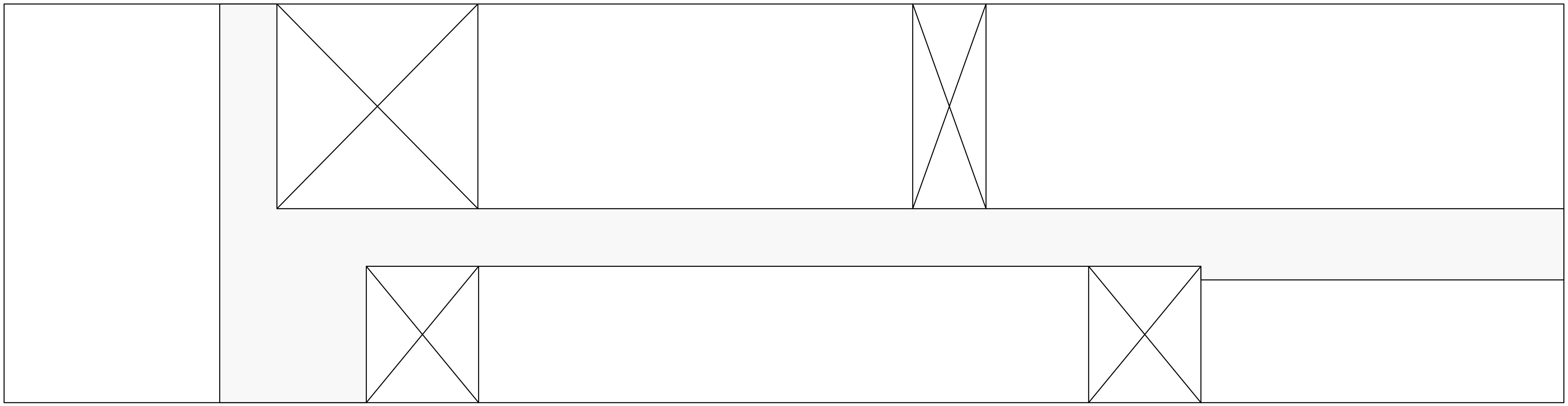
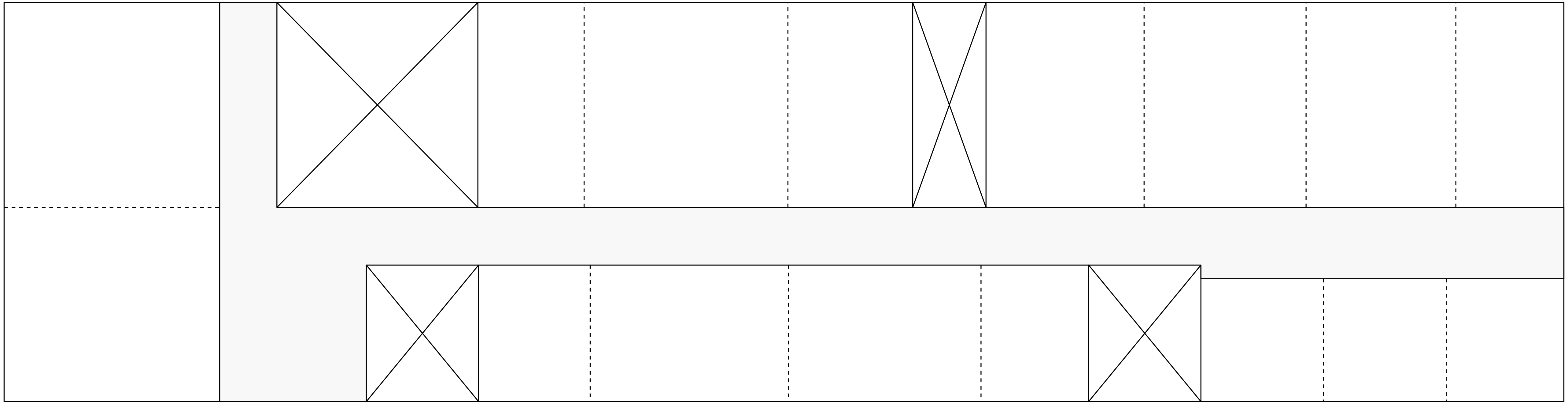
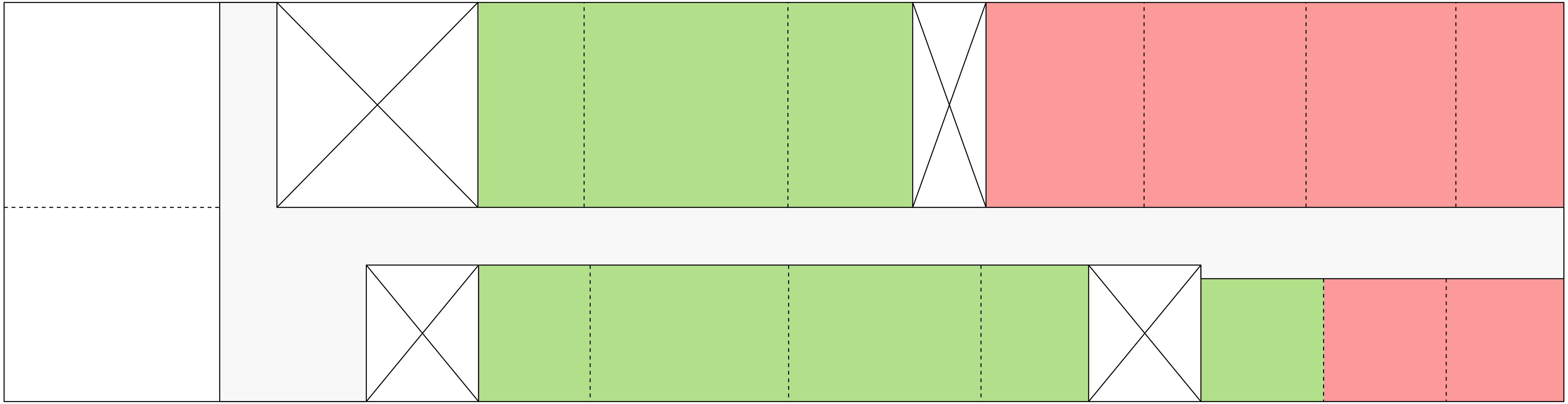
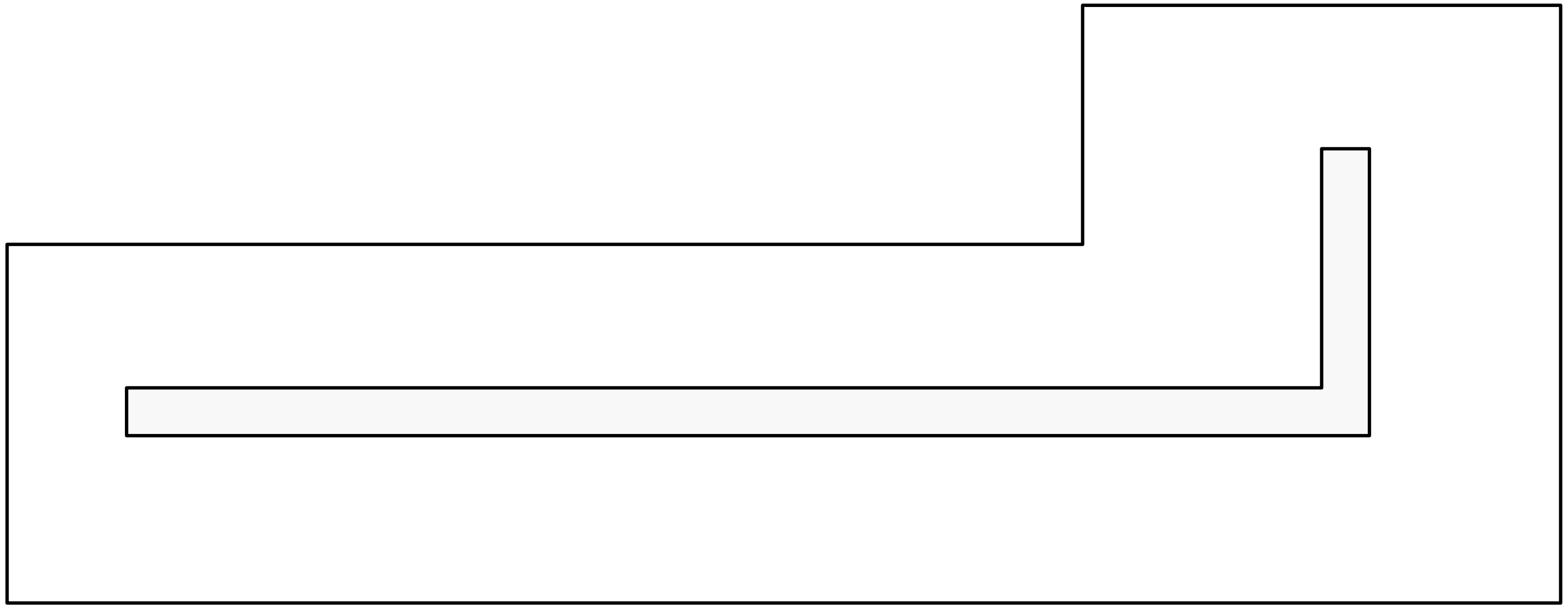
Algorithms for Automated Floor Planning

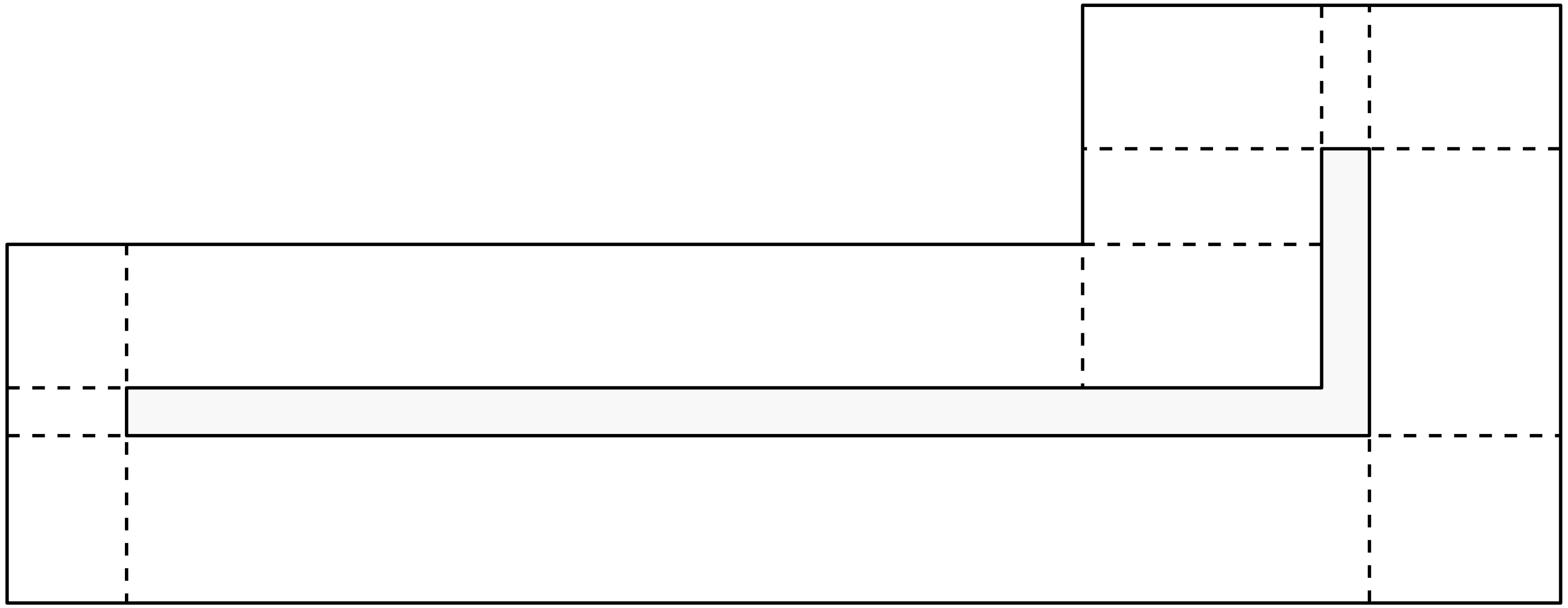
Felix Klesen

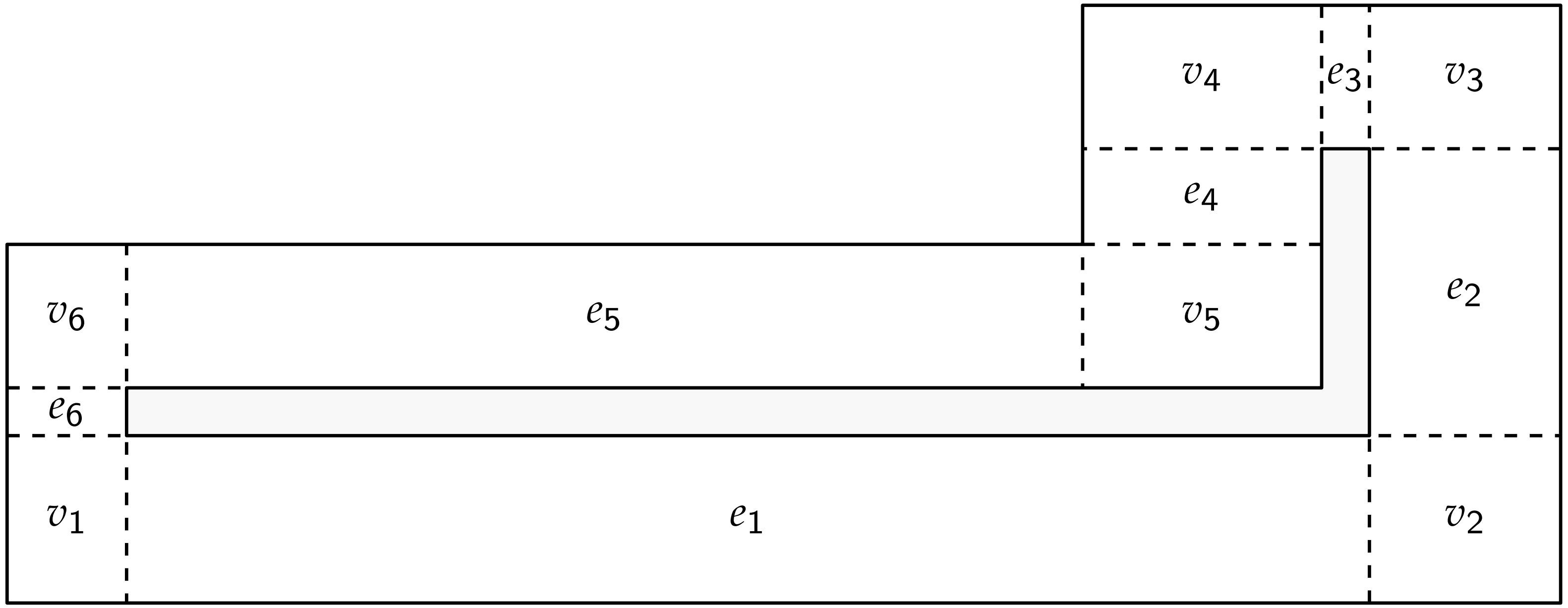


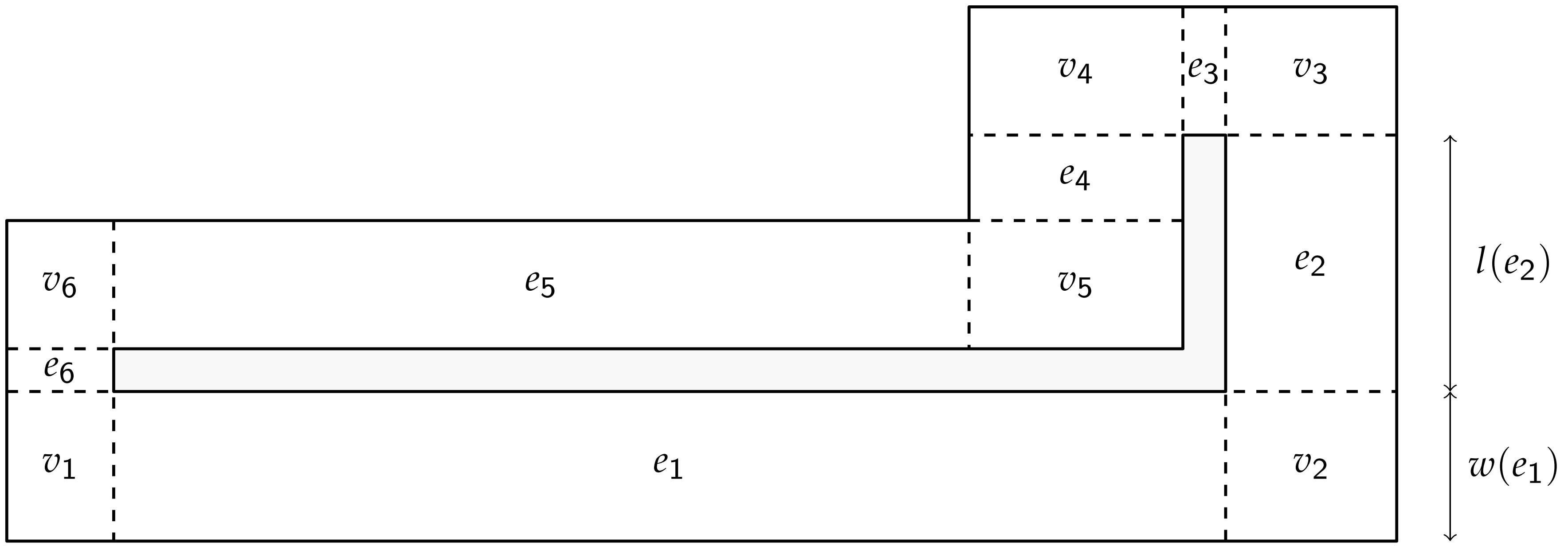




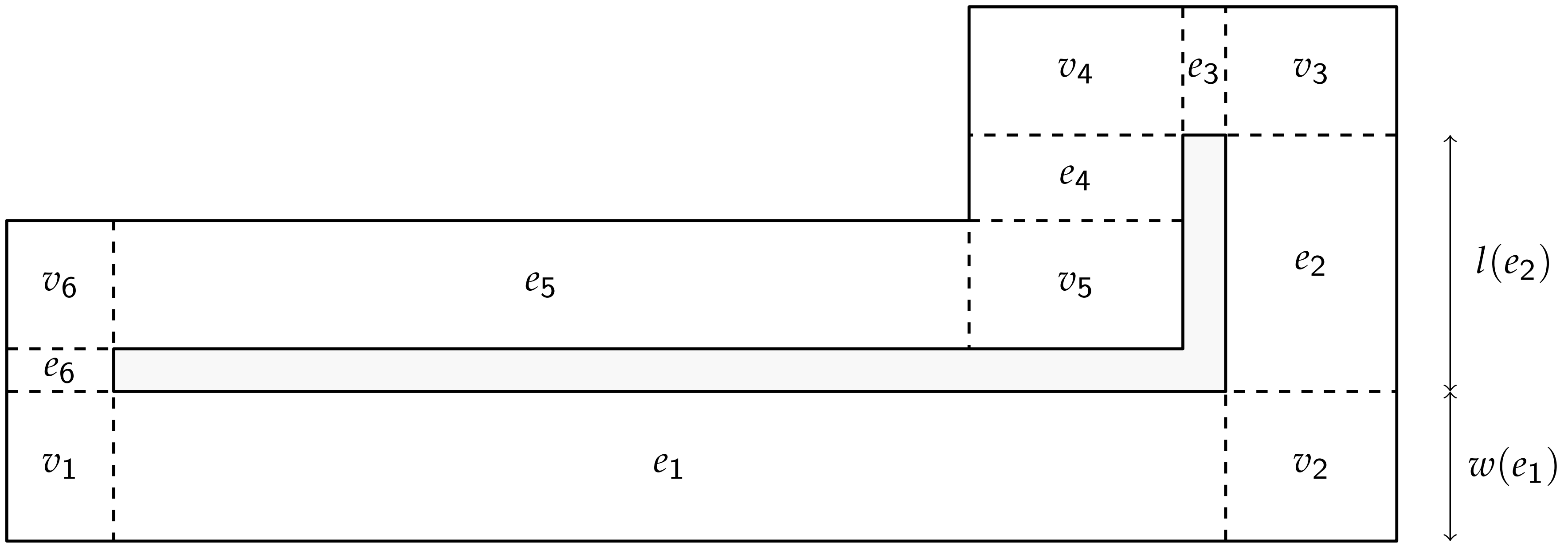




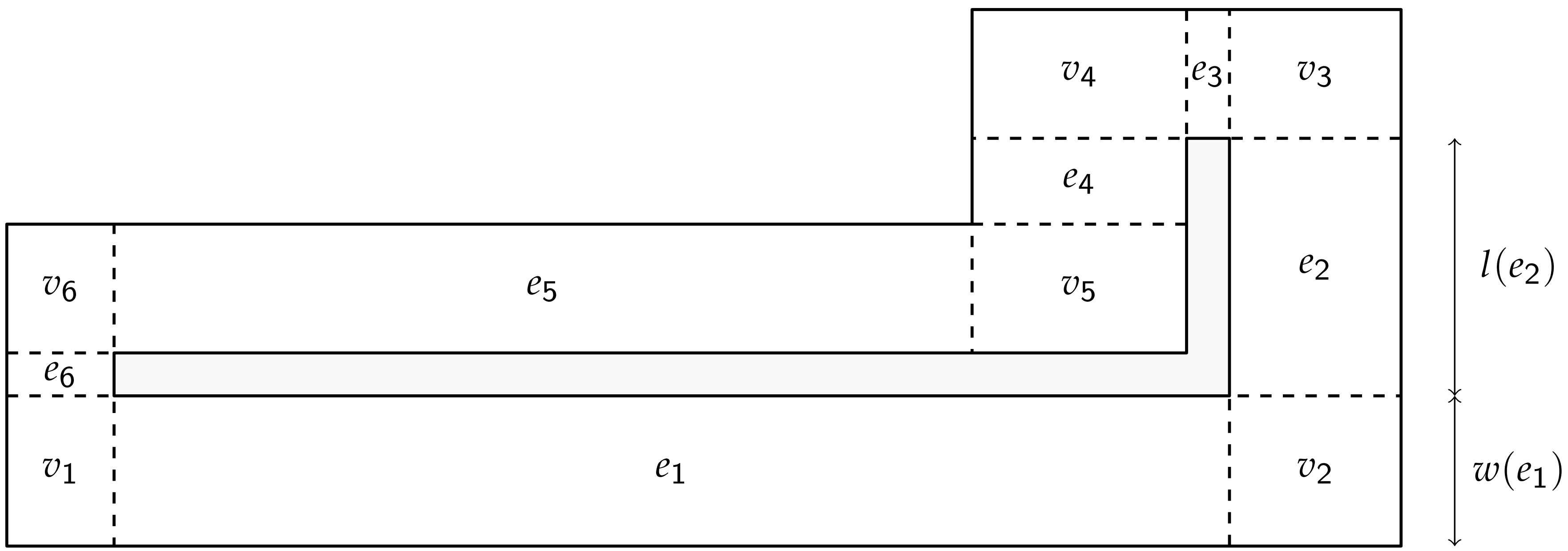

v_4
e_3
v_3
e_4
v_6
e_5
v_5
e_2
e_6
v_1
e_1
v_2



v_4
e_3
v_3
e_4
e_2
$l(e_2)$
v_6
e_5
v_5
e_6
v_1
e_1
v_2
$w(e_1)$



$$g(r,e) = \begin{cases} 1 & \alpha \geq \frac{s(r)}{w(e)^2} \geq \frac{1}{\alpha} \\ 0 & \text{otherwise} \end{cases}$$



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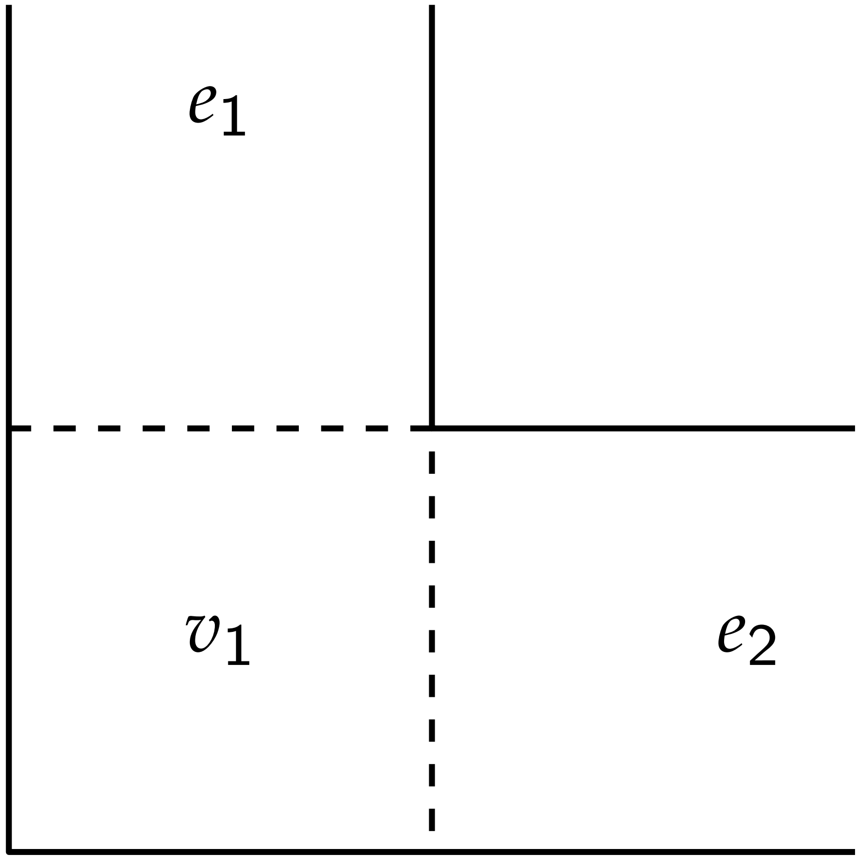

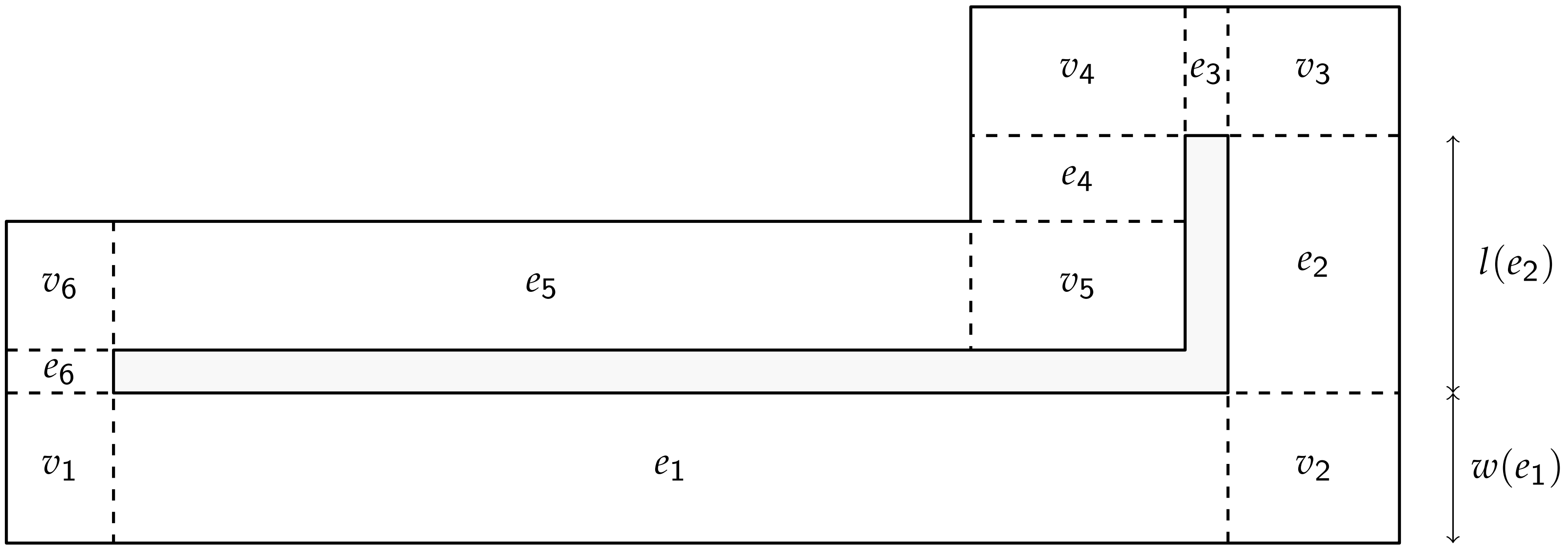

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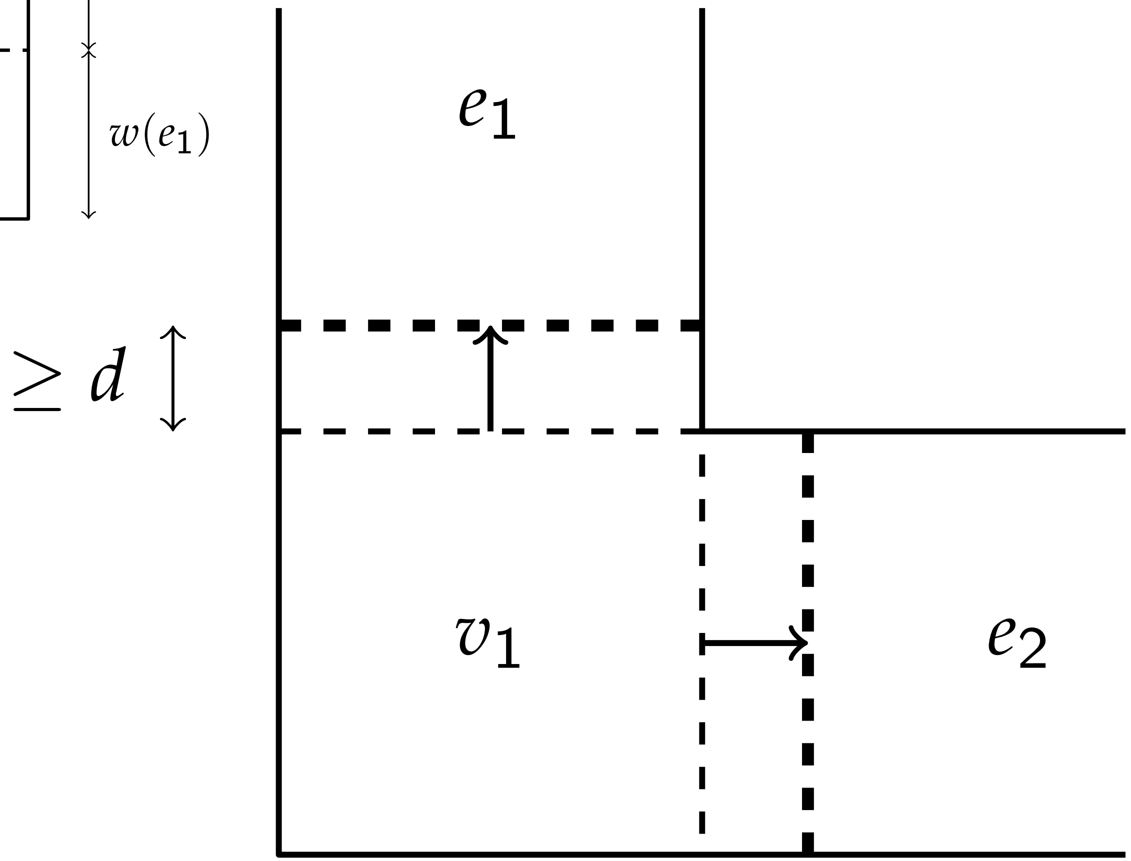

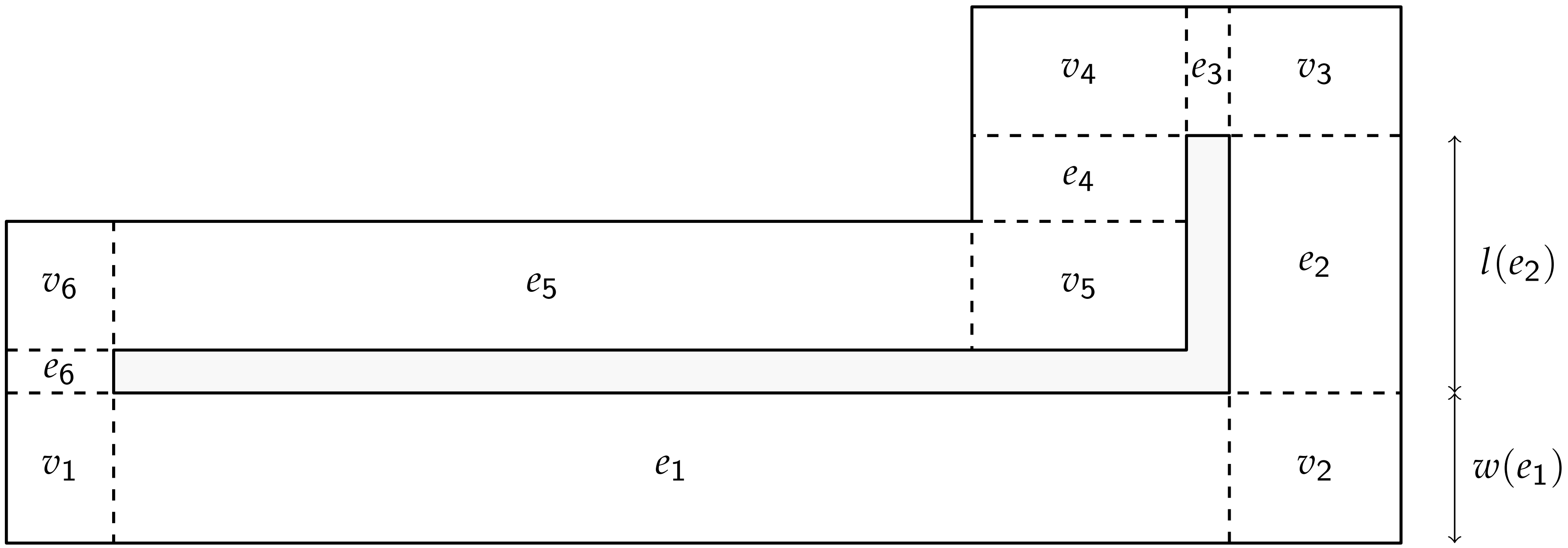

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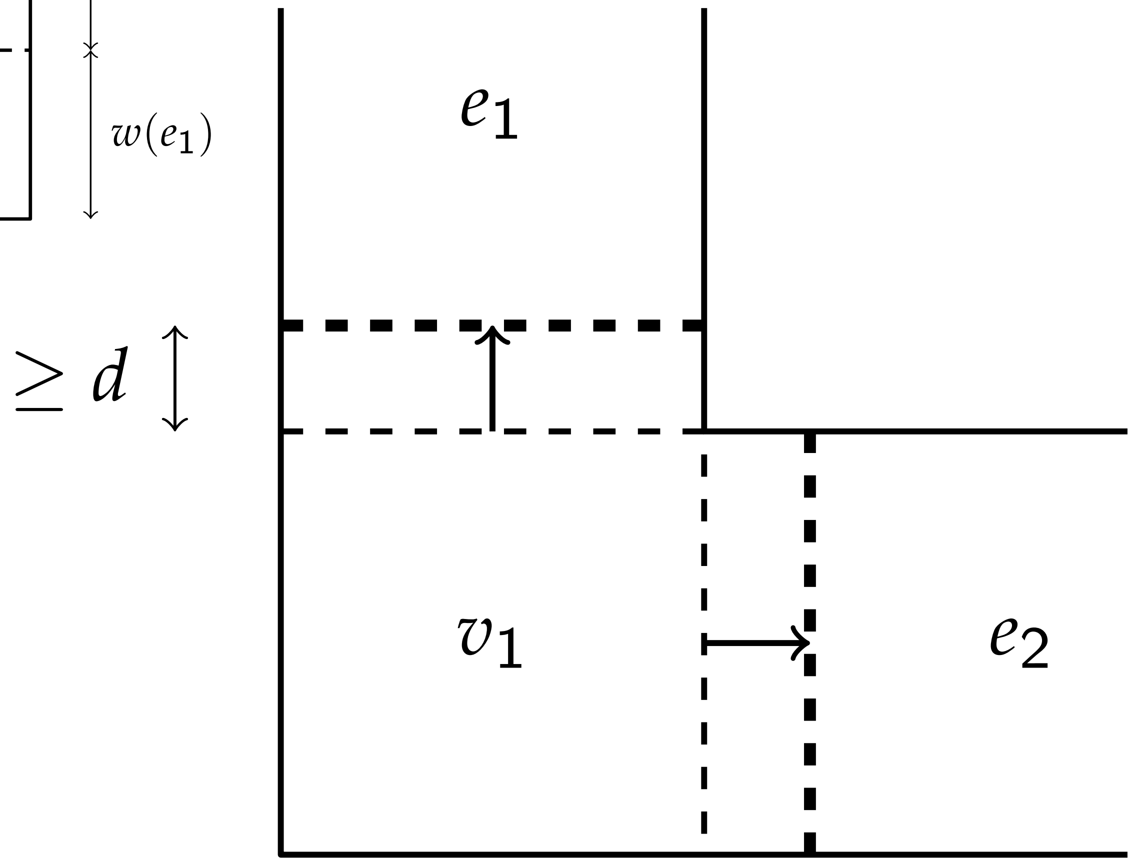
$$q(r,e,v) = \begin{cases} 1 & s(r) \geq s(v) + w(e) \cdot d \\ 0 & \text{otherwise} \end{cases}$$



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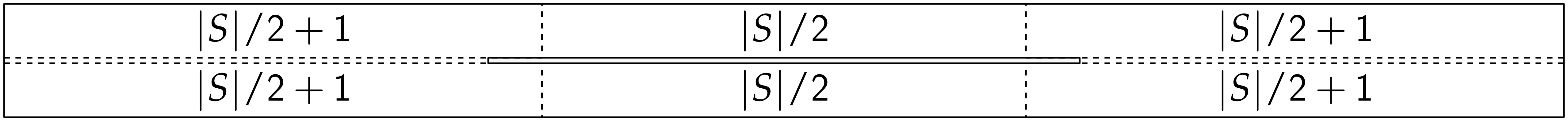

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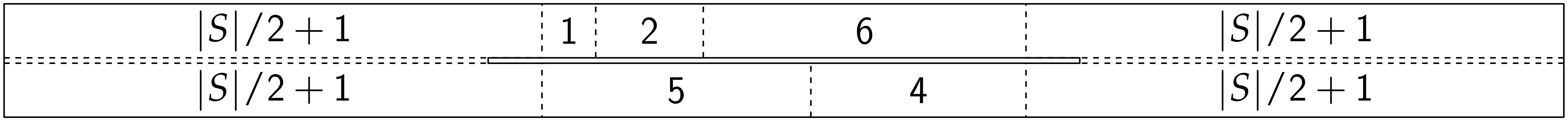

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$$y_{r,e,v} \leq q(r,e,v) \quad \forall r \in R, e \in E, v \in N(e)$$

$$\sum_{r \in R} \left[x_{r,e} \cdot s(r) + \sum_{v \in N(e)} y_{r,e,v} \cdot \big(s(r) - s(v)\big) \right] \leq s(e) \quad \forall e \in E$$

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$$\left(x_{r,e} + \textstyle\sum_{v \in N(e)} y_{r,e,v}\right) \cdot \gamma(r, c) \leq z_{e,c} \quad \forall r \in R, e \in E, c \in C$$

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minimize $\sum_{c \in C} \sum_{e \in E} z_{e,c}$

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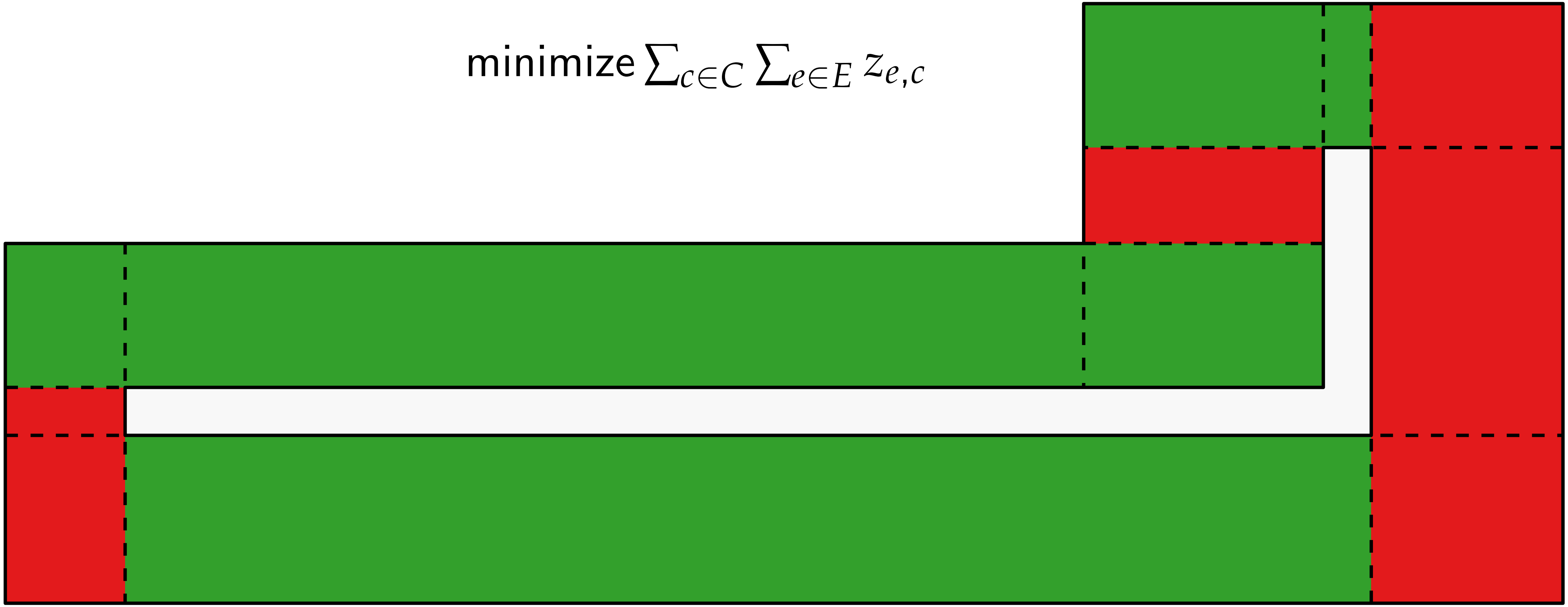
$$\text{minimize} \sum_{c \in C} \sum_{e \in E} z_{e,c}$$



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z_{e,c} &\in \{0,1\} && \forall e \in E, c \in C \\
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$$
\text{minimize} \sum_{c \in C} \sum_{e_1 \in E} \left[\sum_{e_2 \in E} z_{e_1,c} \cdot z_{e_2,c} \cdot \delta(e_1, e_2) + \sum_{v \in V} z_{e_1,c} \cdot z_{v,c} \cdot \delta(e_1, v) \right]
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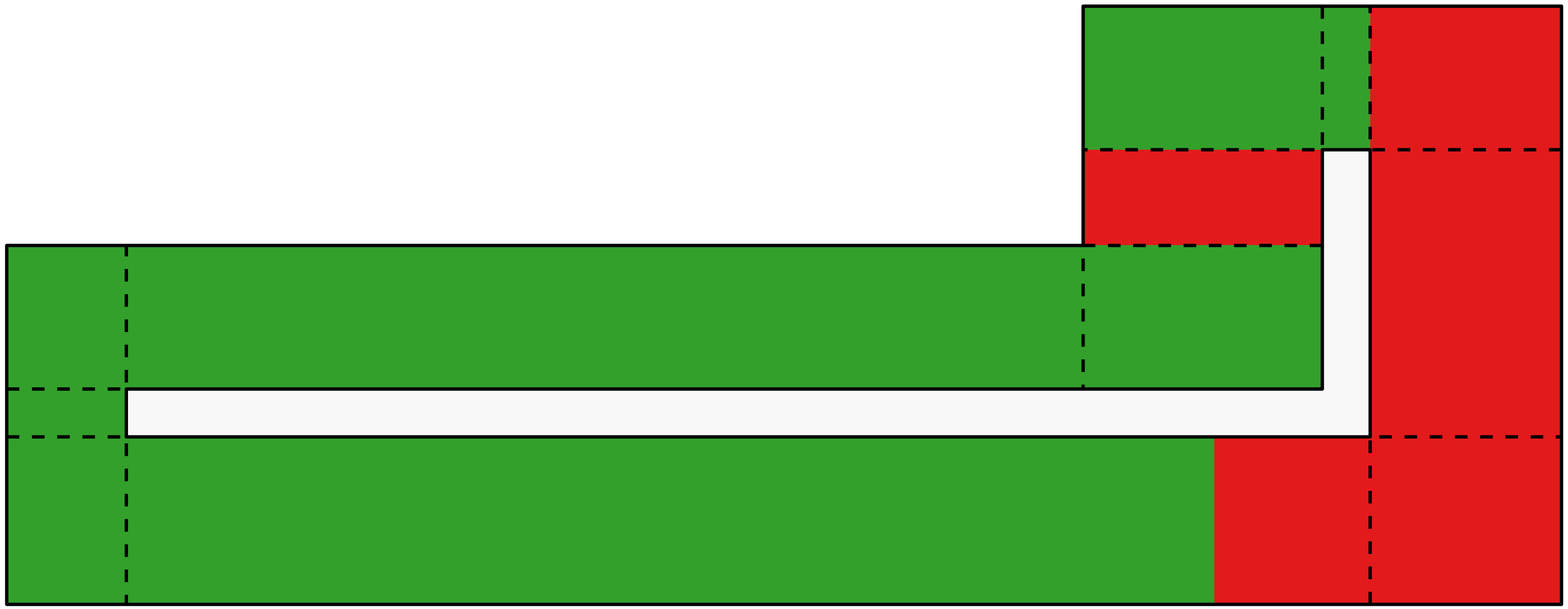

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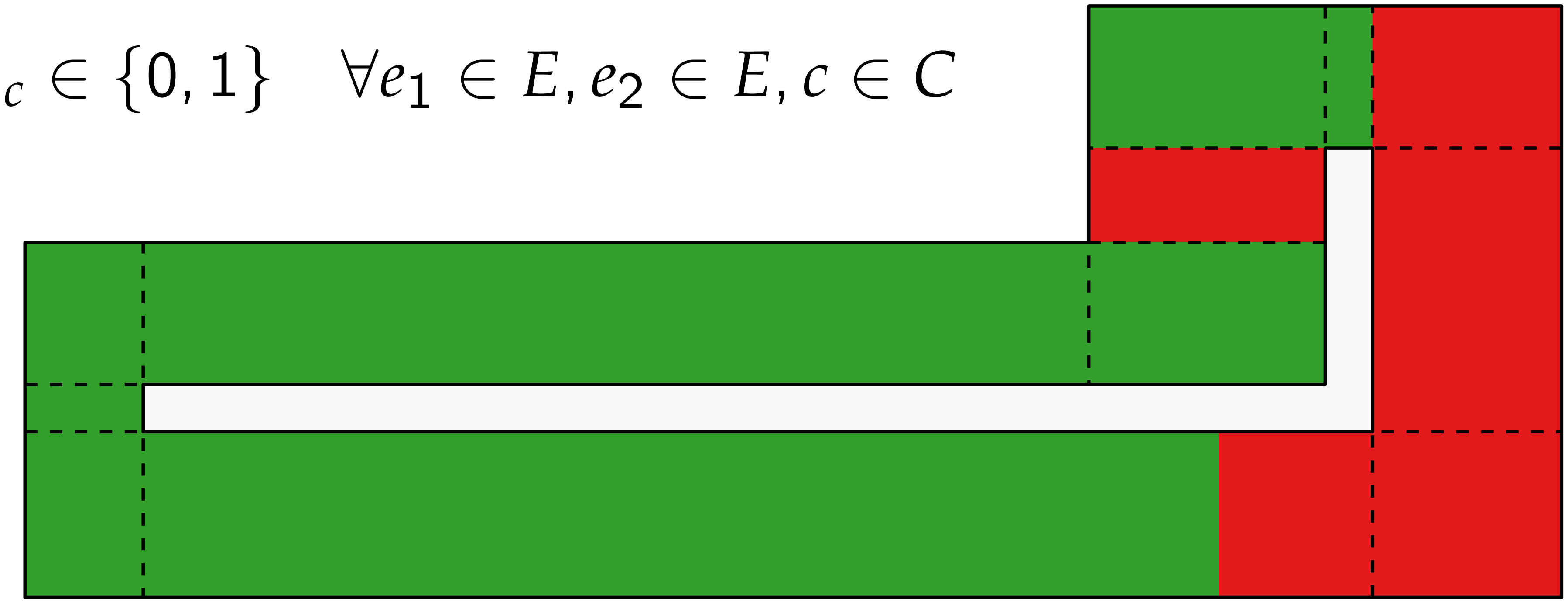
$$u_{e_1,e_2,c} \in \{0,1\} \quad \forall e_1 \in E, e_2 \in E, c \in C$$



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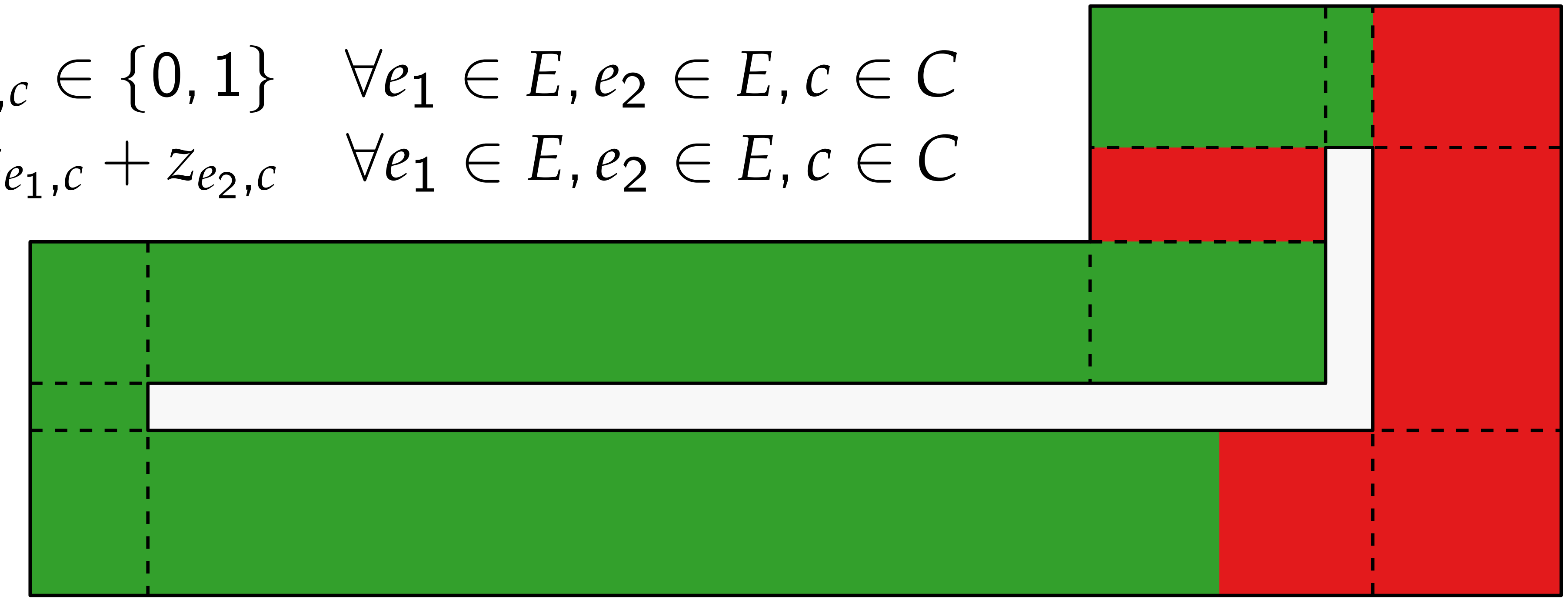
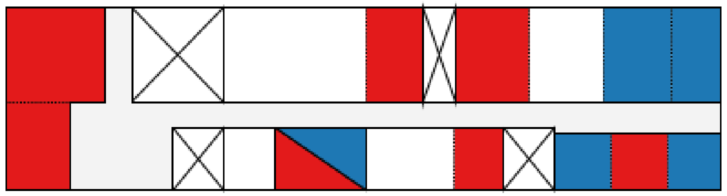
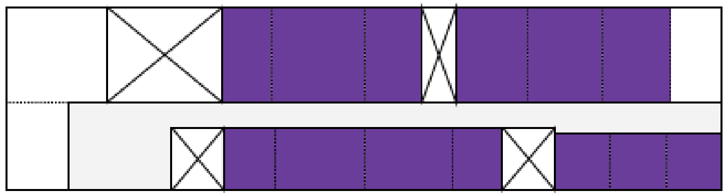
$$u_{e_1,e_2,c} \in \{0,1\} \quad \forall e_1 \in E, e_2 \in E, c \in C$$
$$u_{e_1,e_2,c} + 1 \geq z_{e_1,c} + z_{e_2,c} \quad \forall e_1 \in E, e_2 \in E, c \in C$$

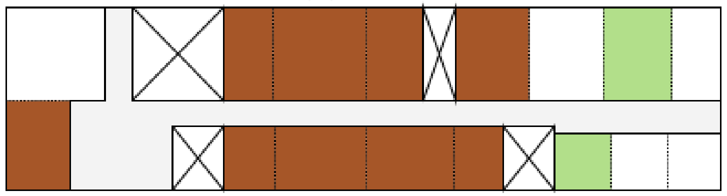




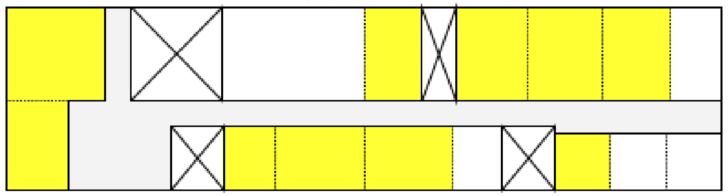
(a) 30.03



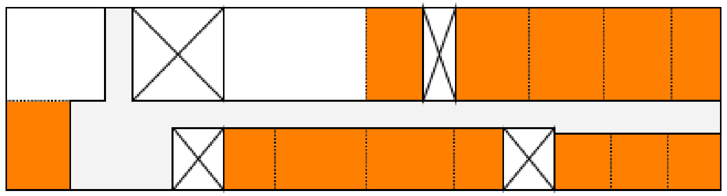
(b) 40.03



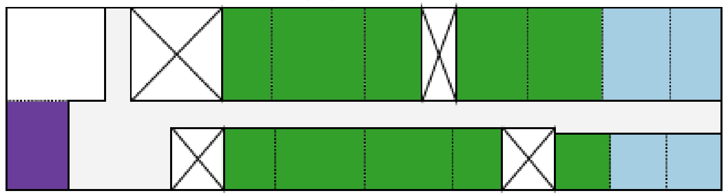
(c) 30.02



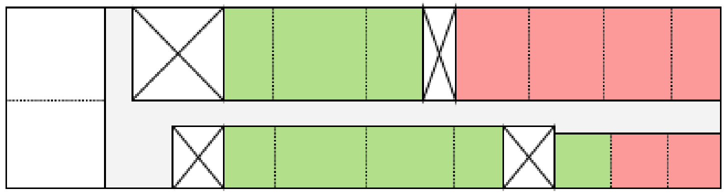
(d) 40.02



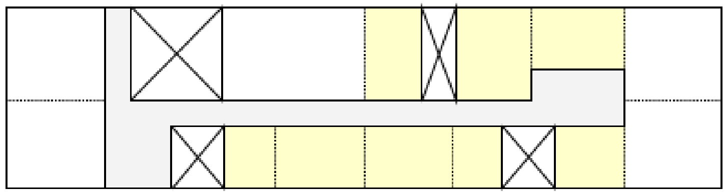
(e) 30.01



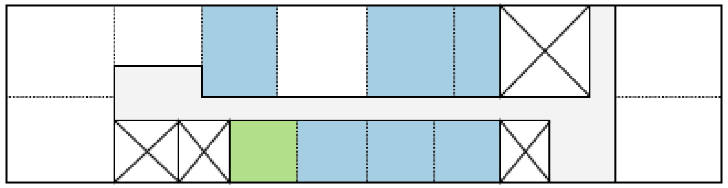
(f) 40.01



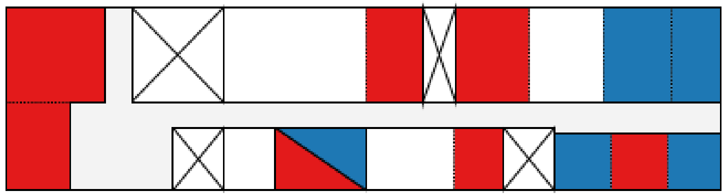
(g) 30.00



(h) 40.00



(i) 31.00



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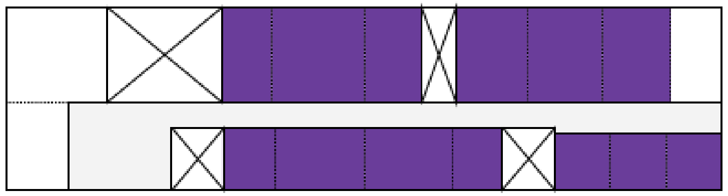
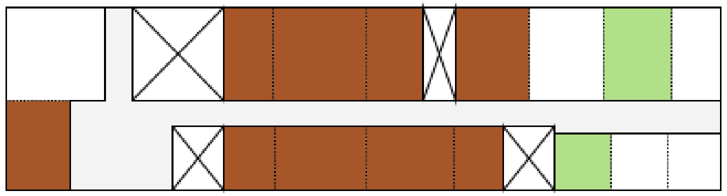
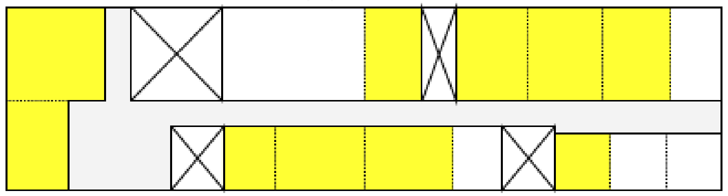


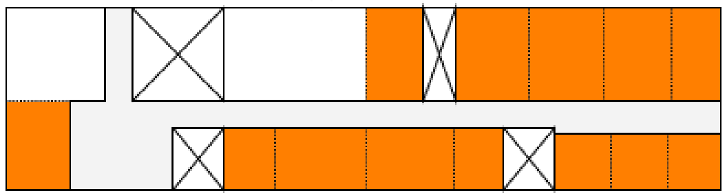
(b) 40.03



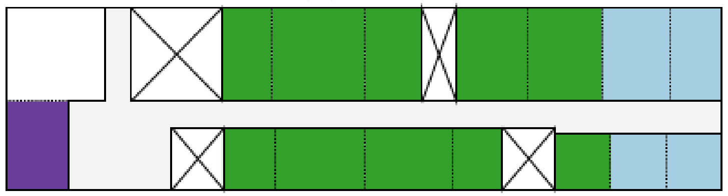
(c) 30.02



(d) 40.02



(e) 30.01



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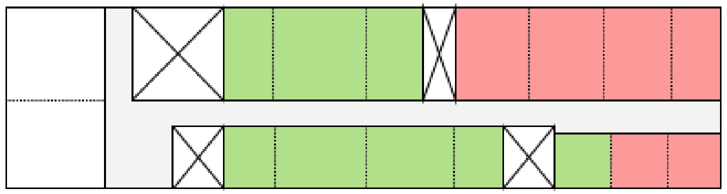

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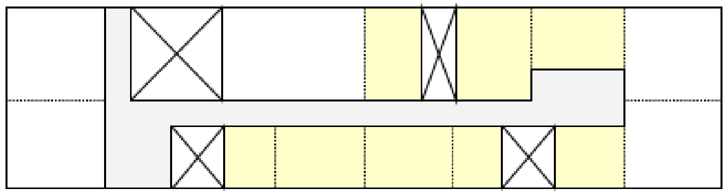

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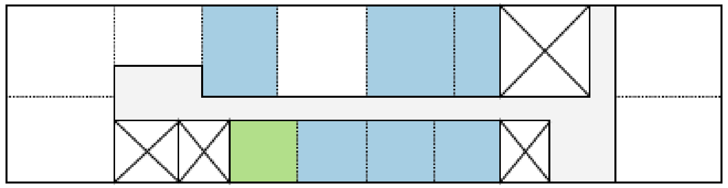

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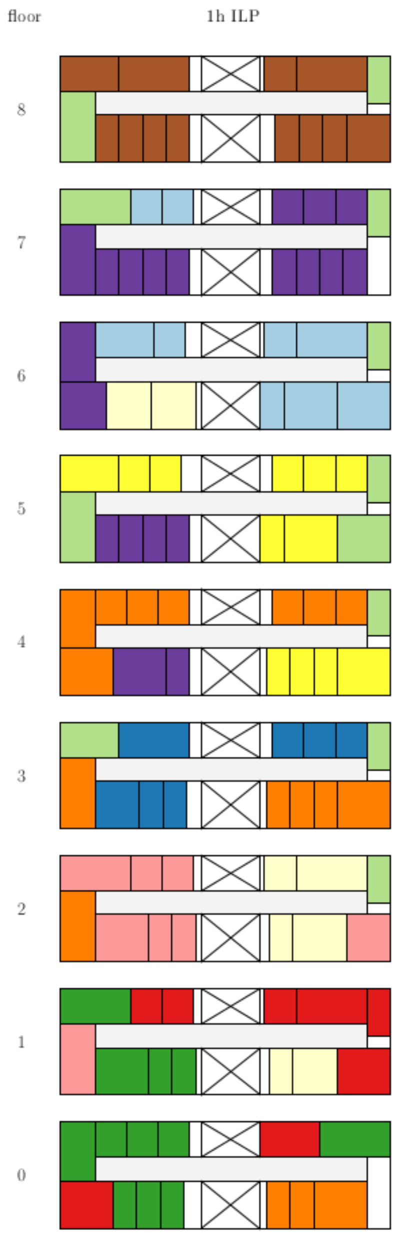
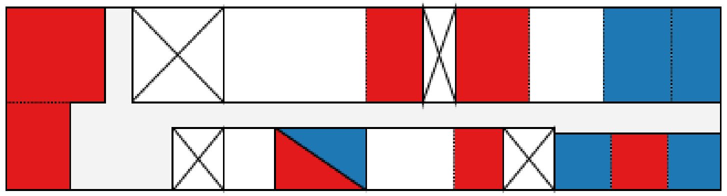
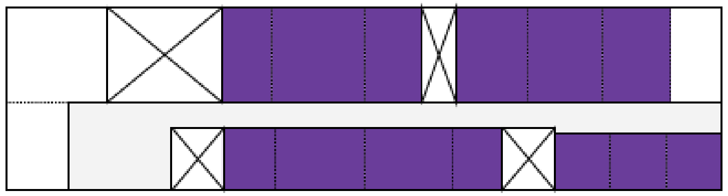




(a) 30.03



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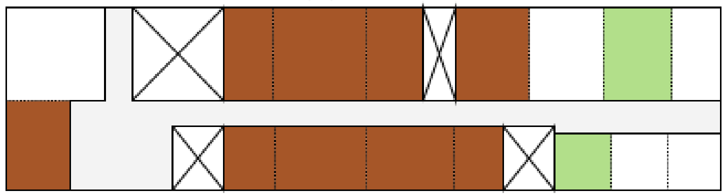
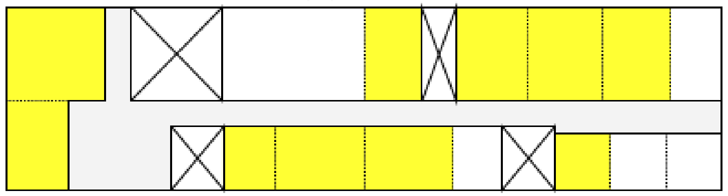


(c) 30.02



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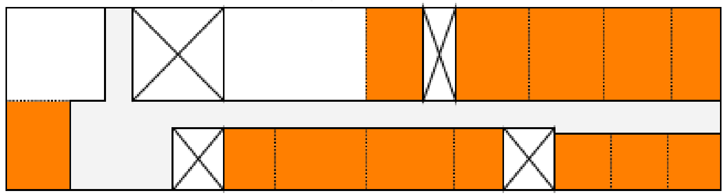

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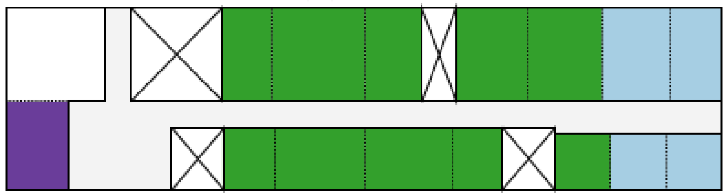

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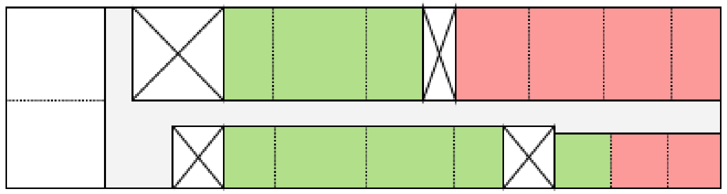
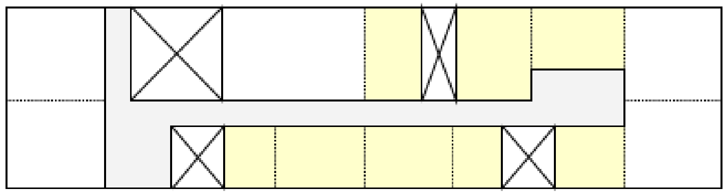
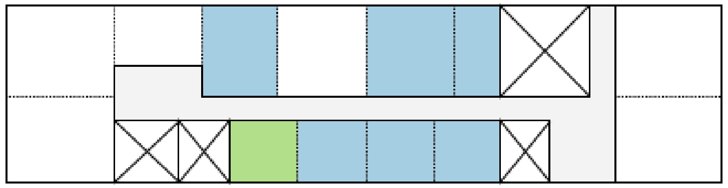


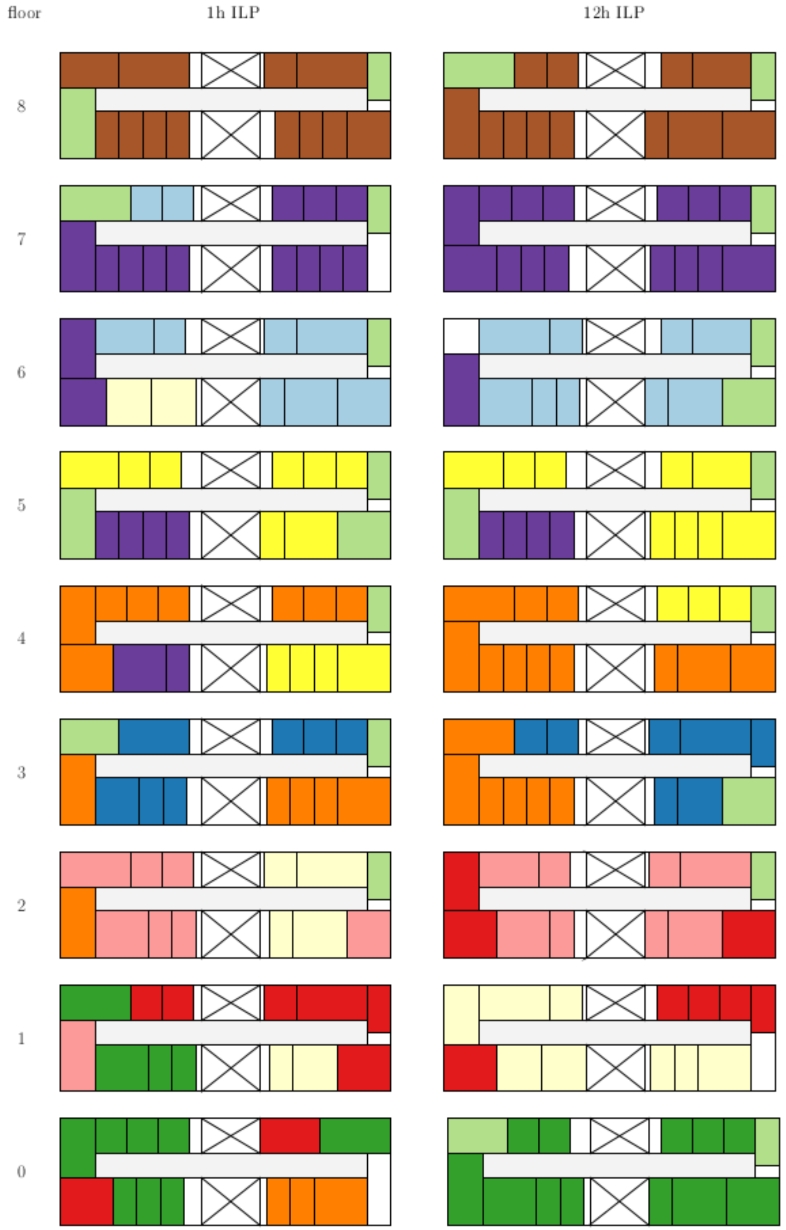
(g) 30.00



(h) 40.00



(i) 31.00



Problem ROOM-ASSIGNMENT

Item: A set R of n rooms, a size function $s\colon R \to \mathbb{N}$, a function $\gamma\colon R \to \{1, \ldots, |C|\}$ assigning each room a colour (group), and a set F of floors, each with an integer capacity m

Question: How to find an assignment of rooms to floors $a\colon R \to F$, that is a valid bin-packing solution while optimizing an objective function $\mathrm{cost}\colon (a, \gamma) \to \mathbb{R}$?

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$$x_{c,f} = \begin{cases} 1 & \bigvee_{r \in R} \left[\Big(a(r) = f \Big) \wedge \Big(\gamma(r) = c \Big) \right] \\ 0 & \text{otherwise} \end{cases}$$

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Room-assignment is NP-hard to approximate.

Problem AREA-DISTRIBUTION

Item: A set C of colours, a size function $s\colon C \to \mathbb{N}$ and a set F of floors, each with an integer capacity m.

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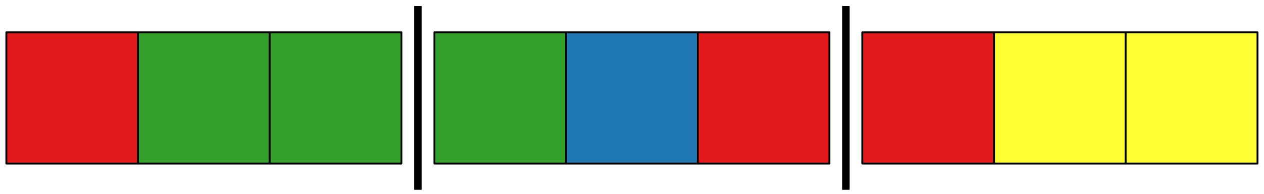
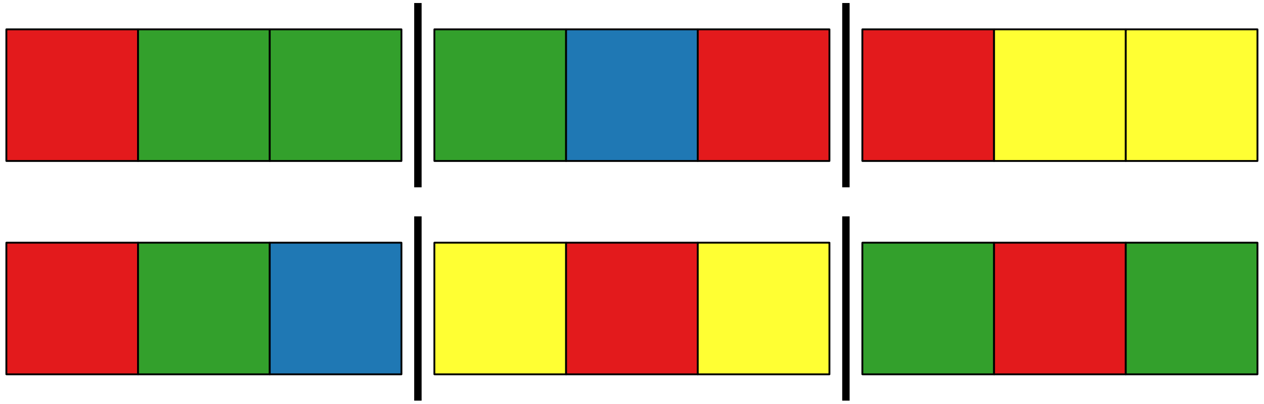
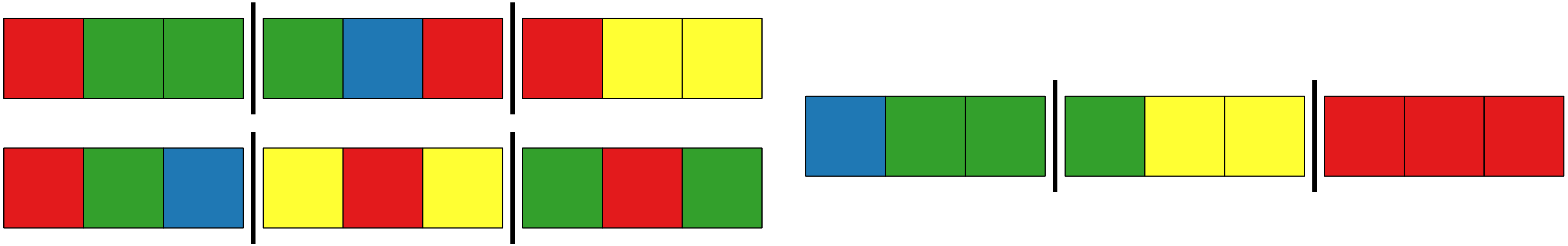
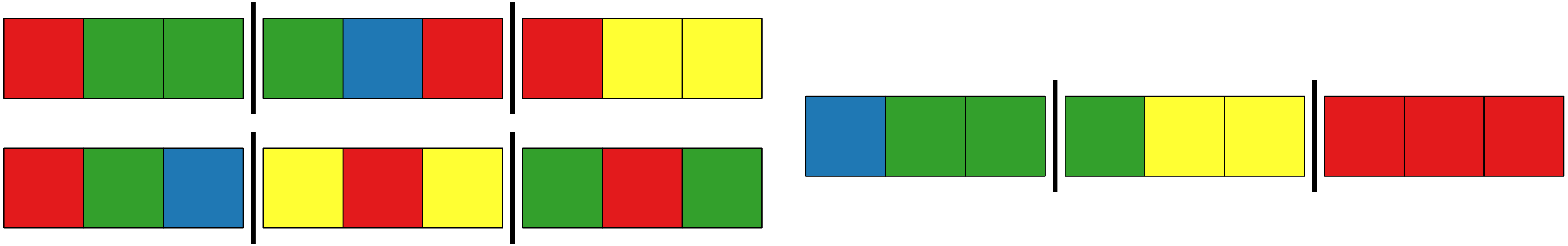
Solving area-distribution is NP-hard.

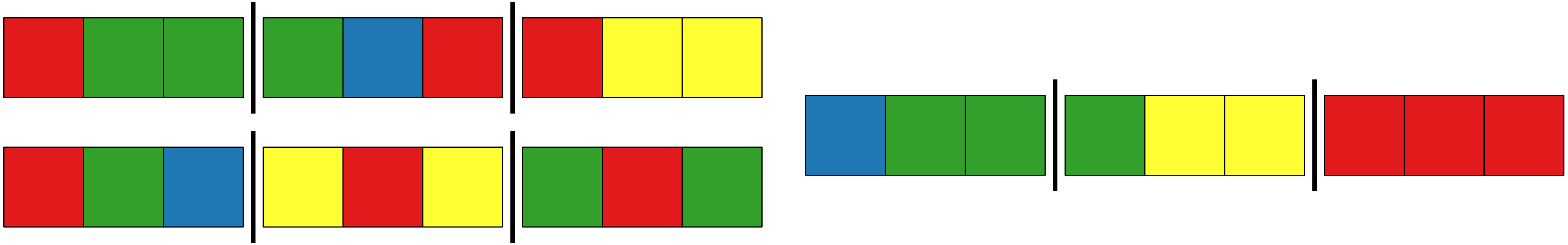




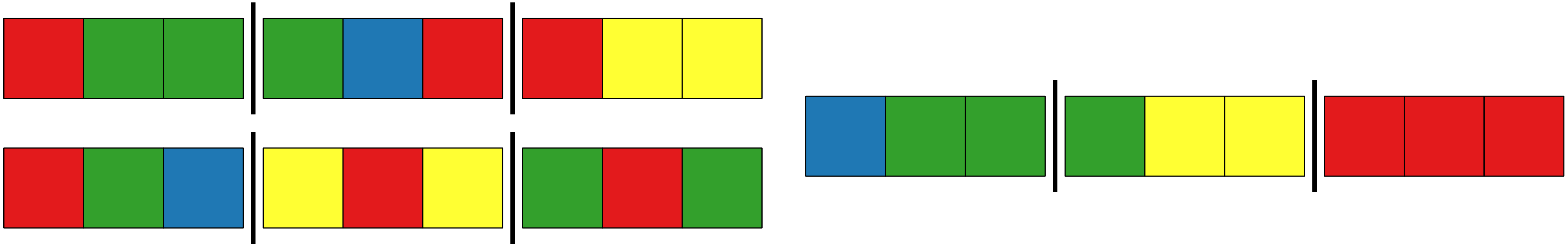




- For any instance of area-distribution there is an optimal solution admitting a nice sequence.



- For any instance of area-distribution there is an optimal solution admitting a nice sequence.
- For any instance of area-distribution we can compute a nice sequence in $O(|C| + |F|)$ time.



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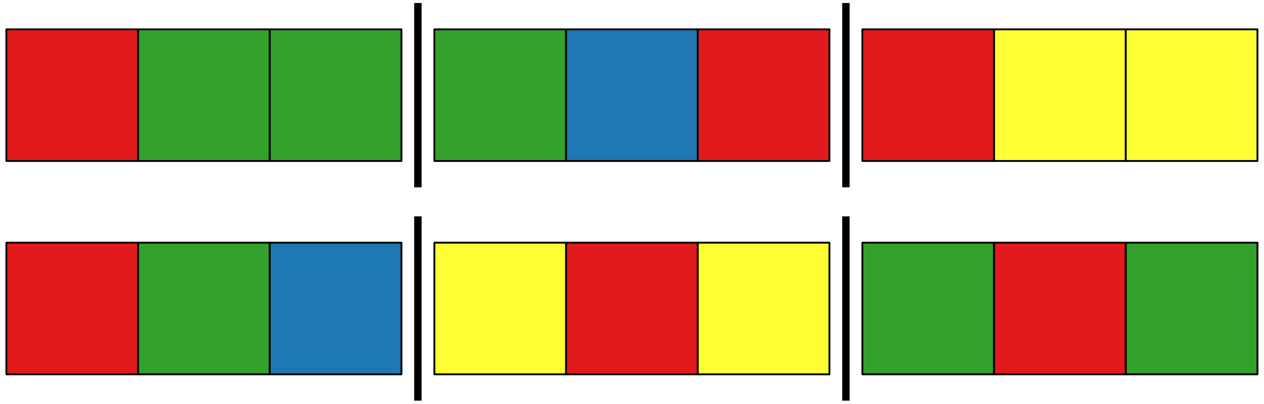
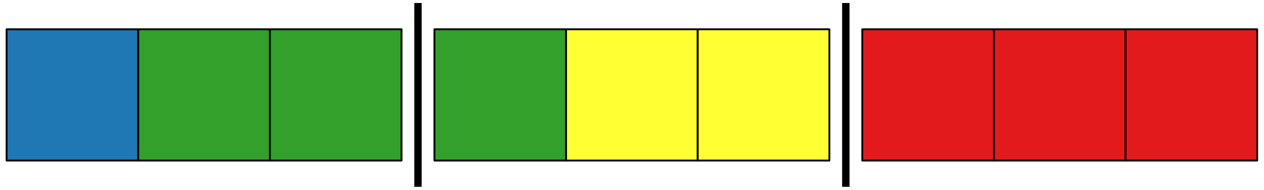
- For any instance of area-distribution we can compute a nice sequence in $O(|C| + |F|)$ time.
- For any instance of area-distribution the solution represented by any nice sequence is a 2-approximation.





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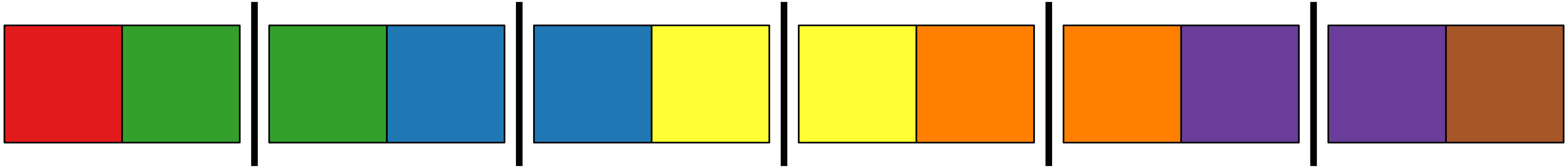

Problem AREA-DISTRIBUTION IN SINGLE BUILDING

Item: A set C of colours, a size function $s\colon C \to \mathbb{N}$, a set F of floors, each with an integer capacity m and a distance function $\delta\colon (F, F) \to \mathbb{R}$.

Question: How to find a distribution of the colour's items to floors $a\colon (C, F) \to \mathbb{N}$, that is a valid bin-packing solution while optimizing an objective function $\mathrm{cost}\colon (a, \delta) \to \mathbb{R}$?

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$$\text{minimize } \mathrm{cost}(a, \delta) = \sum_{c \in C} \left[1 + \max_{f_1, f_2 \in F} x_{c,f_1} \cdot x_{c,f_2} \cdot \delta(f_1, f_2)\right]$$

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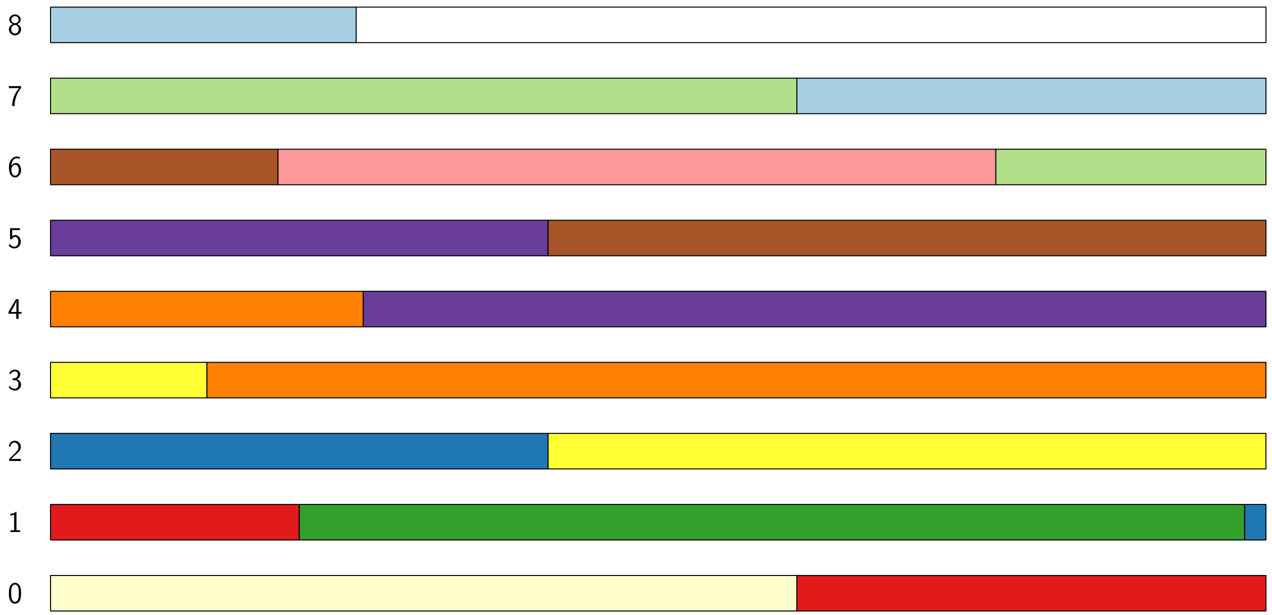
Any optimal solution for an instance of area-distribution in single building can be represented by a nice sequence.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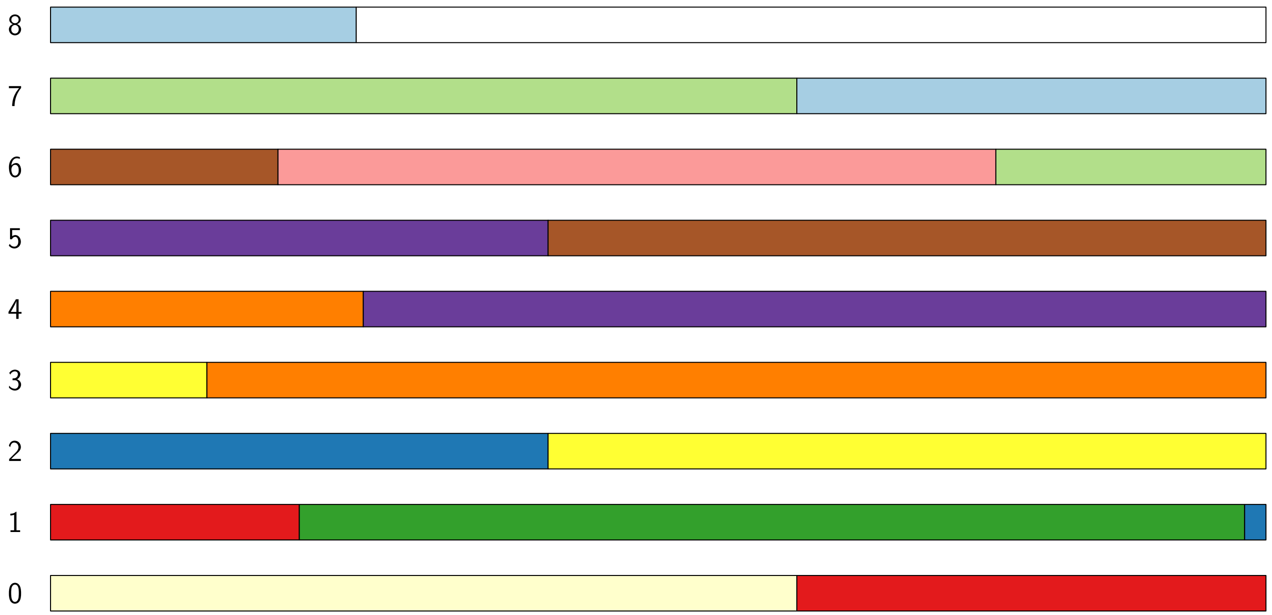
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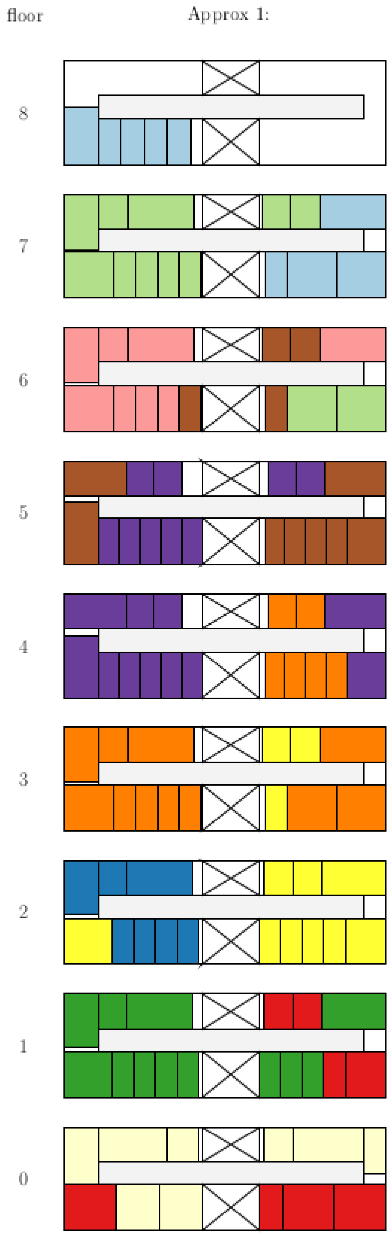
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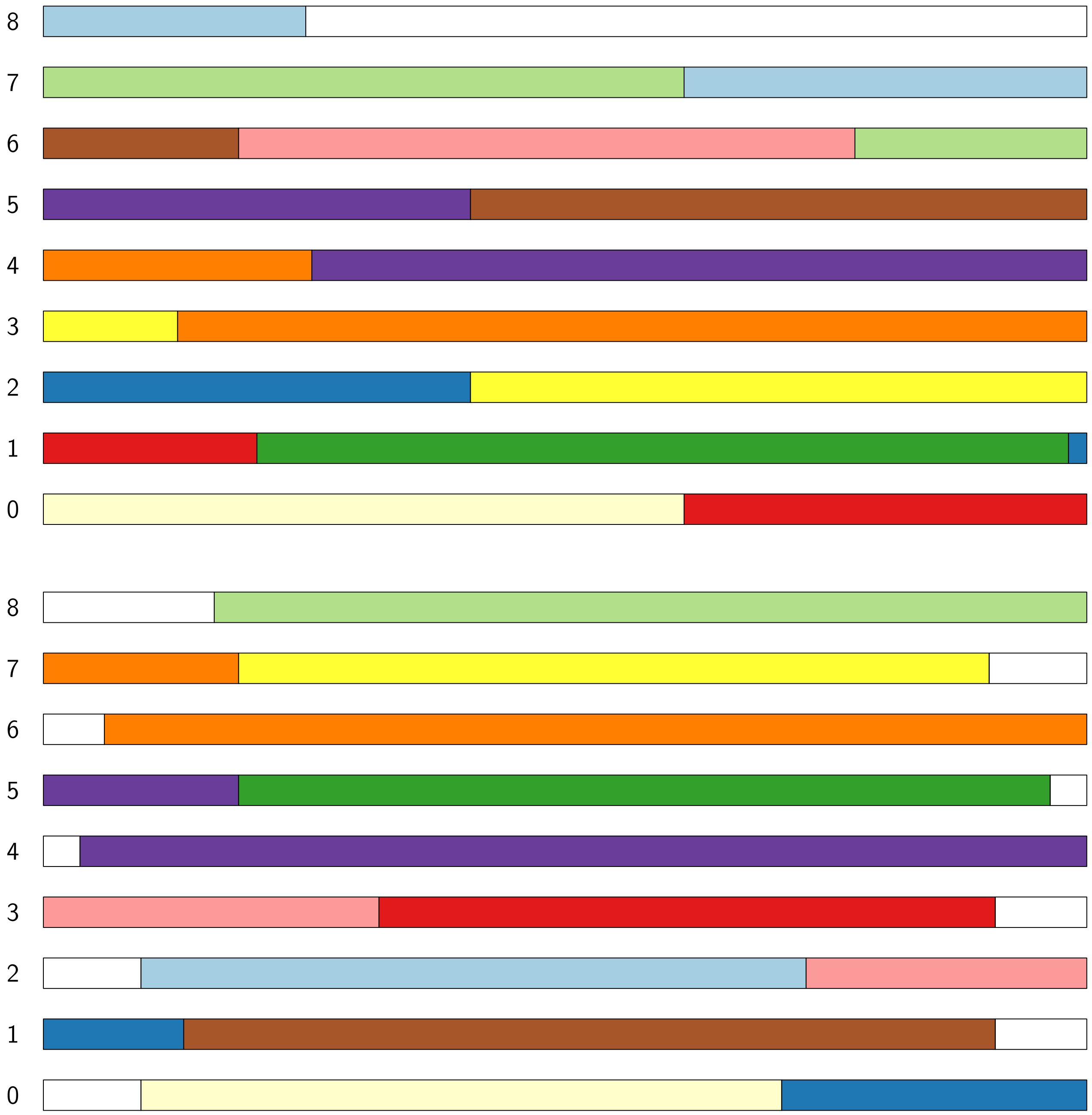
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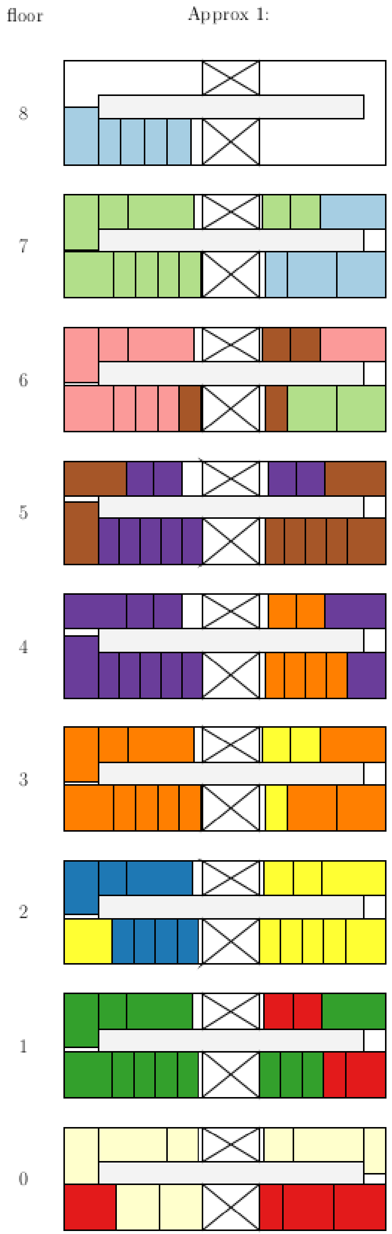
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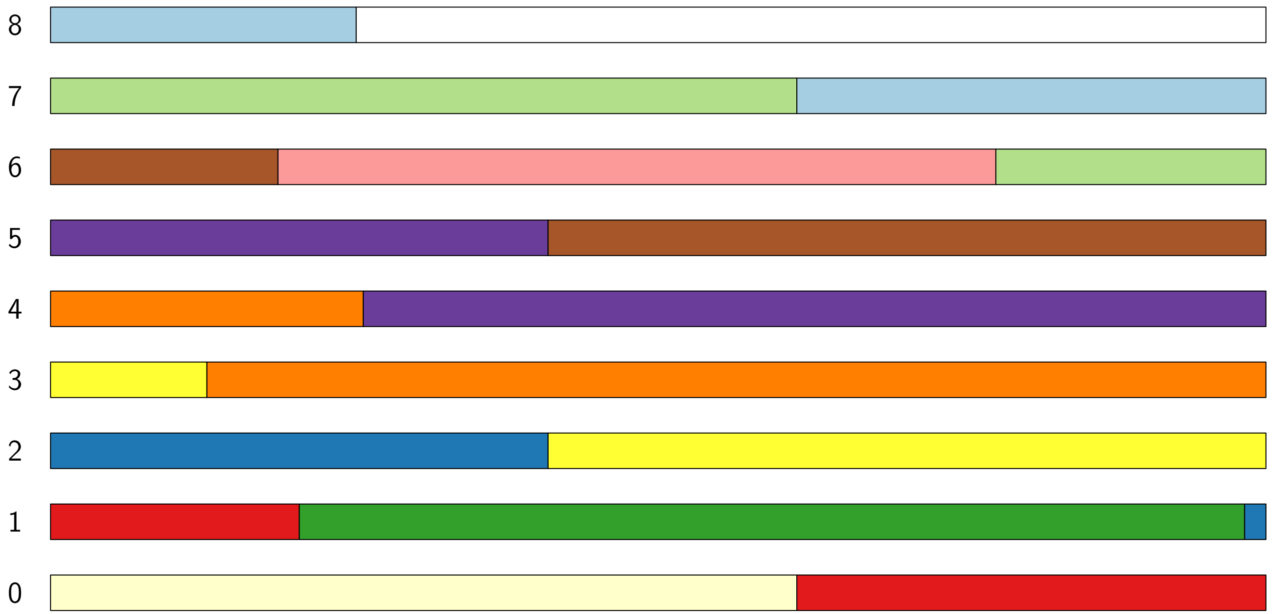
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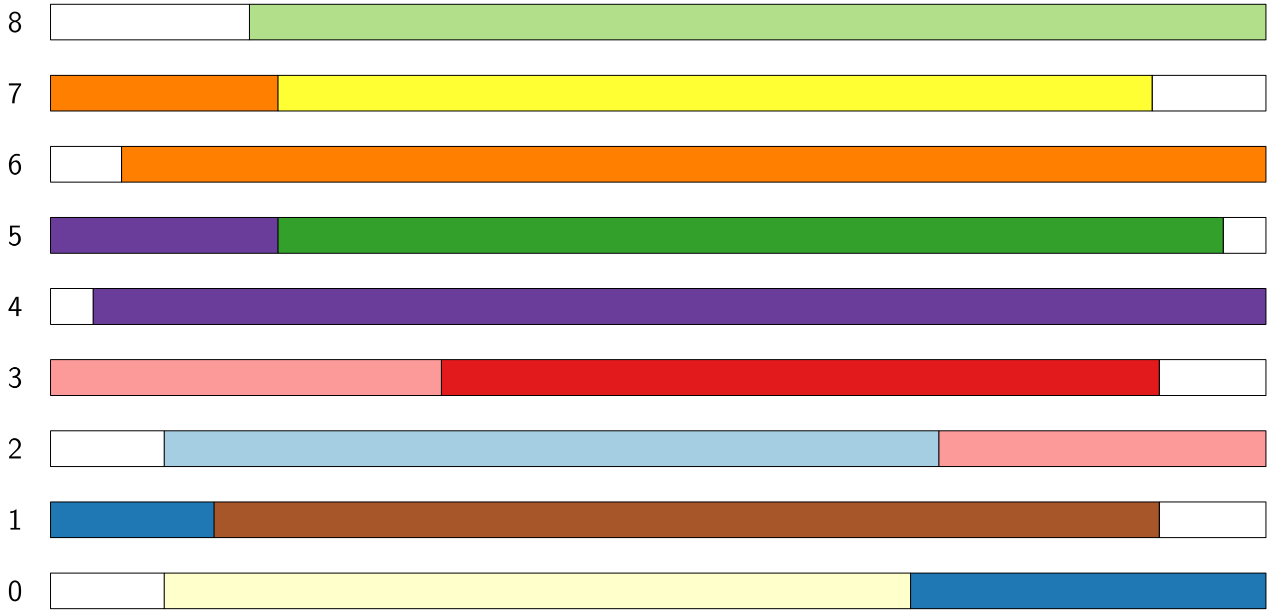
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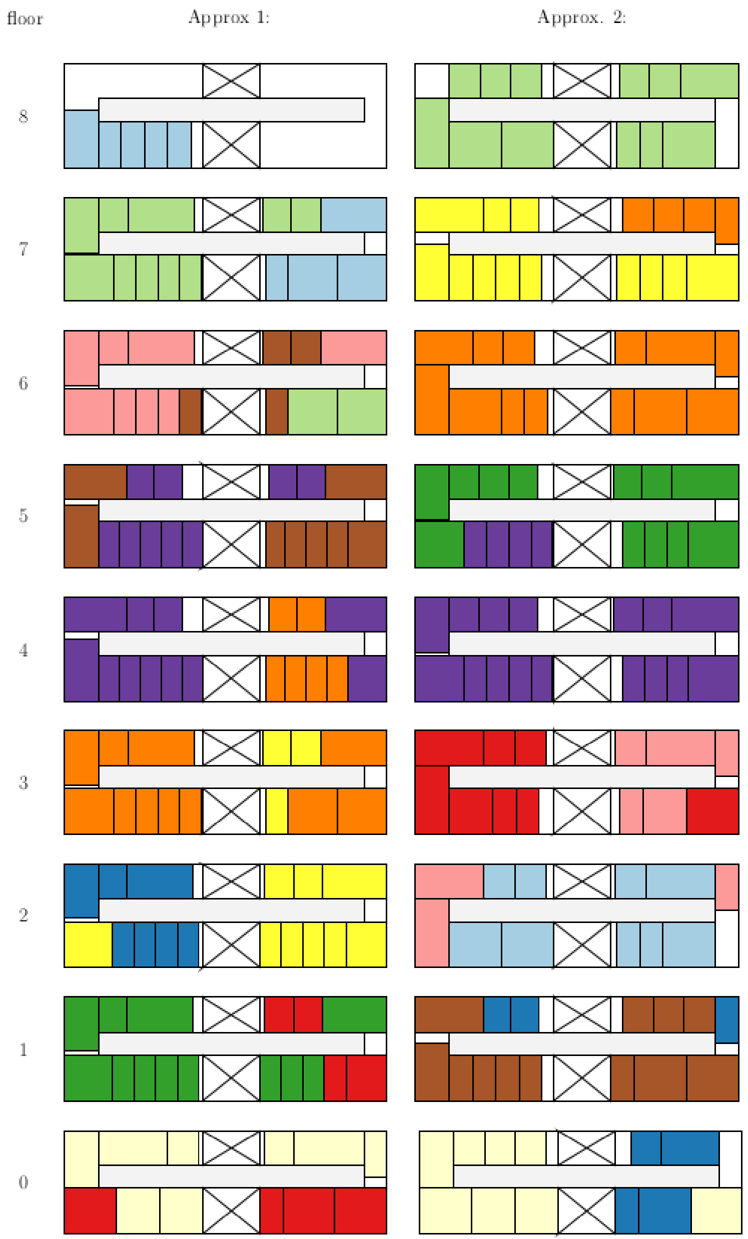
floor
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Approx. 2:
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