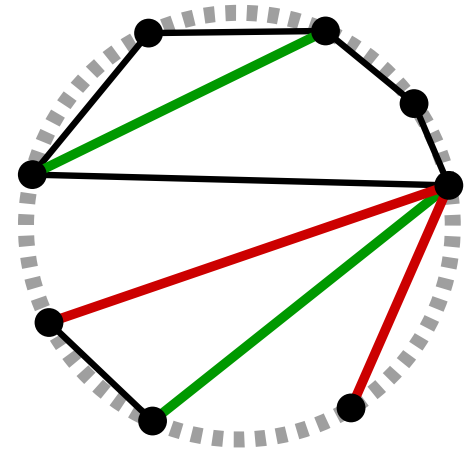
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Output:

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all edges inside P , one bend, planar



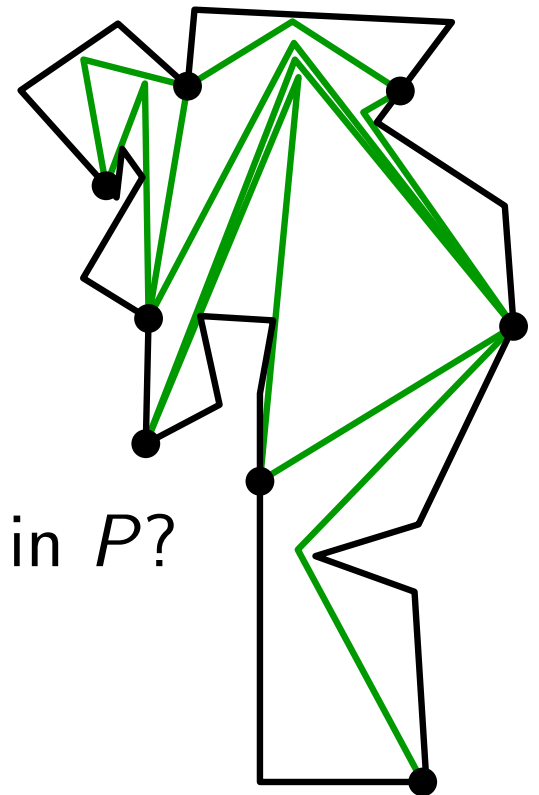
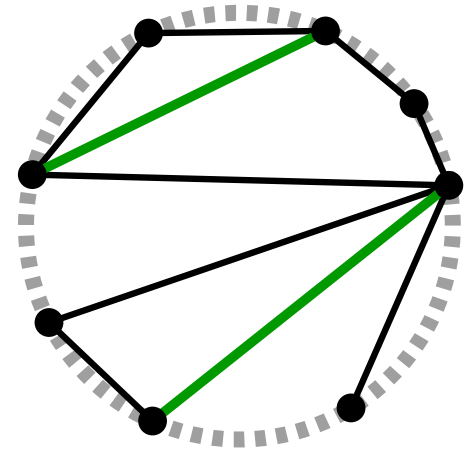
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- Is there a one-bend planar drawing of G in P ?



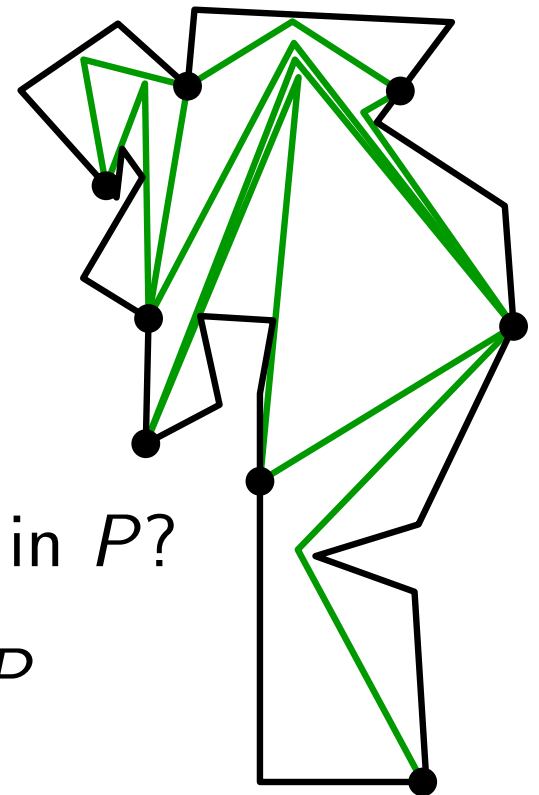
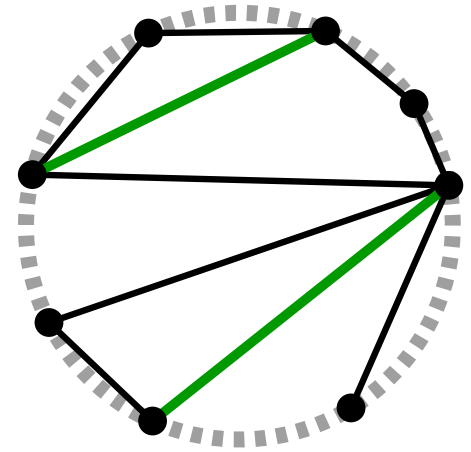
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⇒ Partial Drawing Extension with host P



Motivation & Related Work

- Prescribing the **outer face**? \Rightarrow a graph drawing classic
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- Topological drawings? \Rightarrow polynomial time

[Angelini et al., '15]

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- Linear number of bends per edge suffices ($72|P|$)

[Angelini et al., '15] [Chan et al., '15]

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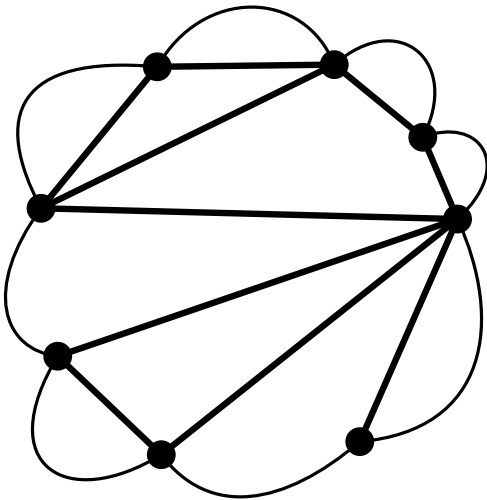
\Rightarrow Let's try **non-straight** edges!

- Topological drawings? \Rightarrow polynomial time
- Linear number of bends per edge suffices ($72|P|$)
- P star-shaped? \Rightarrow **one bend** suffices

[Angelini et al., '15] [Chan et al., '15] [Mchedlize & Urhausen, '18]

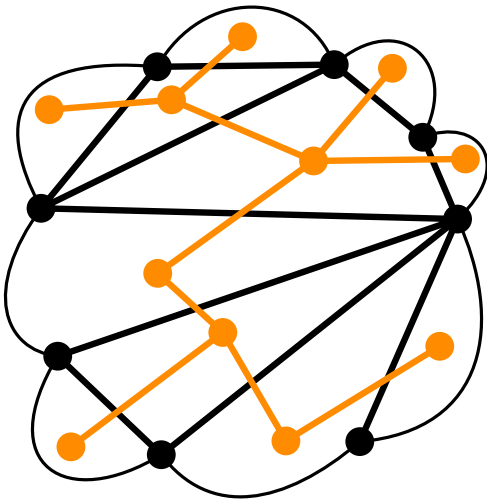
Algorithm Overview

- $G \cup P$ is a biconnected outerplanar graph



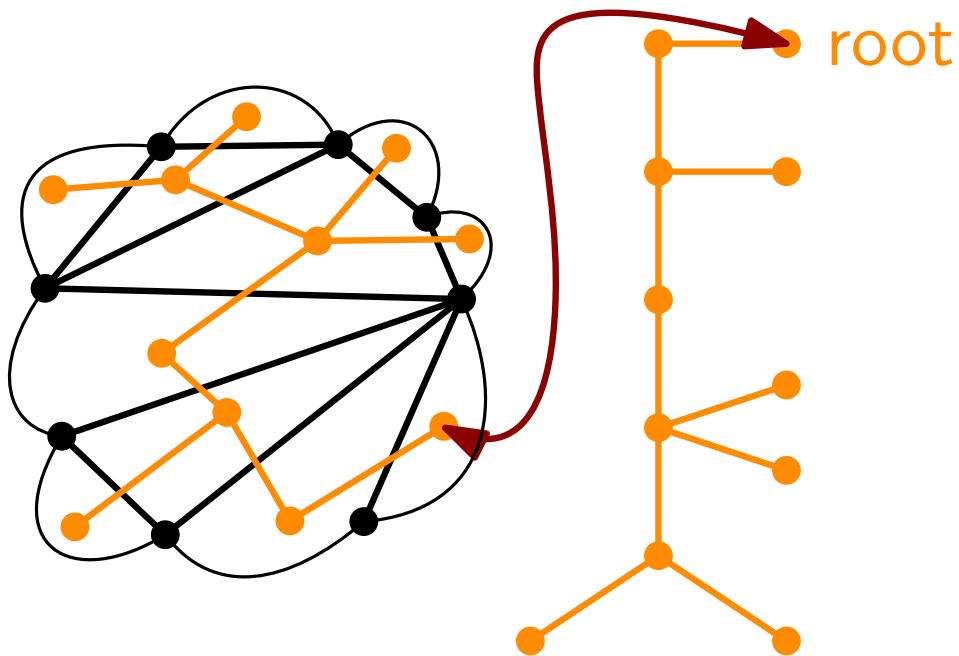
Algorithm Overview

- $G \cup P$ is a biconnected outerplanar graph \Rightarrow use dual tree



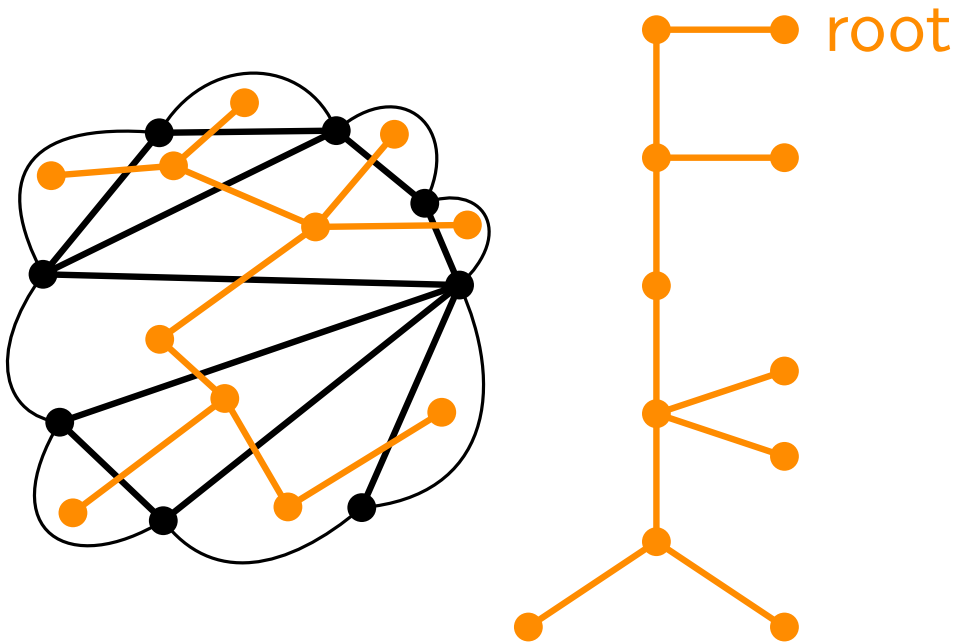
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- $G \cup P$ is a biconnected outerplanar graph \Rightarrow use dual tree
- Traverse that tree bottom-up, iteratively refining P



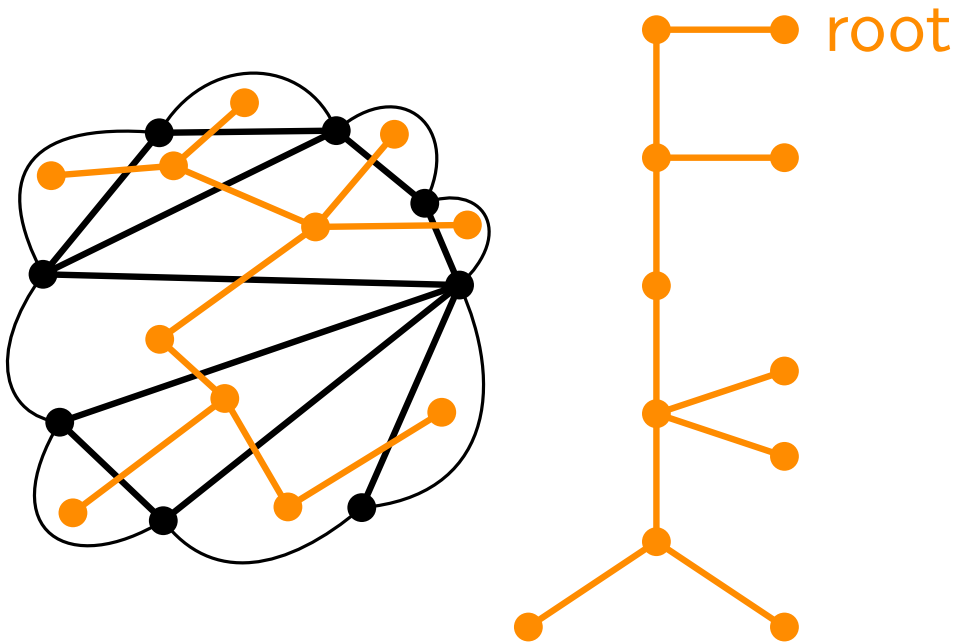
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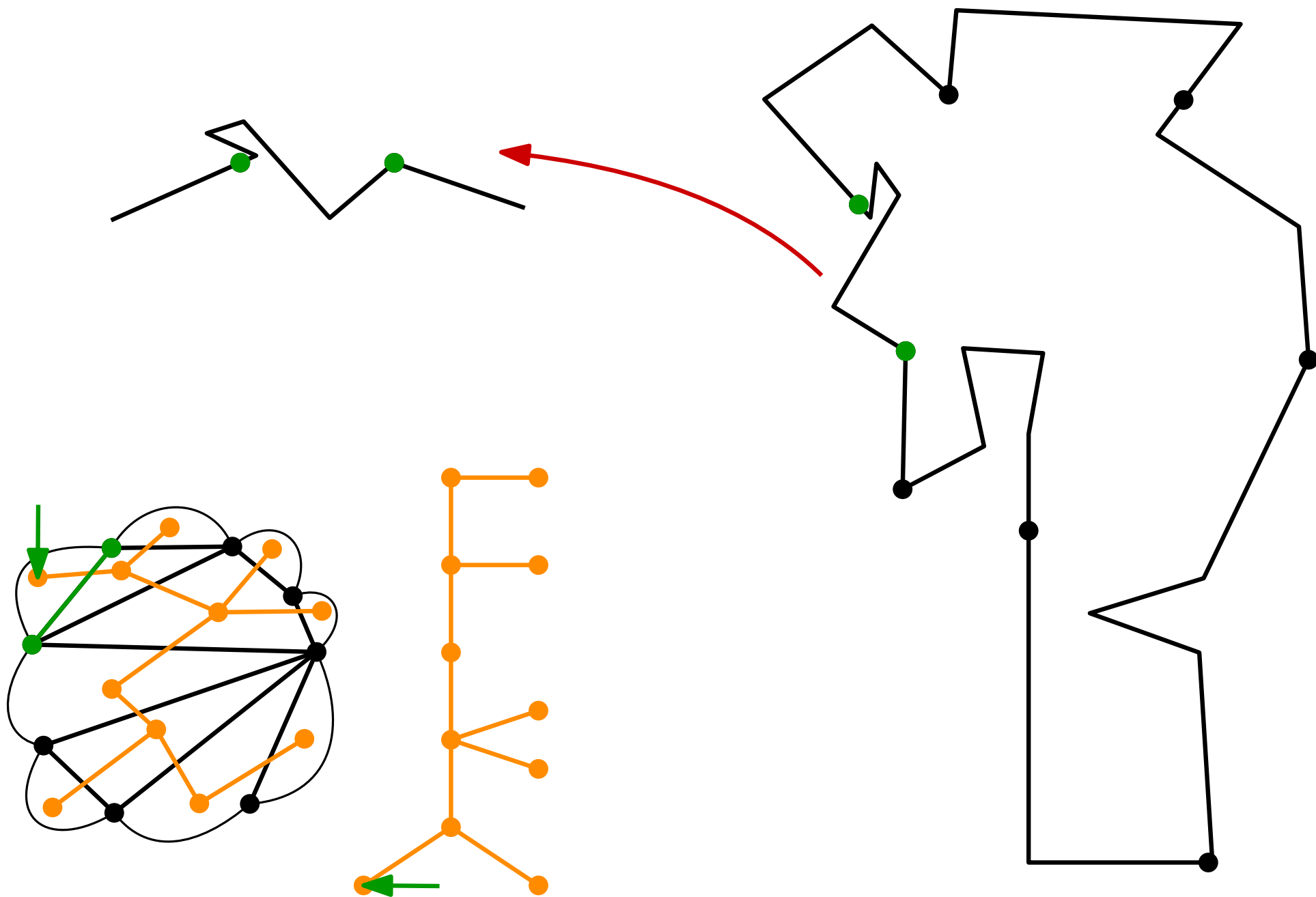


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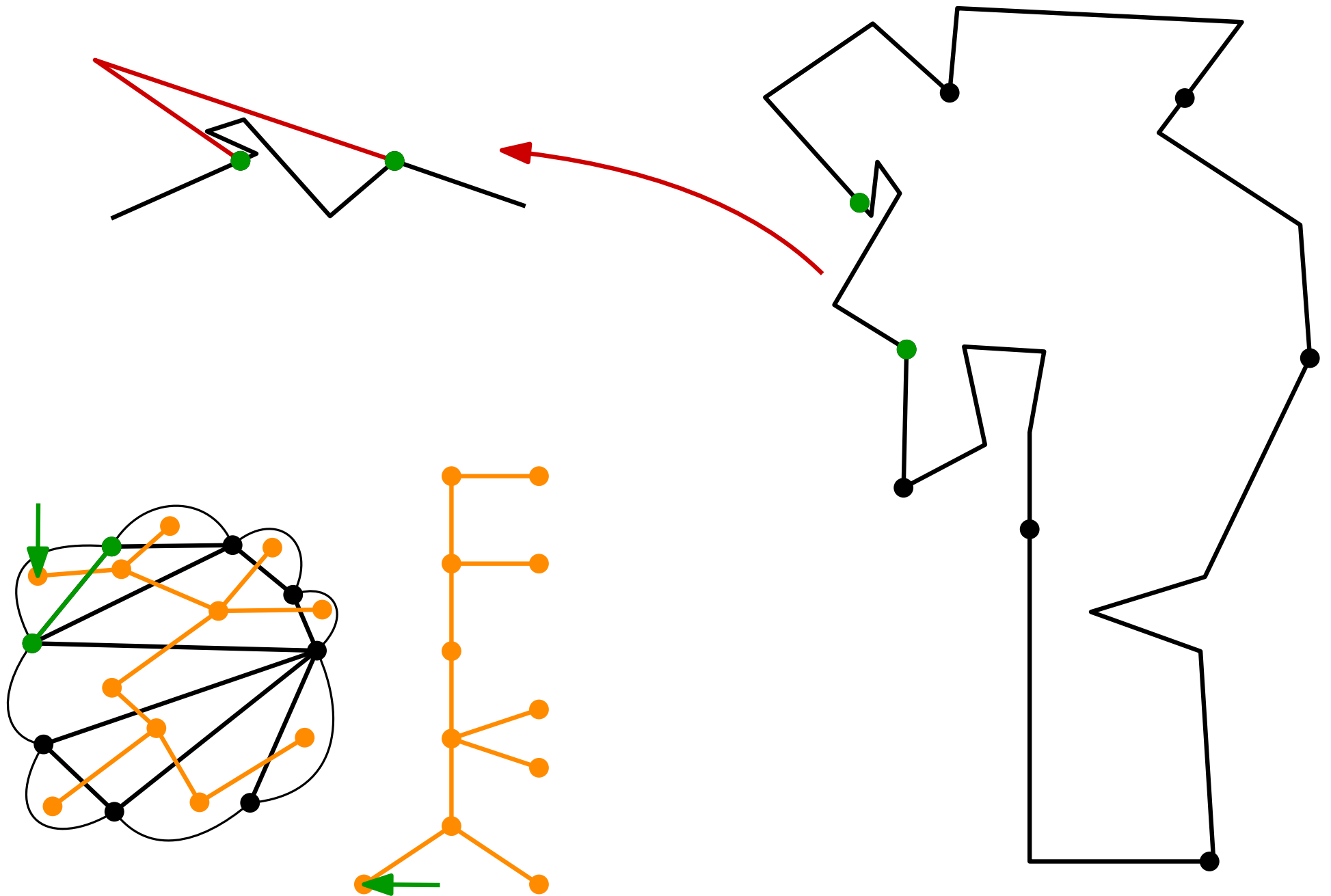
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- Once all “safe” refinements are done, fix the remaining edges top-down.



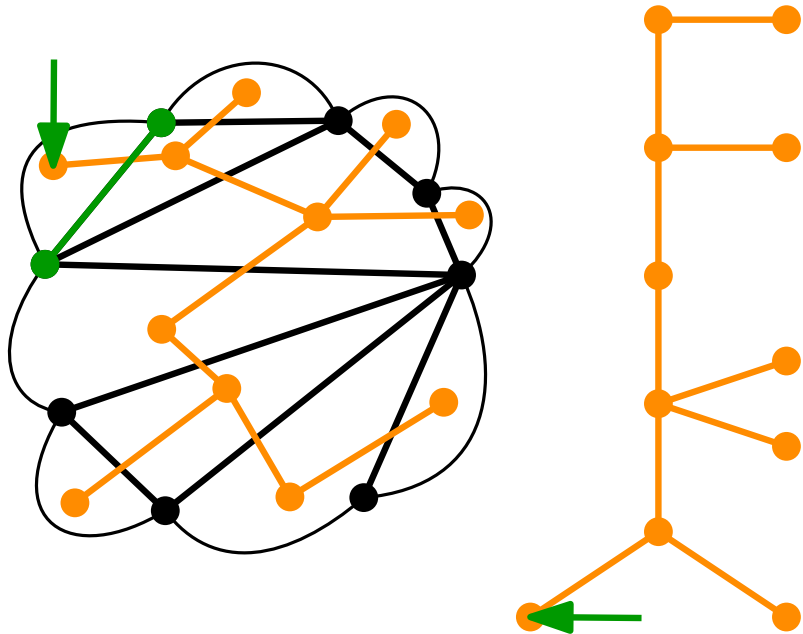
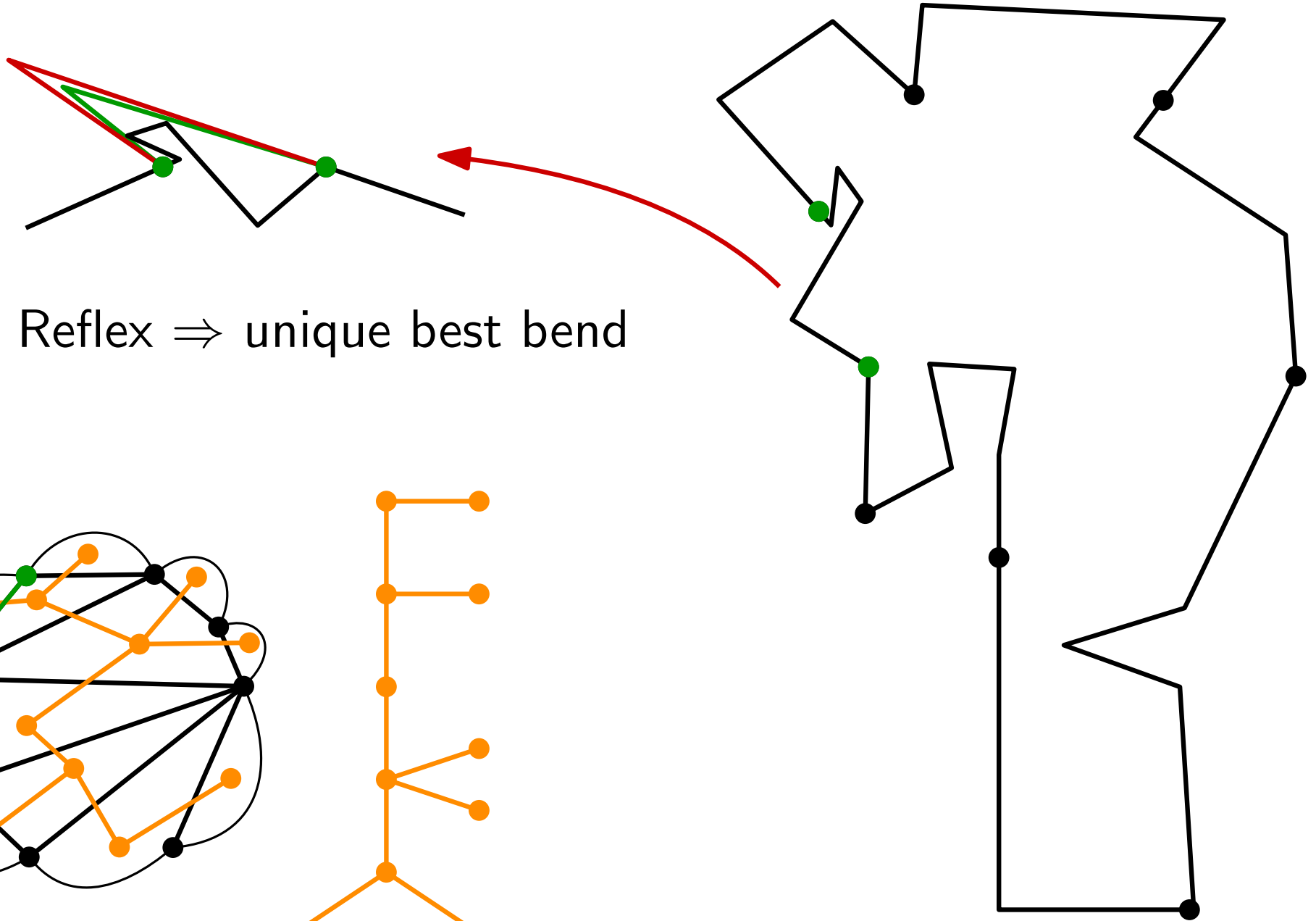
Refinement Types – Reflex



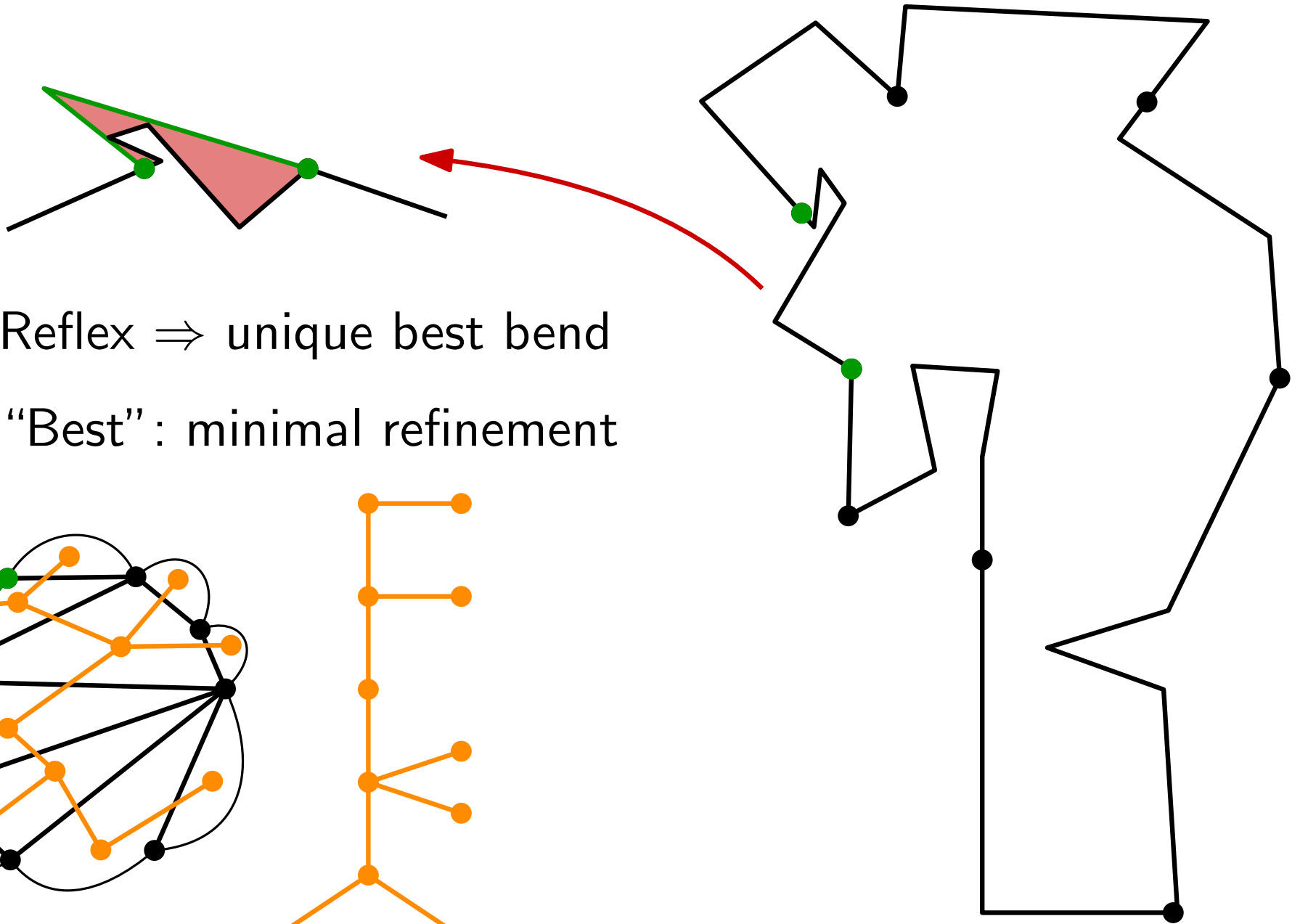
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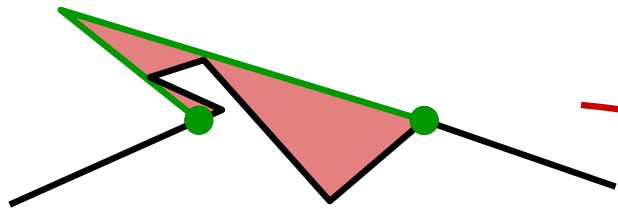


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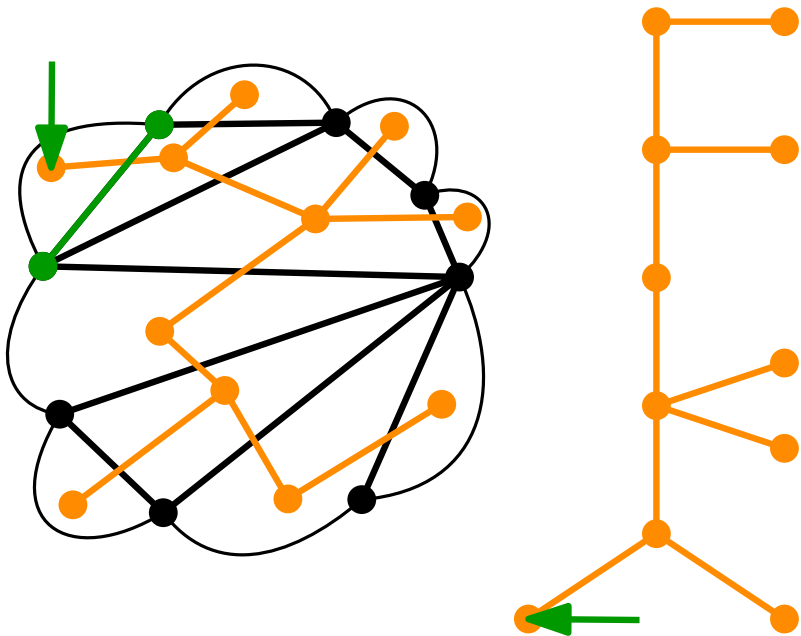
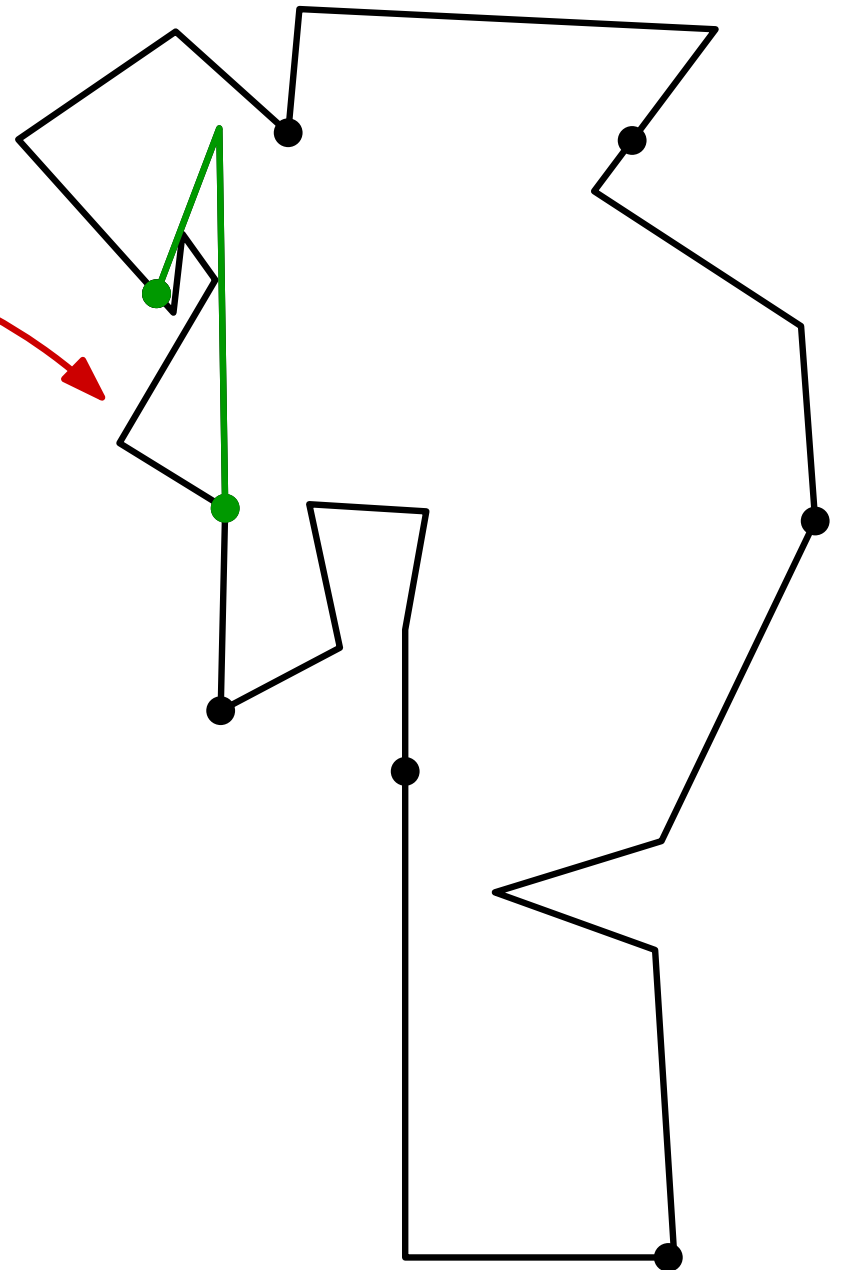


- Reflex \Rightarrow unique best bend
- “Best”: minimal refinement

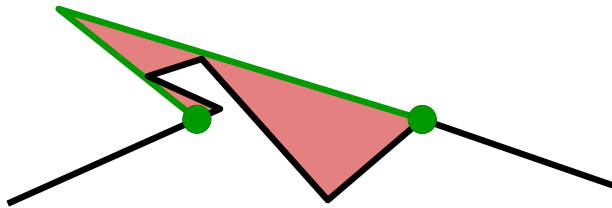
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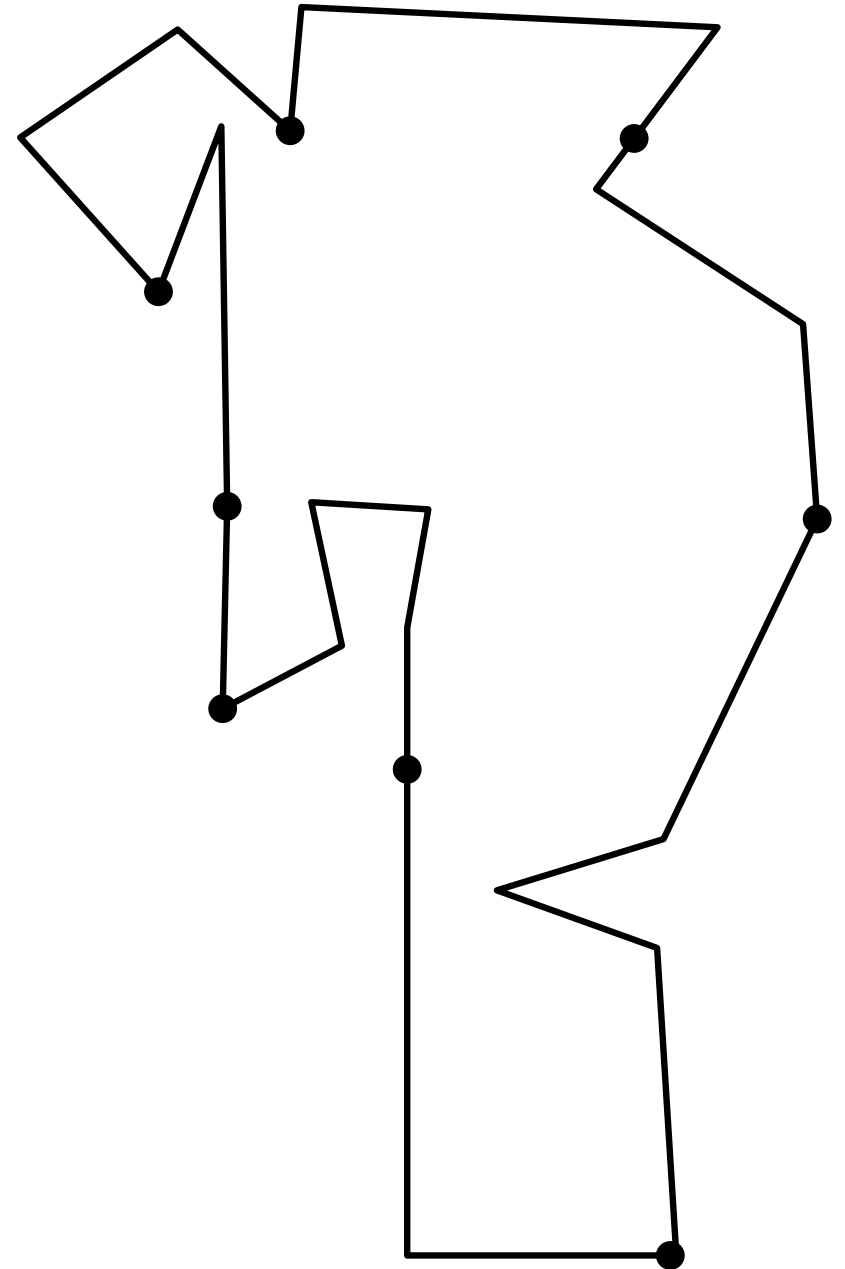
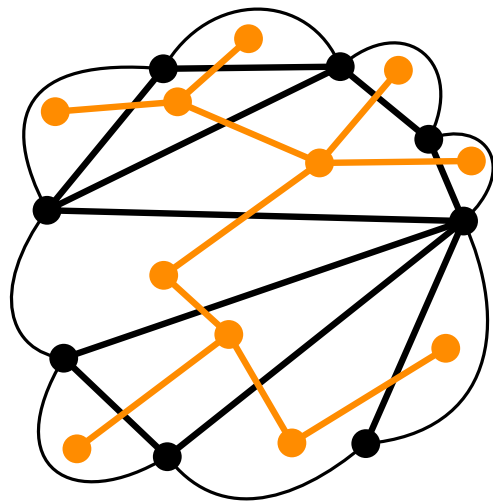
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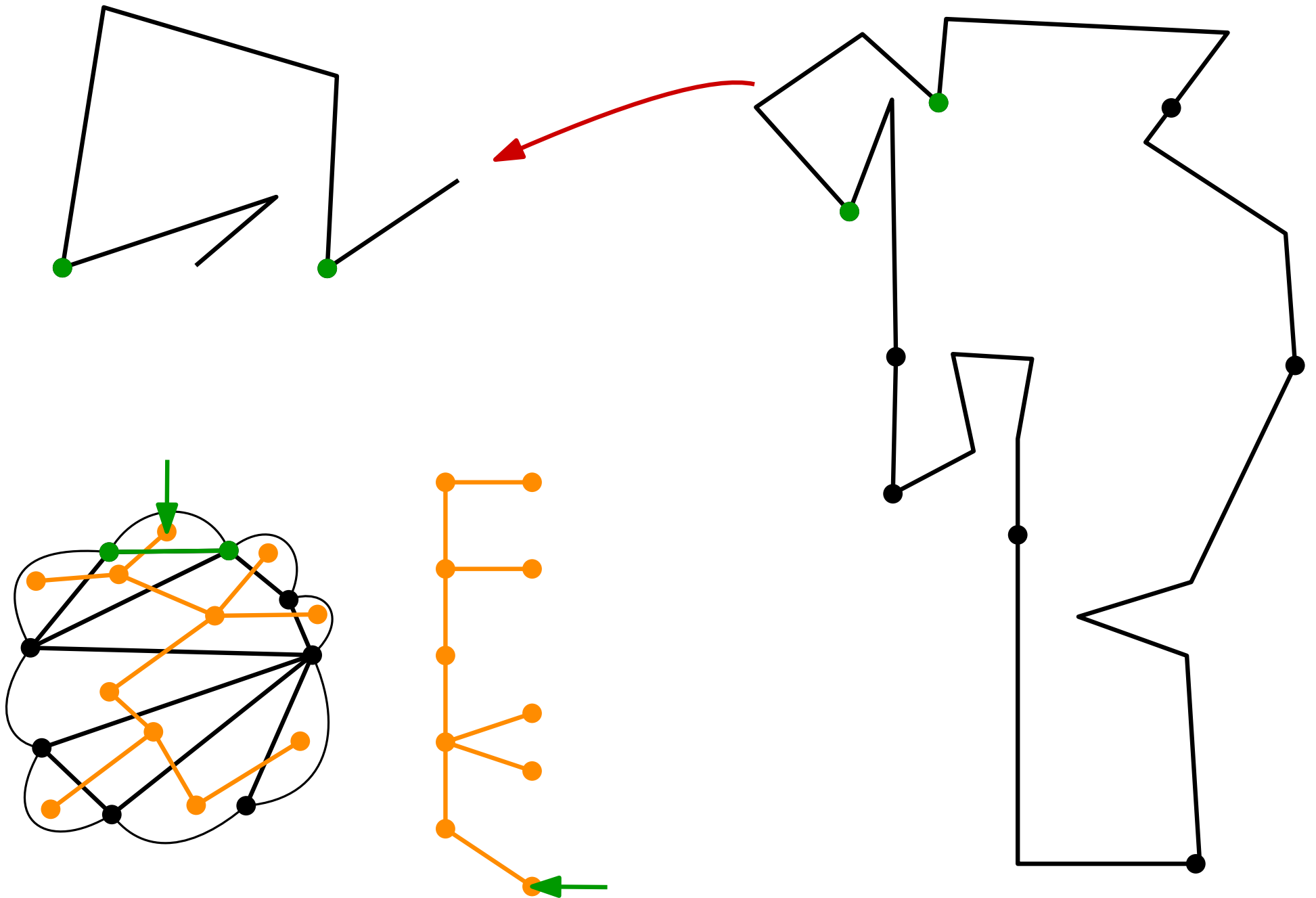
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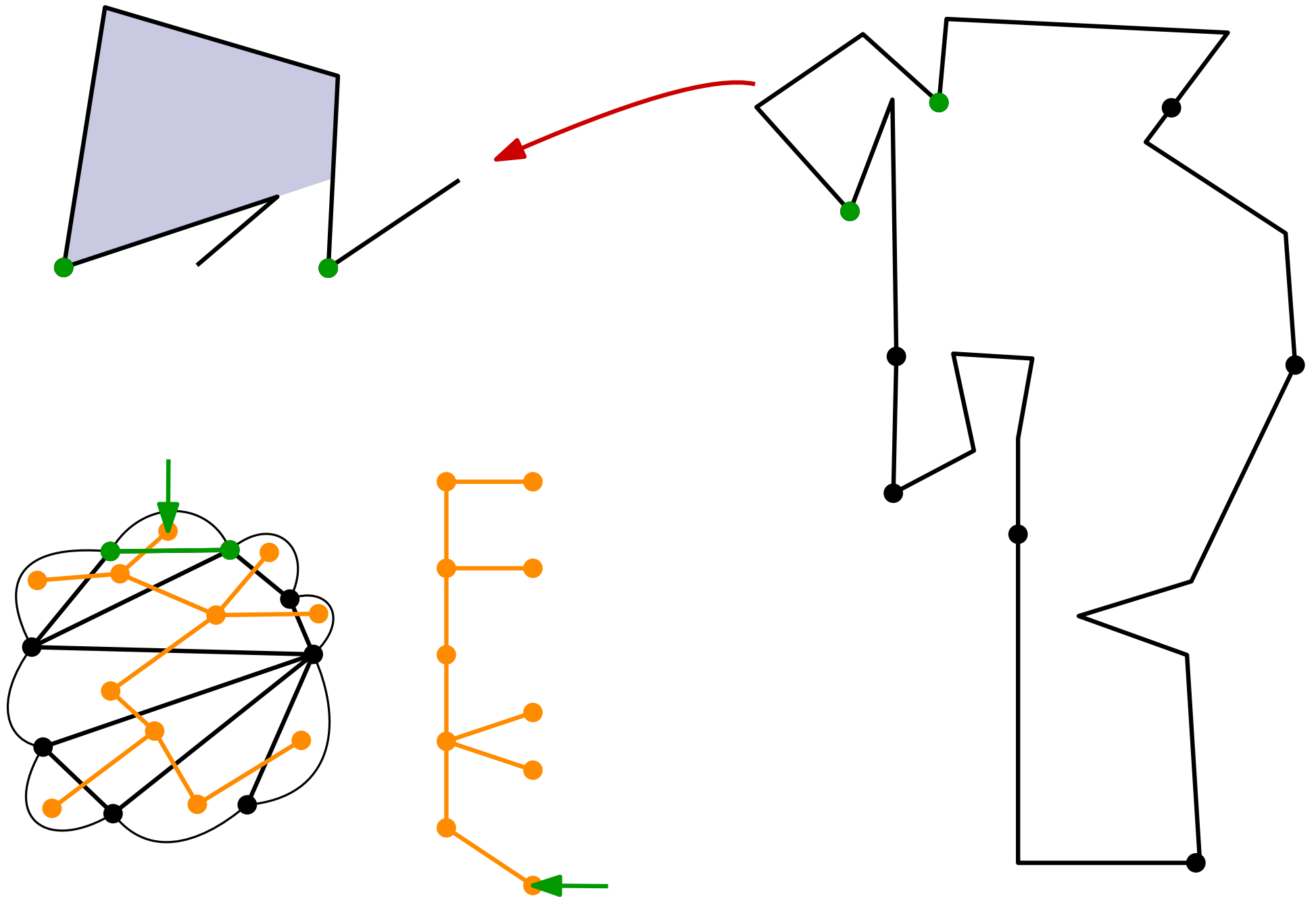
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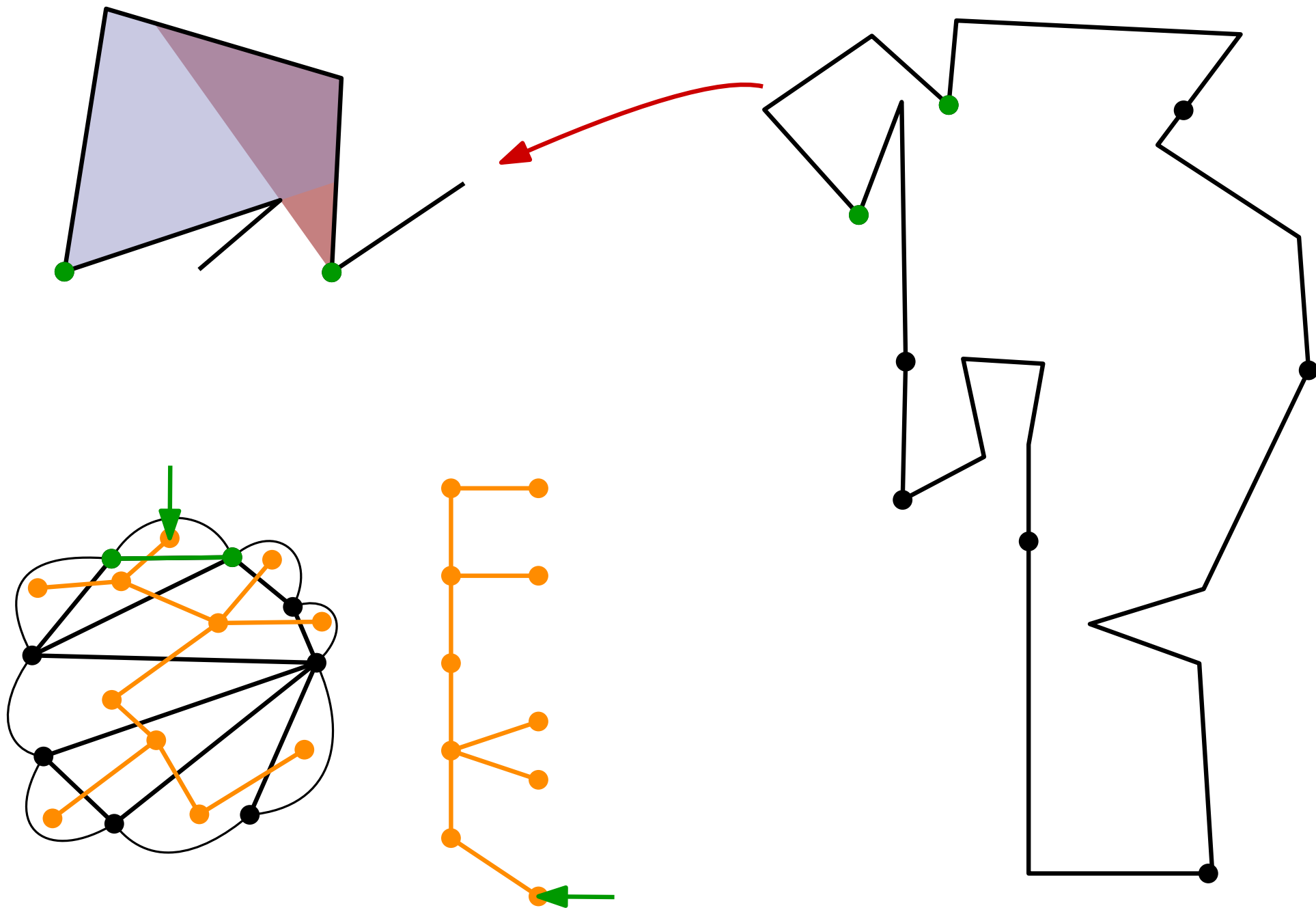
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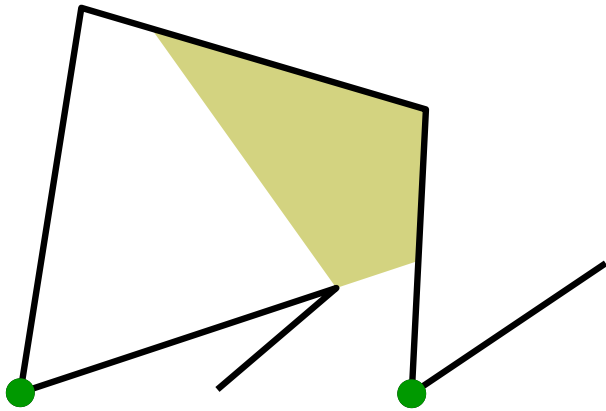
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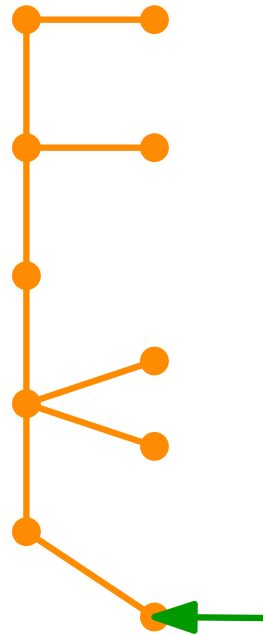
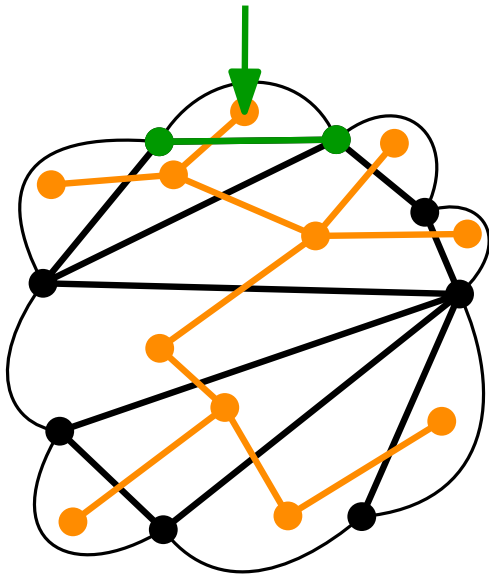
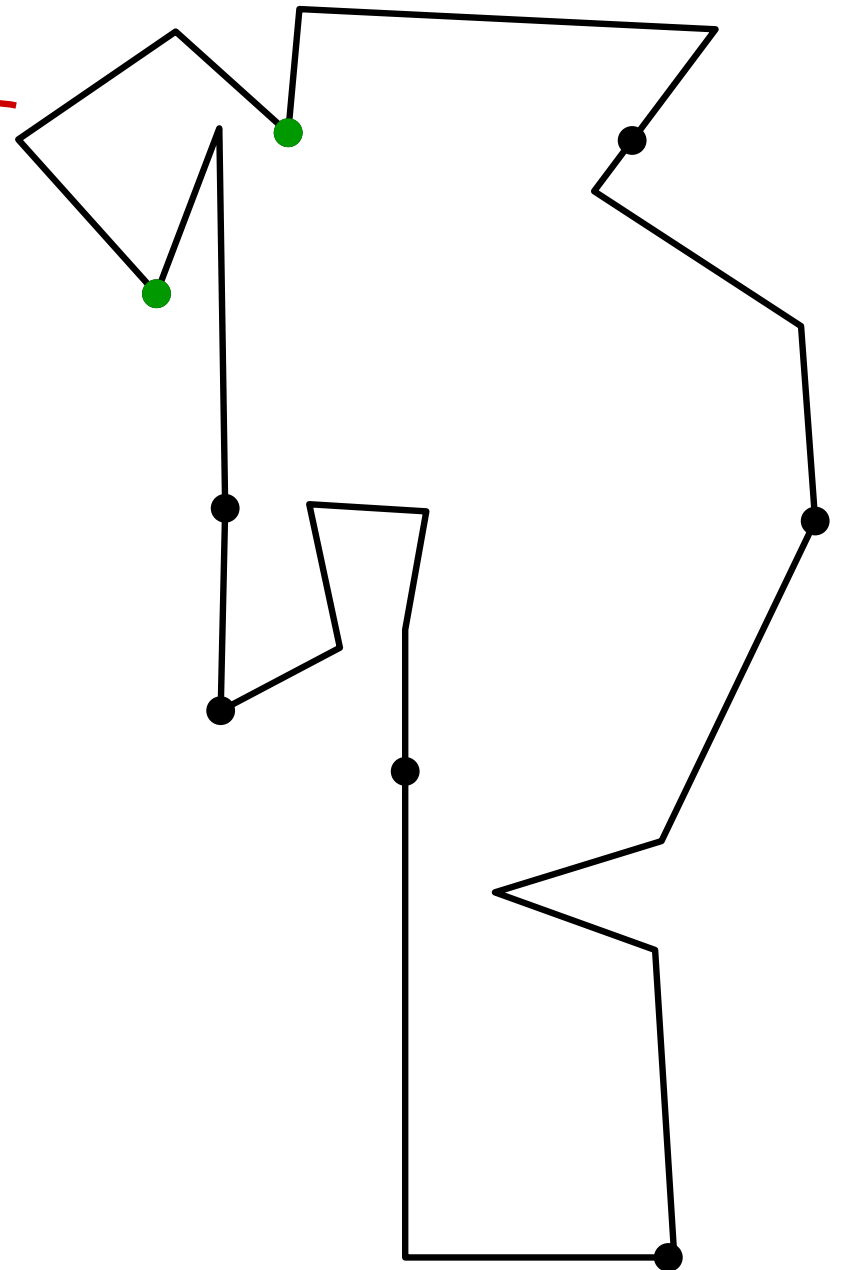
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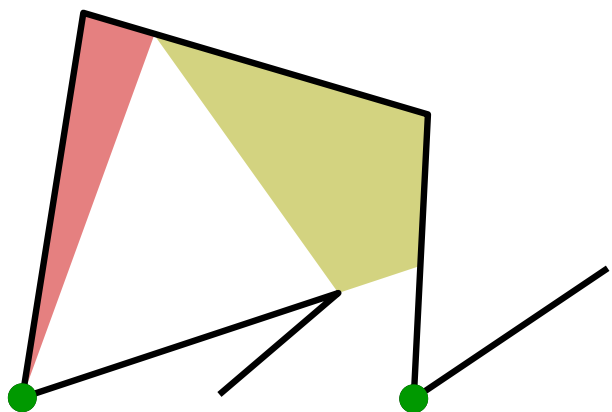
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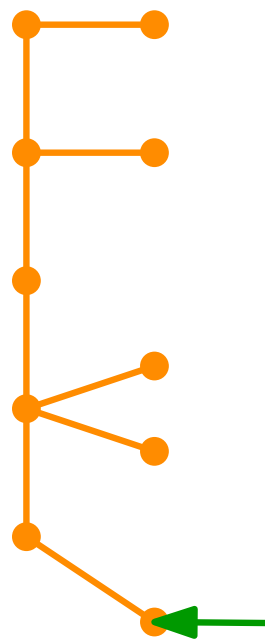
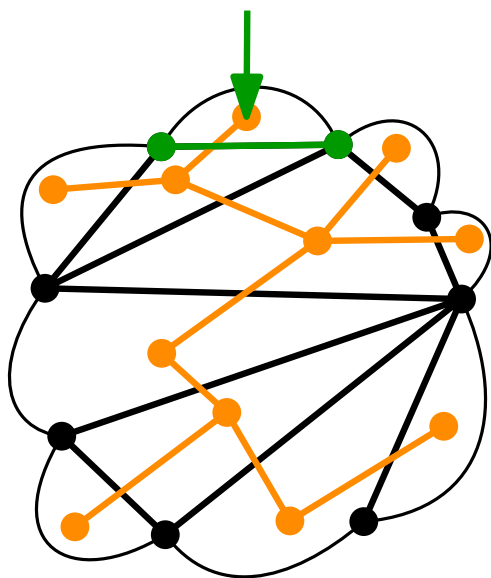
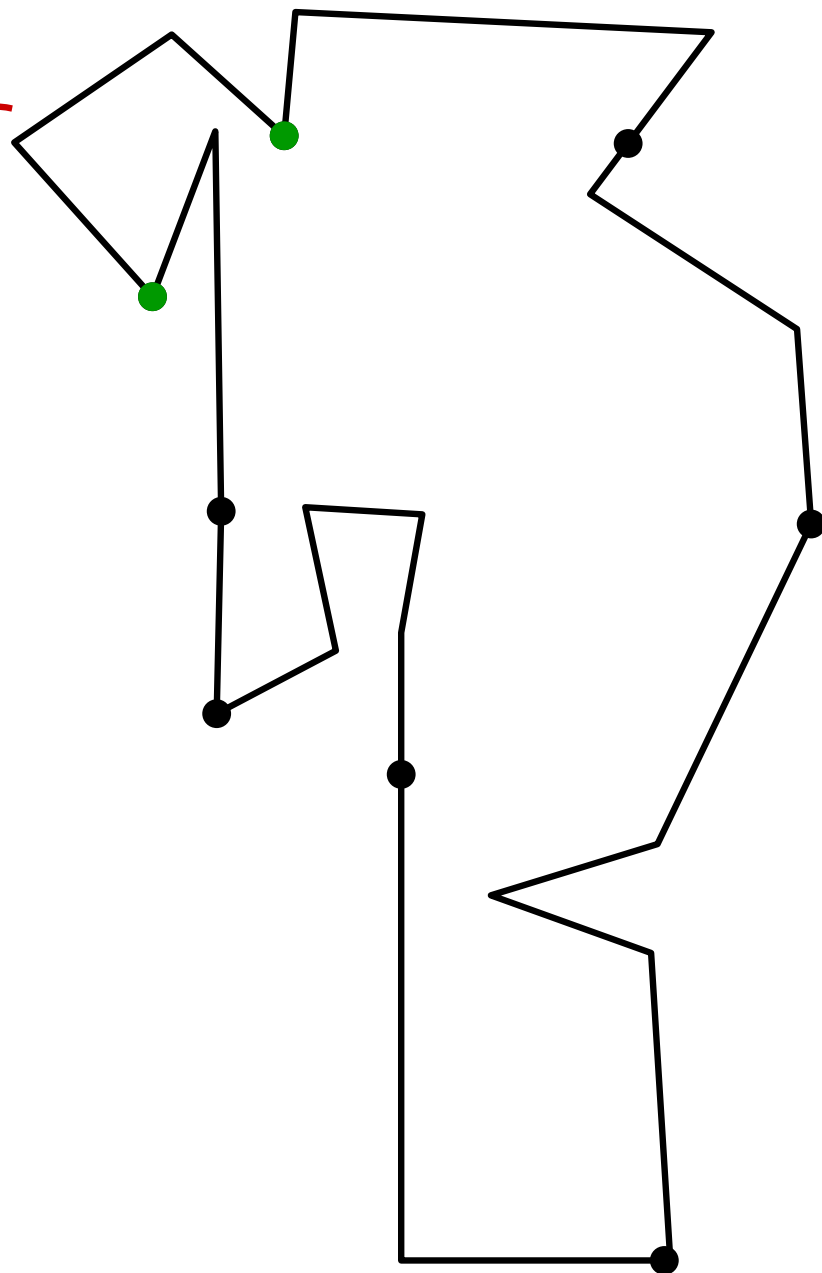
- Convex \Rightarrow intersect visibilities



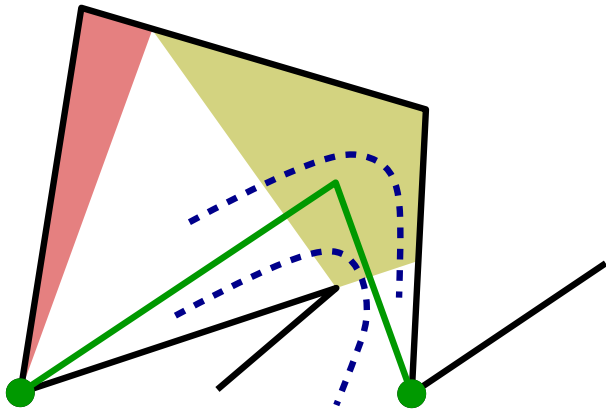
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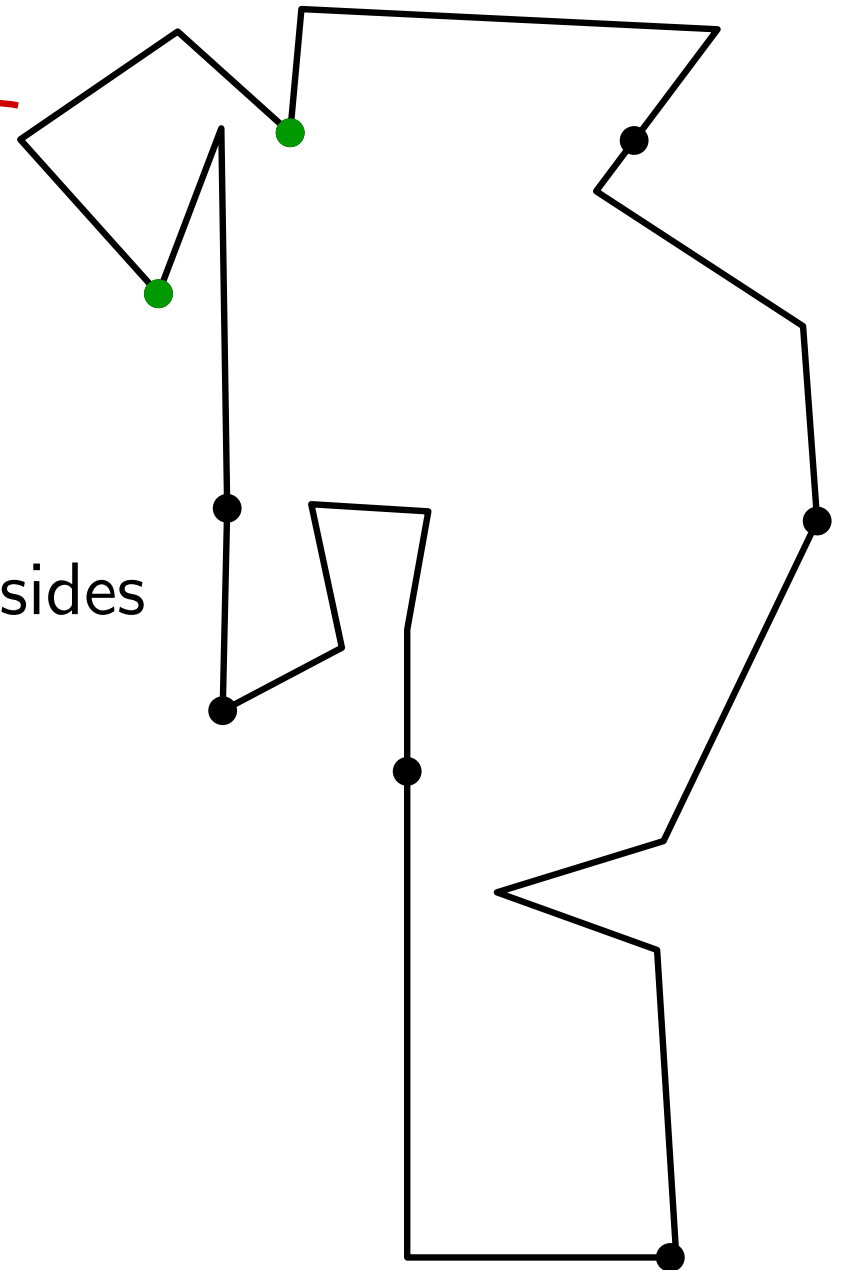
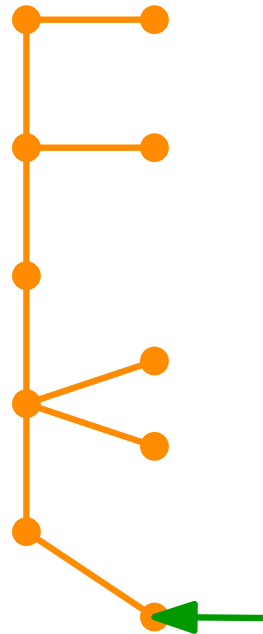
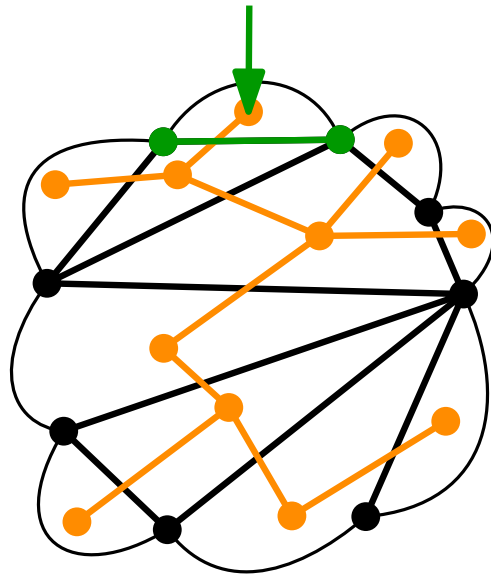
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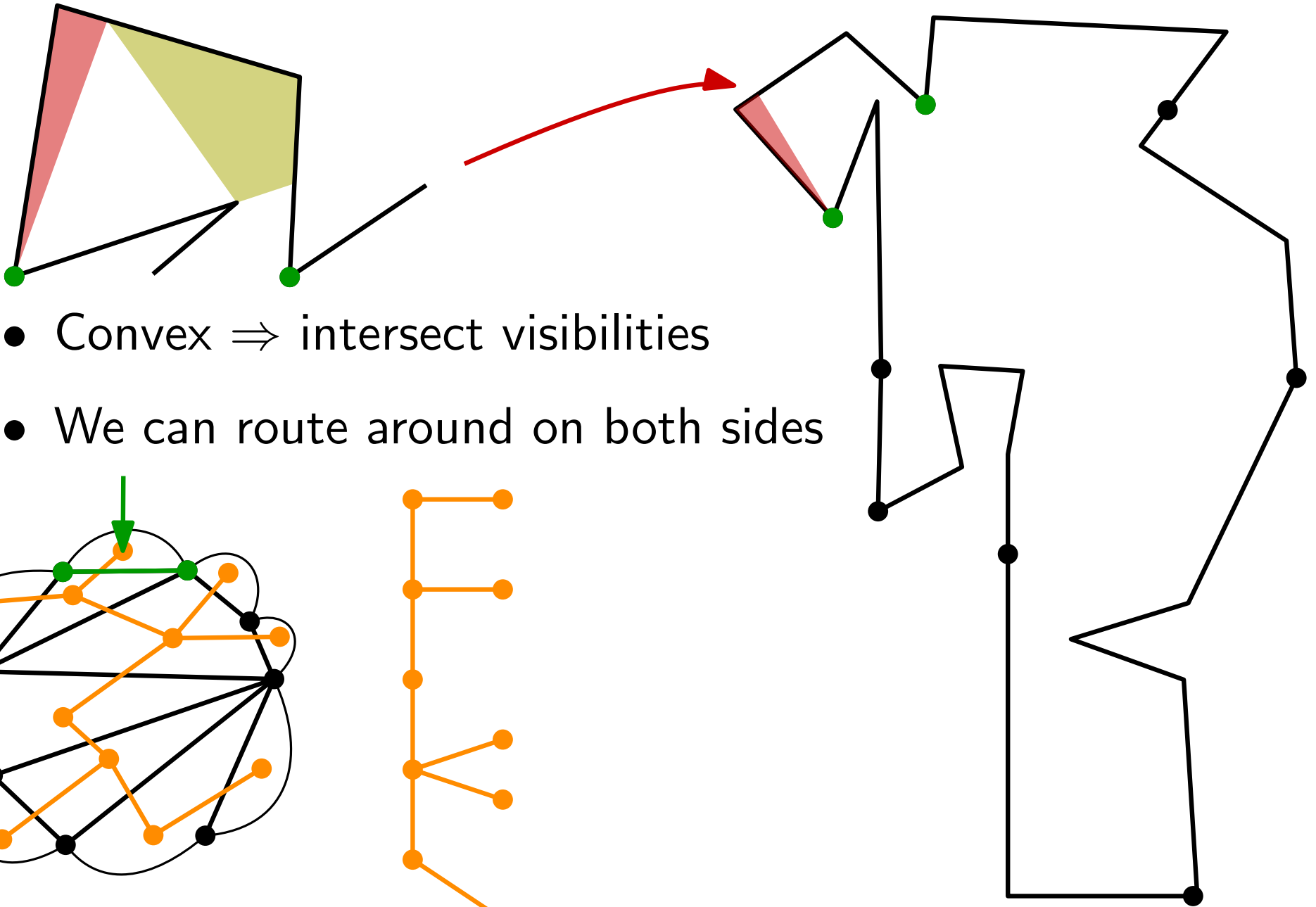
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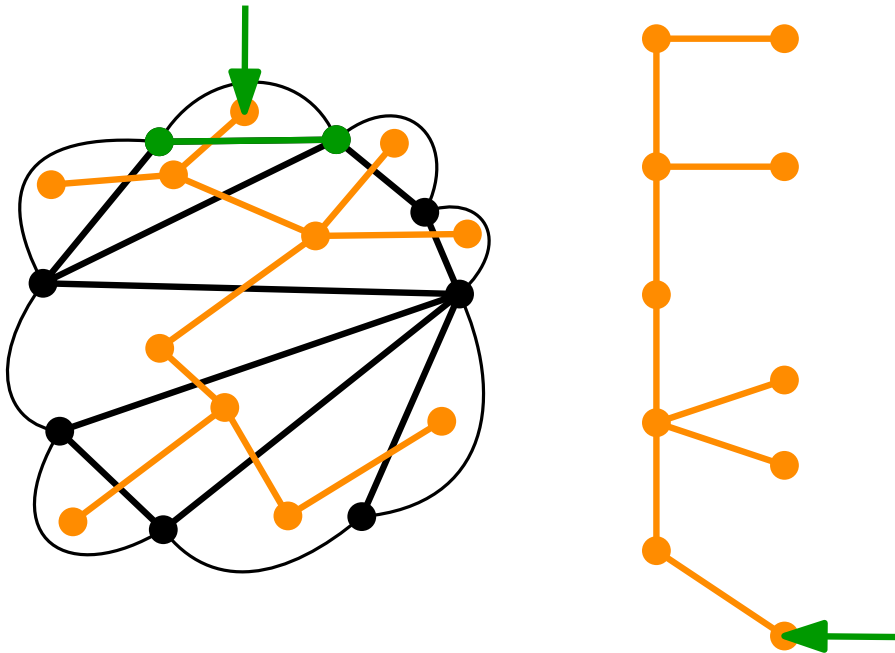
- Convex \Rightarrow intersect visibilities
- We can route around on both sides



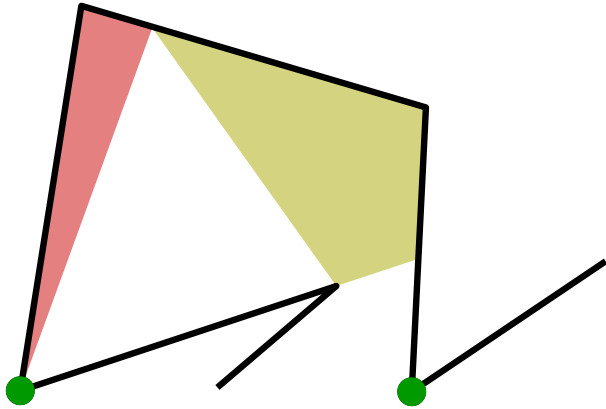
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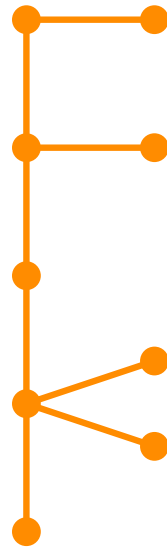
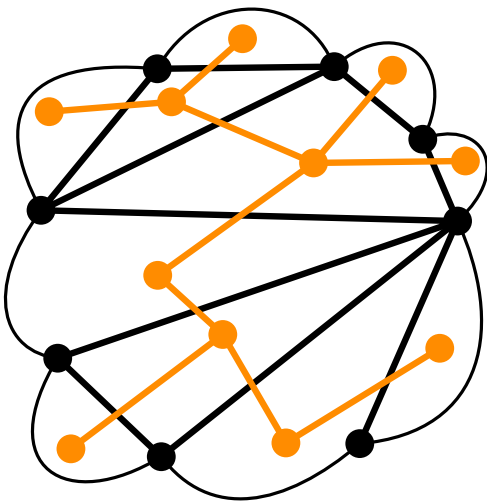
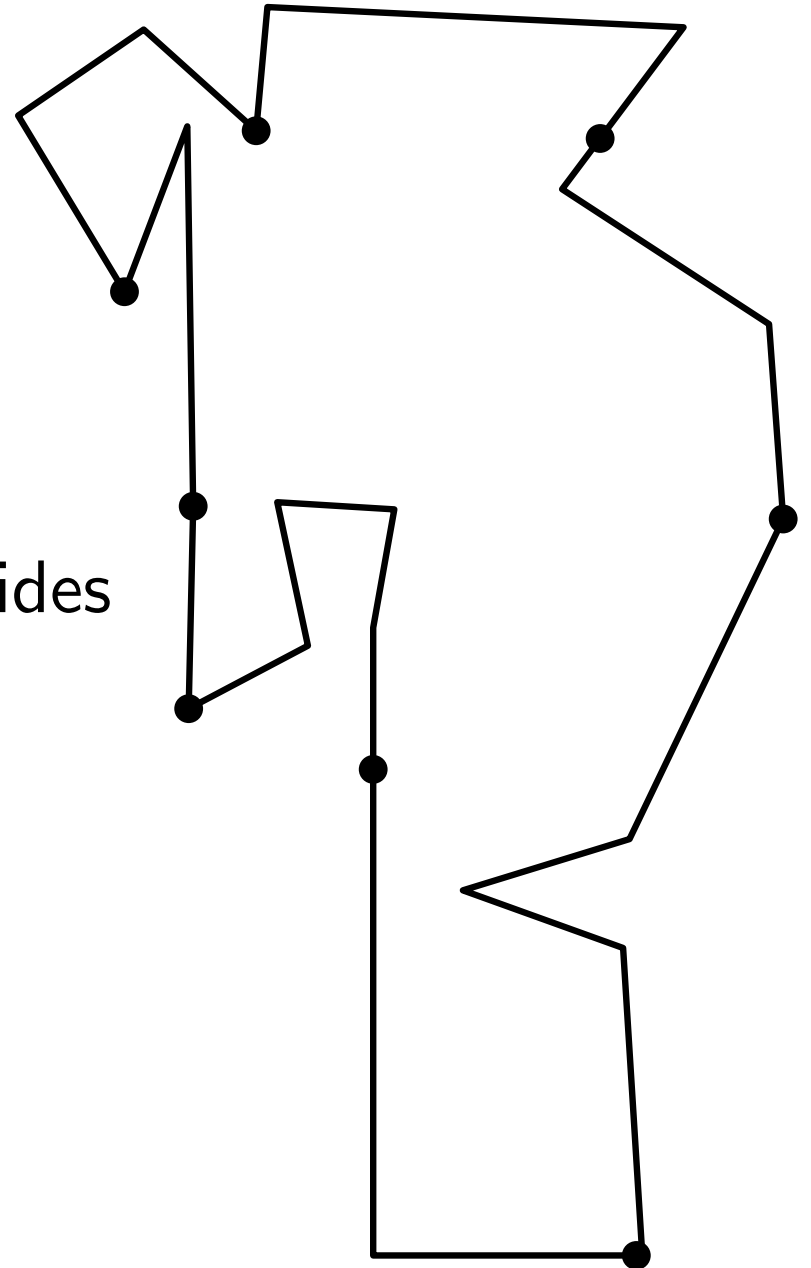
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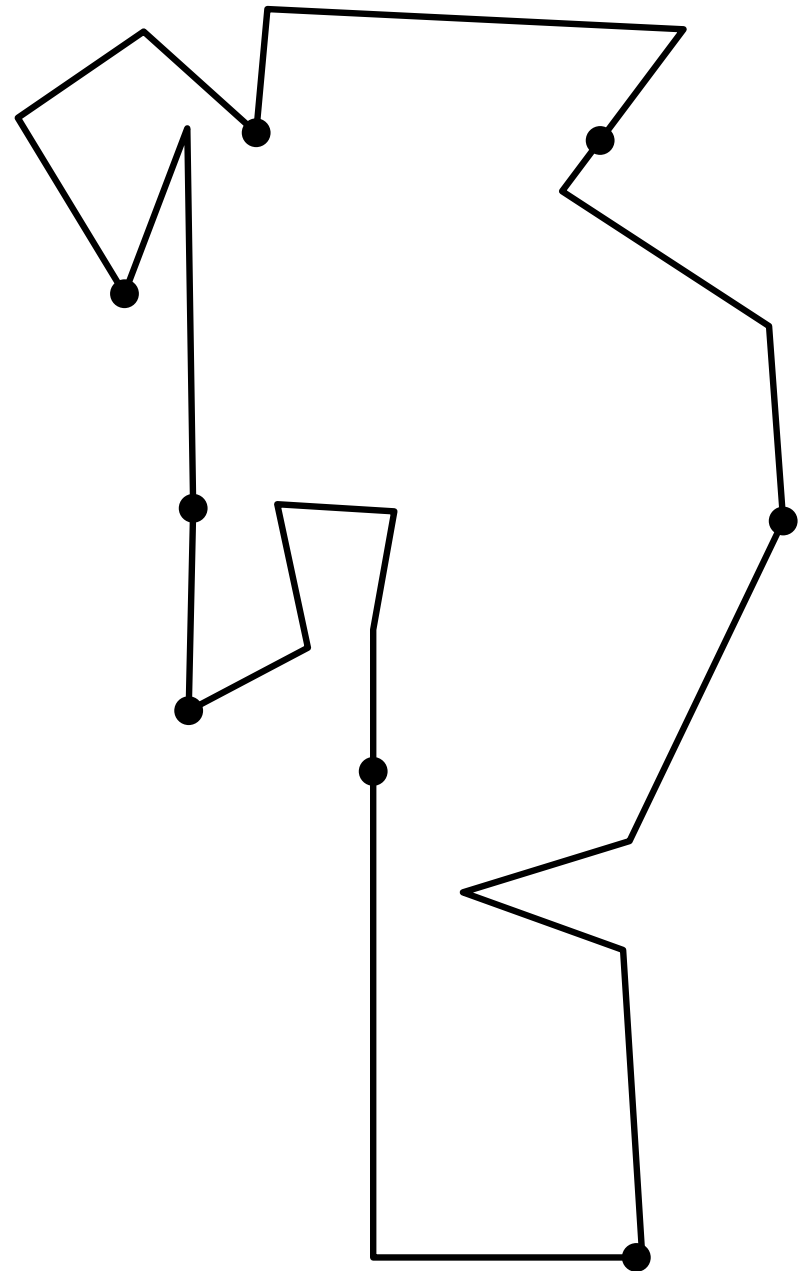
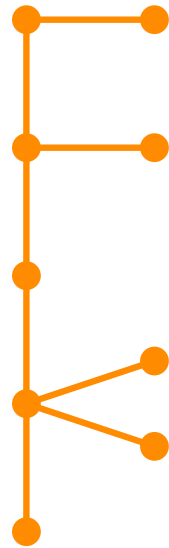
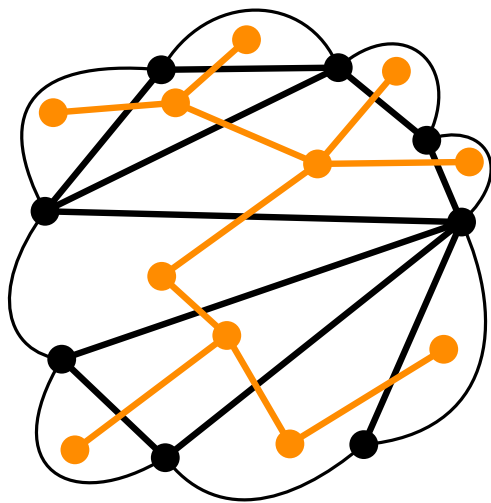
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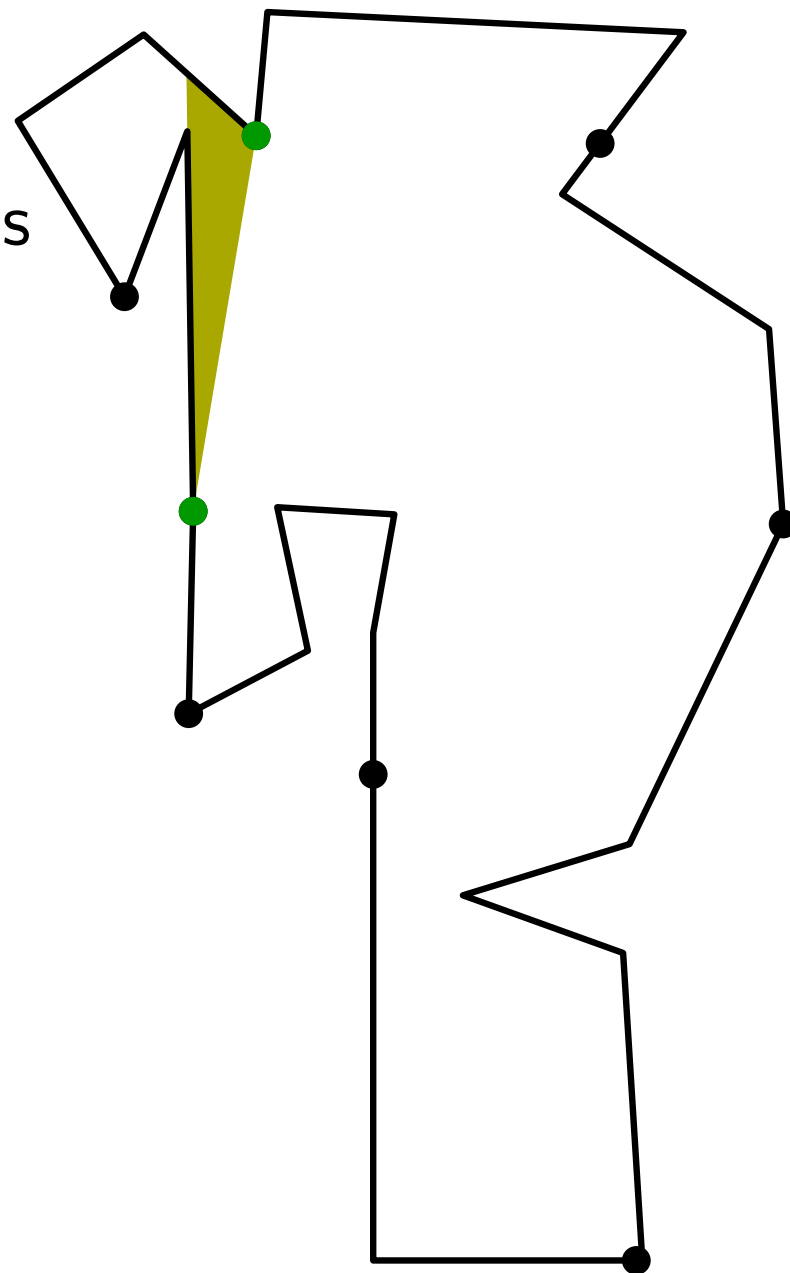
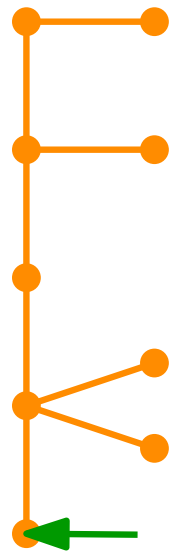
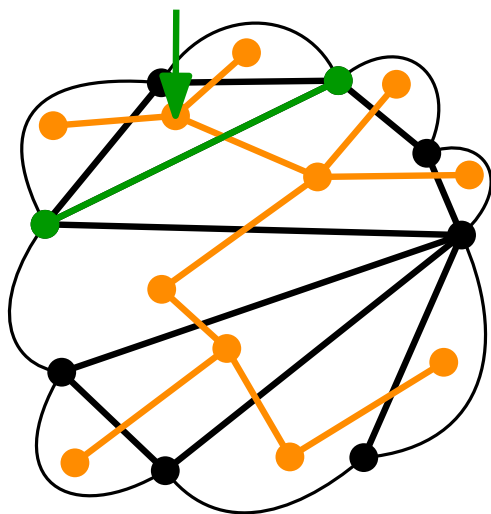


Refinement Types – Inner Nodes



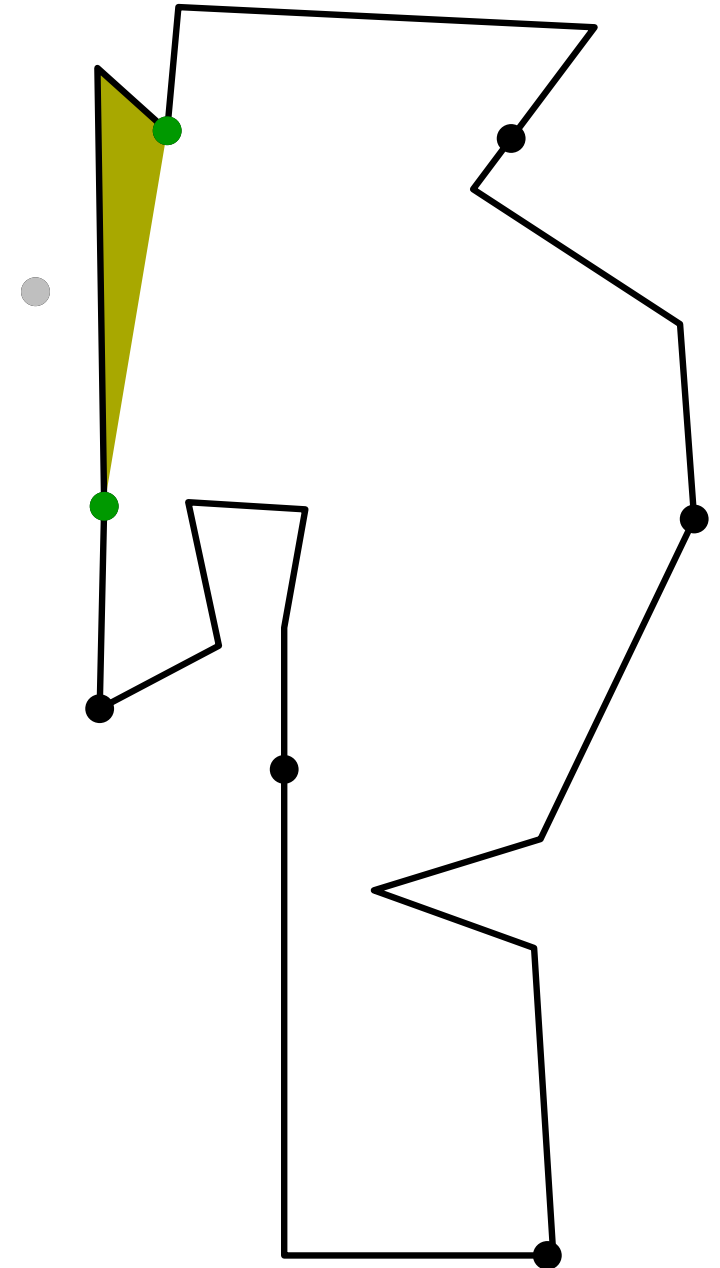
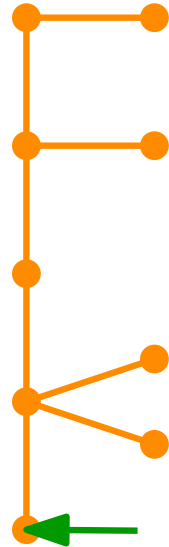
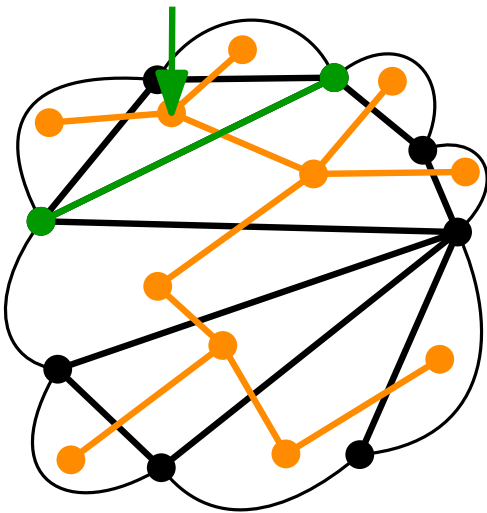
Refinement Types – Inner Nodes

- Refinements can cut stored regions

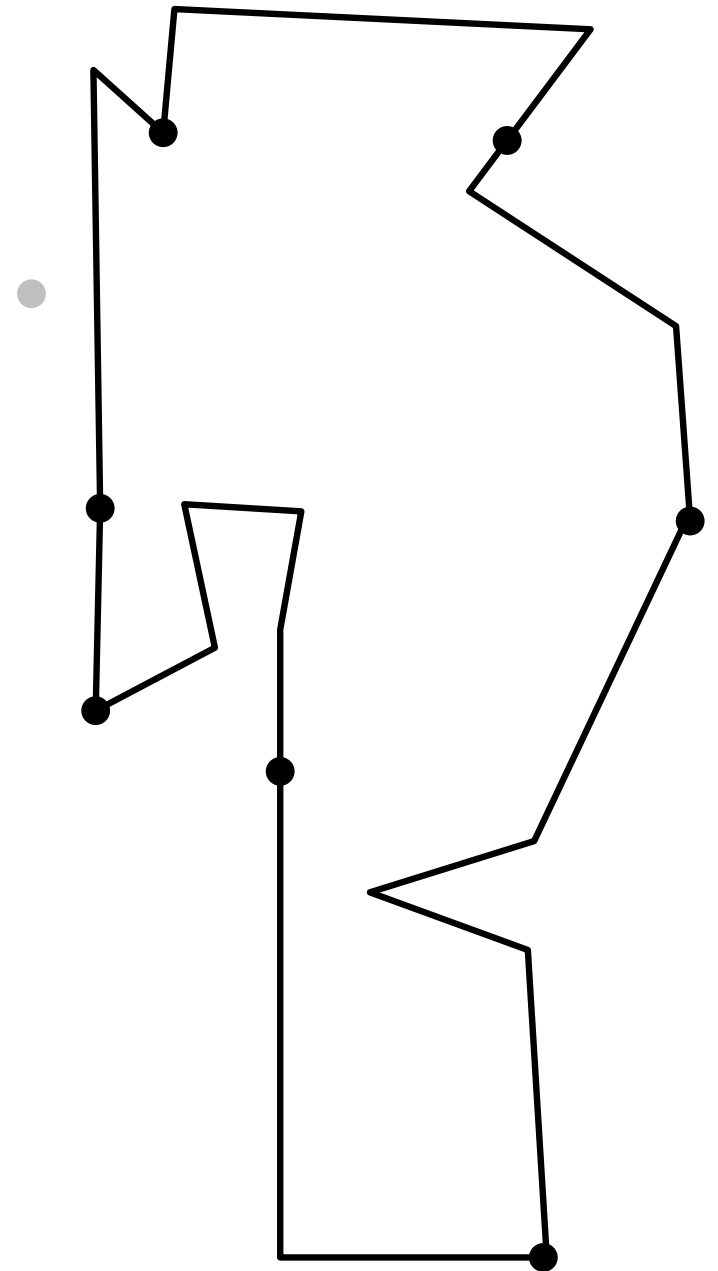
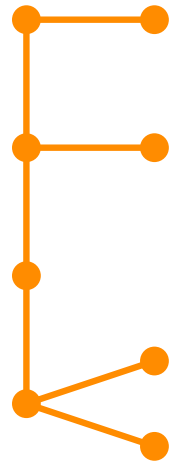
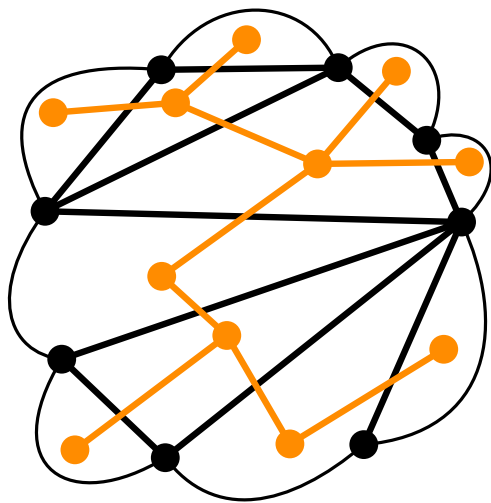


Refinement Types – Inner Nodes

- Refinements can cut stored regions
- Store & solve at backtracking

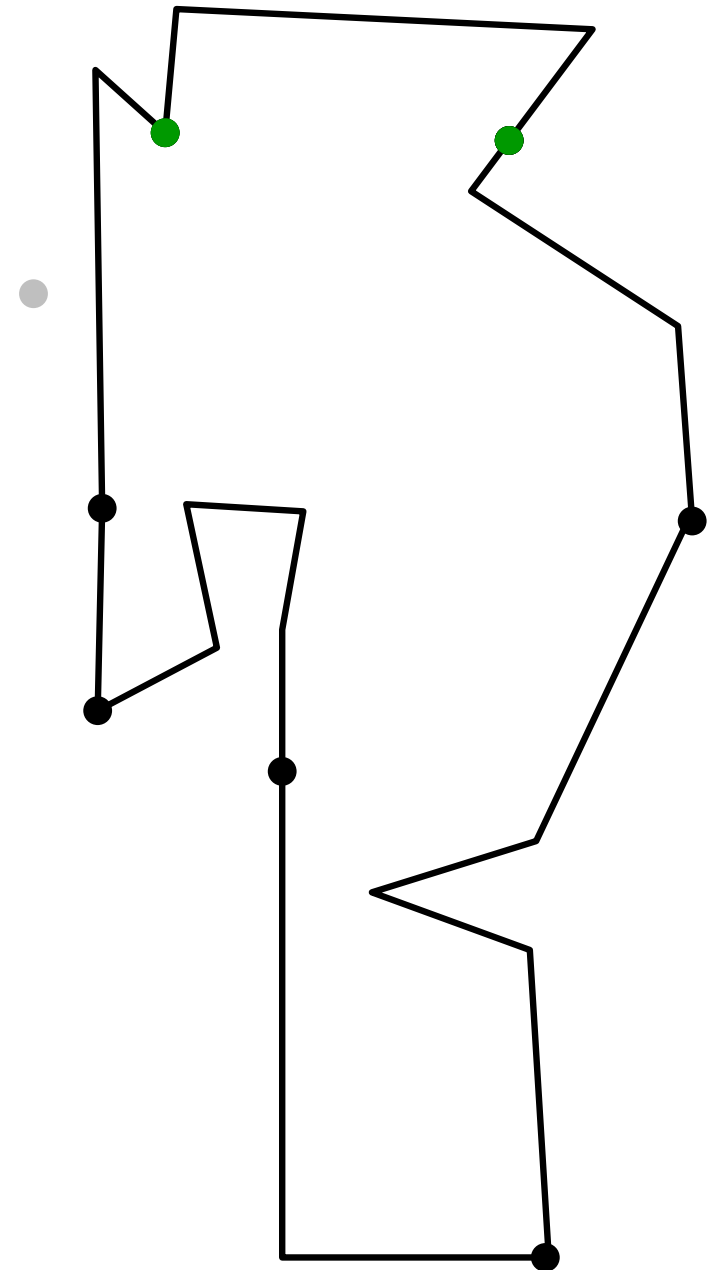
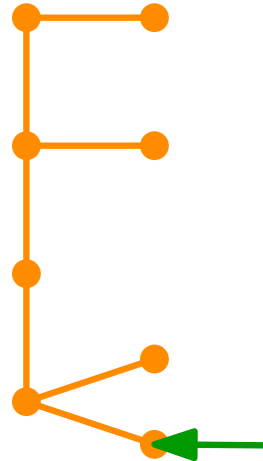
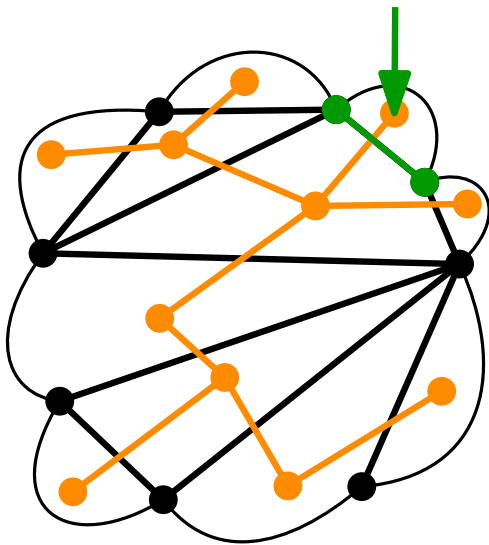


Example – Rest



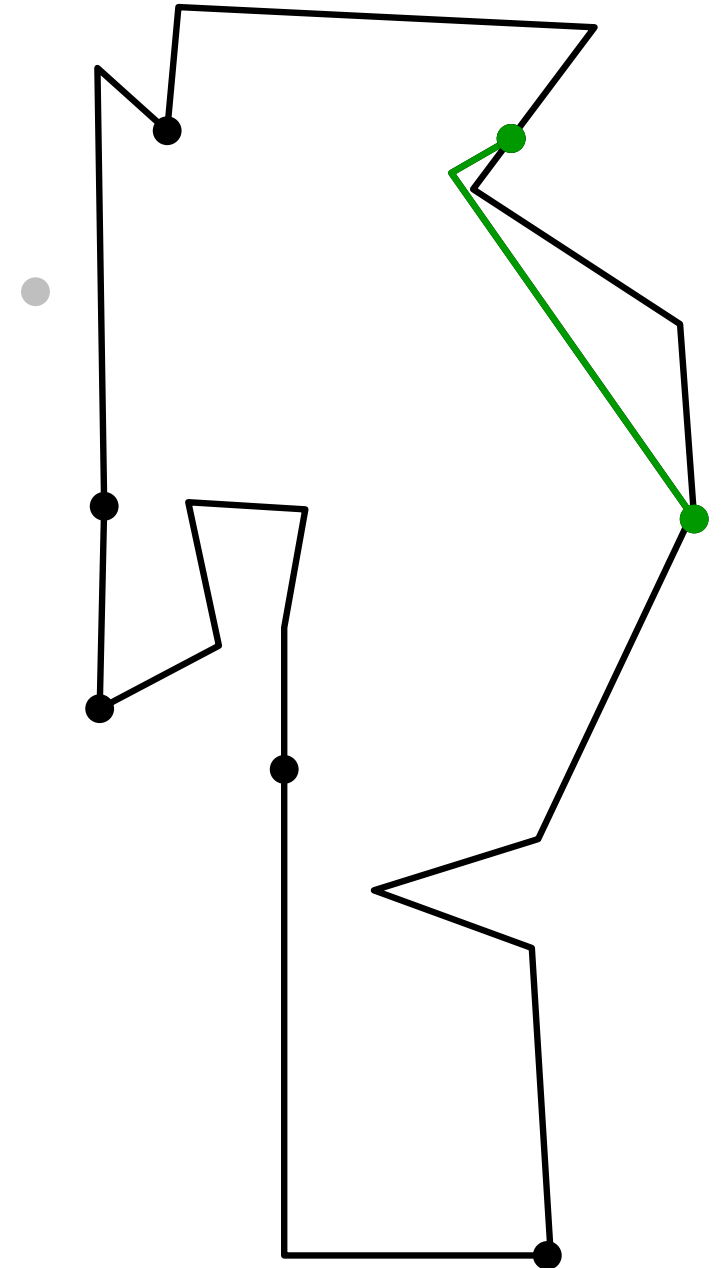
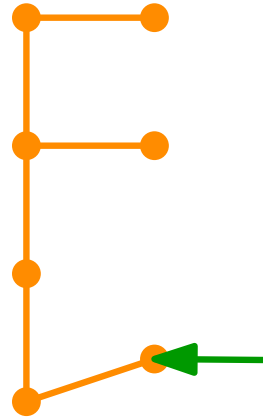
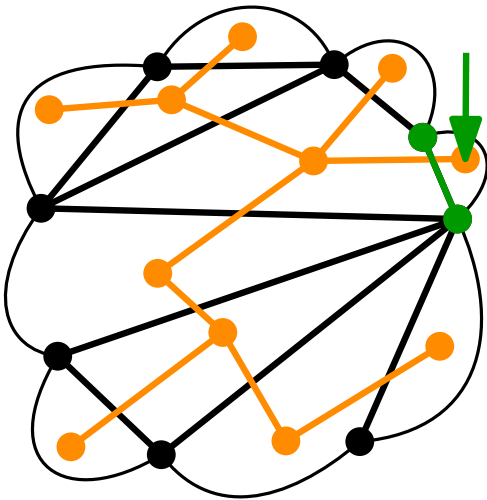
Example – Rest

- Step 4: convex



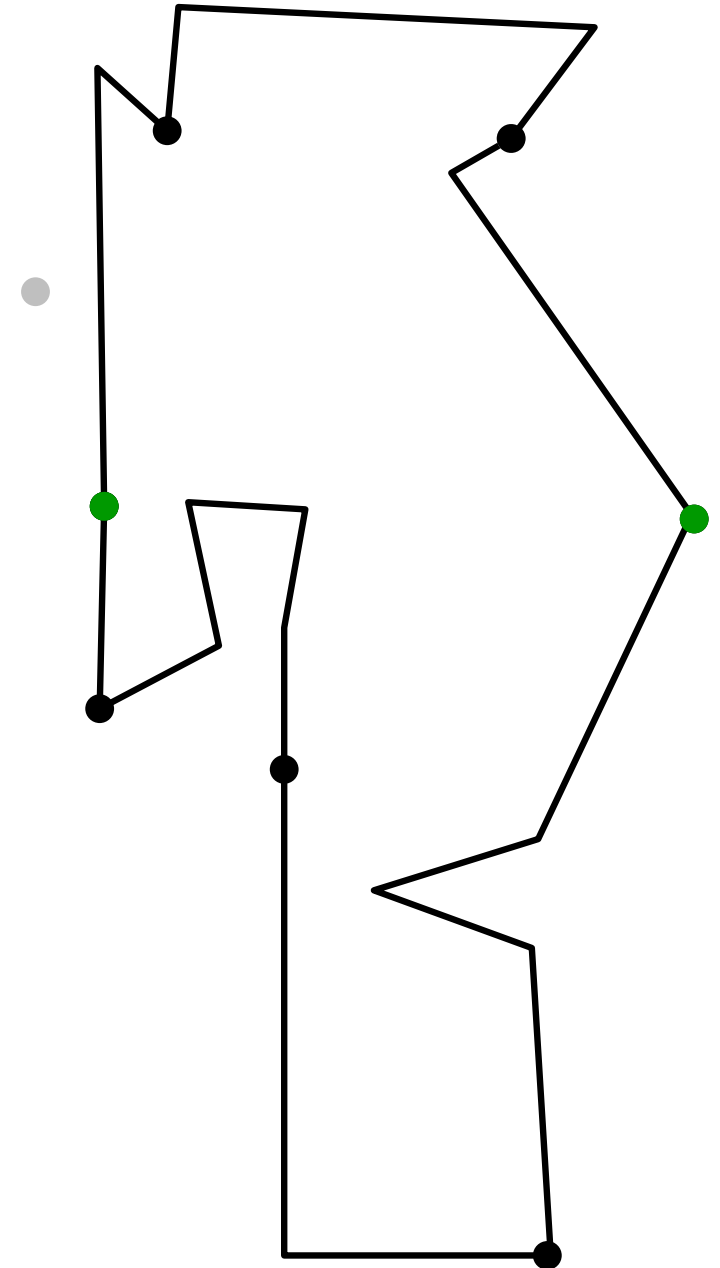
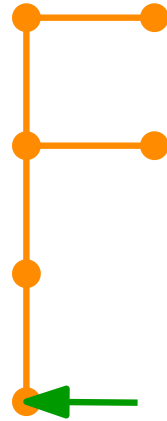
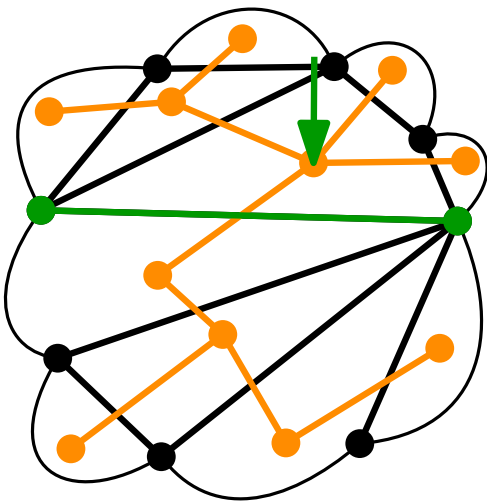
Example – Rest

- Step 4: convex
- Step 5: reflex



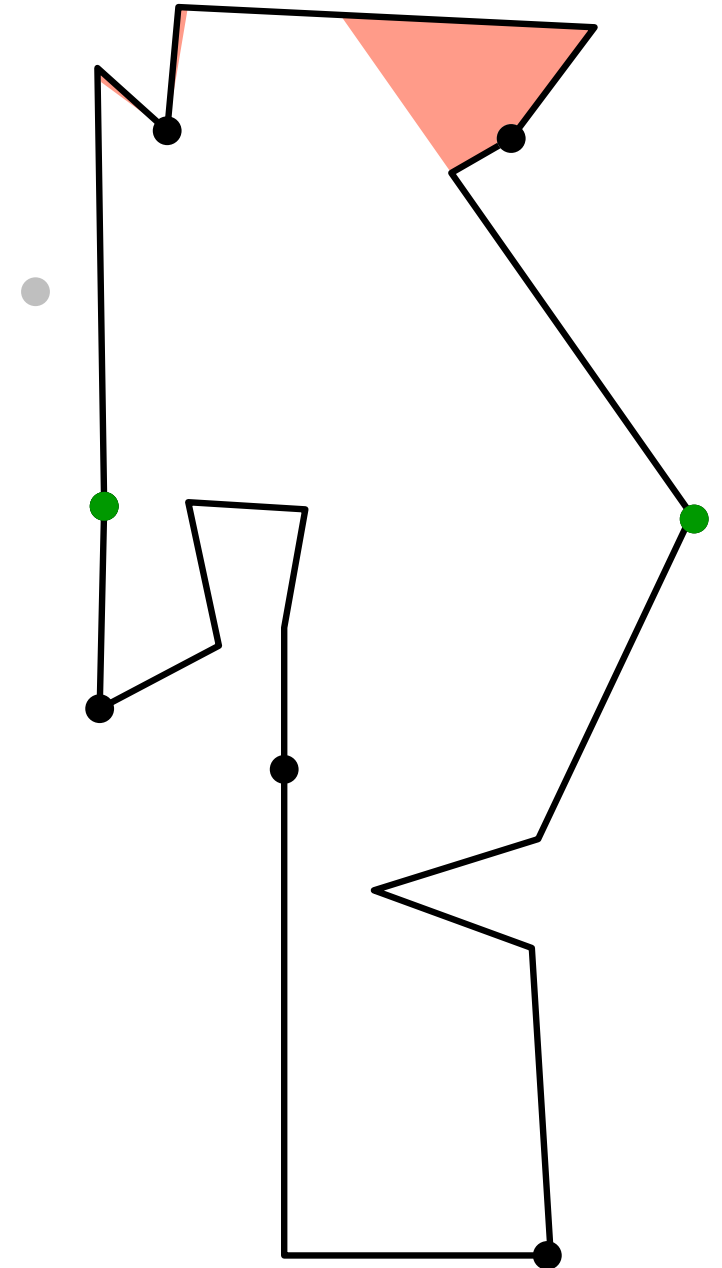
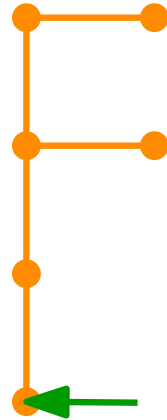
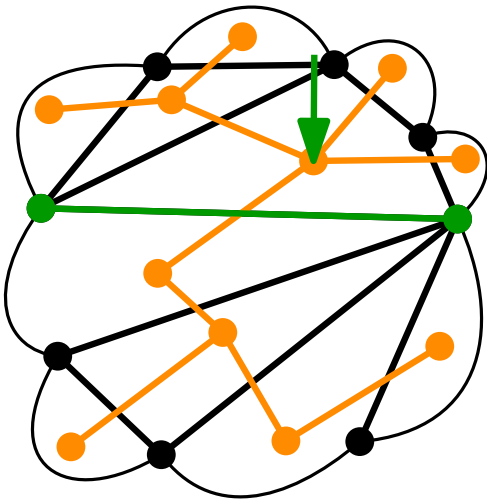
Example – Rest

- Step 4: convex
- Step 5: reflex



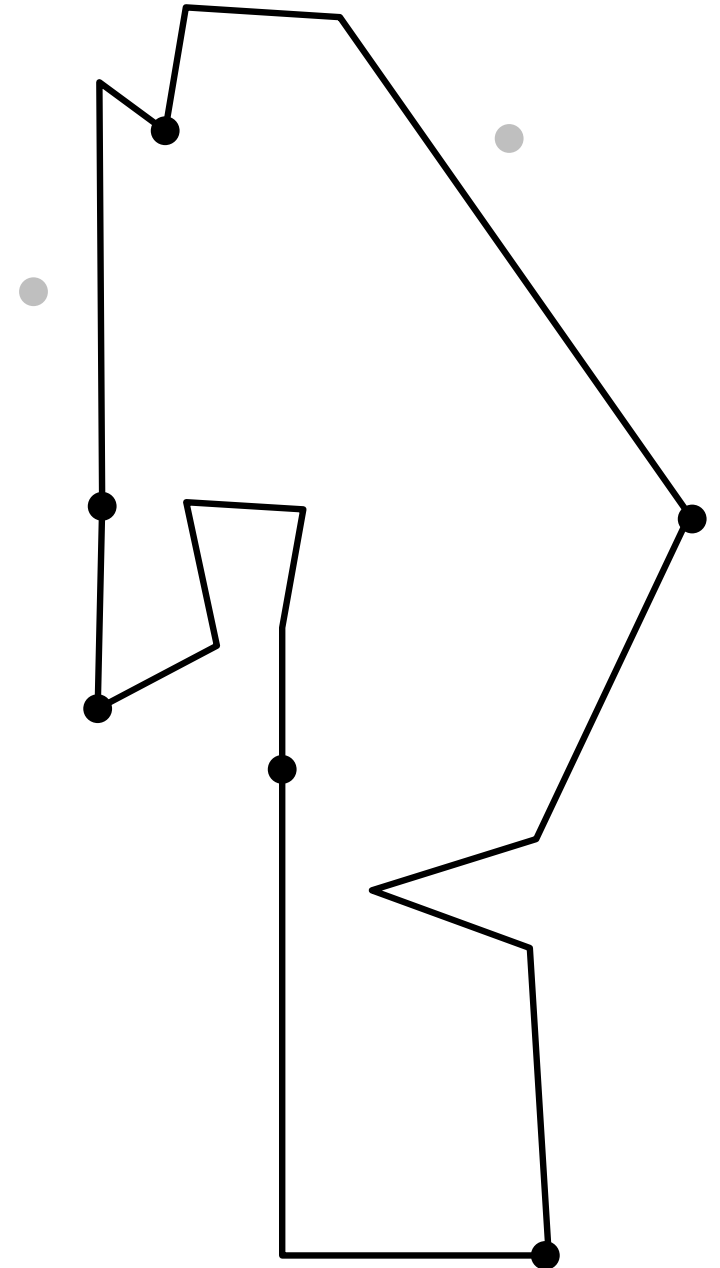
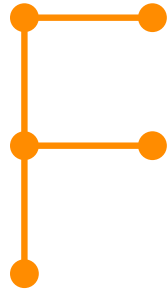
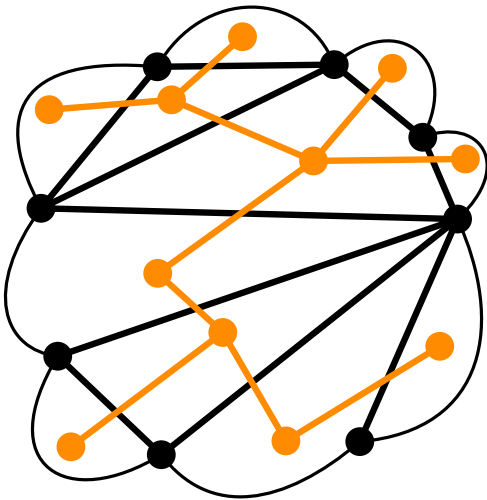
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex



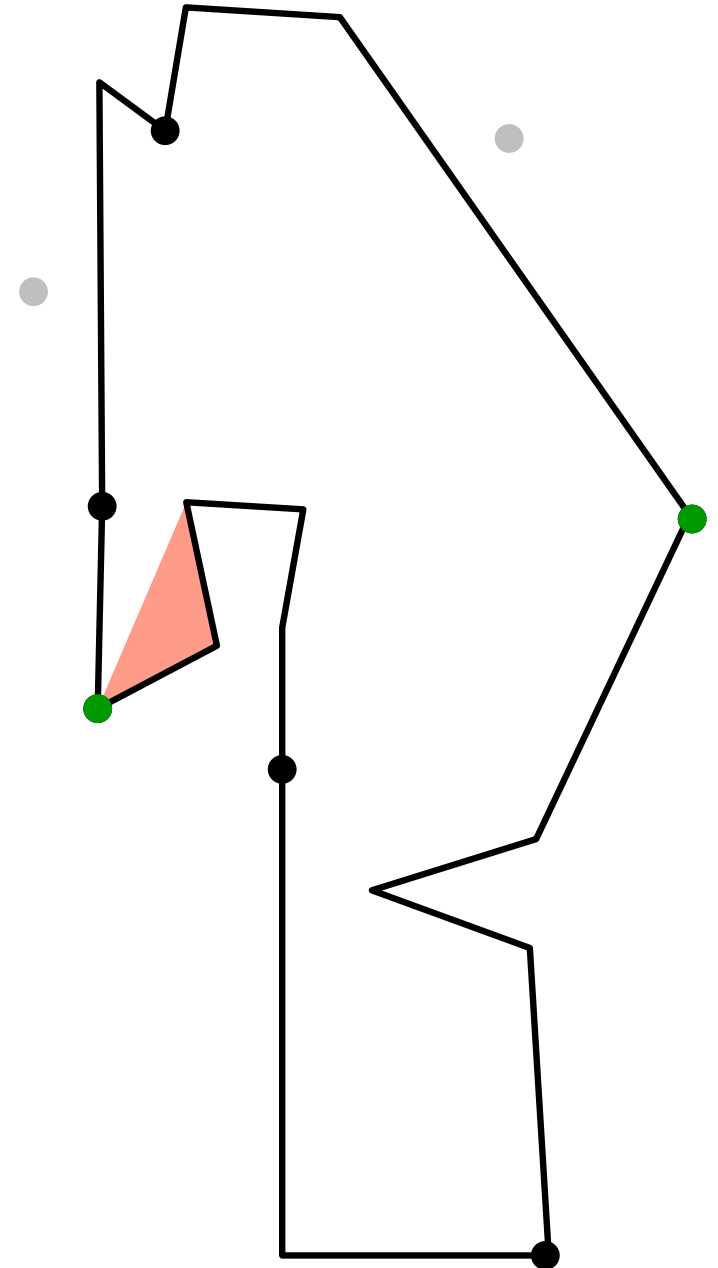
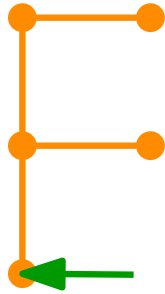
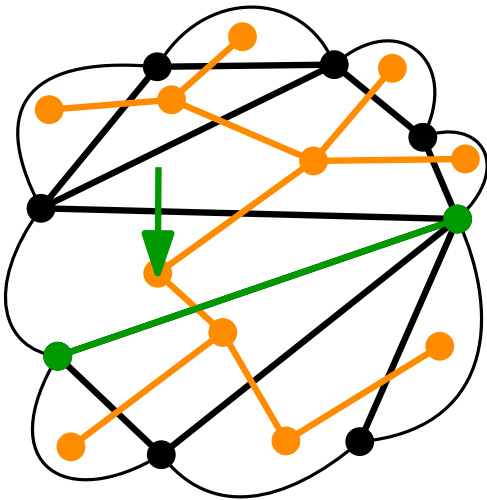
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex



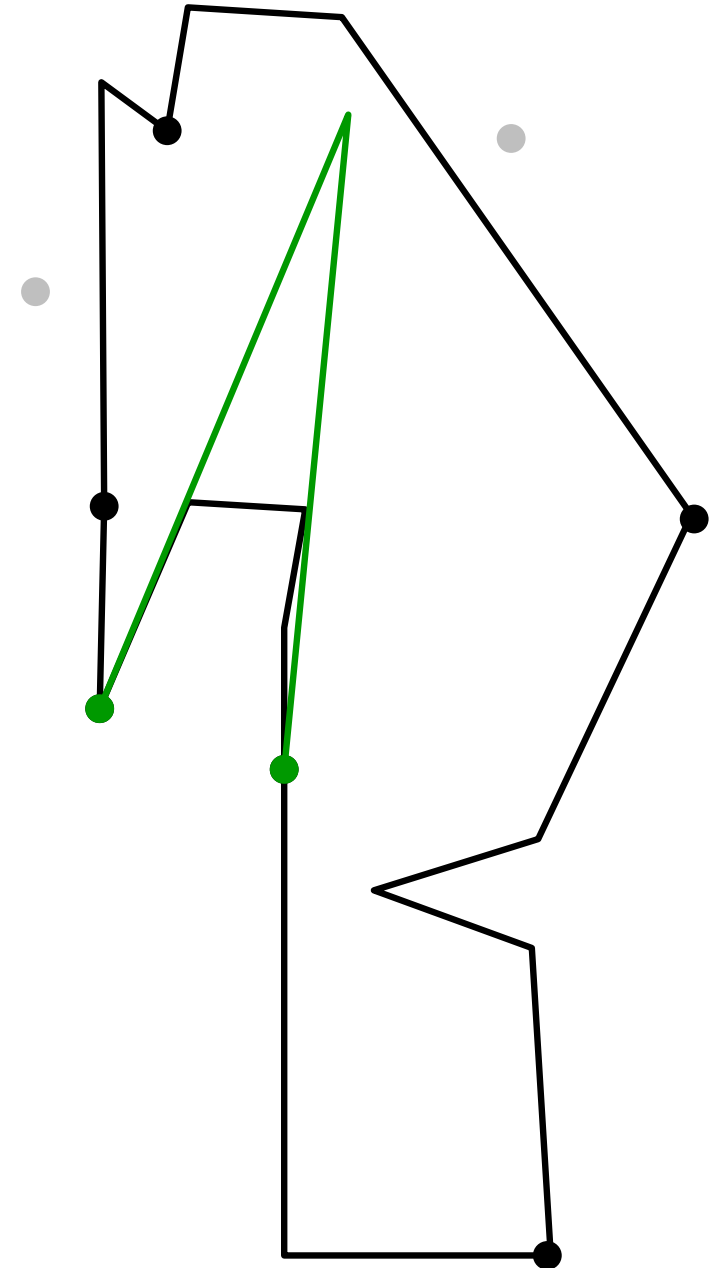
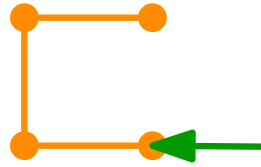
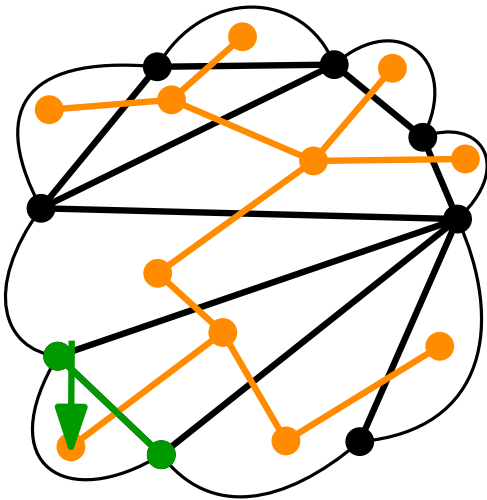
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
- ...



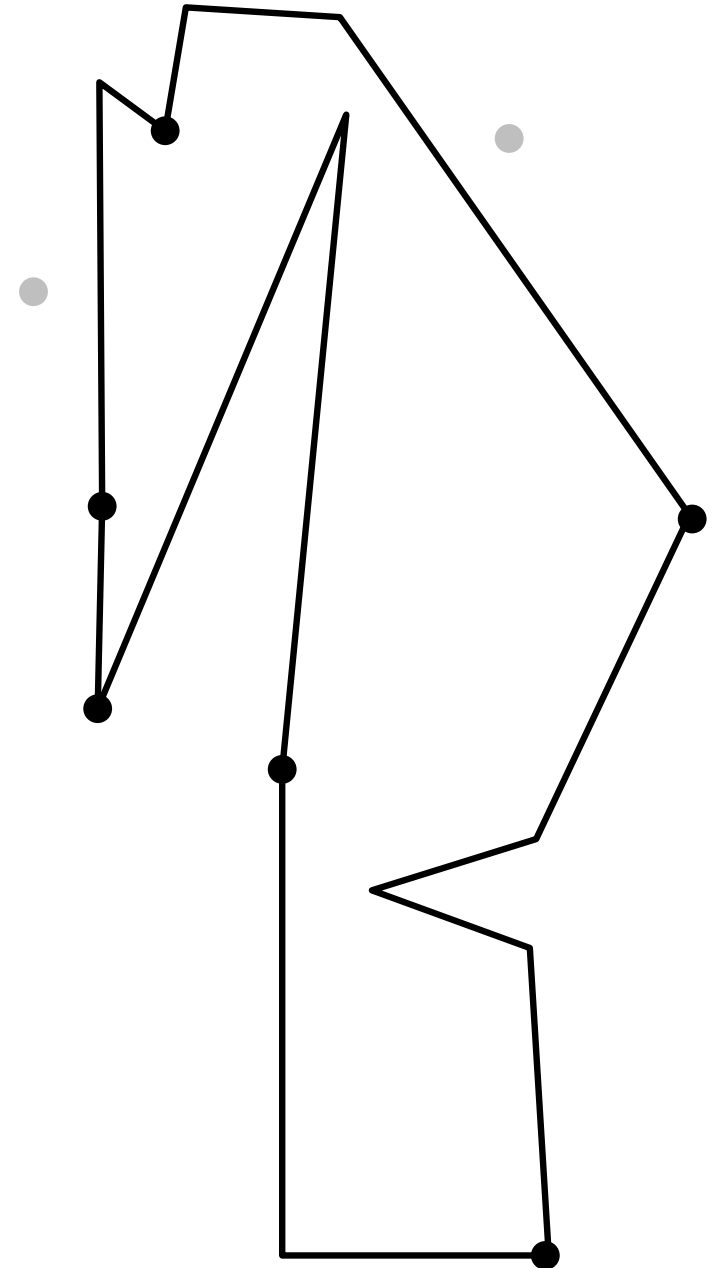
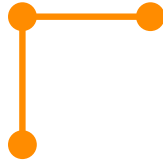
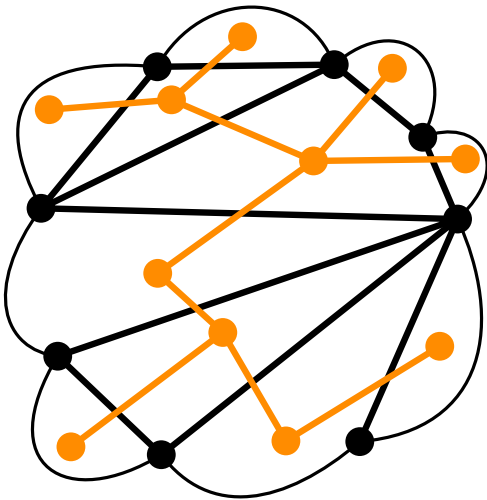
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
-



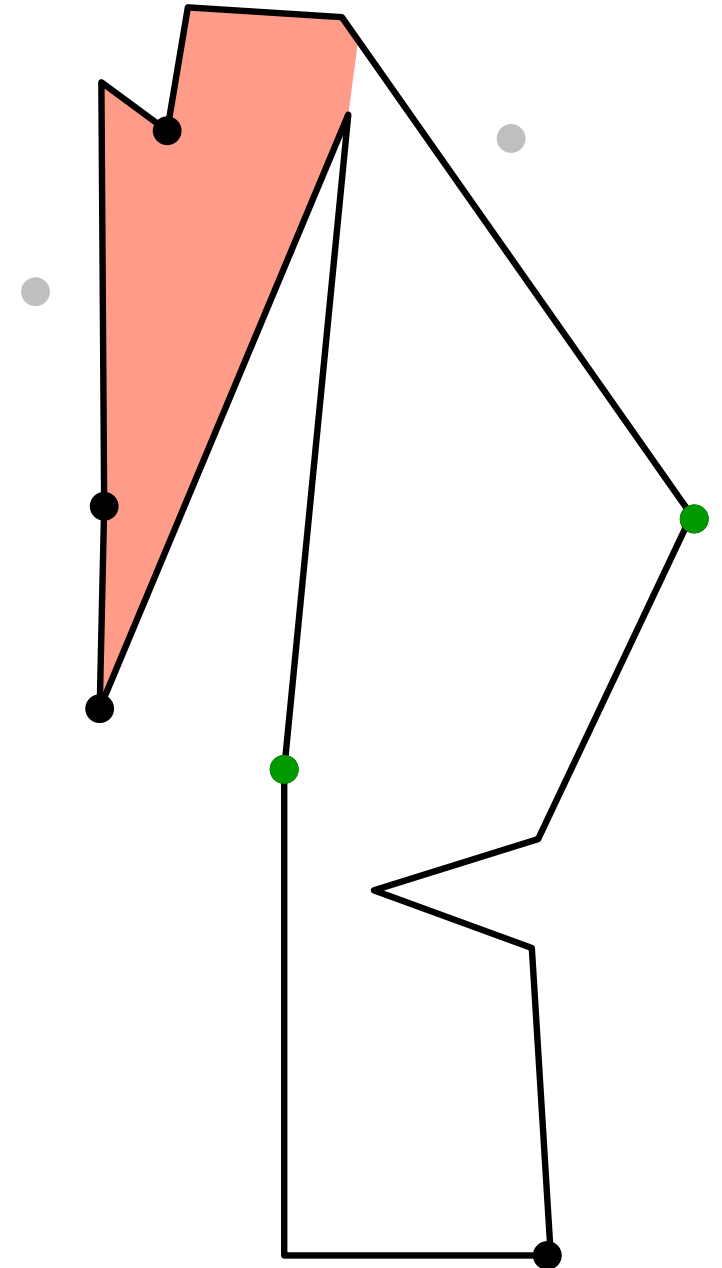
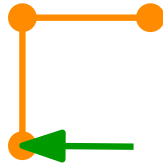
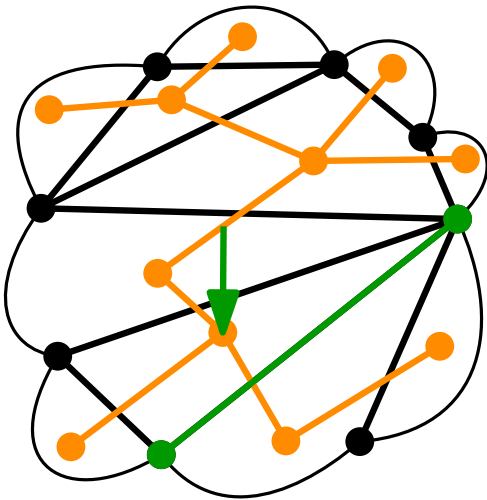
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
-



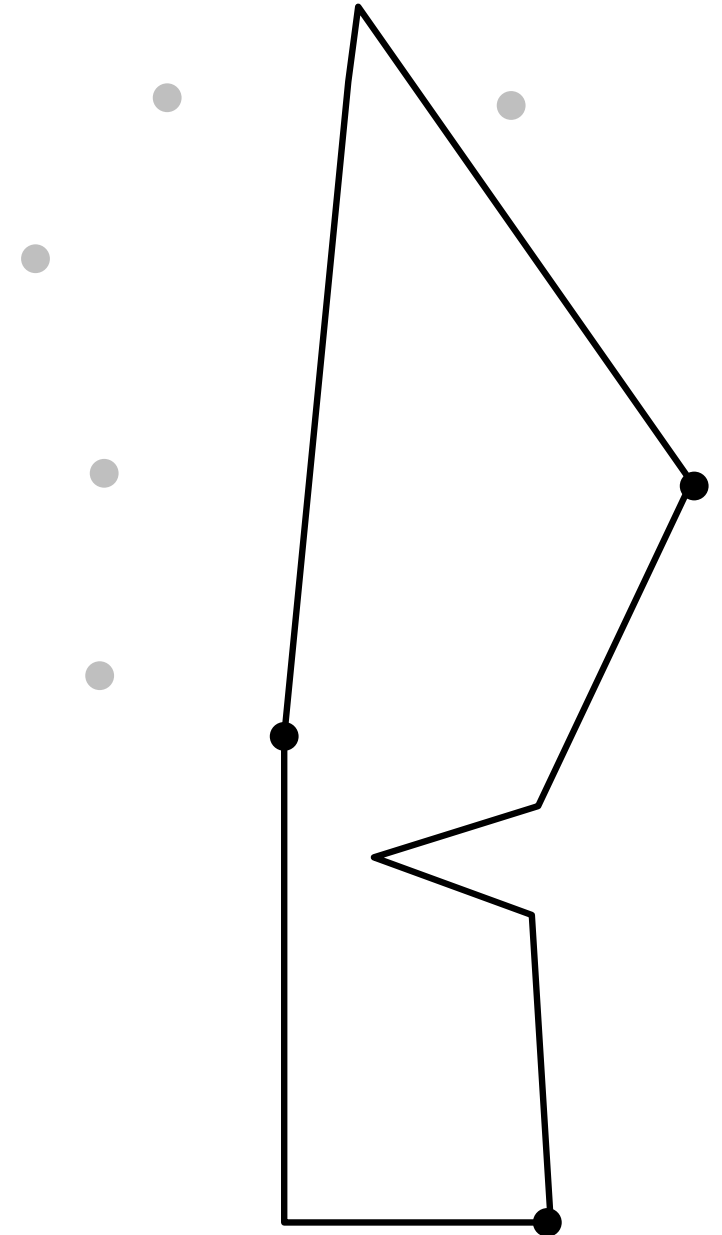
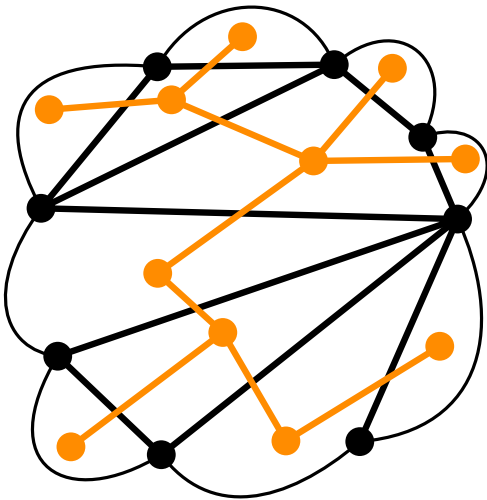
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
- ...



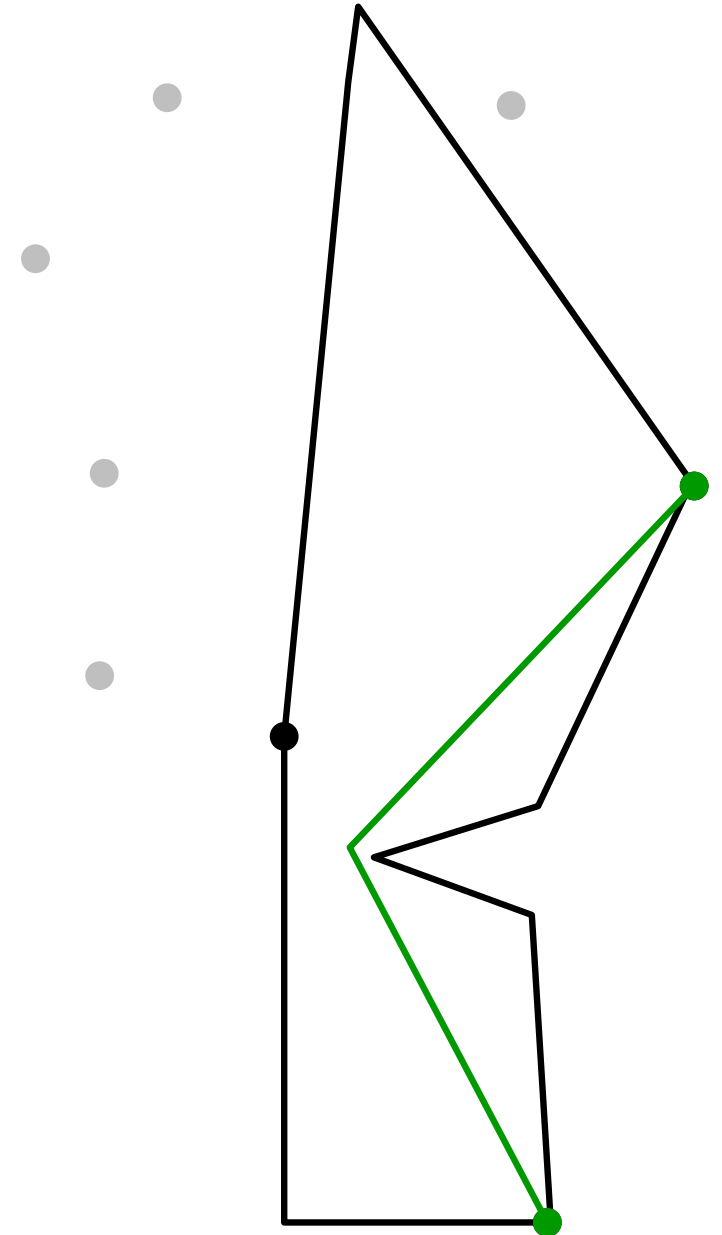
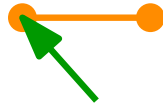
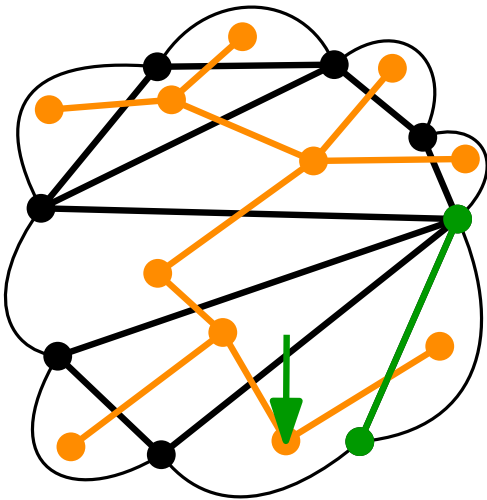
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
-



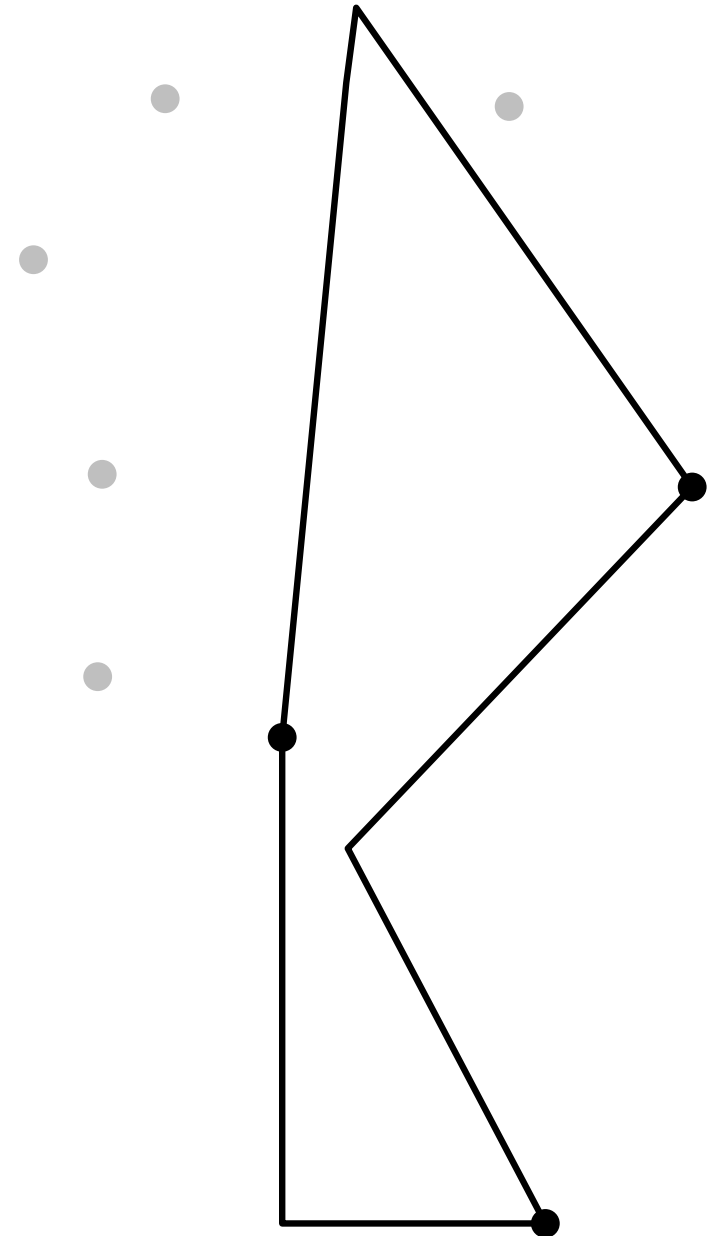
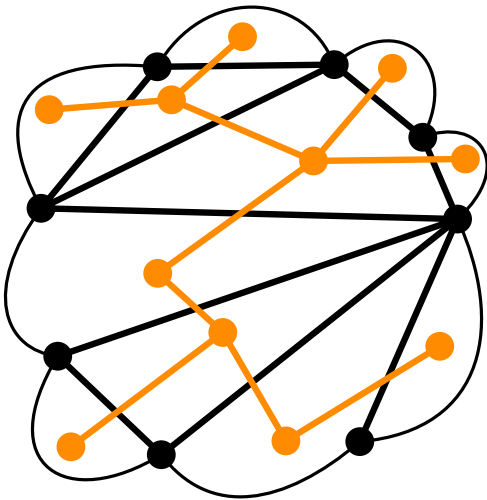
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
-



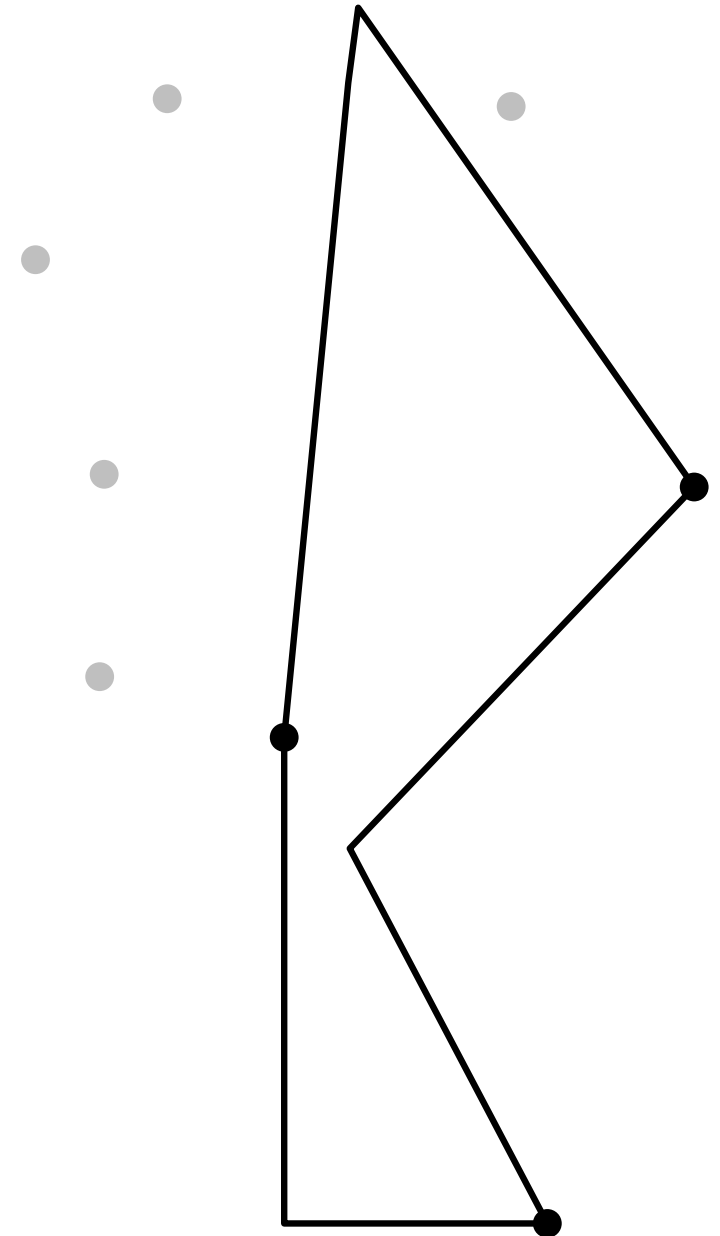
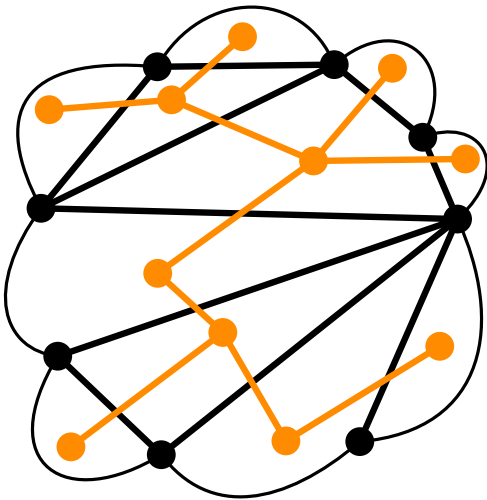
Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
- ...

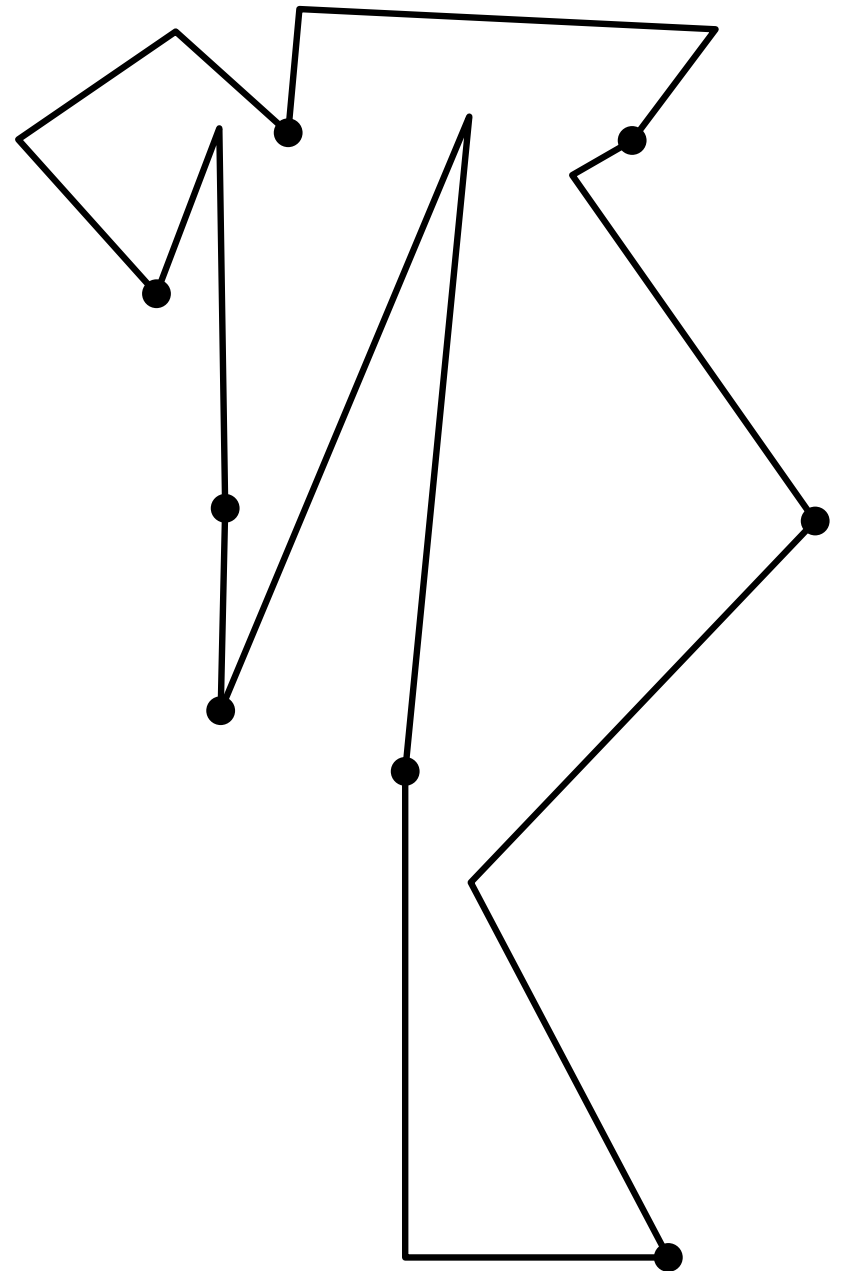
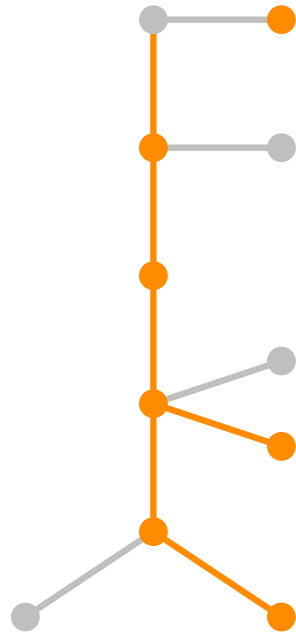
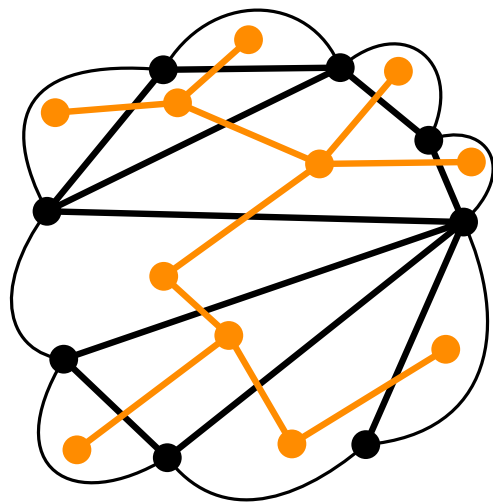


Example – Rest

- Step 4: convex
- Step 5: reflex
- Step 6: convex
-

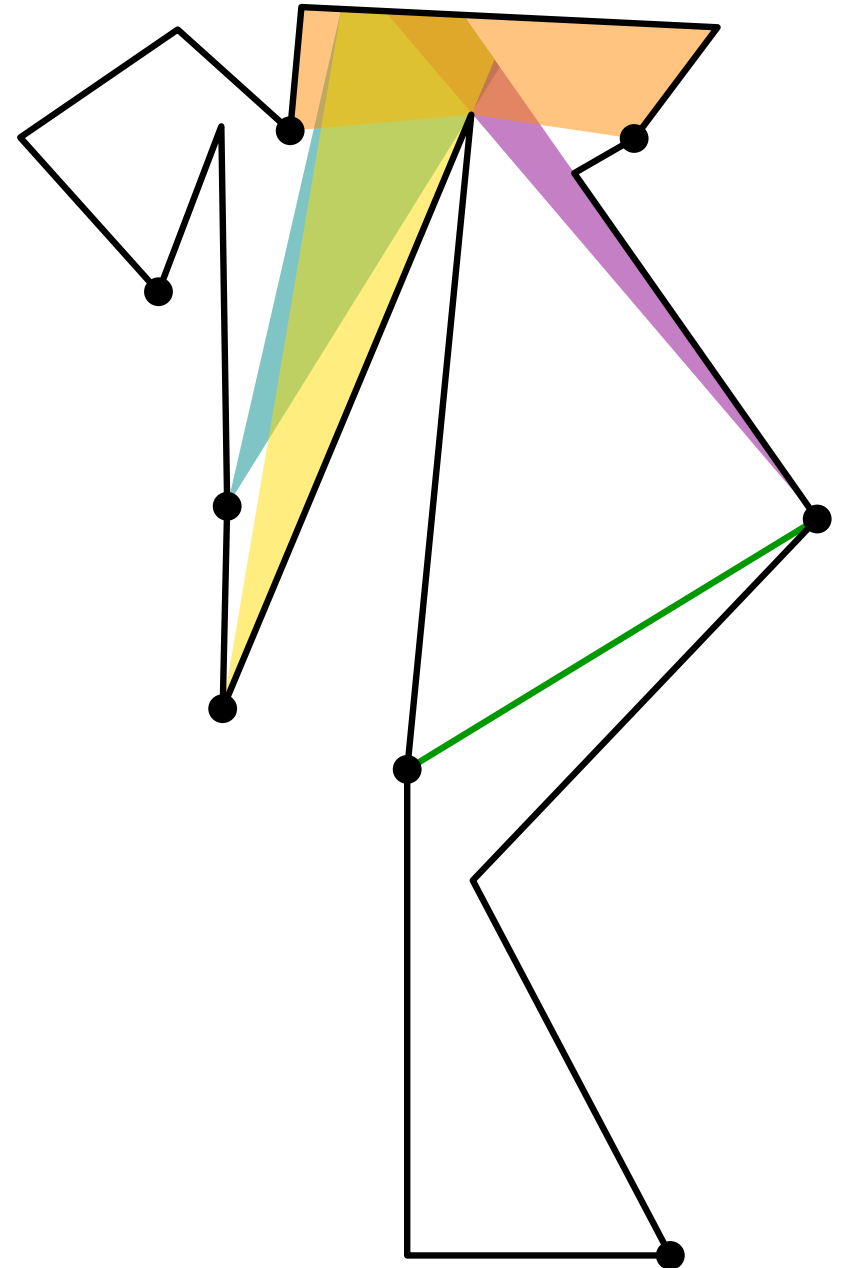
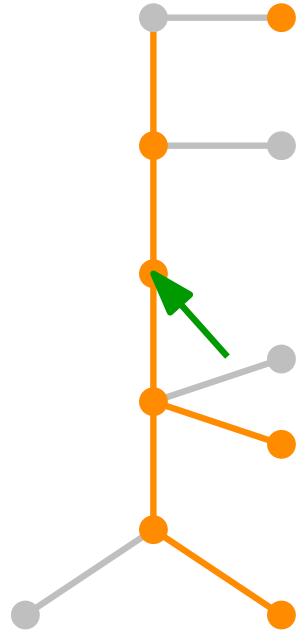
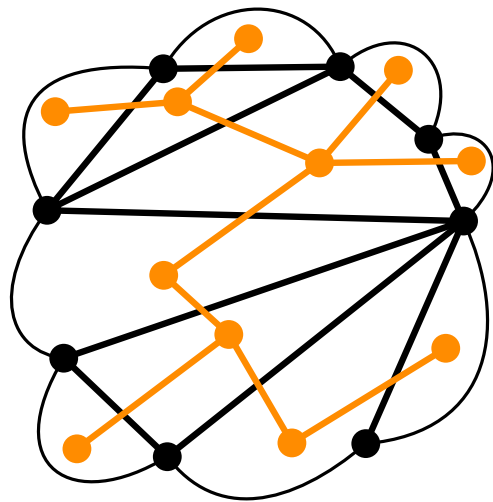


Example – Fixing Convex Edges



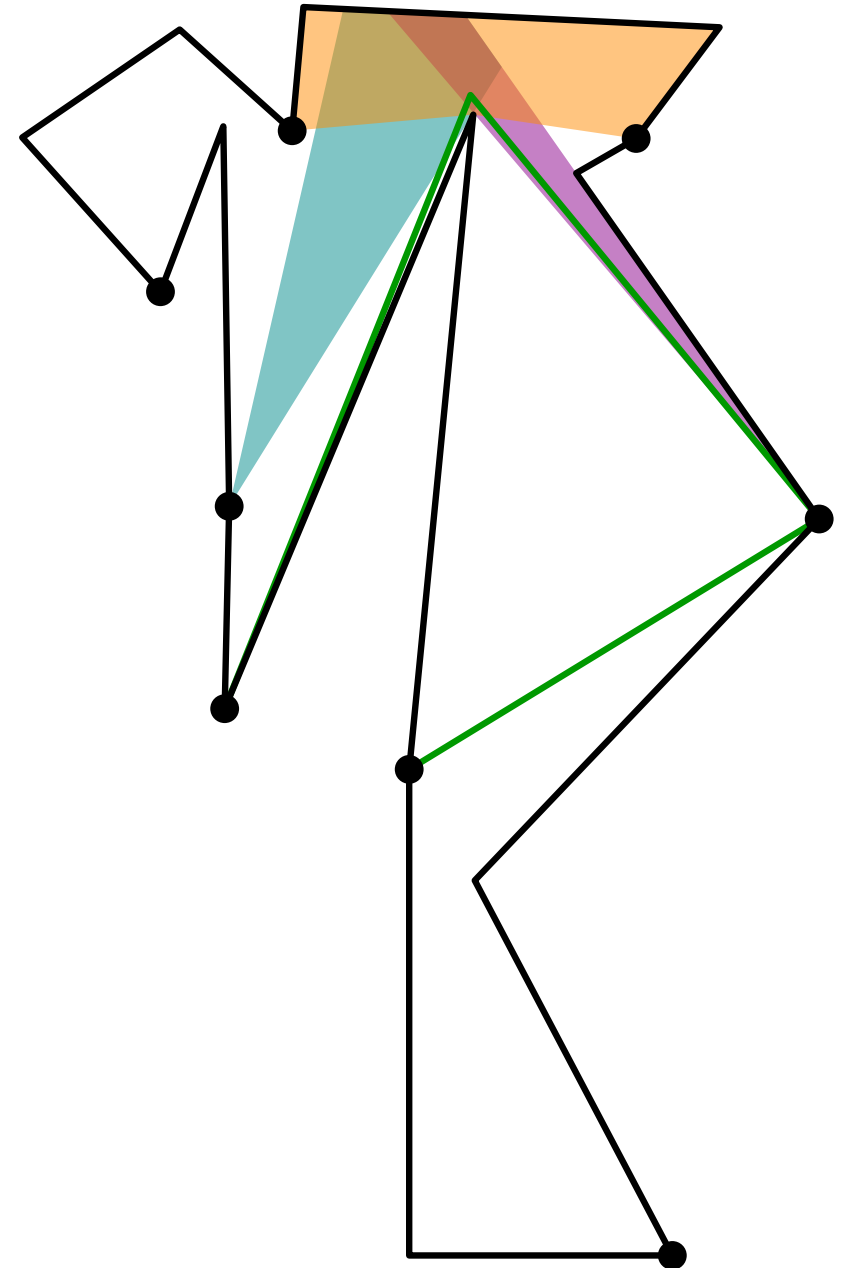
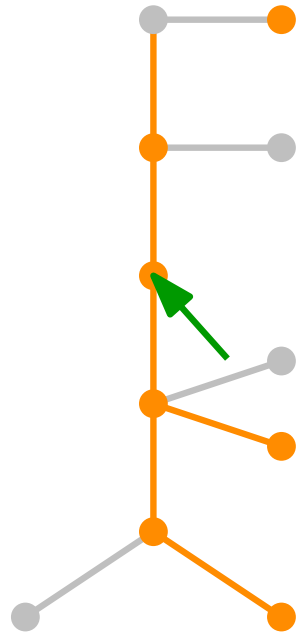
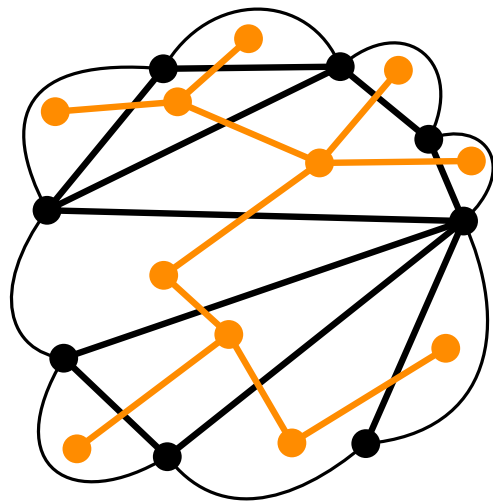
Example – Fixing Convex Edges

- Stored regions can overlap



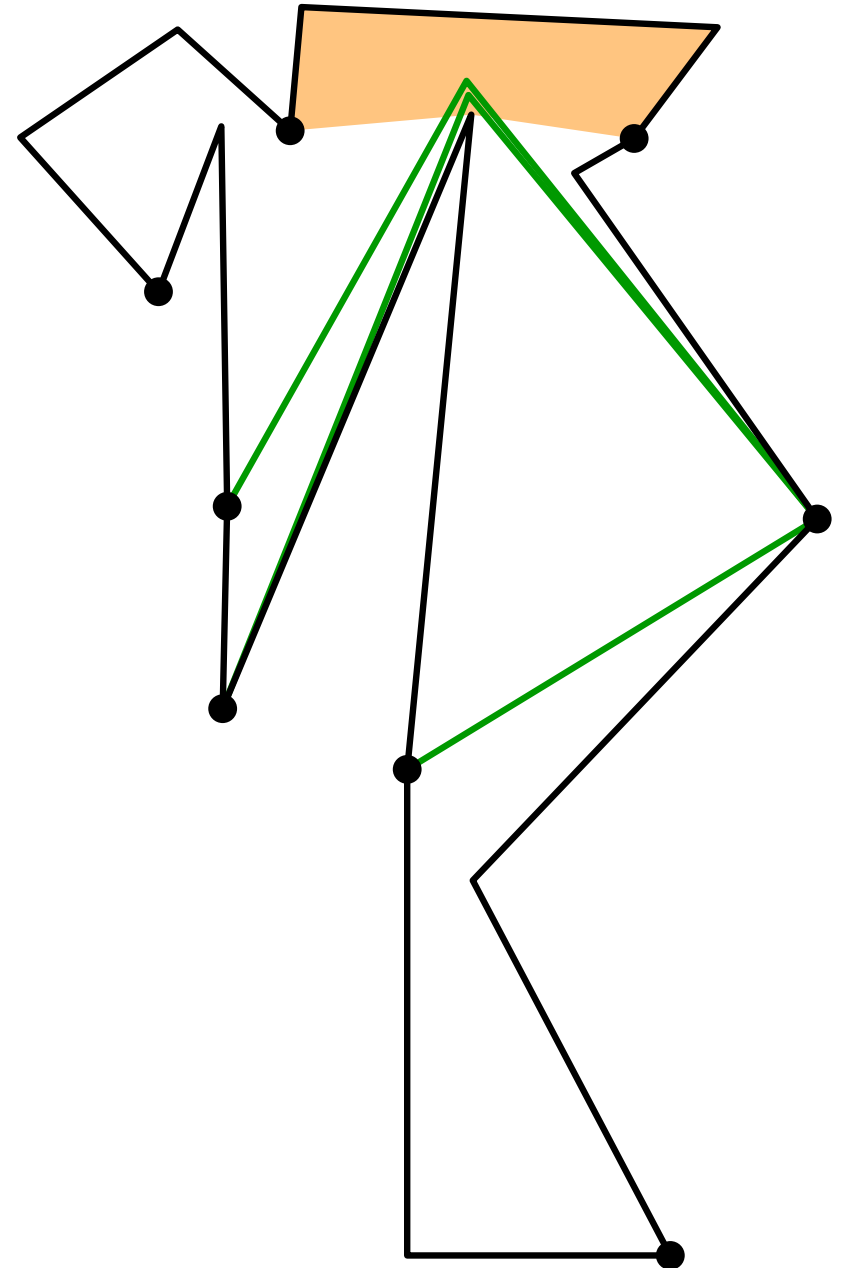
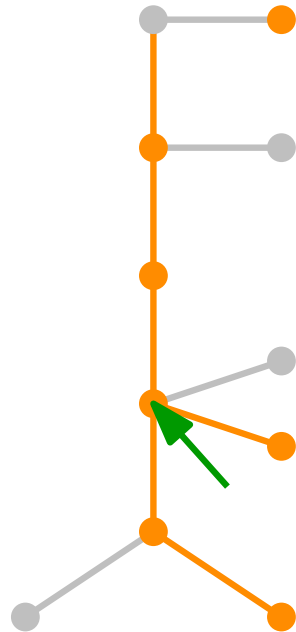
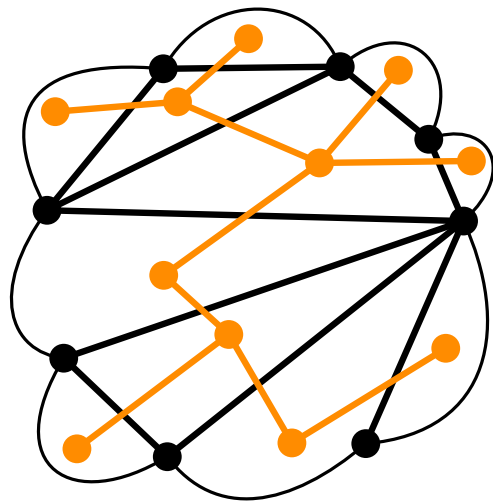
Example – Fixing Convex Edges

- Stored regions can overlap
- Route around bends in sequence



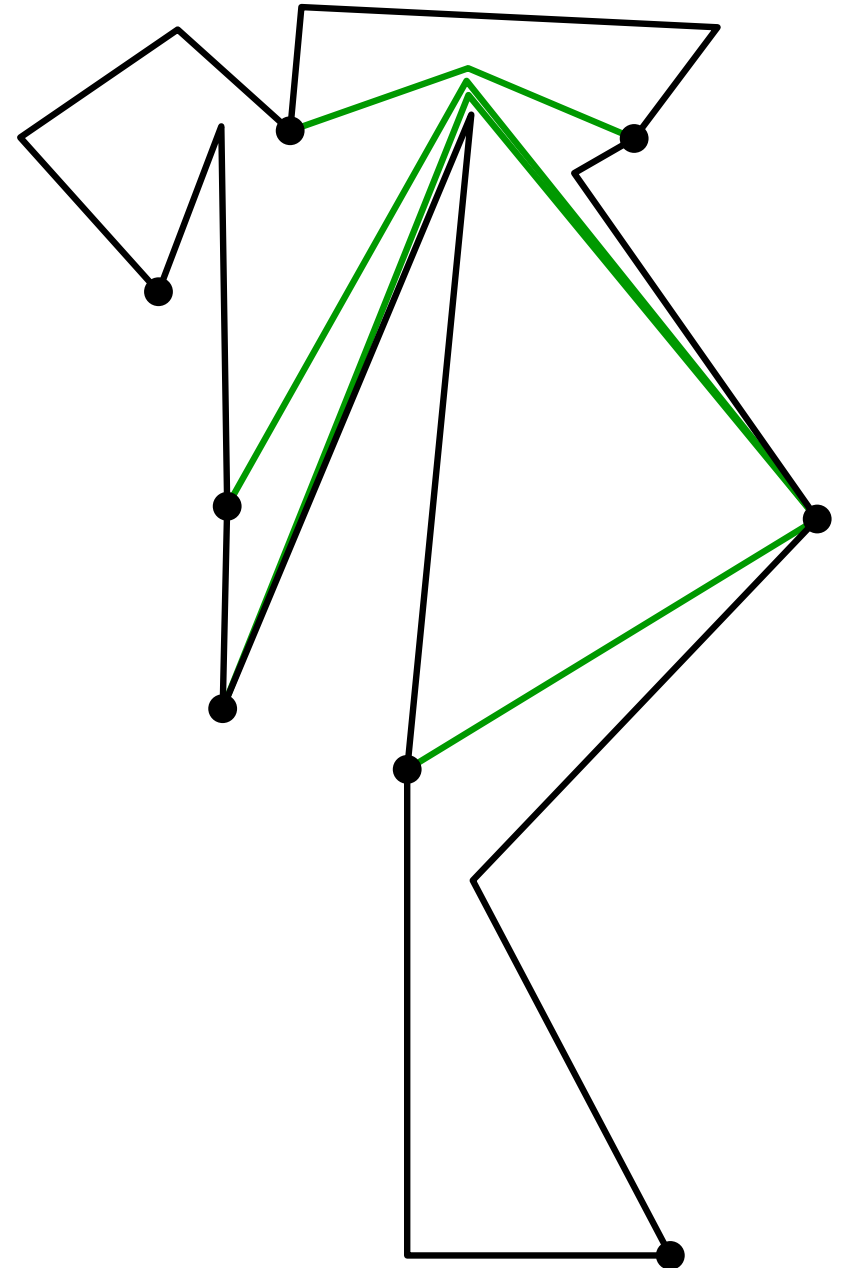
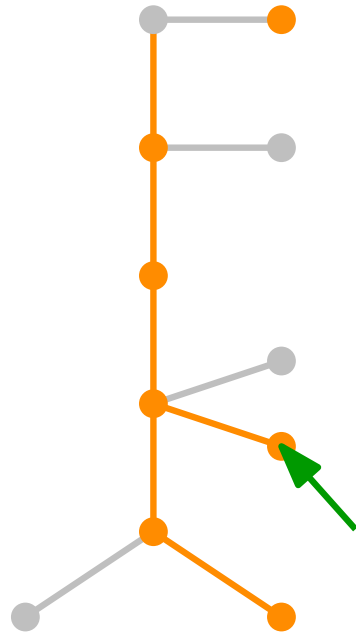
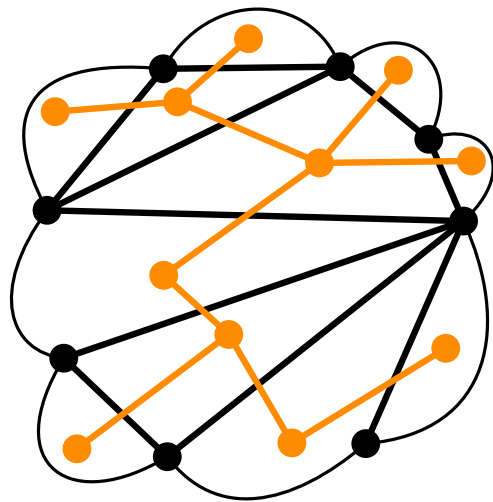
Example – Fixing Convex Edges

- Stored regions can overlap
- Route around bends in sequence



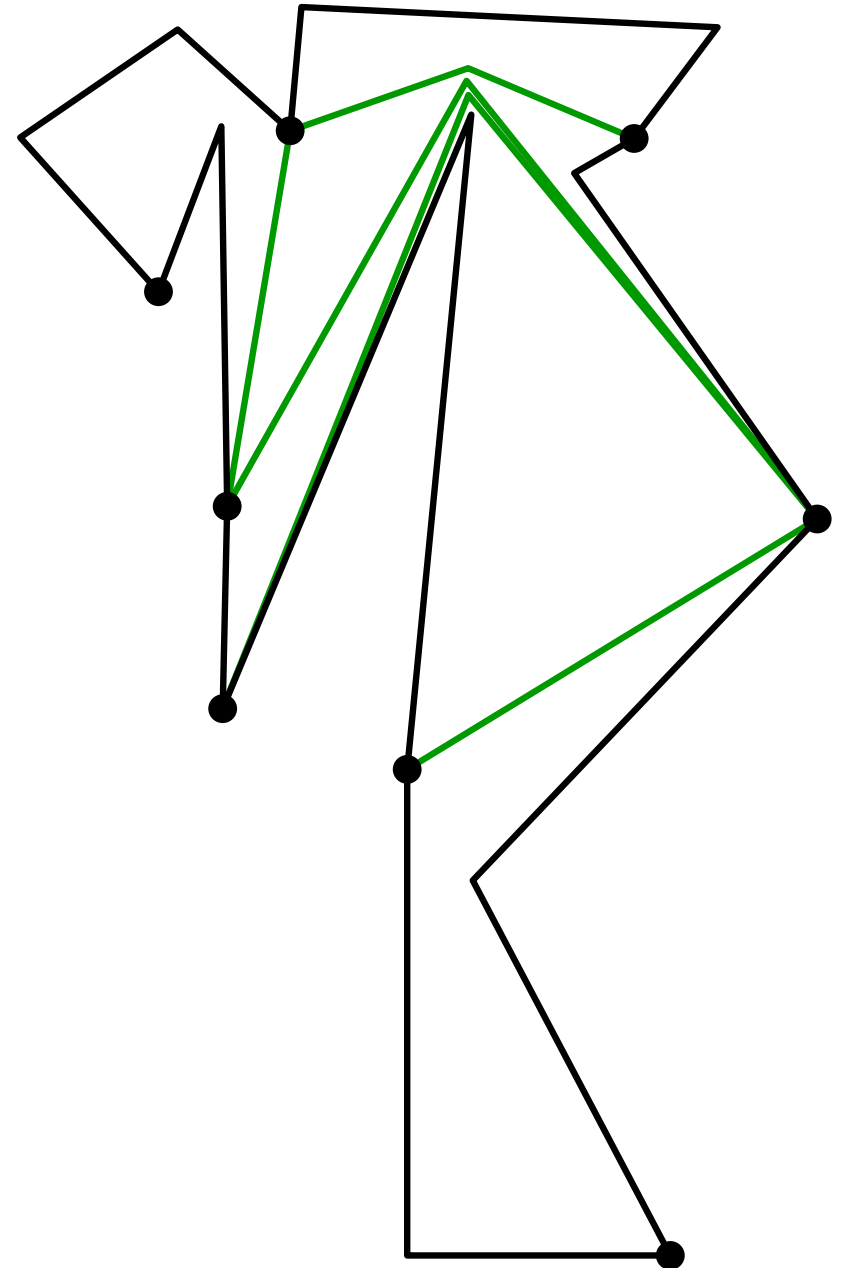
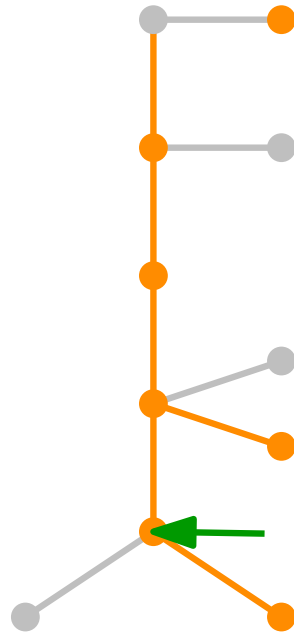
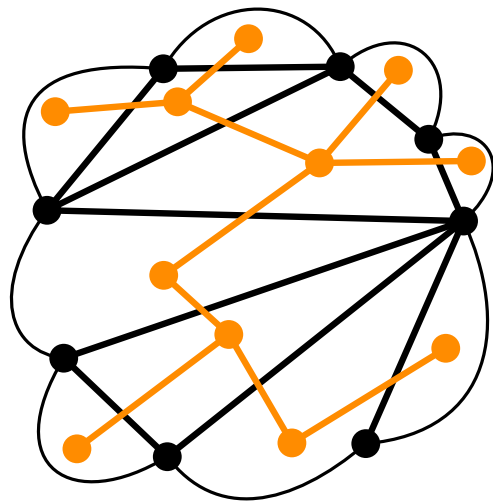
Example – Fixing Convex Edges

- Stored regions can overlap
- Route around bends in sequence



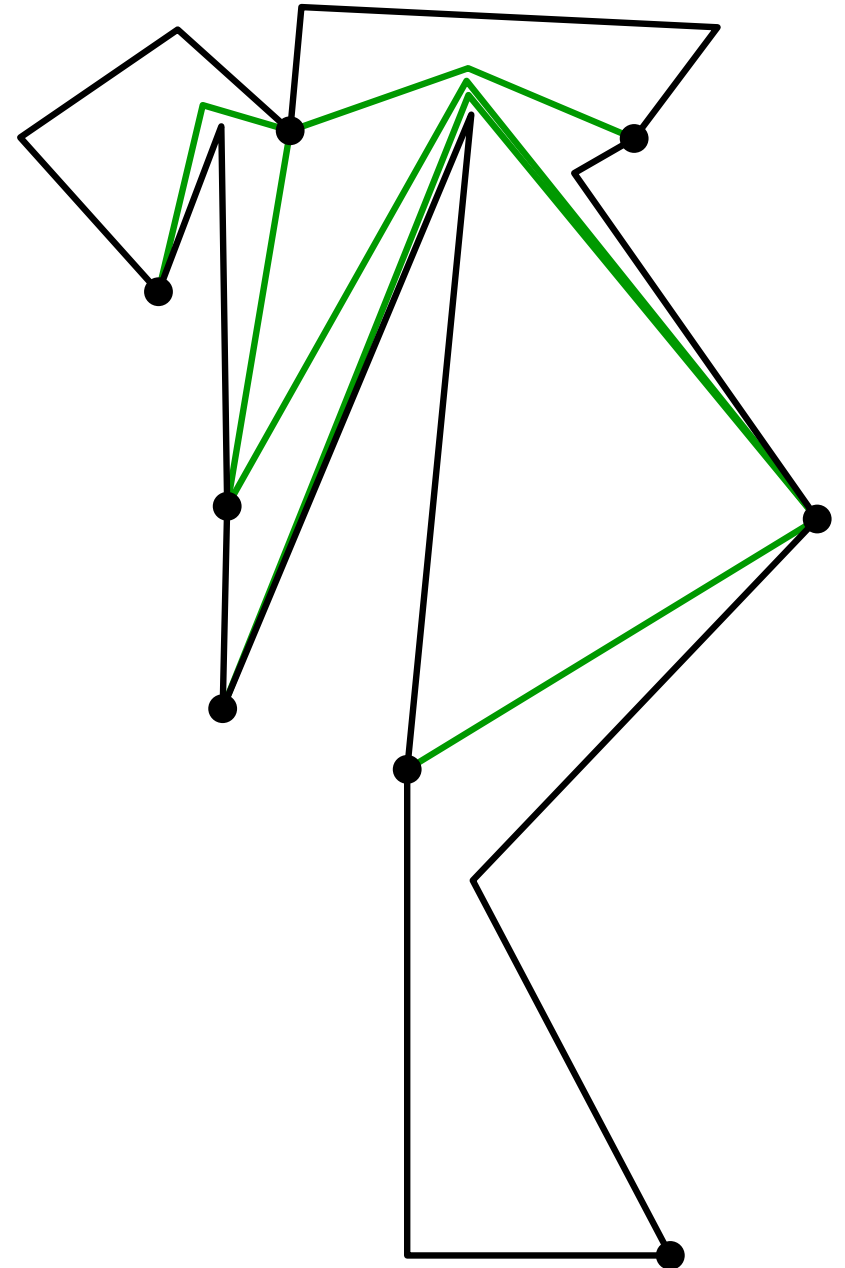
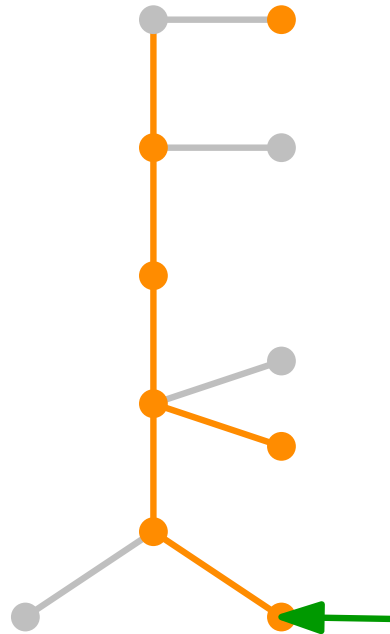
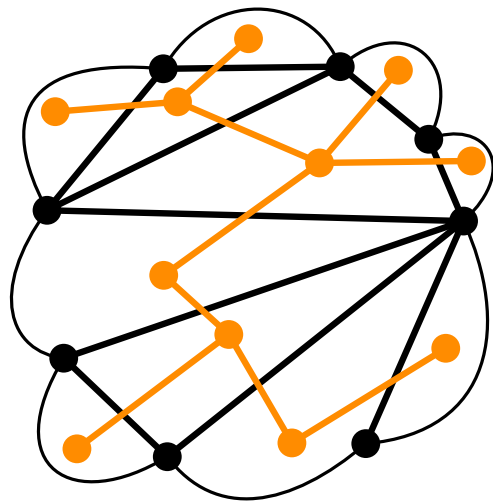
Example – Fixing Convex Edges

- Stored regions can overlap
- Route around bends in sequence



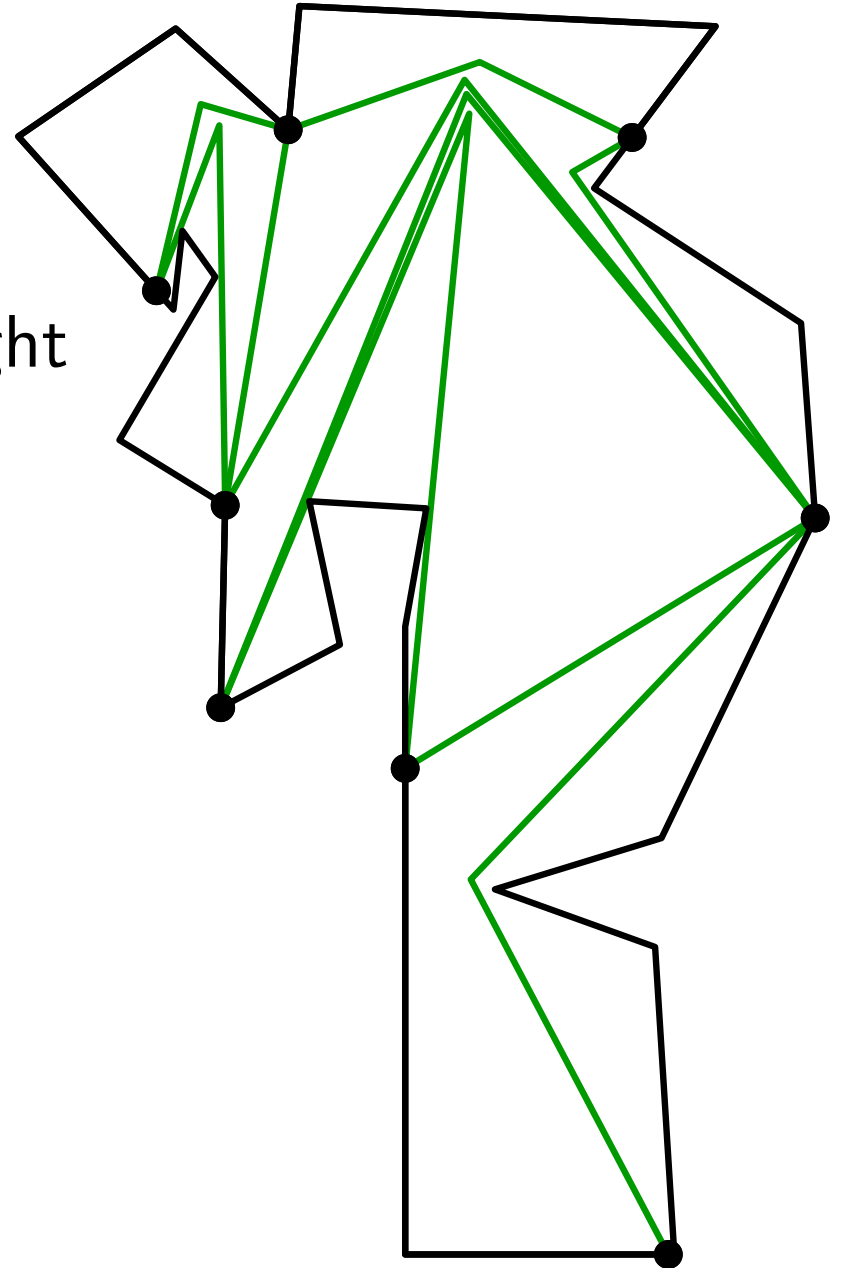
Example – Fixing Convex Edges

- Stored regions can overlap
- Route around bends in sequence



Conclusion

- Dual tree gets the sequencing right



Conclusion

- Dual tree gets the sequencing right
- Visibility region is/becomes empty
 \Rightarrow NO-instance
- Runtime $O(|V||P|)$

