On the edge-length ratio of 2-trees

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Graph drawings

We want straight-line planar drawings

$$\text{vertices: } \{a, b, c, d, e, f\}$$
$$\text{edges: } \big\{\{a,b\}, \{b,c\}, \dots \big\}$$
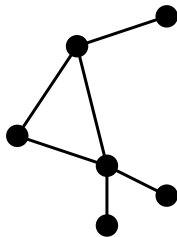
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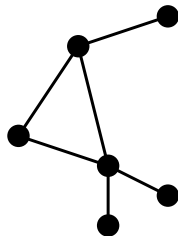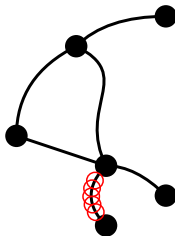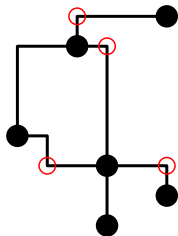
straight-line drawing = edges are line segments

Graph drawings

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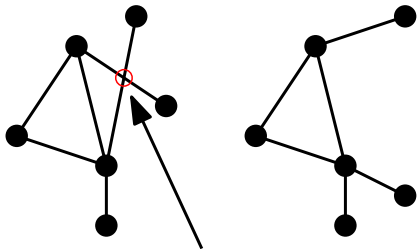
Planar drawing = crossings are forbidden

Graph drawings

Position changes matter to us because we care about edge lengths.
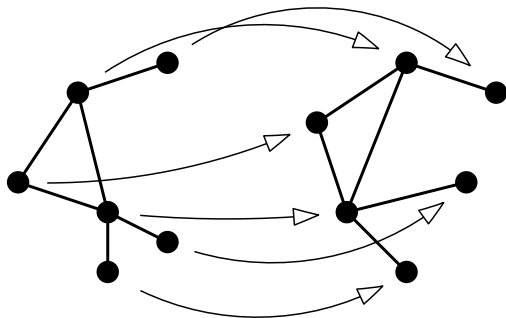
Graph drawings

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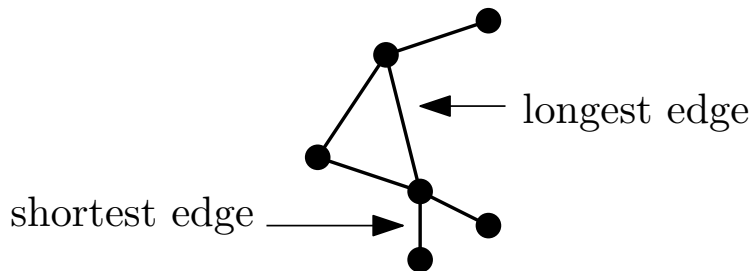
$$\text{edge-length ratio} = \frac{|\text{longest edge}|}{|\text{shortest edge}|}$$

Edge-length – Previous work

- If given edge-lengths, then NP-complete [Eades, Wormald]

NP-complete
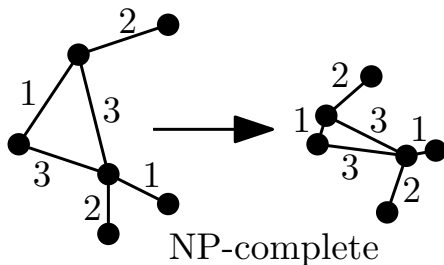
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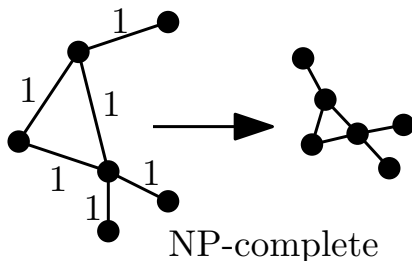
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- if degree-4 trees on integer grid with all edge-lengths equal, then NP-complete [Bhatt and Cosmadakis]
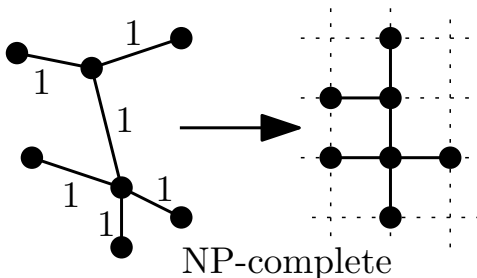
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Edge-length – Previous work

- Hoffmann, Van Kreveld, Kusters, Rote proposed relaxation: edge-length ratio

longest edge

shortest edge

Figure: edge-length ratio is between the longest and the shortest edge

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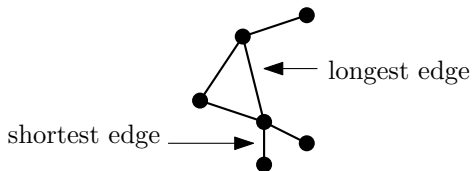- minimizing edge-length ratio is hard for general graphs [Chen, Jiang, Kanj, Xia, Zhang]

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- outerplanar graph have bounded edge-length ratio 2 [Lazard, Lenhart, Liotta]

Figure: edge-length ratio 2 for outerplanar graphs

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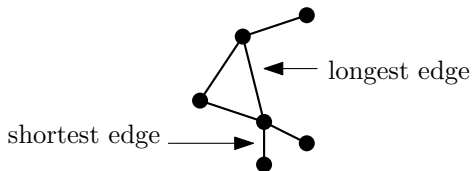- minimizing edge-length ratio is hard for general graphs [Chen, Jiang, Kanj, Xia, Zhang]
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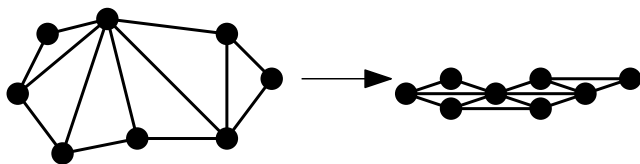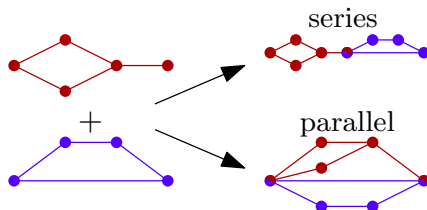

series

parallel

$+$

Can series-parallel graphs be drawn with constant edge-length ratio?

Edge-length ratio

Can series-parallel graphs be drawn with constant edge-length ratio? NO – they have unbounded edge-length ratio! Because subclass of series-parallel graphs called 2-trees have unbounded edge-length ratio. 2-trees definition:
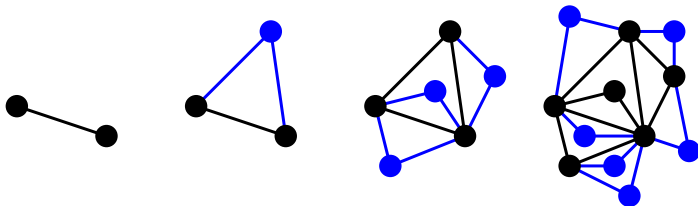

Figure: 2-trees are defined constructively

Edge is a 2-tree; adding a vertex connected to two neighboring vertices to a 2-tree is still a 2-tree.

Result 1: 2-trees have unbounded edge-length ratio

Outline of the proof:

- Start with a big 2-tree,
- consider its (fixed) drawing,
- shrinking area chain of triangles,
- shrinking perimeter chain,
- small perimeter
  $\implies$ short edges
  $\implies$ small ratio.

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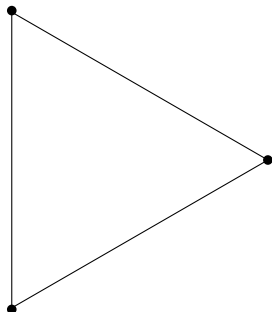# Result 1: 2-trees have unbounded edge-length ratio

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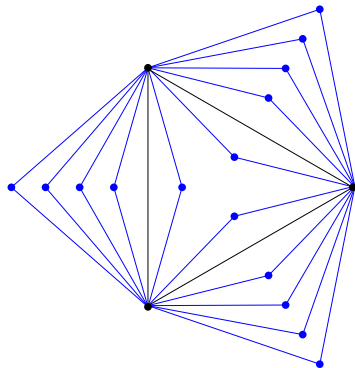
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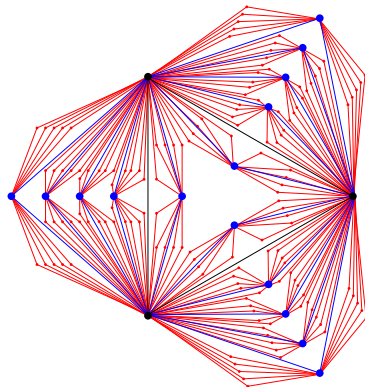# Result 1: 2-trees have unbounded edge-length ratio

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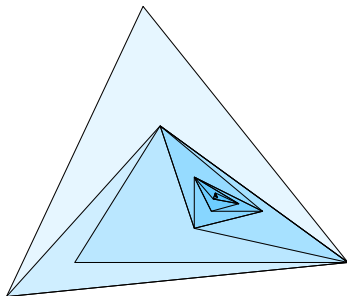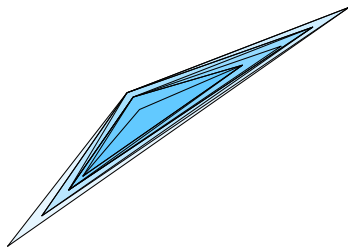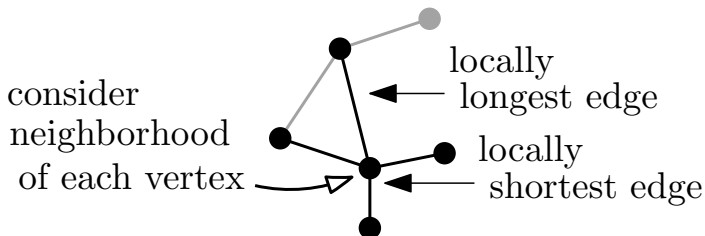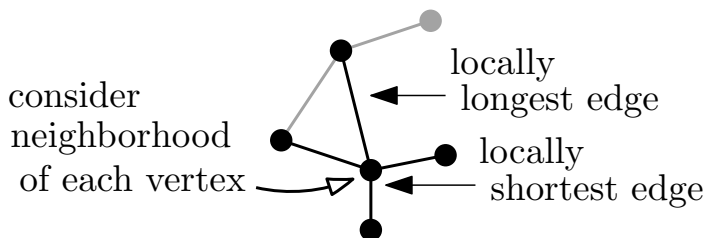# Result 1: 2-trees have unbounded edge-length ratio

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Local edge-length ratio

consider
neighborhood
of each vertex

locally
longest edge

locally
shortest edge

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$$\text{local edge-length ratio} = \max \frac{|A|}{|B|}$$

where edges A and B are incident.

Result 2: 2-trees have bounded local edge-length ratio

Outline of the proof:

- Find graph layers (BFS),
- decompose it into parts,
- draw each part separately and guarantee its children can be drawn.

Figure: graph to be decomposed

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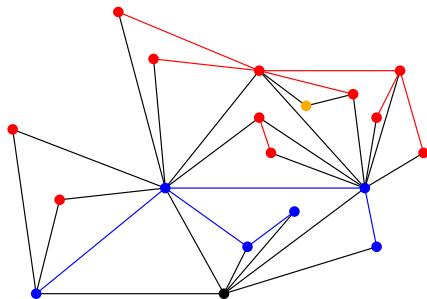

Figure: graph decomposition into parts

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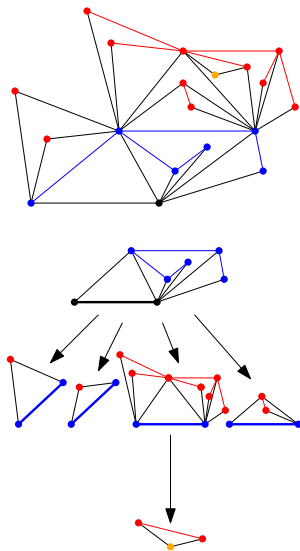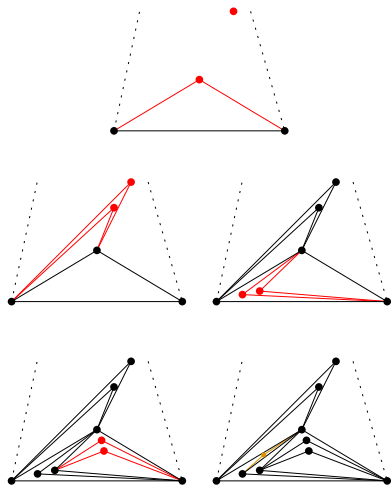

Figure: drawing a graph part

Results and open problems

1.R: 2-trees have unbounded edge-length ratio
ratio $= \Omega(\log(\text{graph size}))$.

- Close the gap between edge-length ratio lower (logarithmic) and upper bound (linear) of 2-trees.

2.R: Local edge-length ratio of 2-trees is upper bound by 4.

- Is 4 tight local edge-length ratio for 2-trees?
- Investigate interplay of edge-length ratio with other parameters, such as angular resolution, to make the graph drawings readable.

Thanks for watching!