Fréchet Distance Between Uncertain Trajectories

Computing Expected Value and Upper Bound

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Eindhoven University of Technology

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Utrecht University

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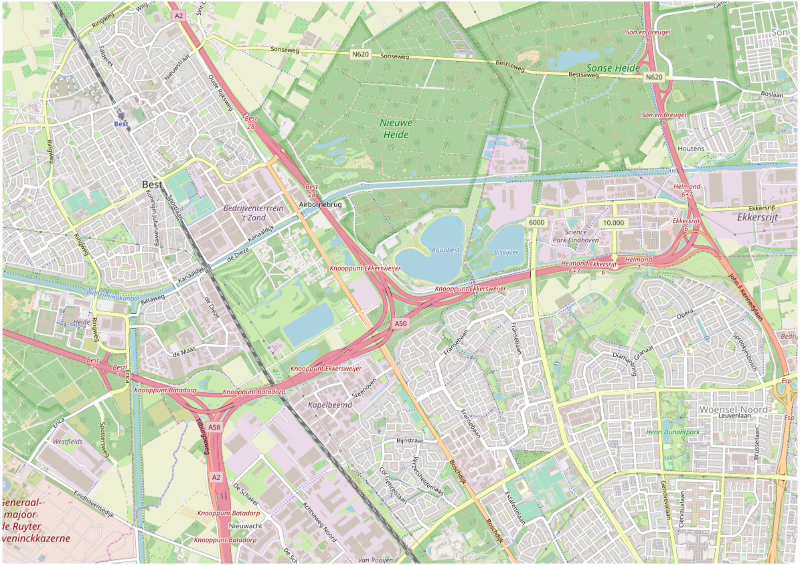
Why (uncertain) trajectories?



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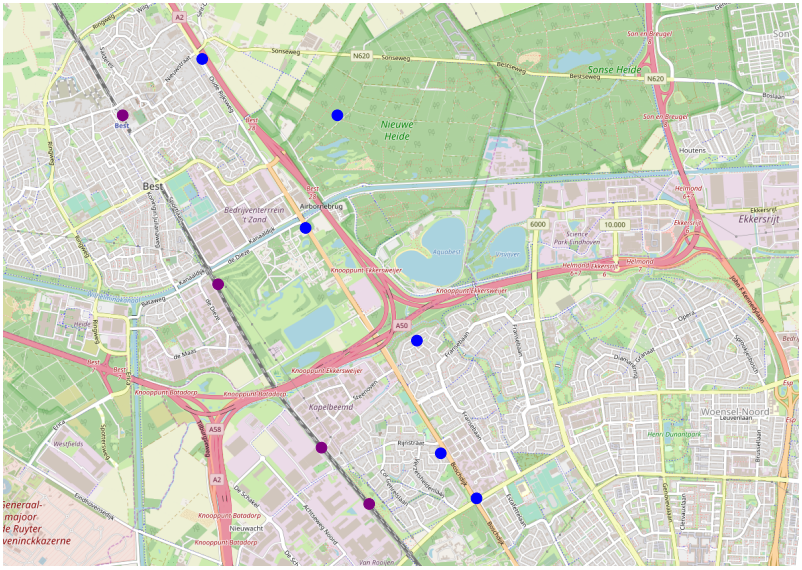

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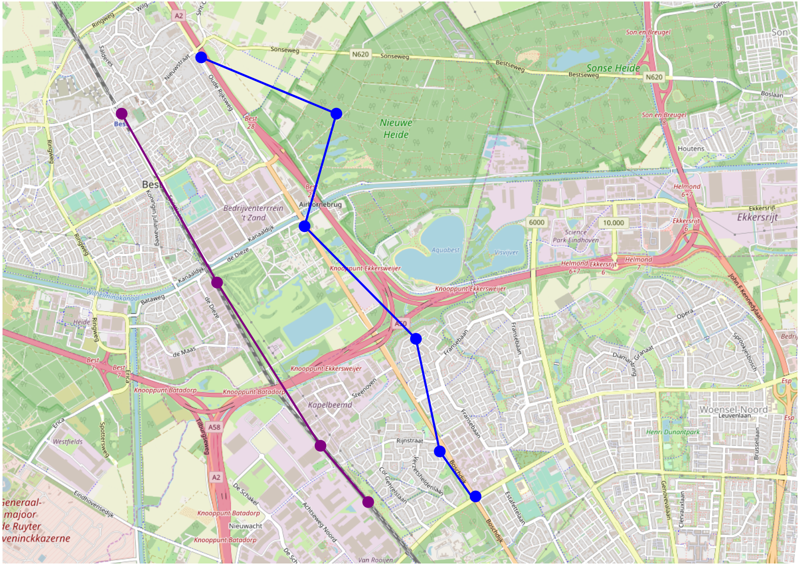

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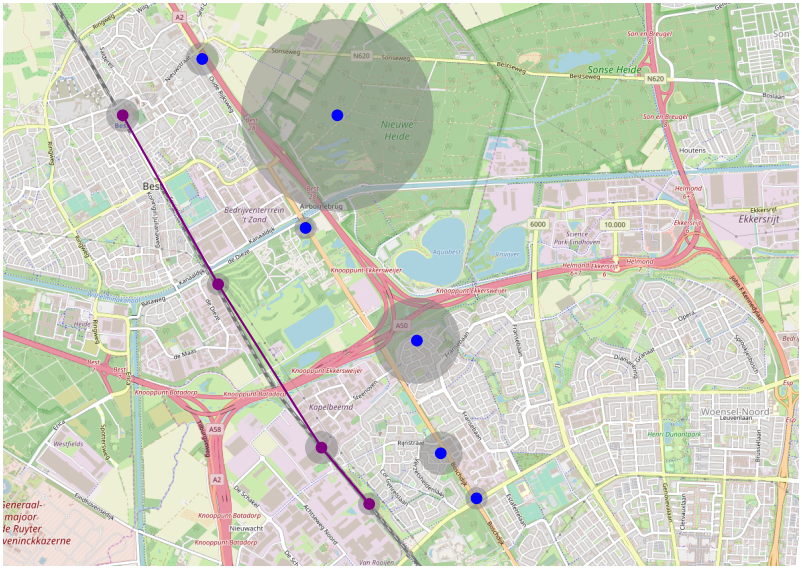

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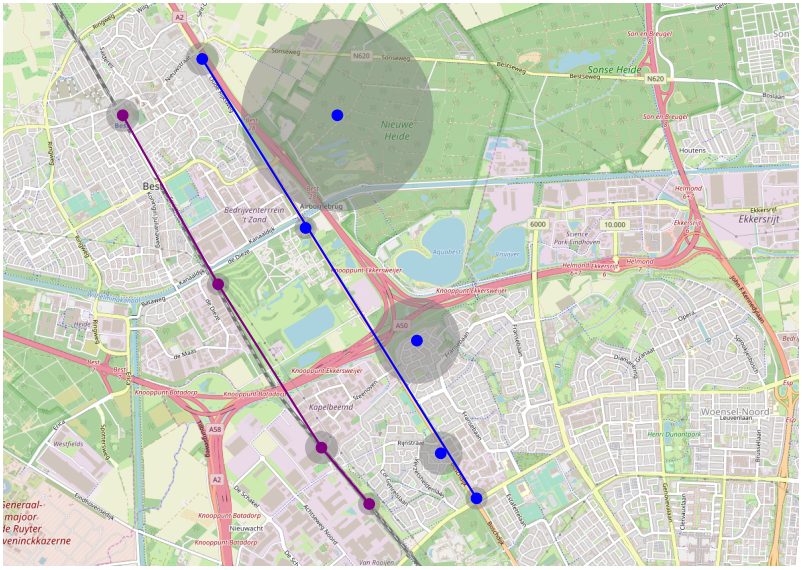

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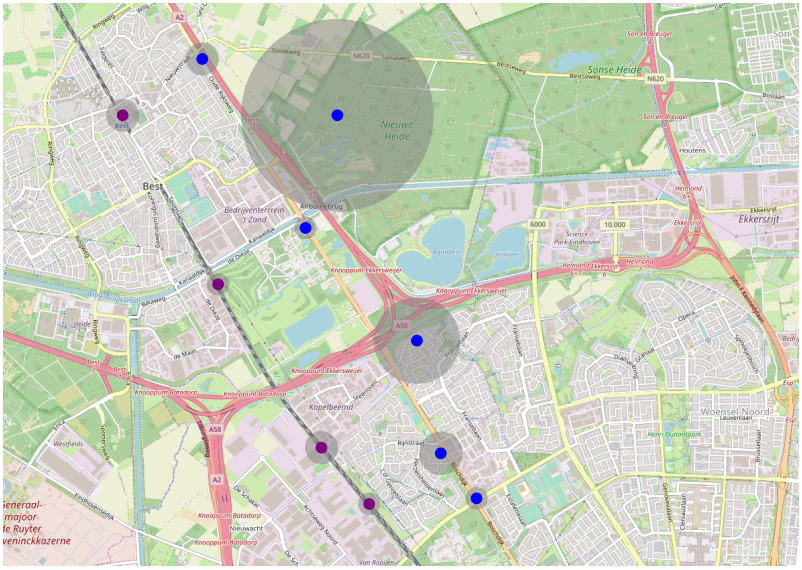

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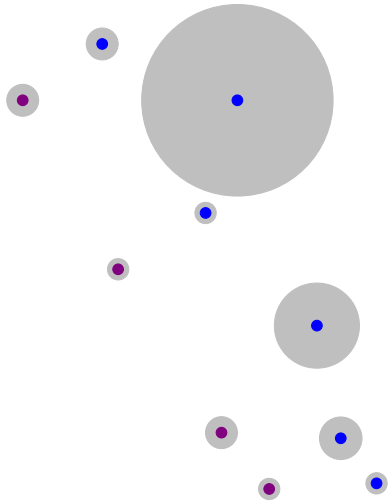

Uncertain trajectories

Measurement uncertainty, regions connected with line segments.

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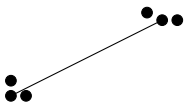
Indecisive point:
ℓ options per point



$\ell = 3$

Uncertain trajectories

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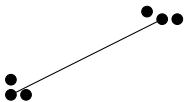
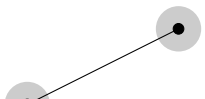


$\ell = 3$

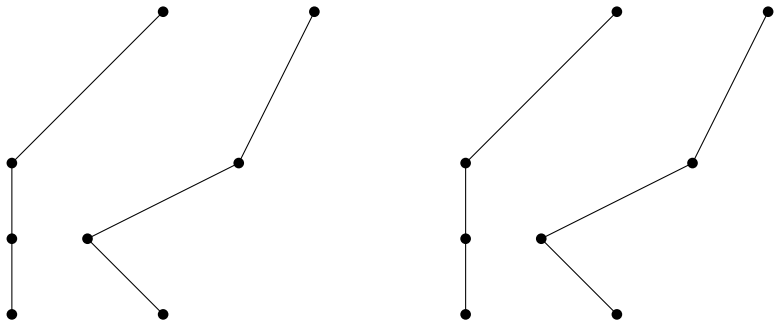
Imprecise point:
Connected region



circle

Similarity measures

Discrete Fréchet distance and continuous Fréchet distance:



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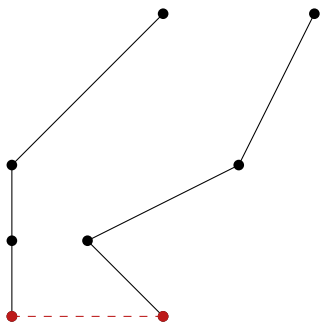

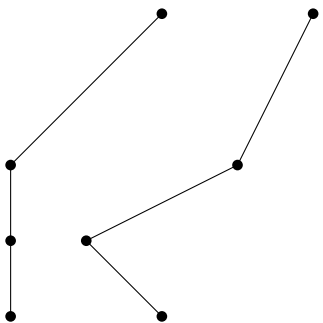

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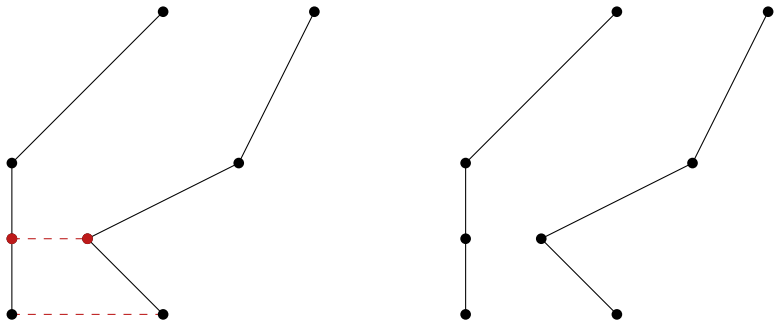

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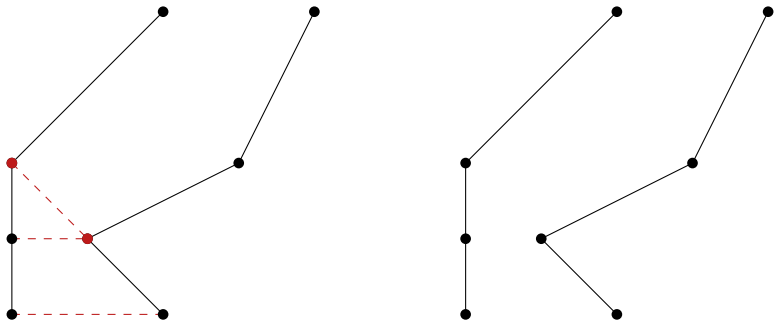

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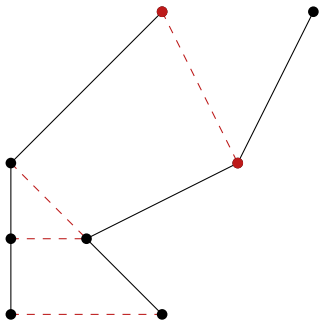

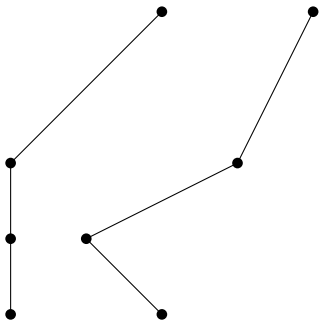

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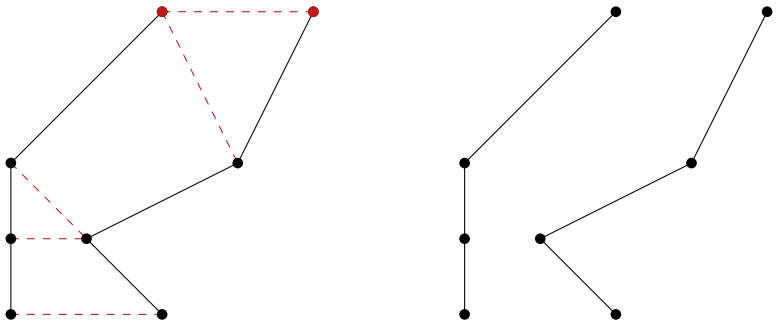

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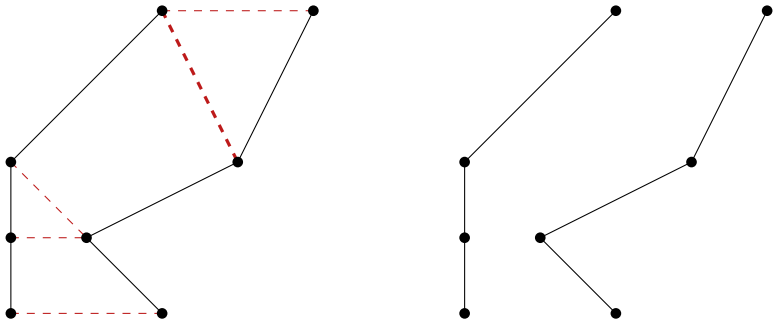

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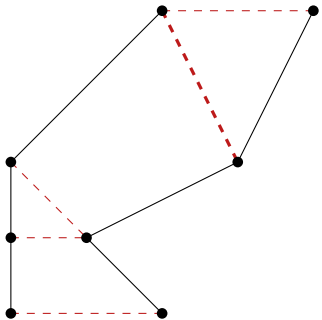
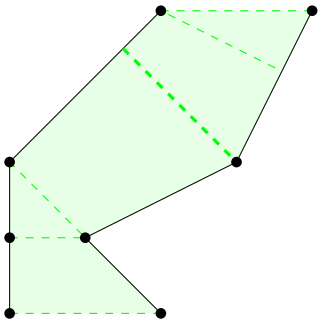
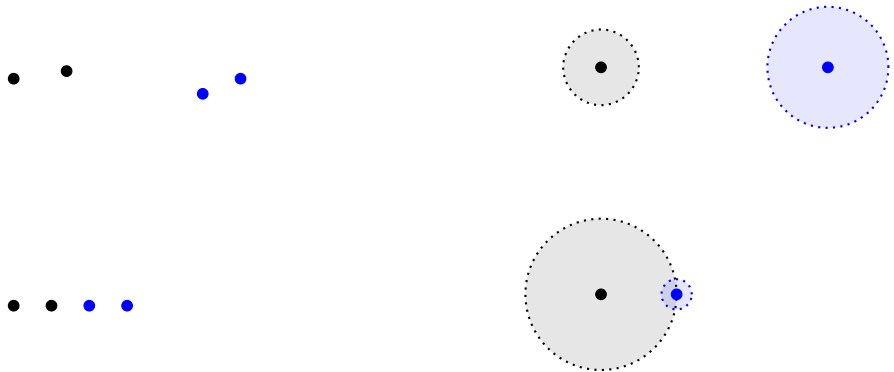




Fréchet distance on uncertain trajectories

Lower bound? Upper bound? Expected value?



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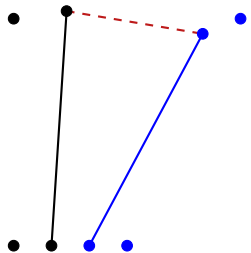

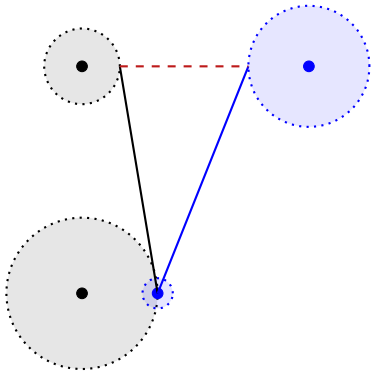

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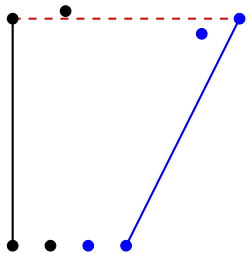

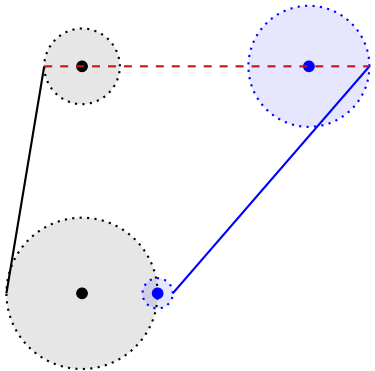

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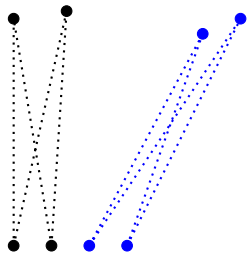

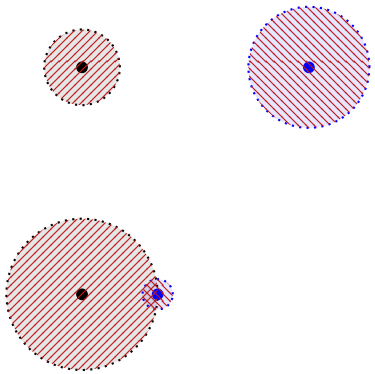

Overview

		indecisive	imprecise	
			disks	line segments
discrete FD	LB	Polynomial*	Polynomial*	Polynomial*
	UB	NP-complete	NP-complete	NP-complete
	Exp	#P-hard	—	#P-hard
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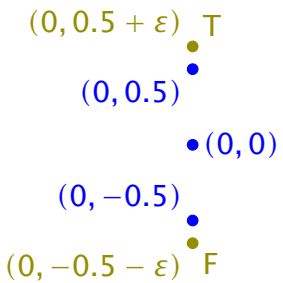
Indecisive: Polynomial-time algorithms with time bands for all cases.

Hardness results

Construction: Indecisive, discrete FD, upper bound

Reduction from SAT.
Two curves:

- Precise curve: formula structure;
- Indecisive curve: variable assignment.



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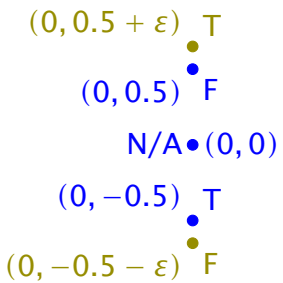

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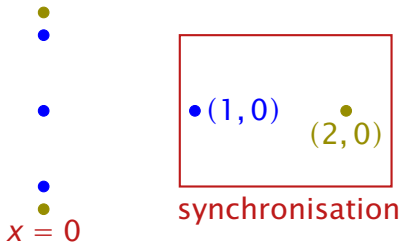

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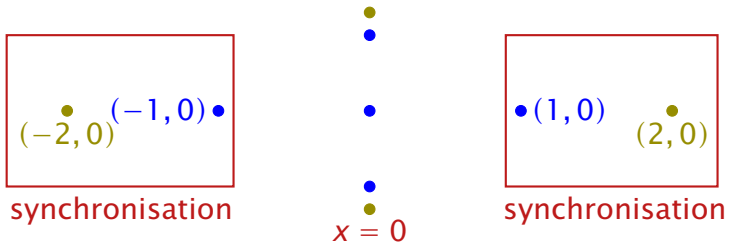
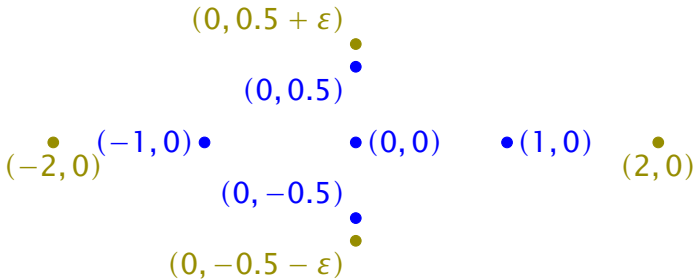


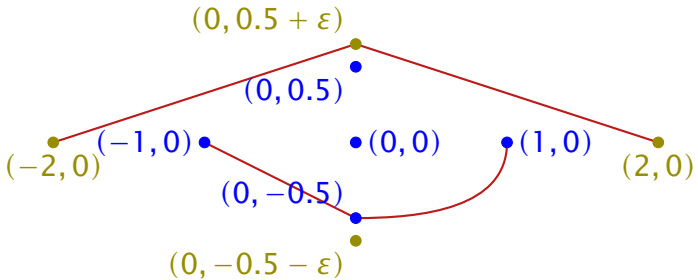
Construction: example



$C = x_1 \vee x_2$

Realisation: $x_1 = \mathsf{T}$, $x_2 = \mathsf{F}$, $x_3 = \mathsf{F}$.

Construction: example



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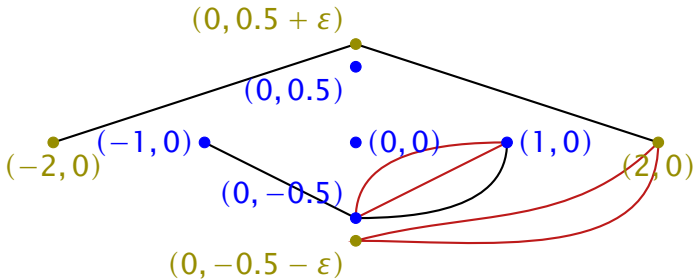

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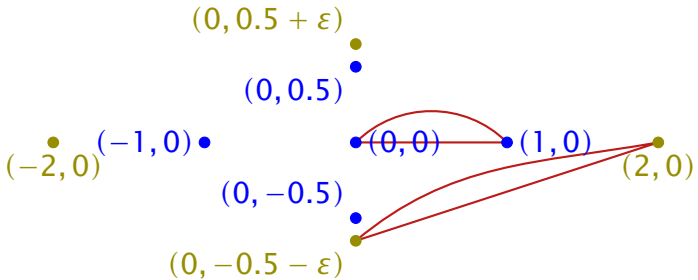

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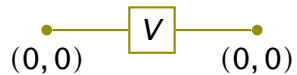

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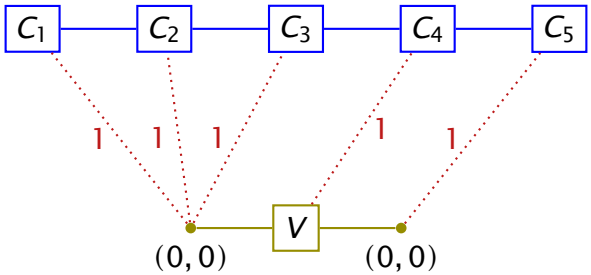
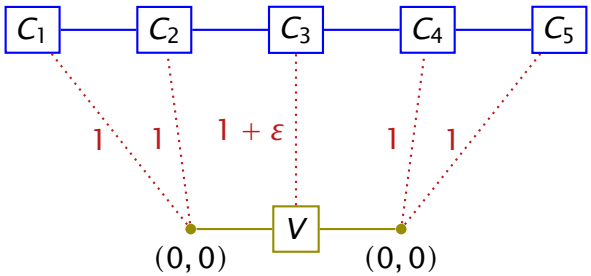


$C_4 = \mathsf{F}$

Construction: Indecisive, discrete FD, upper bound



$C_i = \mathsf{T}$ for all i

Hardness results

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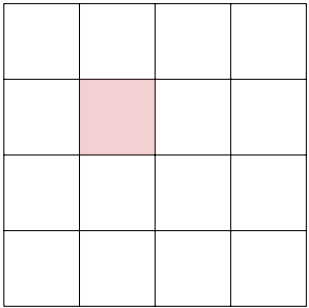
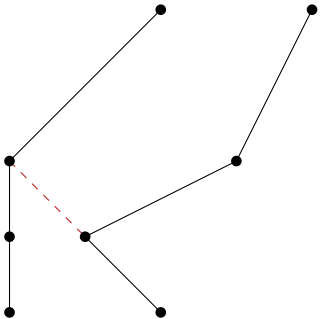
Algorithms using time bands

Time bands: idea





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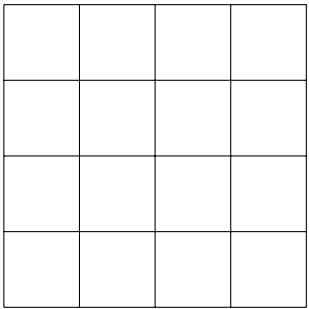

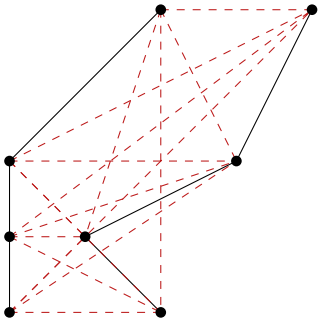

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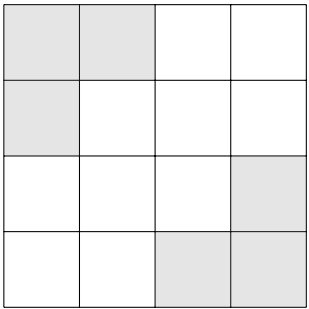
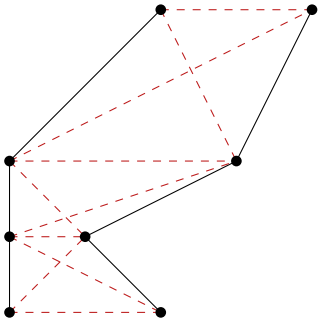


$w = 1$



Time bands: results

Time band width w, indecisive trajectories with ℓ options, length n:

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- UB/Exp FD: polynomial in n.

Summary

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FD	UB	NP-complete	NP-complete	NP-complete
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Time bands:
Polynomial-time algorithms for constant-width bands for UB and Exp discrete FD and FD on indecisive trajectories.