Current Results and Open Problems on Reconfiguration of Modular Robots
Exercises & Open Problems

Reconfiguring edge-connected sets of triangles

1. Geometric model on a triangular grid [easy]

a) Analyze in detail the possible moves of a robot module in a triangular grid, depending on *i)* the obstructions it can encounter and *ii)* the shape of the module.

b) Determine which of your moves are equivalent and which are substantially different.

2. The reconfiguration graph for edge-connected sliding triangles [easy] Once the sliding move has been well defined in Question 1, study the reconfiguration graph G_n whose vertices are all the edge-connected configurations with n modules, and an edge exists between two vertices if one slide move transforms one into the other. More precisely:

a) Is G_n connected?

b) If not, how many connected components does it have?

c) What are their sizes?

3. A reconfiguration algorithm for edge-connected sliding triangles [medium] Can we connect the reconfiguration graph G_n by using helpers? If so, what is the minimum number of helpers that are always sufficient and sometimes necessary?

4. The reconfiguration graph for edge-connected pivoting triangles [easy] Once the pivoting move has been well defined in Question 1, study the reconfiguration graph G_n whose vertices are all the edge-connected configurations with n modules, and an edge exists between two vertices if one pivot move transforms one into the other. More precisely:

a) Prove that here exist rigid configurations of edge-connected pivoting triangles.

b) What is the number and sizes of the connected components of G_n?

5. A reconfiguration algorithm for edge-connected pivoting triangles [medium?]

a) Find necessary conditions for reconfigurability. Are they sufficient?

b) Is it possible to use helpers to connect the reconfiguration graph? How many?

Improving the current results on sliding robots

6. Sliding squares [medium] Let I and G be two edge-connected configurations of sliding squares in the plane, not necessarily disjoint. Let I' and G' respectively be their 1-offsets, i.e., I' (resp. G') is the Minkowski sum of I (G) and one lattice square. Find an algorithm to reconfigure I into G, such that all moves happen within $I' \cup G'$.

7. Sliding cubes [medium, maybe difficult?]

a) Does the algorithm by Abel & Kominers really need $\Omega(n^3)$ slide moves?

b) In the affirmative, can we come up with an algorithm using $o(n^3)$ slide moves?

c) If not, can we al least determine a large subclass of shapes for which we can solve the problem using $O(n^2)$ slide moves?

8. Sliding modules [difficult]

a) Find a reconfiguration algorithm for edge-connected 2D sliding robots producing many simultaneous moves in parallel.

b) Distribute it (or maybe directly solve the previous question by coming up with a distributed algorithm from scratch).

Pivoting cubes in 3D

9. Pivoting cubes [difficult]

a) Study the reconfiguration graph for facet-connected pivoting cubes.

b) Is it possible to reconfigure between any two shapes with the same number of modules by using helpers?

c) If so, what is the minimum number of helpers that are always sufficient and sometimes necessary?

Exploring how to detect and react to failures

10. Squeezing modular robots [easy-medium] Let I and G be two edge-connected configurations of squeezing modular robots on a grid. Assume that I and G are disjoint except for one module r, that is the lexicographically larger module of I and the lexicographically smaller module of G. Consider the reconfiguration algorithm from I to G that tunnels modules from I into G through (one of) their spanning trees rooted at r. Propose strategies to survive to the failure of one of the robot modules (different from r). Take into account different failure reasons. What if more than one module simultaneously fail?

Observation

The difficulty level of each questions is only indicative, as some of these problems are currently open and I cannot be sure of their real difficulty. I have labelled as "easy" the questions that are solvable in minutes. In the "medium" category fall all problems that I think can be solved with a reasonably small amount of effort and time. I have classified as "difficult" the problems that I think most probably need a collaborative effort sustained in time to be solved. But, as I said, I may be wrong.