On the complexity of the middle curve problem

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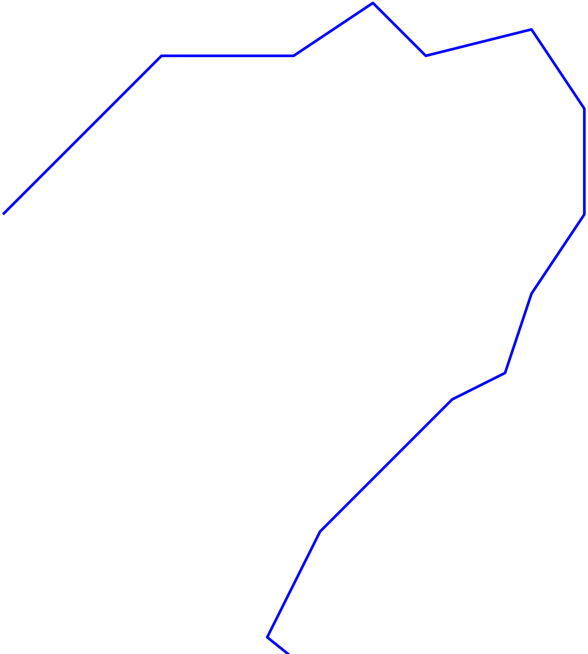
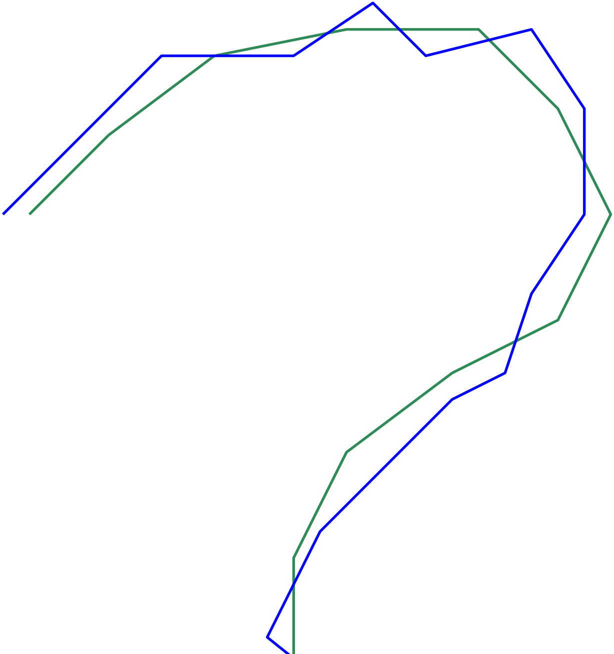
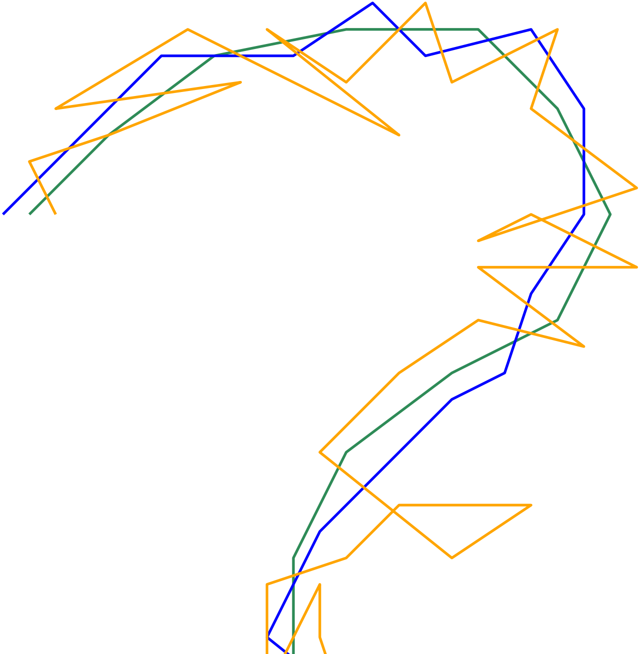
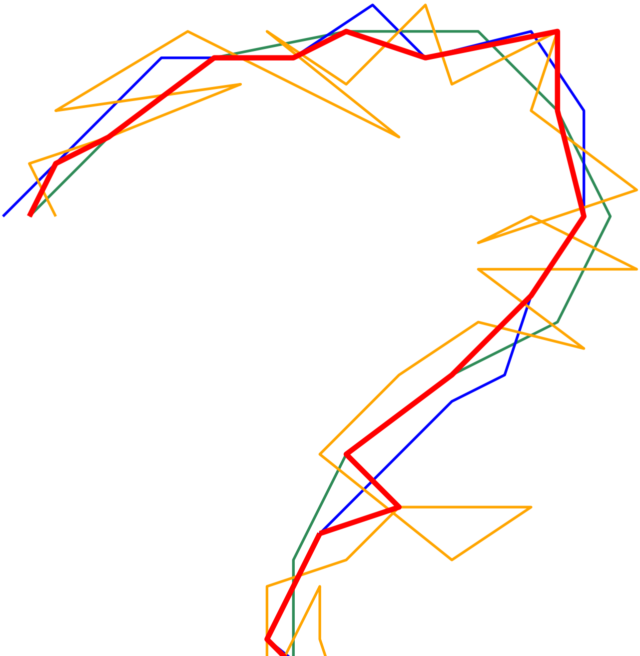
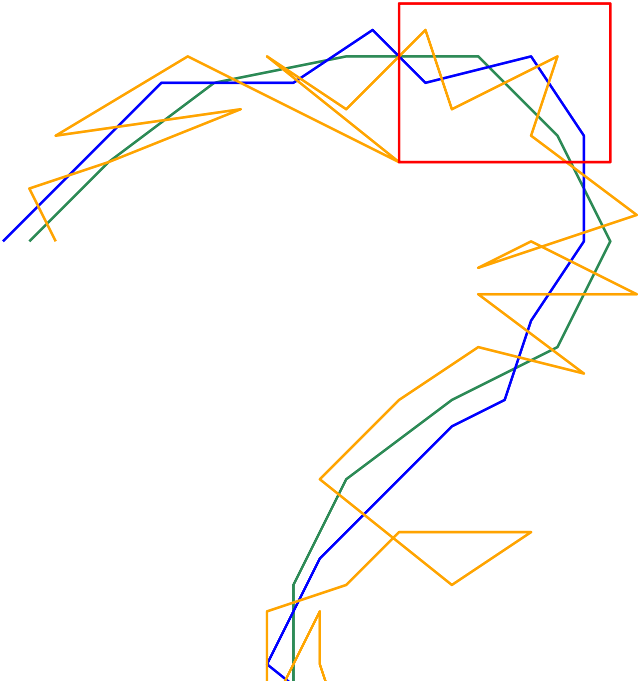
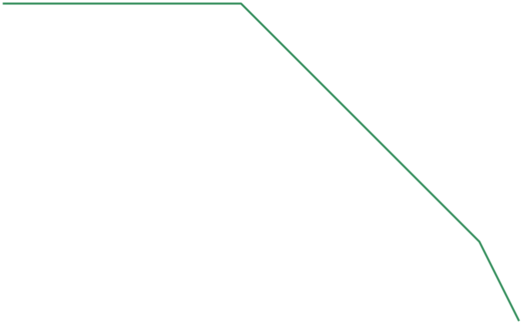


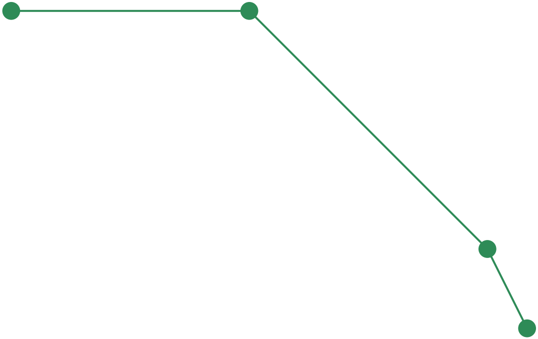






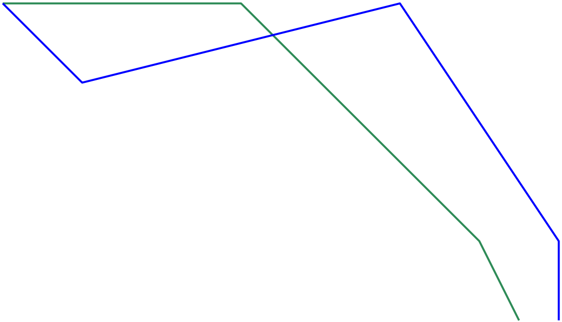


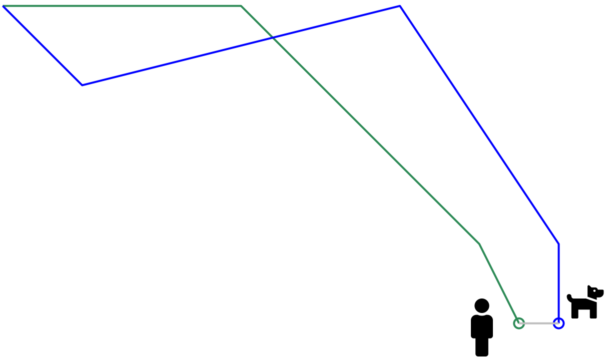




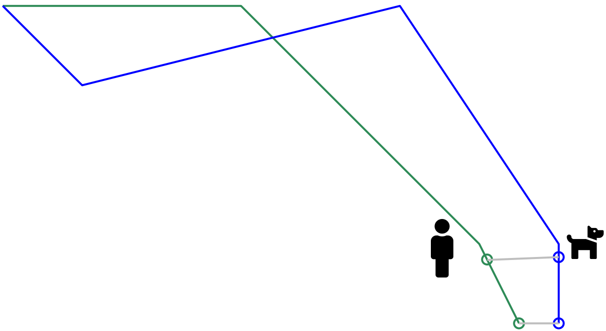
Definition polygonal curve P

- sequence of vertices $\langle p_1, \ldots, p_\ell \rangle$ in $\mathbb{R}^d$, connected by line segments
- ℓ : complexity of P





continuous Fréchet distance d_{cF}



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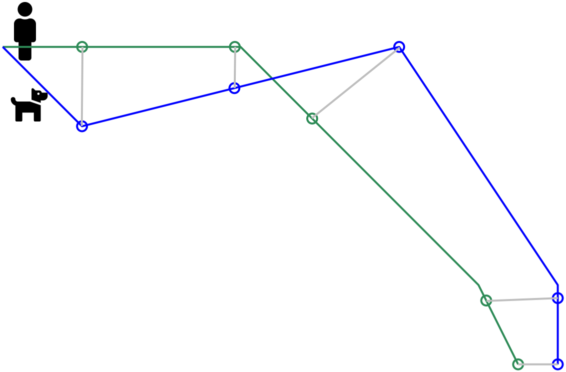

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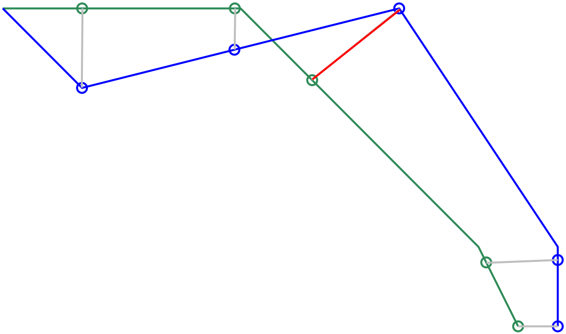

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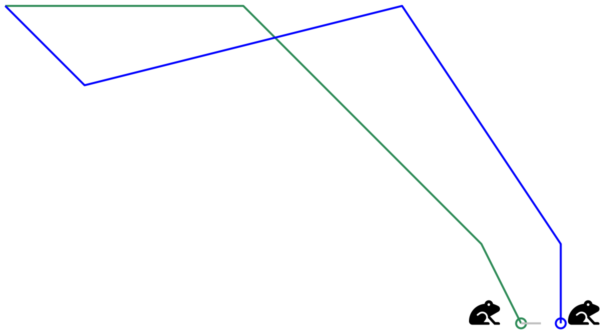

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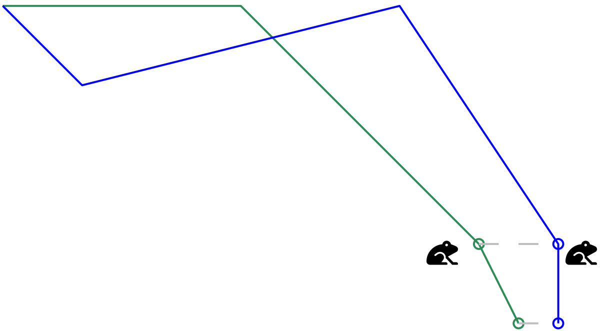

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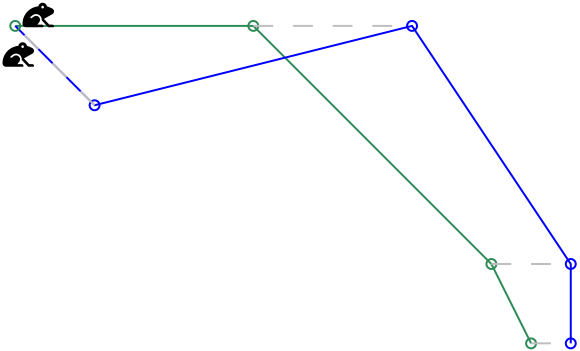

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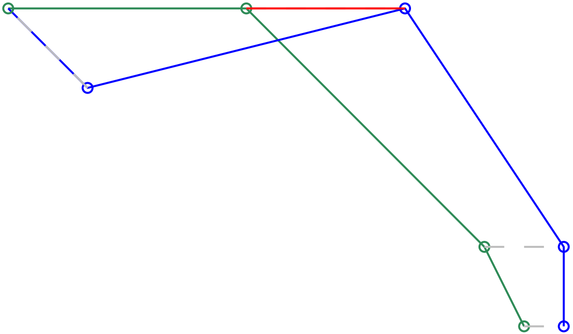

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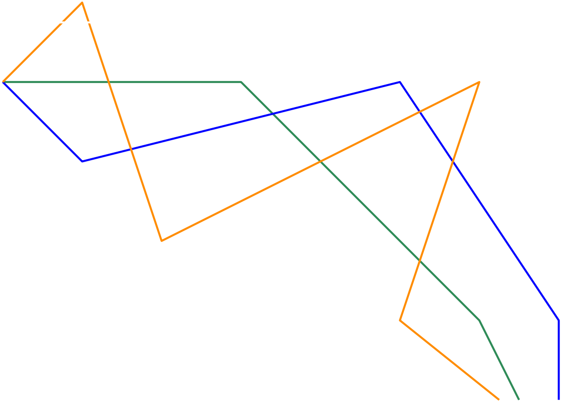
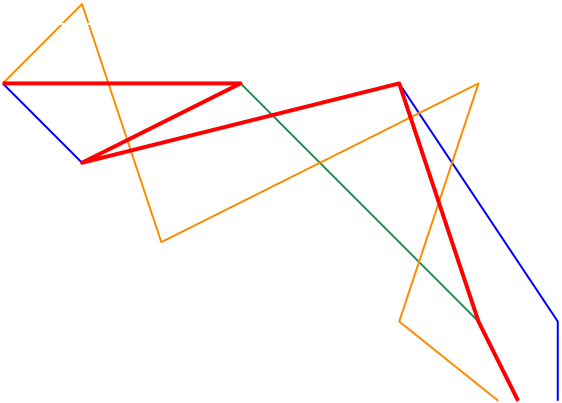


$\mathcal{P} = \{P_1, \ldots, P_n\}$ set of polygonal curves

$\delta \geq 0$

$d_F \in \{d_{dF}, d_{cF}\}$



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Definition middle curve with distance δ

$M = \langle m_1, \ldots, m_\ell \rangle \quad m_i \in \bigcup_{P_j \in \mathcal{P}} P_j$

s.t. $\max\{d_F(M, P_j) | P_j \in \mathcal{P}\} \leq \delta$

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Problem MIDDLE CURVE [Ahn et al. '16]

Given $\mathcal{P} = \langle p_1, \ldots, p_n \rangle$ *with complexity* $\leq m$ *and* $\delta > 0$

unordered MIDDLE CURVE
middle curve with distance δ*?*

ordered MIDDLE CURVE
unordered MIDDLE CURVE
+ vertices respect the order in their original curves?

restricted MIDDLE CURVE
ordered MIDDLE CURVE
+ vertices get matched to themselves in their original curve?

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running time of algorithm [Ahn et al. '16]:

- **ordered case**: $O(m^{2n})$
- **unordered case**: $O(m^n \log m)$
- **restricted case**: $O(m^n \log^n m)$

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- reduction from Shortest Common Supersequence

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Given a set of sequences S over binary alphabet $\Sigma = \{A, B\}$

positive integer t

Exists a sequence s^* of length at most t, that is a supersequence of all sequences in S?

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positive integer t

Exists *a sequence* s^* *of length at most* t*, that is a supersequence of all sequences in* S*?*

example:

$$S = \{AB, BB\}$$
$$t = 3$$
$$s^* = ABB$$

Given SCS-instance (S, t), construct MIDDLE CURVE instance for $i + j = t$

$$(G \cup \{A^i, B^j\}, 1)$$

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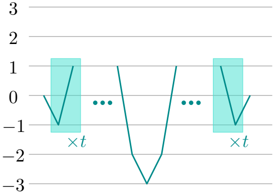
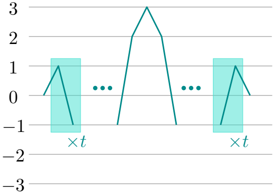
$A \rightarrow$

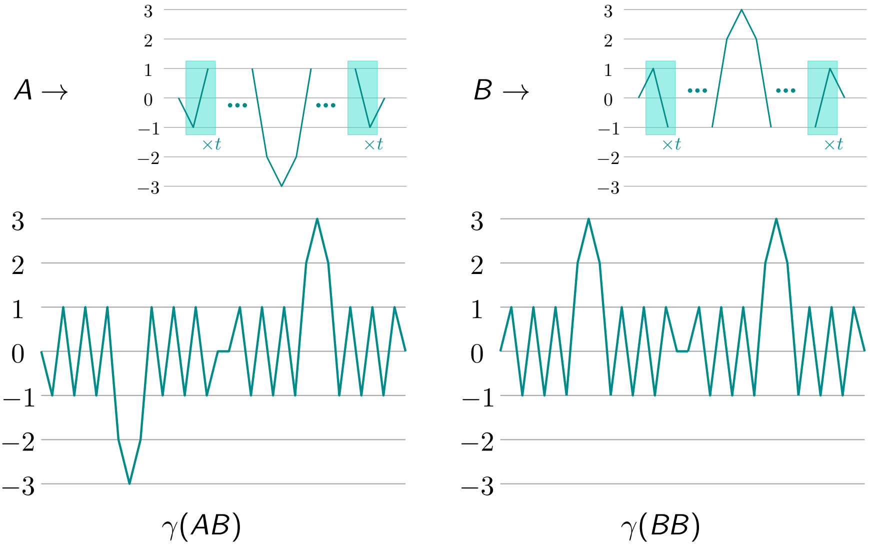


$B \rightarrow$



Given SCS-instance (S, t), construct Middle Curve instance for $i + j = t$

$$(G \cup \{A^i, B^j\}, 1)$$



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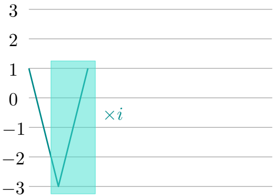

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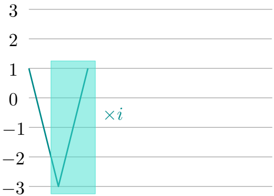

$B^j =$

A^1

B^2

Given SCS-instance (S, t), construct MIDDLE CURVE instance for $i + j = t$

$$(G \cup \{A^i, B^j\}, 1)$$

SCS $\Leftarrow$ unordered MIDDLE CURVE

SCS $\Rightarrow$ restricted MIDDLE CURVE

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Theorem

Middle Curve for the discrete Fréchet distance is NP-hard.

- holds for every variant

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*MIDDLE CURVE for the **continuous** Fréchet distance is NP-hard.*

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Theorem

*MIDDLE CURVE for the **continuous** Fréchet distance is NP-hard.*

- holds for every variant

test Fréchet in $O(m\ell \log(m\ell))$ [Alt and Godau '95]

Theorem

MIDDLE CURVE is NP-complete for the discrete and continuous Frechet distance.

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Given $\mathcal{P} = \langle p_1, \ldots, p_n \rangle$ *and* $\delta > 0$

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Problem PARAM MIDDLE CURVE

Given $\mathcal{P} = \langle p_1, \ldots, p_n \rangle$ *and* $\delta > 0$ ***and a parameter*** $\ell > 0$

unordered PARAM MIDDLE CURVE

middle curve with distance δ ***and complexity*** ℓ*?*

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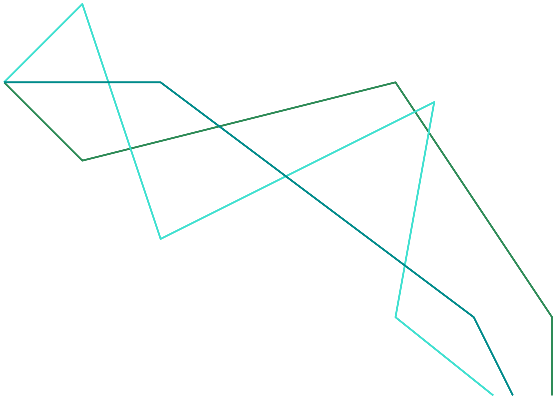
Theorem

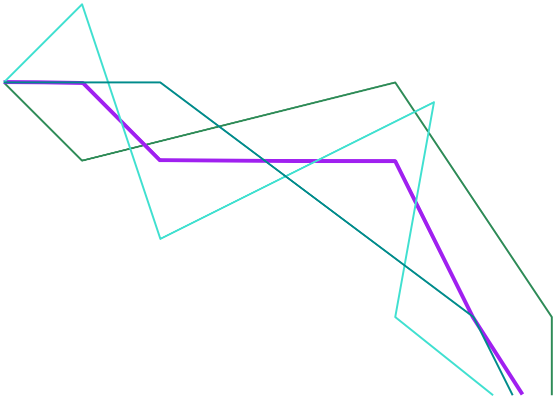
Every variant of the PARAM MIDDLE CURVE Instance can be decided in $O((mn)^{\ell} nm\ell \log(m\ell))$ *time via brute force.*

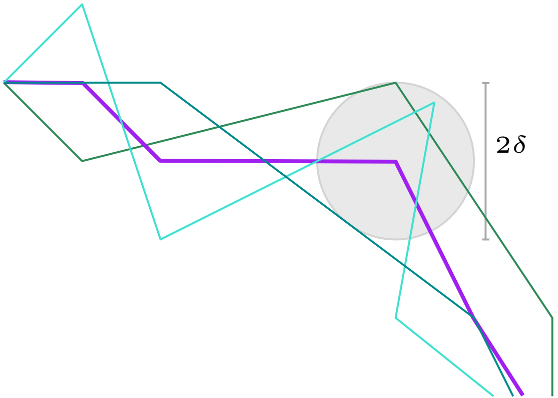
Problem (k, ℓ)-CENTER

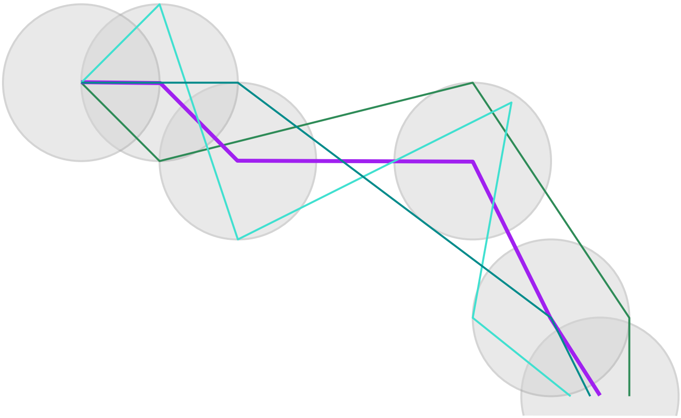
Given *polygonal curves* $\mathcal{G} = \{G_1, \ldots, G_n\}$ *and distance measure* d

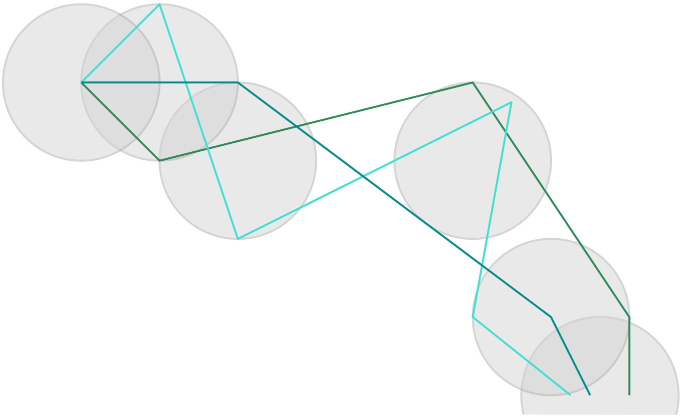
Find *set of curves* $\mathcal{C} = \{C_1, \ldots, C_k\}$, *each of complexity of at most* ℓ, *that minimizes* $\max_{G \in \mathcal{G}} \min_{i=1}^{k} d(C_i, G)$

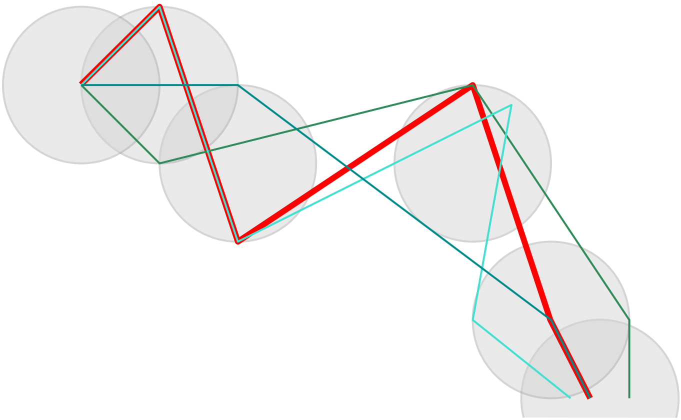


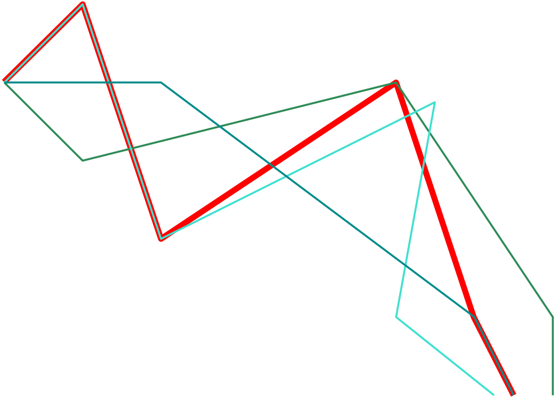


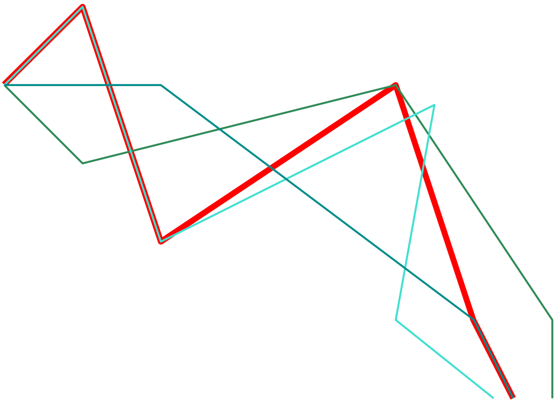

2δ









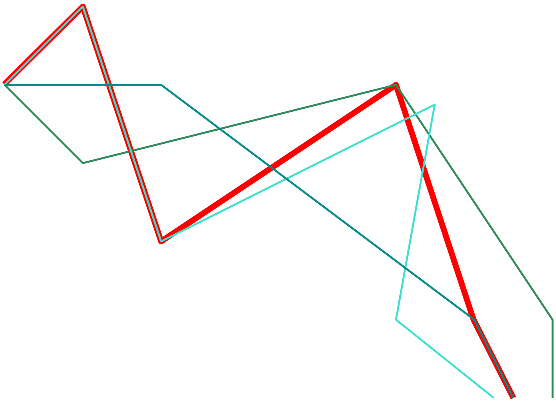


- n curves
- m input complexity
- δ > 0
- ℓ middle curve complexity

Theorem

Given α-*approximation algorithm for* $(1, \ell)$-*CENTER with running time* T

We can compute a (2α)-*approximation of the unordered* PARAM MIDDLE CURVE *in* $O(\ell mn + T)$ *time.*



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Corollary using [Buchin, Driemel and Struijs 19]

We can compute a $(2+\epsilon)$-approximation of the unordered Param Middle Curve *in $O((((\frac{4\sqrt{d}}{\epsilon}+1)^d\ell)^\ell + \log(\ell+n))\ell mn)$ time.*

Open question

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  Guesses:

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  Guesses:
  XP ✓ hard ✗

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XP	✓	hard	✗
paraNP	✓	hard	✗

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W[i]	✗		