

Expected Complexity of Routing in Θ_6 and Half- Θ_6 graphs

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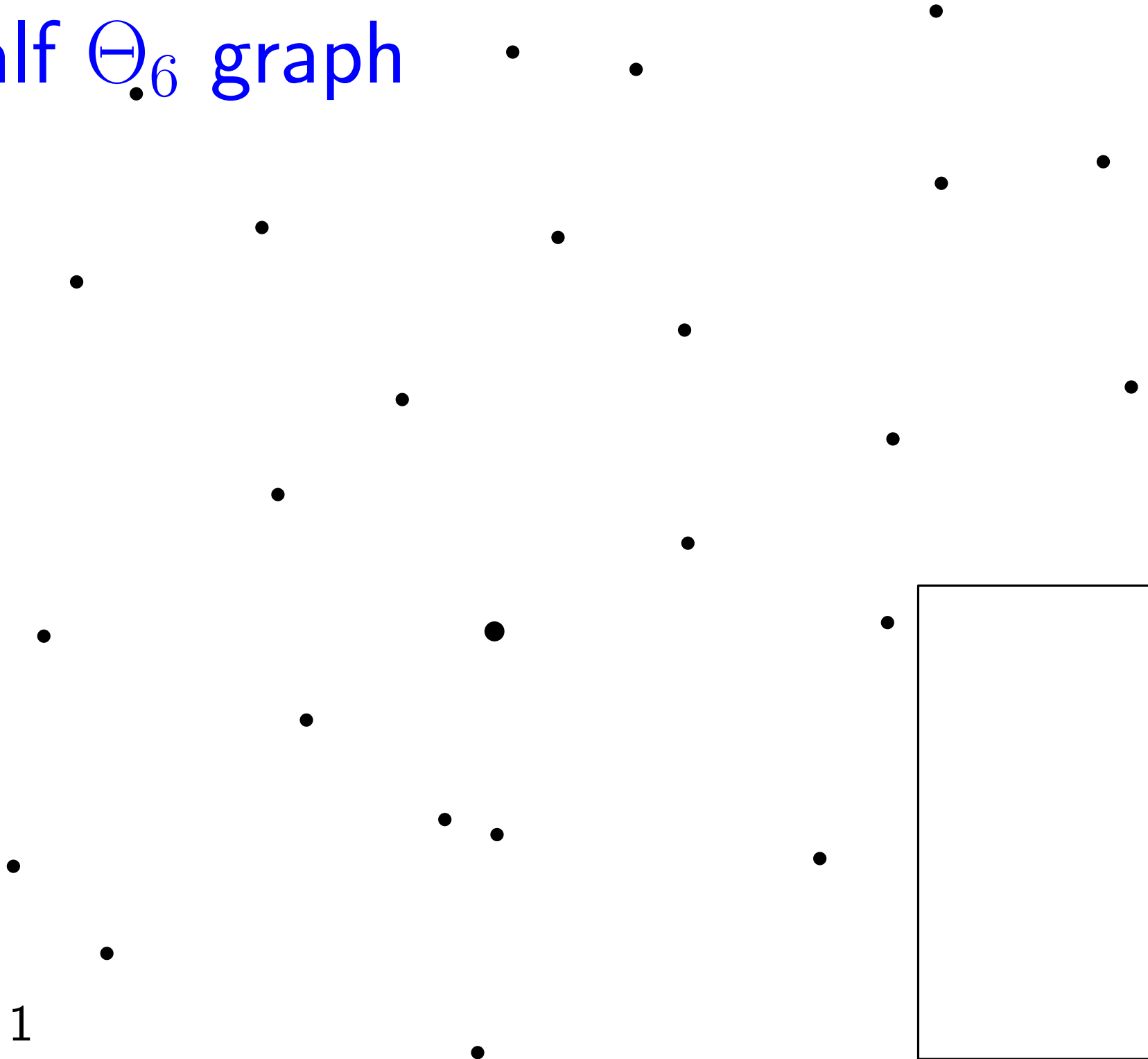
Carleton
UNIVERSITY

C G L A B



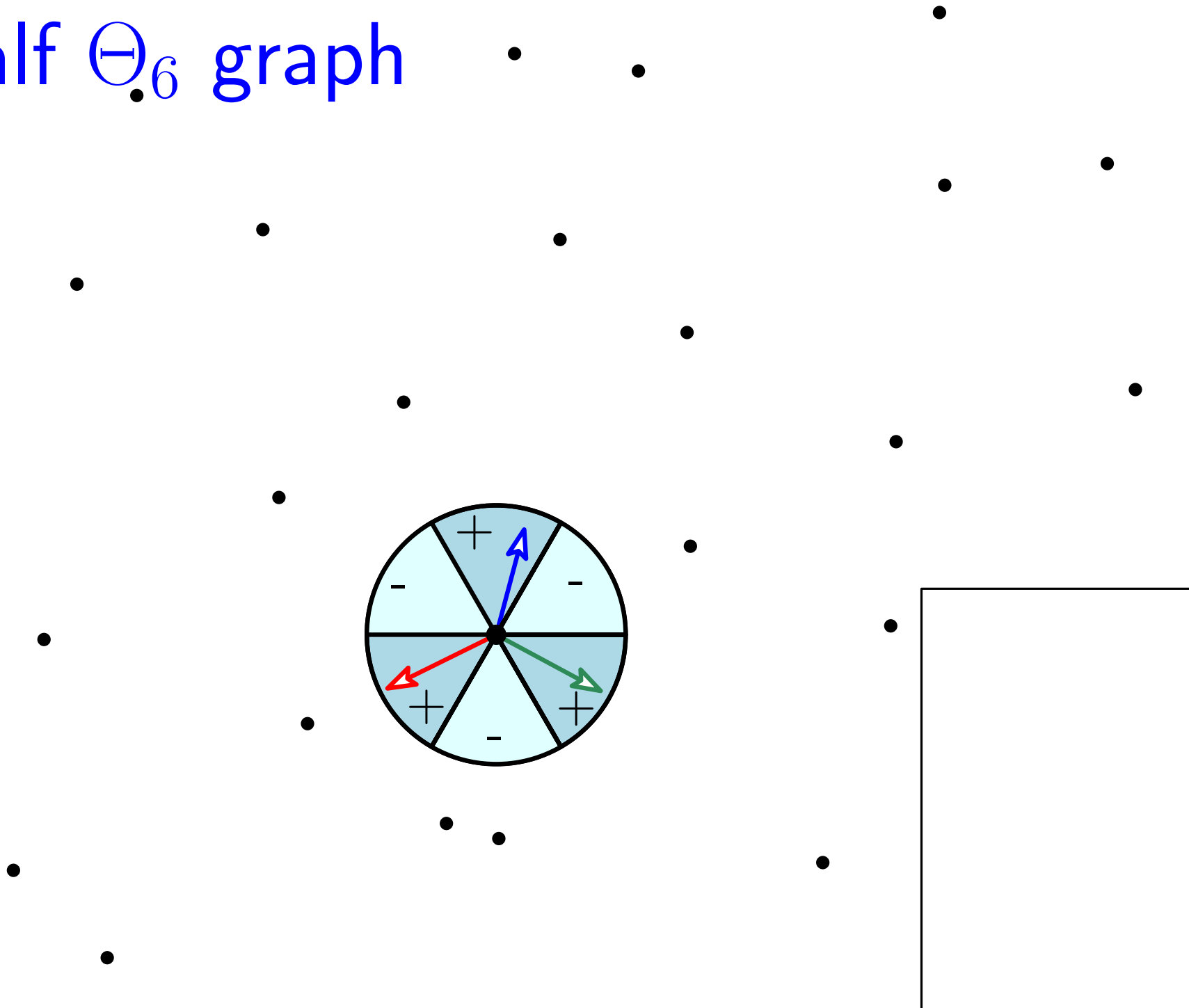
uOttawa

Half Θ_6 graph

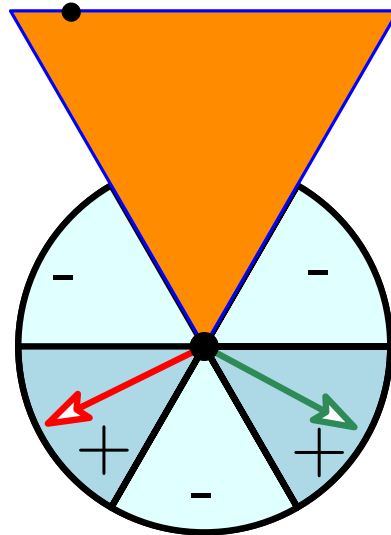


2 - 1

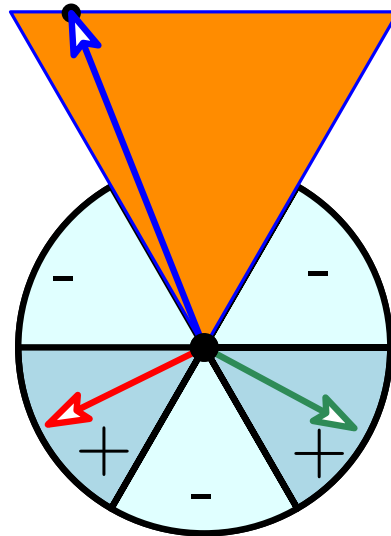
Half Θ_6 graph



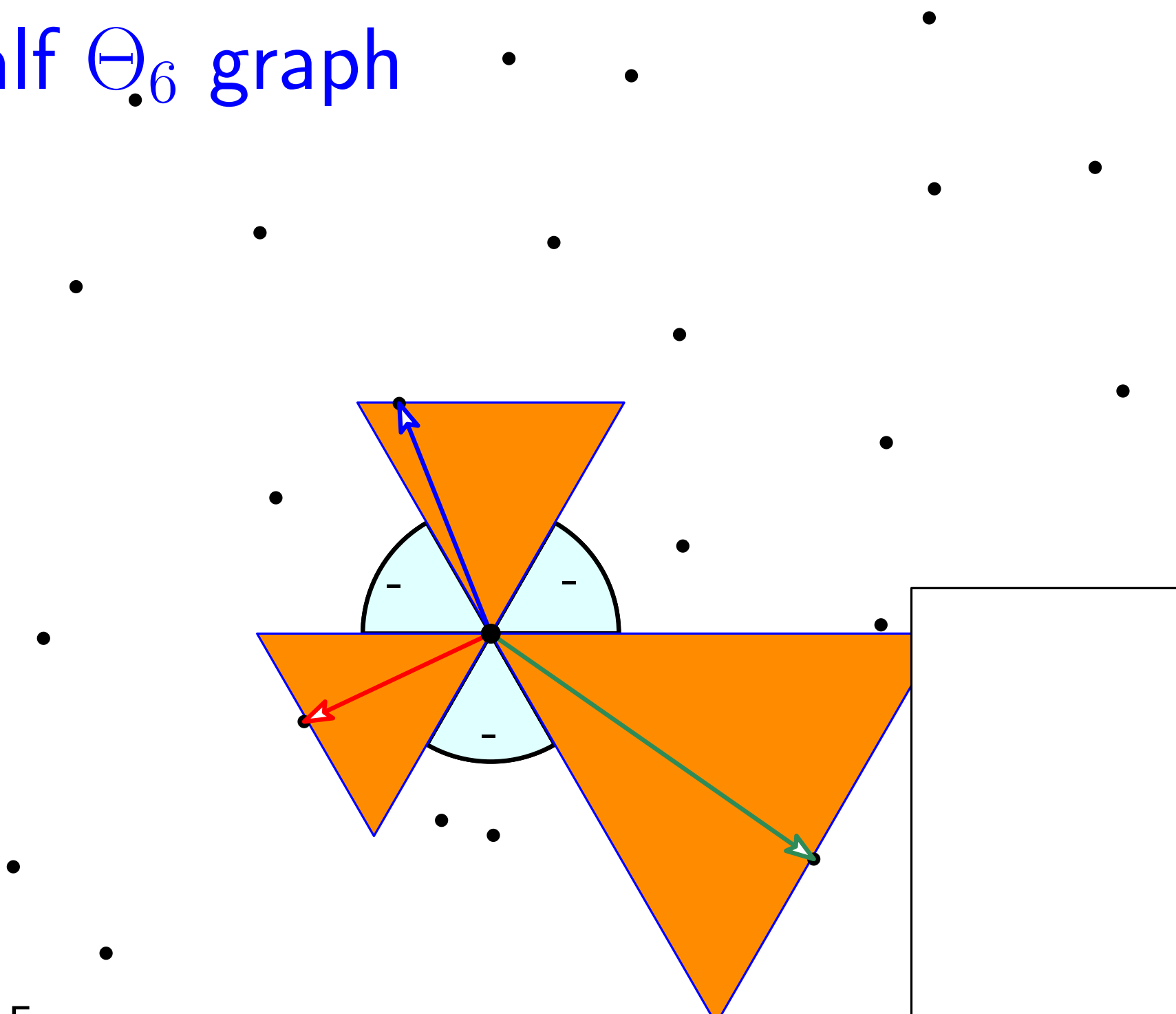
Half Θ_6 graph



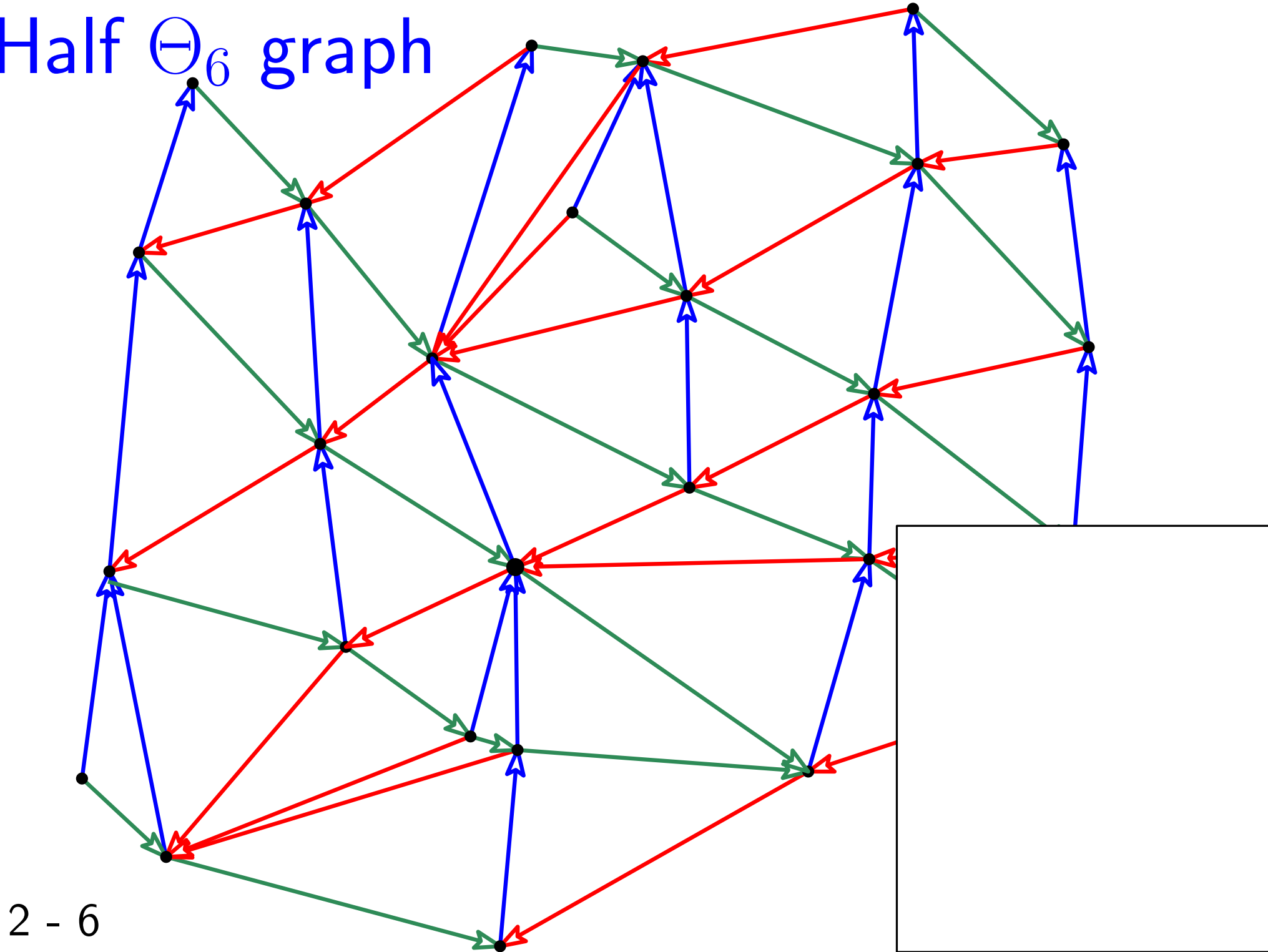
Half Θ_6 graph



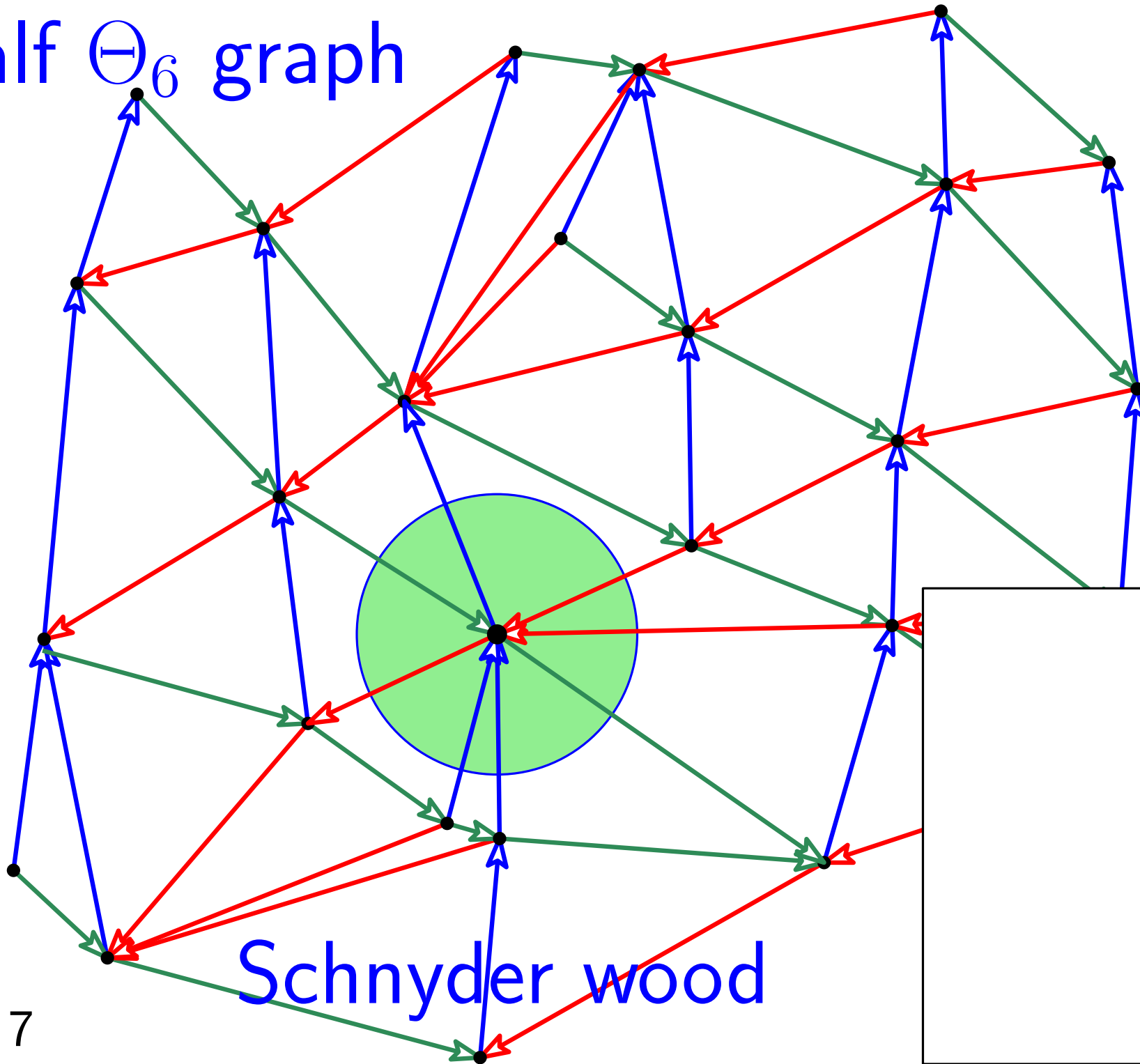
Half Θ_6 graph



Half Θ_6 graph



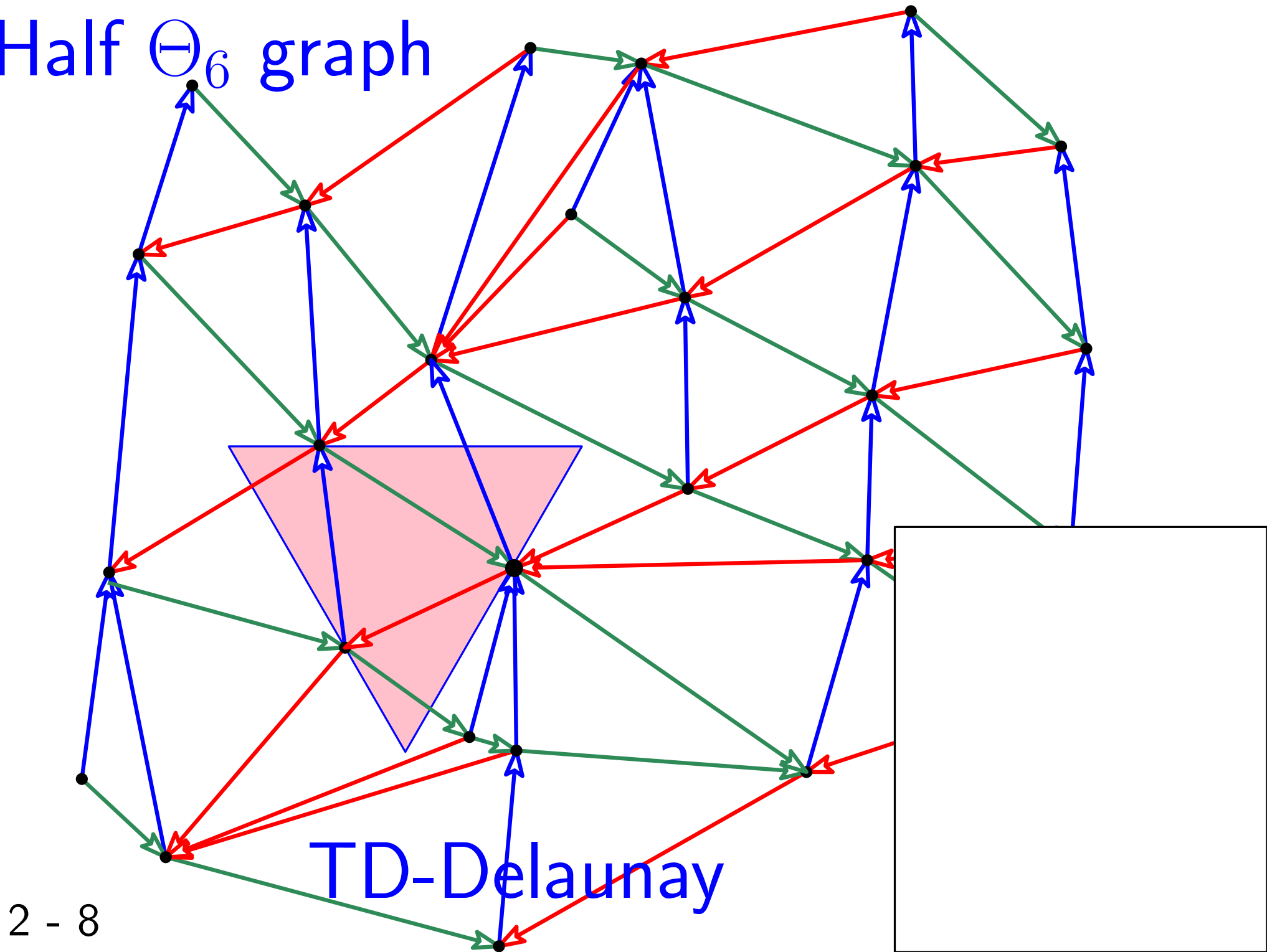
Half Θ_6 graph



2 - 7

Schnyder wood

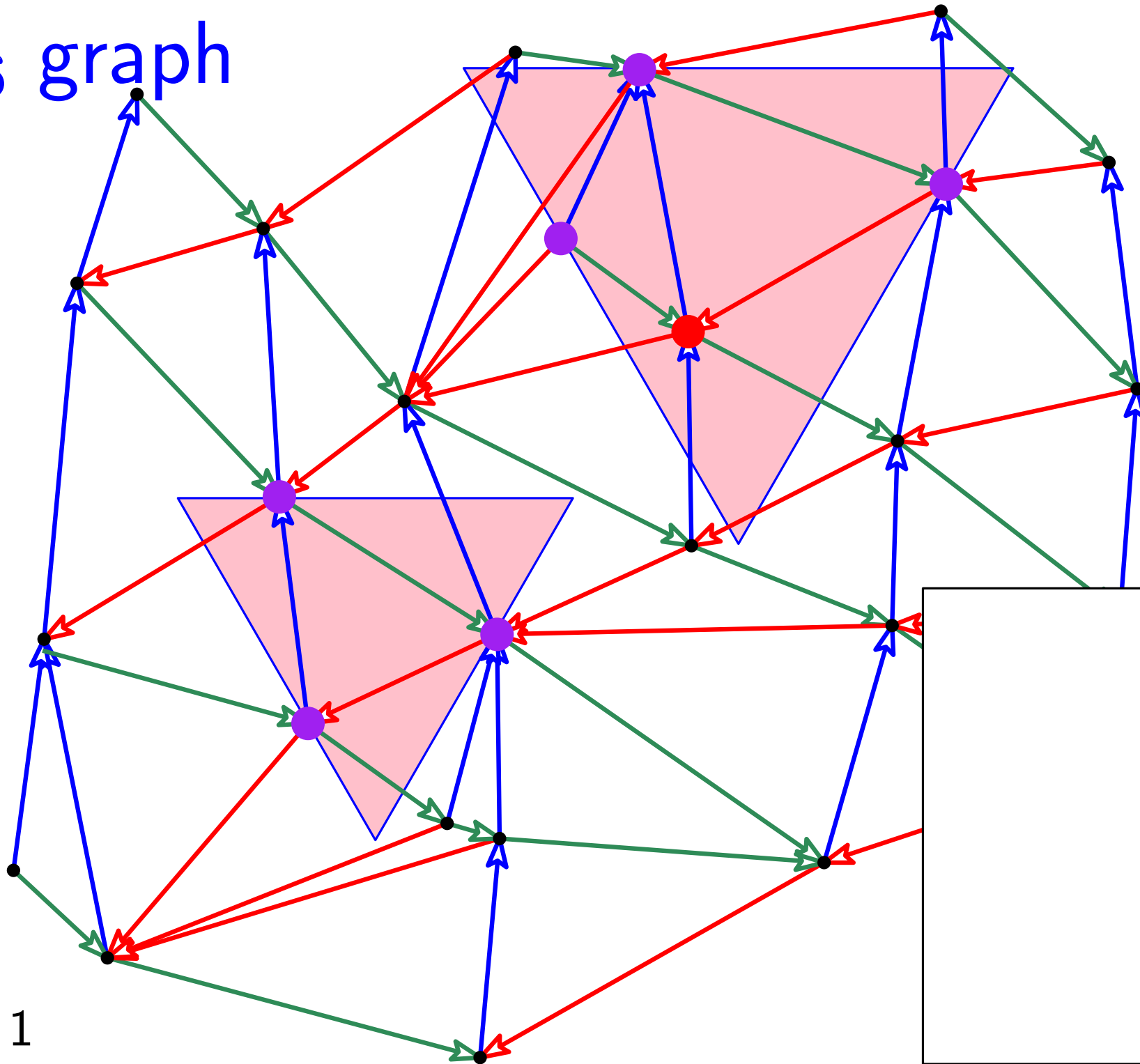
Half Θ_6 graph



TD-Delaunay

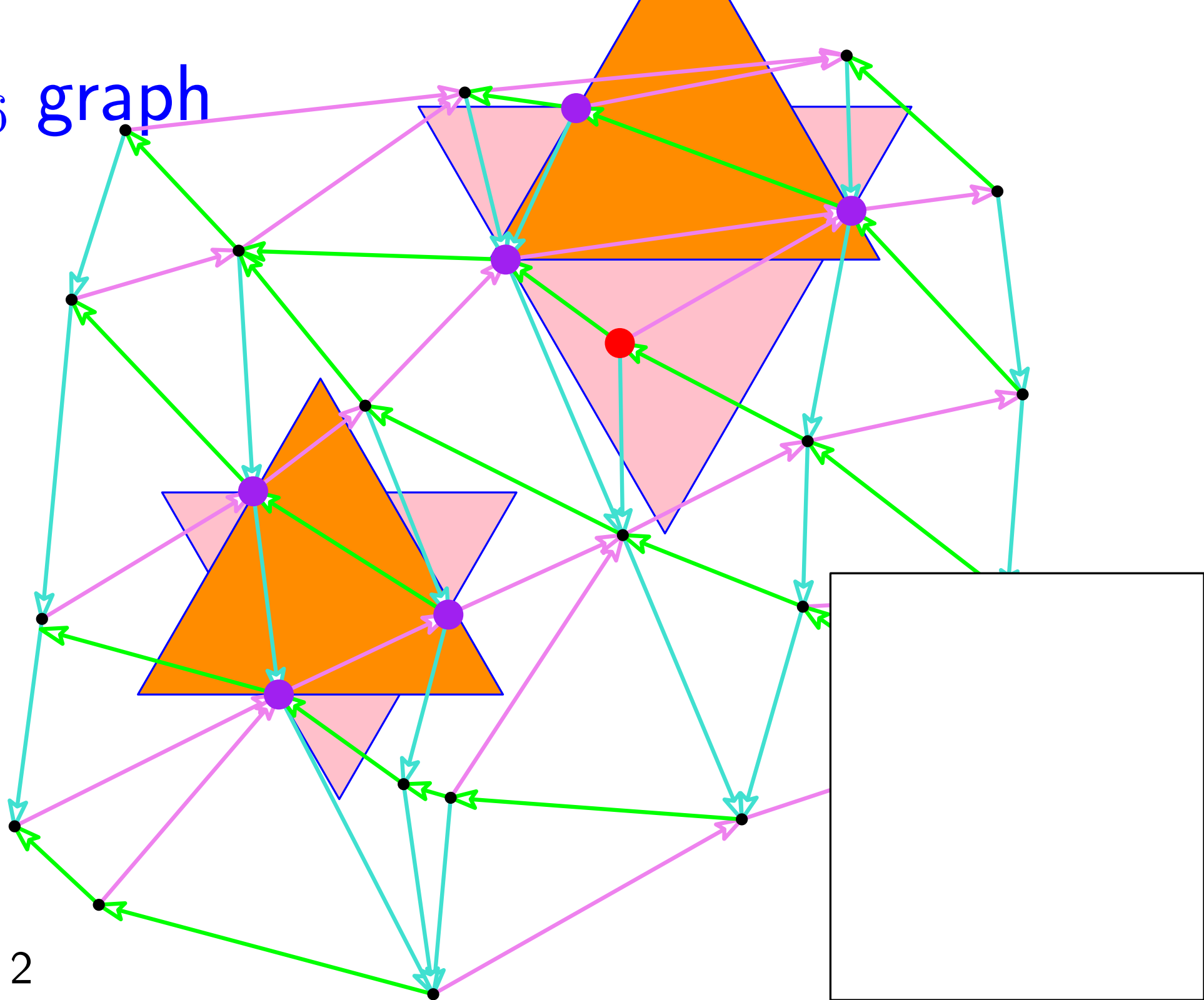
2 - 8

Θ_6 graph



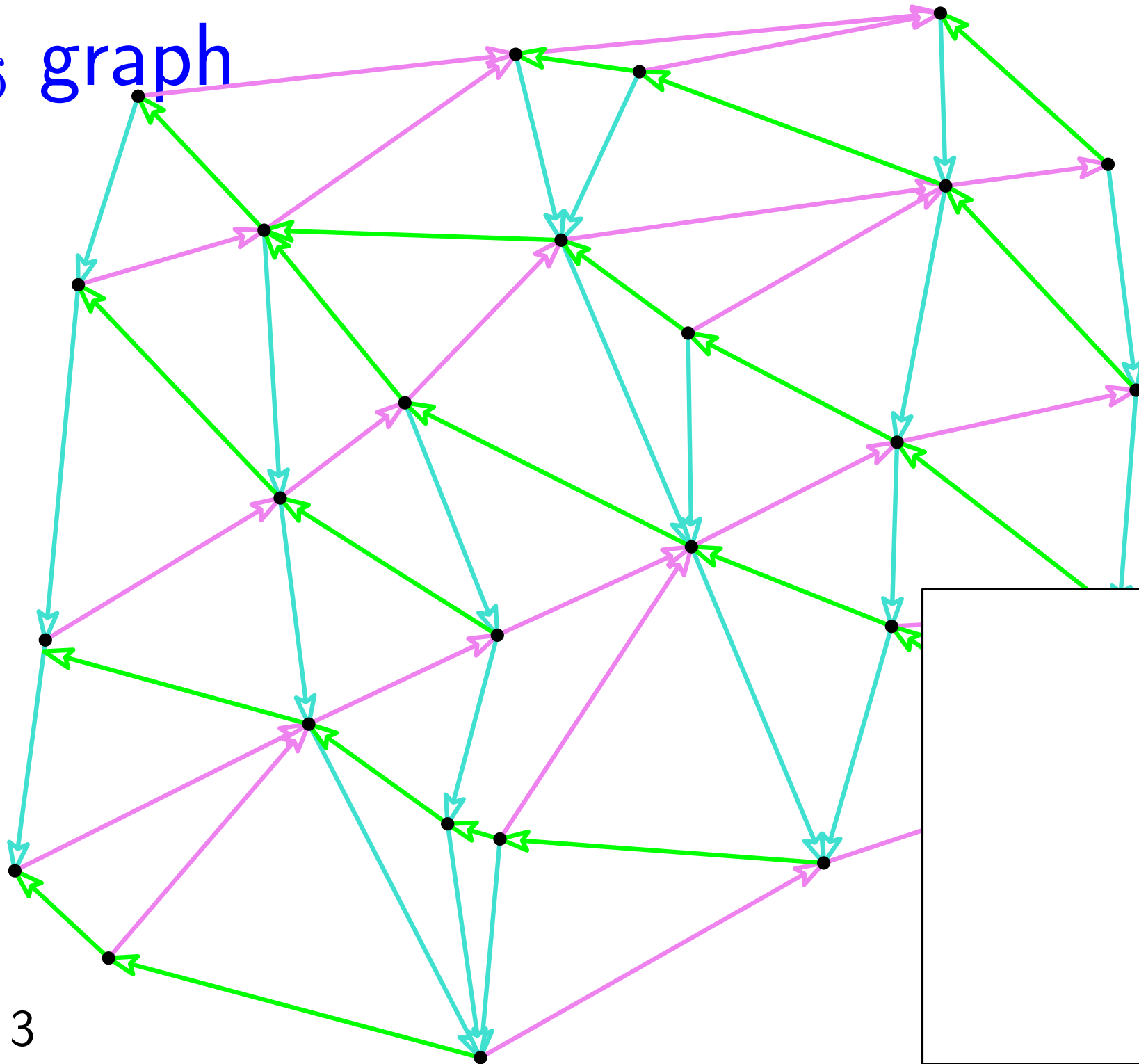
3 - 1

Θ_6 graph

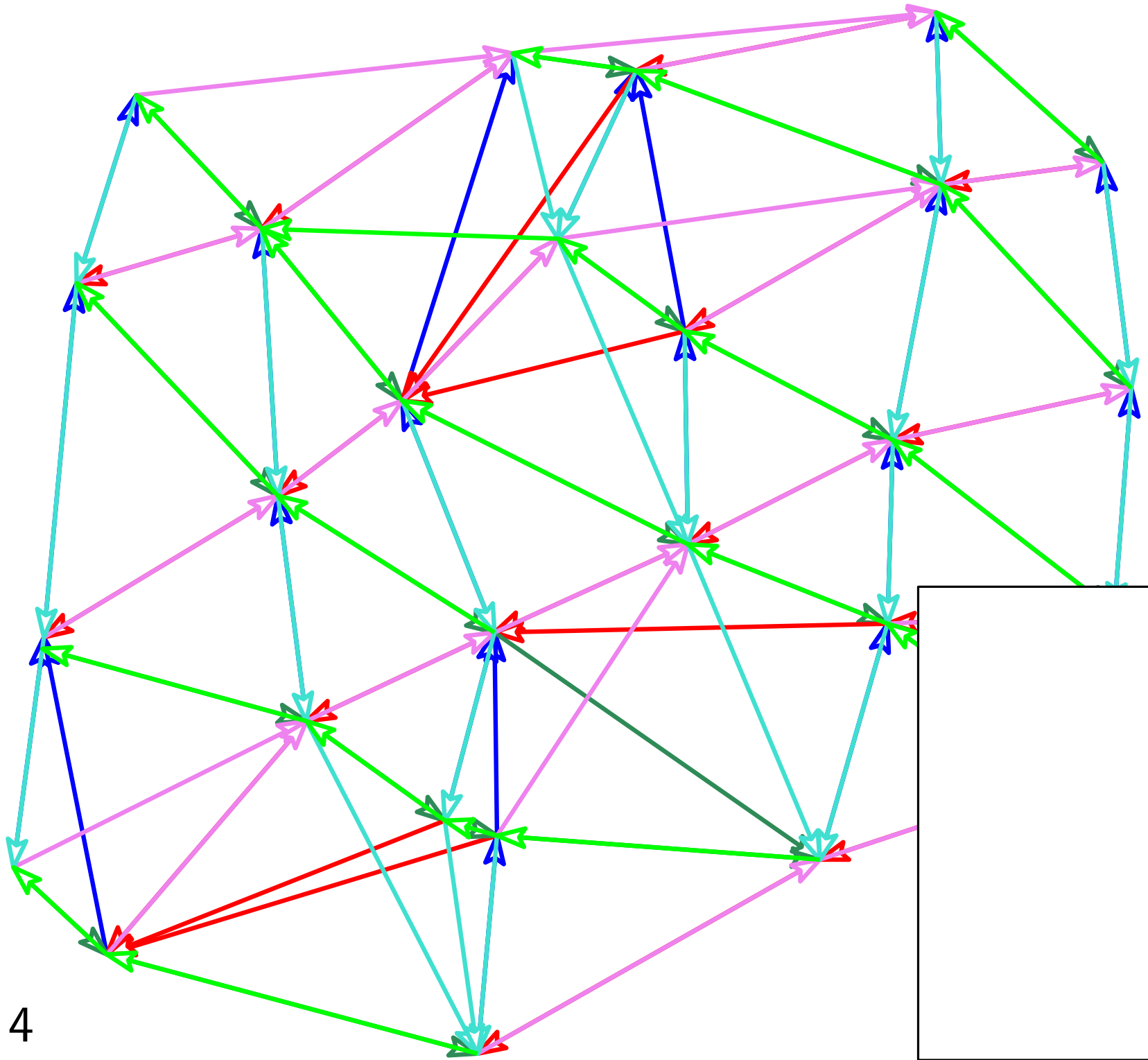


3 - 2

Θ_6 graph

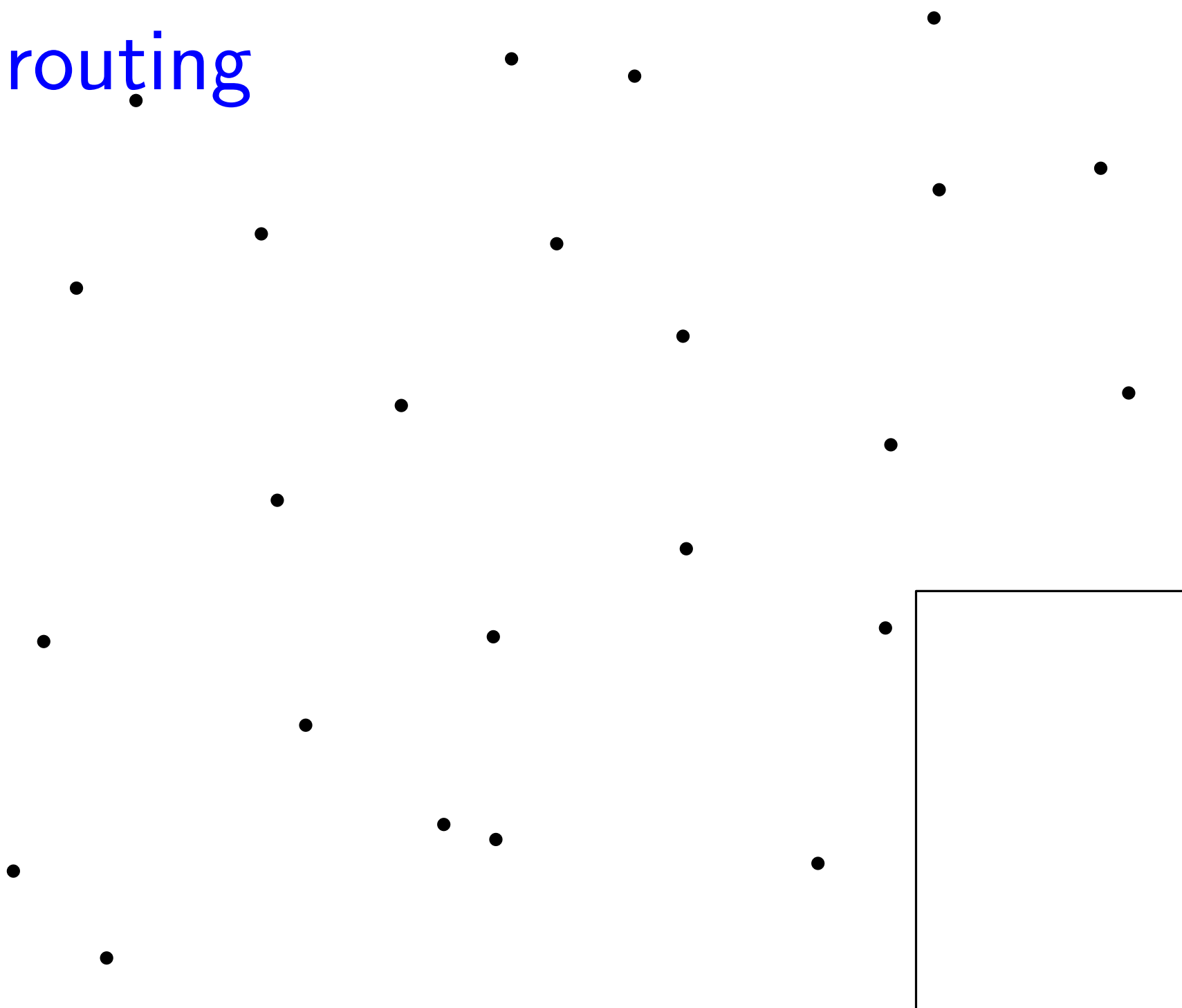


3 - 3



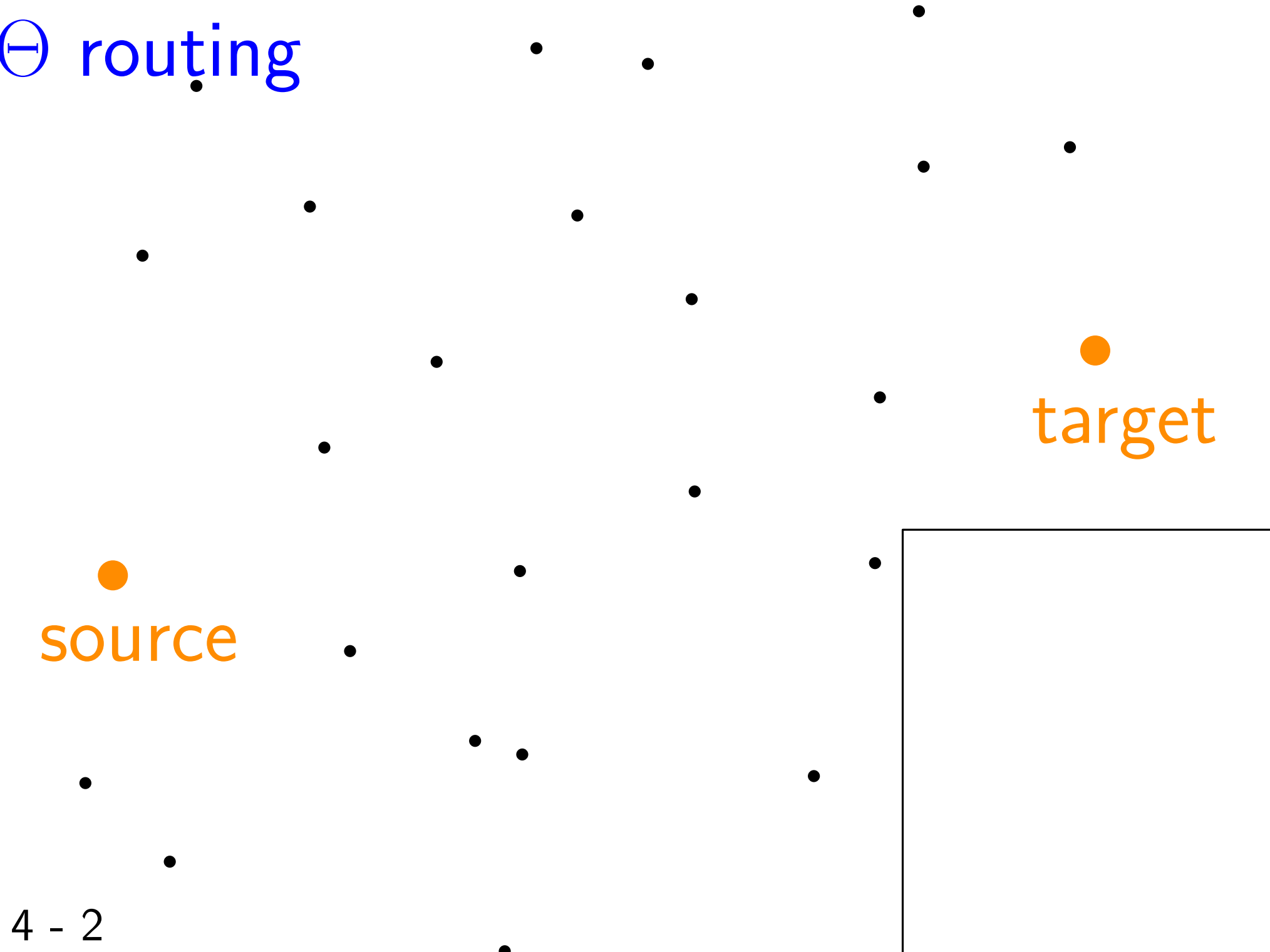
3 - 4

⊖ routing



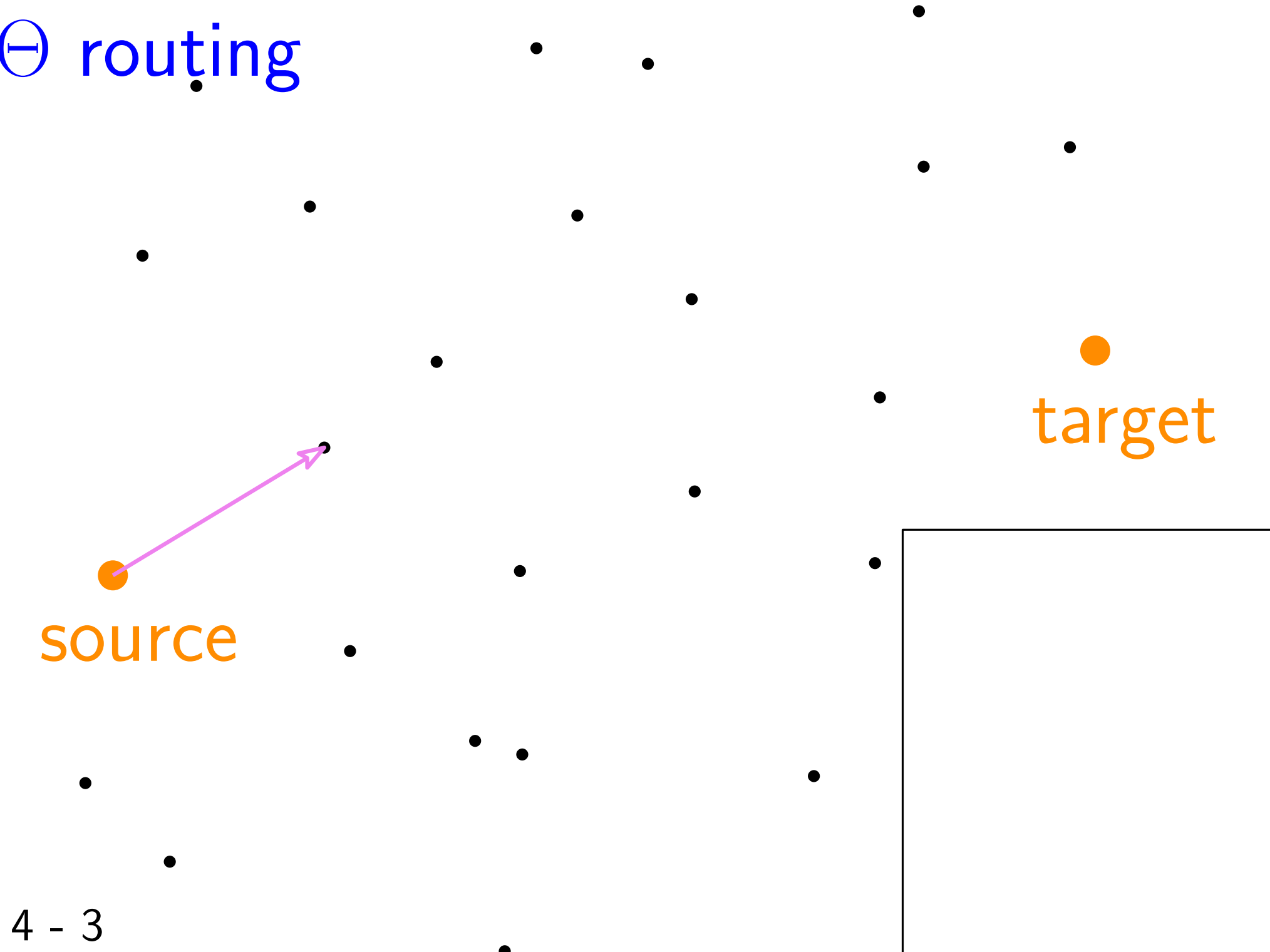
4 - 1

⊖ routing



4 - 2

⊖ routing



target

source

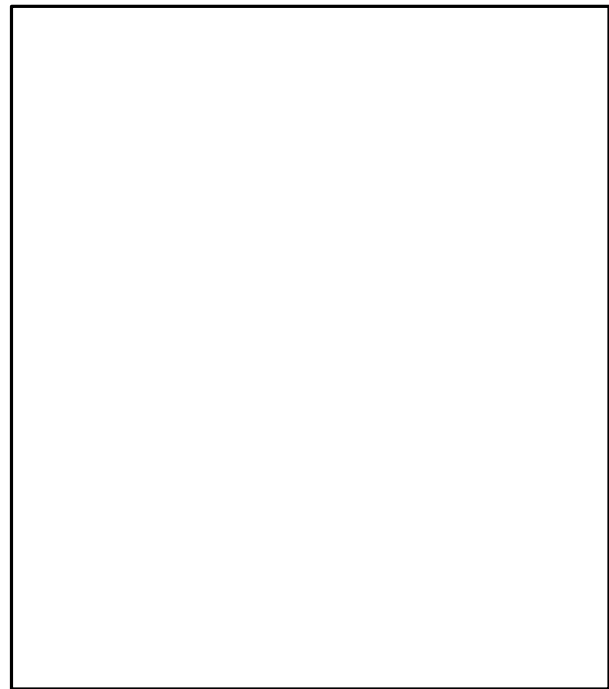
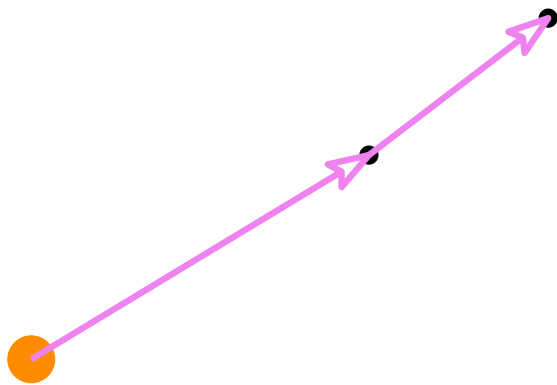
4 - 3

⊖ routing

source

target

4 - 4

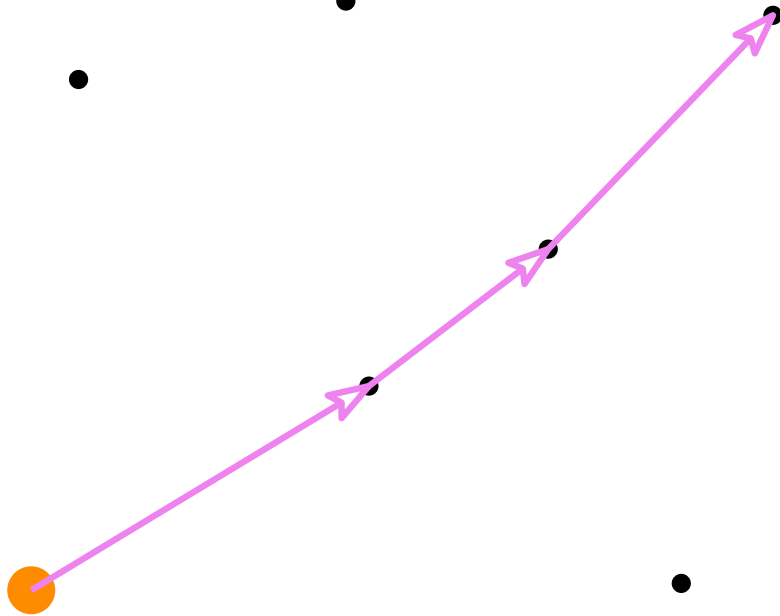


⊖ routing

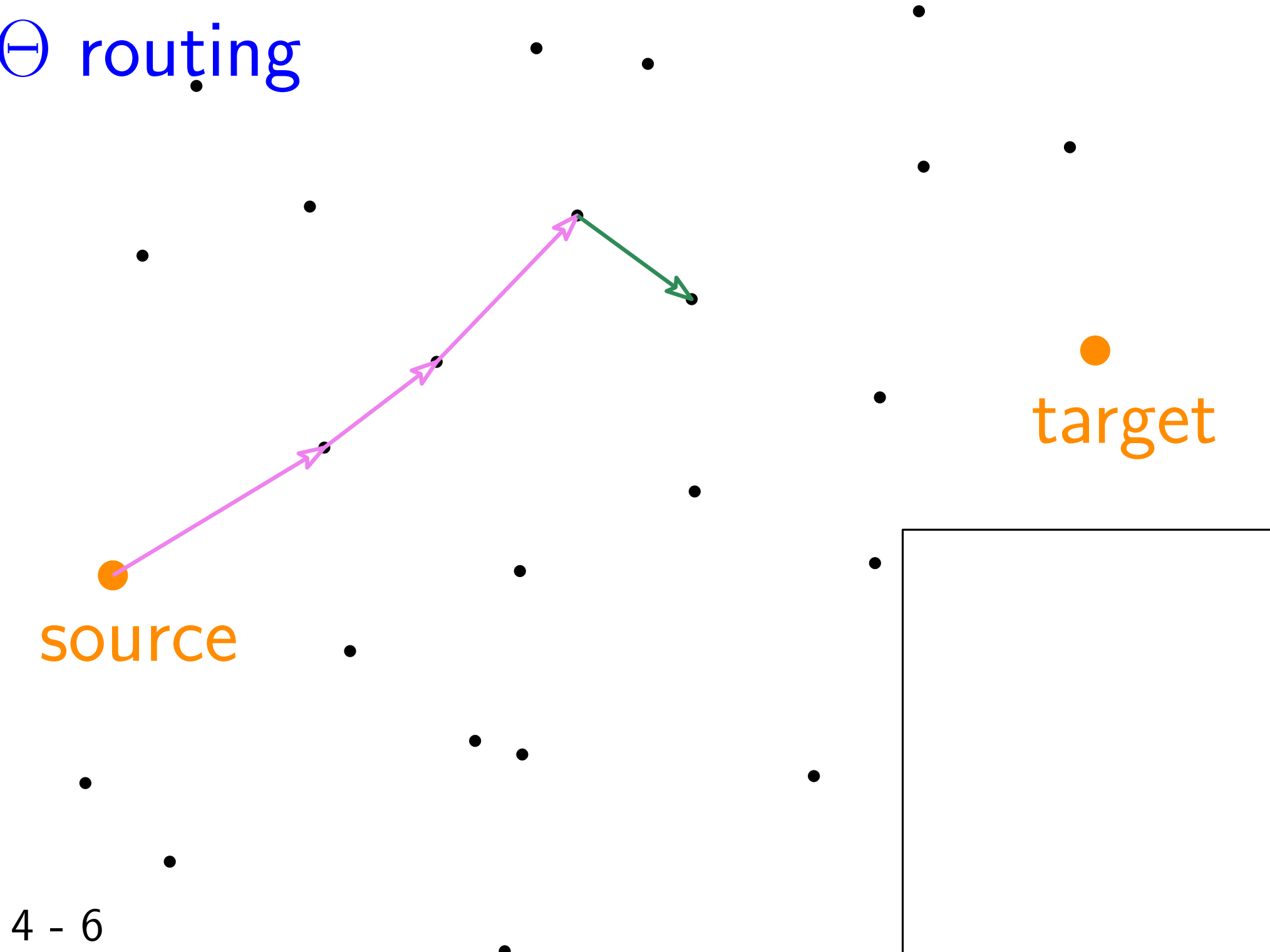
source

target

4 - 5



⊖ routing



source

target

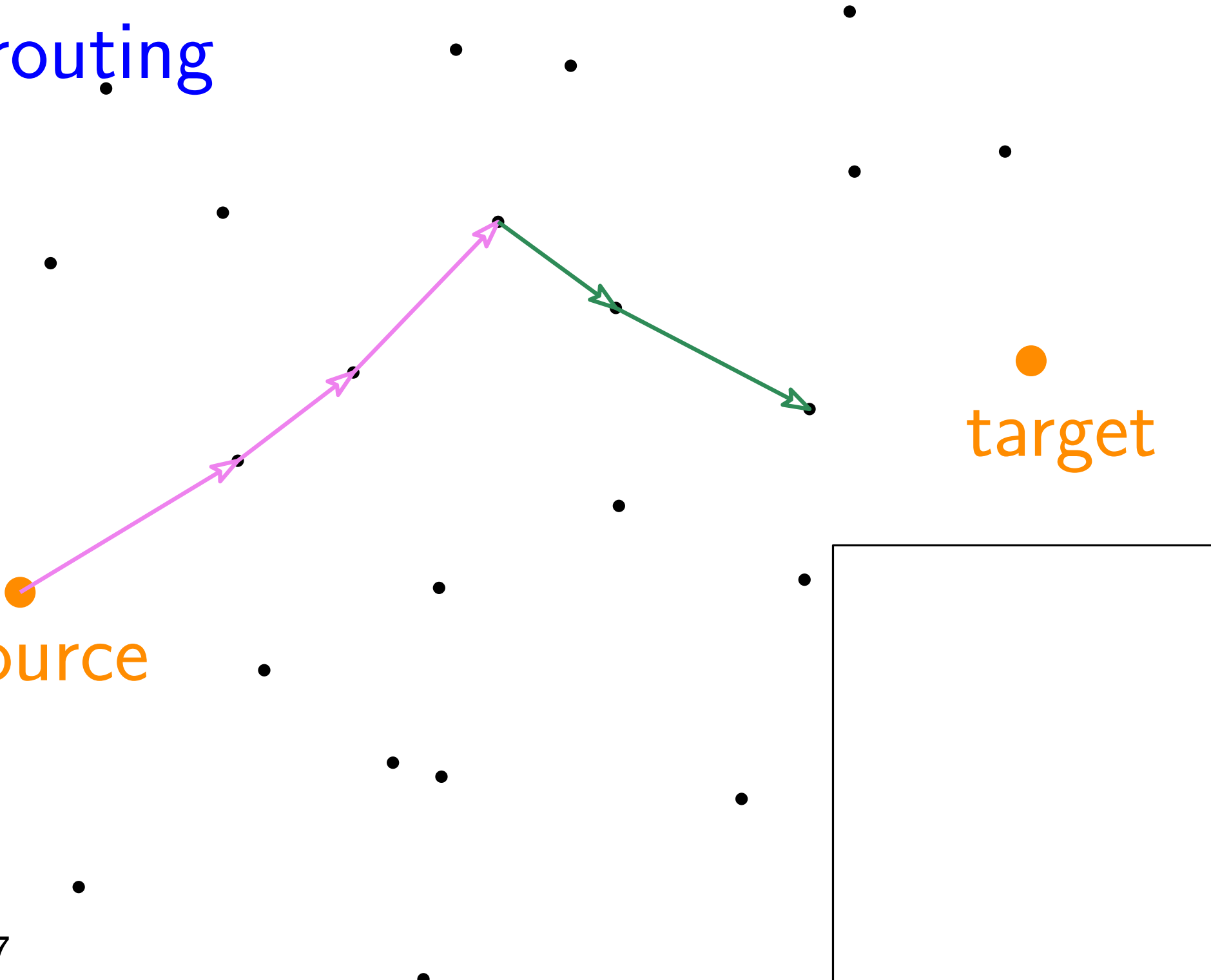
4 - 6

⊖ routing

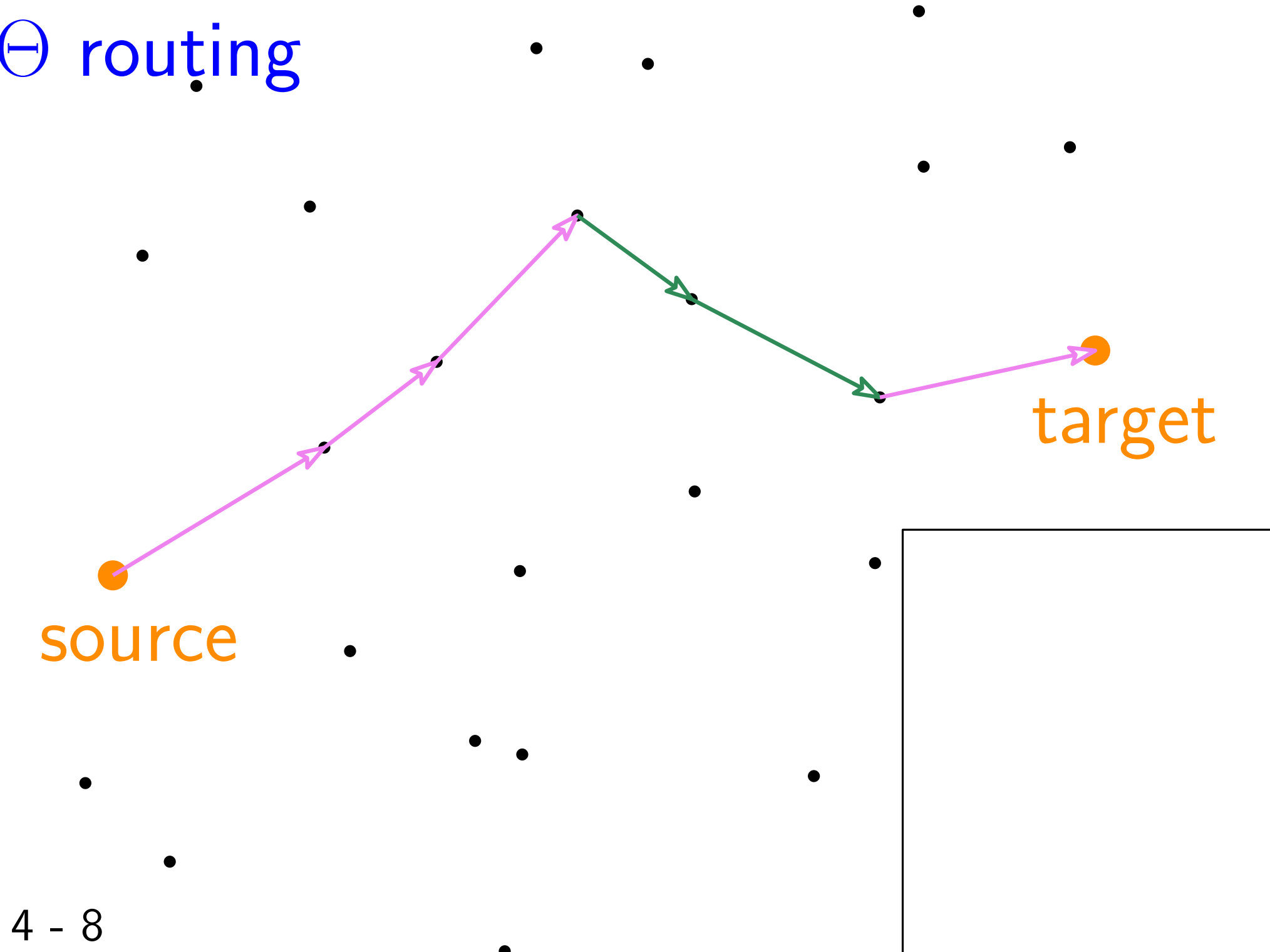
source

target

4 - 7



⊖ routing

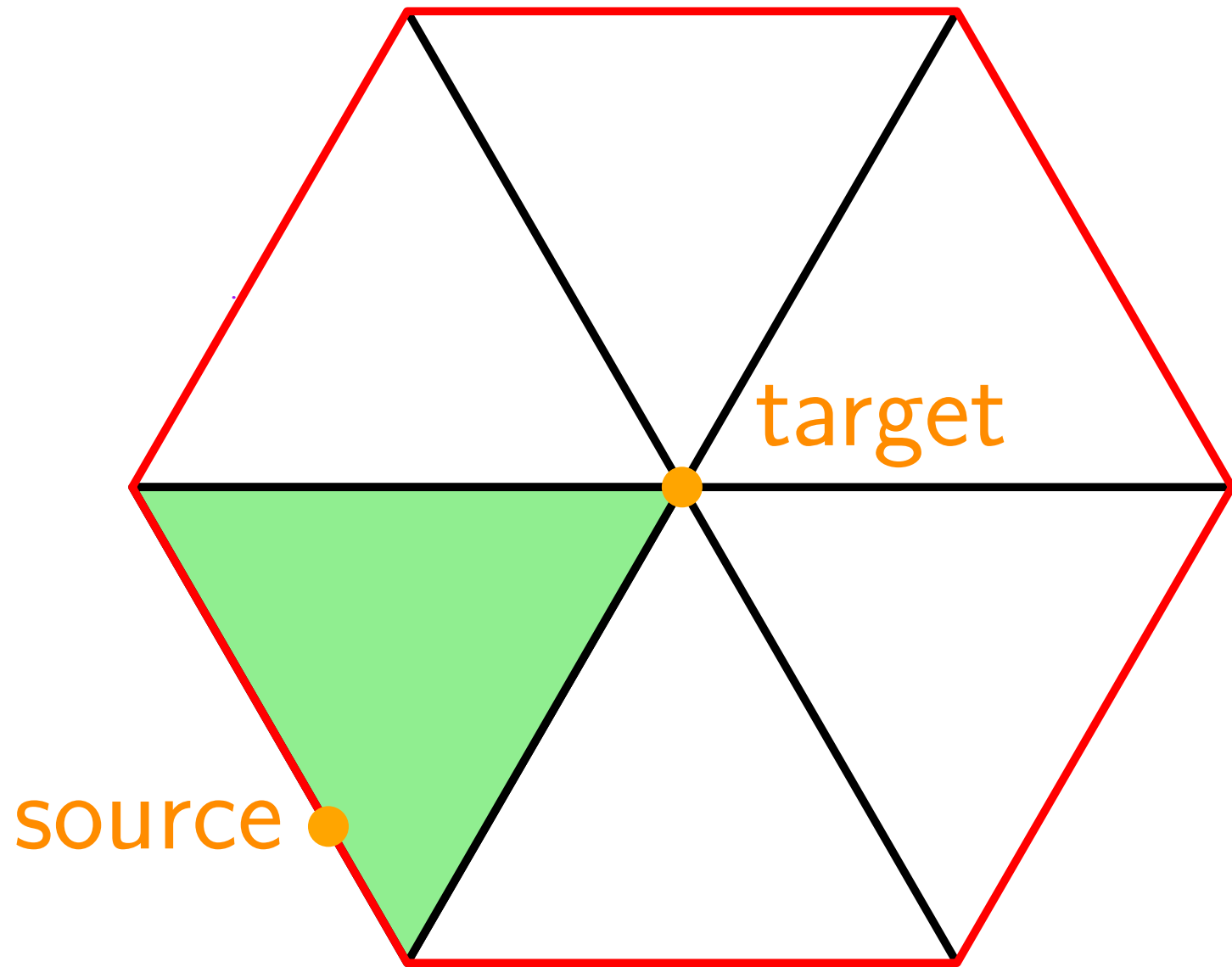


source

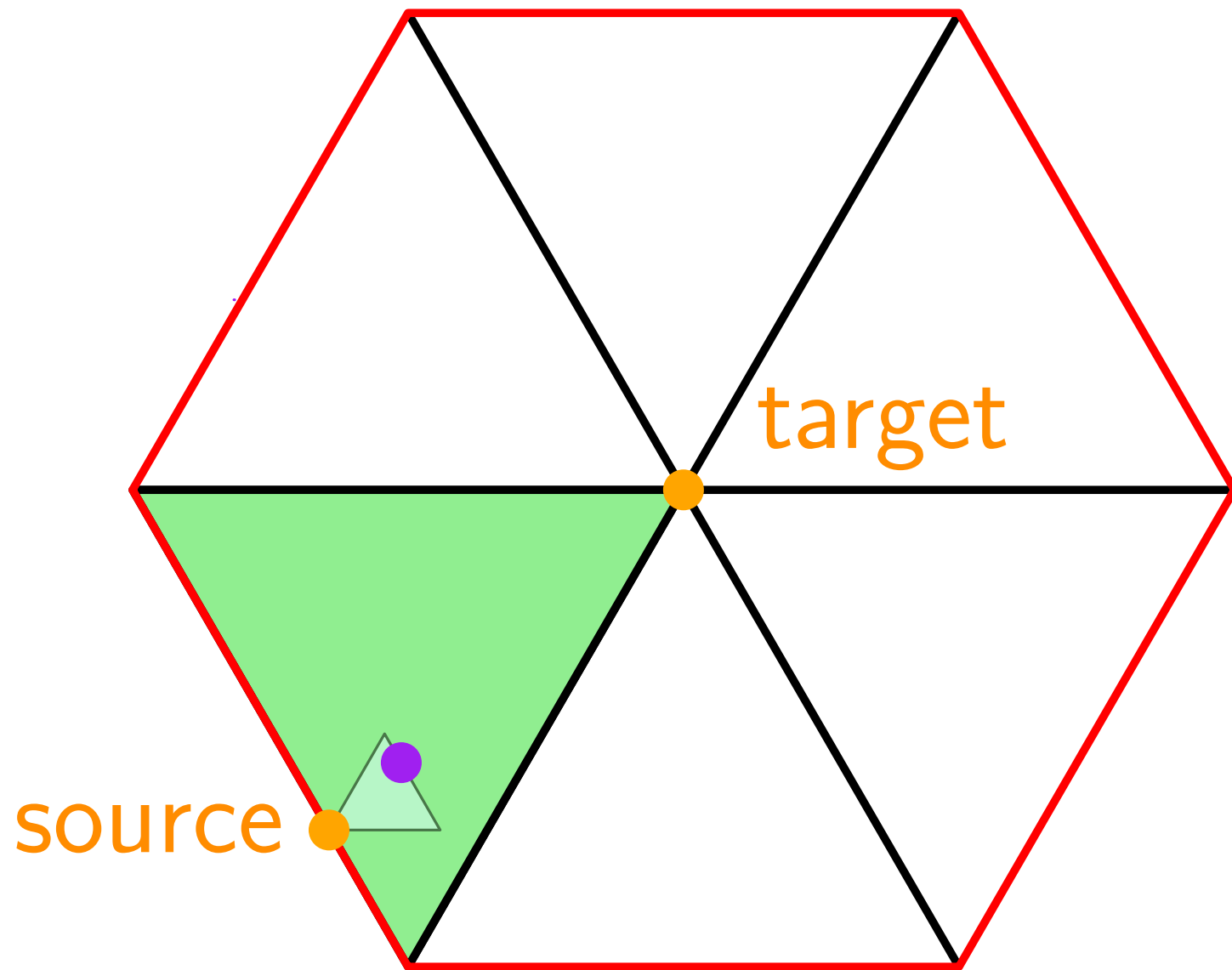
target

4 - 8

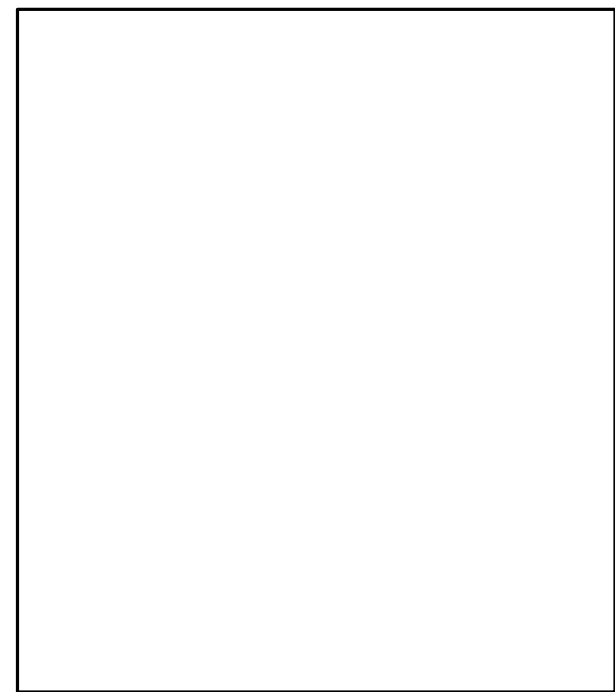
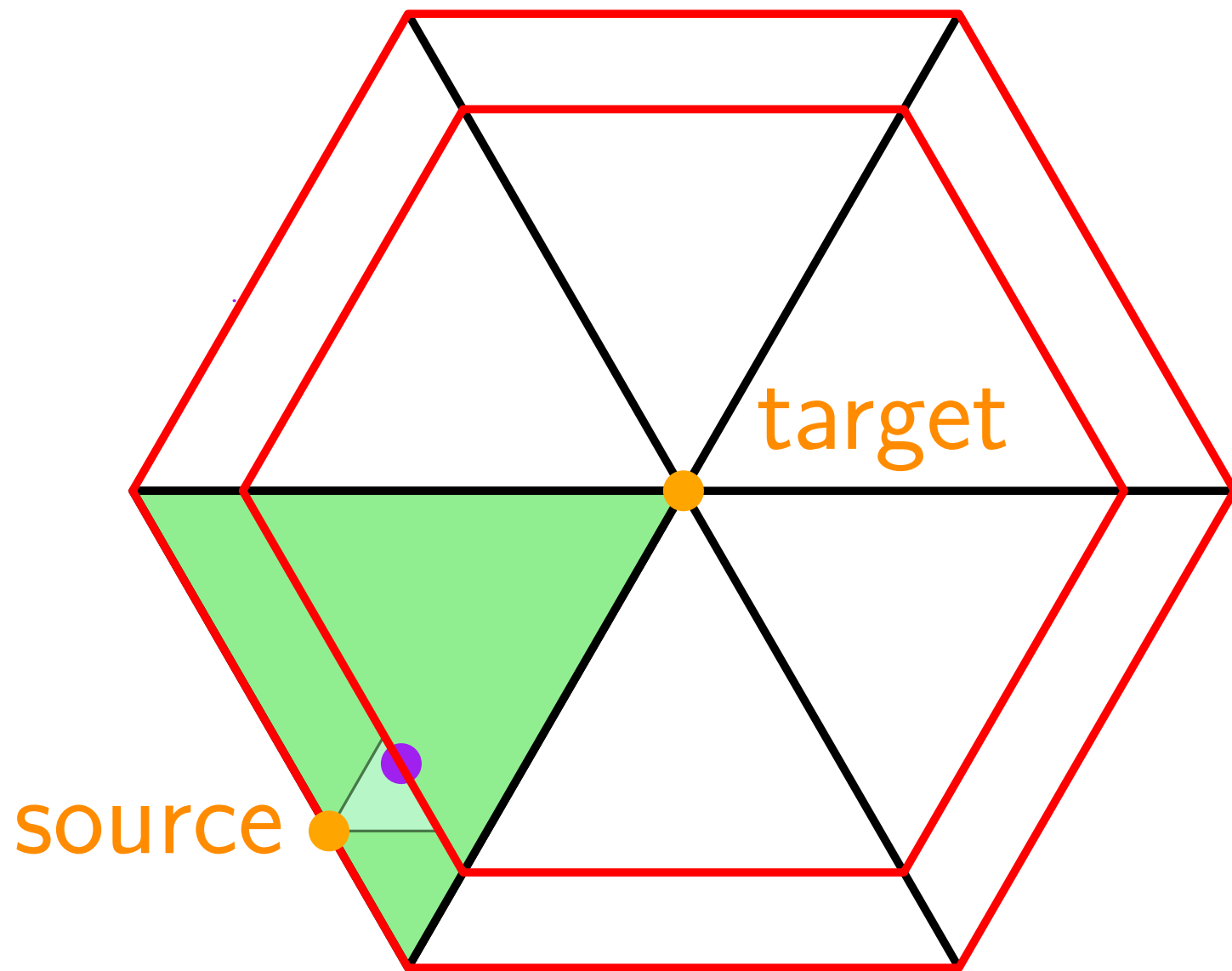
⊖ routing



⊖ routing



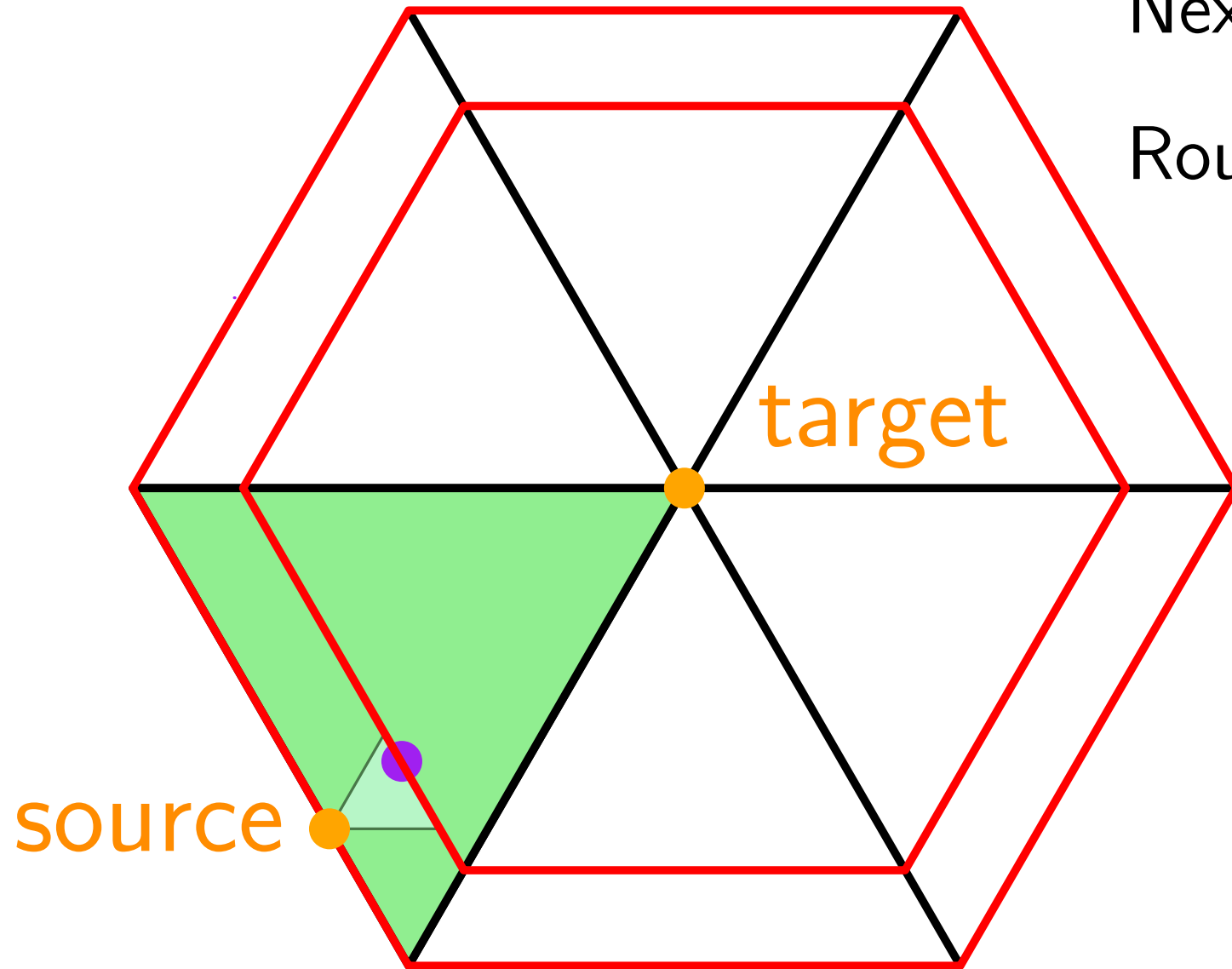
⊖ routing



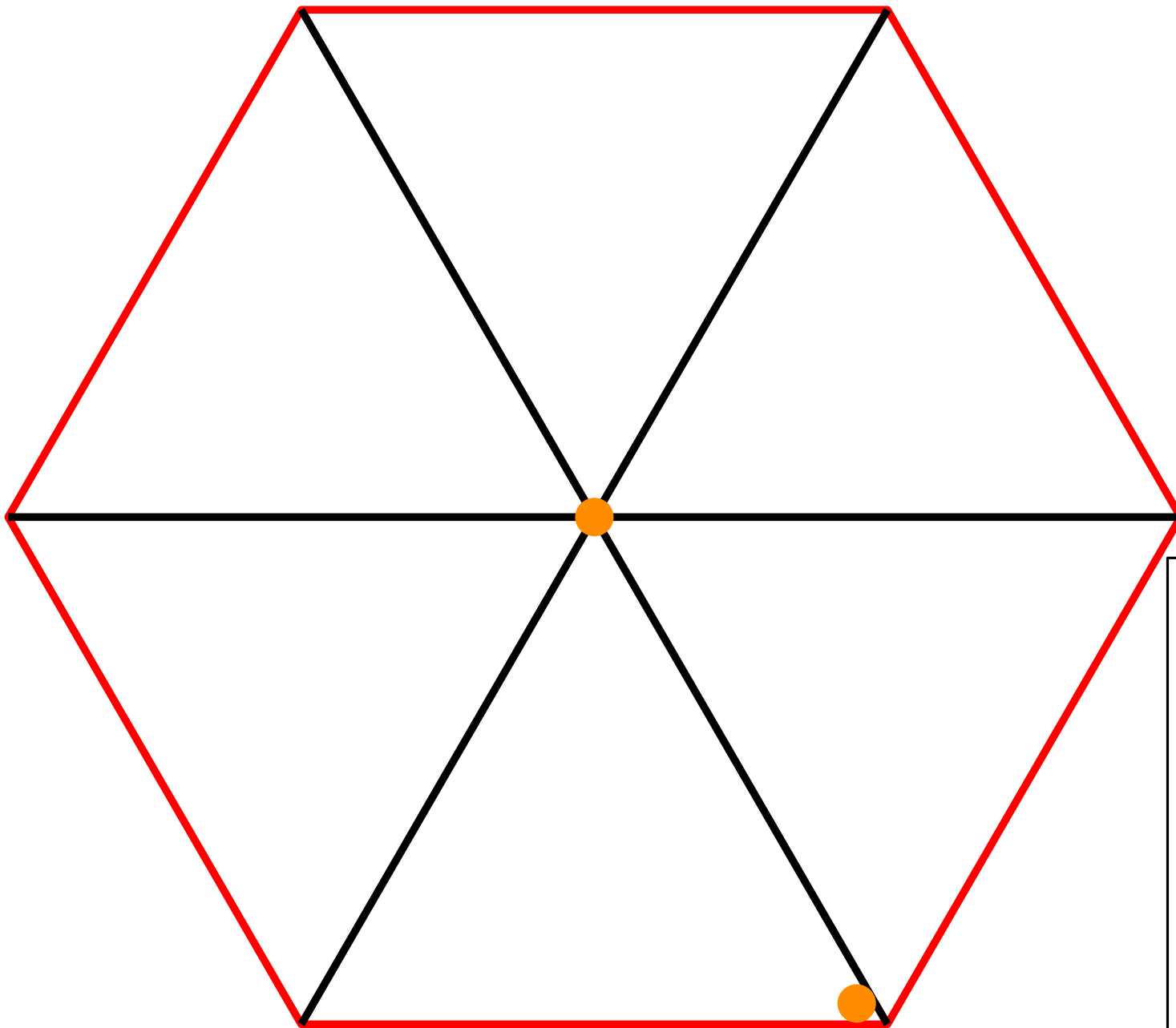
⊖ routing

Next point is closer

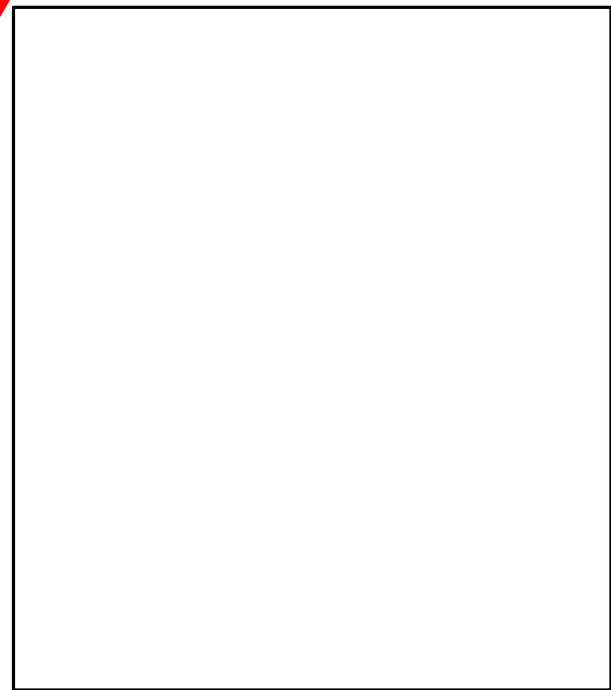
Routing terminates



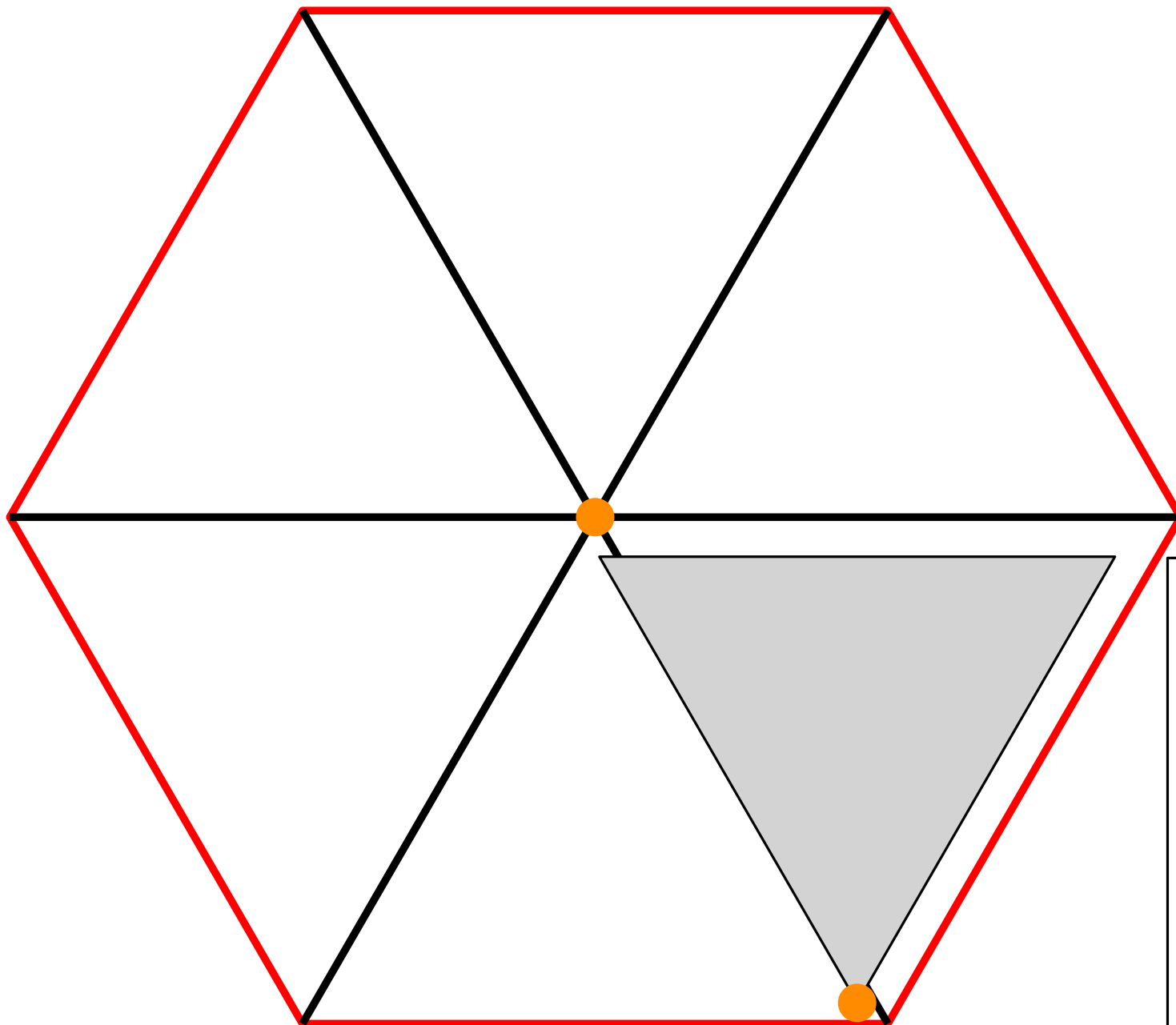
⊖ routing



6 - 1

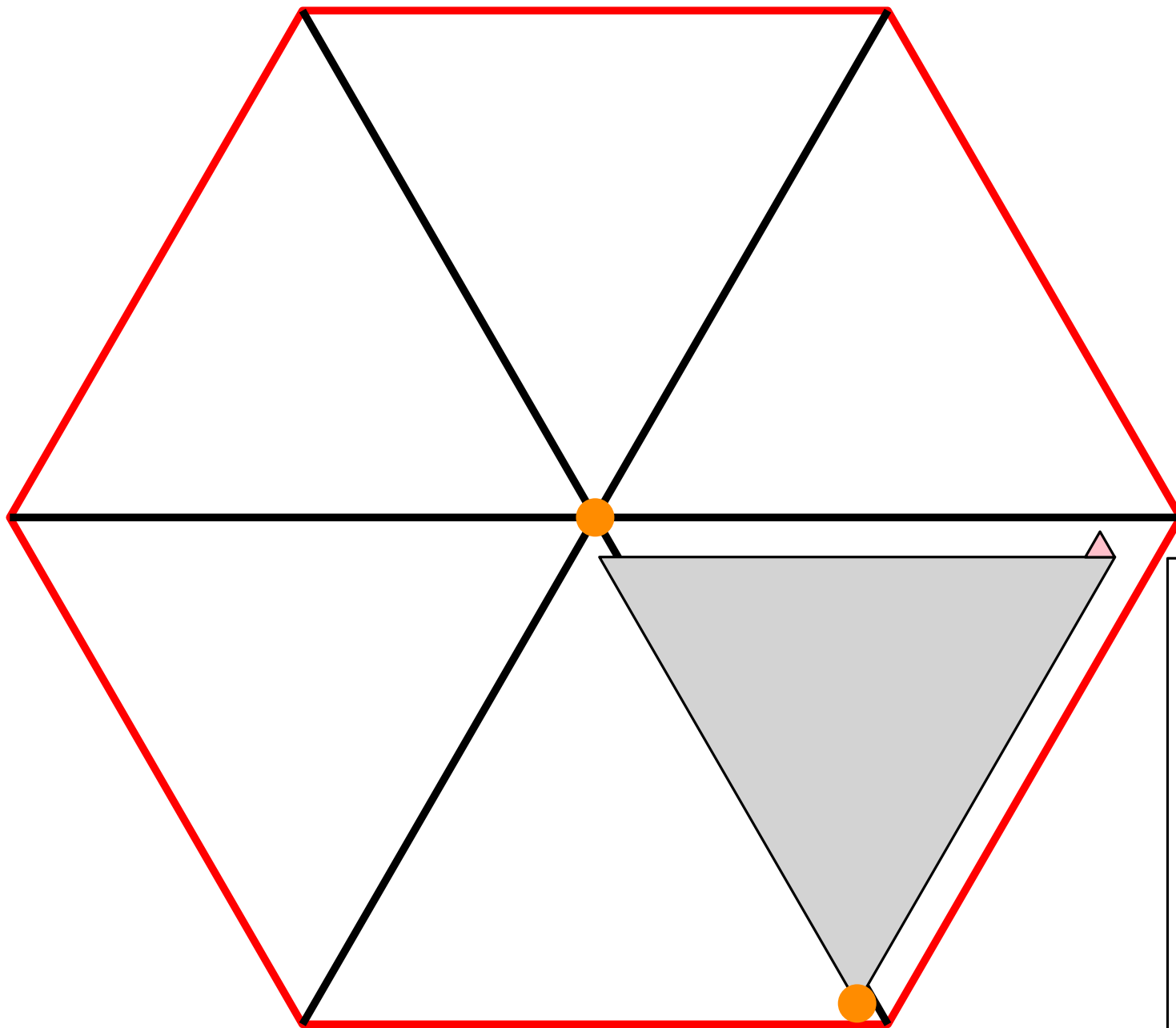


⊖ routing

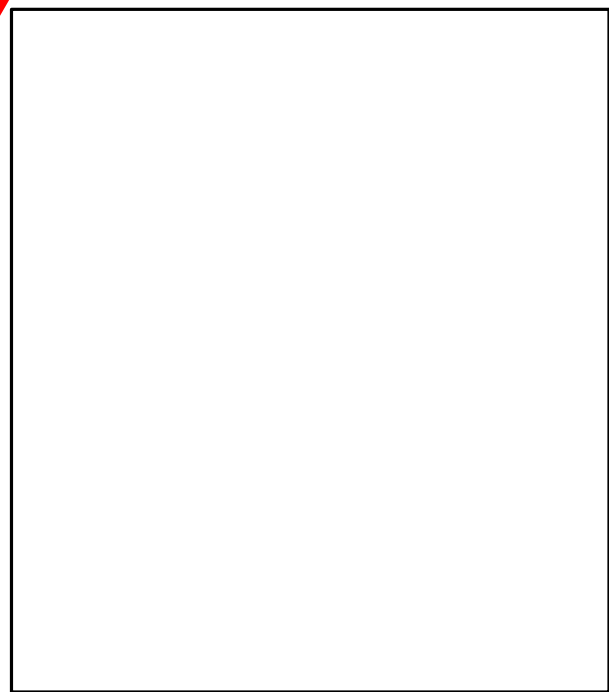


6 - 2

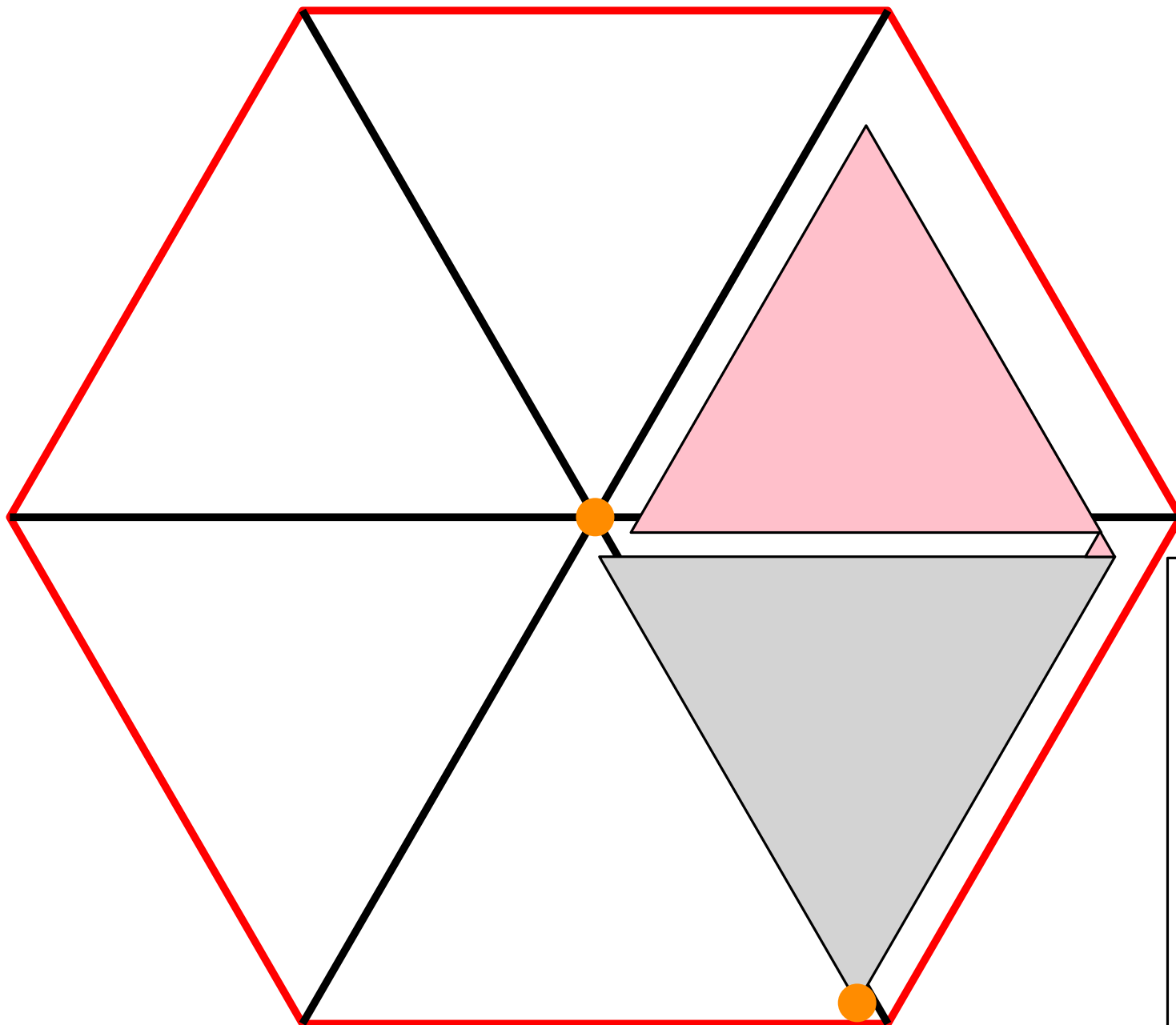
⊖ routing



6 - 3

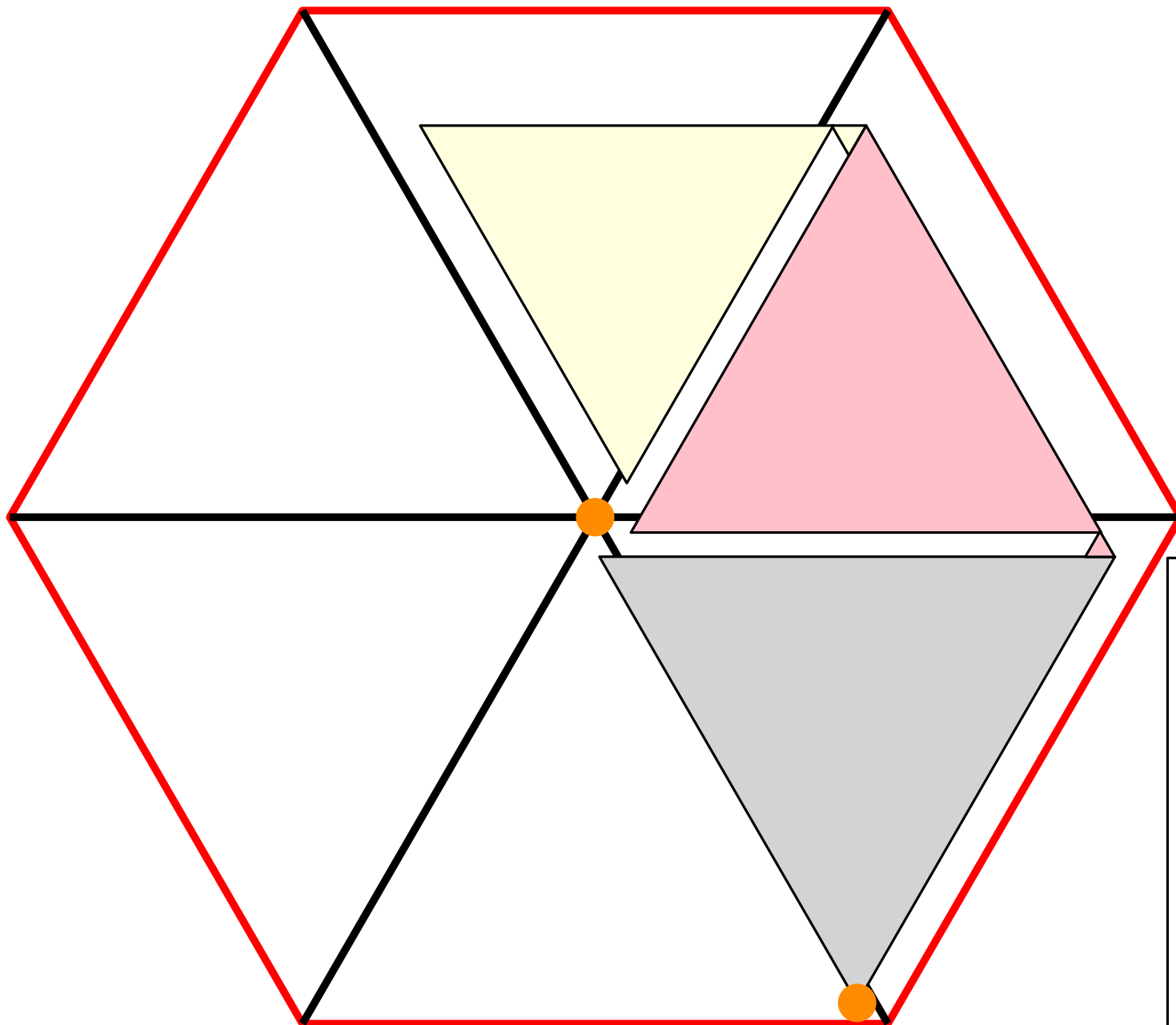


⊖ routing



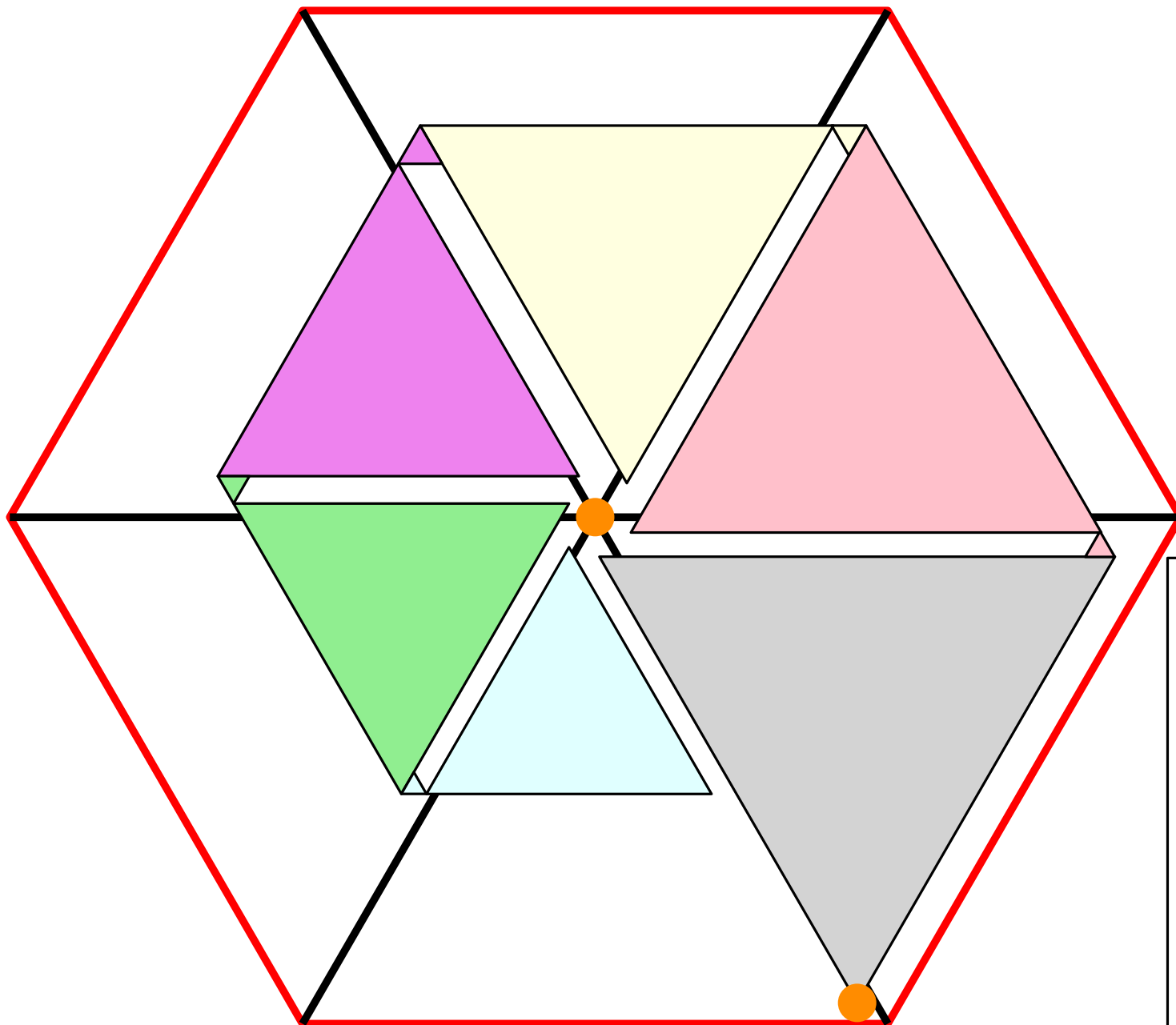
6 - 4

⊖ routing



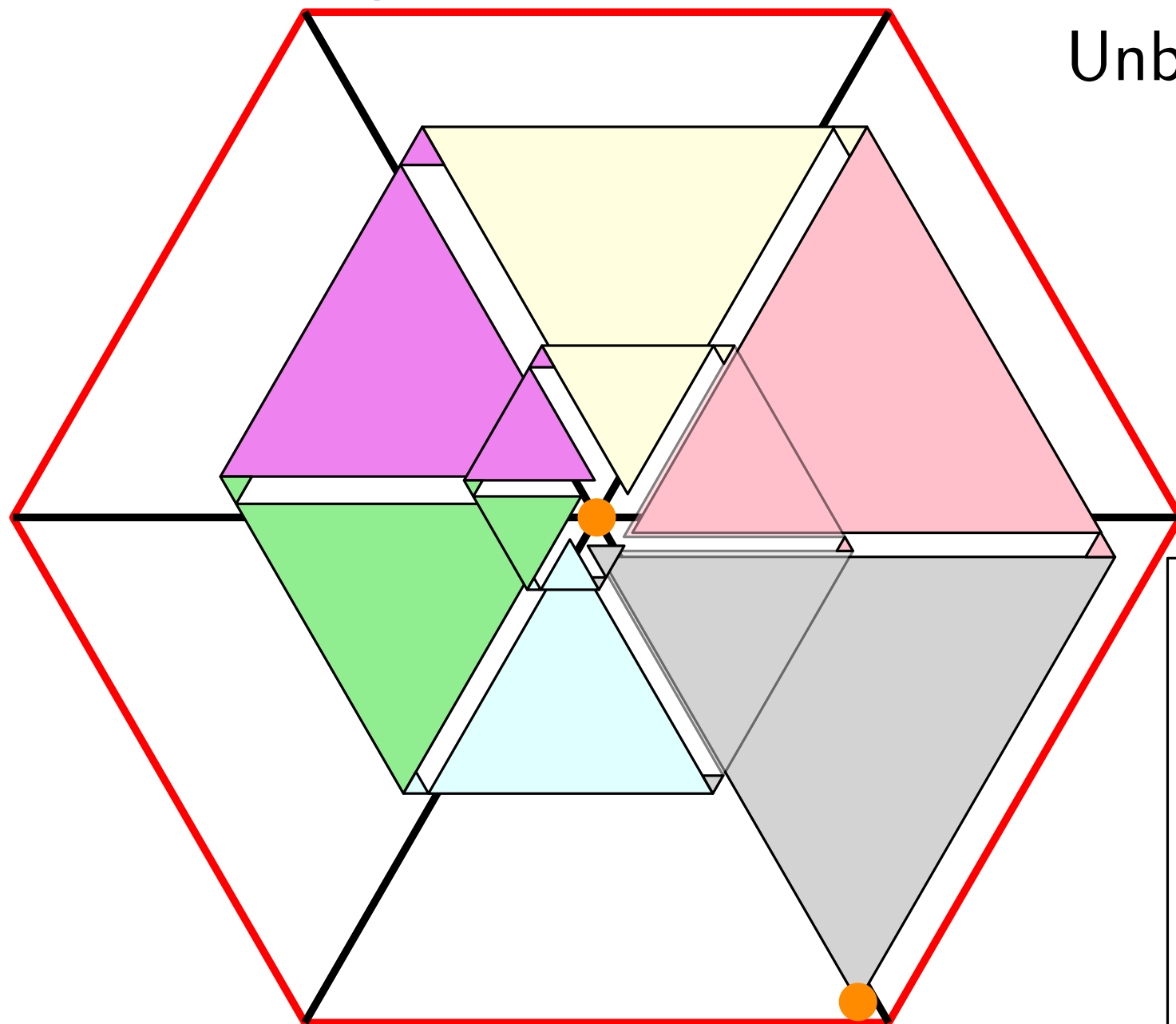
6 - 5

⊖ routing



6 - 6

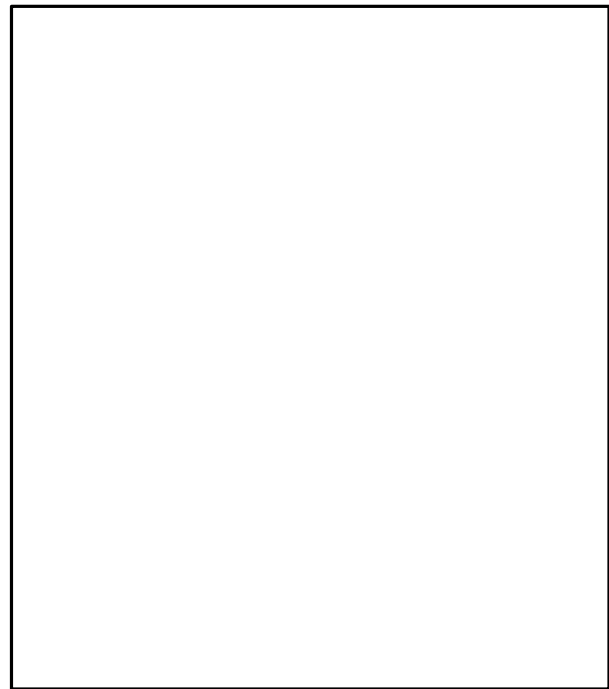
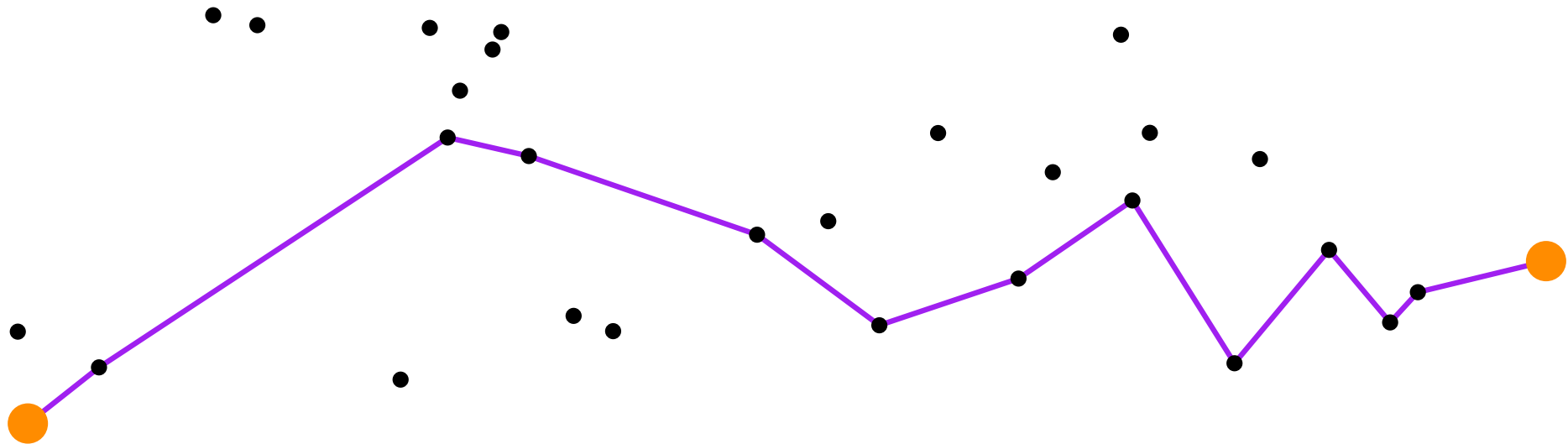
⊖ routing



Unbounded length

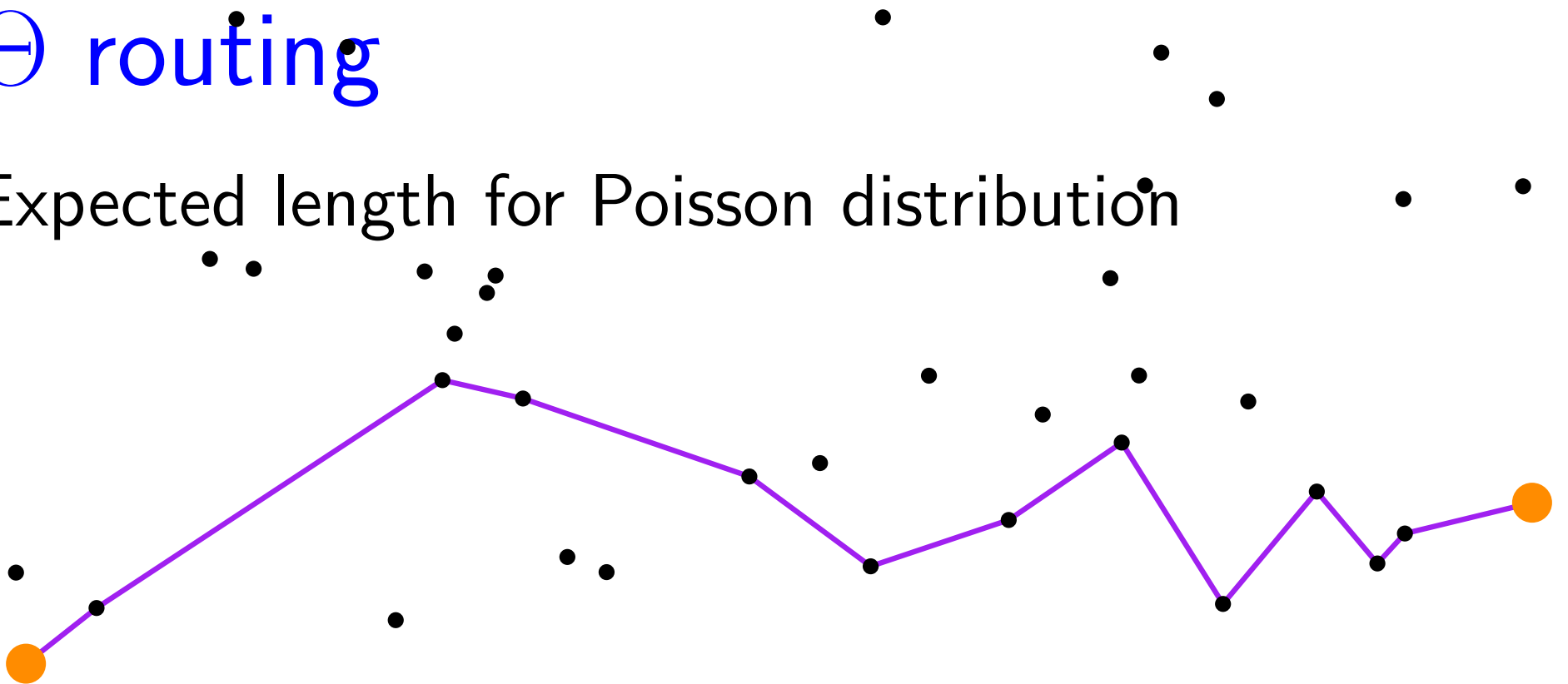
⊖ routing

Expected length for Poisson distribution



⊖ routing

Expected length for Poisson distribution



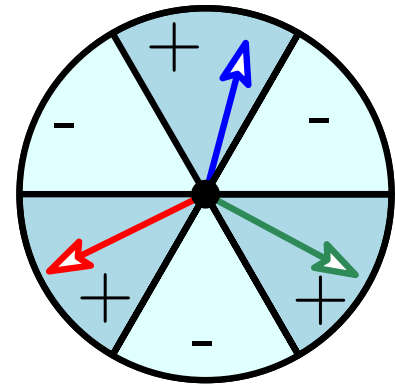
$$\leq \frac{1}{6}(3 \ln 3 + 4) \simeq 1.216$$

[Bonichon Marcket]

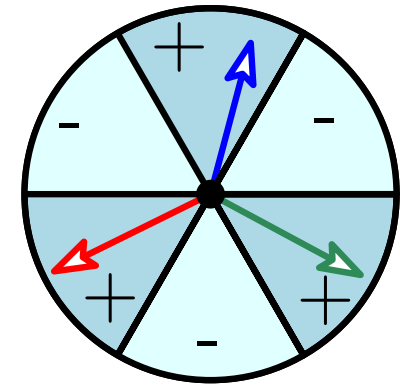
Routing in half Θ_6

Positive routing

Negative routing



Routing in half Θ_6



Positive routing ≤ 2

Negative routing $\leq \frac{5}{\sqrt{3}} \simeq 2.89$

[Bose, Fagerberg, van Renssen, Verdonschot]

Our contribution

Two tools: forward routing and side routing

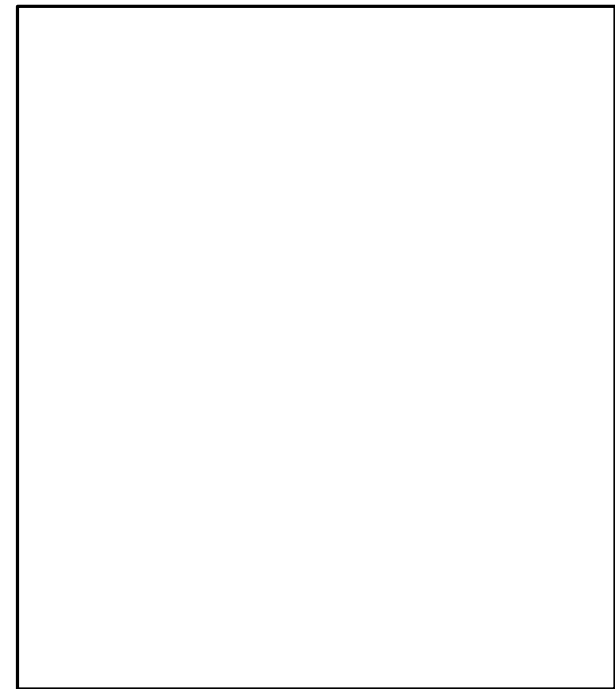
New negative routing algorithms

same worst-case bound

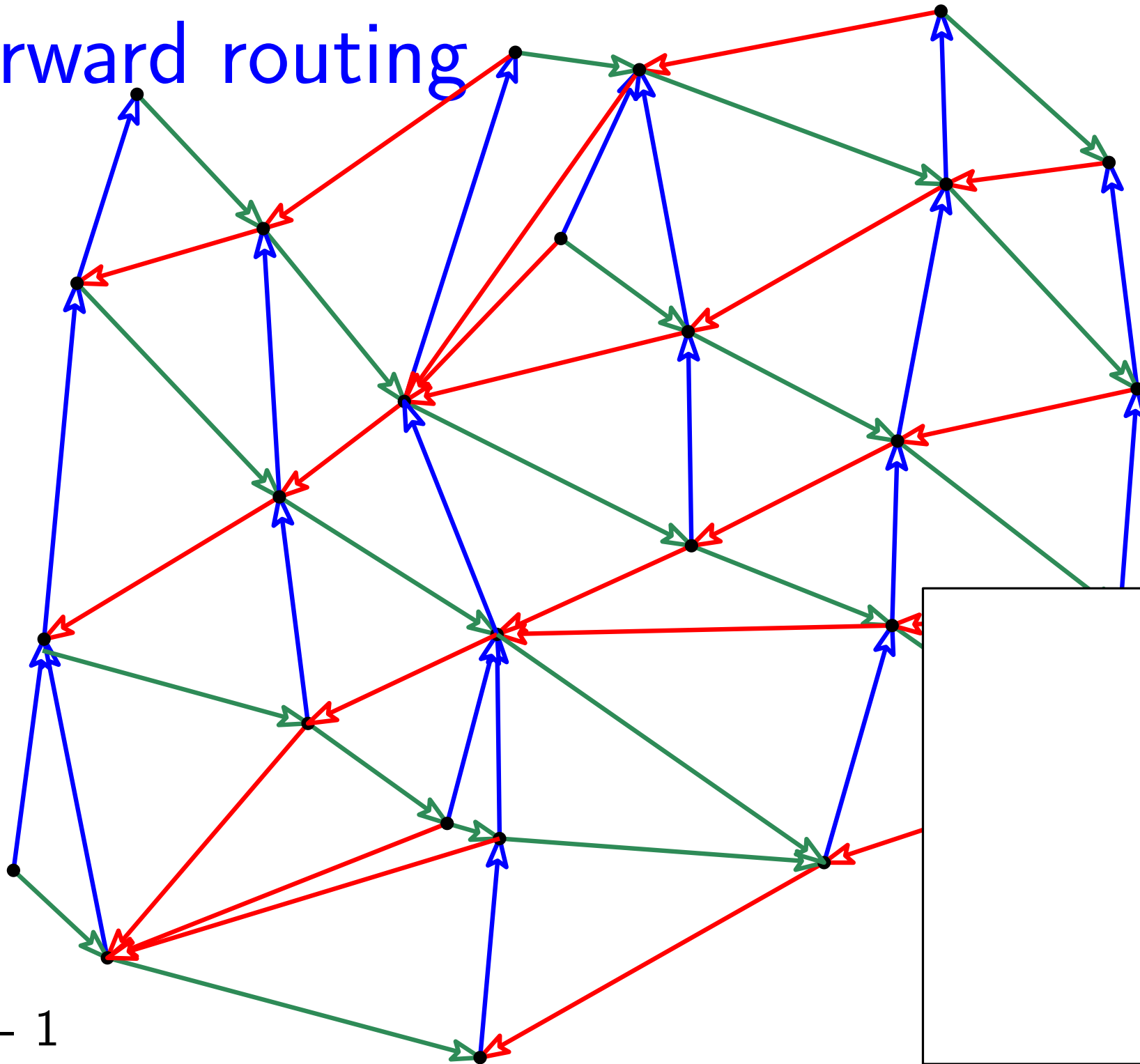
Probabilistic analysis for Poisson distribution

positive routing

new negative routings

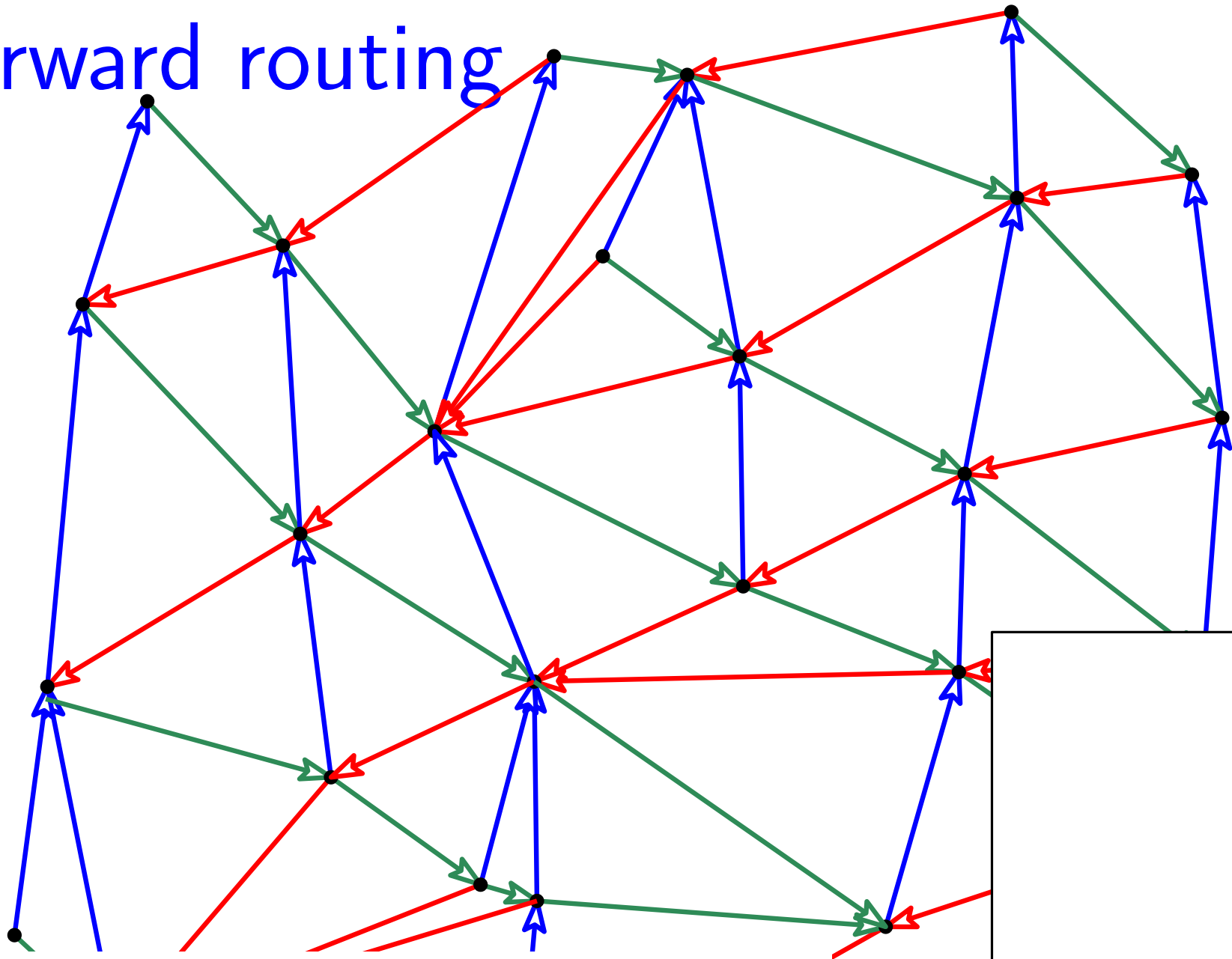


Forward routing



10 - 1

Forward routing



Given source and color (e.g., blue)

10 - 2

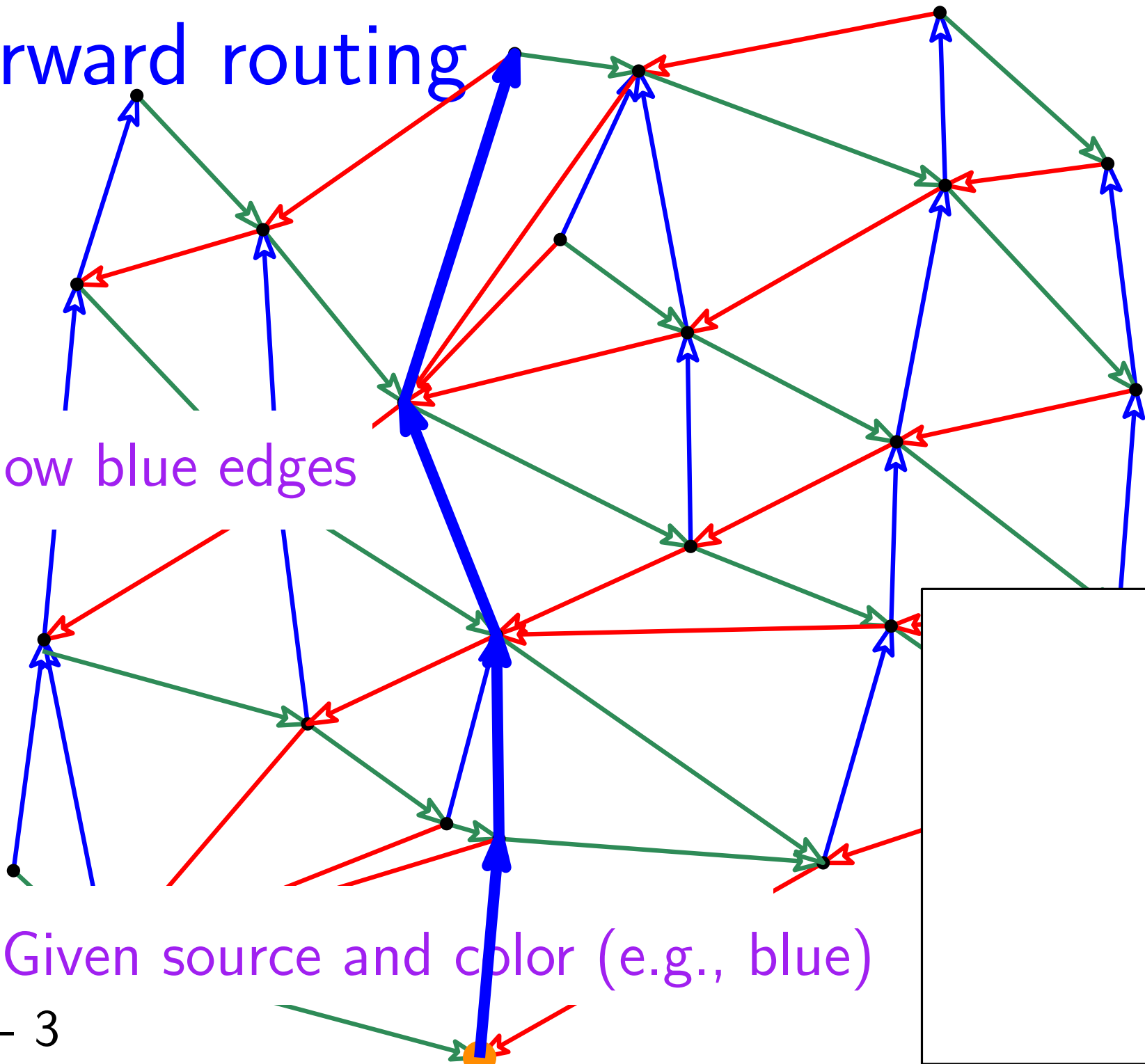


Forward routing

Follow blue edges

Given source and color (e.g., blue)

10 - 3



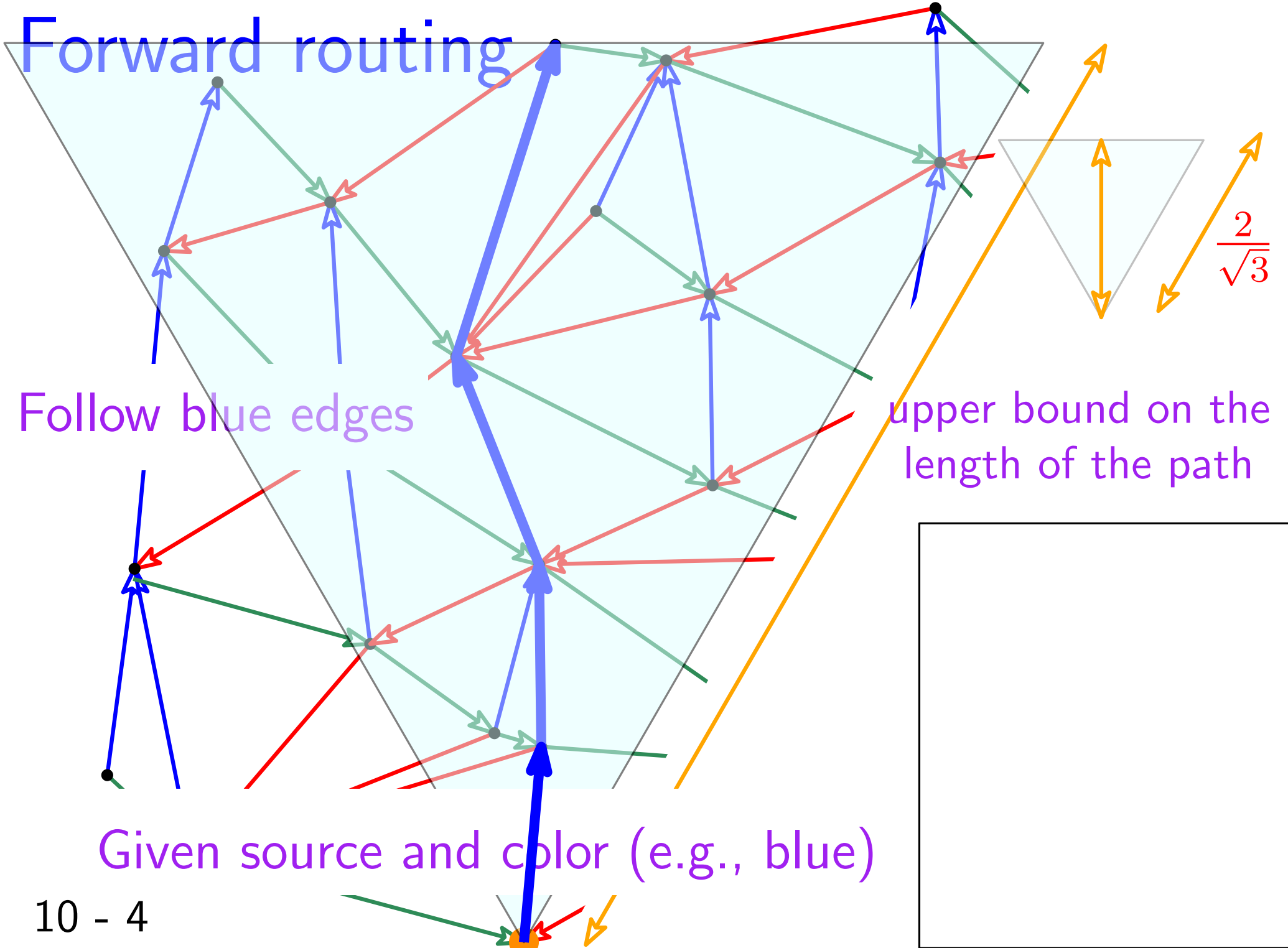
Forward routing

Follow blue edges

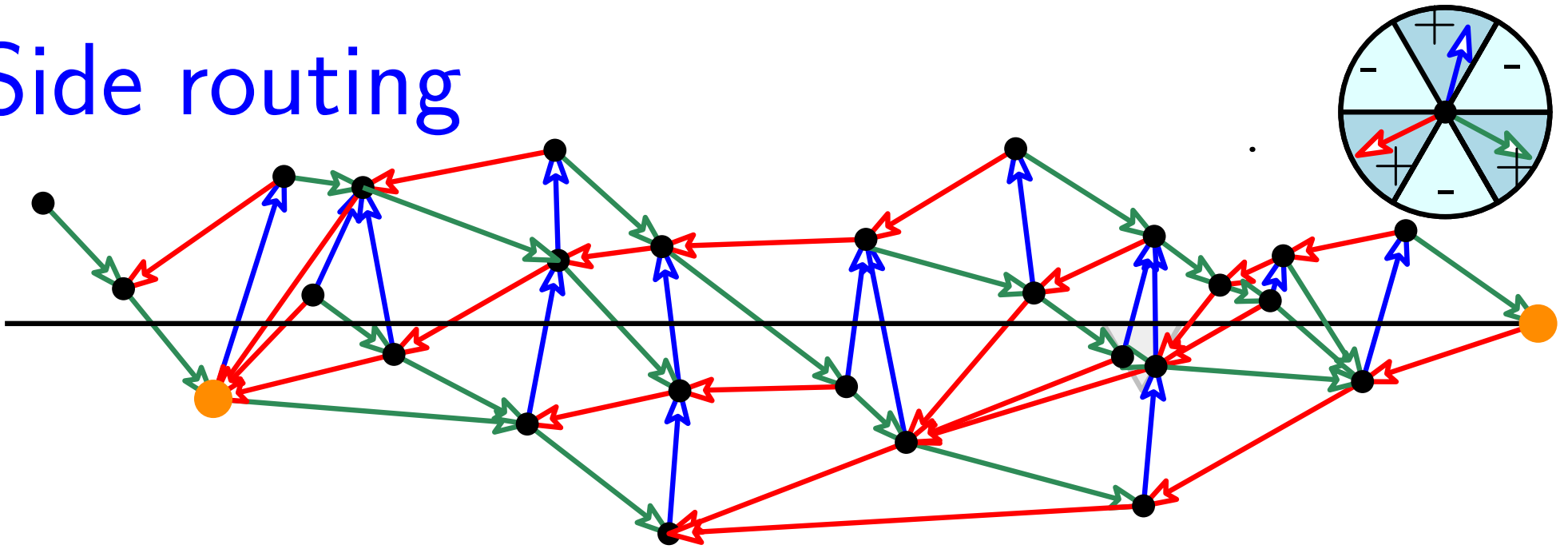
upper bound on the length of the path

Given source and color (e.g., blue)

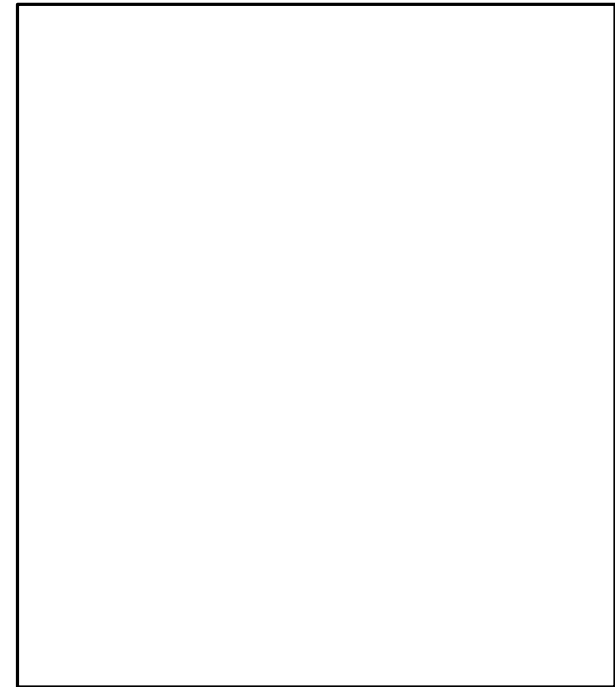
10 - 4



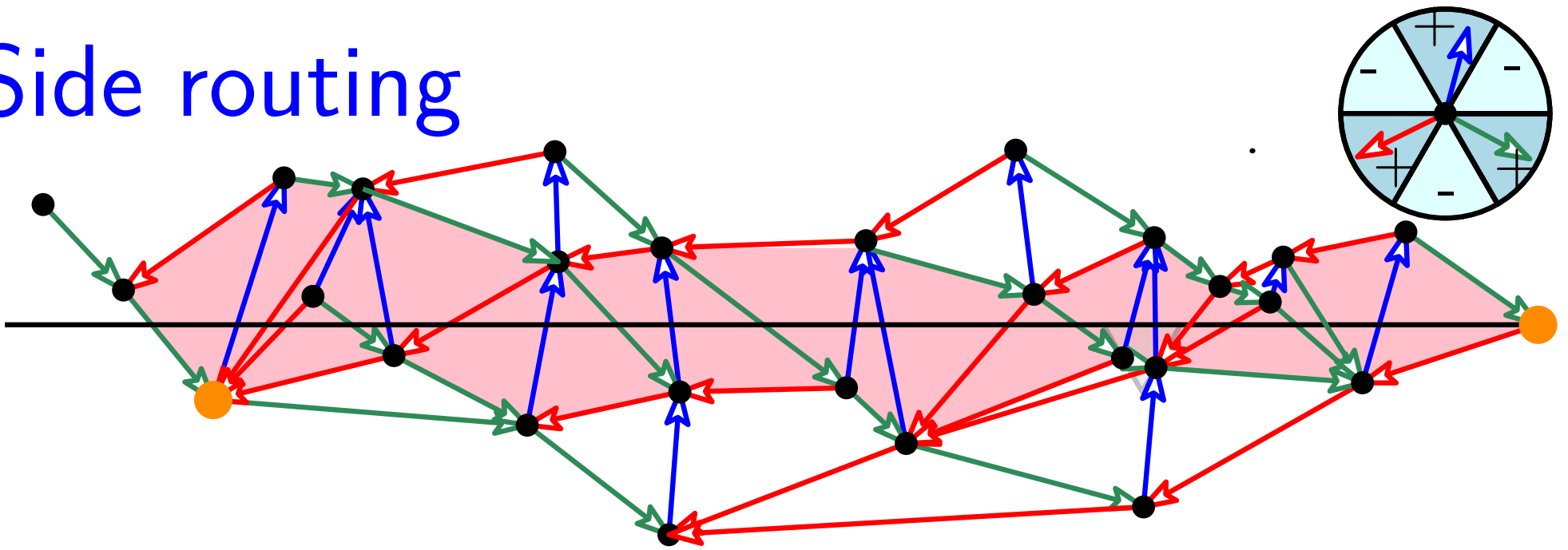
Side routing



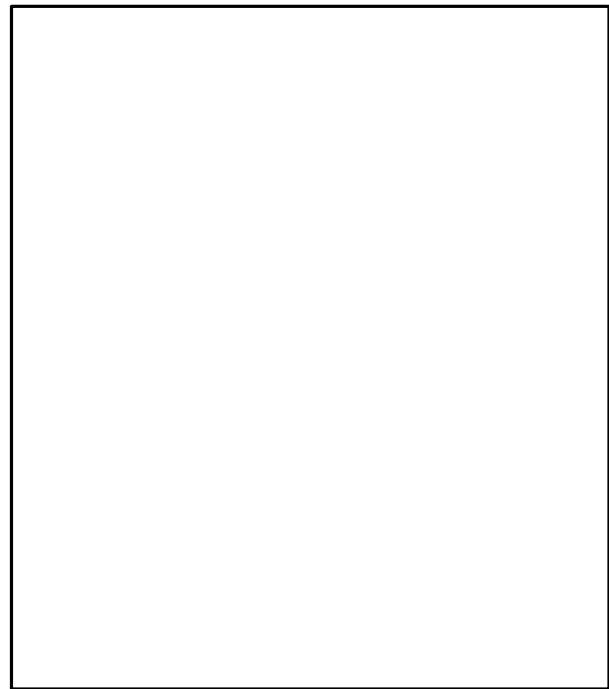
Given target, direction, edge crossing from - to +



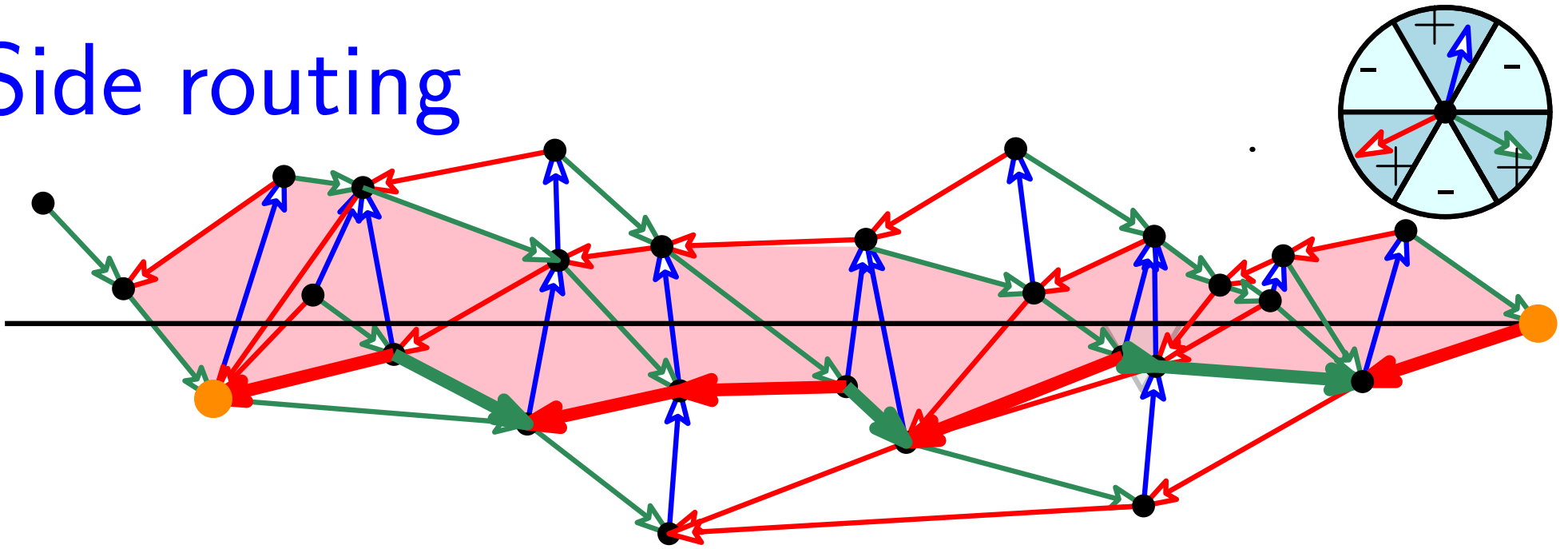
Side routing



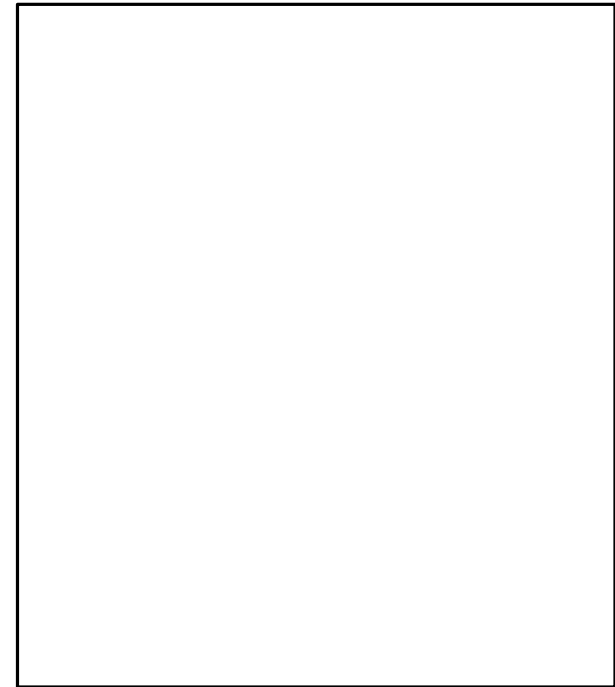
Given target, direction, edge crossing from - to +
Use boundary of triangles crossing the ray



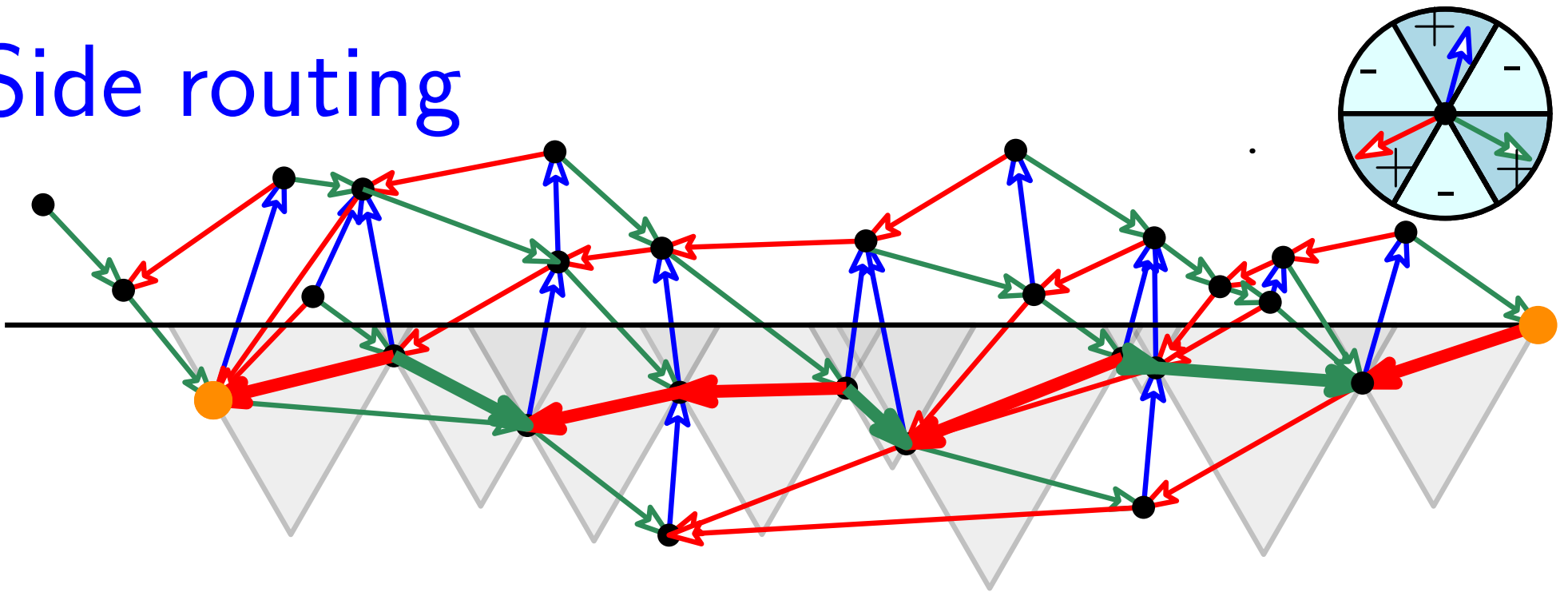
Side routing



Given target, direction, edge crossing from - to +
Use boundary of triangles crossing the ray



Side routing

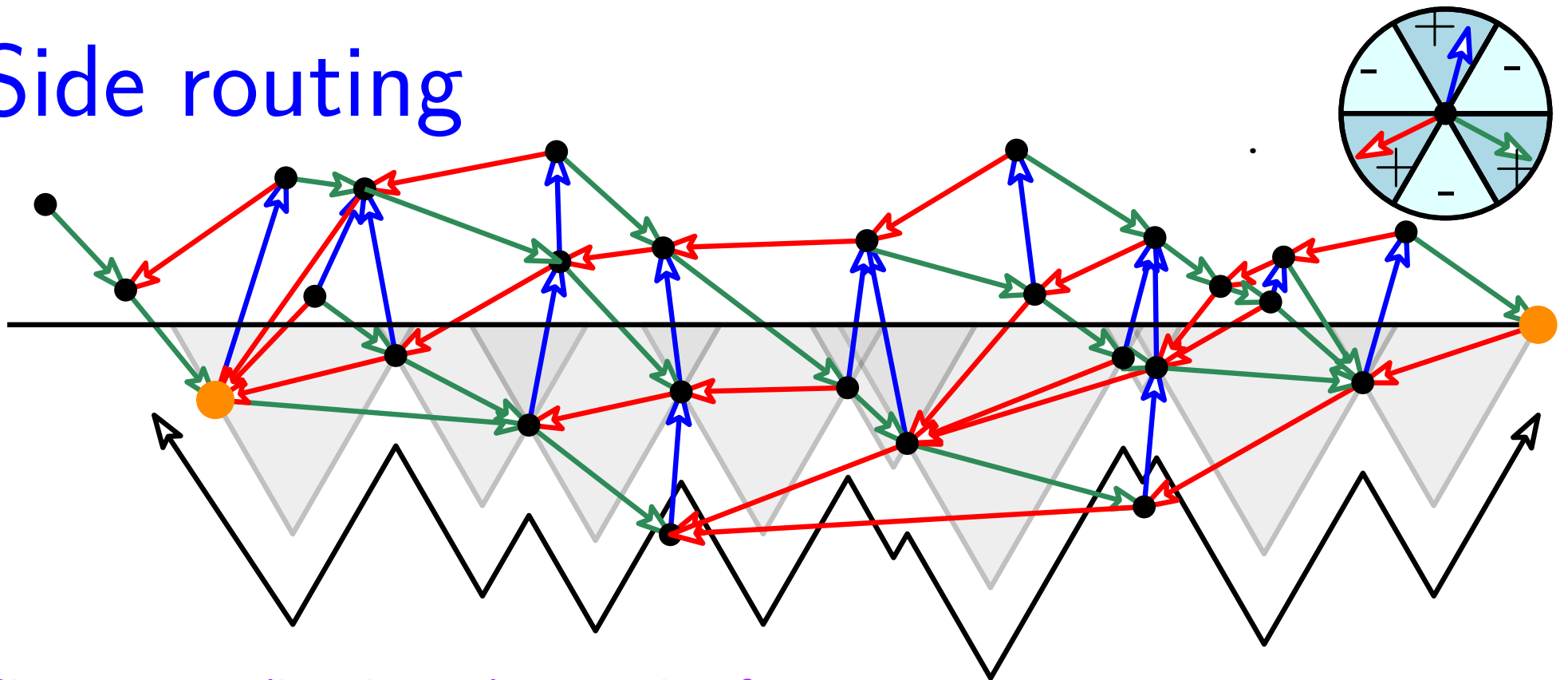


Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray

Bicolor path, empty certifying triangles aligned on ray

Side routing

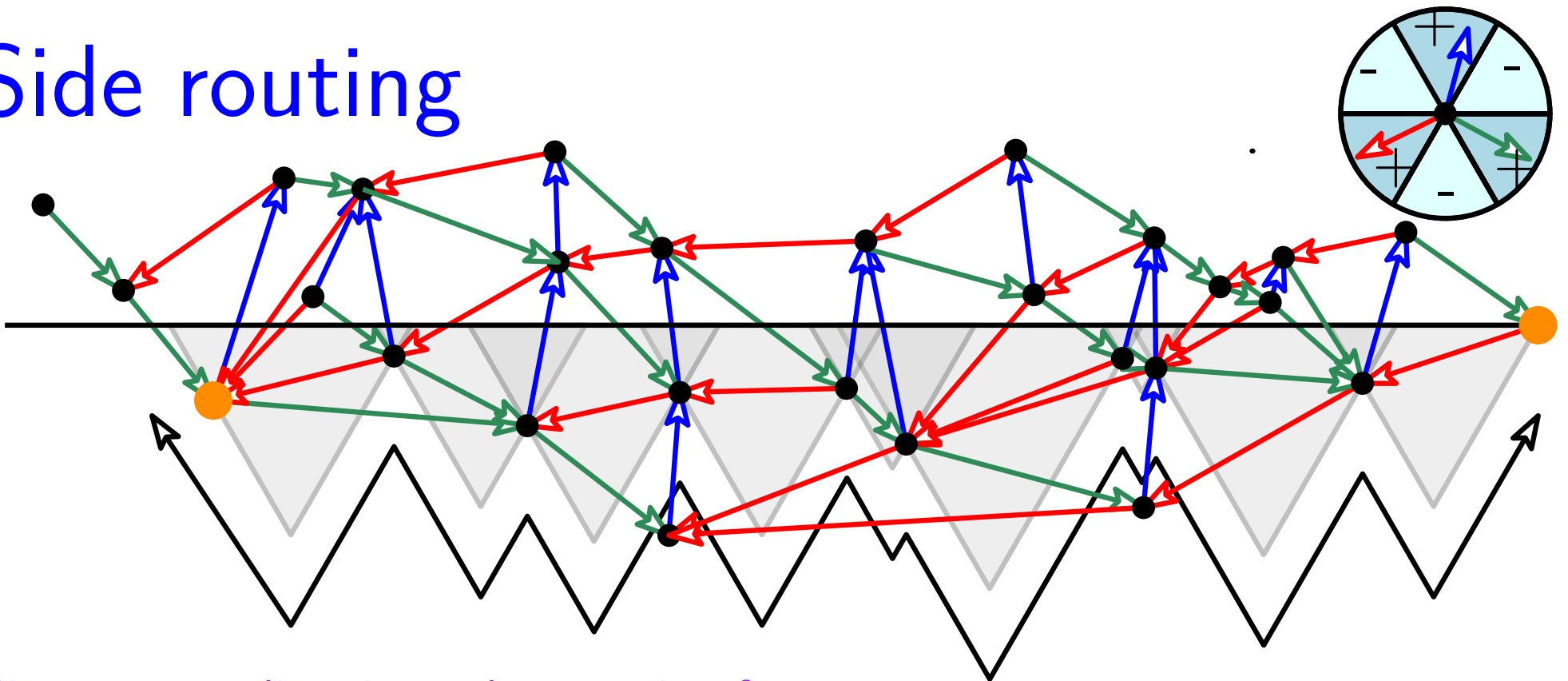


Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray

upper bound on the length of the path

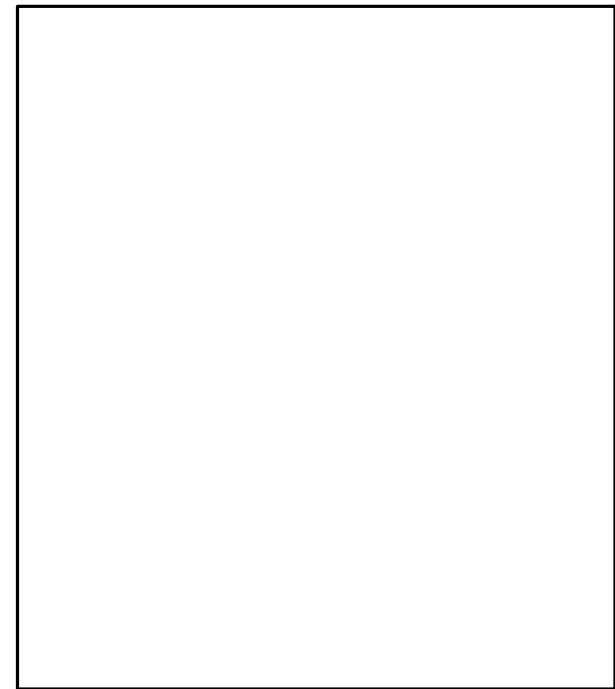
Side routing



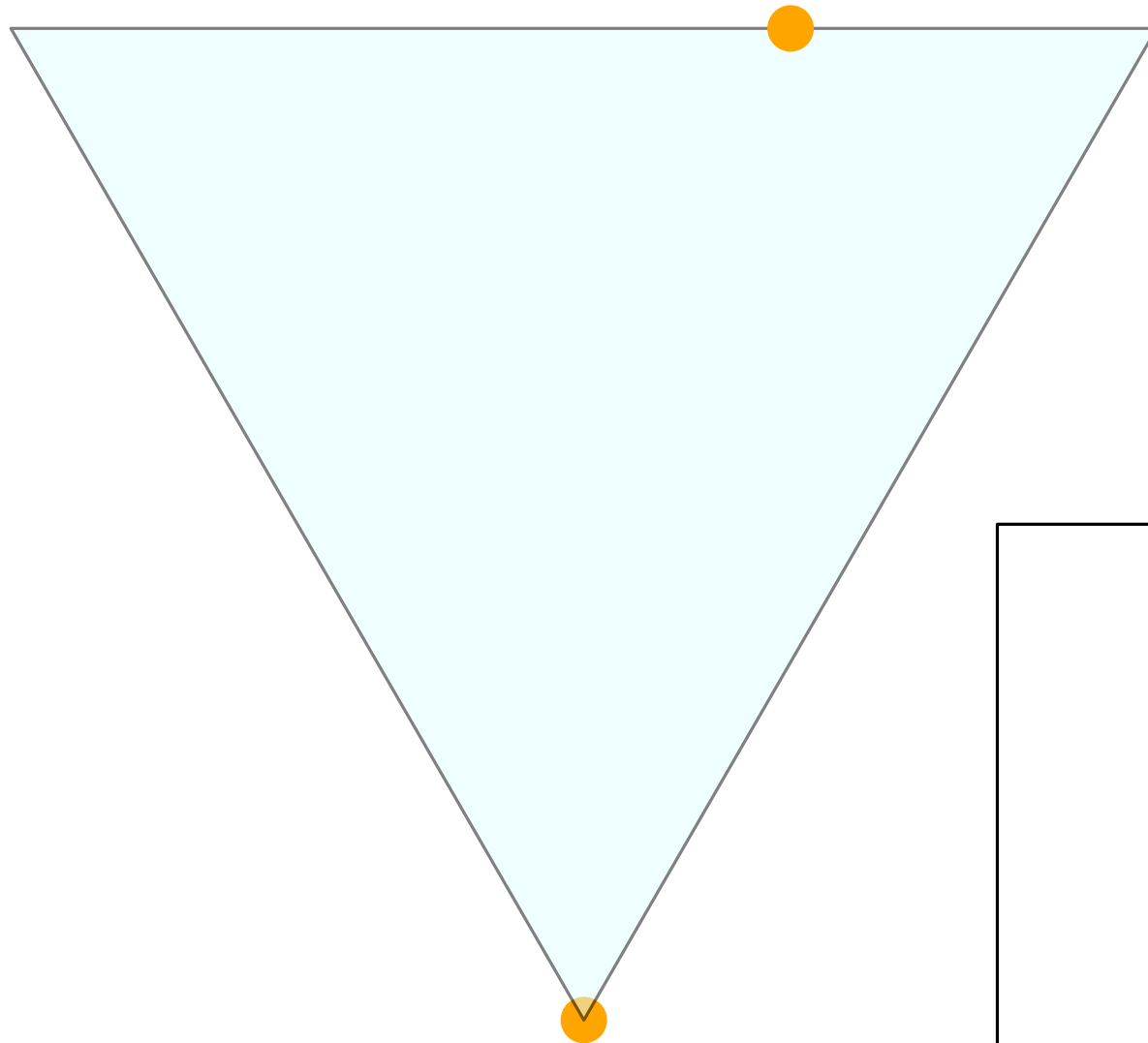
Given target, direction, edge crossing from - to +
Use boundary of triangles crossing the ray

upper bound on the length of the path

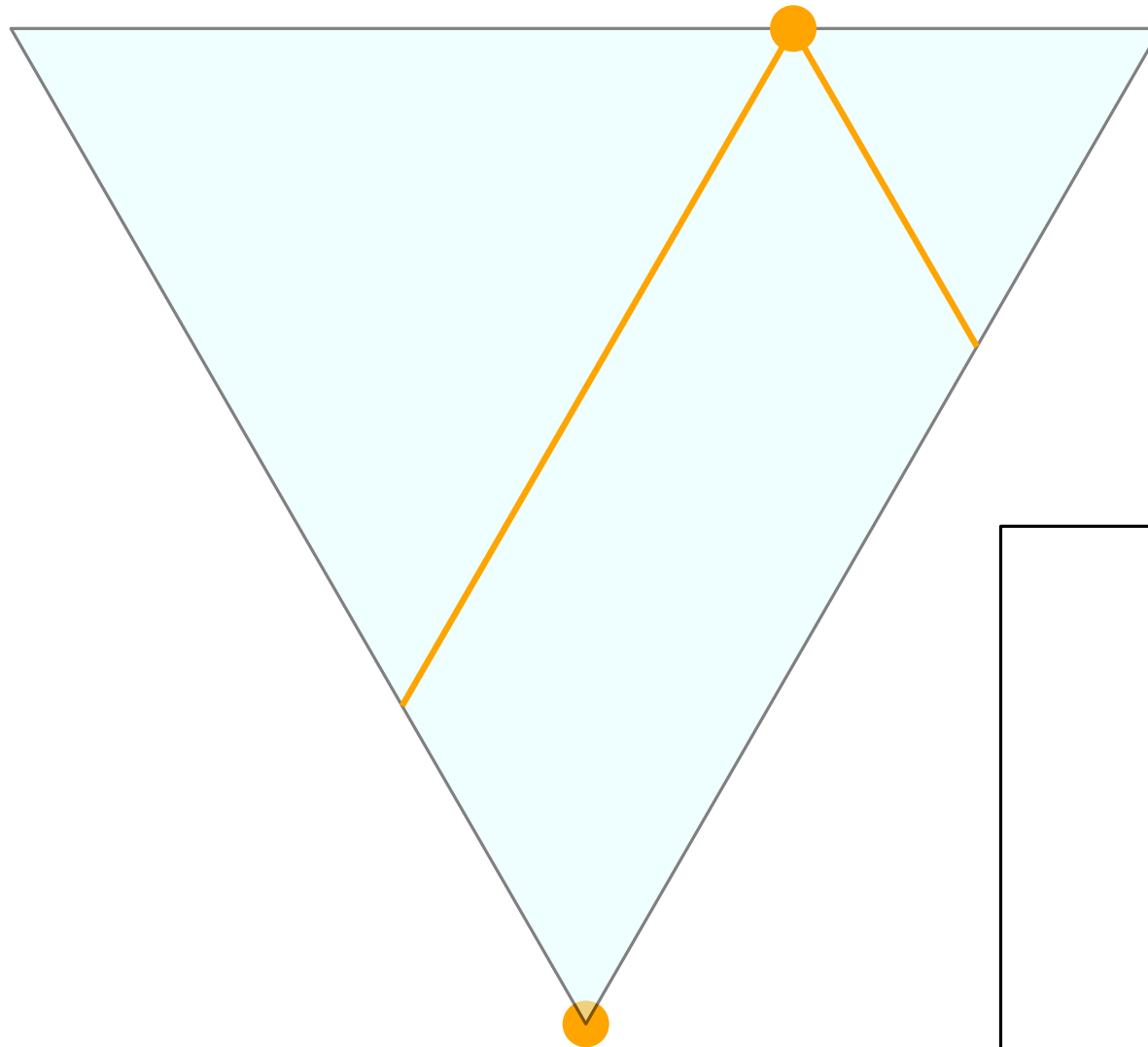
$$\leq 2$$



Positive routing

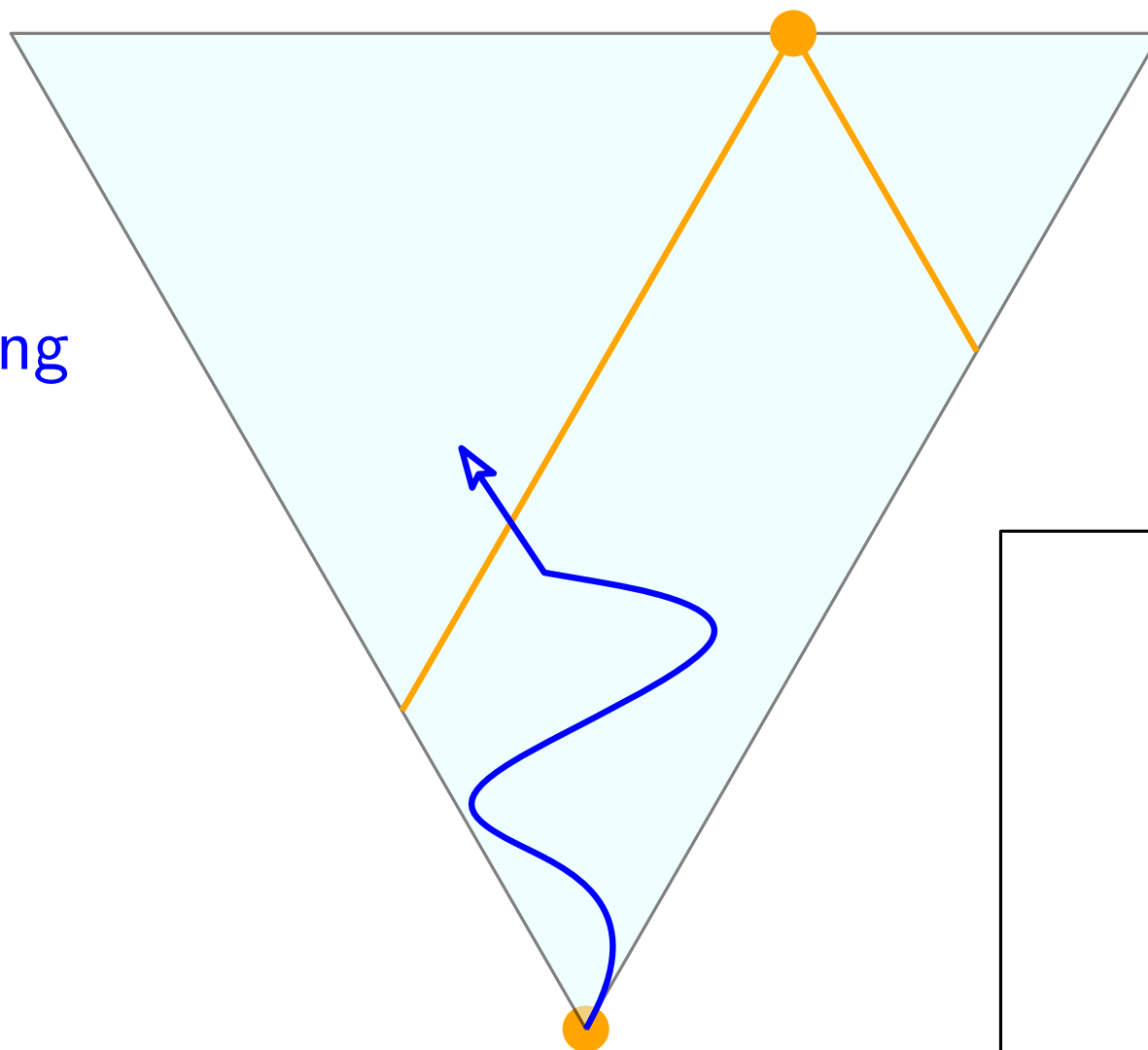


Positive routing



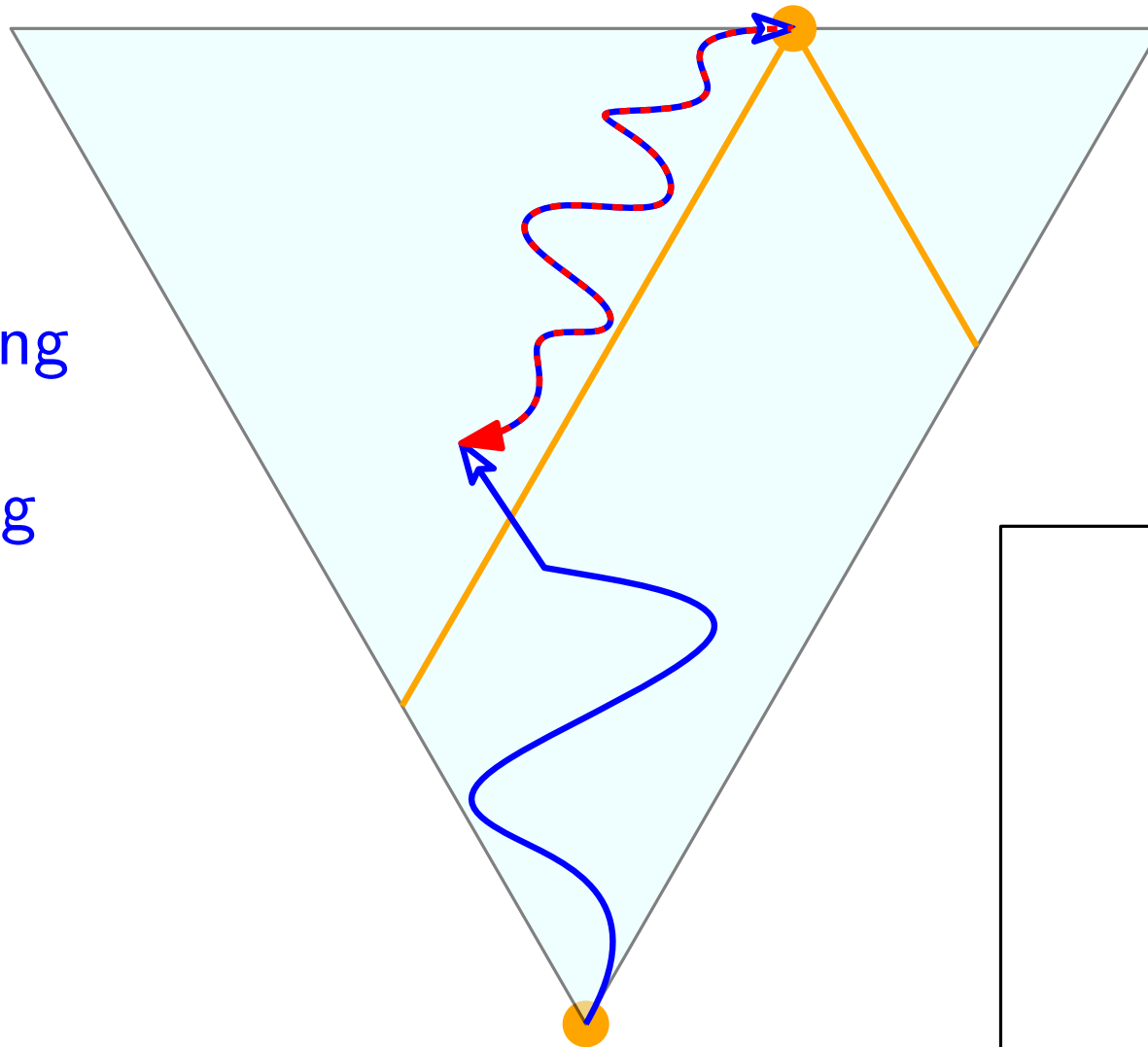
Positive routing

Forward routing

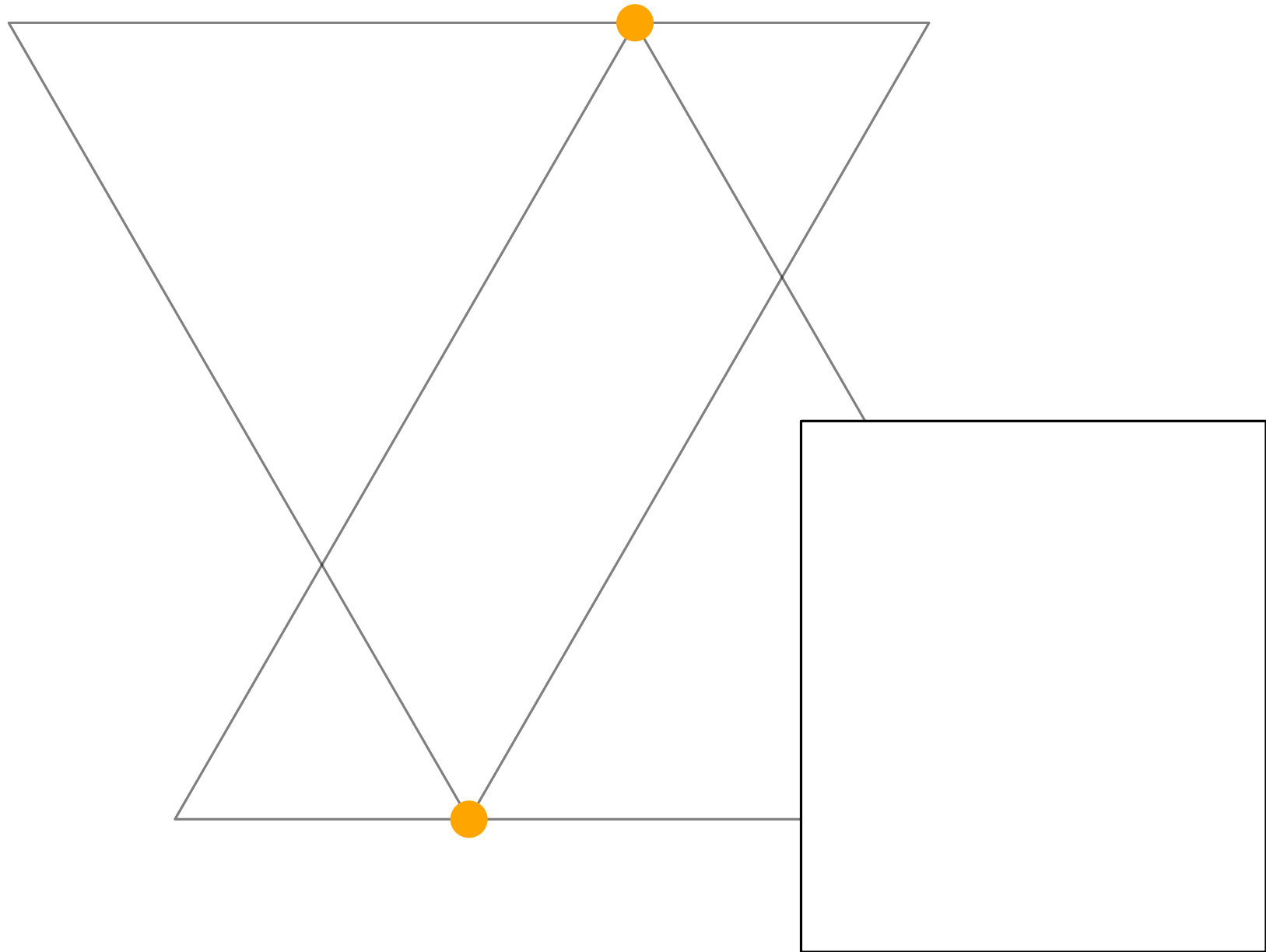


Positive routing

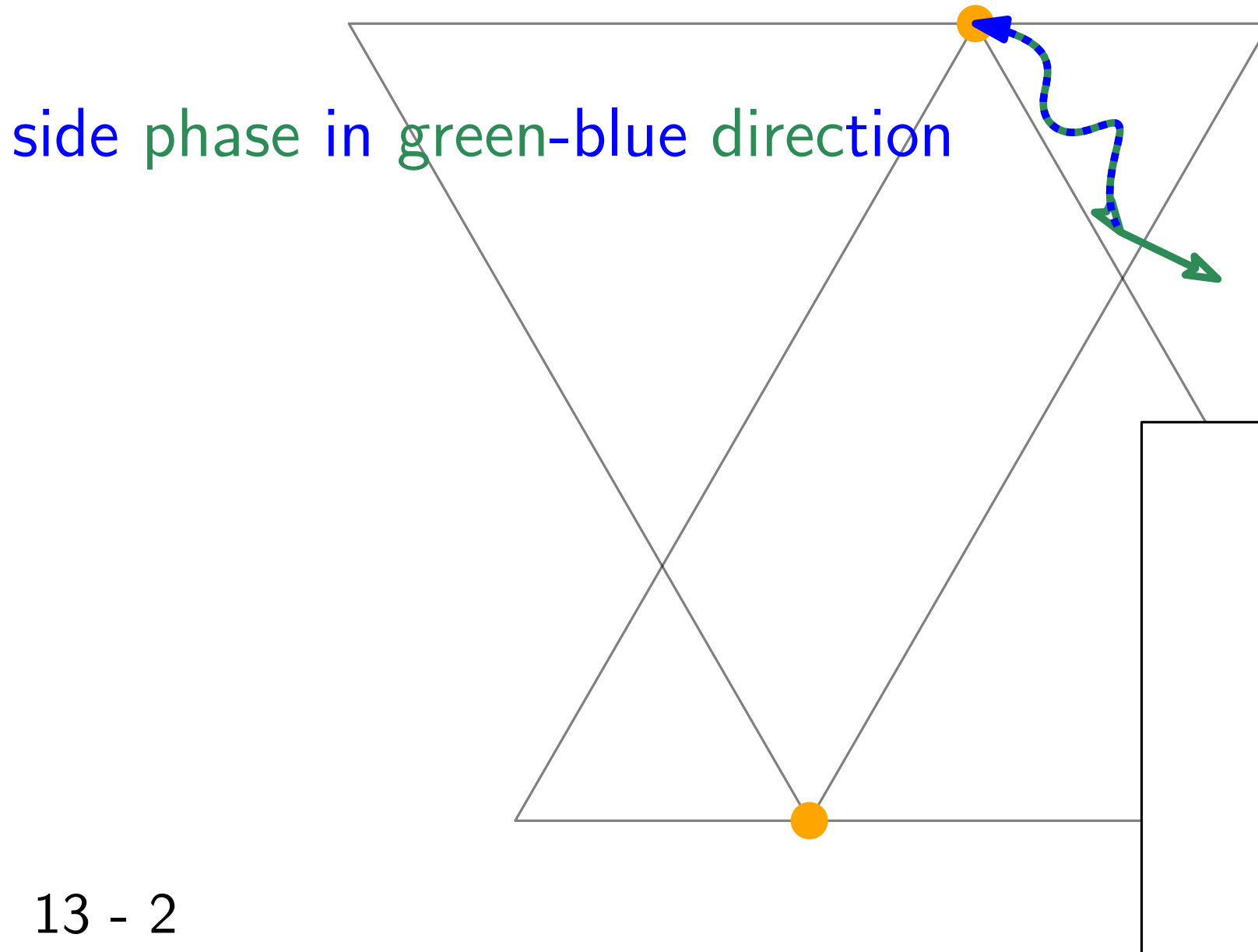
Forward routing
+ Side routing



1-Memory negative routing



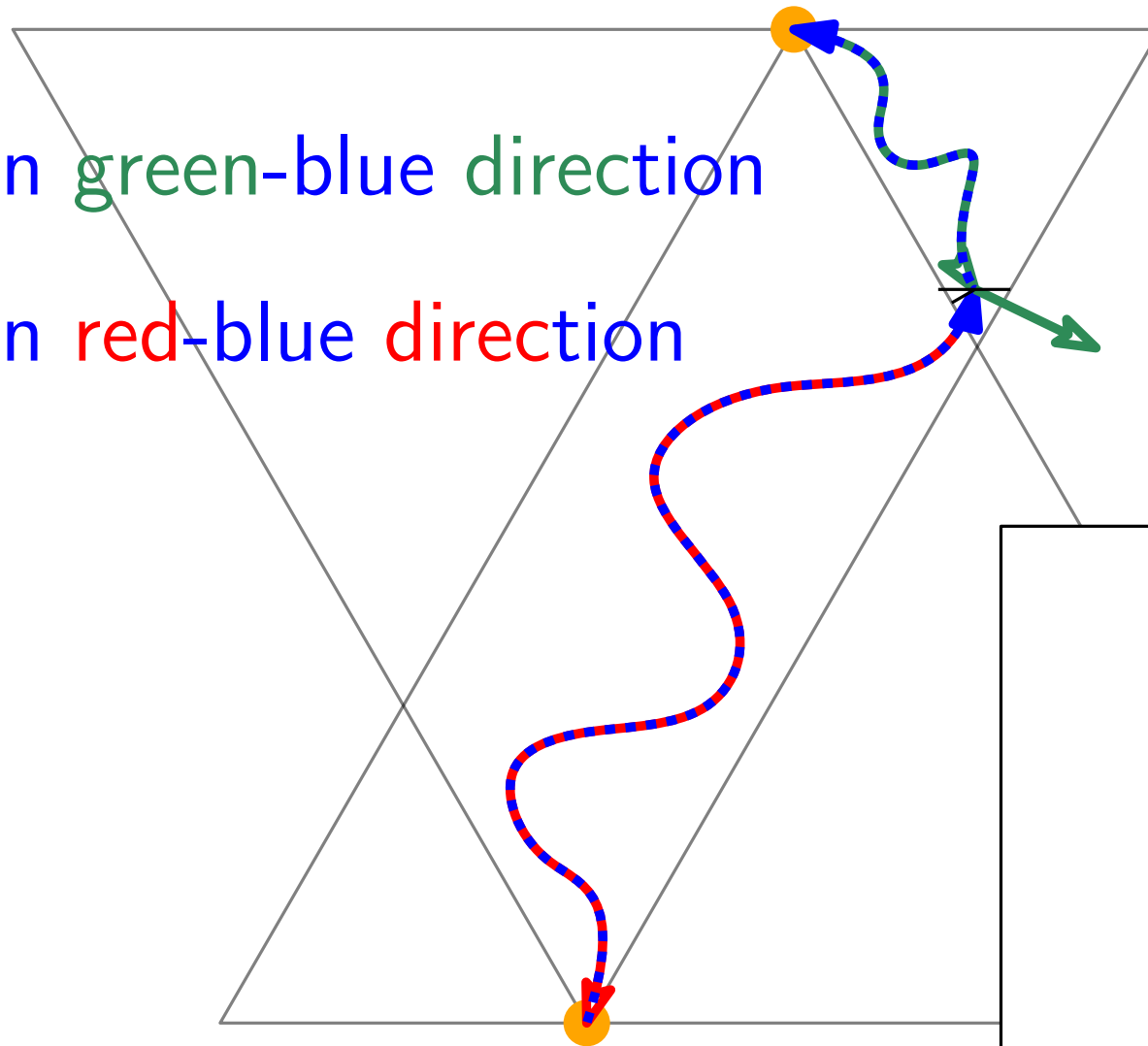
1-Memory negative routing



1-Memory negative routing

side phase in green-blue direction

side phase in red-blue direction

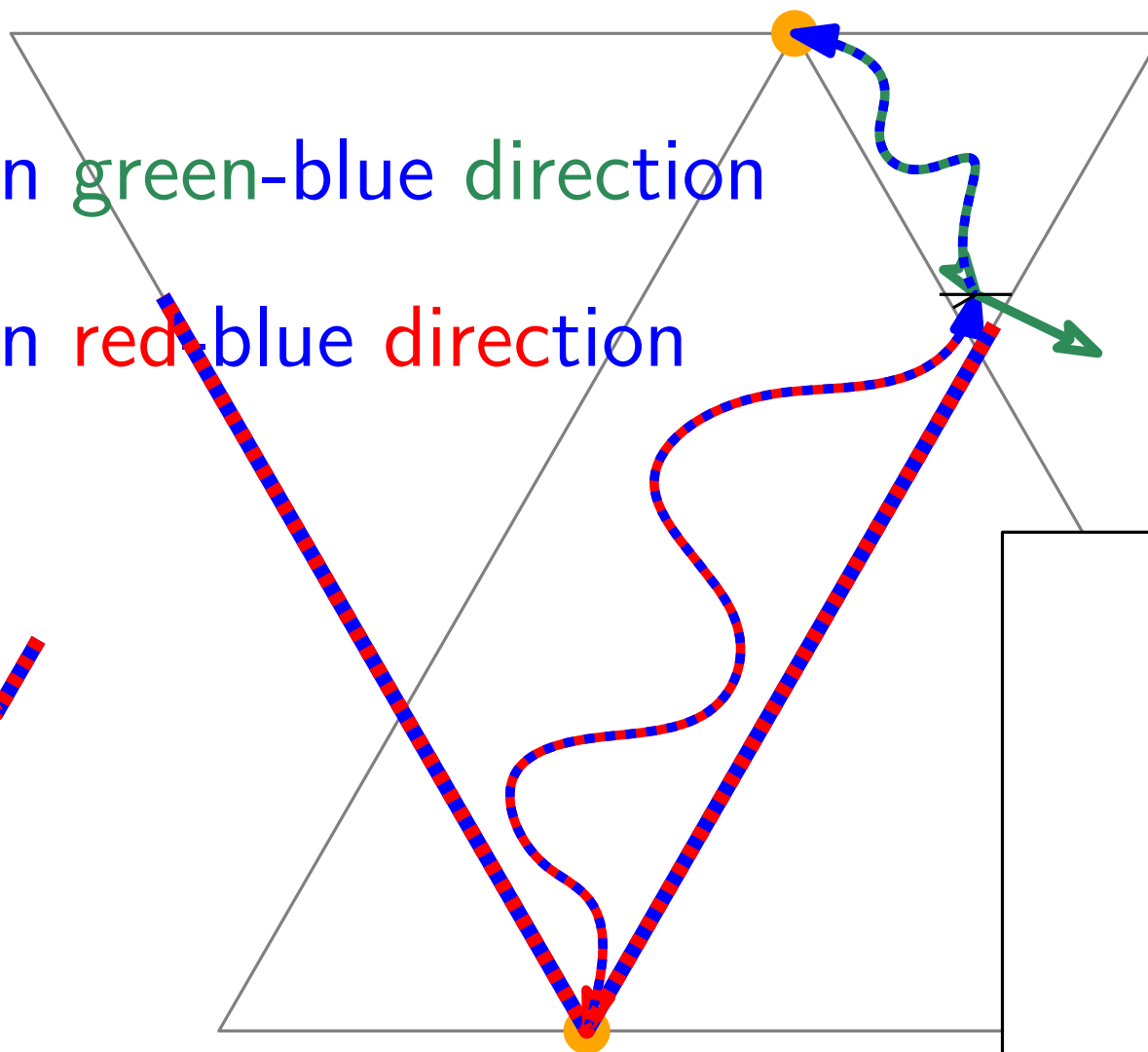


1-Memory negative routing

side phase in green-blue direction

side phase in red-blue direction



Length $\leq \sqrt{v}$

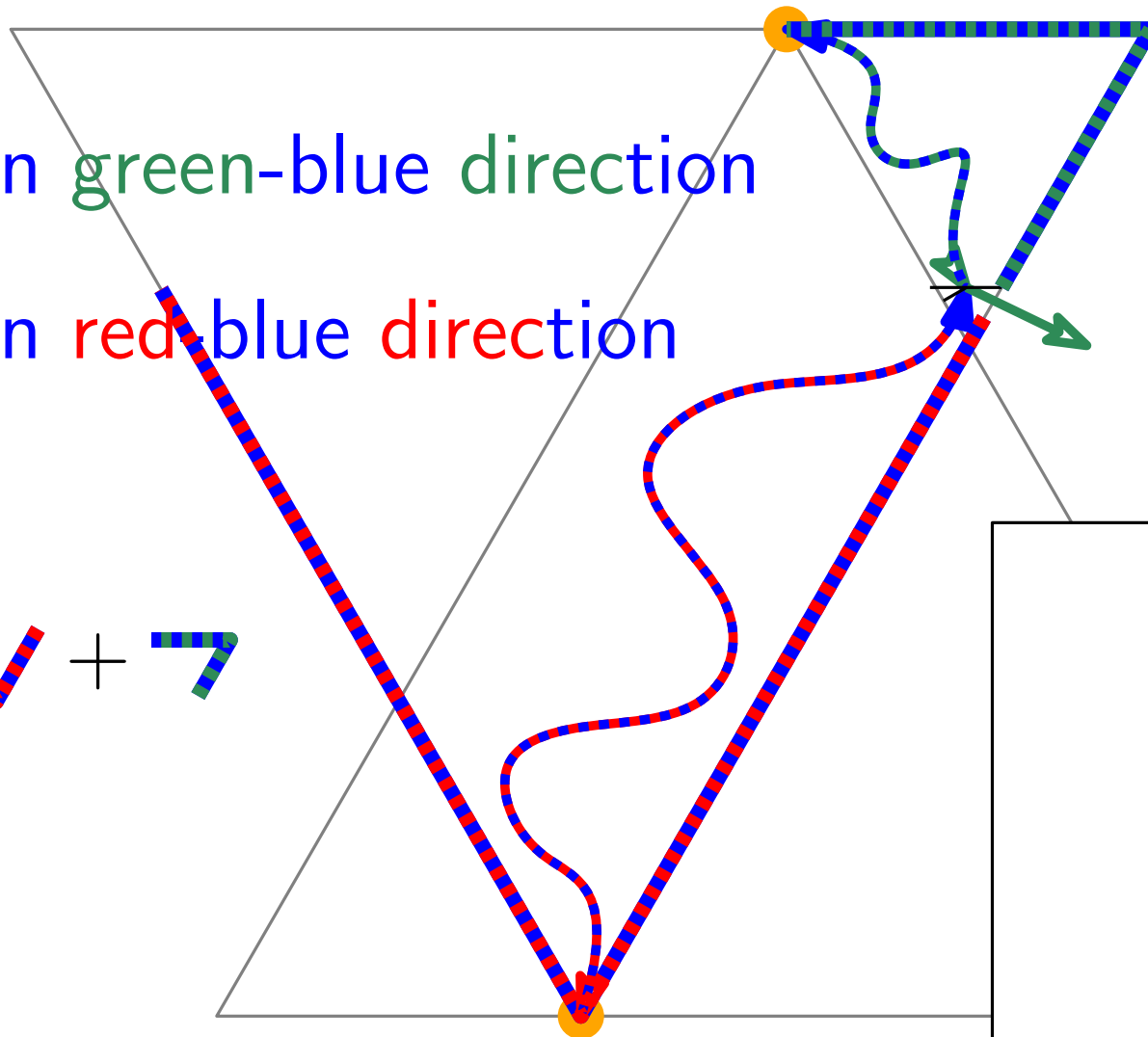


1-Memory negative routing

side phase in green-blue direction

side phase in red-blue direction

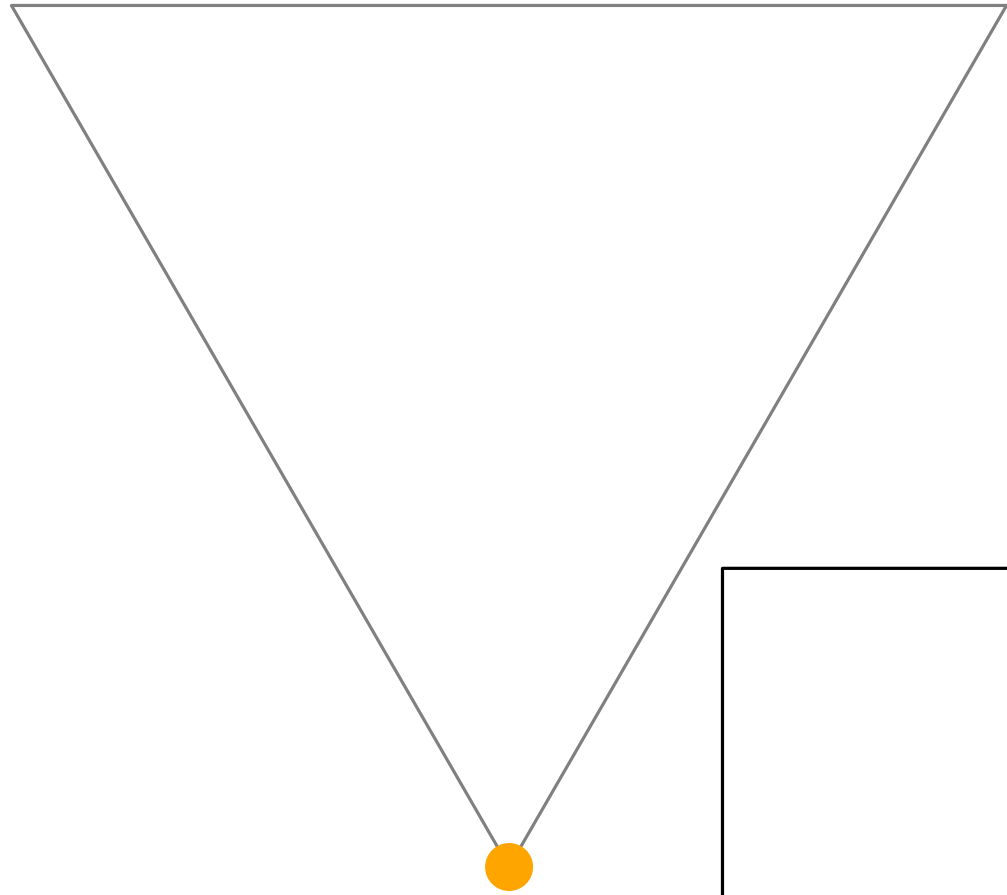
Length \leq  + 



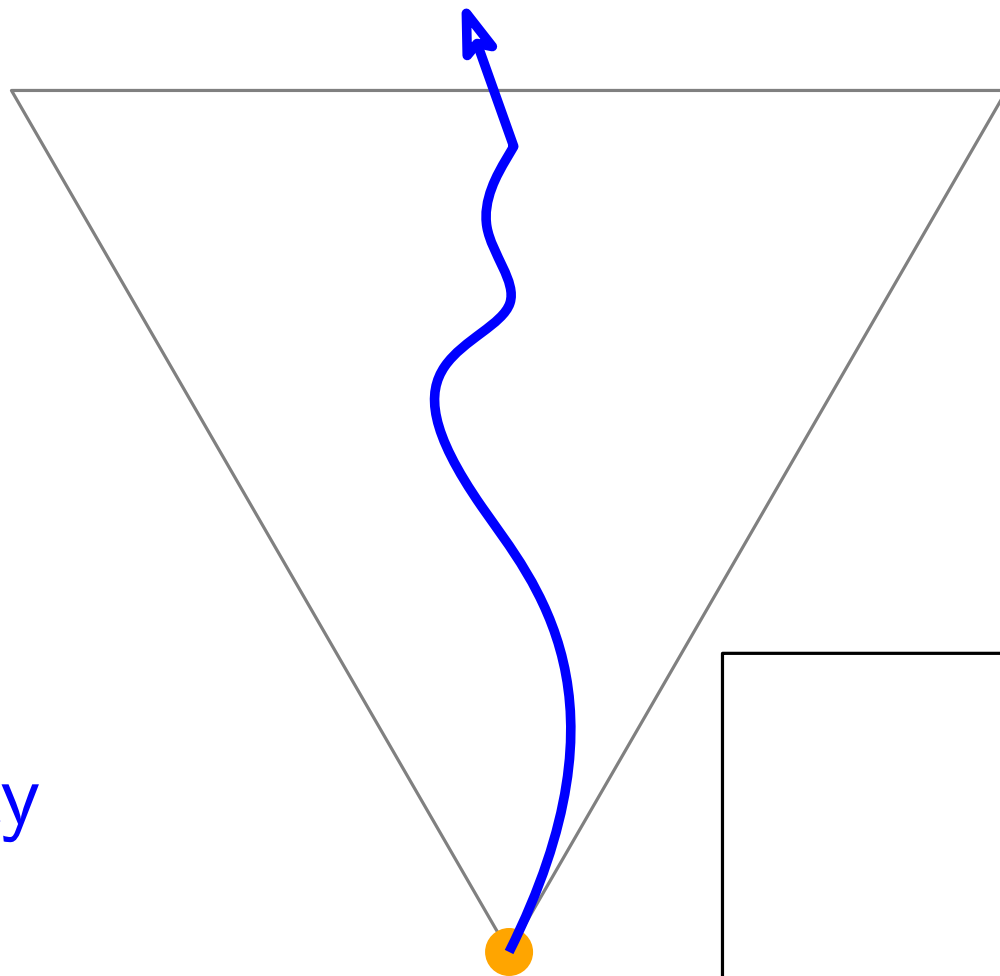
Probabilistic analysis

Expected length ?

Forward routing

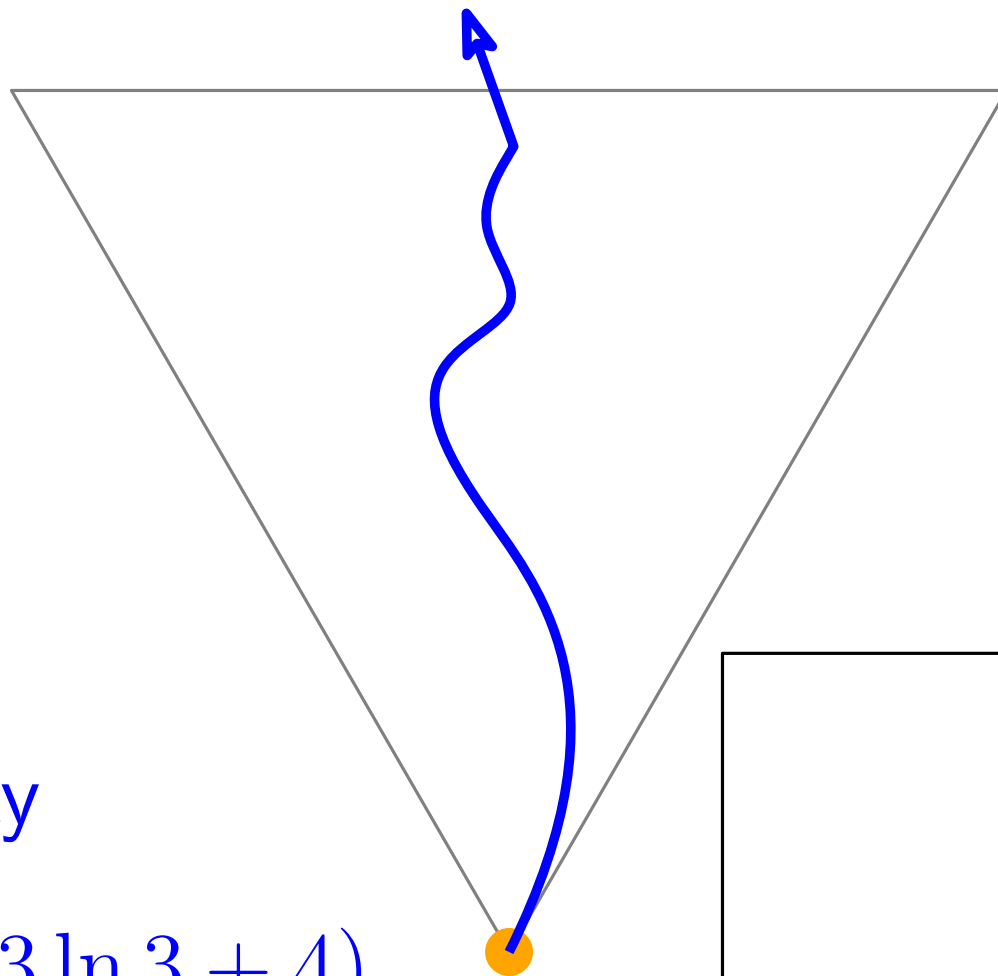


Forward routing



With high probability

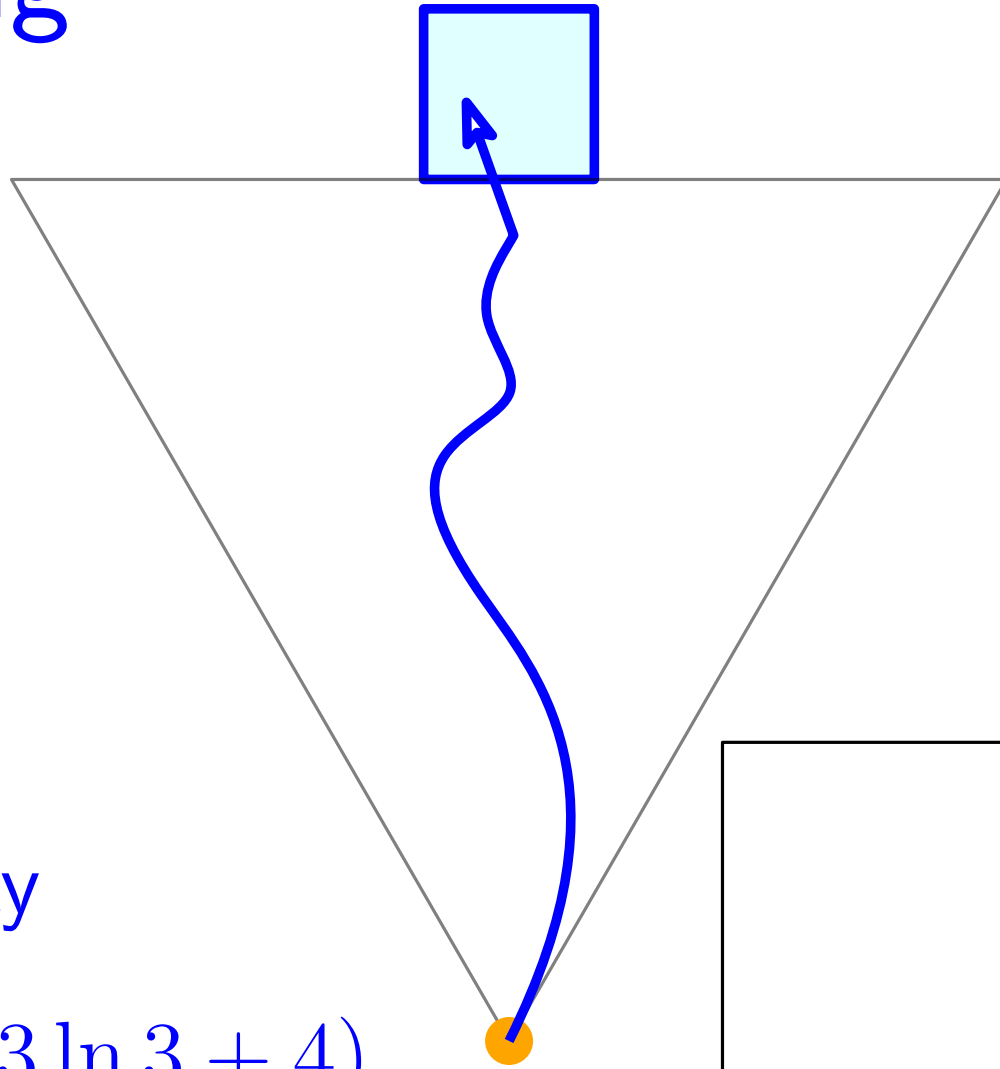
Forward routing



With high probability

stretch close to $\frac{\sqrt{3}}{12}(3 \ln 3 + 4)$

Forward routing



With high probability

stretch close to $\frac{\sqrt{3}}{12}(3 \ln 3 + 4)$

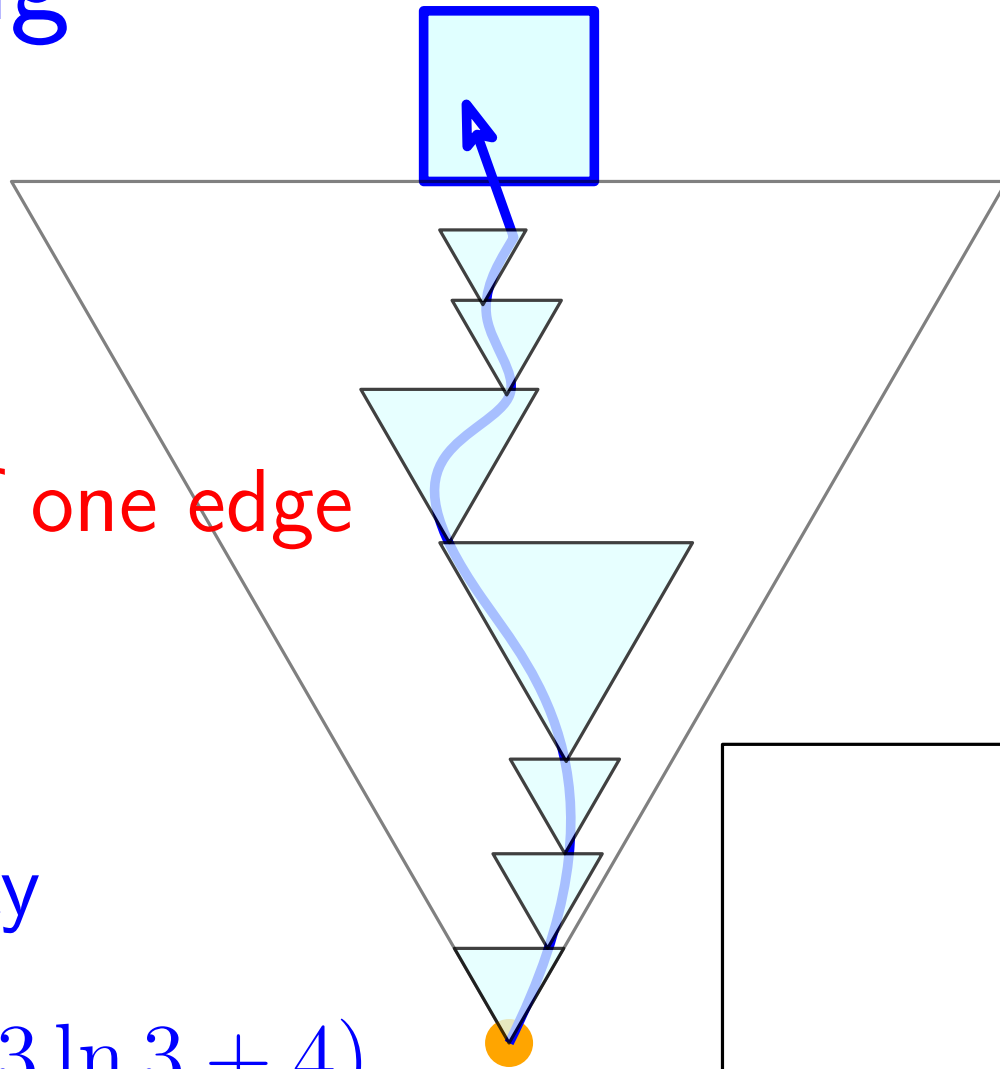
path terminates in a square of side $\tilde{O}(\lambda^{\frac{1}{4}})$

Forward routing

Proof:

Independance

Expected slope of one edge

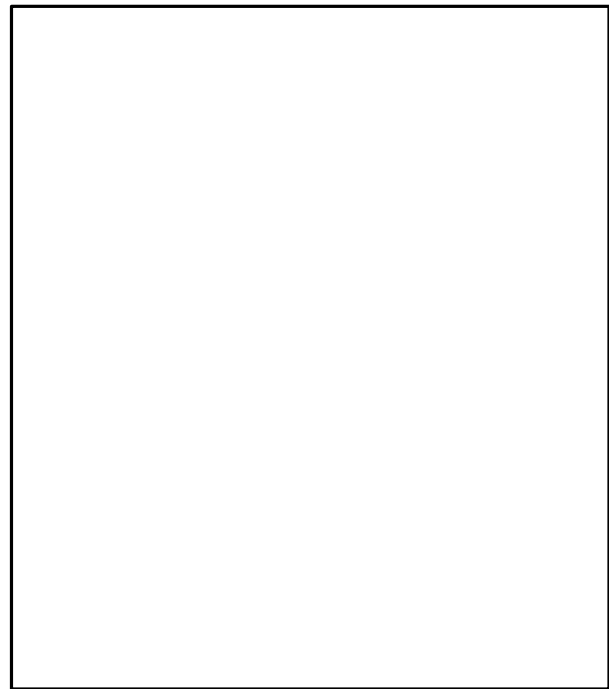
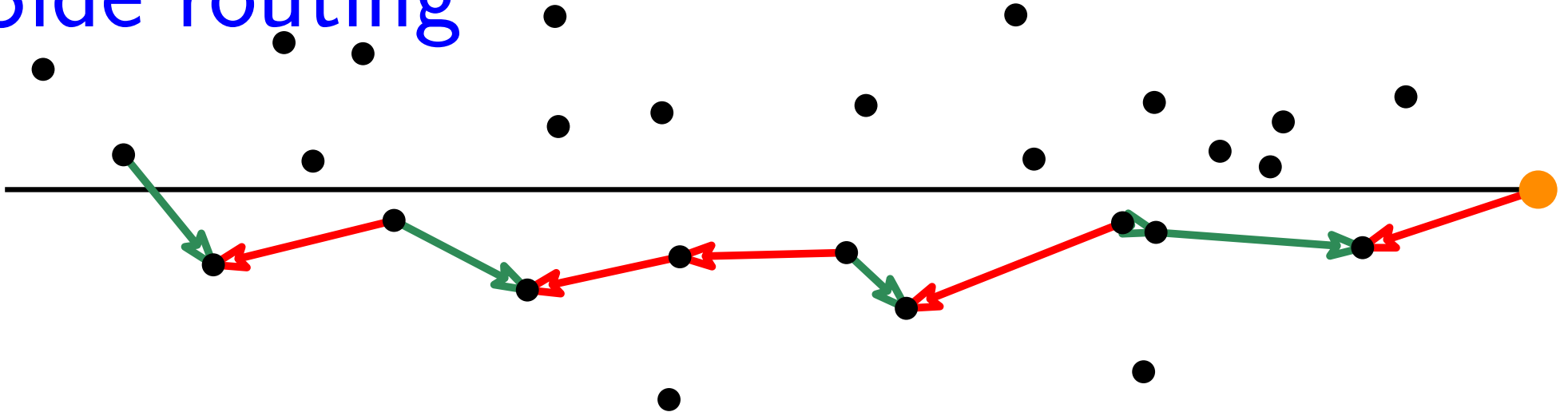


With high probability

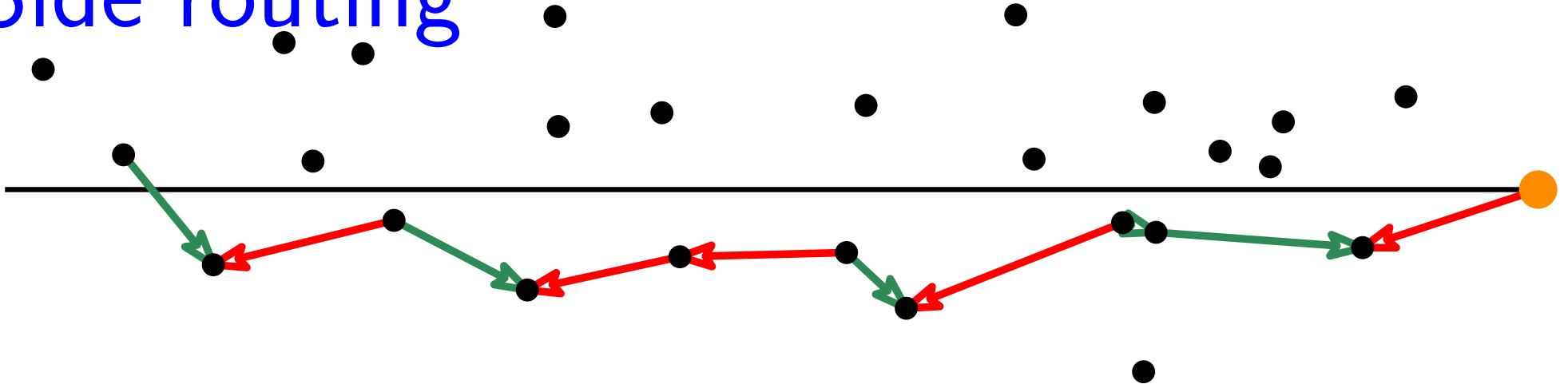
stretch close to $\frac{\sqrt{3}}{12}(3 \ln 3 + 4)$

path terminates in a square of side $\tilde{O}(\lambda^{\frac{1}{4}})$

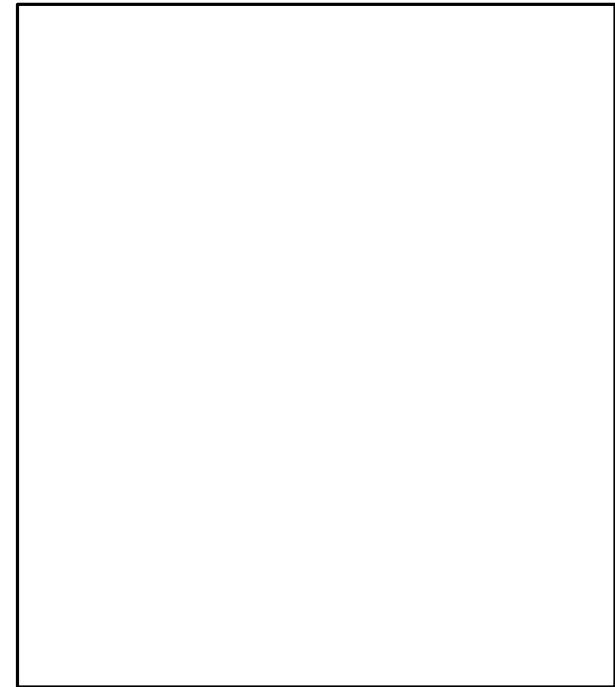
Side routing



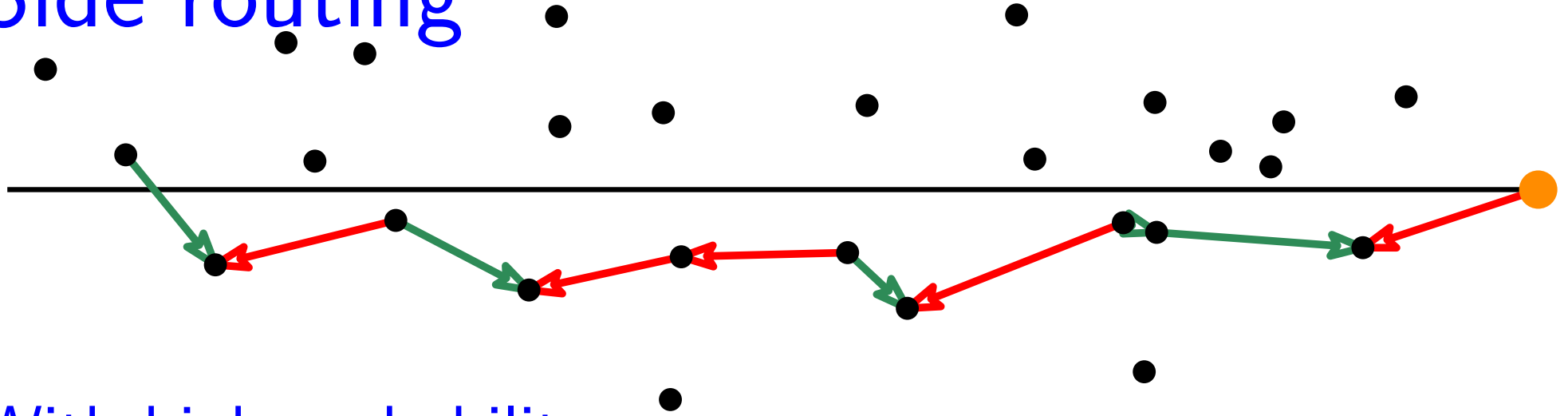
Side routing



With high probability

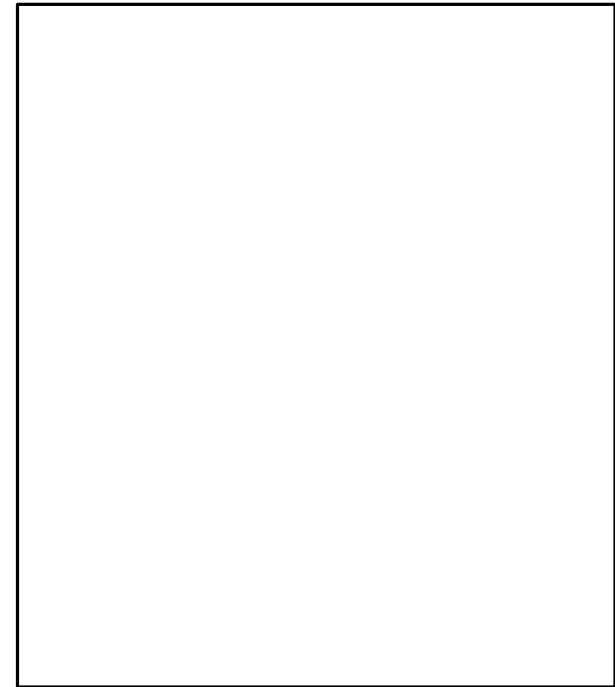


Side routing

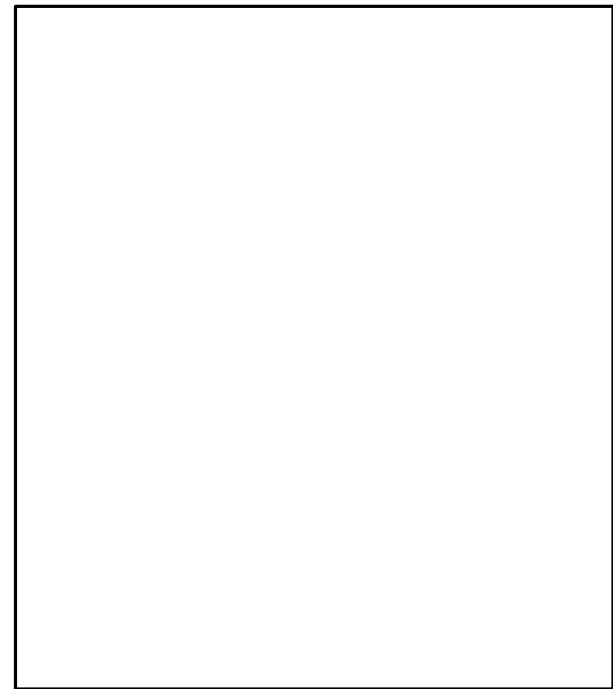
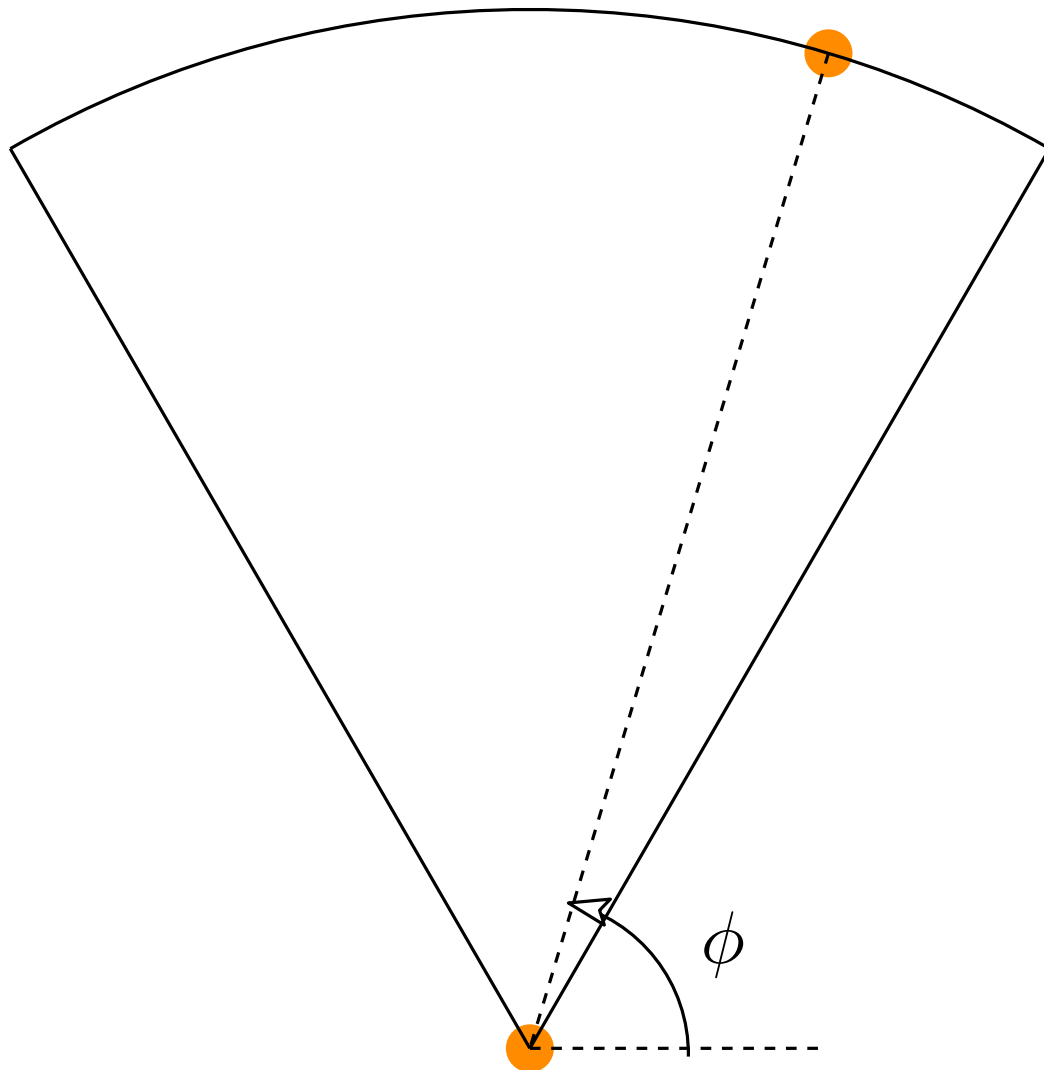


With high probability

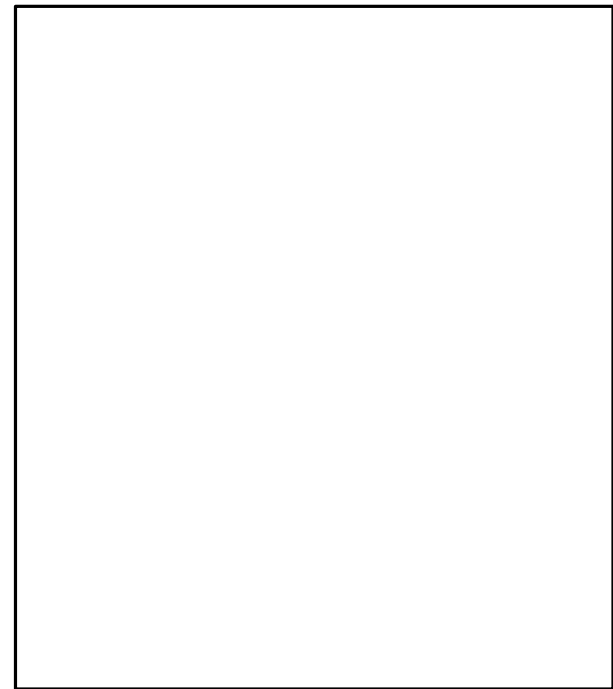
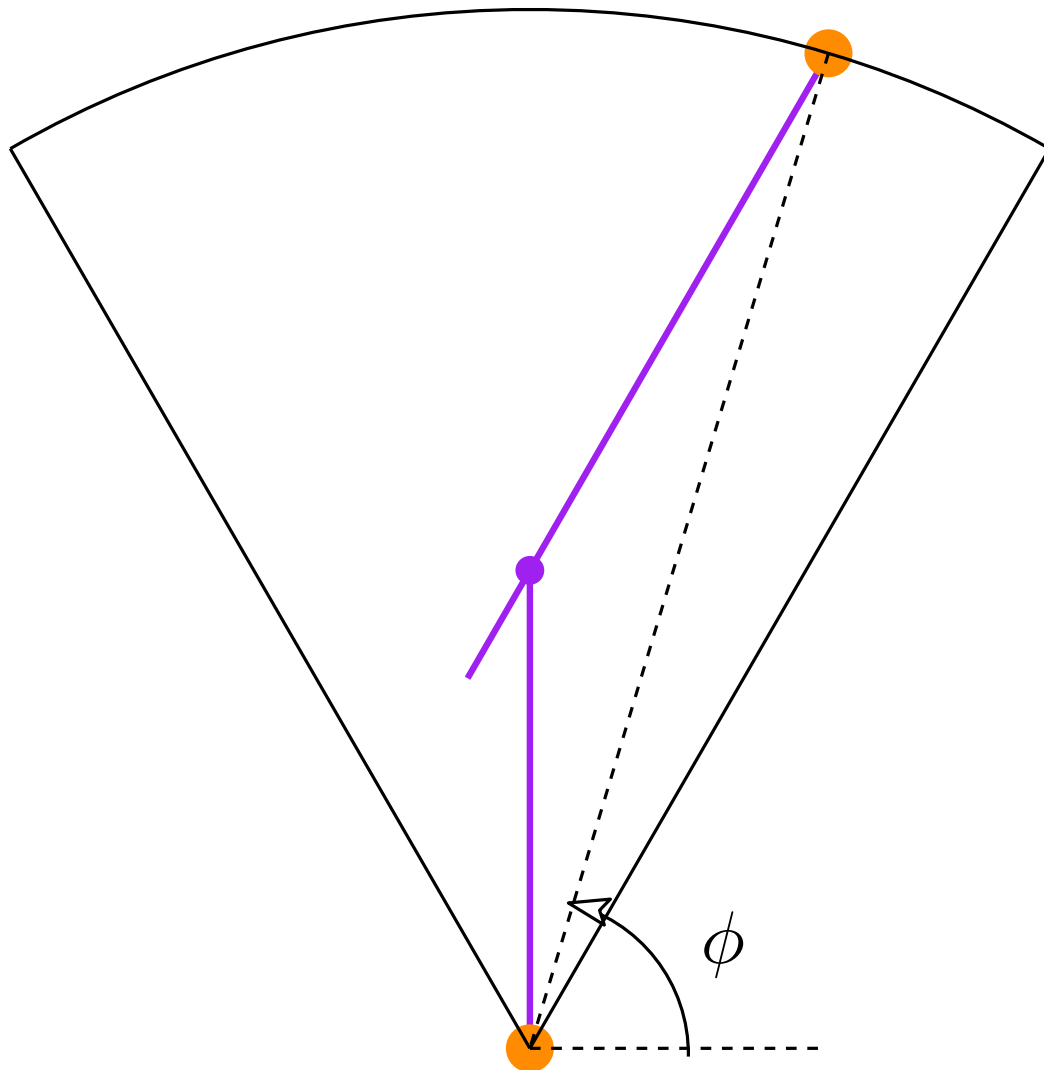
stretch close to $\frac{\sqrt{3}}{12} (3 \ln 3 + 4)$



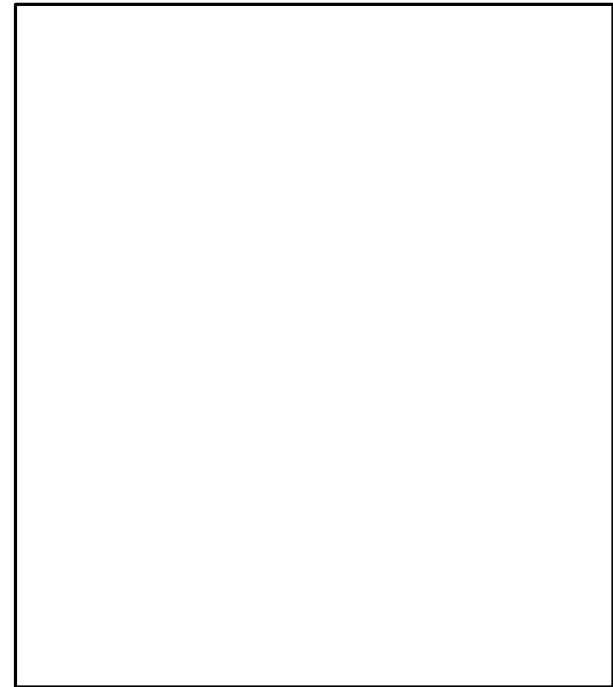
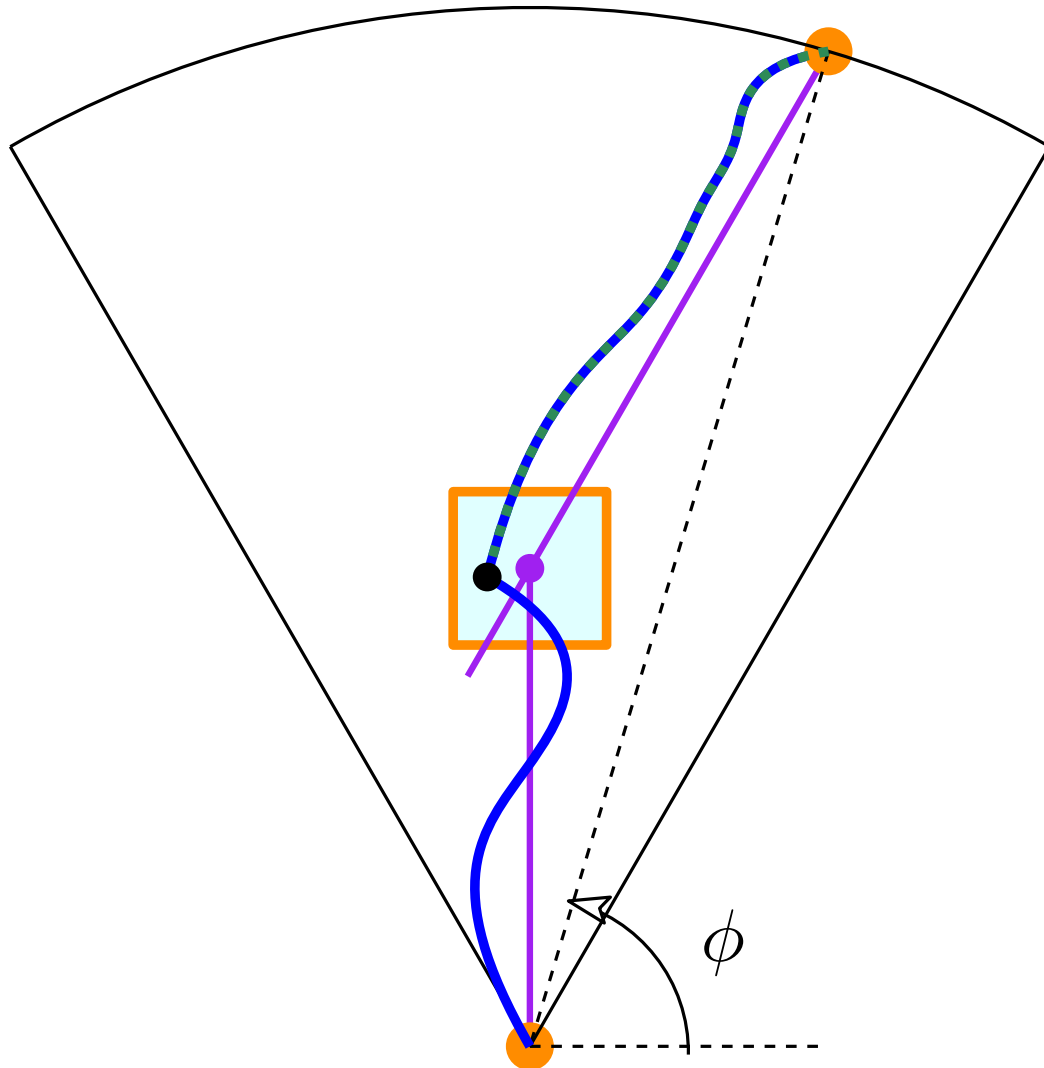
Positive routing



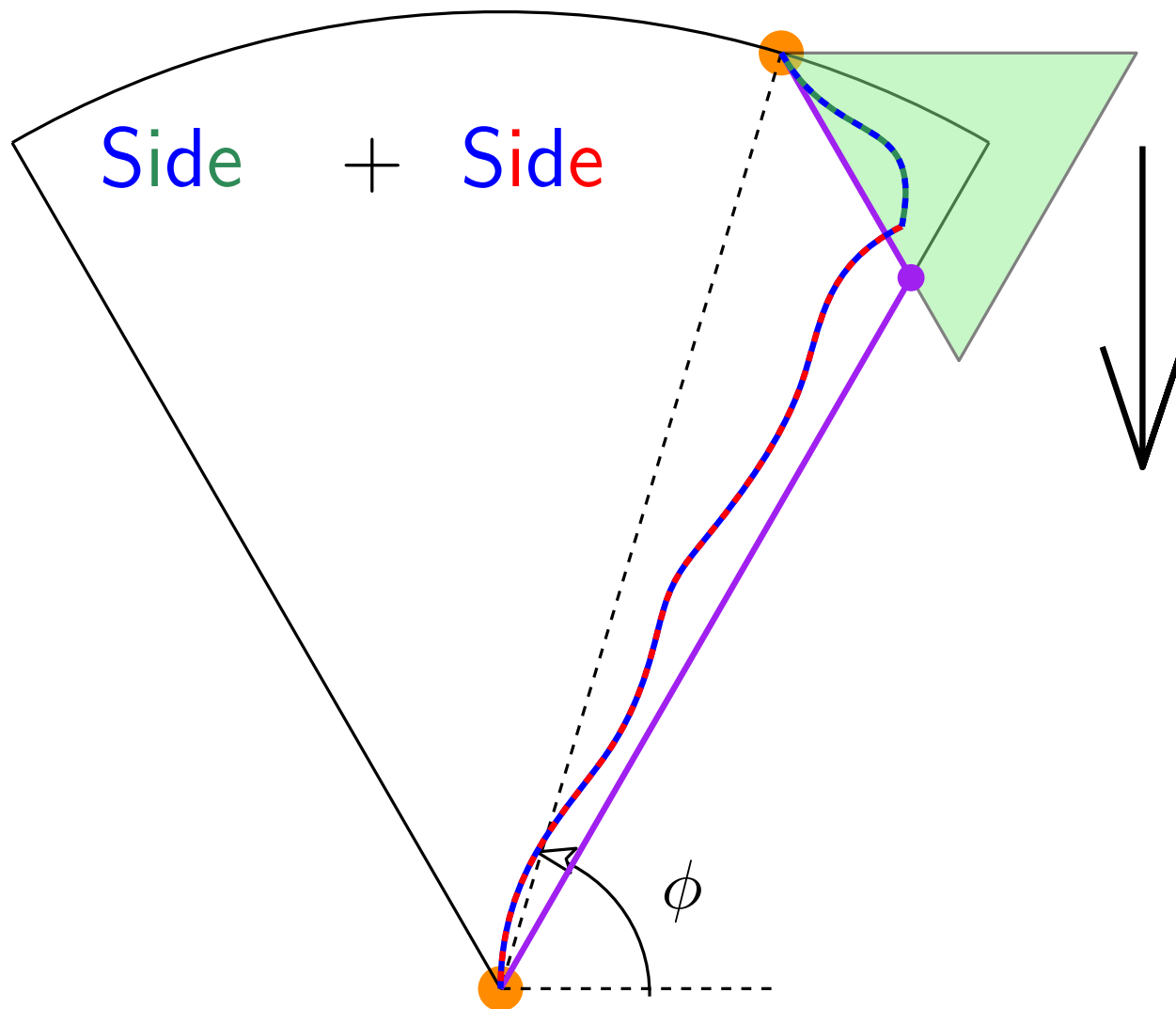
Positive routing

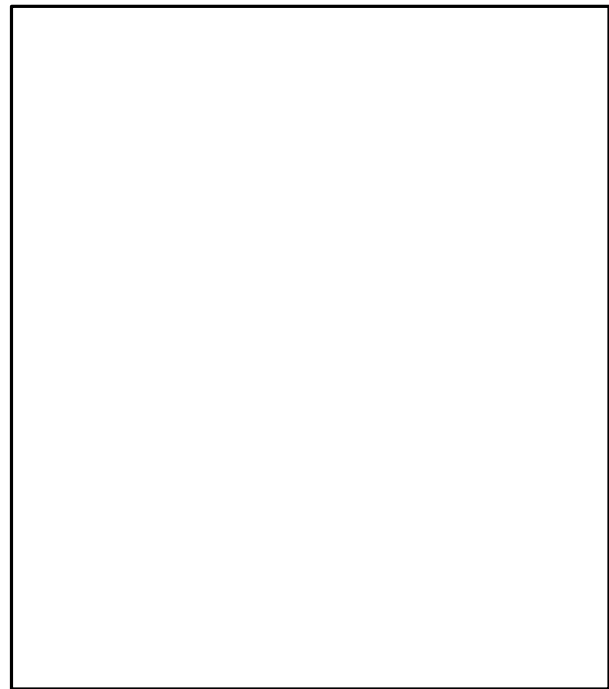
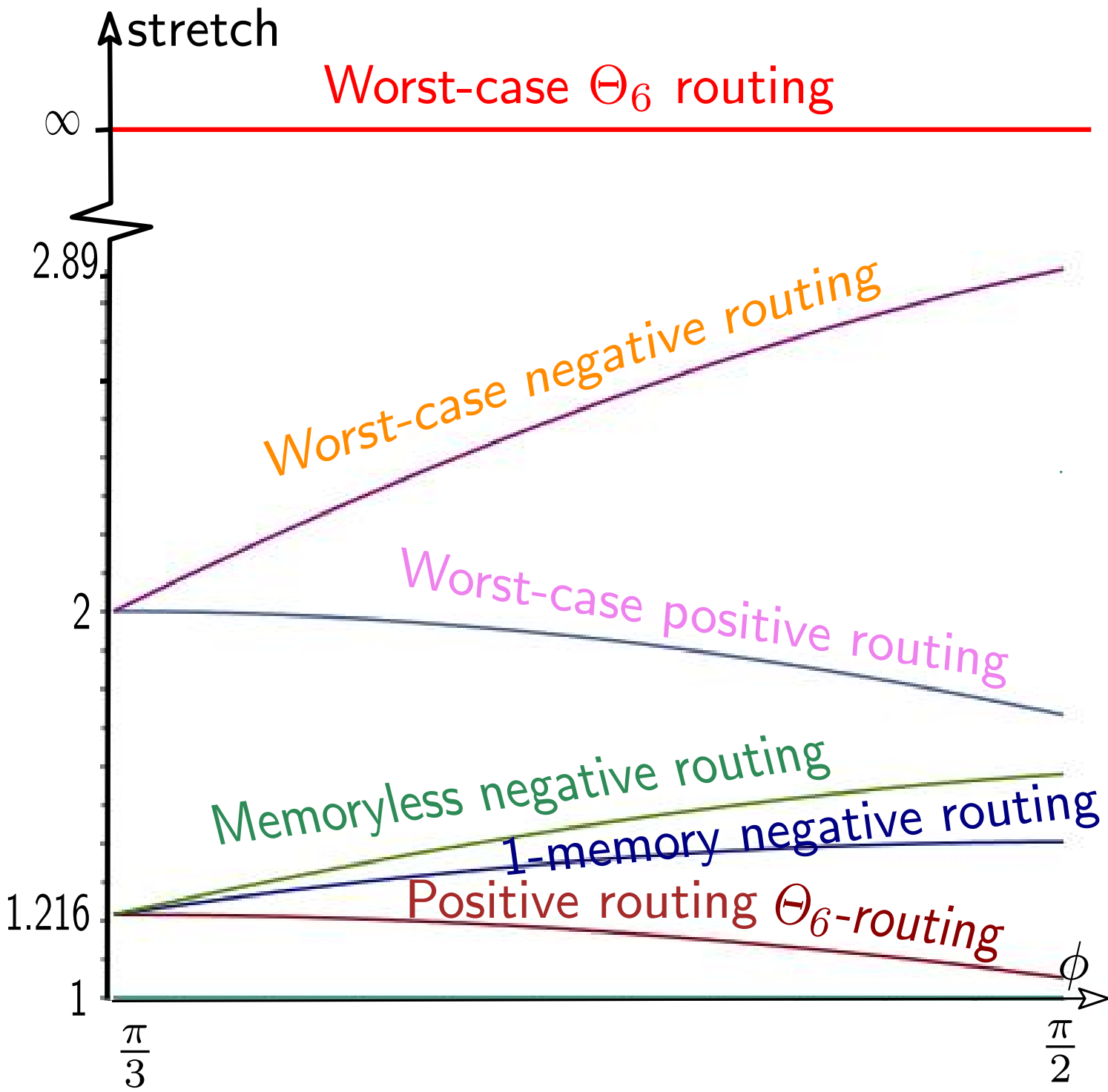


Positive routing



1-Memory negative routing





ARXIV : 1910.14289

