Enumerating tilings of triply-periodic minimal surfaces

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Overview

Preliminaries
- Motivation

Minimal surfaces and orbifolds
- Triply periodic minimal surfaces
- Orbifolds

Enumerating isotopy classes of tilings
- Isotopic tiling theory
- Theoretical results
- Algorithm
- Implementations

Conclusion

Graphs in nature and materials science

- Chemical structures are often modelled as (locally finite, simple) graphs in $\mathbb{R}^3$.

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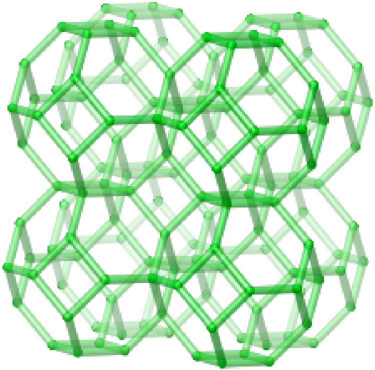

Sodalite, an aluminosilicate, a real Zeolite in $\mathbb{R}^3$. Image courtesy of Stephen Hyde.

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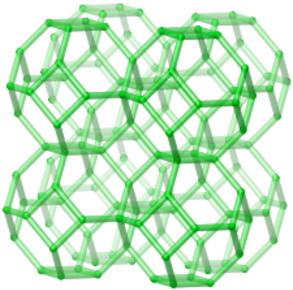
- Many molecular structures grow in restricted environments, modelled as neighbourhoods of surfaces of CMC or minimal surfaces.



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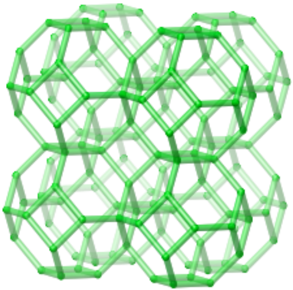

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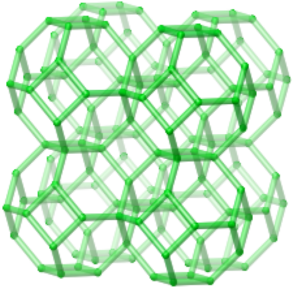

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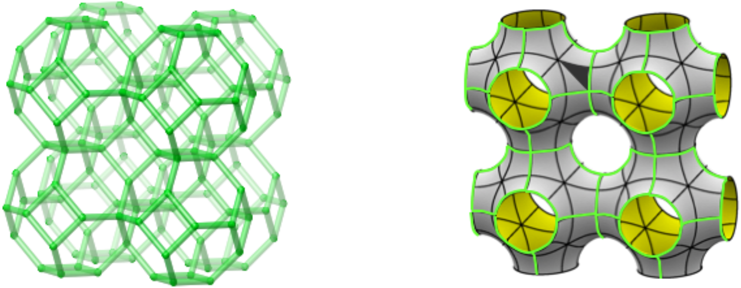

Figure: Sodalite on the primitive surface

The central idea of the EPINET approach

- Study entangled graphs by explicit constructions of graphs on surfaces.
- **The EPINET approach:** Investigate *nets* by drawing graphs symmetrically on an embedded hyperbolic surface in $\mathbb{R}^3$. Then forget about the surface.
- Use special surfaces so that produced graphs are symmetric in $\mathbb{R}^3 \implies \mathbb{H}^2$

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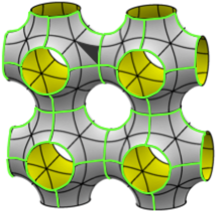

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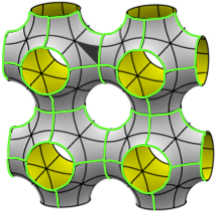

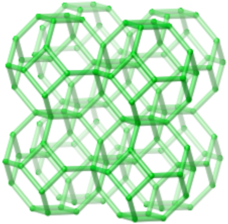

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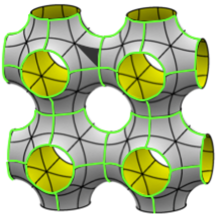

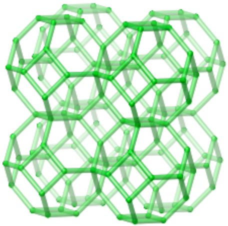

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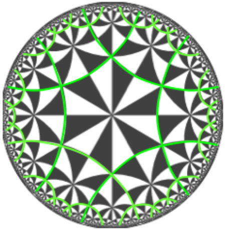

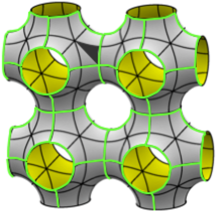

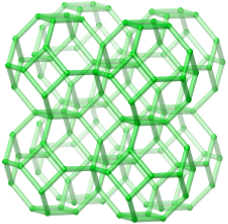

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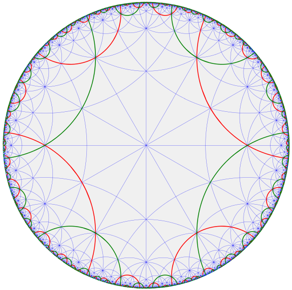

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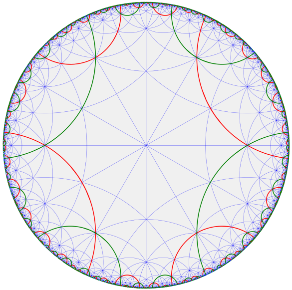

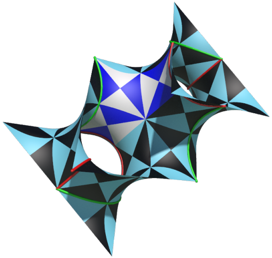

Figure: Hyperbolic Tiling and the corresponding drawing on the diamond surface in $\mathbb{R}^3$.

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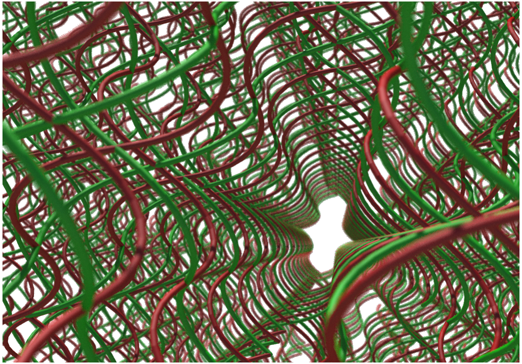
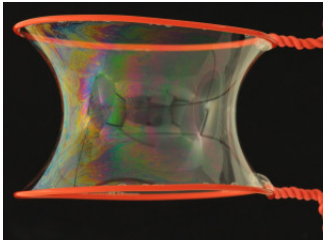


Figure: The corresponding net in $\mathbb{R}^3$, representing a molecular structure grown on the diamond surface with two distinct strands.

Minimal Surfaces

- Locally minimize their surface area relative to the boundary of a small neighbourhood of any point.
- The soap film bounded by a wire is a minimal surface;
  many equipotential surfaces in nature are (close to) minimal;
  many membranes found in living tissue.



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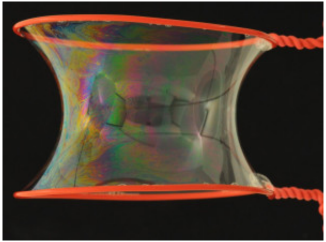

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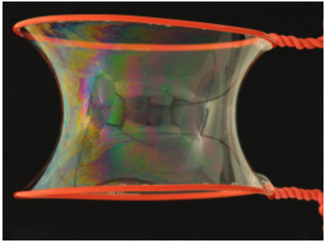

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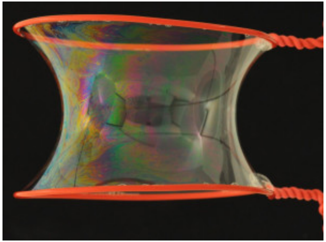

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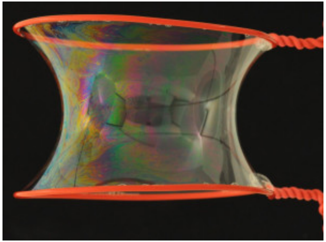

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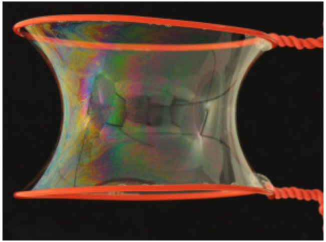

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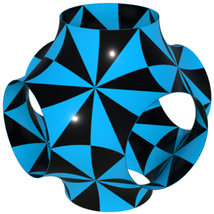

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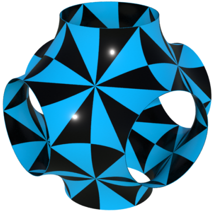

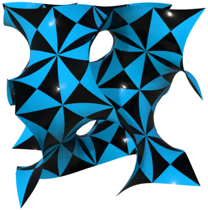

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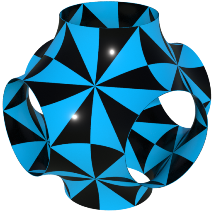

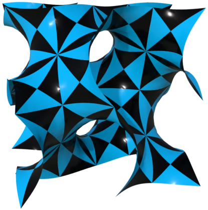

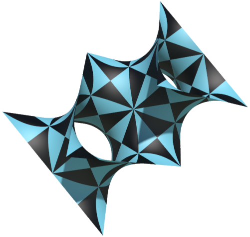

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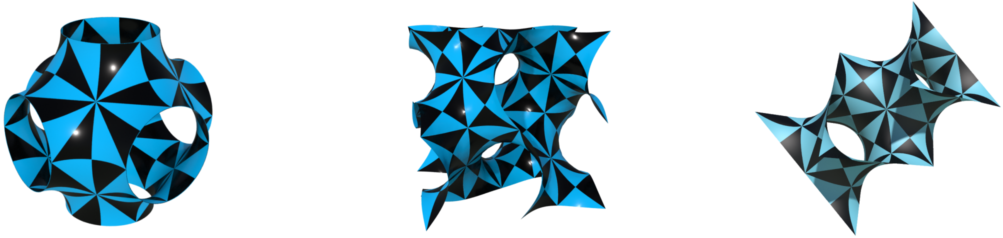
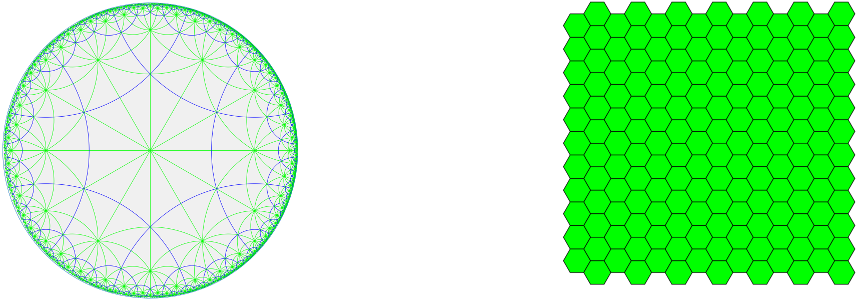


- The translations are a result of more refined symmetries.
- These symmetries yield a *hyperbolic orbifold* ($\star 246$).

Central definition - Orbifolds

Definition - developable orbifolds for equivariant tiling theory

Let $X = \mathbb{H}^2, \mathbb{E}^2$ and $\Gamma \subset \mathrm{Iso}(X)$ discrete. The set of data (X, Γ) associated with the quotient map $\pi : X \to X/\Gamma$ is a (metric) orbifold.



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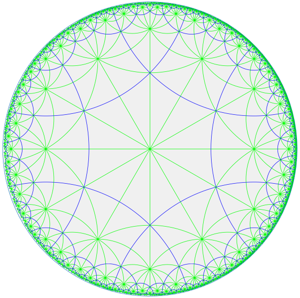

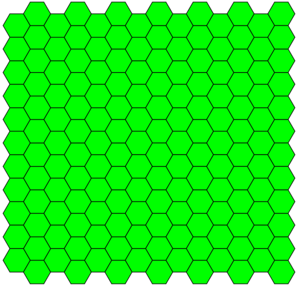

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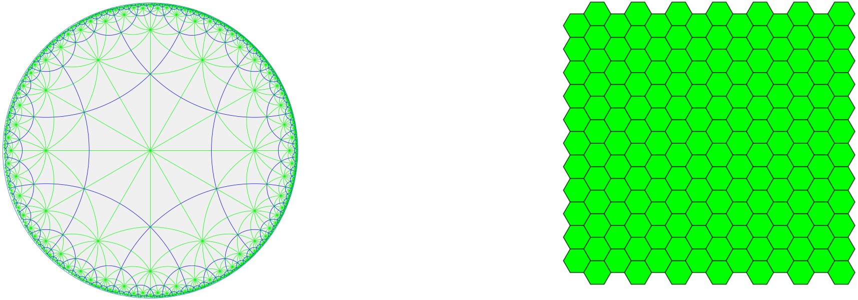

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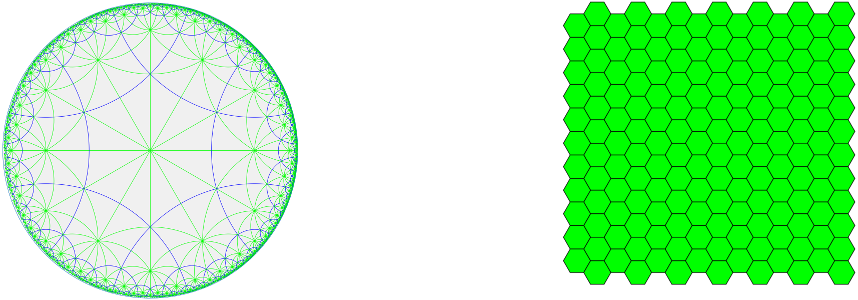

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Take-Home Message I

- Graphs in $\mathbb{R}^3$ can be studied (effectively) by exploring graphs on TPMS
- These are covered by the hyperbolic plane $\mathbb{H}^2$.
- Prominent TPMS exhibit a high degree of symmetry.
- Orbifolds are generalisations of surfaces that account for symmetries.
- Orbifolds with quotient space of finite area are determined, up to homeomorphism, by the isomorphism class of their symmetry group.

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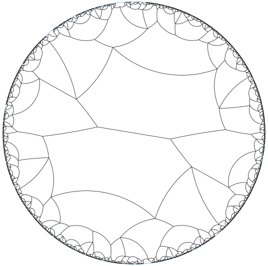
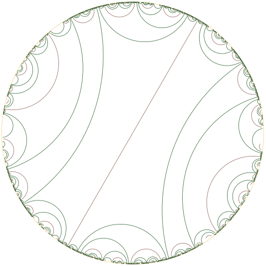


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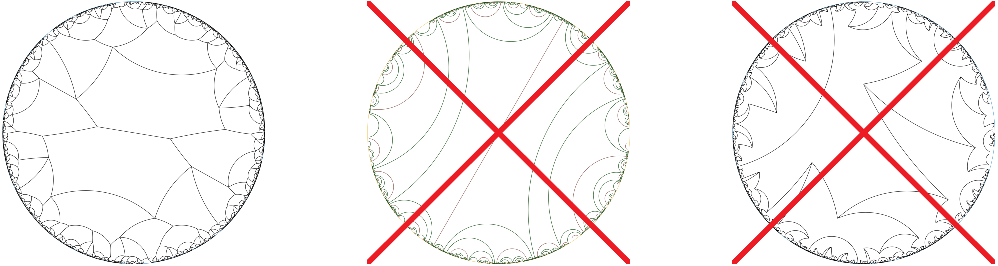

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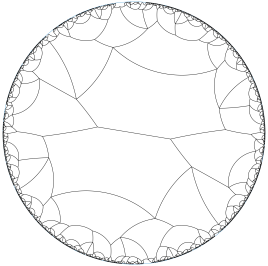

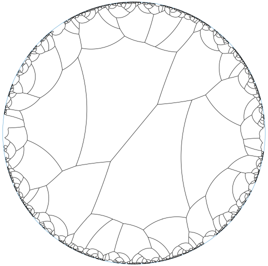

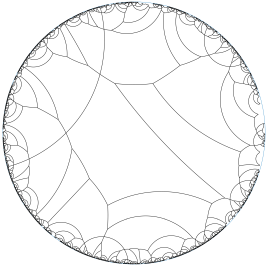

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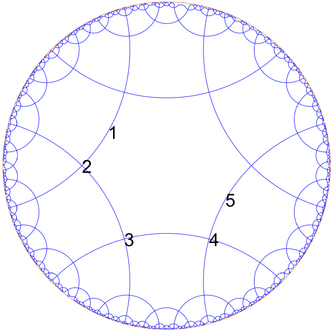

Figure: Tilings with symmetry group 22222 and same combinatorial structure.

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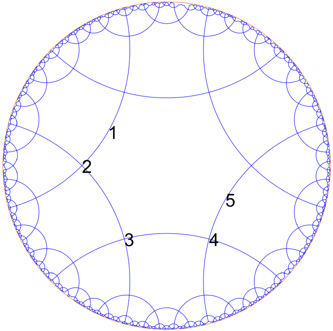

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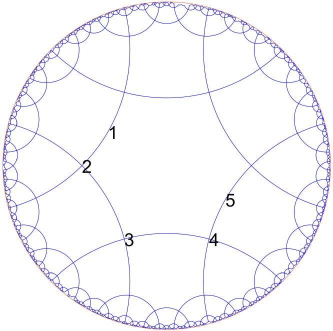

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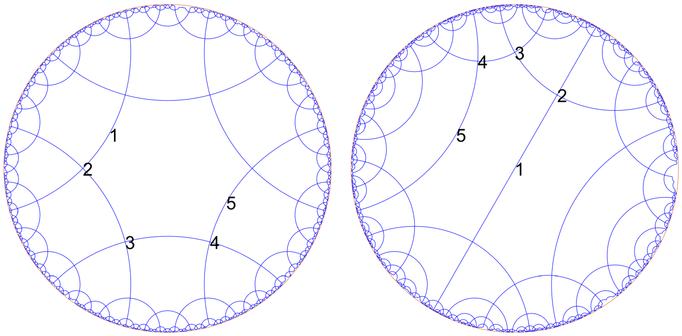

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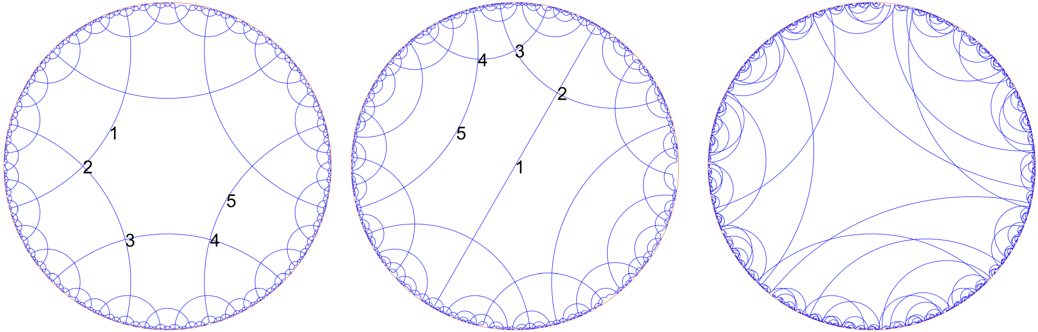

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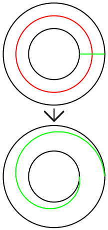

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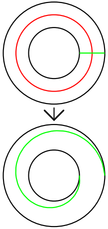

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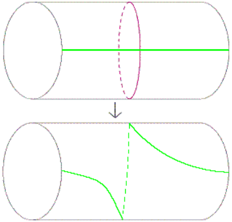

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Central theoretical result for enumerations

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- This means that the MCG $\mathrm{Mod}^{\pm}(\mathcal{O})$ corresponds to special sets of generators of Γ.

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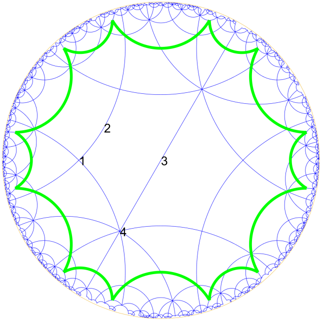

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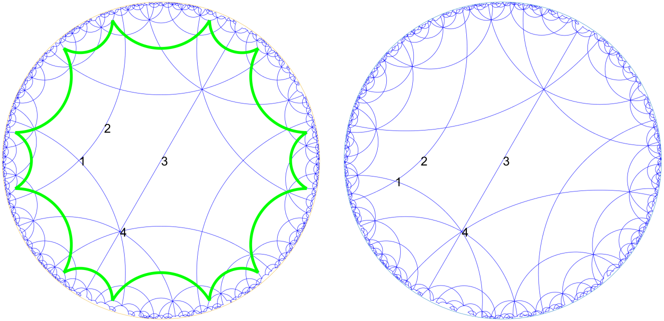

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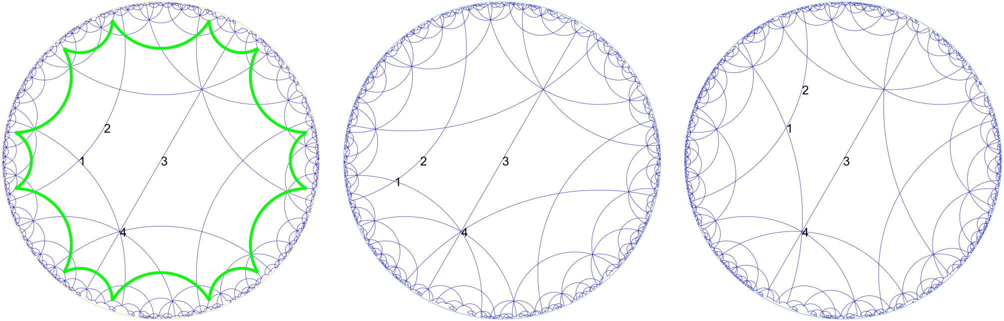

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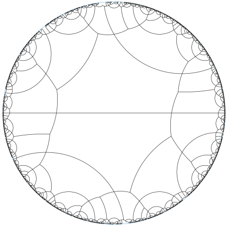
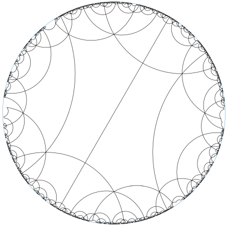
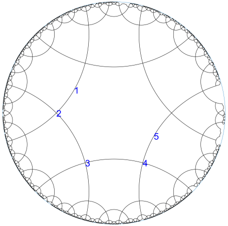
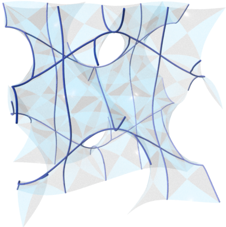
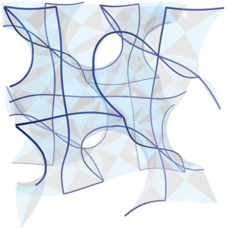
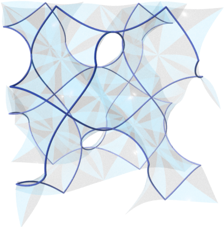
Different isotopy classes of tilings and resulting structures in $\mathbb{R}^3$

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Thanks to my collaborator
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