36th European Workshop on Computational Geometry

Disjoint tree-compatible plane perfect matchings

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Compatibility of graphs

Setting: set S of $2n$ points in the plane in general position

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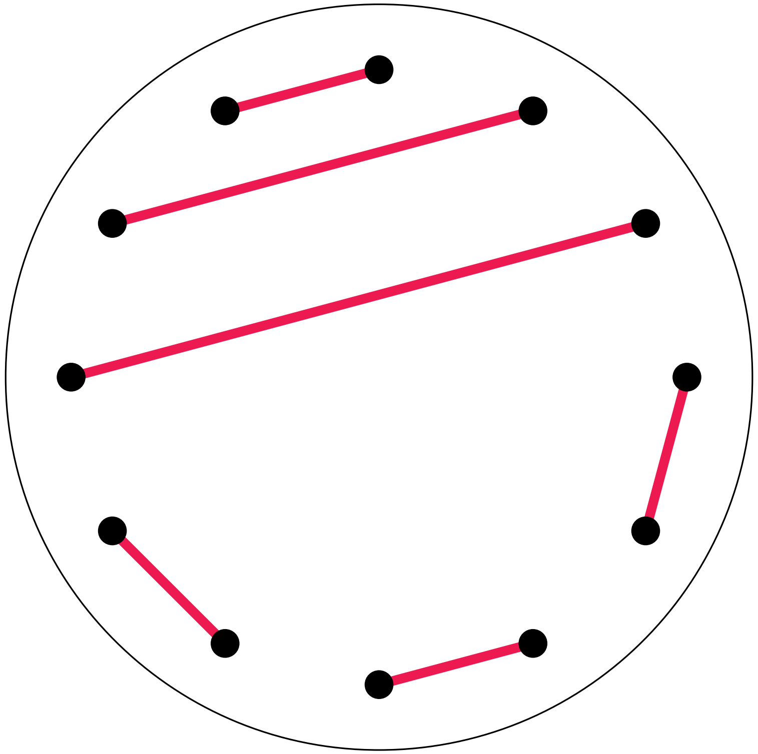

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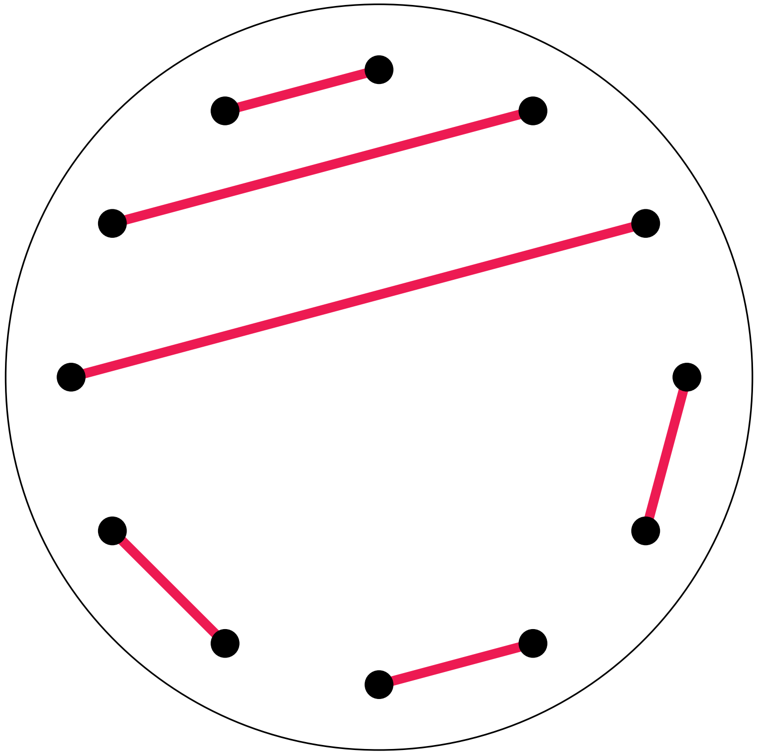

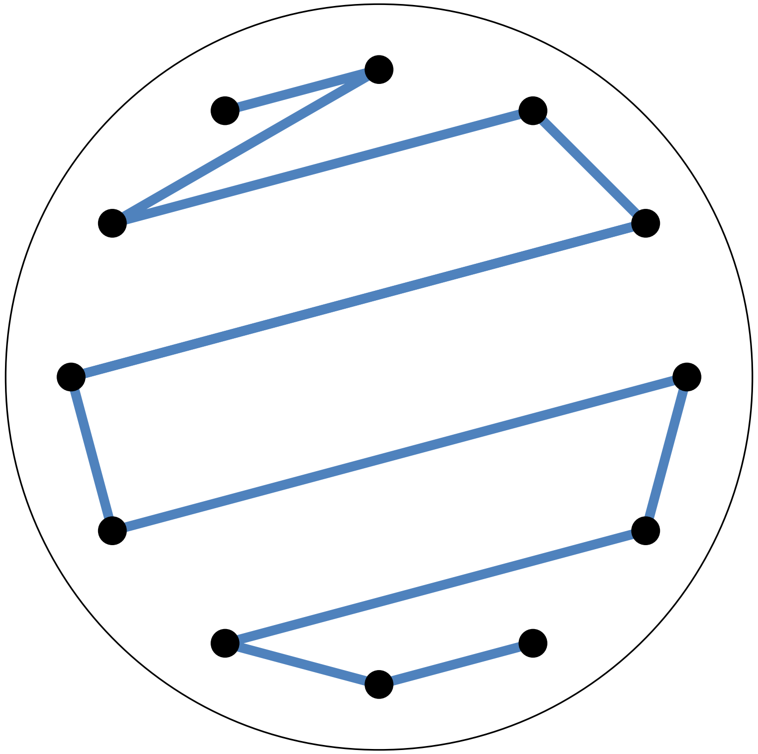

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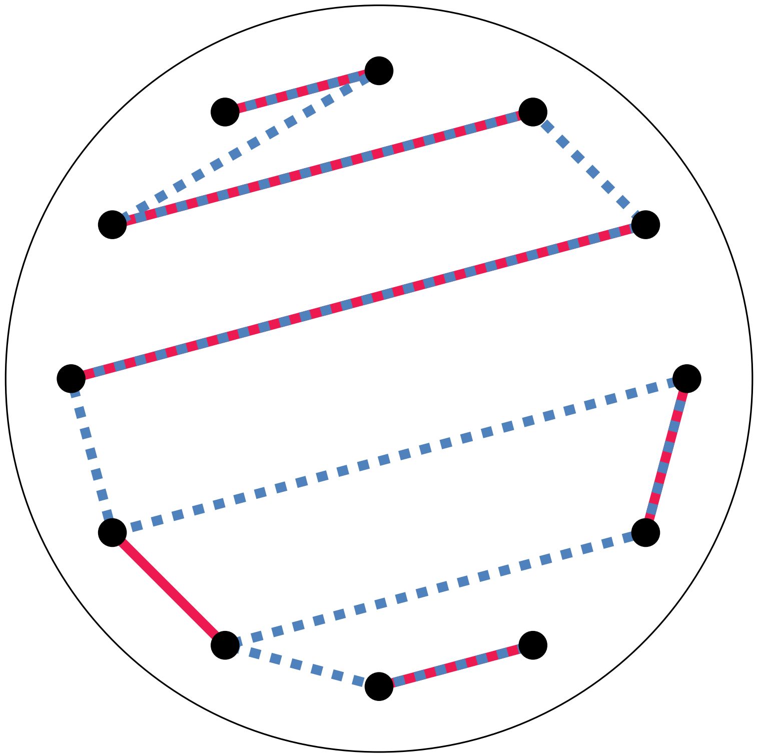

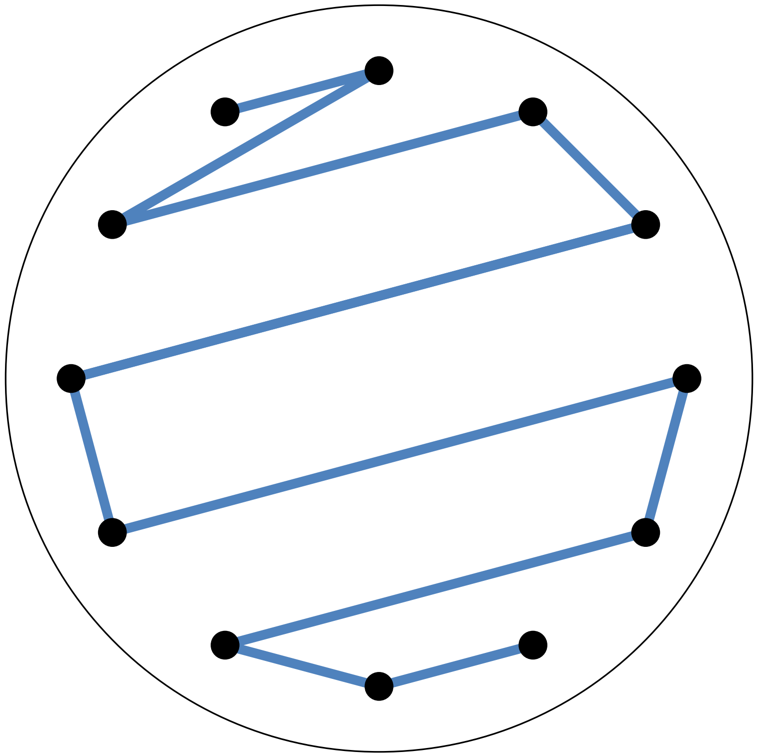

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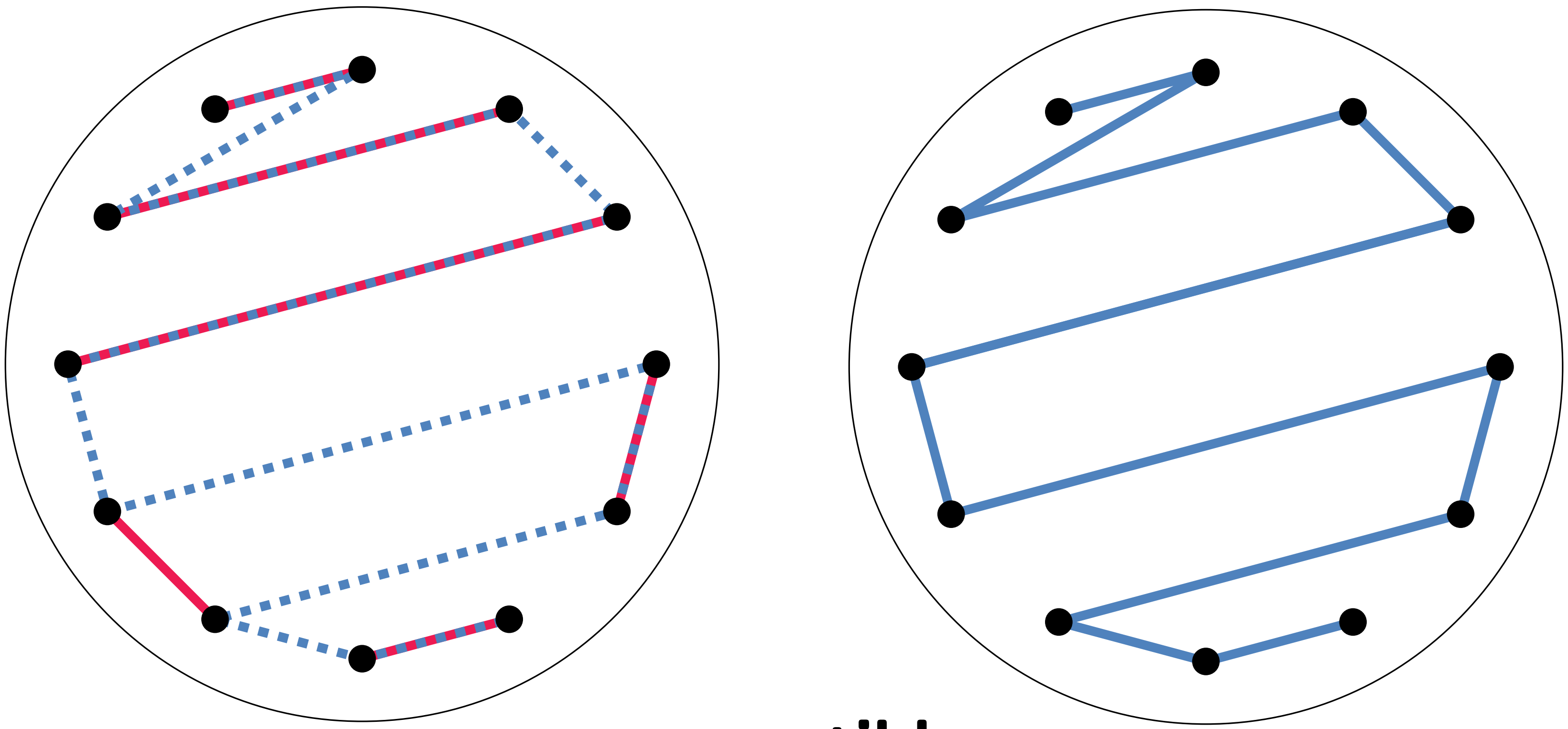

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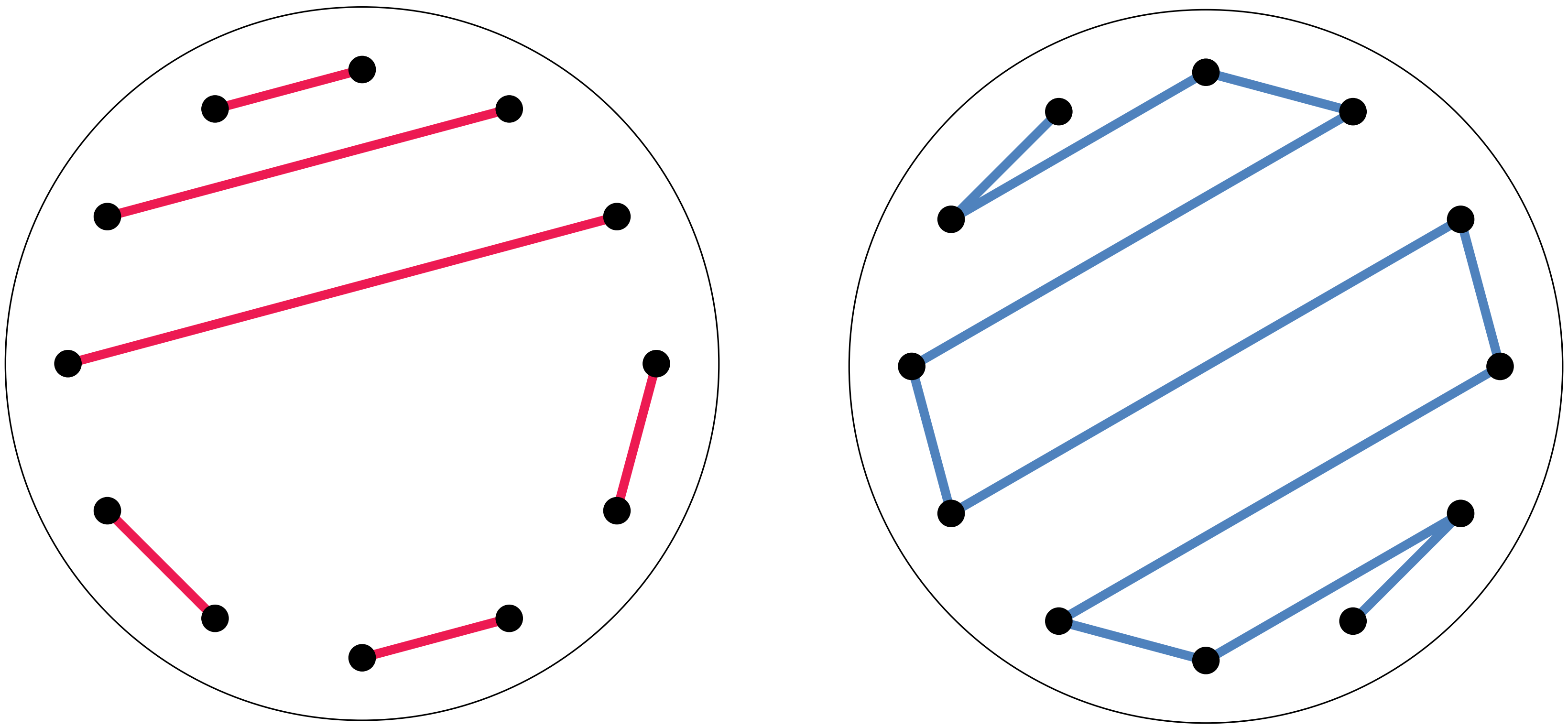

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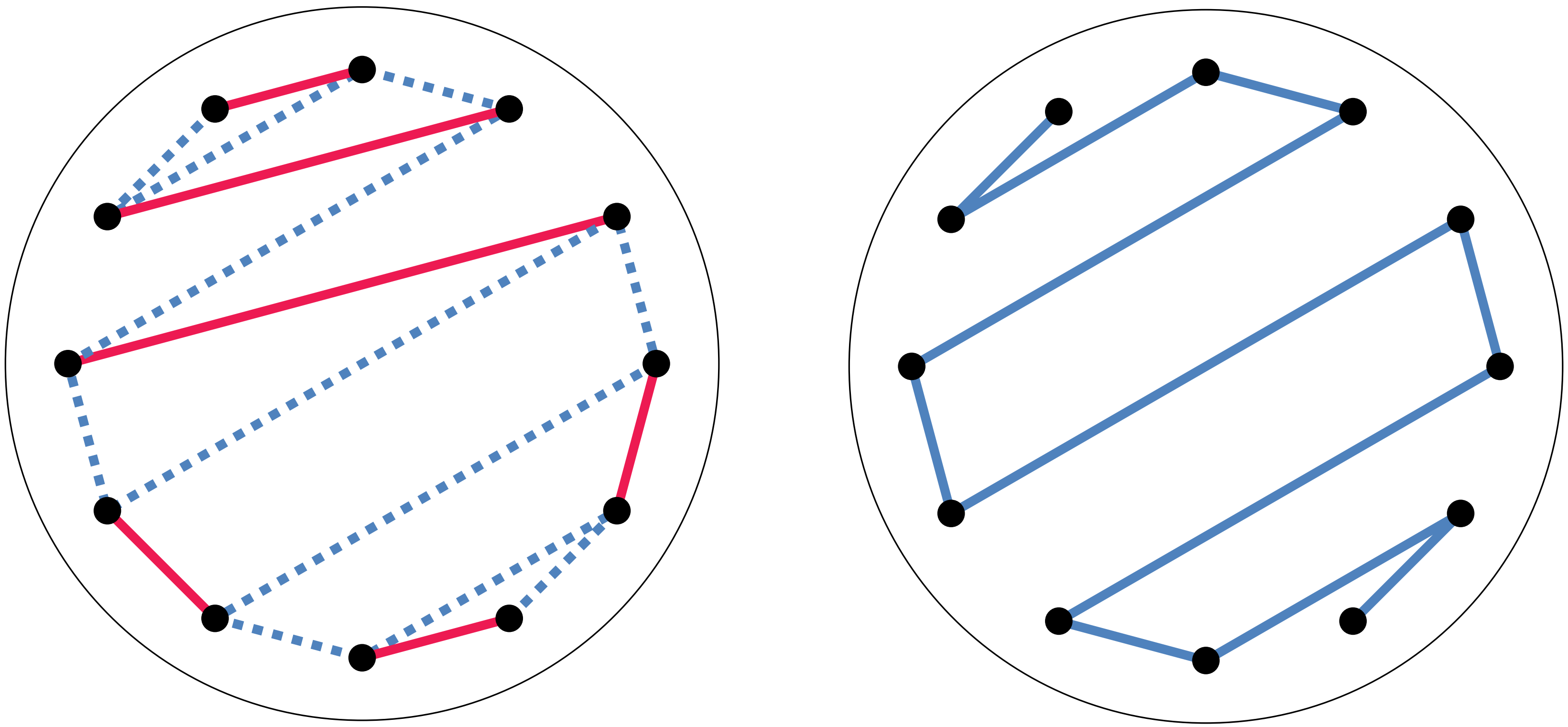

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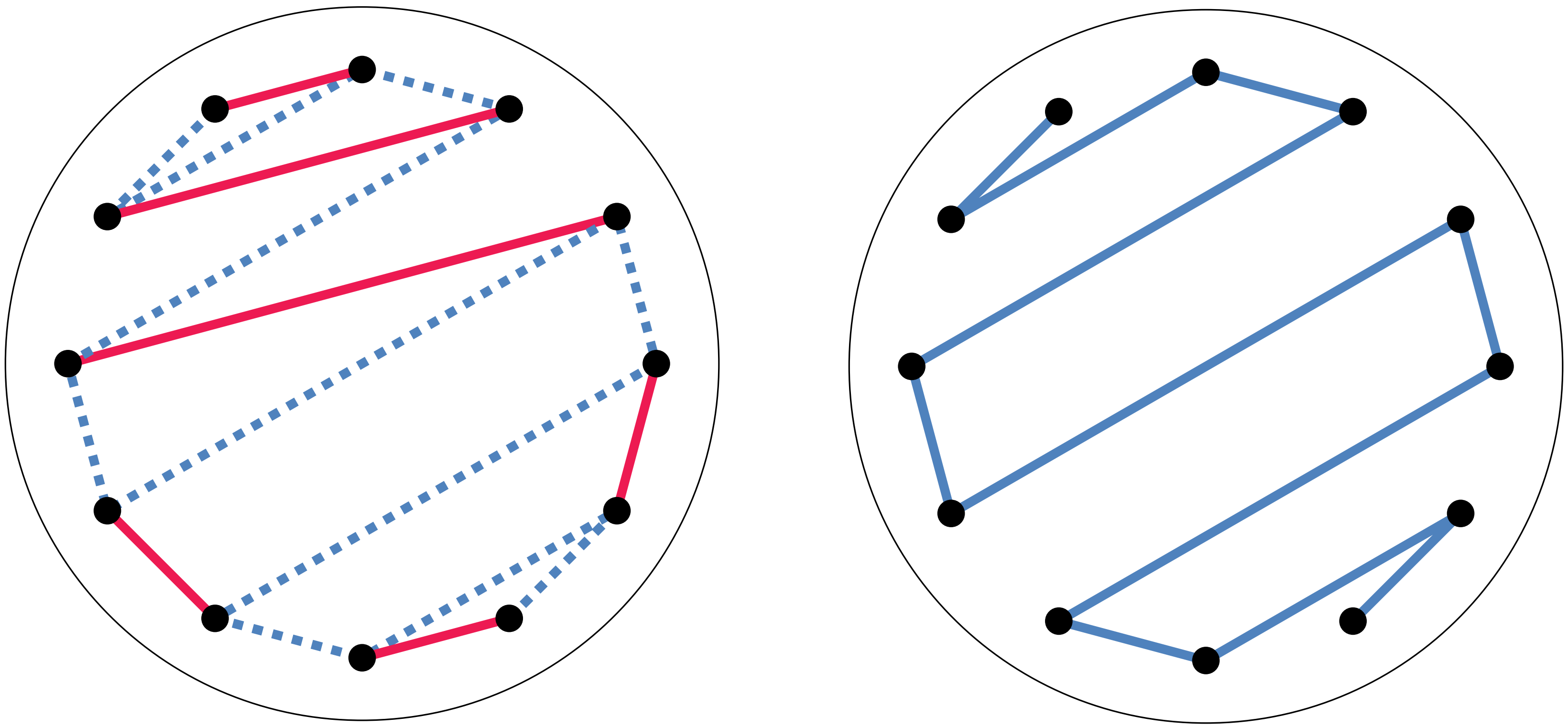

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- diameter is $O(\log n)$ [ABDGHHKMRSSUW; 2009.]
  and $\Omega(\log n / \log\log n)$ [A.Razen; 2008.]

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Alternative way of defining compatibility?

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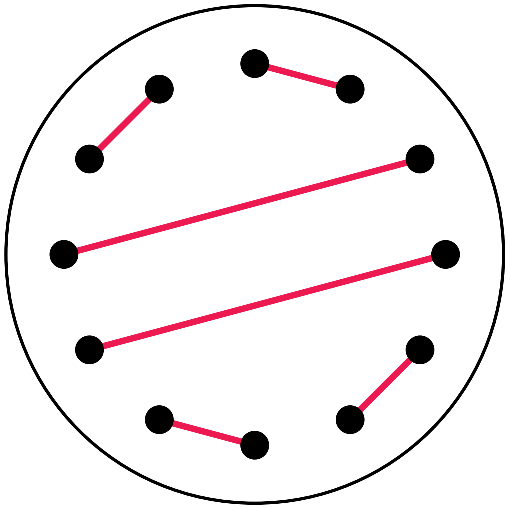

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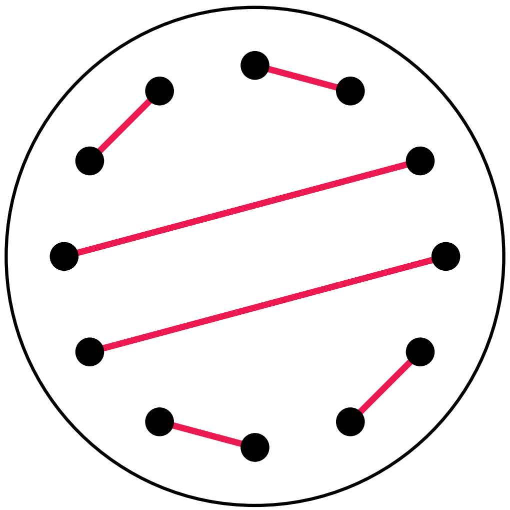

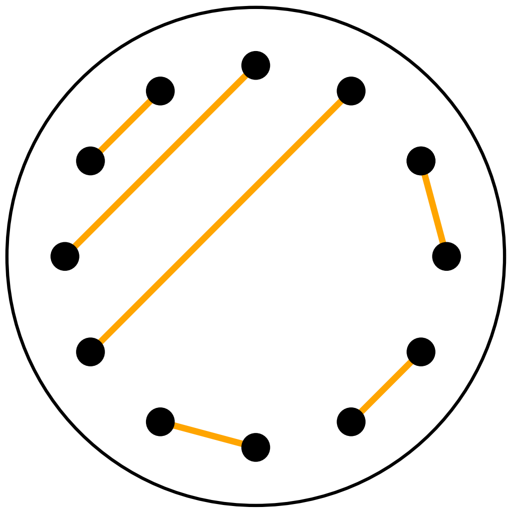

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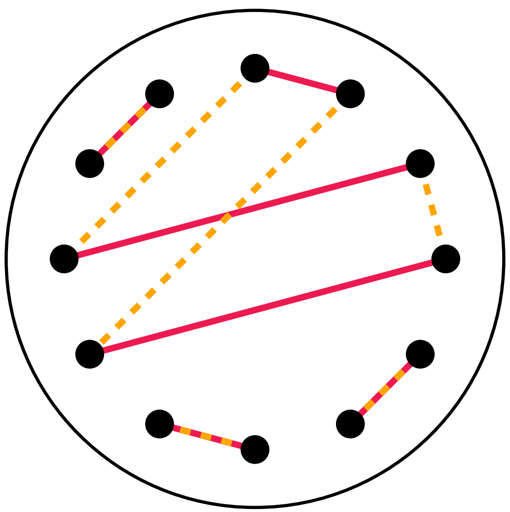
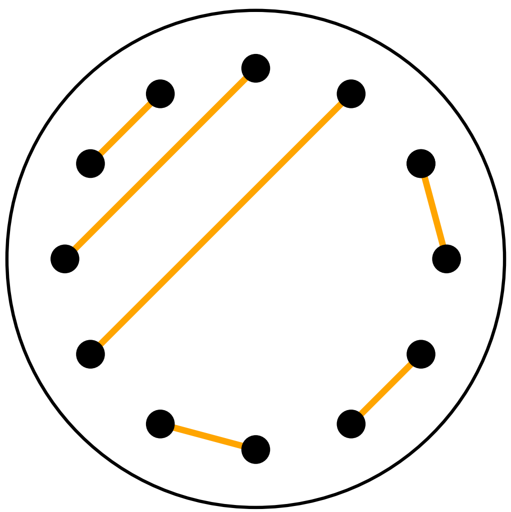


not compatible!



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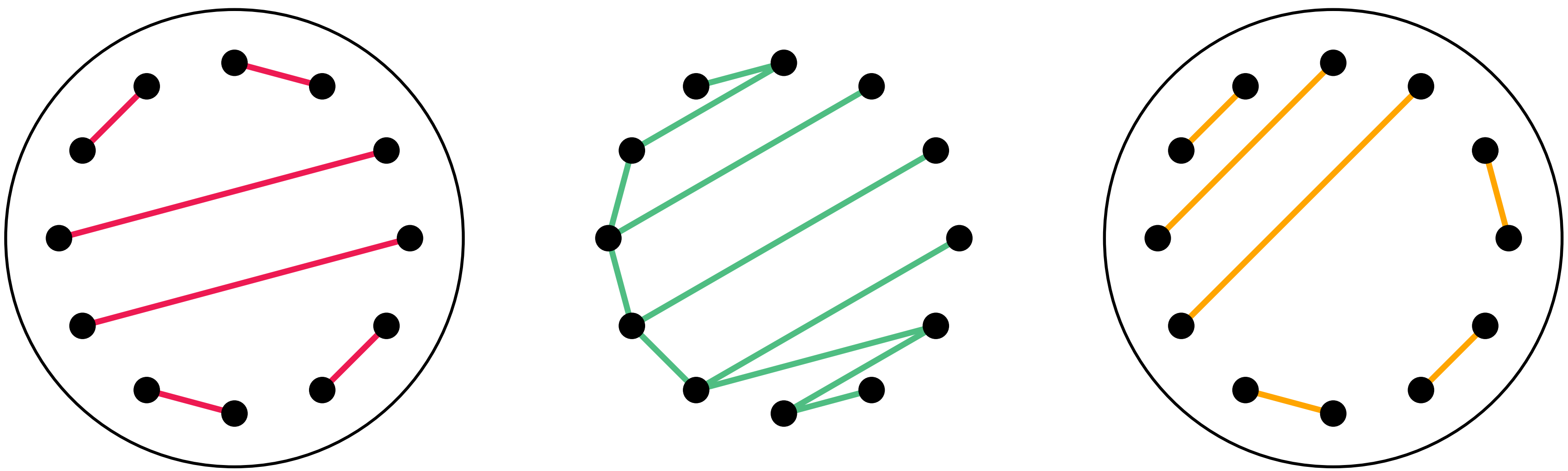

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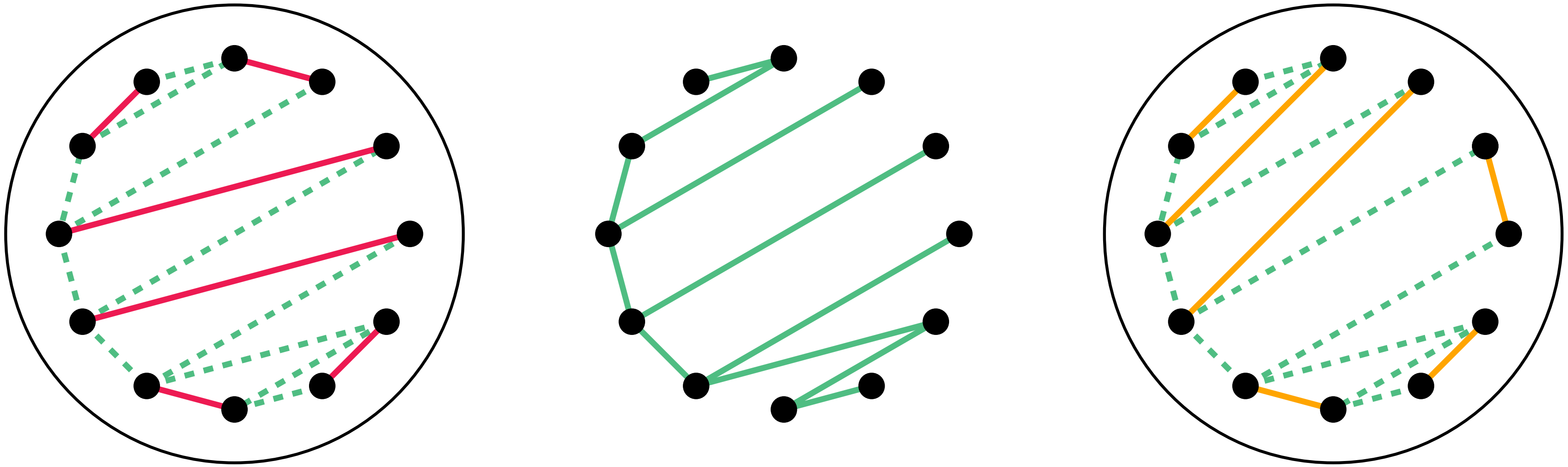

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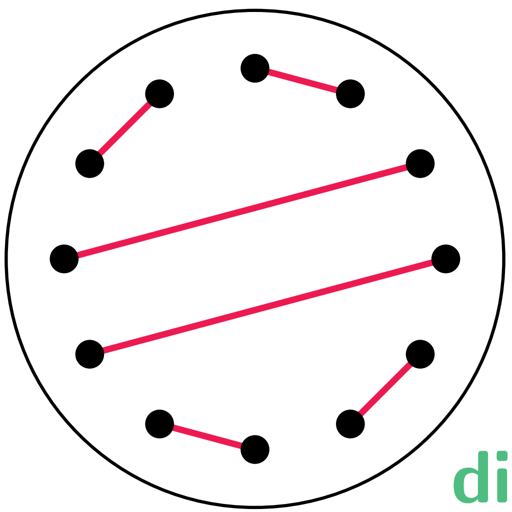

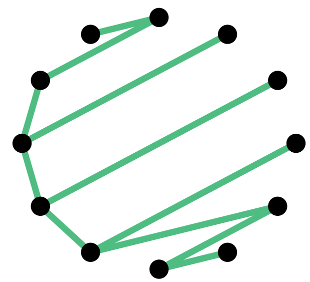

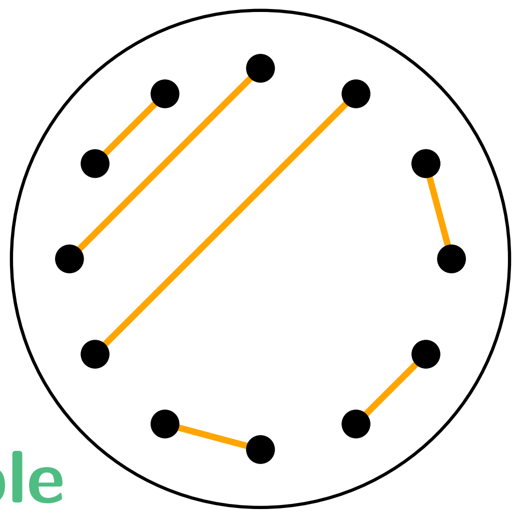

matchings are disjoint tree-compatible

(for short: tree-compatible)

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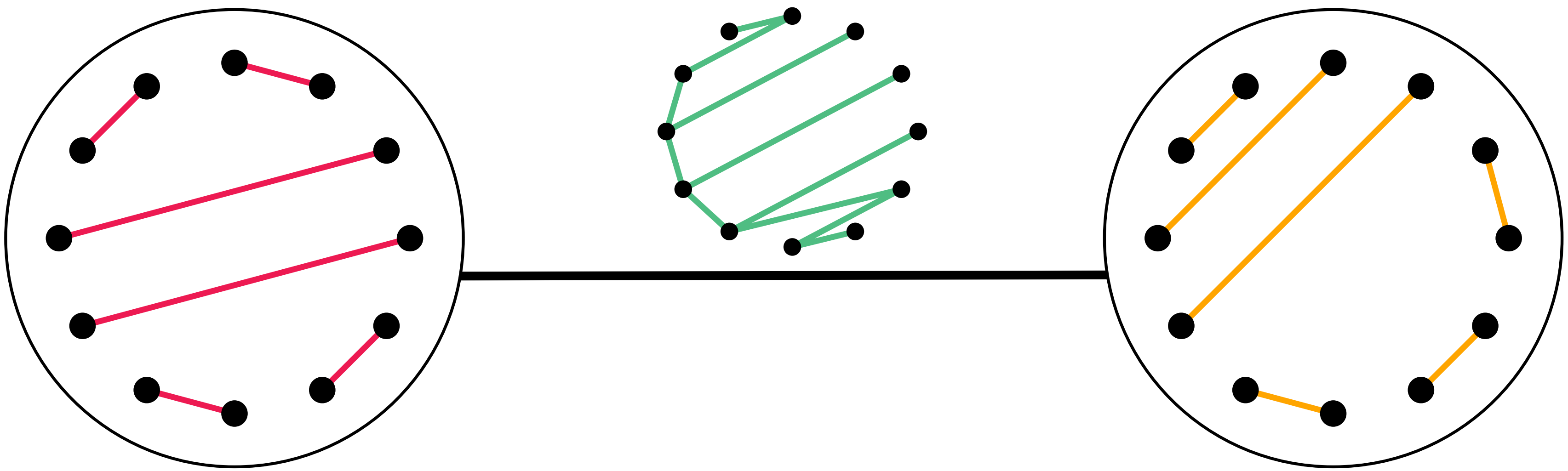

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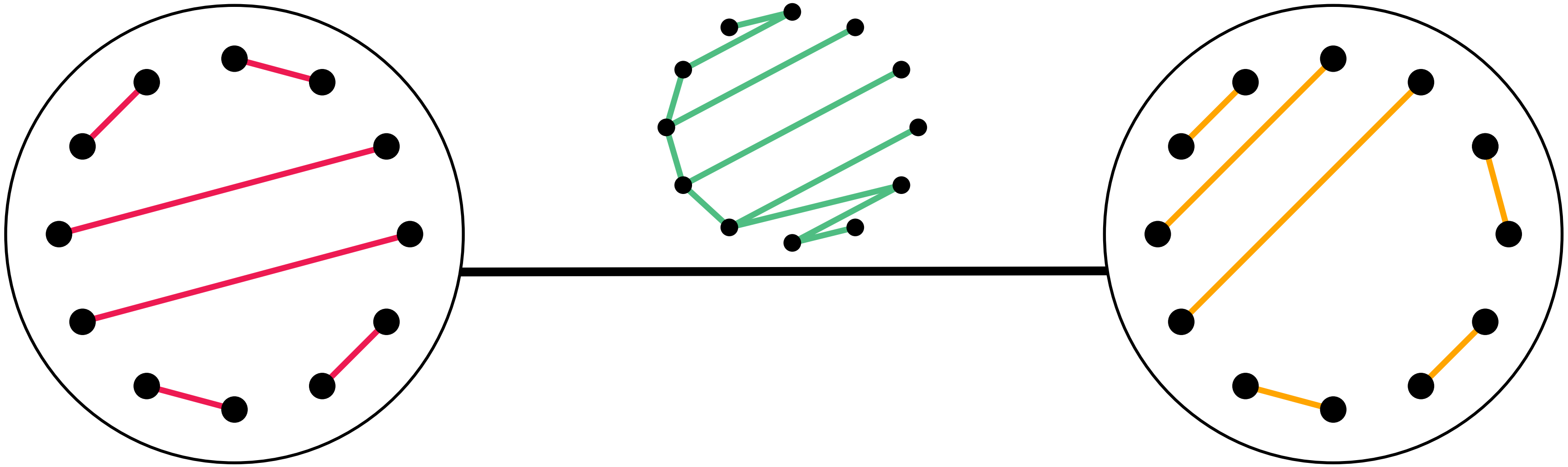

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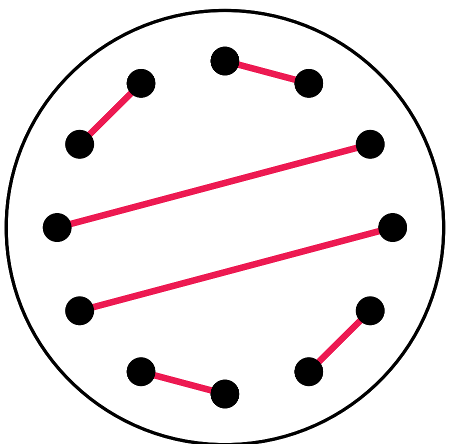

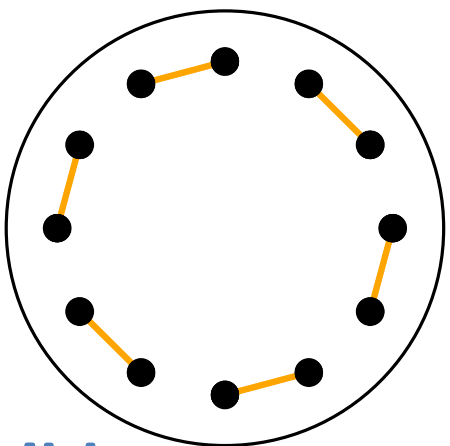

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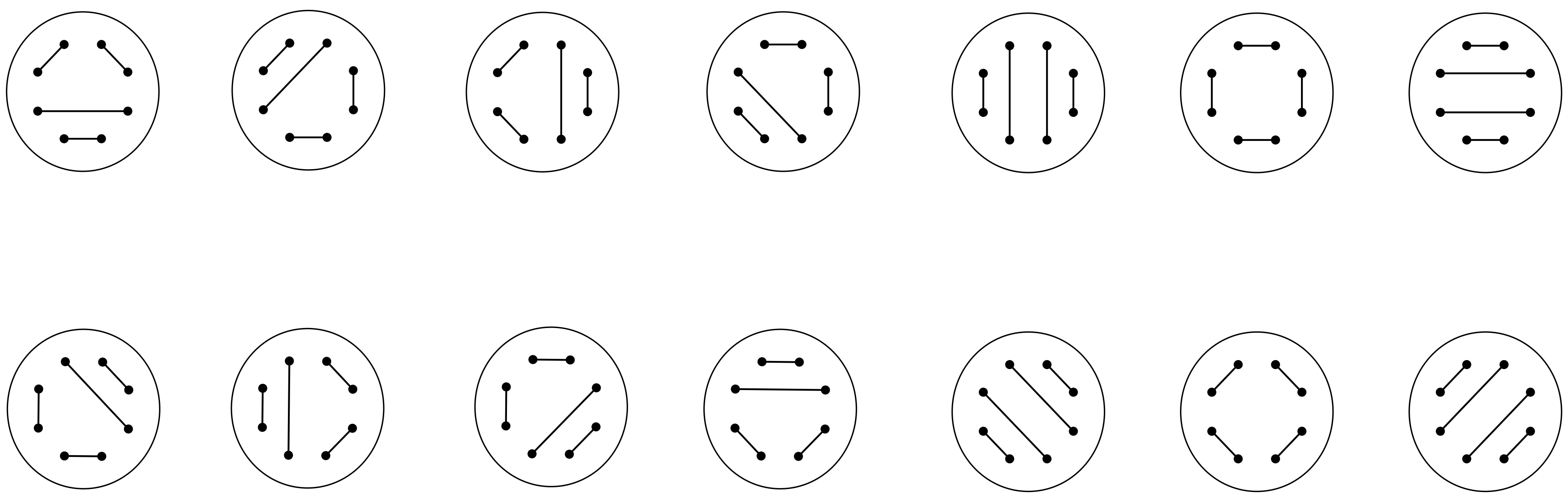

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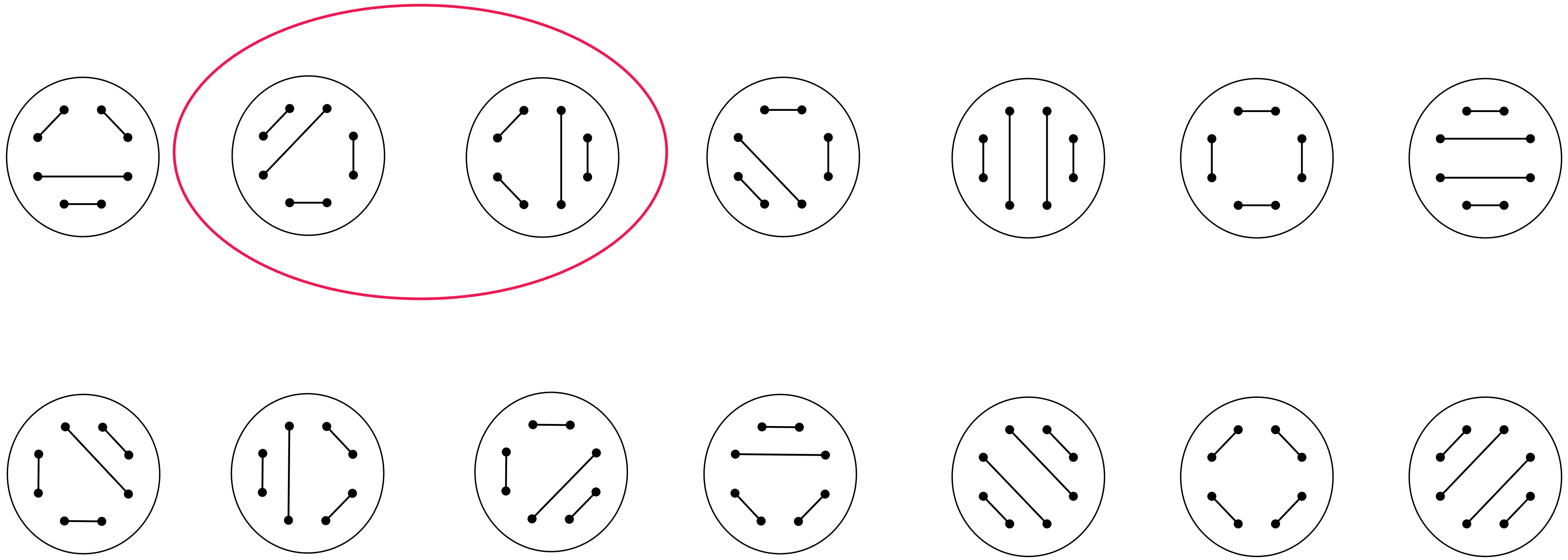

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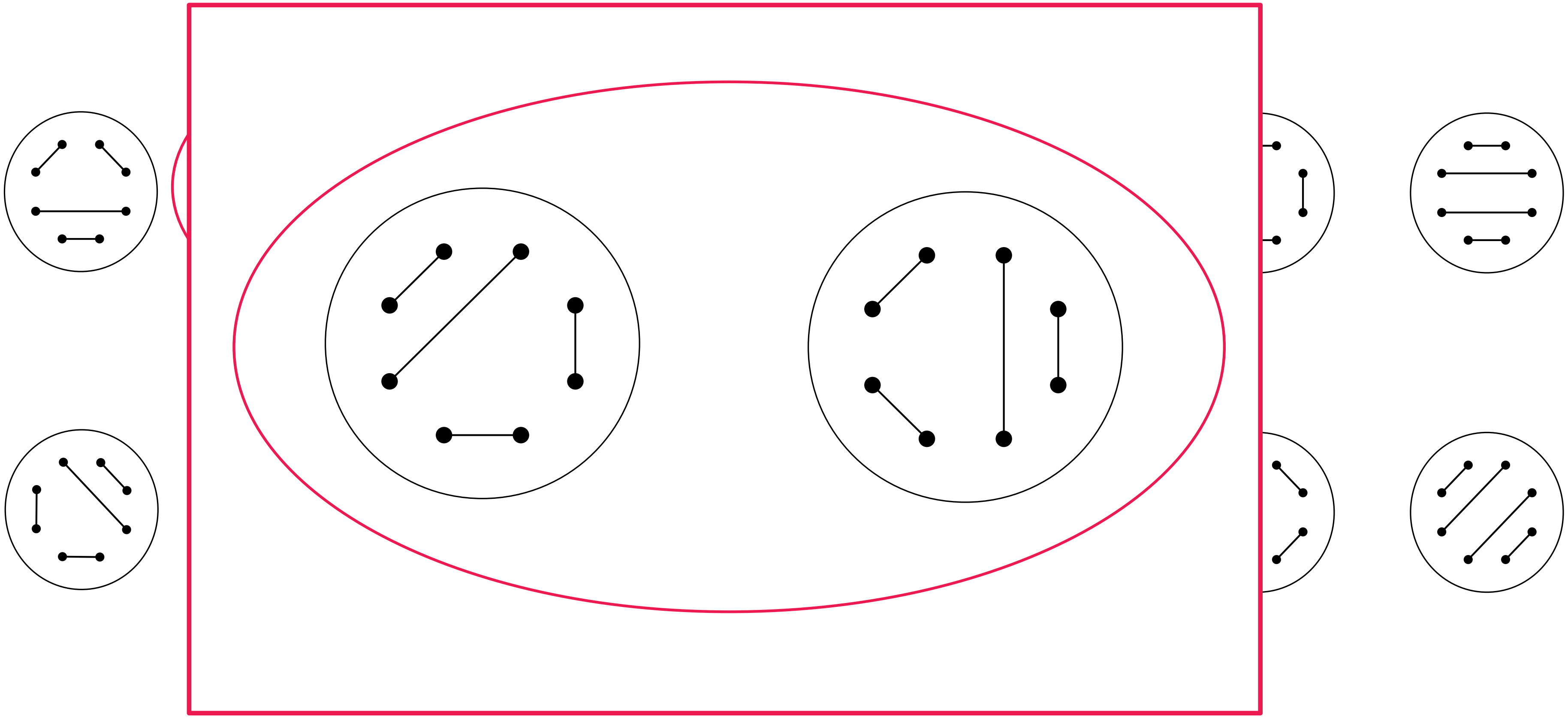

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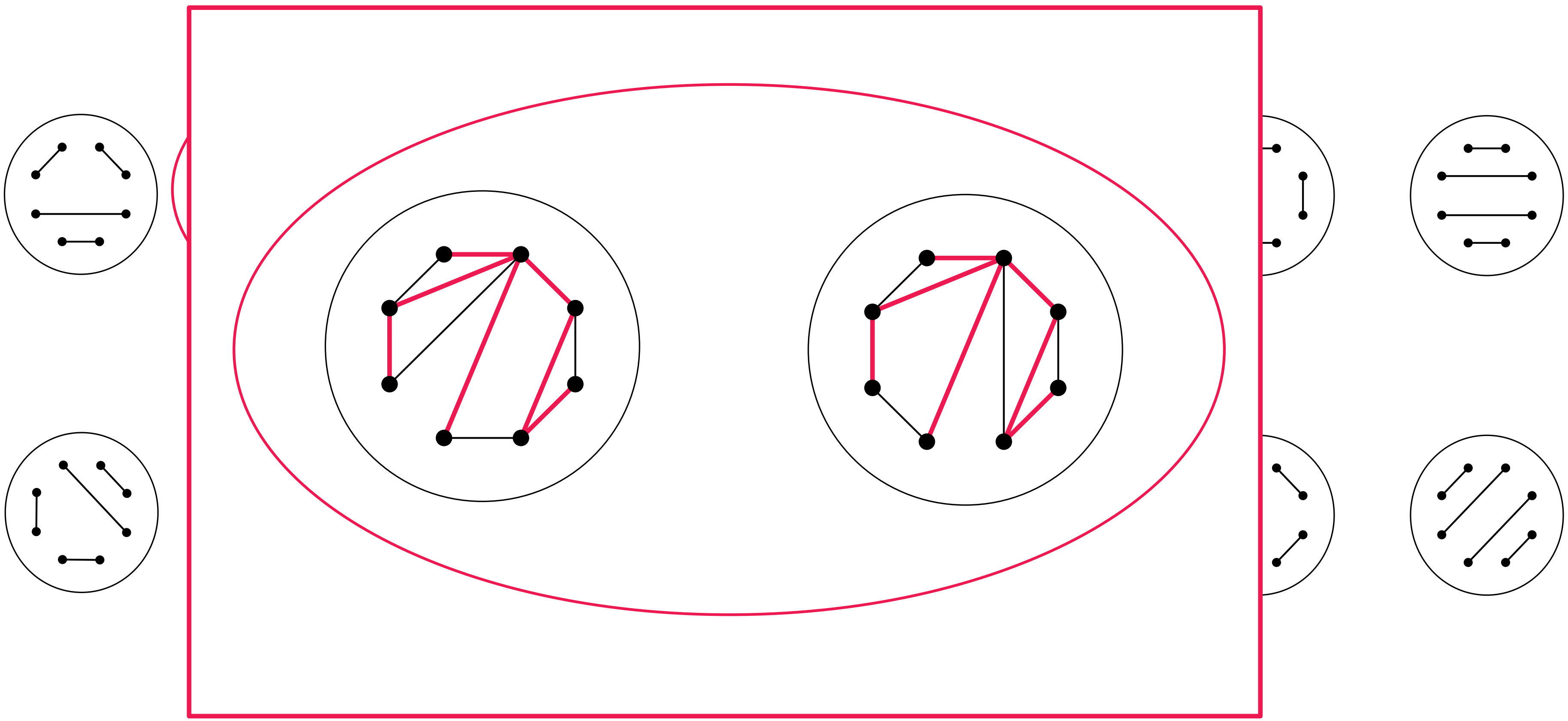

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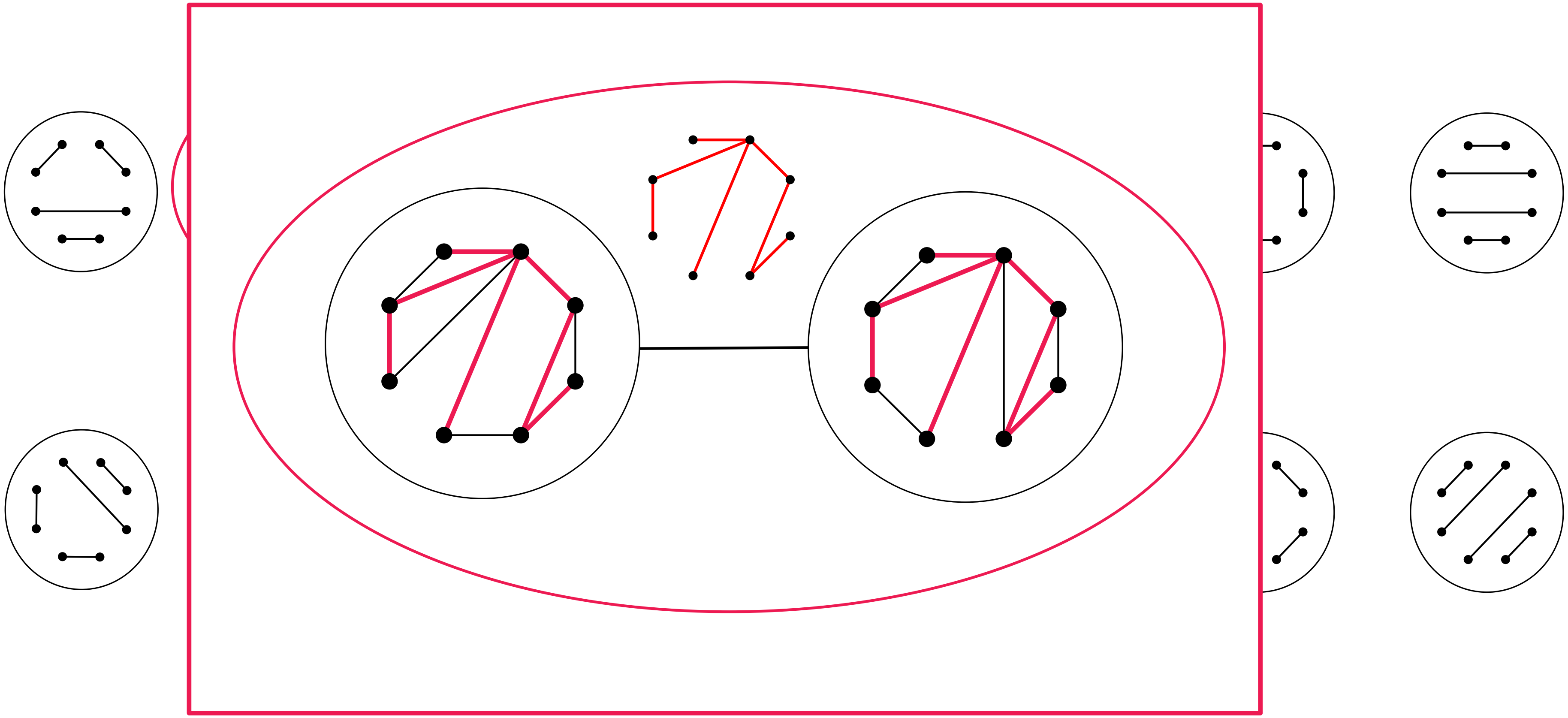

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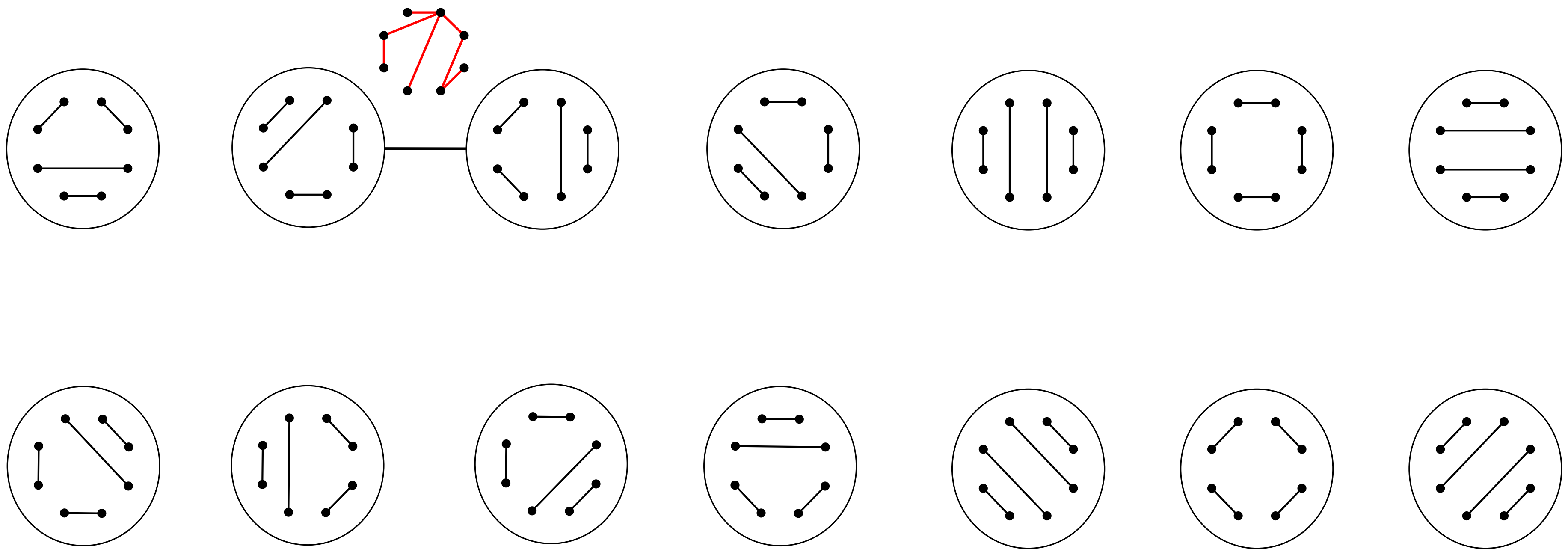

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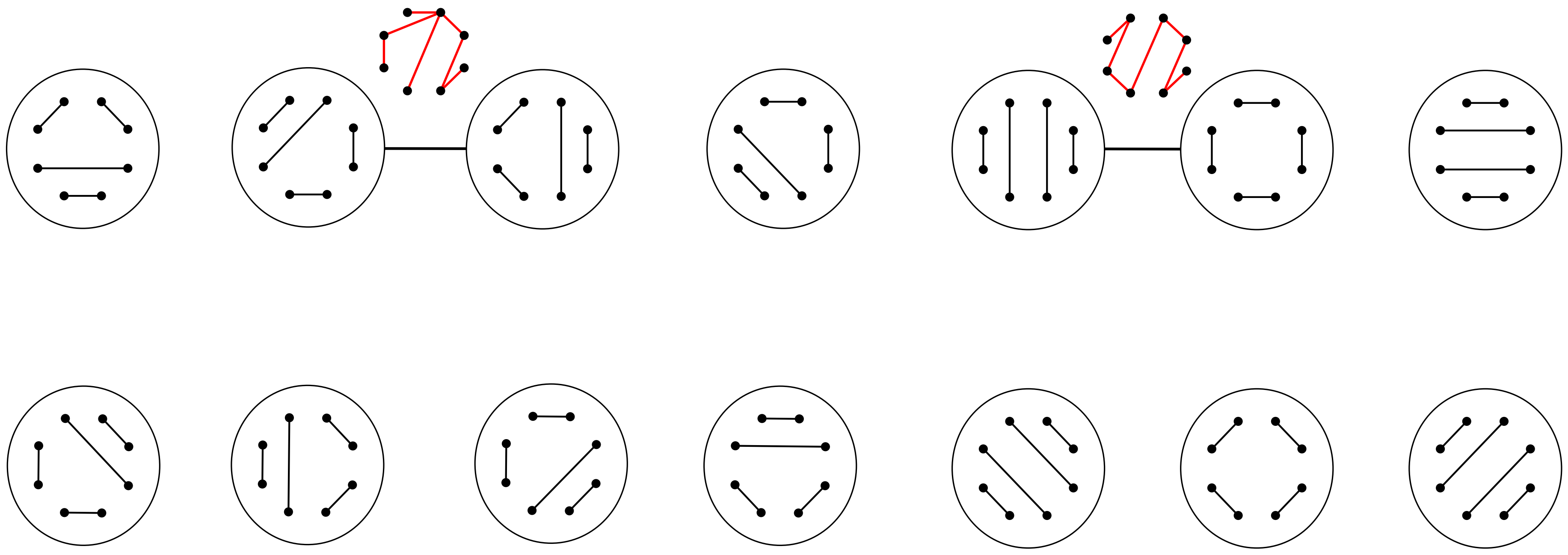

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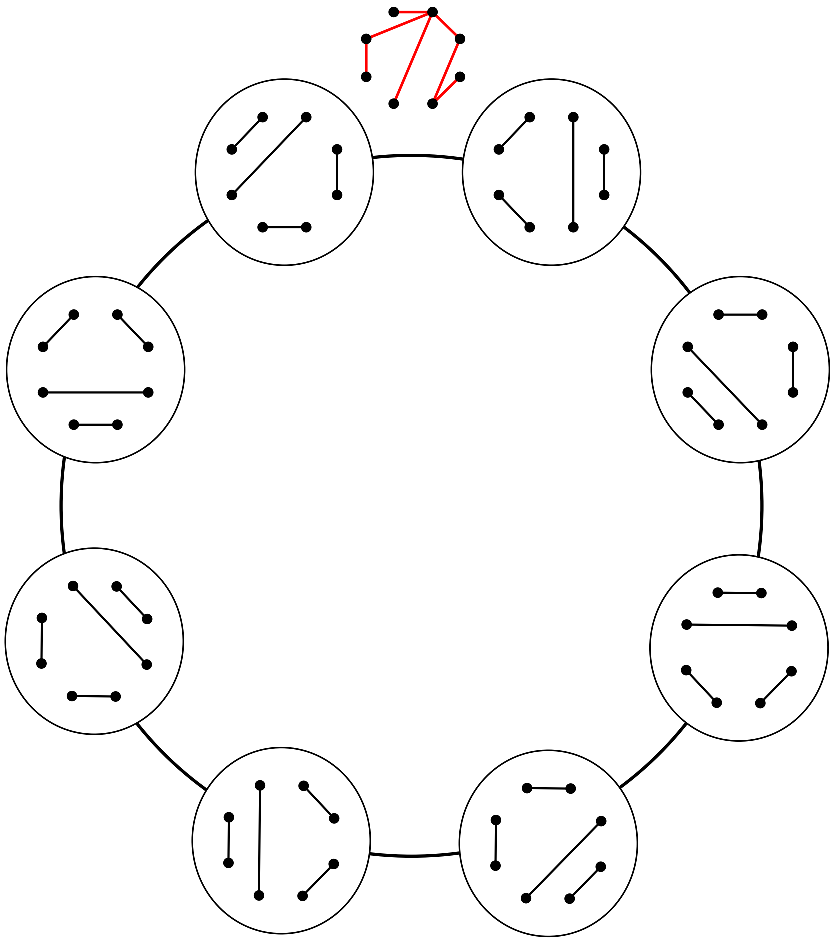

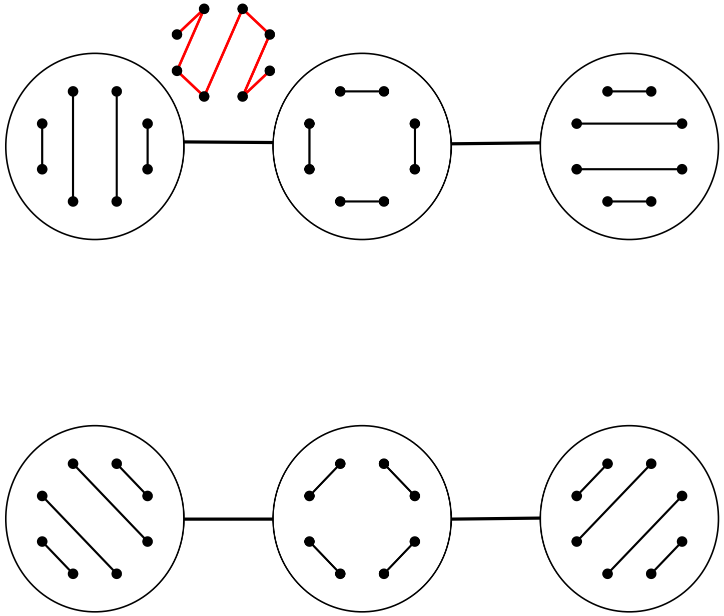

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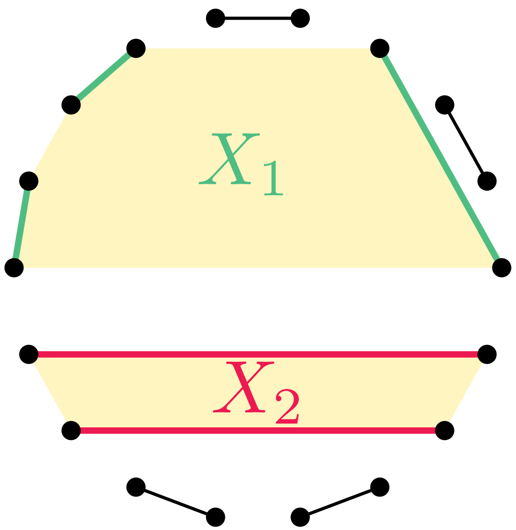

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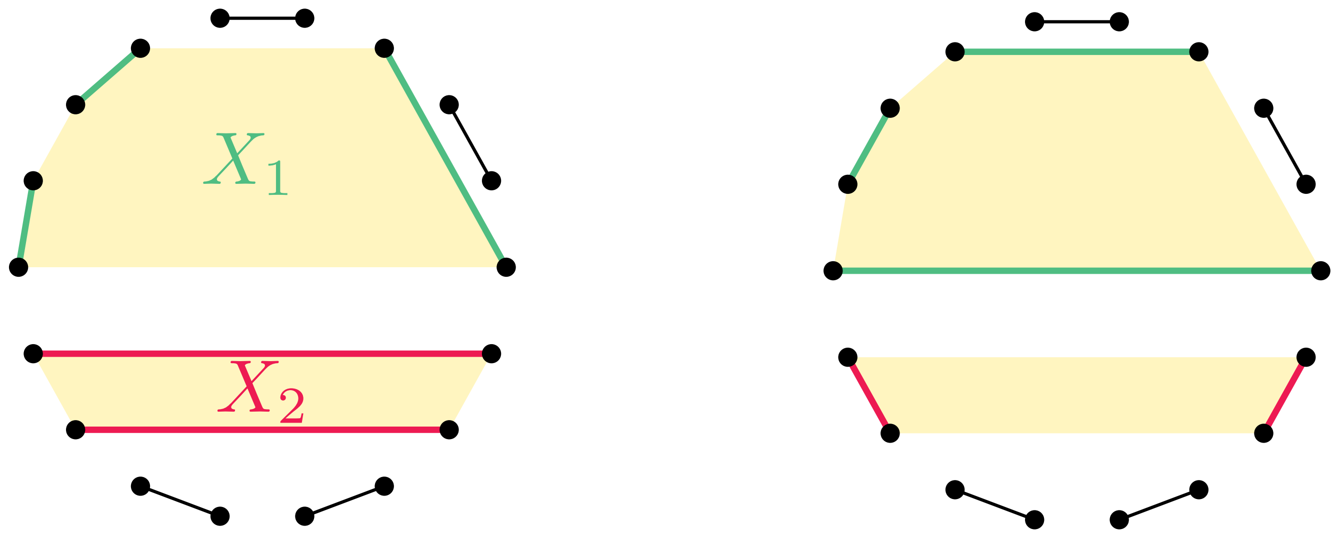

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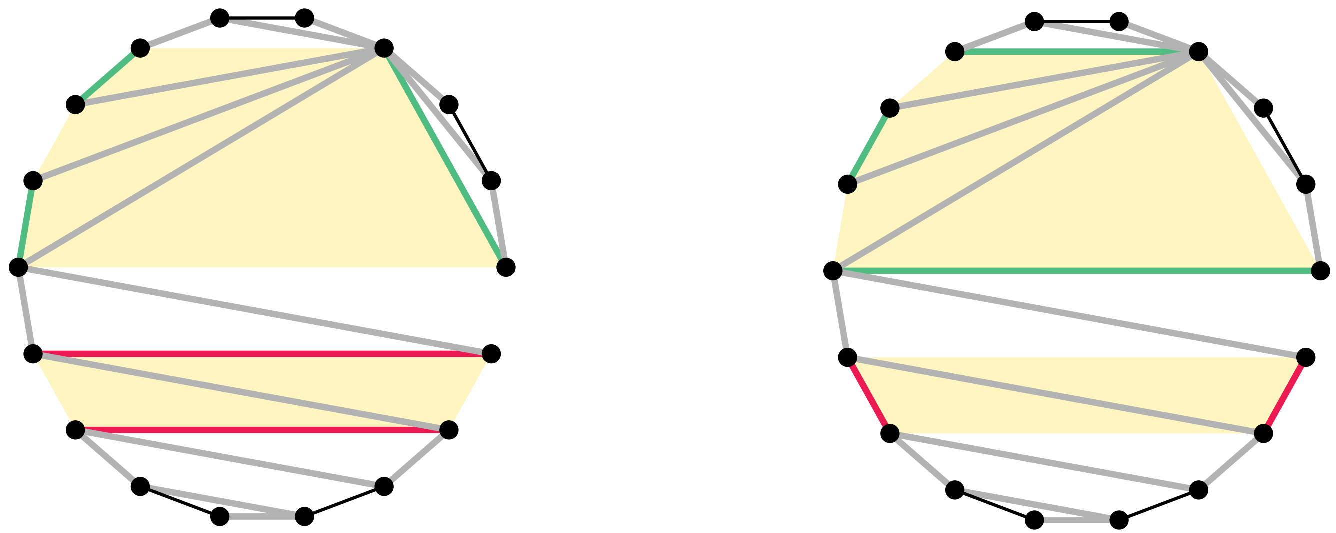

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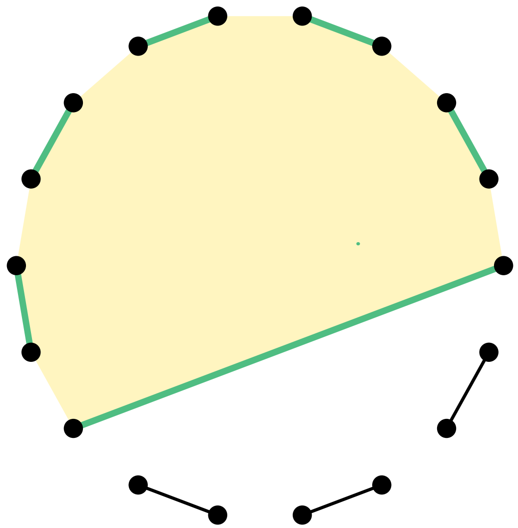

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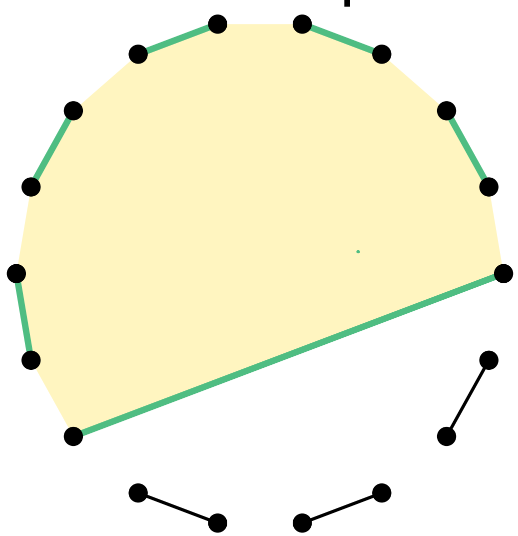

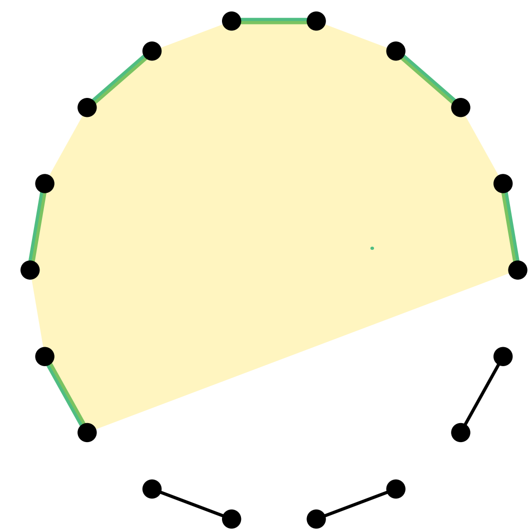

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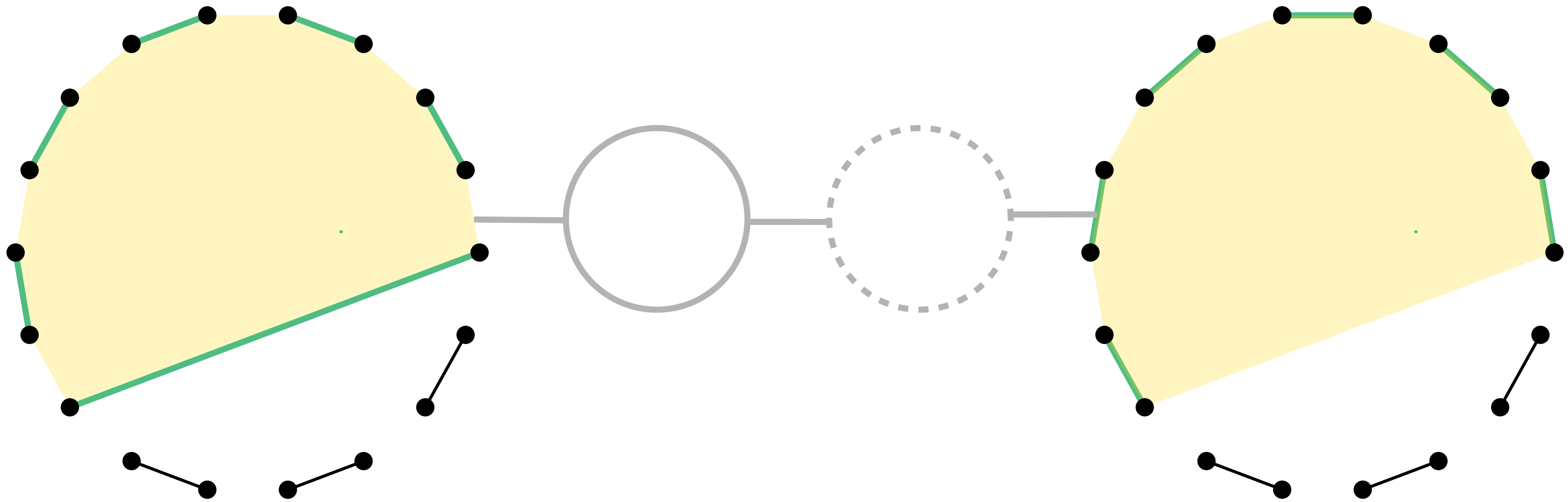

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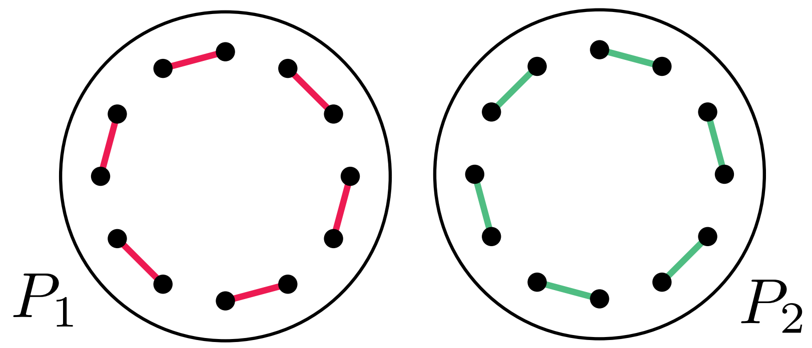

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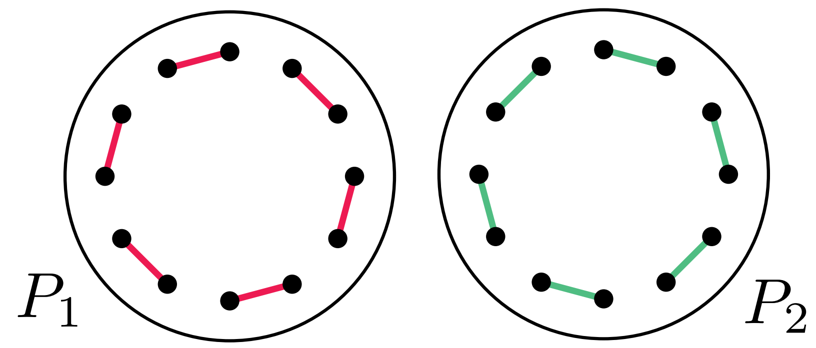

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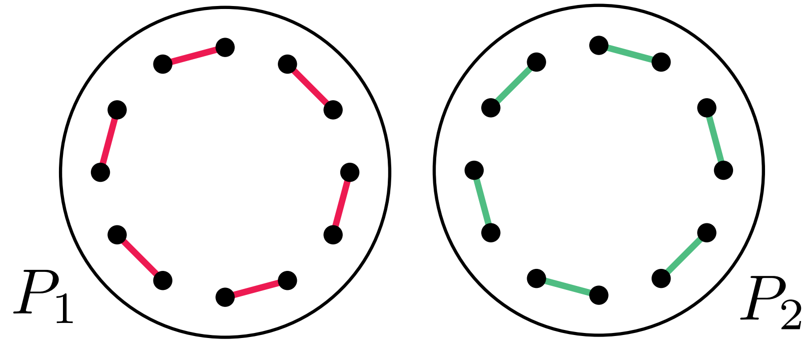

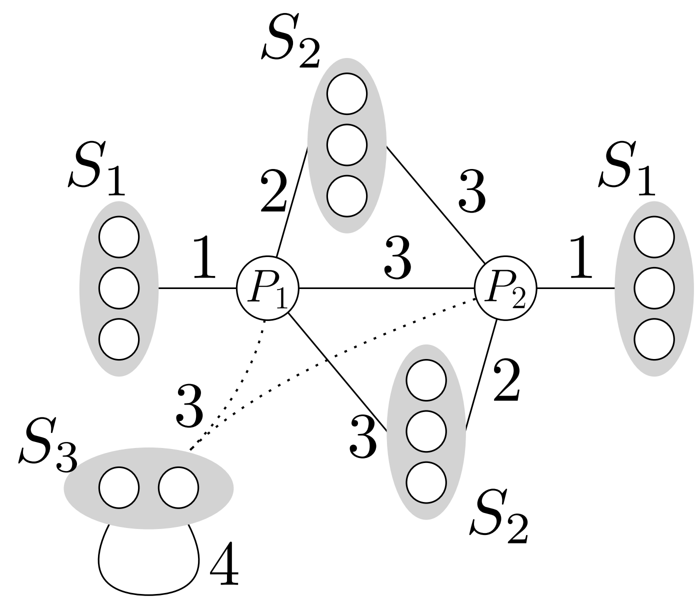

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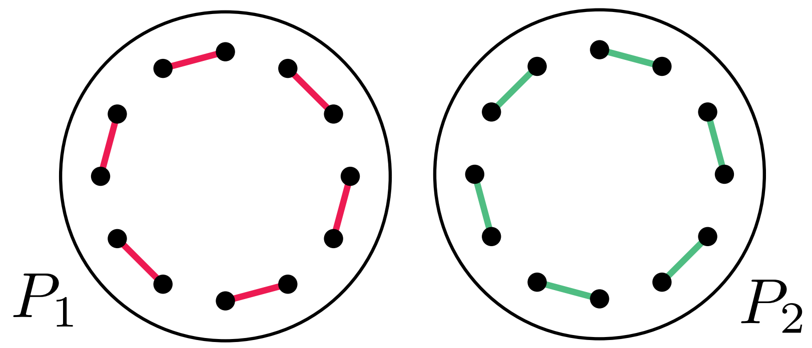
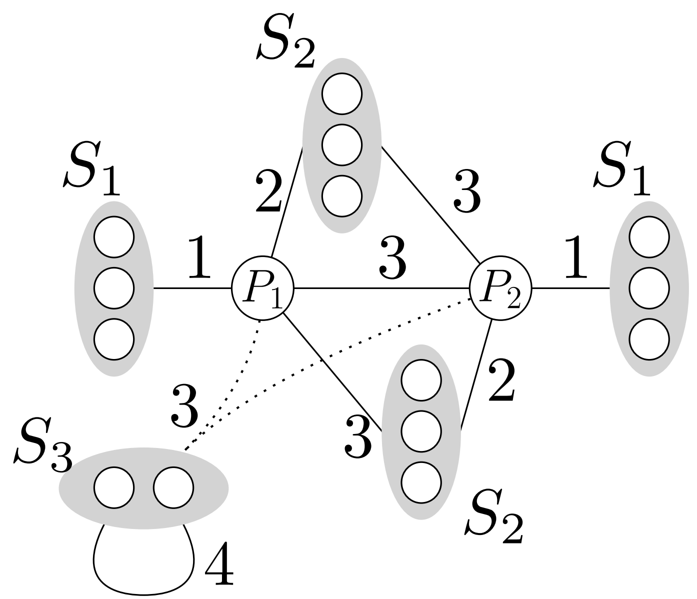
- more sophisticated arguments yield bound 5





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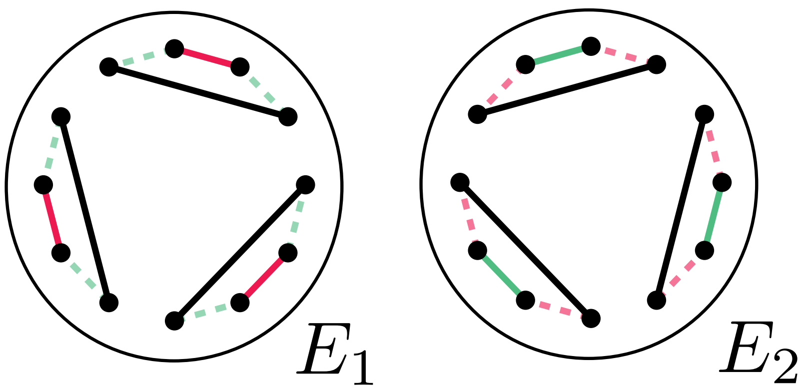

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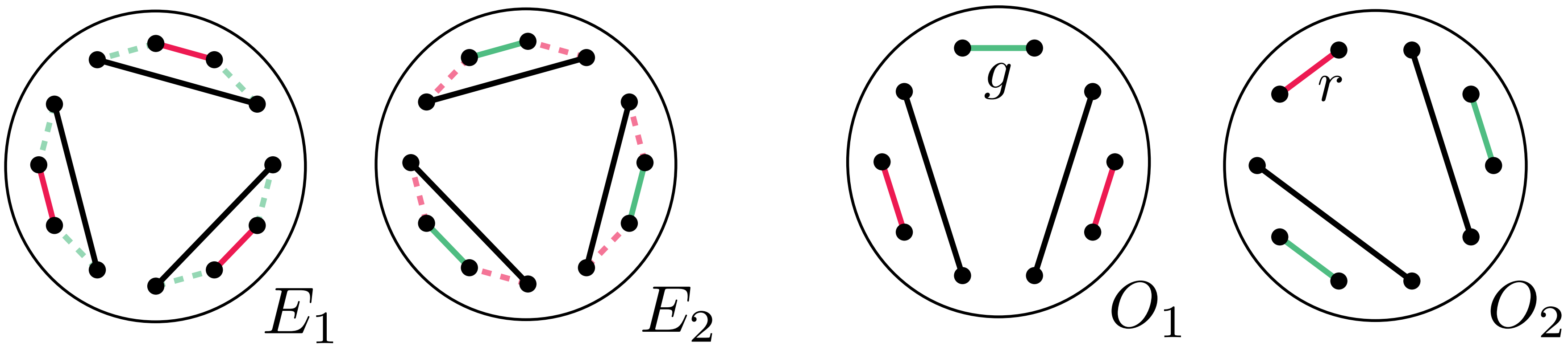

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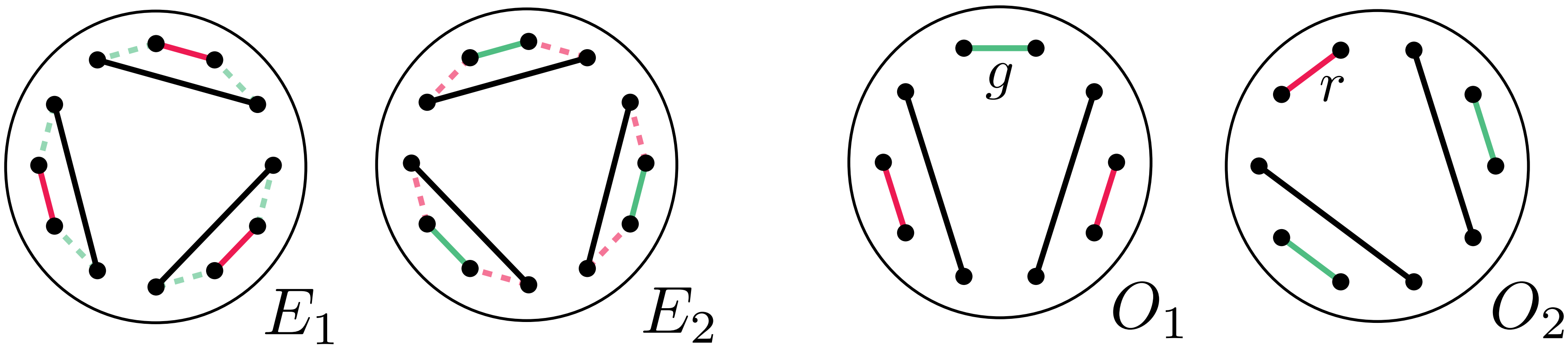

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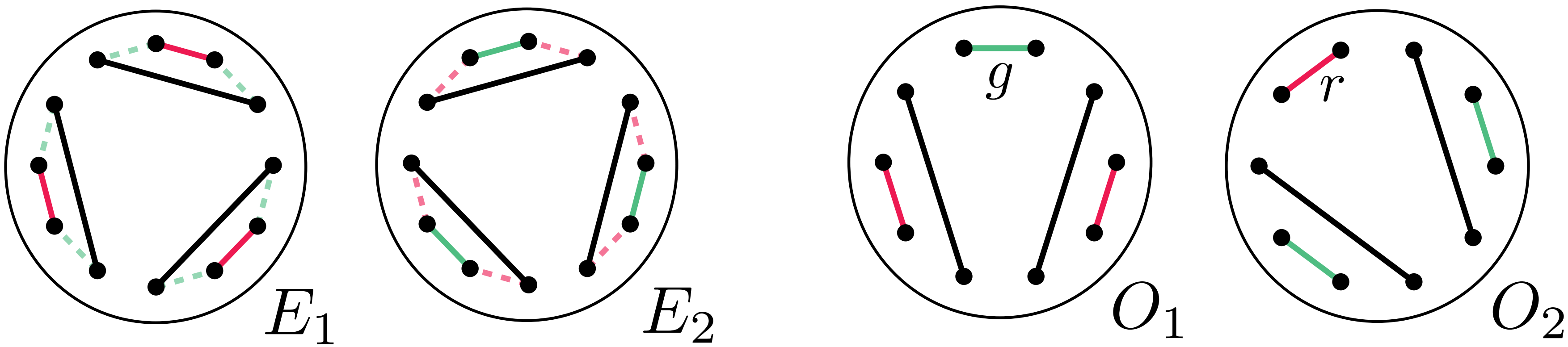

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(ii) M tree-compatible to $O_1 \Rightarrow$ at most one green perimeter edge, which is g (analogously for O_2 and r)

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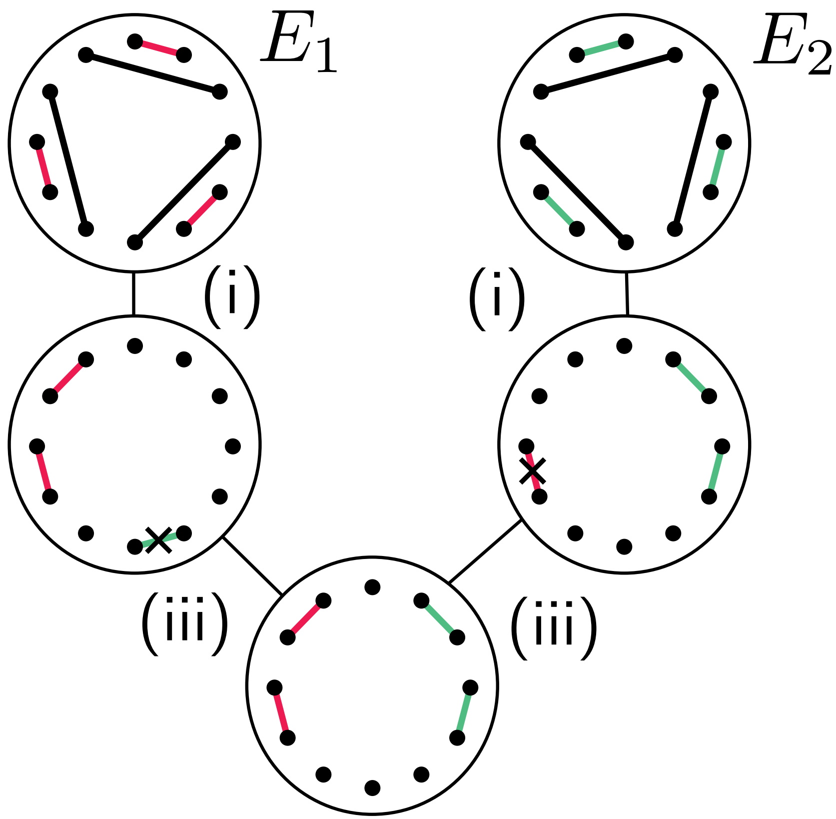
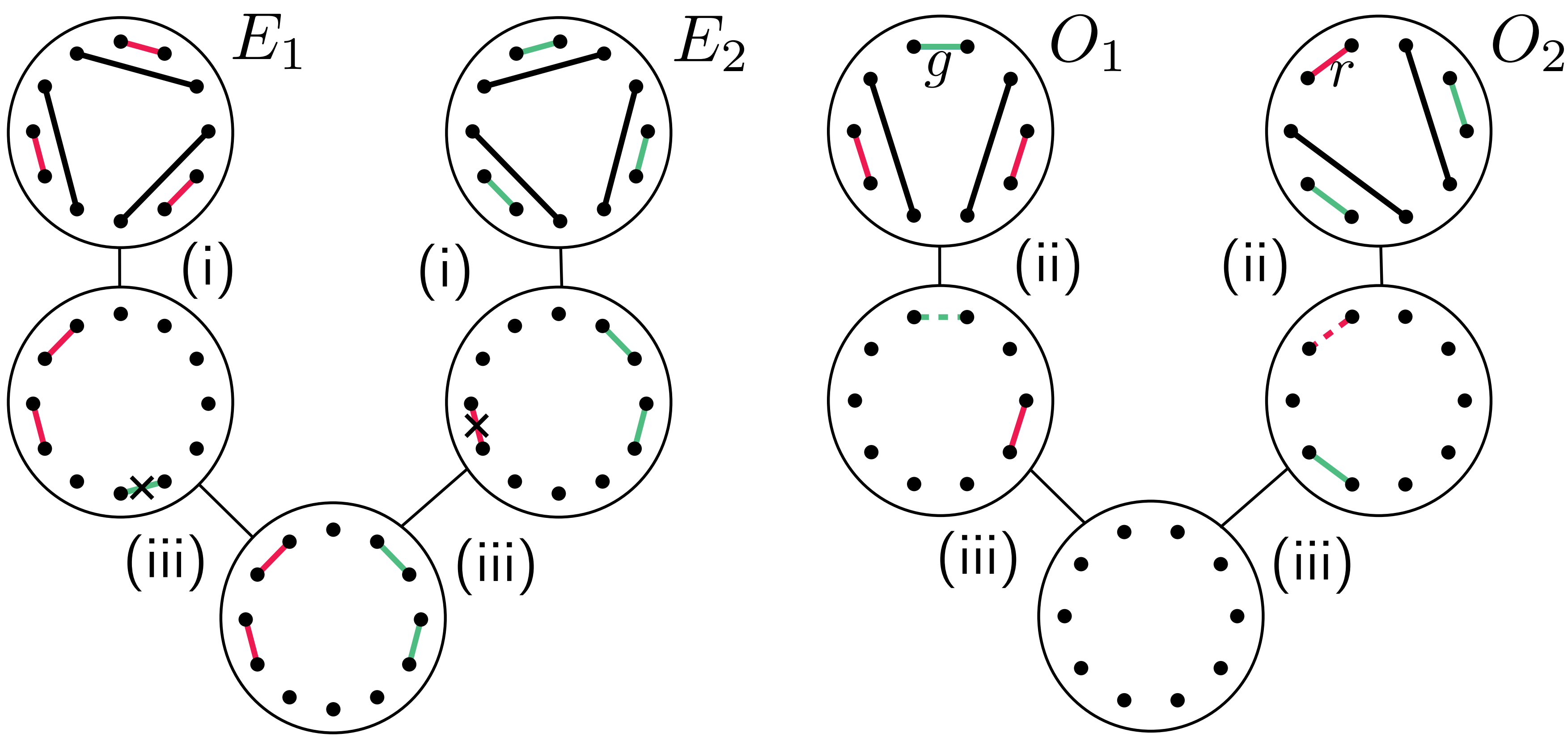


Lower bound

(iii) M and M' tree-compatible $\Rightarrow$ at least two perimeter edges in common



Summary / Open problems

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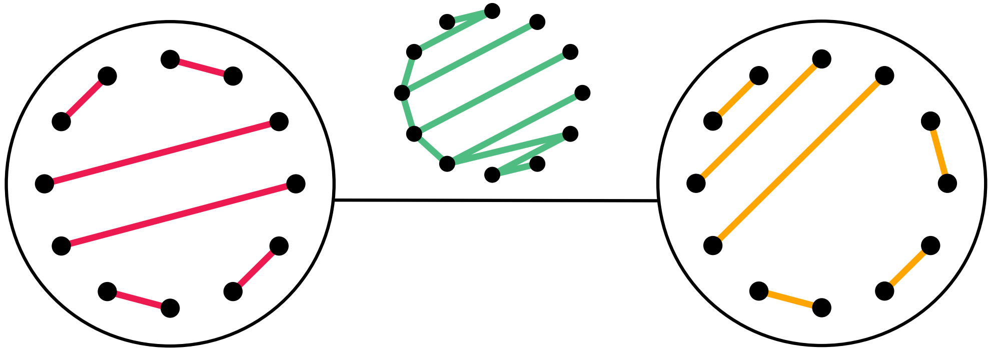

- The disjoint tree-compatibility graph G_{2n} is connected if and only if $2n \geq 10$.
- The diameter in that case is either 4 or 5.
- Conjecture. The diameter for all $2n \geq 18$ is 4.
  (diam(G_{2n})$= 5$ for $n \in \{5, 6, 7, 8\}$ and diam(G_{18})$= 4$)
- Is G_{2n} connected for general point sets (which n)?
- Compatibility via other graph classes?
  Ongoing work: disjoint path-compatibility

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