EINDHOVEN UNIVERSITY OF TECHNOLOGY

Bitonicity of Euclidean TSP in Narrow Strips

EuroCG presentation

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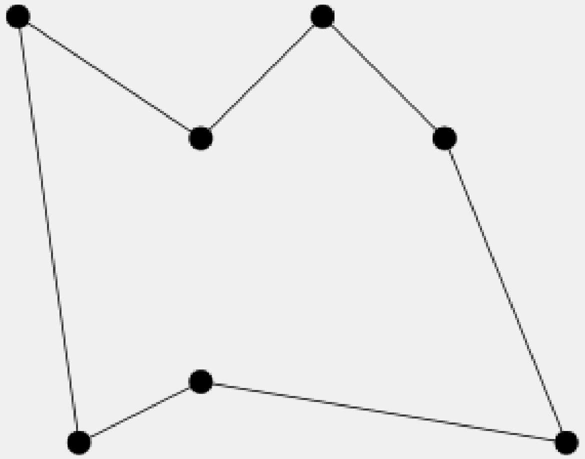
Introduction

The Euclidean Travelling Salesman Problem

In red: added commentary to make the slides readable

The Euclidean Travelling Salesman Problem

Find a shortest tour visiting all points

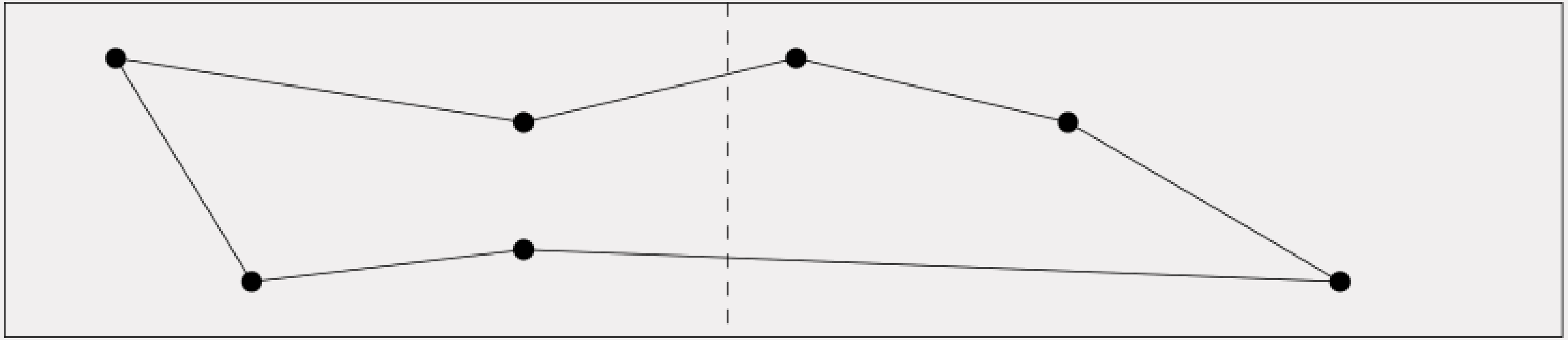


Euclidean TSP in narrow strips

Find a shortest tour visiting all points

Likely to be a *bitonic* tour

A tour is *bitonic* if it crosses any vertical line at most twice



Motivation

d-dimensional Euclidean TSP: NP-hard

Can be solved in $2^{O(n^{1-1/d})}$ time

ETH-tight

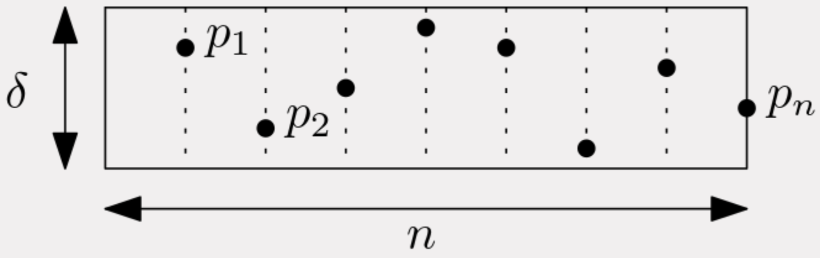
$2^{O(\sqrt{n})}$ for $d = 2$

Bitonic tours: $O(n \log^2 n)$

Problem description

$P = \{p_1, p_2, \ldots, p_n\}$ point set in $[0, n] \times [0, \delta]$

x-coordinate of p_i is exactly i



Bitonicity of Euclidean TSP in Narrow Strips

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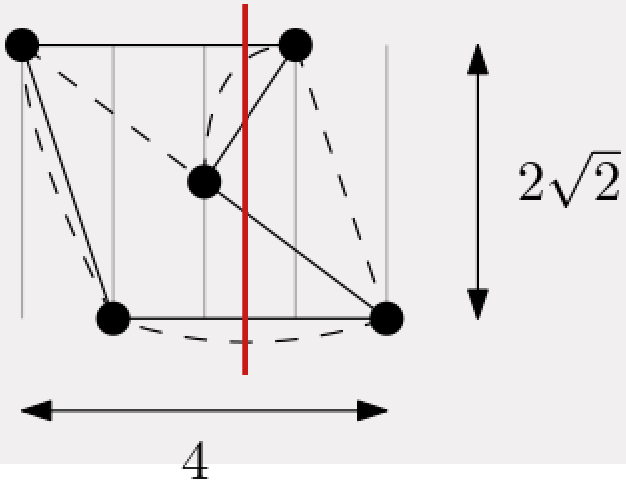
Theorem 1.

If $\delta \leq 2\sqrt{2}$, there exists a shortest tour that is bitonic. This bound is tight.

Construction for $\delta > 2\sqrt{2}$:



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If $\delta \leq 2\sqrt{2}$, there exists a shortest tour that is bitonic. This bound is tight.

Proof sketch:

Transform tour T into (shorter) bitonic tour T'

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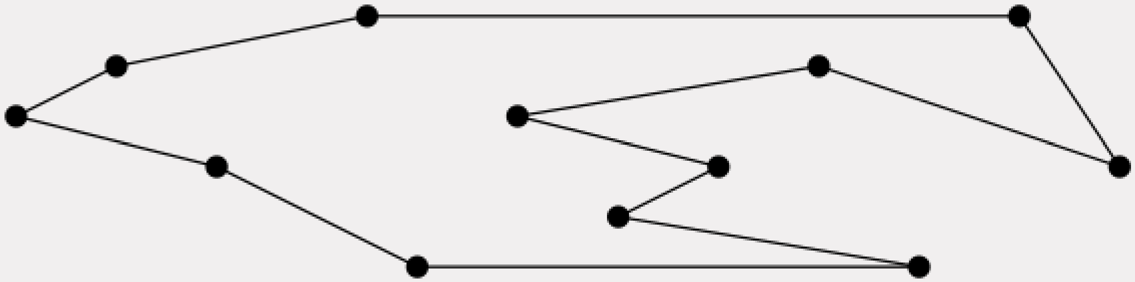

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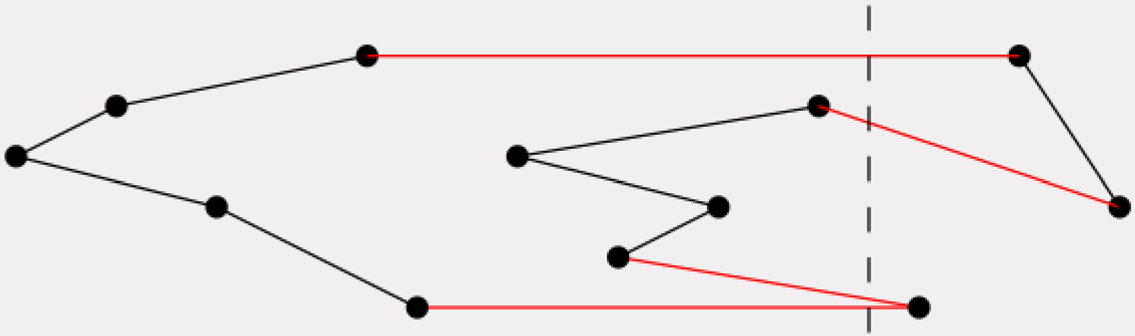

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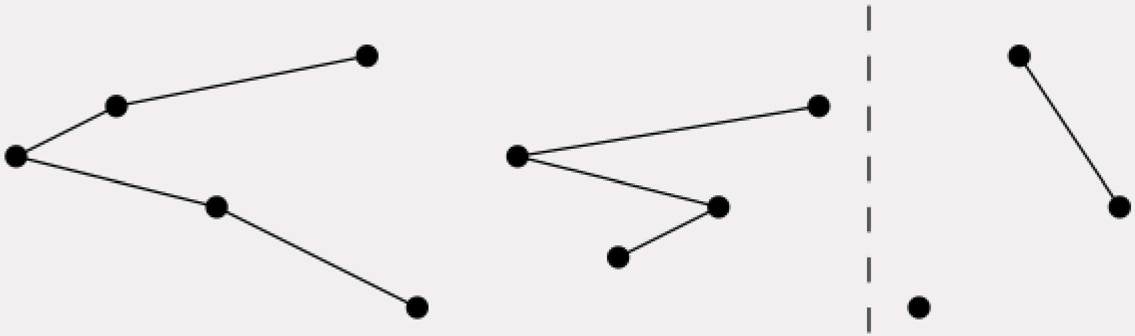

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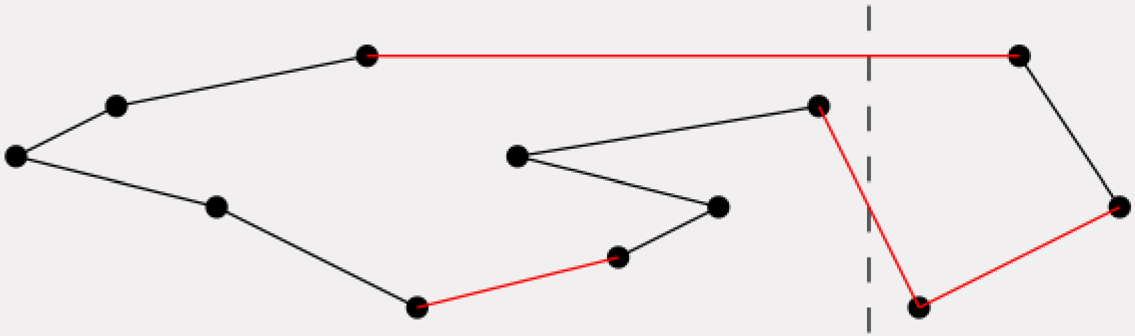

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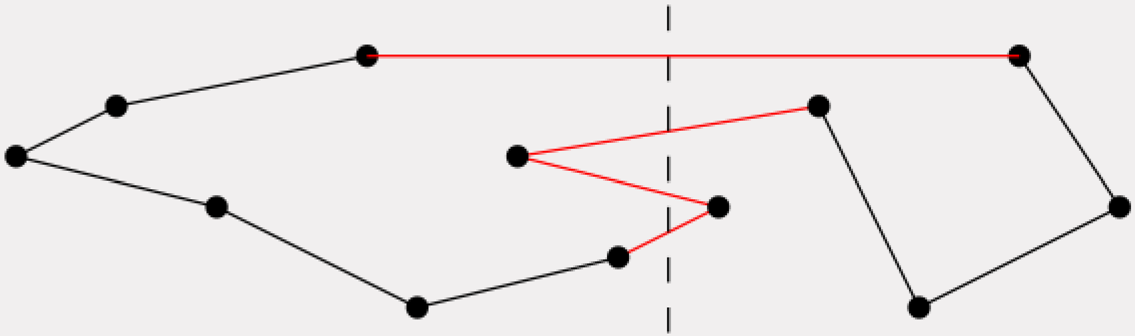

TU/e

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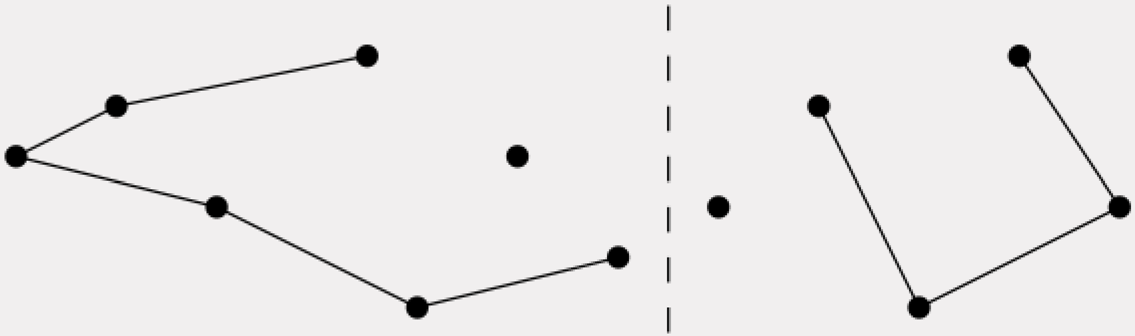

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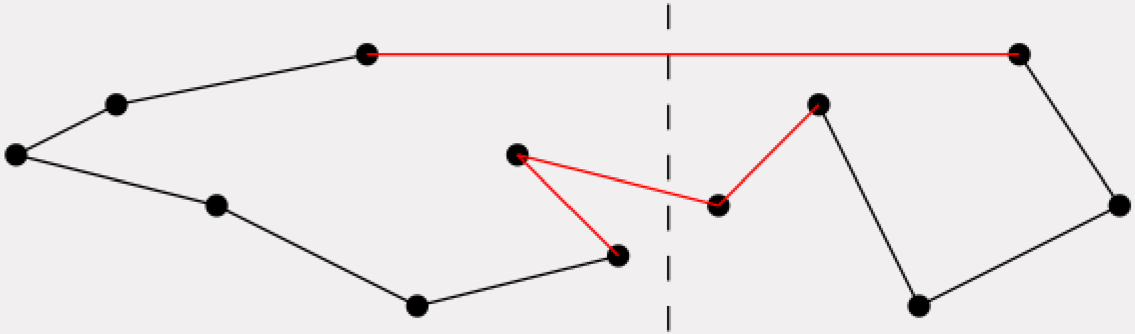

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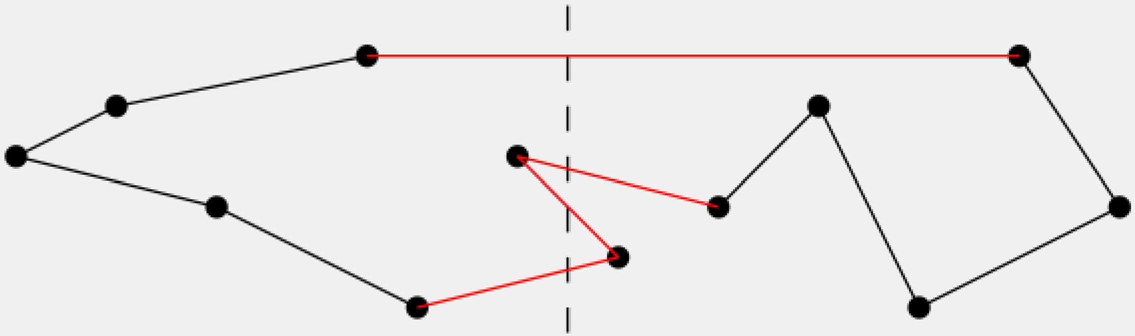

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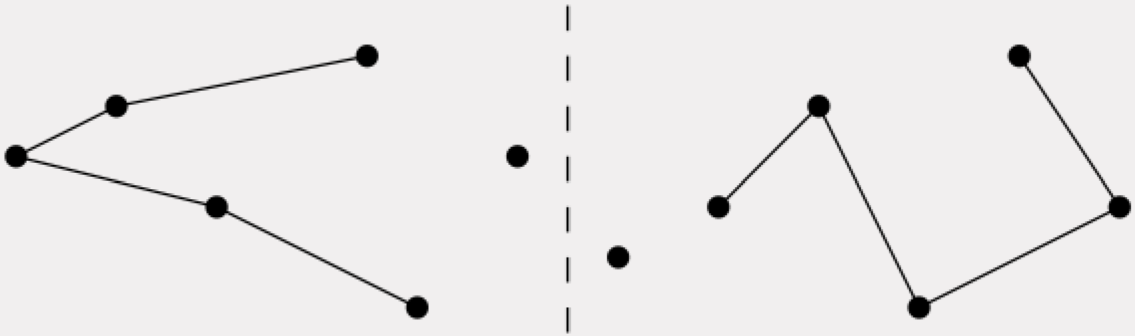

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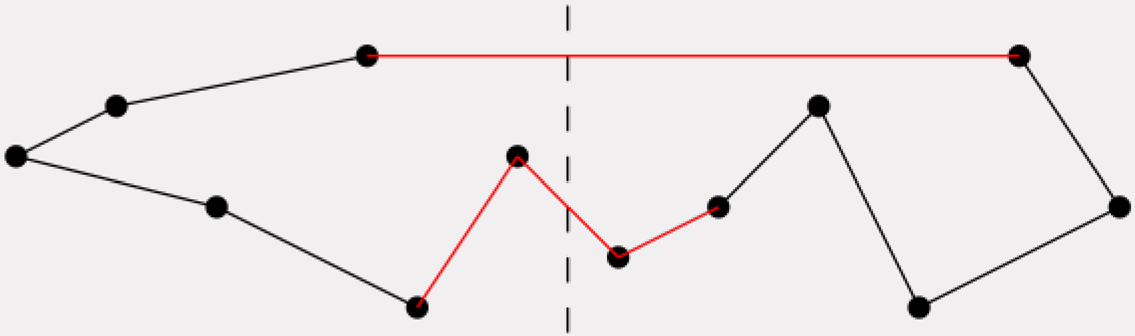

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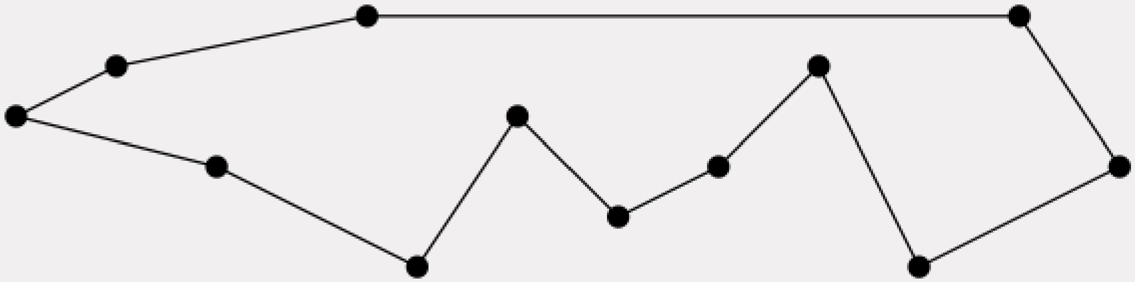

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An edge set E is *superior* to an edge set F if

- The sum of the lengths of the edges of E is strictly less than that of F, or
- The sums are equal, but
  - No vertical line crosses E strictly more times than F, and
  - There exists a vertical line which crosses E strictly fewer times than F

Bitonicity of Euclidean TSP in Narrow Strips

Proof sketch:

Transform tour T into (shorter) bitonic tour T'

Step 1:

Superior edge set exists if ‘interesting’ points have consecutive x-coordinates

$$\Rightarrow$$

A superior edge set always exists

‘Interesting’ points are those which cross the vertical line we are currently looking at during our sweep from right to left

TU/e

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Proof sketch:

Transform tour T into (shorter) bitonic tour T'

Step 1:

Superior edge set exists if 'interesting' points have consecutive x-coordinates

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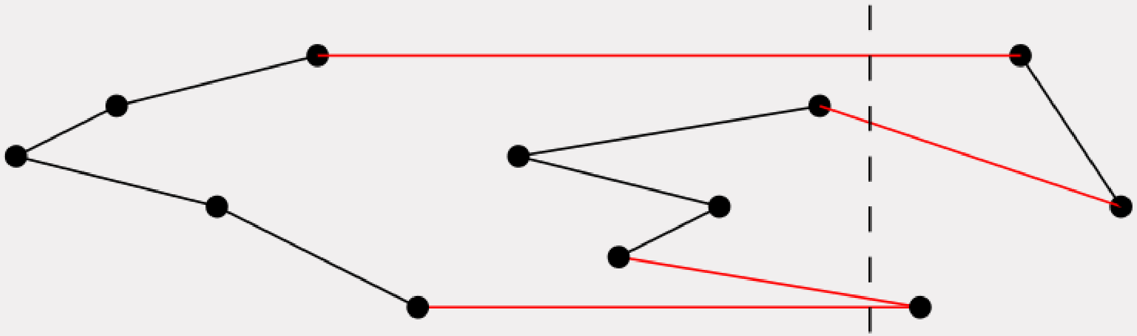

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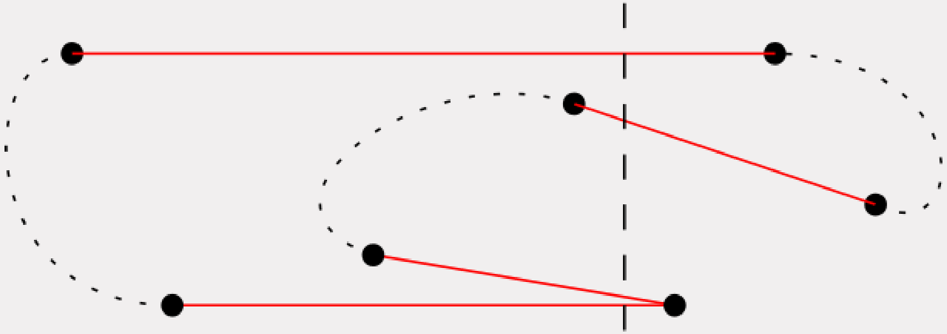
Proof sketch:

Transform tour T into (shorter) bitonic tour T'

The exact connections are unimportant, but their connections are; the new set of edges must still form a tour together with the black edges



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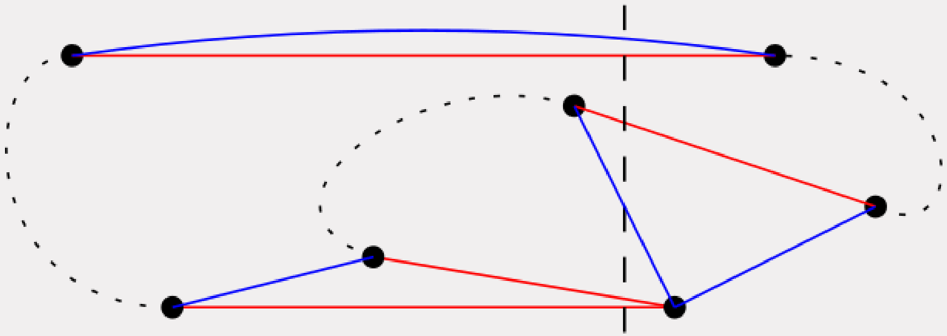
Proof sketch:

Transform tour T into (shorter) bitonic tour T'

In blue, a alternative set of edges. Note that moving points along the red edges can only make

blue 'less' superior



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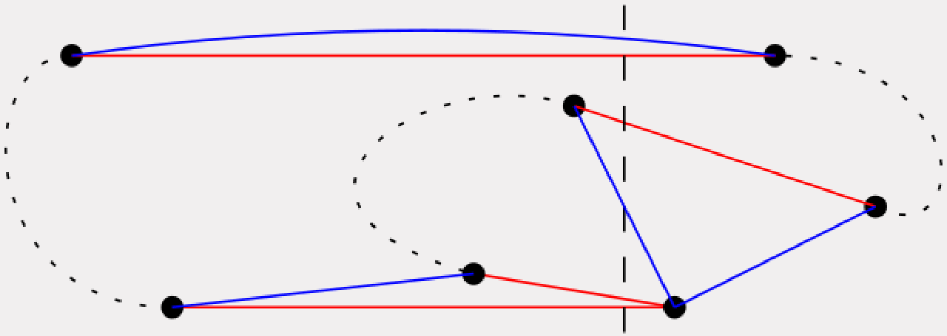

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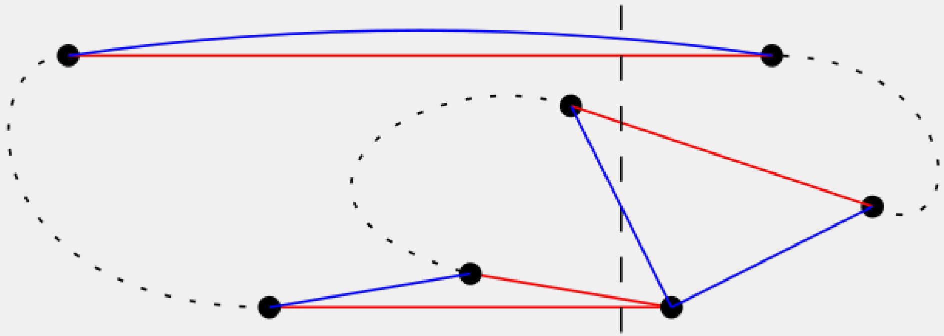

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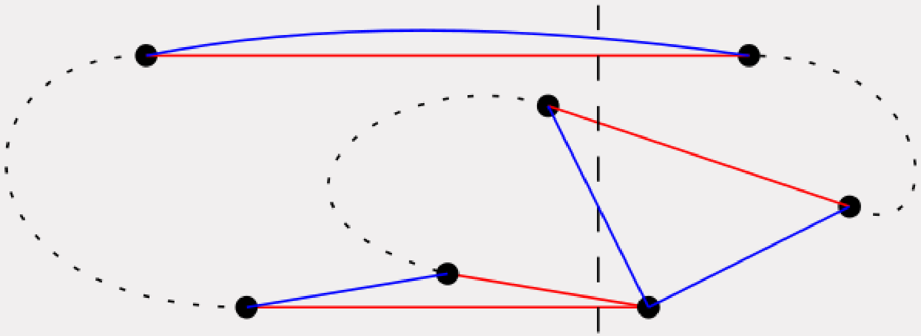

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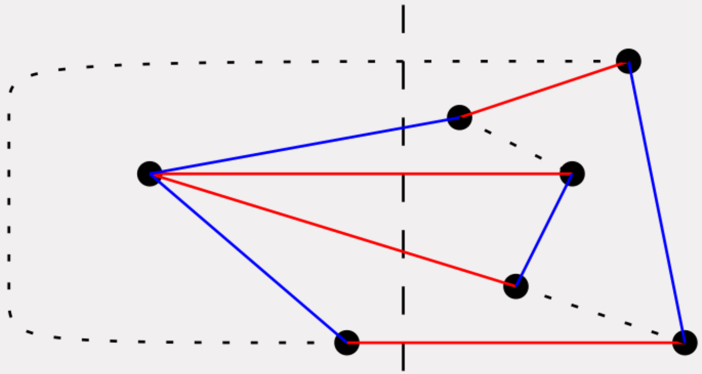
Proof sketch:

Transform tour T into (shorter) bitonic tour T'

If you whish to move a vertex with two red edges connected, things are slightly more complicated…



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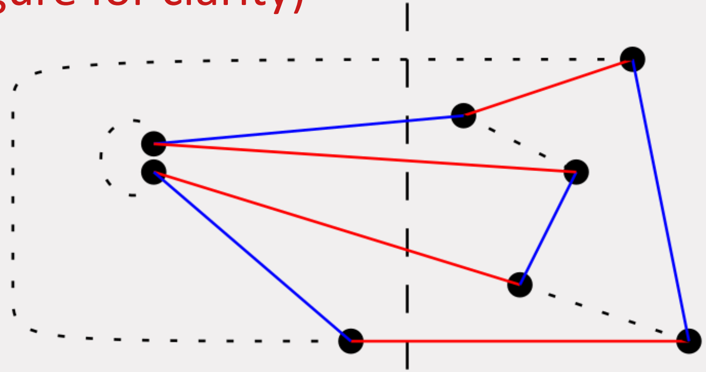
Proof sketch:

Transform tour T into (shorter) bitonic tour T'

Split the vertex into two, adding a connection between them (they are still on the same spot, but slightly displaced in the figure for clarity)



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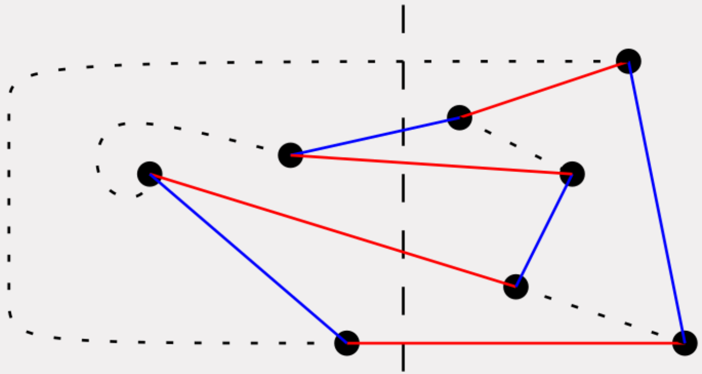
Proof sketch:

Transform tour T into (shorter) bitonic tour T'

Then, you can move one of them as normal



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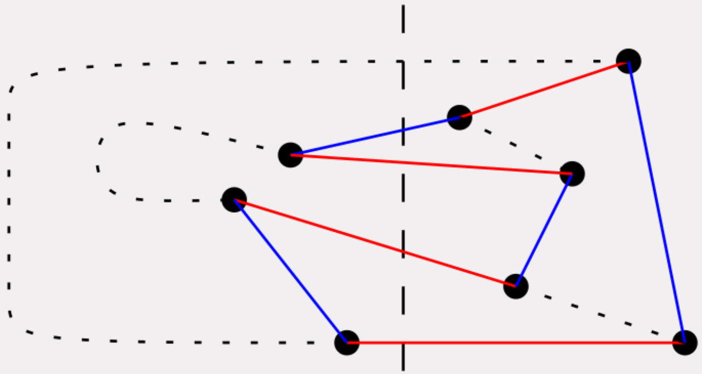

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Step 2:

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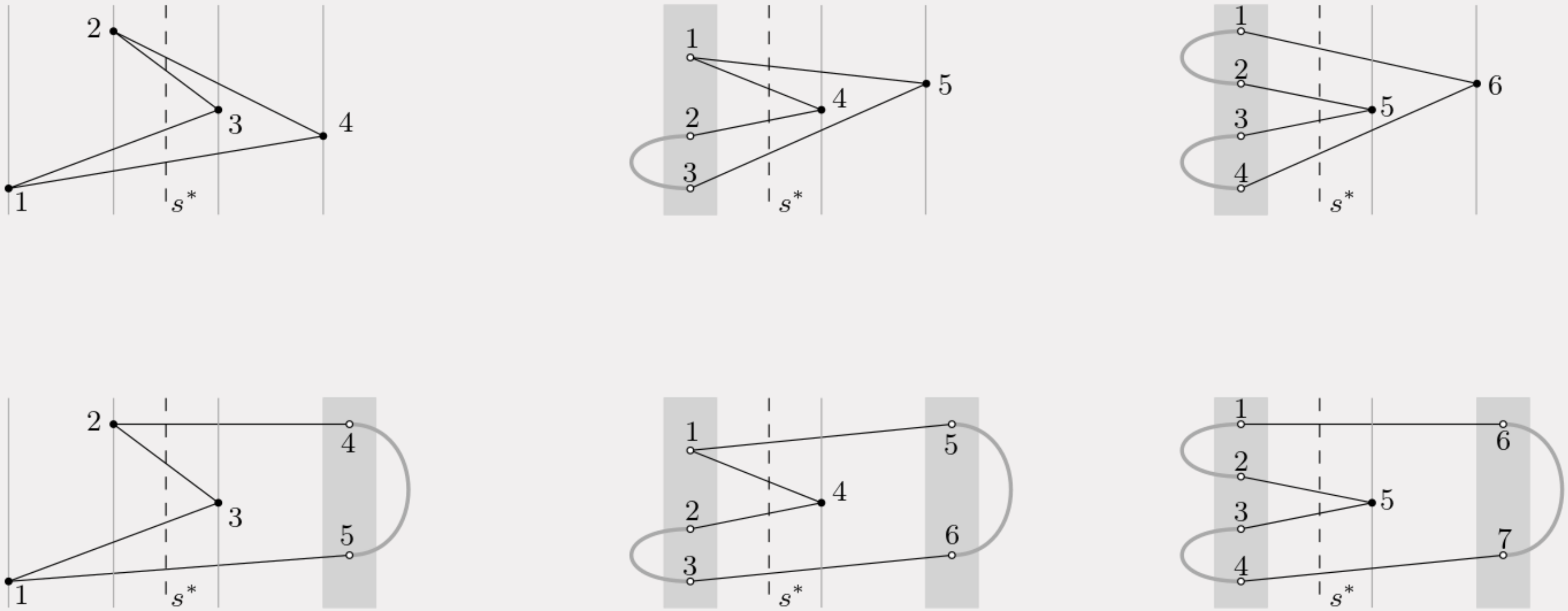
Superior edge set exists if ‘interesting’ points have consecutive x-coordinates

Proof sketch:

Case distinction on the connections between the ‘interesting’ points

Bitonicity of Euclidean TSP in Narrow Strips

All six possible cases. Points in grey blocks can have any horizontal ordering



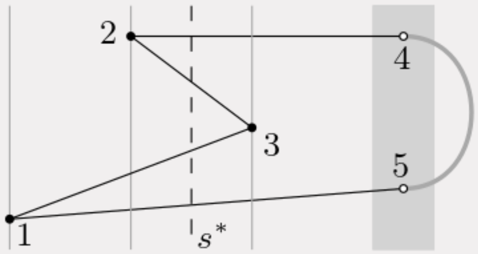
Bitonicity of Euclidean TSP in Narrow Strips

Proof sketch:

Case distinction on the connections between the 'interesting' points

For each case, this can be proven either algebraically, or by computer assistance

The figure to the right gives the bound (both horizontal orderings of points 4 and 5 give the same bound of $2\sqrt{2}$)



Questions?

Feel free to let us know!