Shape Formation in a Three-dimensional Model for Hybrid Programmable Matter

Kristian Hinnenthal, Dorian Rudolph, Christian Scheideler

Paderborn University

Outline

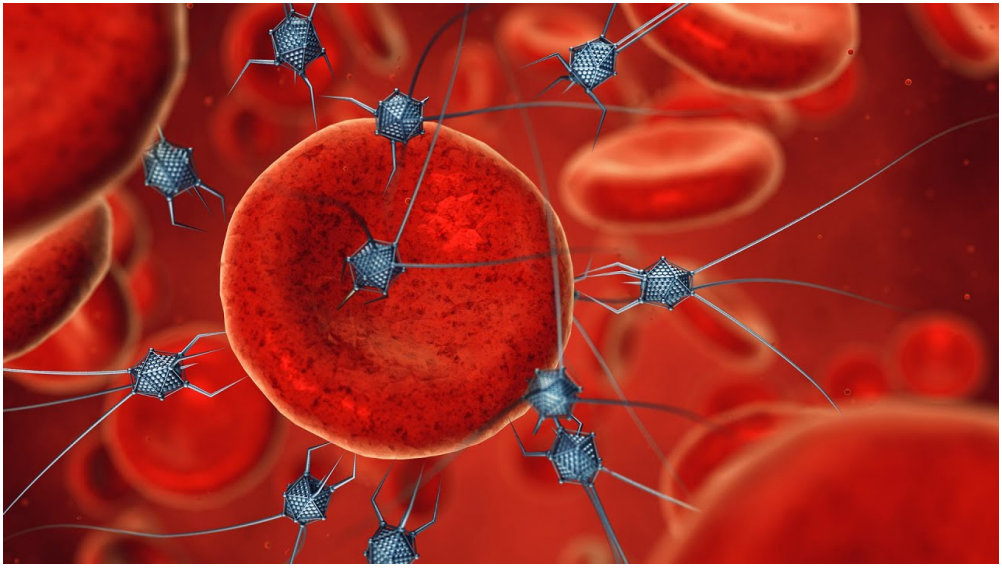
Introduction
- Motivation
- Problem

Model
- 2D
- 3D
- Safely Movable Tiles

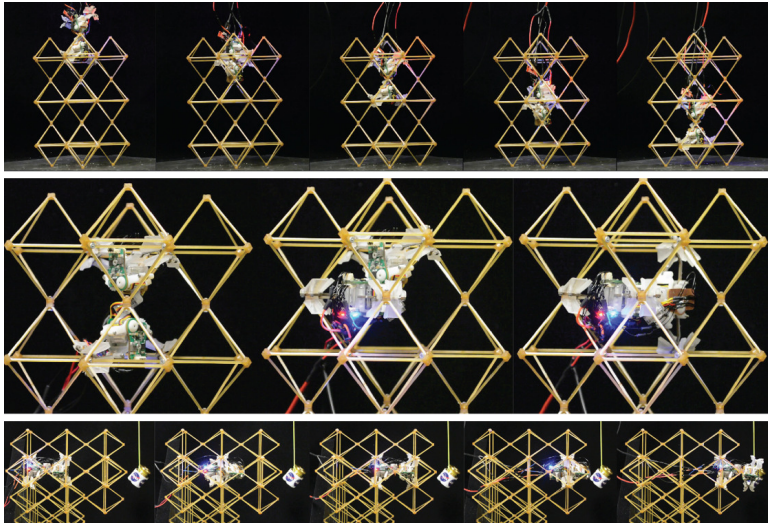
Algorithm

Conclusion

Motiviation I



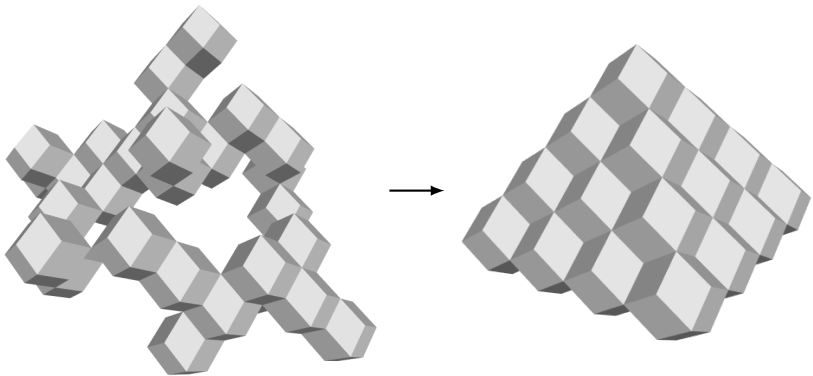
Motivation II



B. Jenett and D. Cellucci, "A mobile robot for locomotion through a 3D periodic lattice environment," 2017 IEEE International Conference on Robotics and Automation (ICRA), Singapore, 2017, pp. 5474-5479.

Problem

- Use a single *finite automaton robot* to reconfigure arbitrary 3D tile structures into a given shape, e.g., a pyramid.



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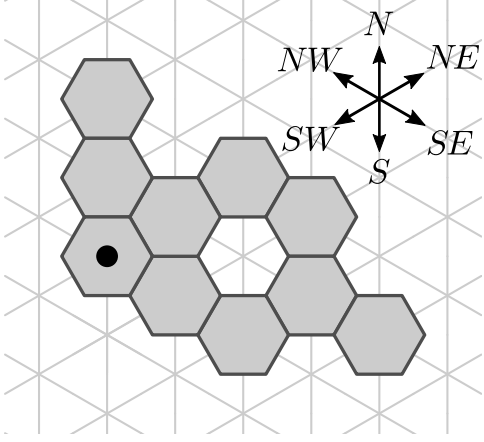

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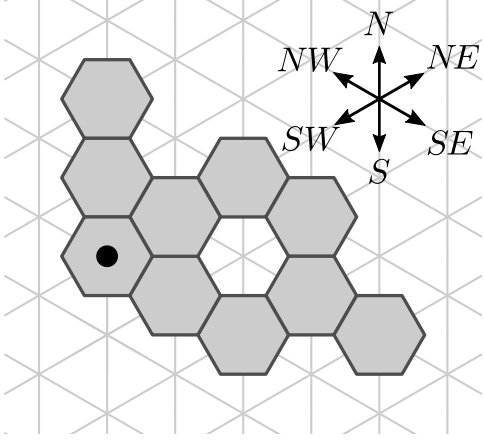

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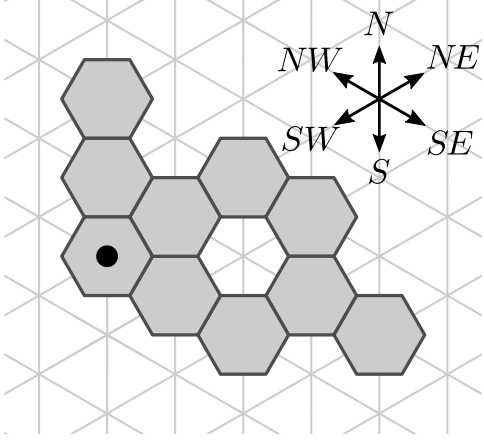

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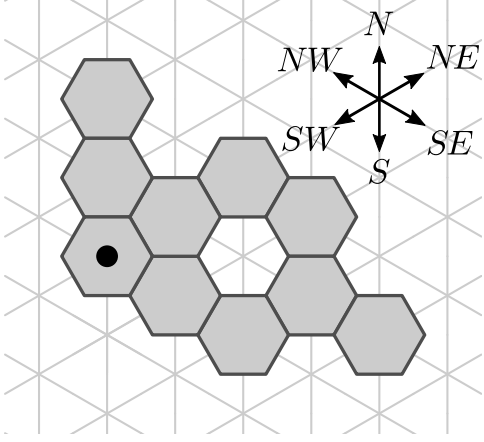

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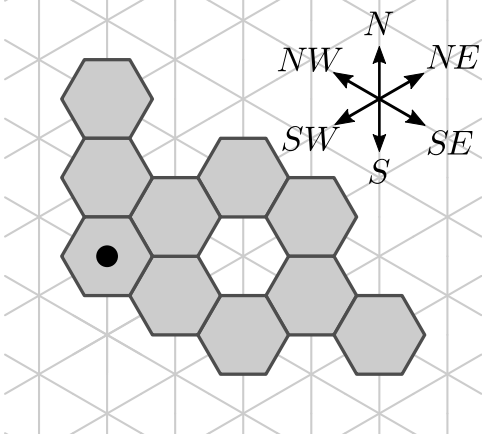
- Tiles (including the robot if it carries a tile) must remain connected.



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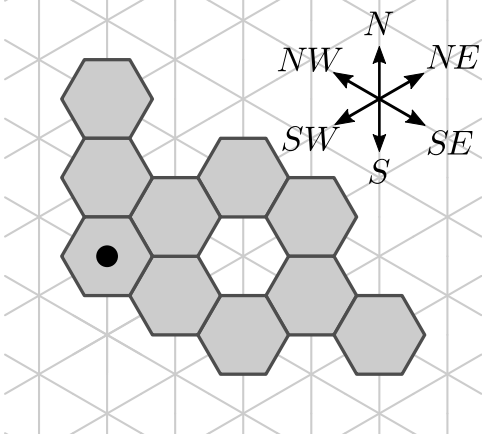

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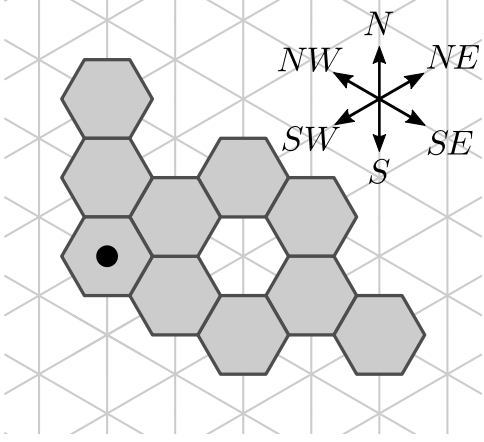

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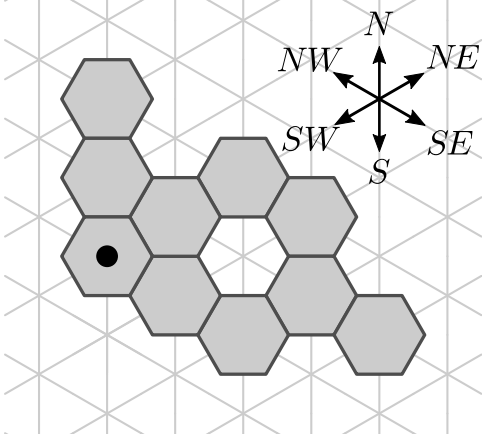

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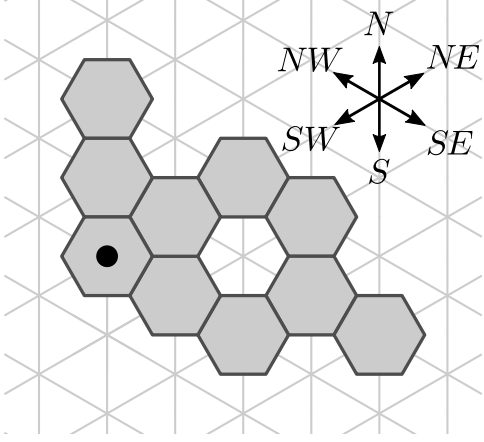

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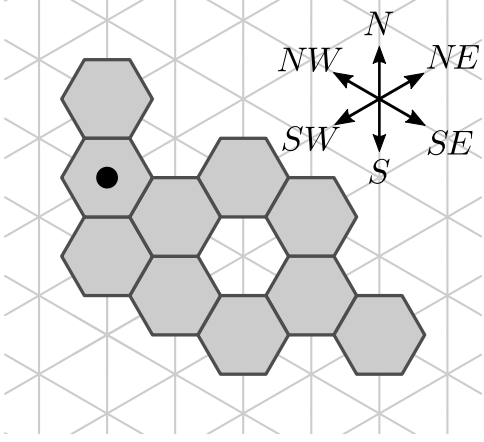

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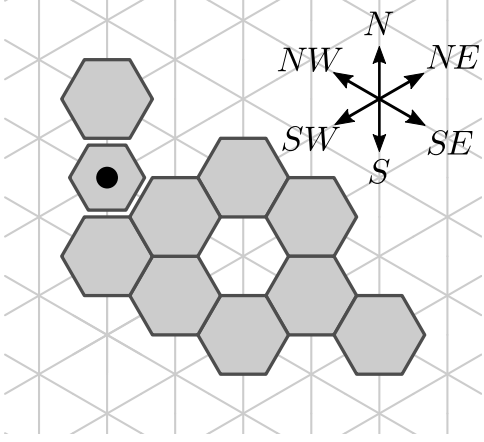

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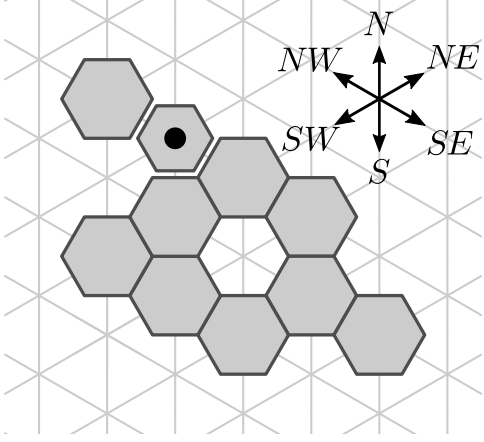

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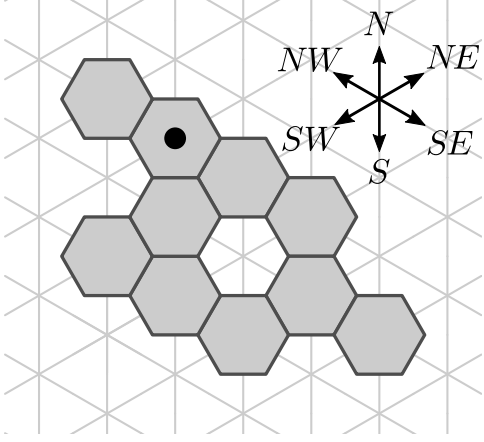

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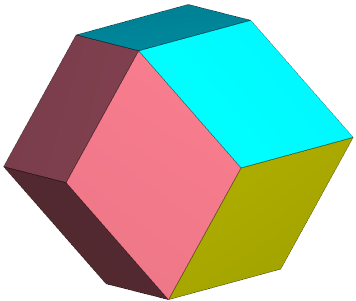

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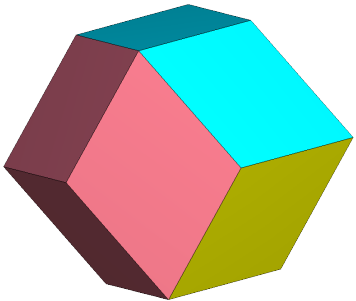

Tomruen [CC BY-SA 4.0]

TED-43 [CC BY 3.0]

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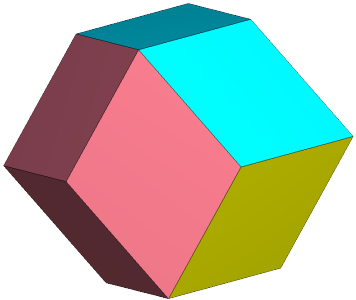

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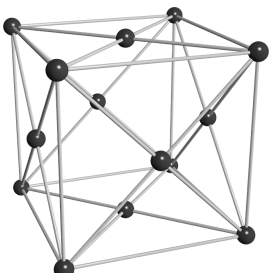
TED-43 [CC BY 3.0]

Didier Descouens [CC BY-SA 4.0]

Lattice

- Rhombic dodecahedral tiles are placed in the *face-centred cubic (FCC)* lattice (i.e., the adjacency graph of the FCC sphere packing)



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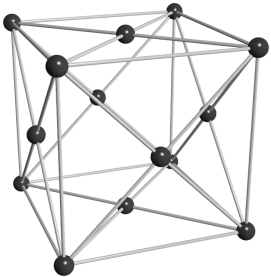

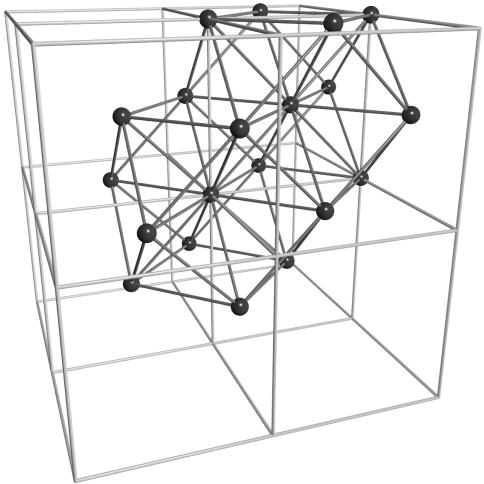

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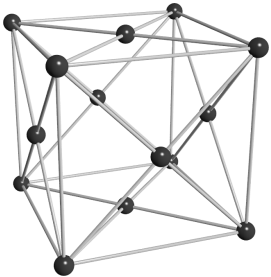
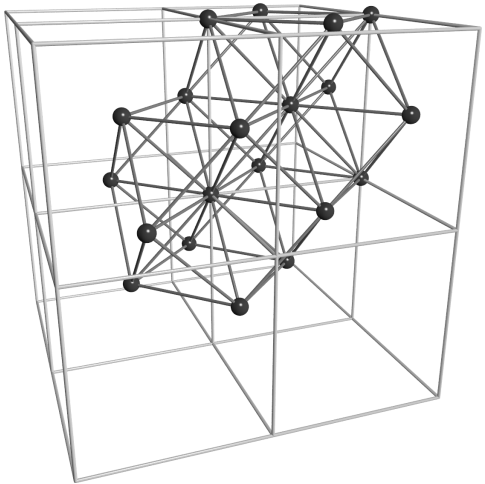
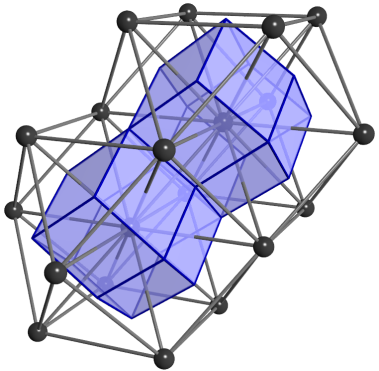
- Voronoi cells of the FCC lattice are rhombic dodecahedra







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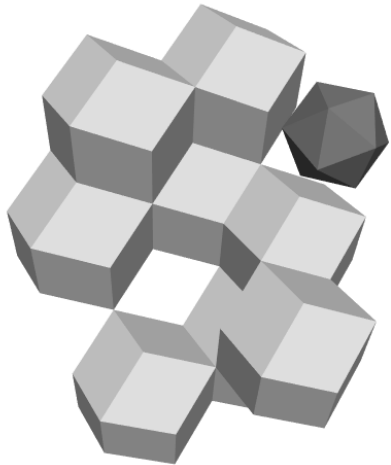

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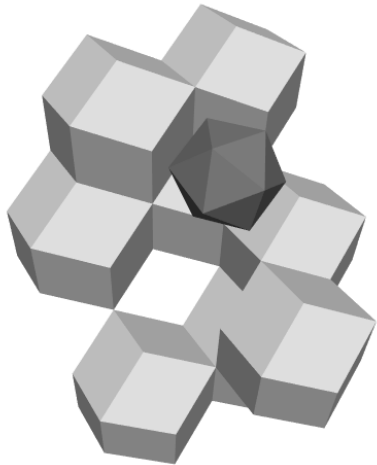

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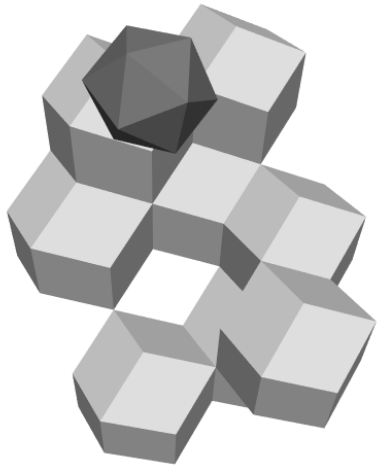

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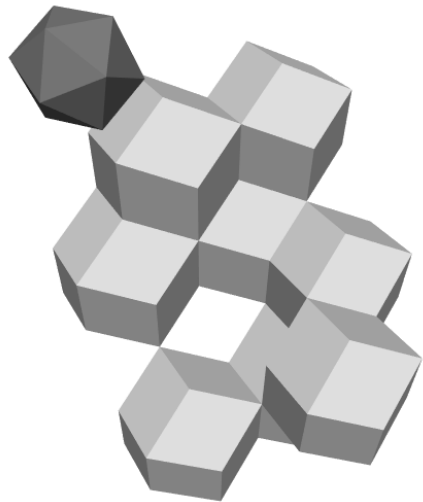

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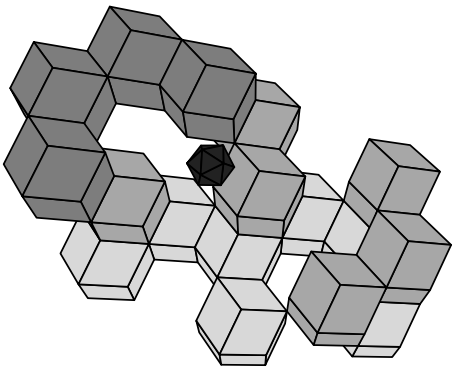
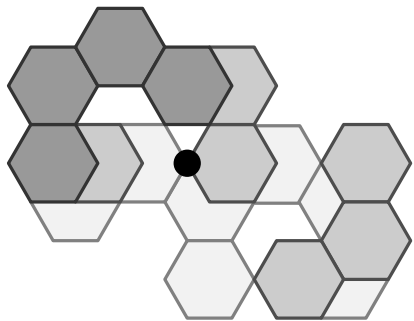
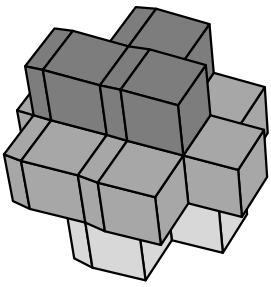
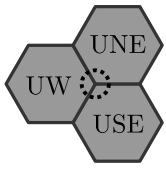
- Can be viewed in terms of hexagonal layers.

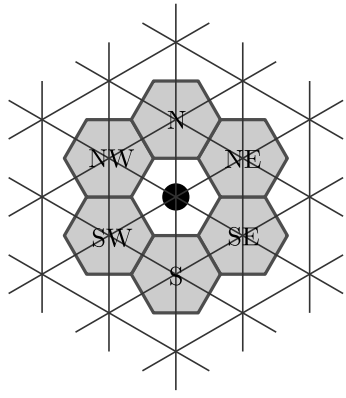


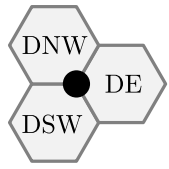


Directions









- Up: UNE/USE/UW
- Same layer: N/NE/SE/S/SW/NW
- Down: DNW/DE/DSW

Safely Movable Tiles

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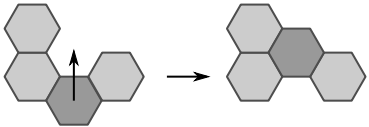

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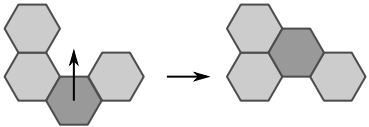

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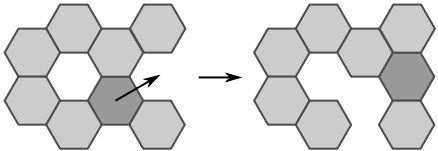

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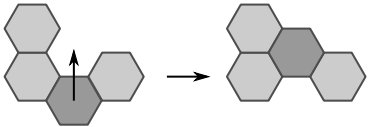

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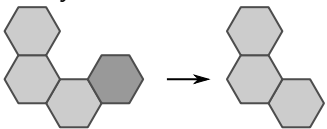

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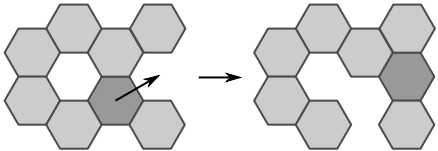

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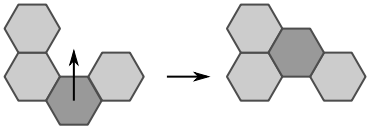

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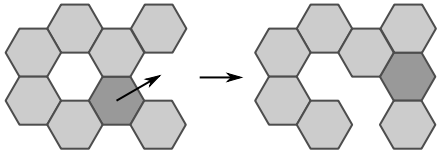

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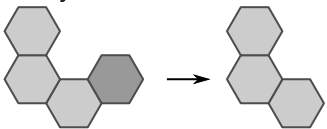

Theorem (2D)

A robot that can always find a safely locally movable tile.

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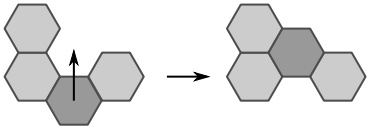

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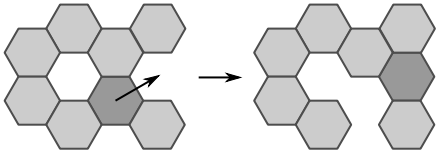

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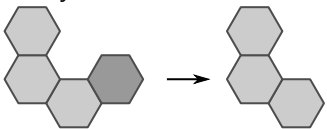
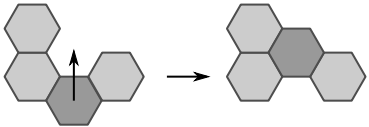


Theorem (2D)

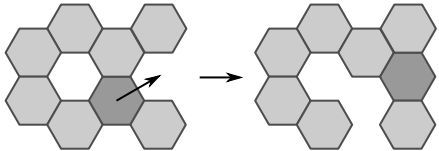
A robot that can always find a safely locally movable tile.
No robot can find a safely removable tile in all configurations.

Safely Movable Tiles

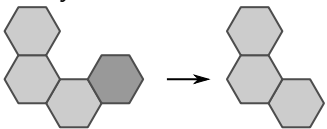
- *Safely locally movable:*



- *Safely movable:*



- *Safely removable:*



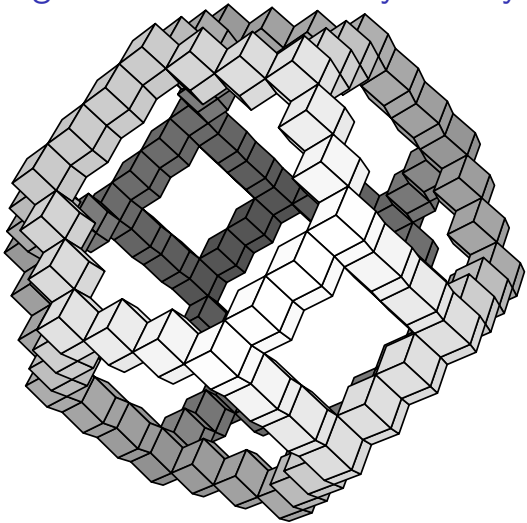
Theorem (2D)

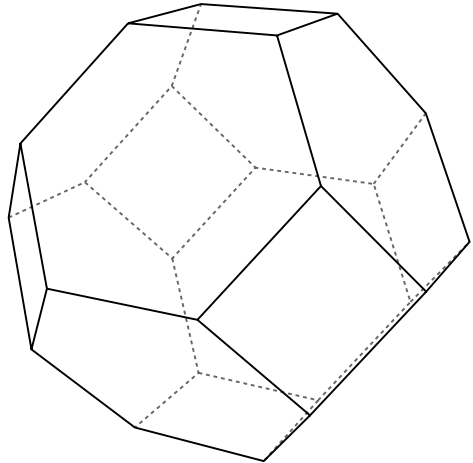
A robot that can always find a safely locally movable tile.
No robot can find a safely removable tile in all configurations.

Theorem (3D)

No robot can find a safely movable tile in all configurations.

Configuration without Safely Locally Movable Tiles





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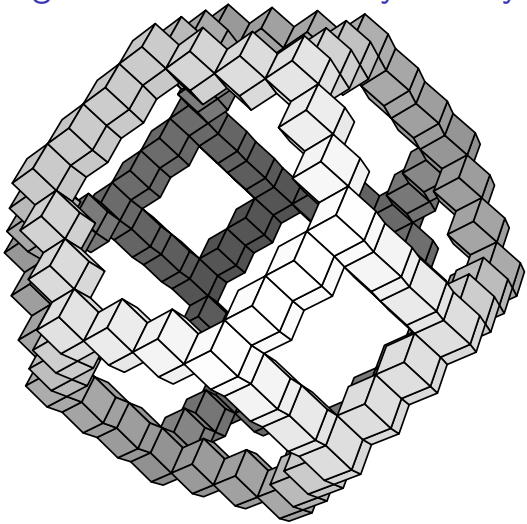
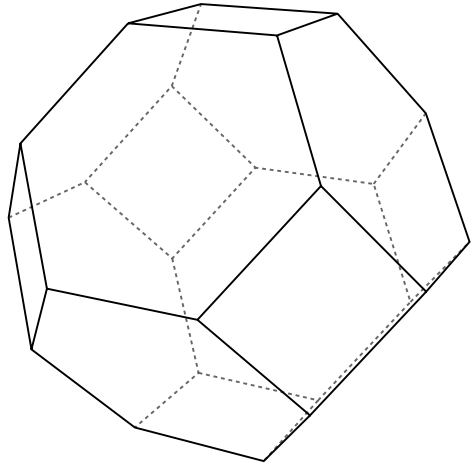




- Solution: The robot initially carries a tile.

Outline

Introduction
- Motivation
- Problem

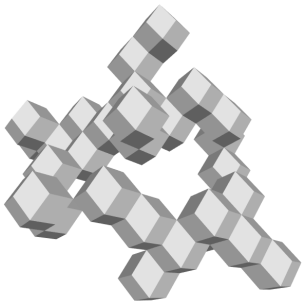
Model
- 2D
- 3D
- Safely Movable Tiles

Algorithm

Conclusion

Intermediate Structure

- Use a line as *intermediate structure* for shape formation.



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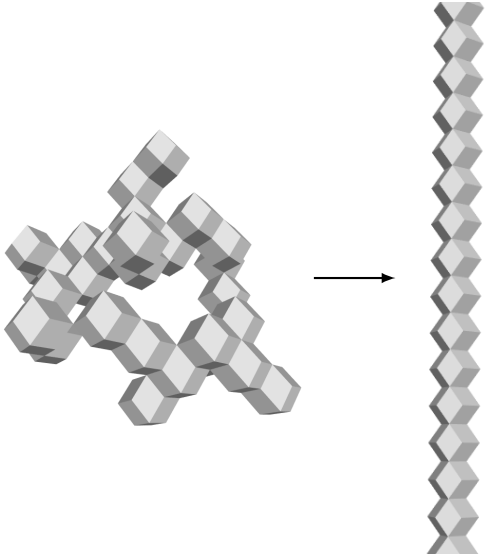

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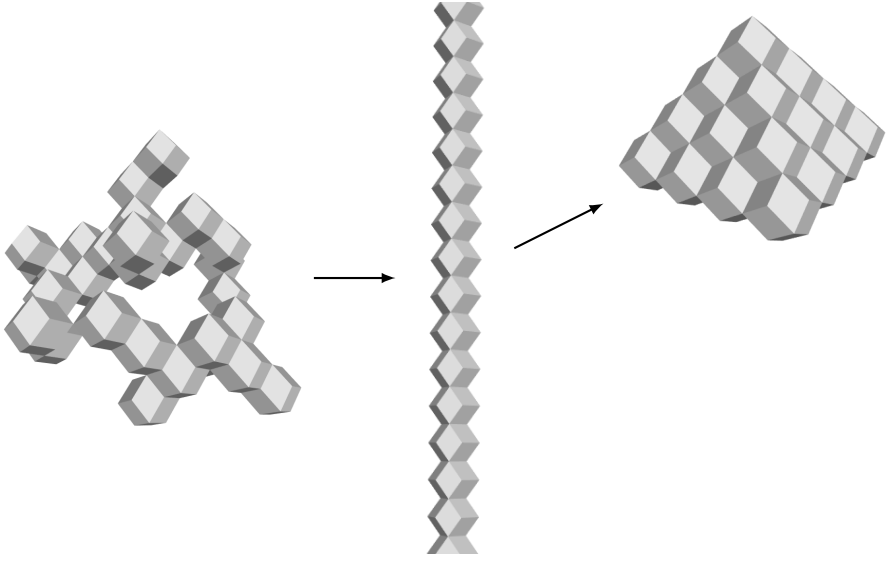

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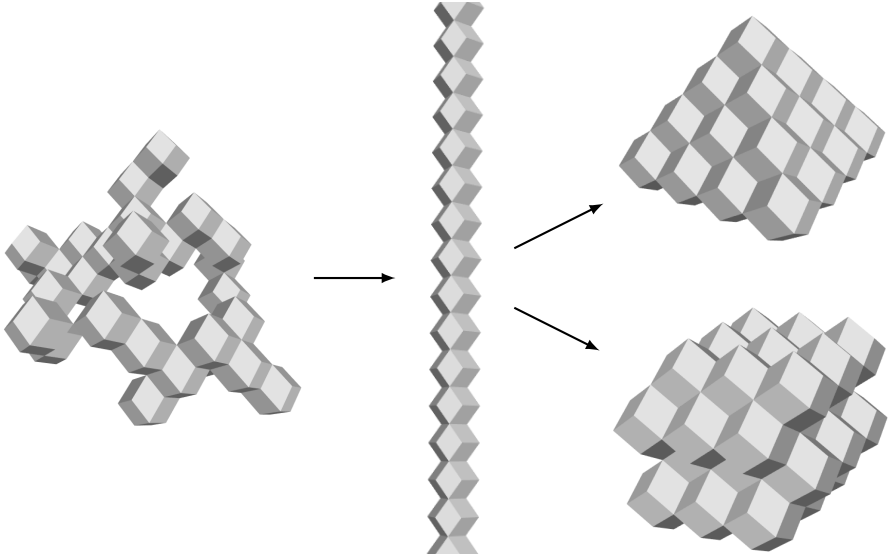

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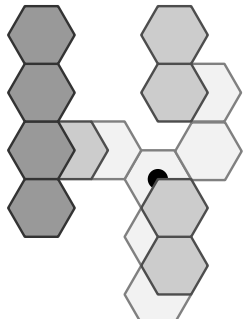

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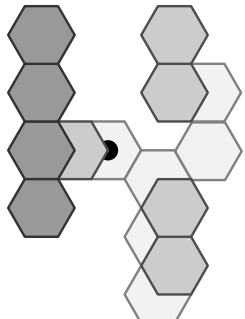

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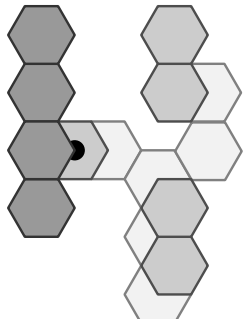

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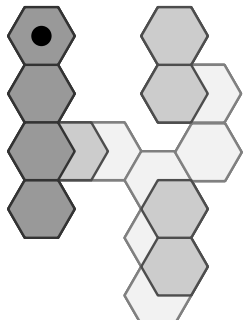

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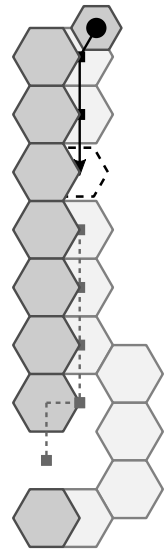

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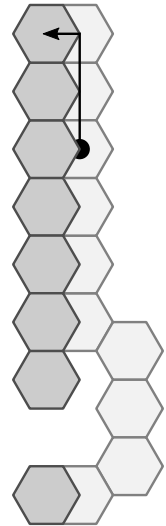

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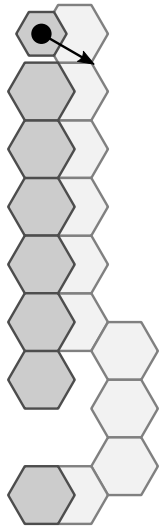

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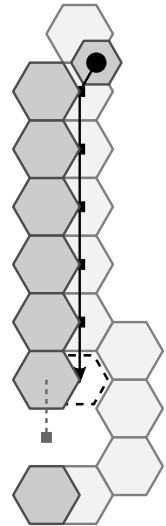

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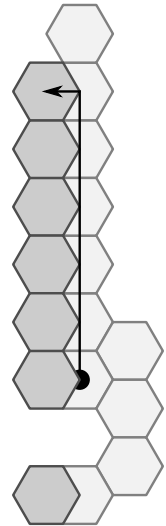

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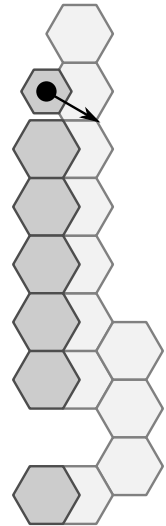

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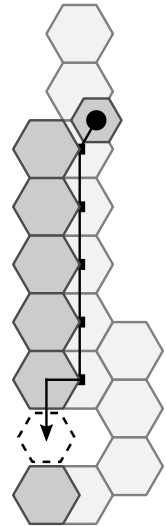

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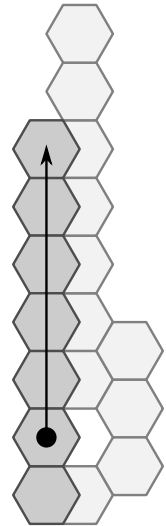

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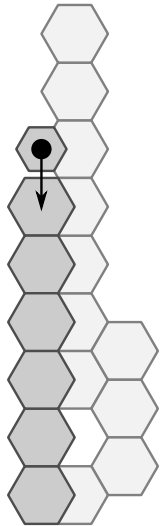

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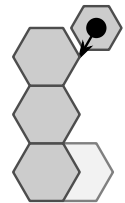

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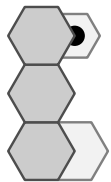

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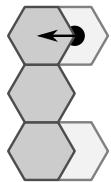

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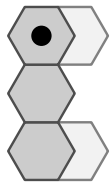

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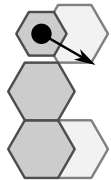

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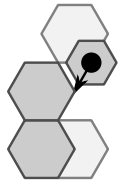

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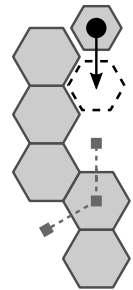

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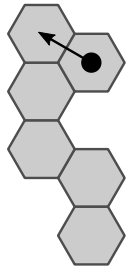

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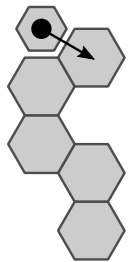

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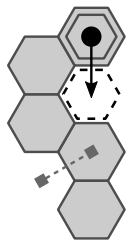

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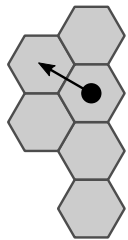

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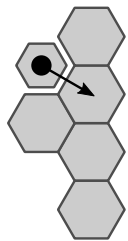

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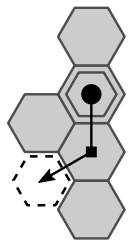

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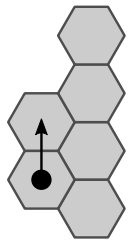

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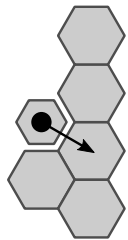

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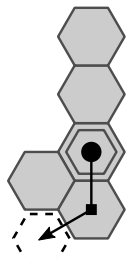

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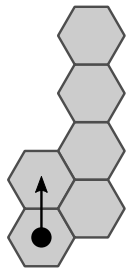

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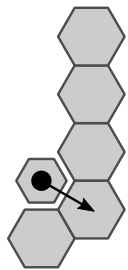

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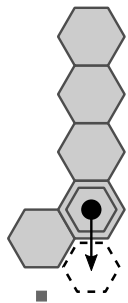

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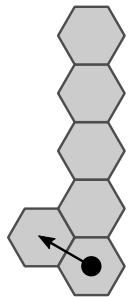

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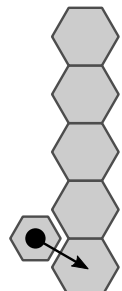

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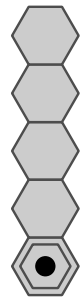

Analysis

Theorem

The robot can build a line in $O(n^3)$ rounds.

Outline

Introduction
- Motivation
- Problem

Model
- 2D
- 3D
- Safely Movable Tiles

Algorithm

Conclusion

Future Work

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[b] Disser, Y., Hackfeld, & J., Klimm, M. (2015). Undirected Graph Exploration with $\Theta(\log \log n)$ Pebbles. In Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms. Society for Industrial and Applied Mathematics.

Try out the Simulator

- `https://go.upb.de/3DHybridSim`
- Browser: Tested with Chrome, Safari does not work