# Public Transportation in Rural Areas: The Clustered Dial-a-Ride Problem

Fabian Feitsch

November 16th, 2018

Attributions of third party images can be found on slide 12.

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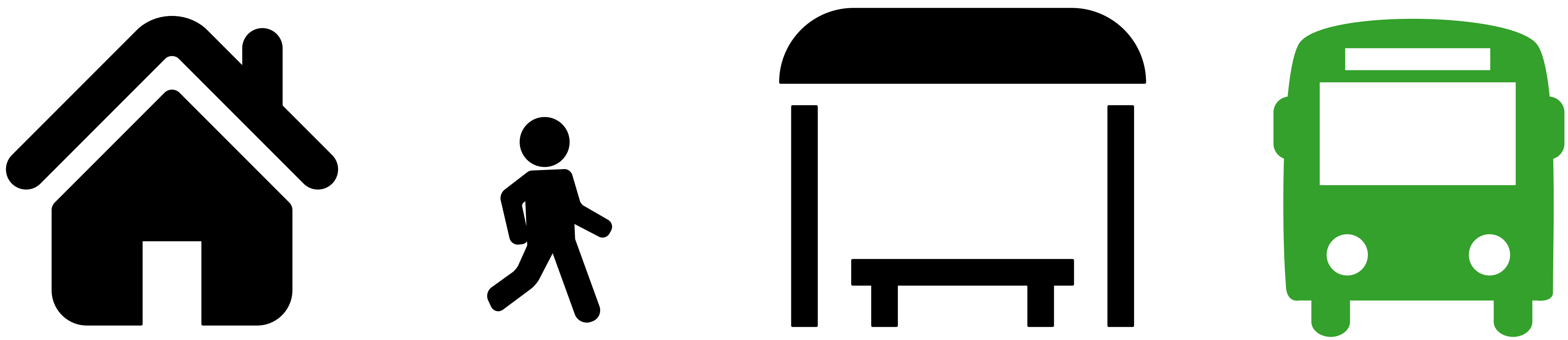

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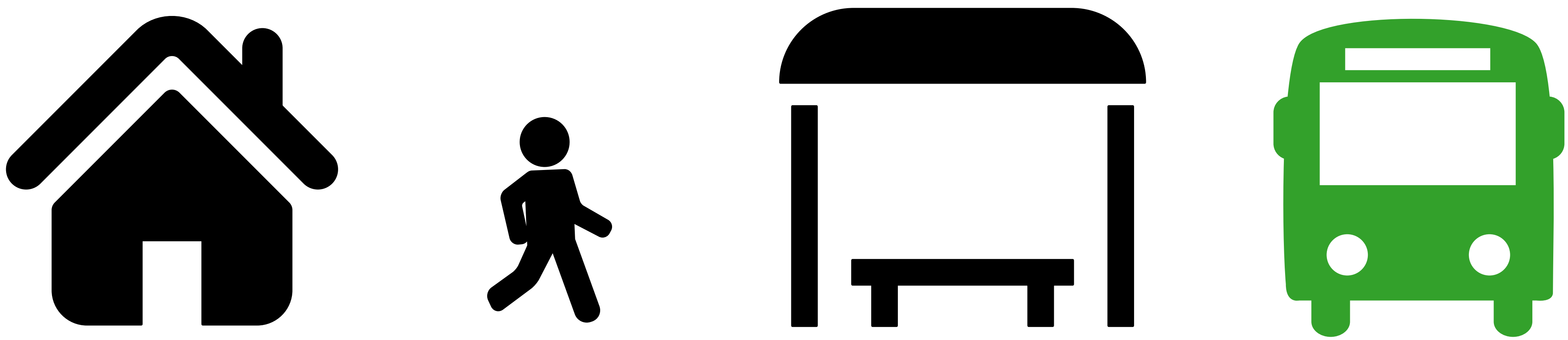

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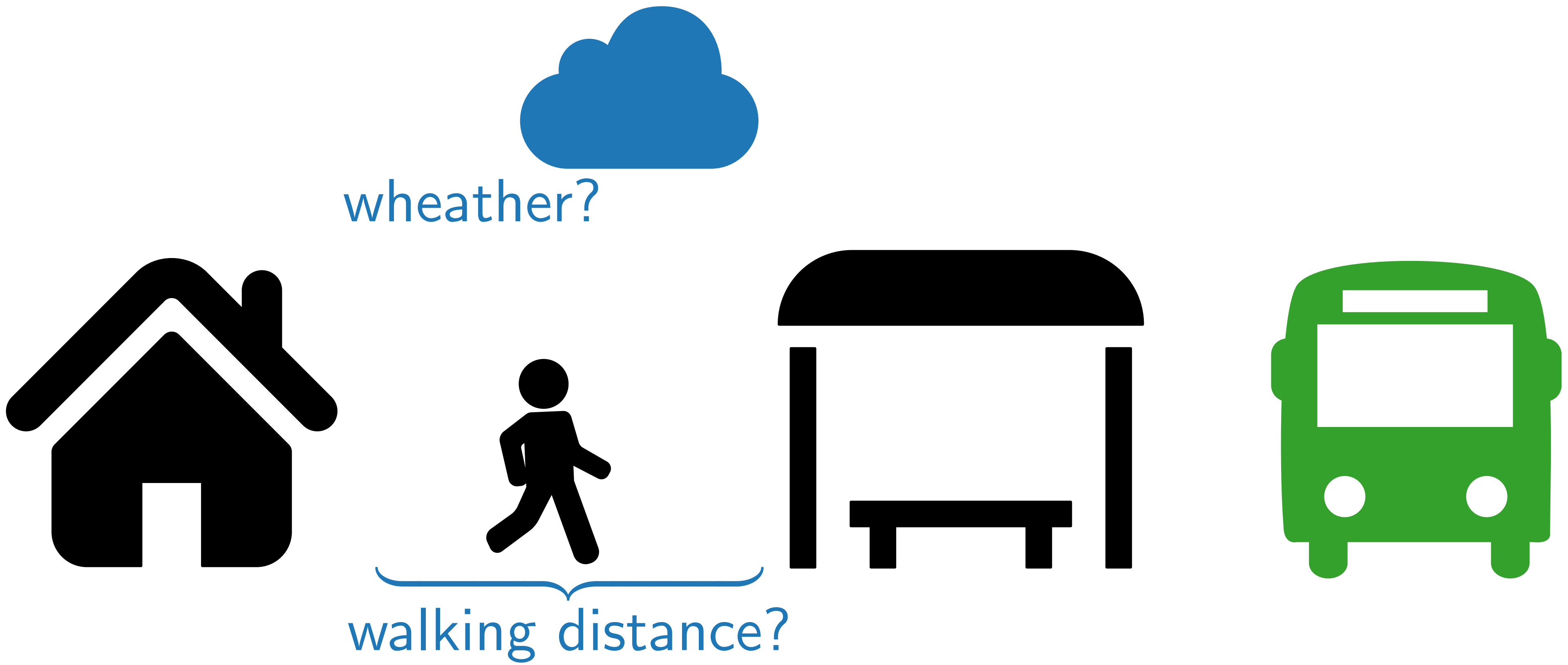


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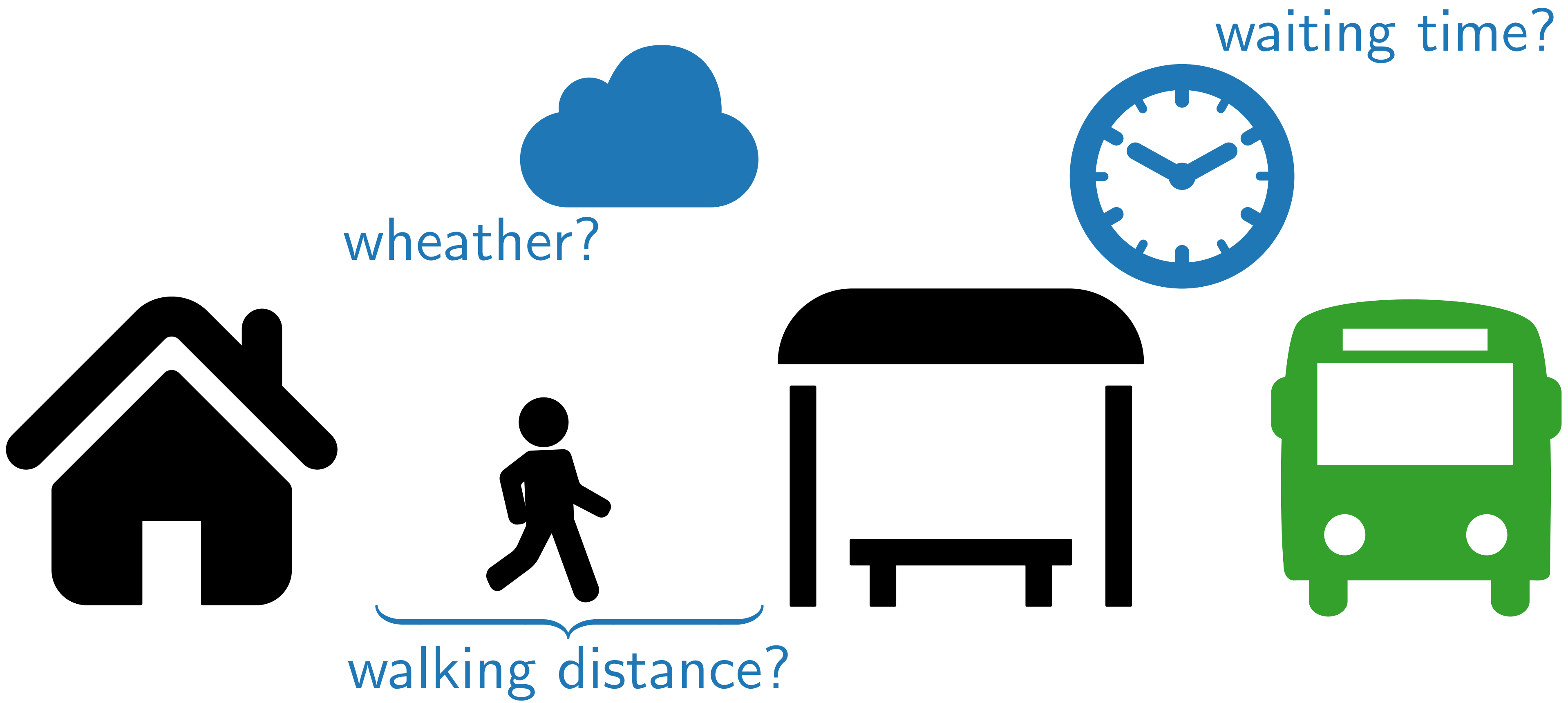


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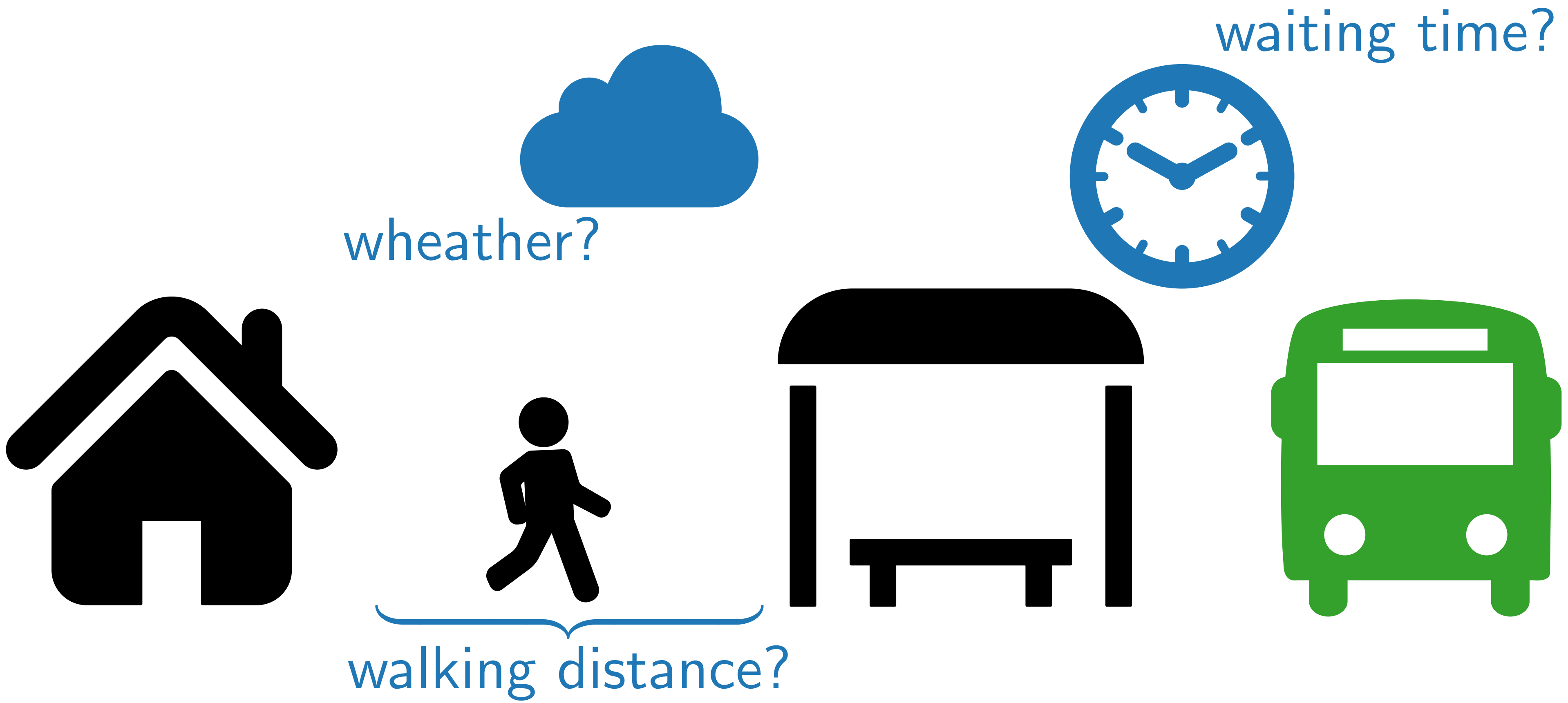


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→ Doorstep Service in Rural Areas

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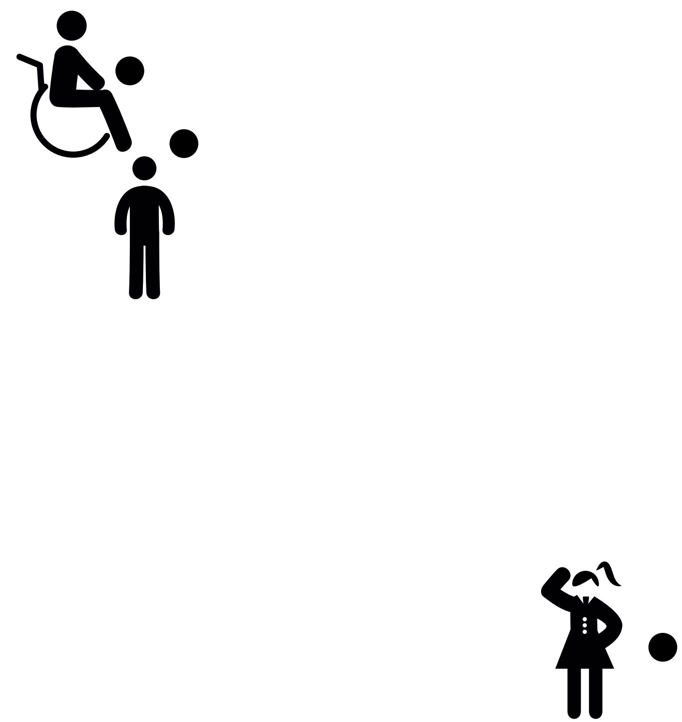

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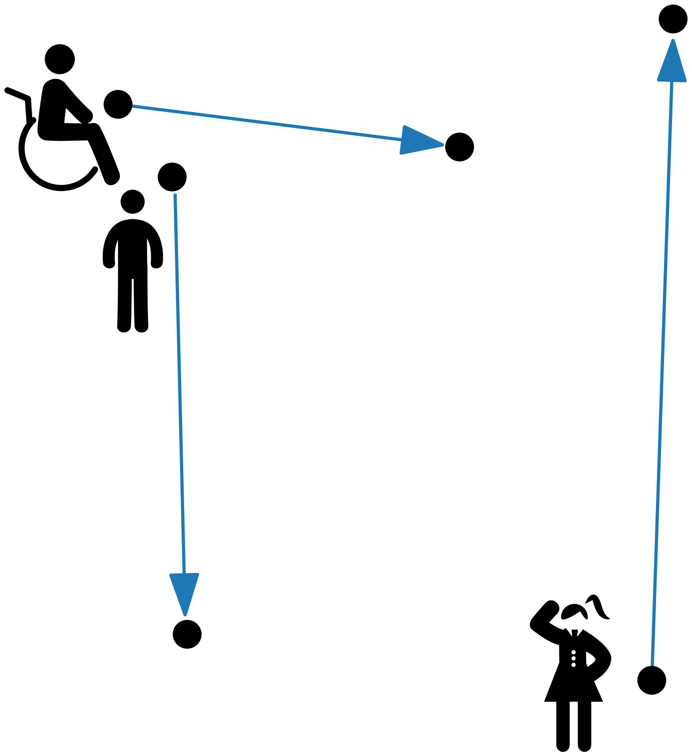

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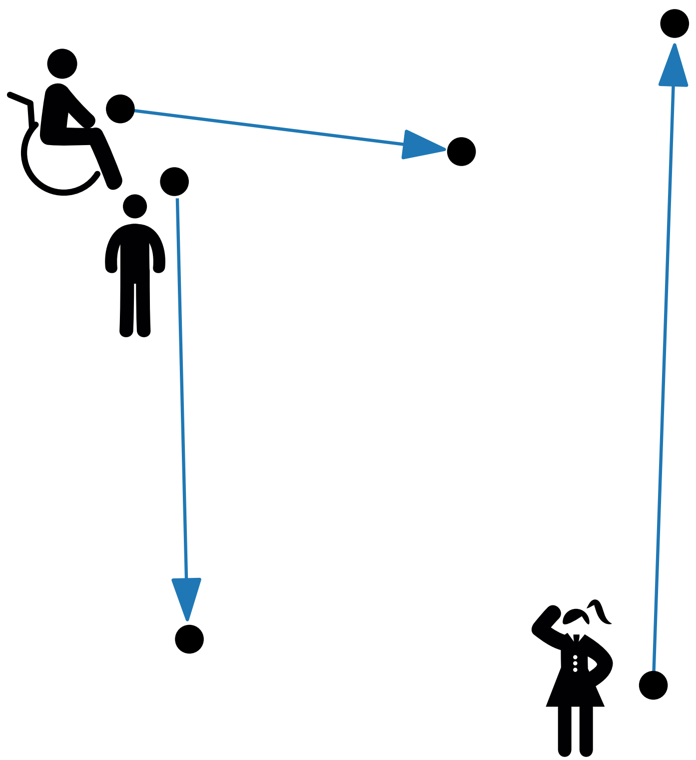

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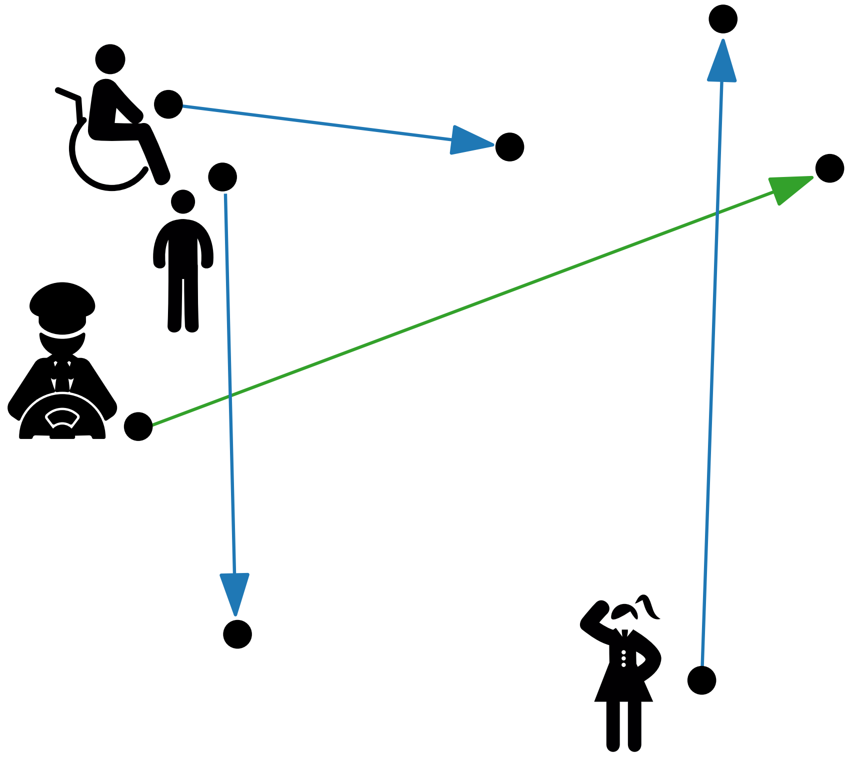

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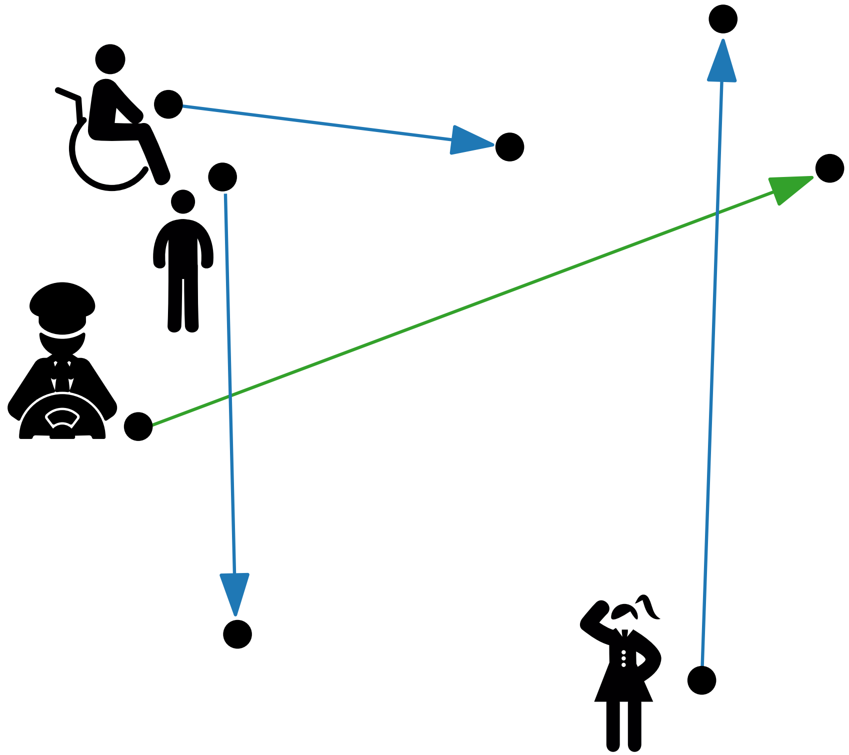

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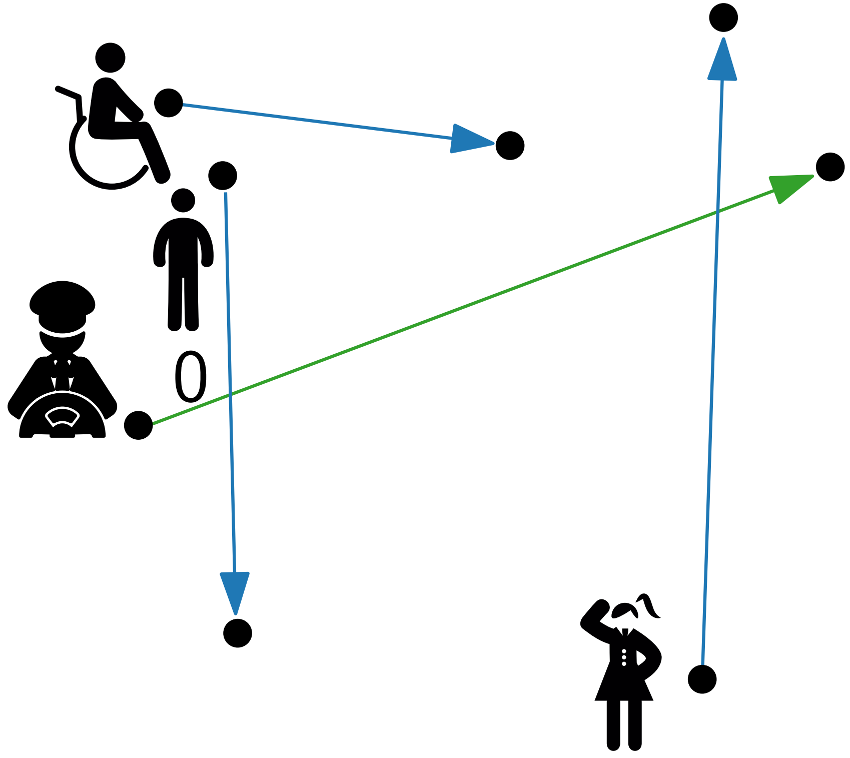

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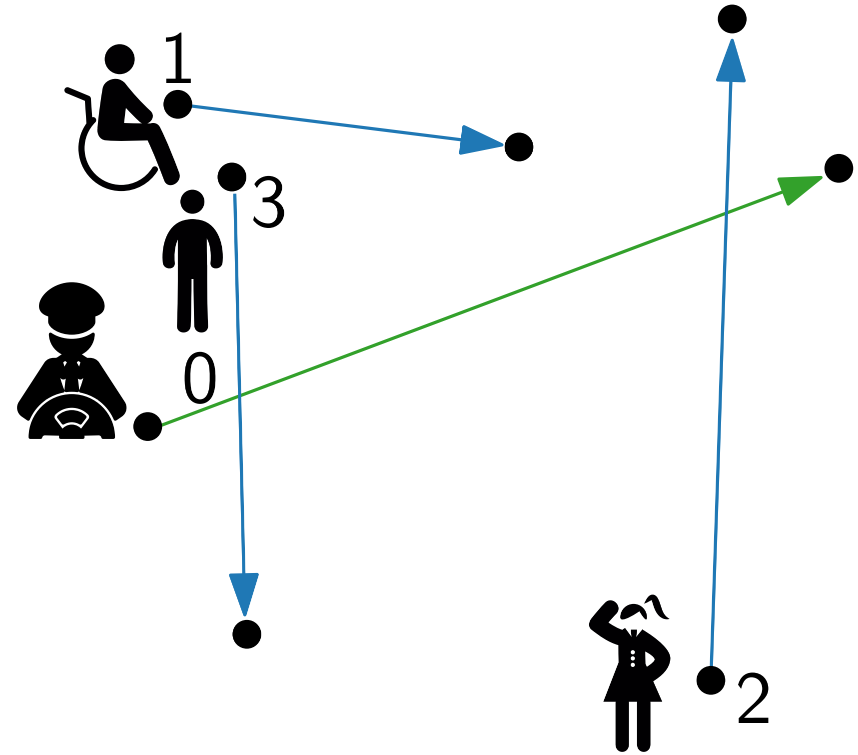

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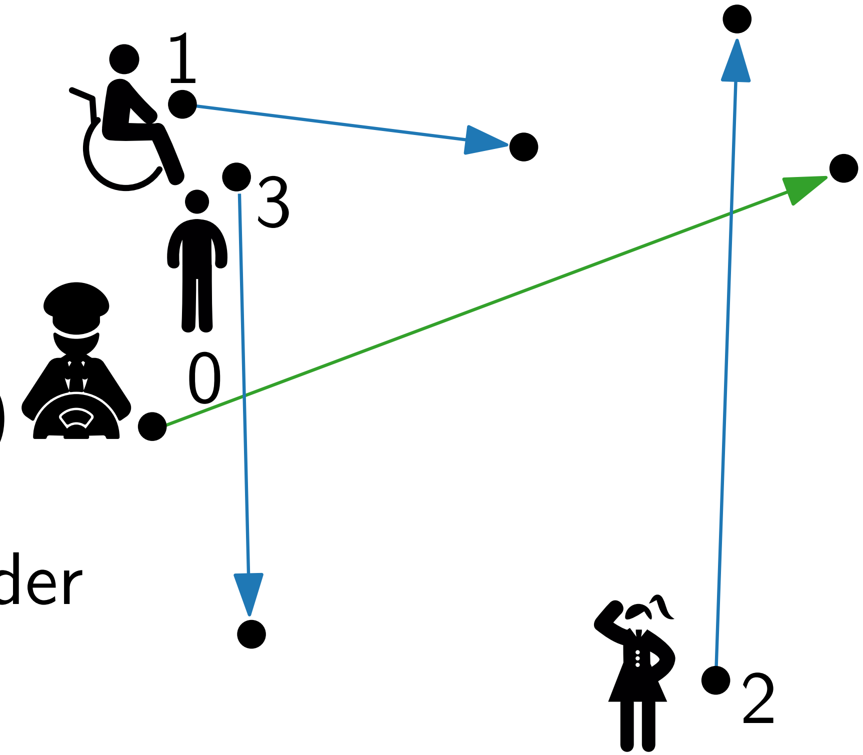

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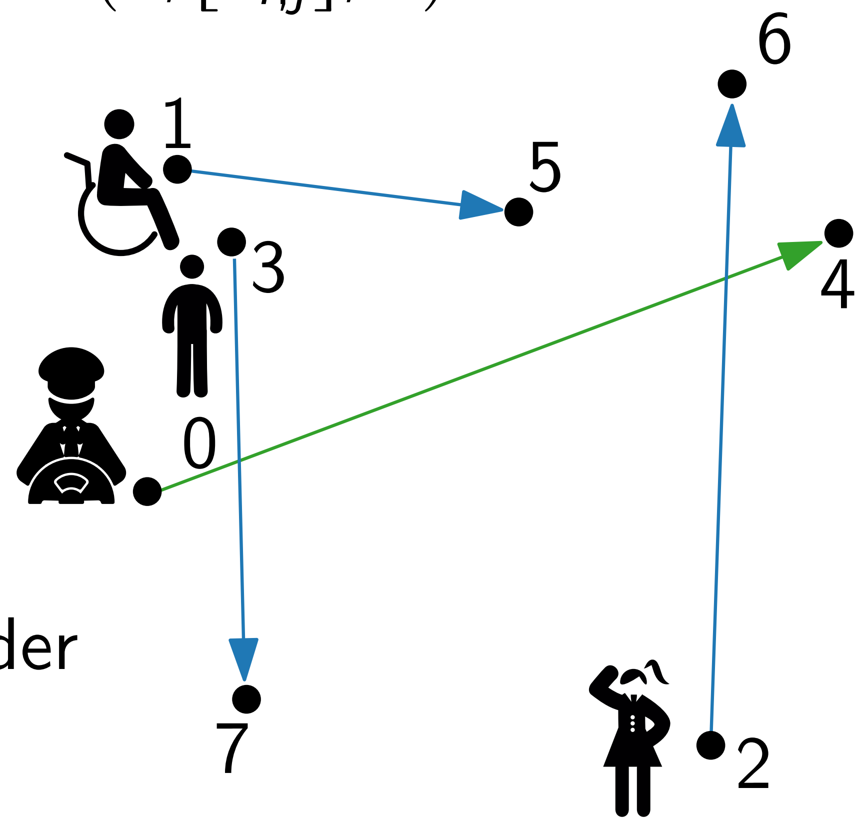

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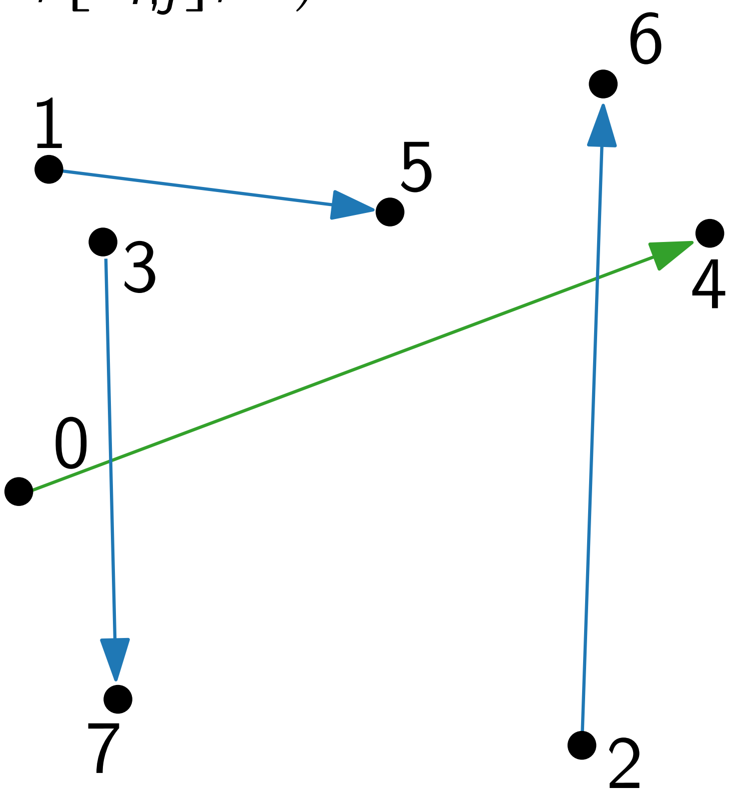

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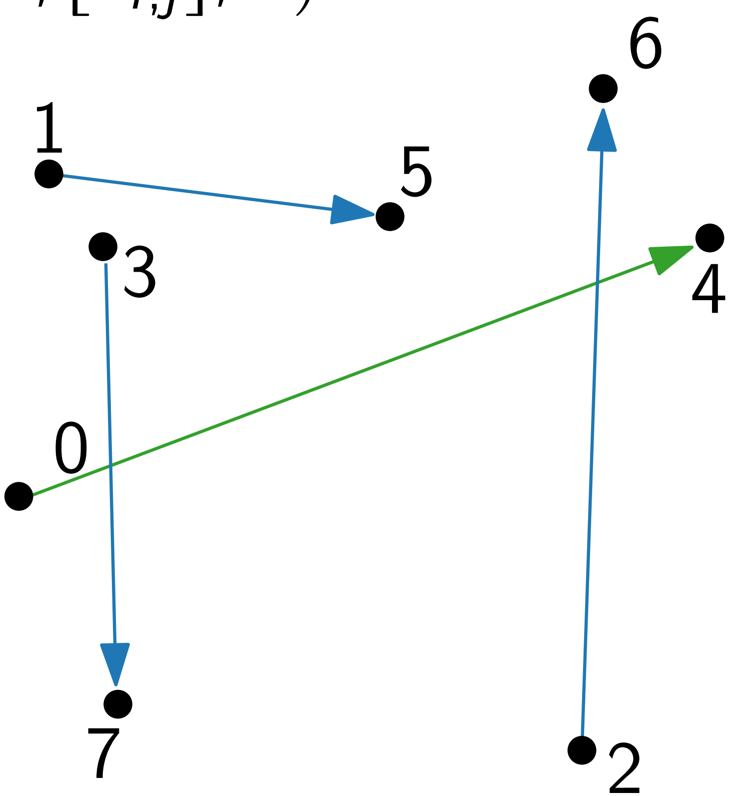

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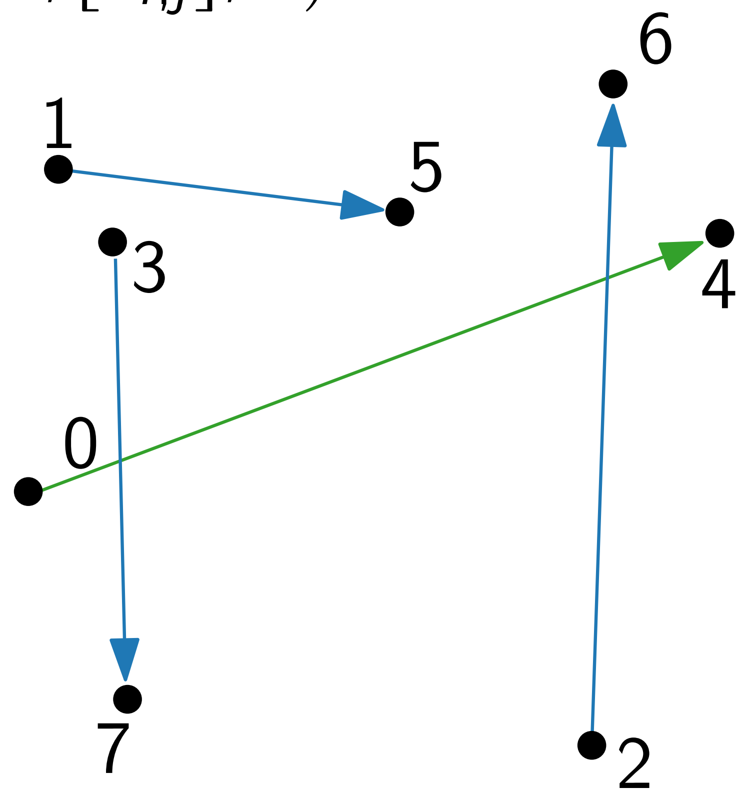

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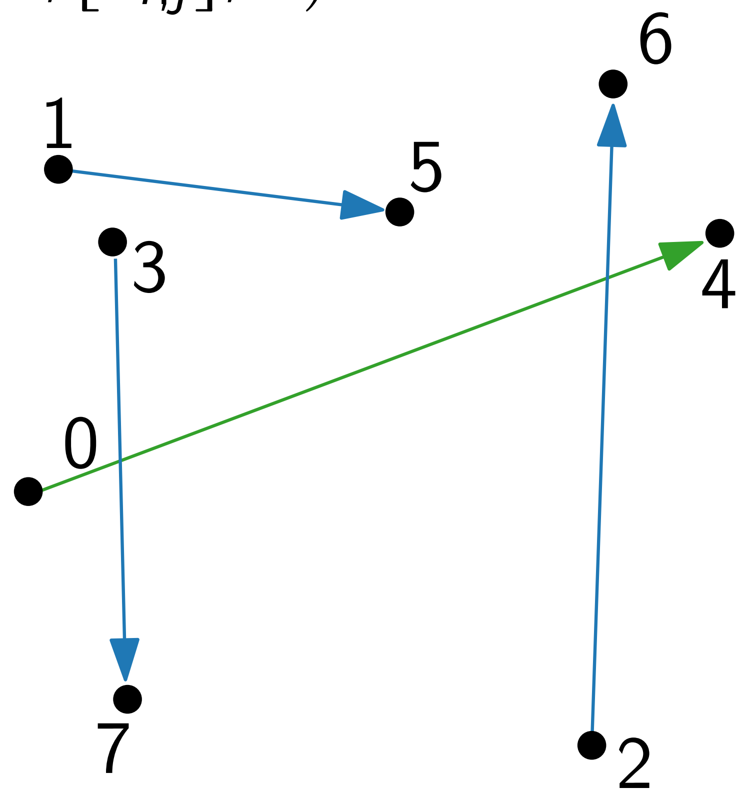


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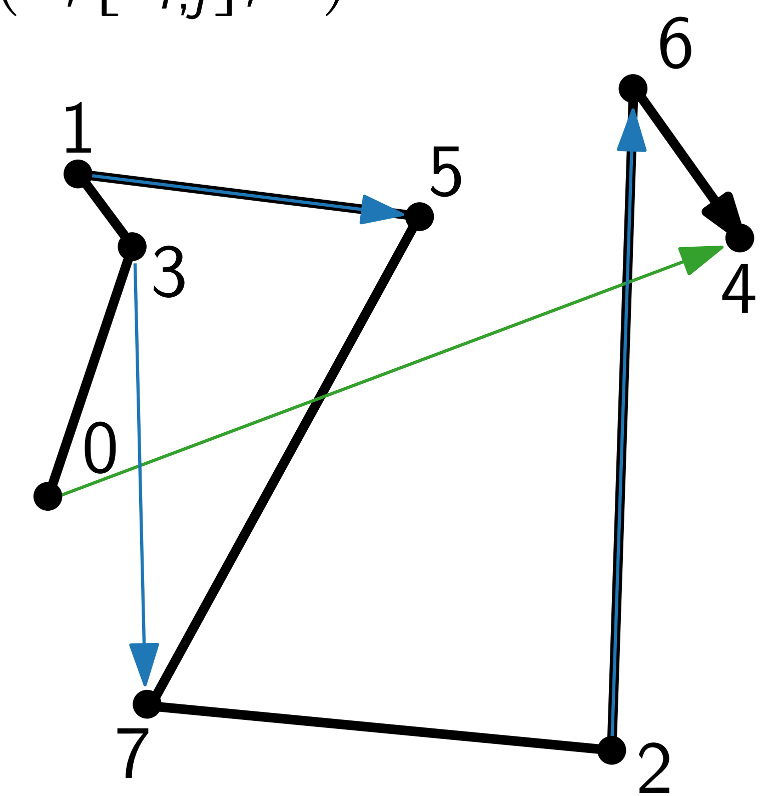


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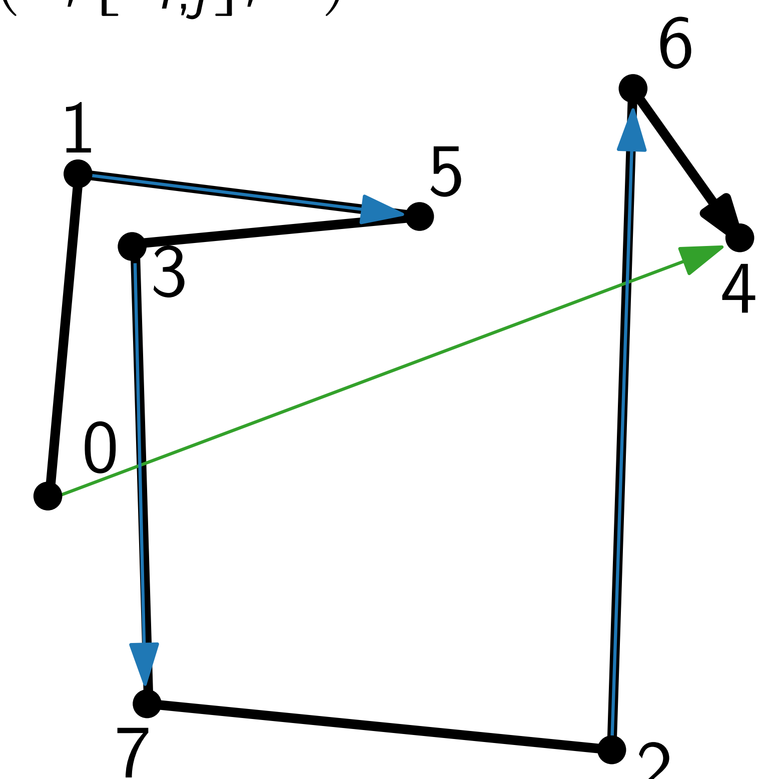


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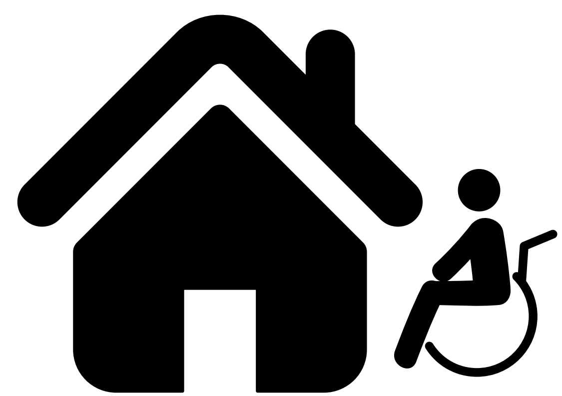
Can be generalized to solve partial instances:

Find best tour such that

a) girl is delivered

b) waiting customer is fetched

c) boy is still on board.



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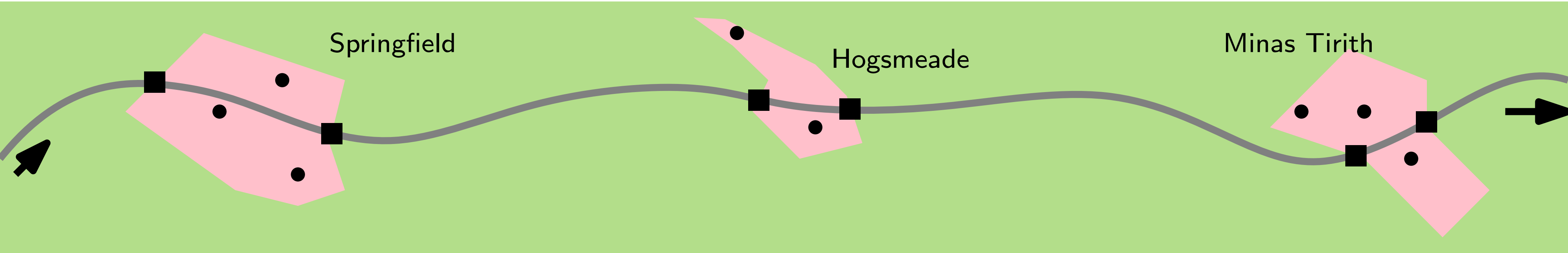

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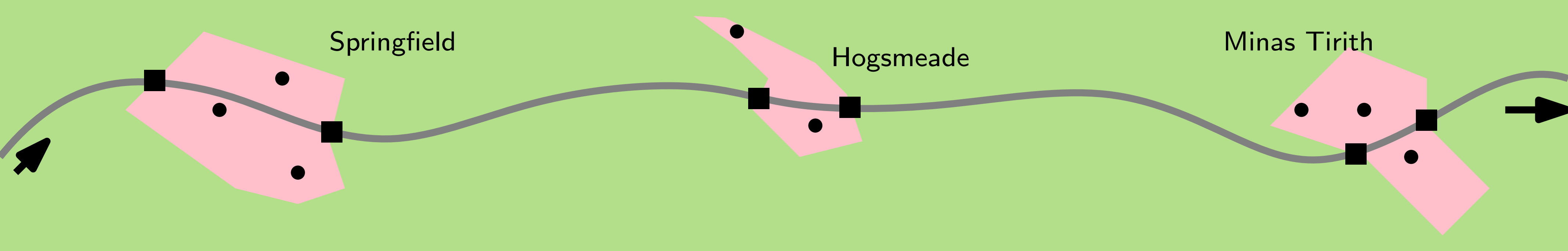

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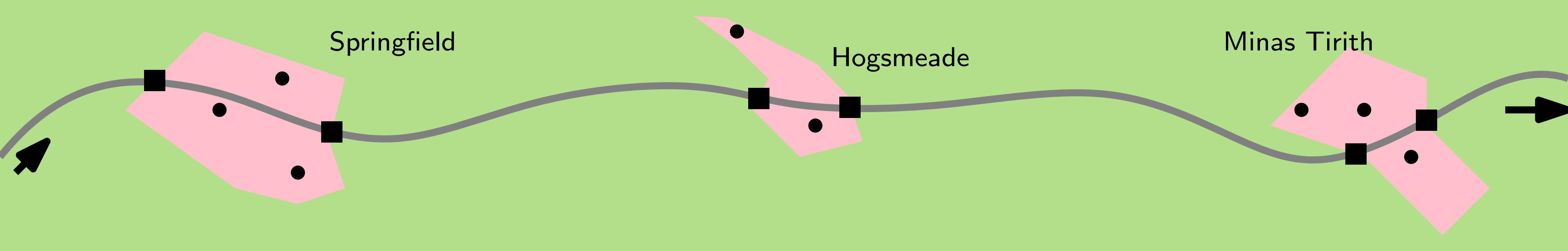


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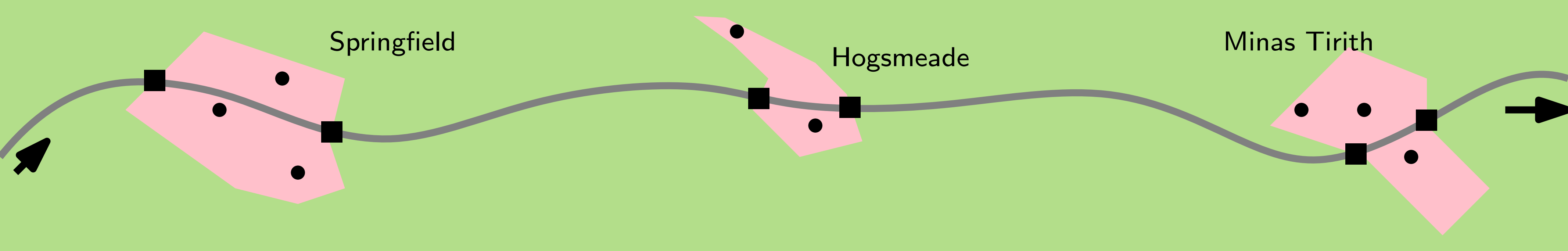


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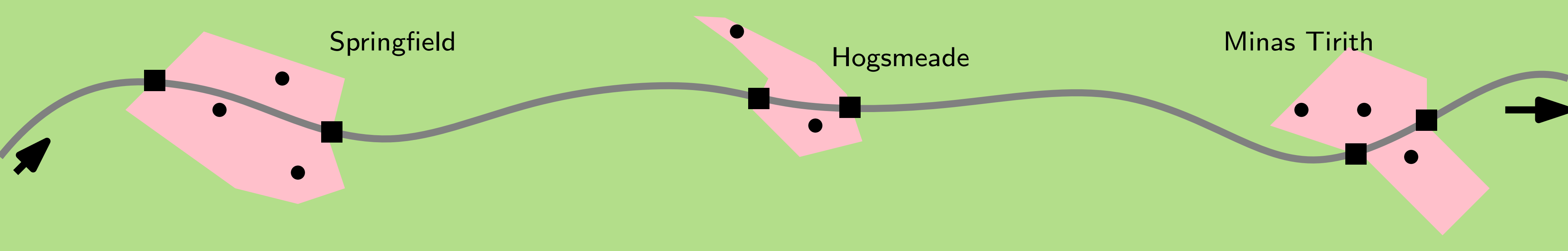


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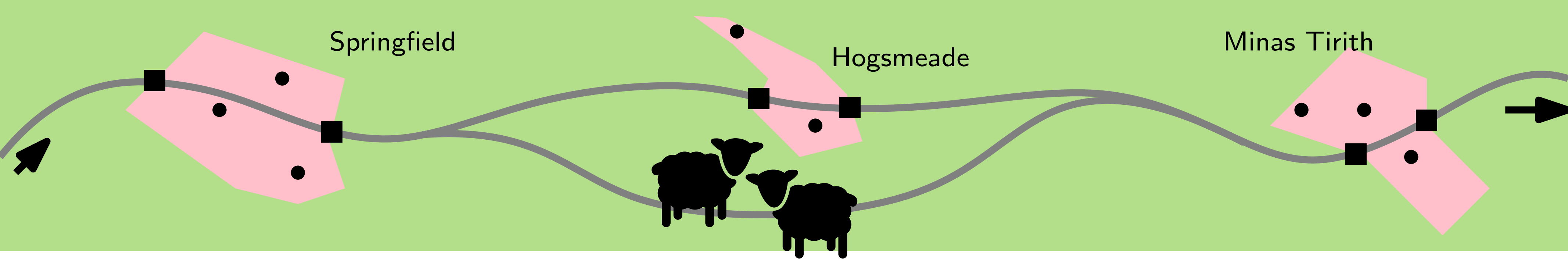


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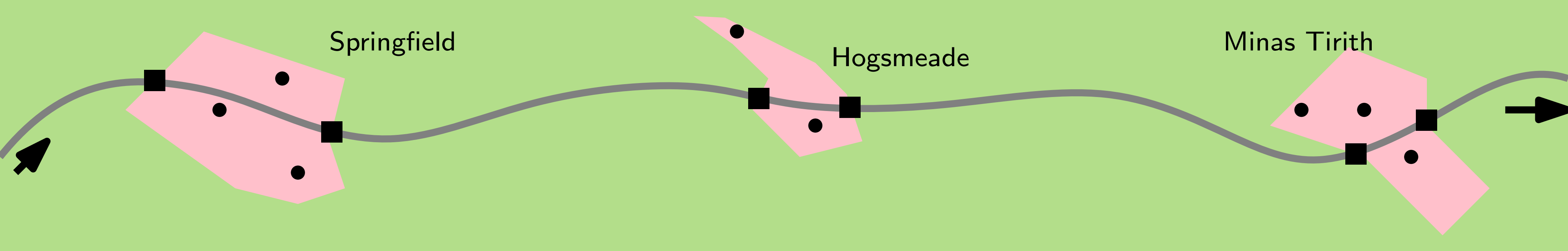


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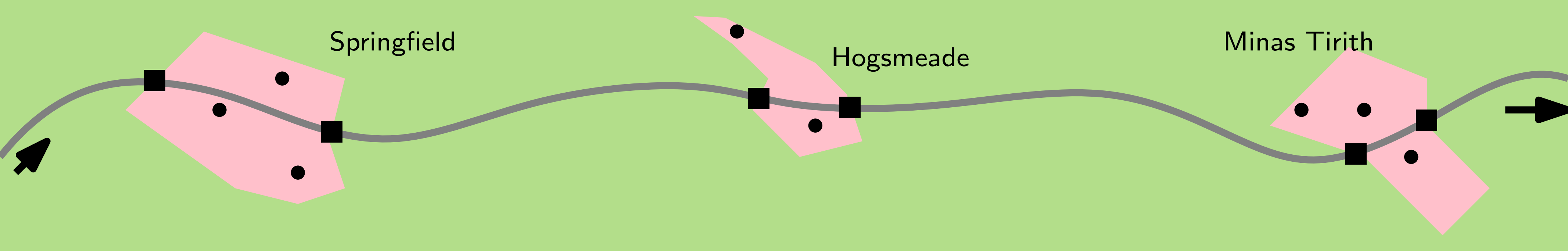


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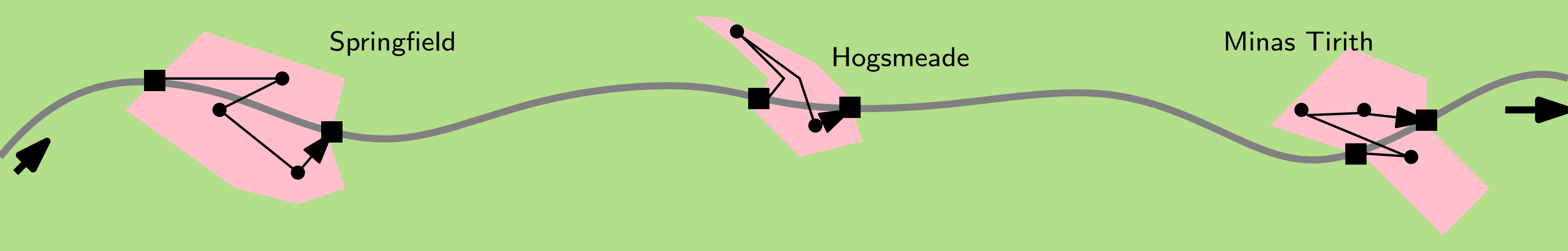


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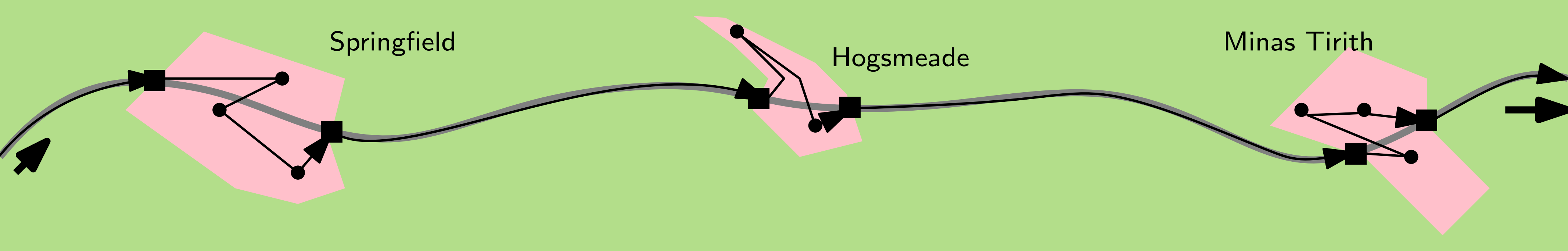


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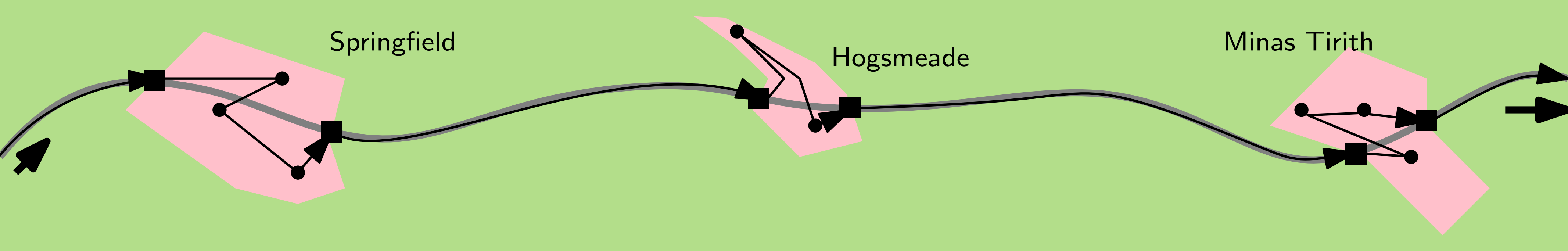


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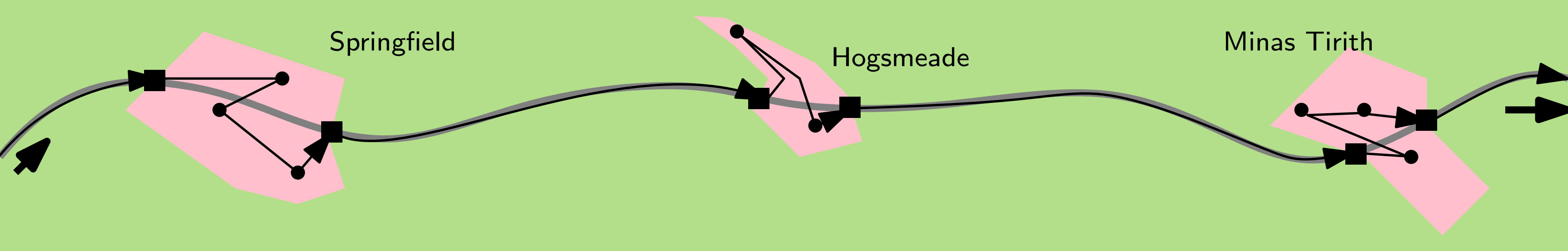


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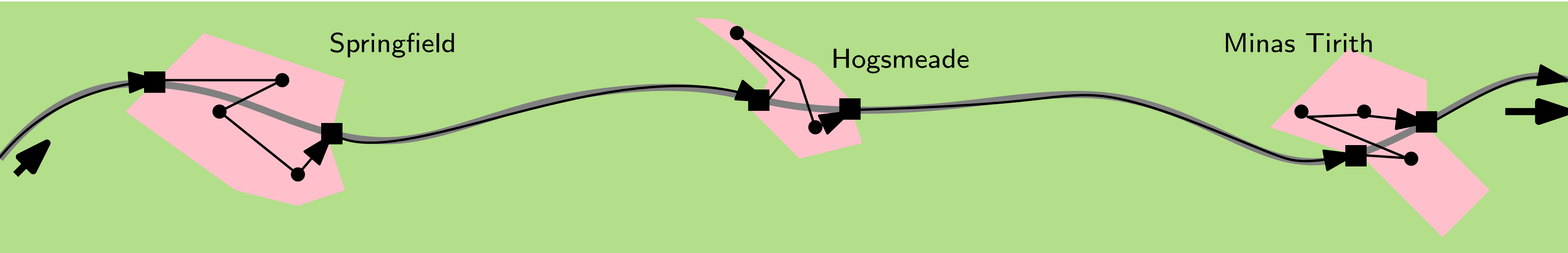


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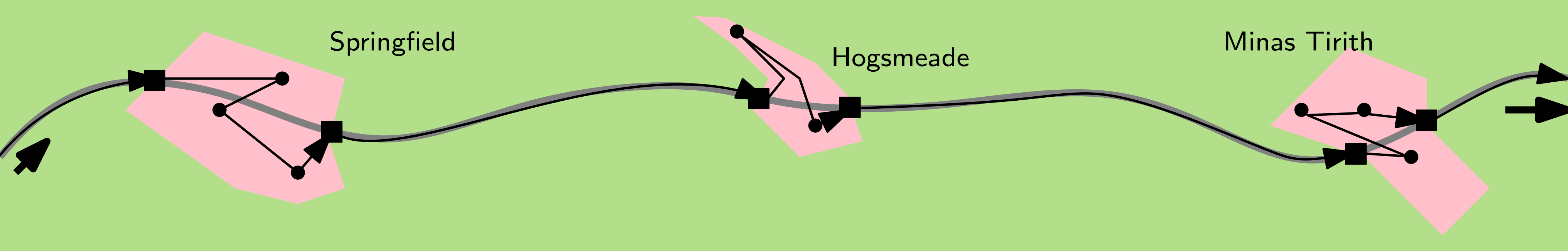


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Classify instances whose optimal tour is unidirectional. (without computing it)

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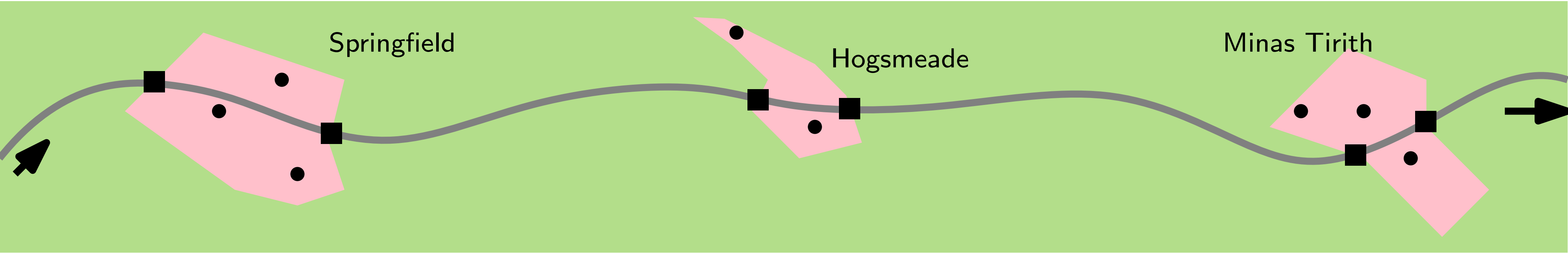

# A Classifier

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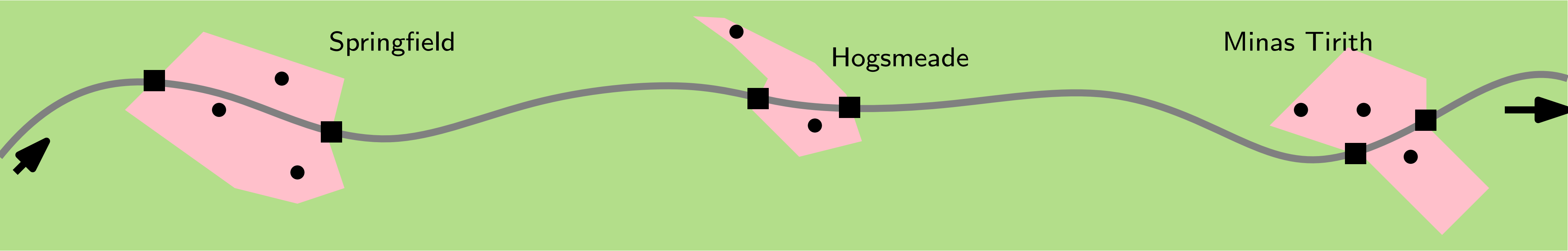

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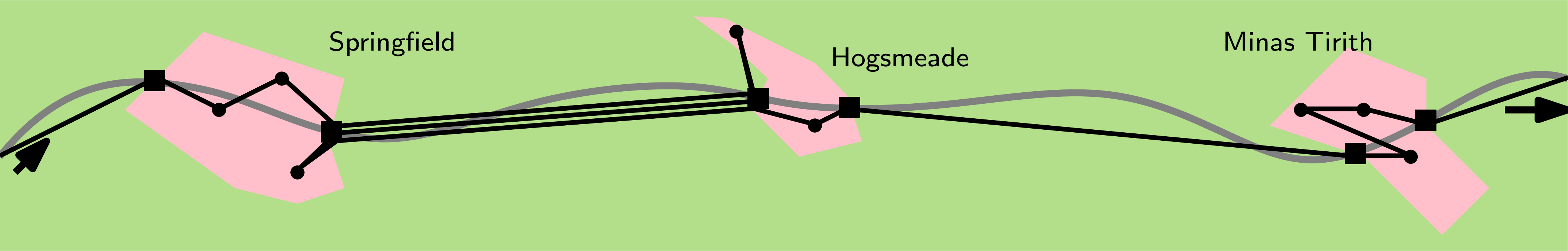

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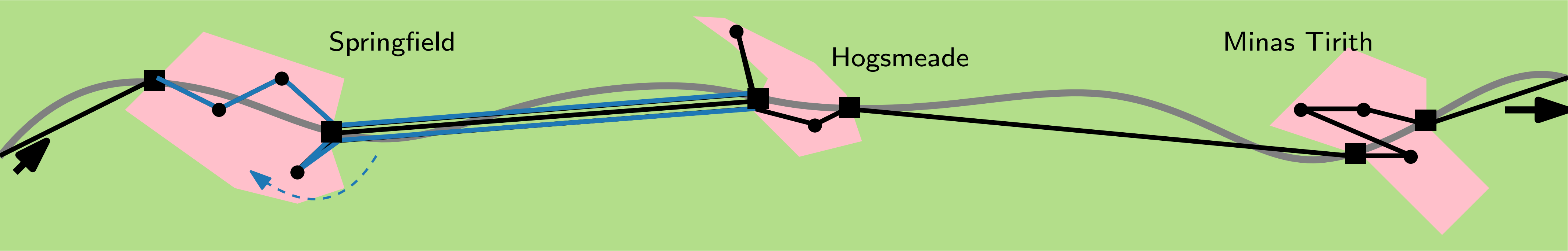

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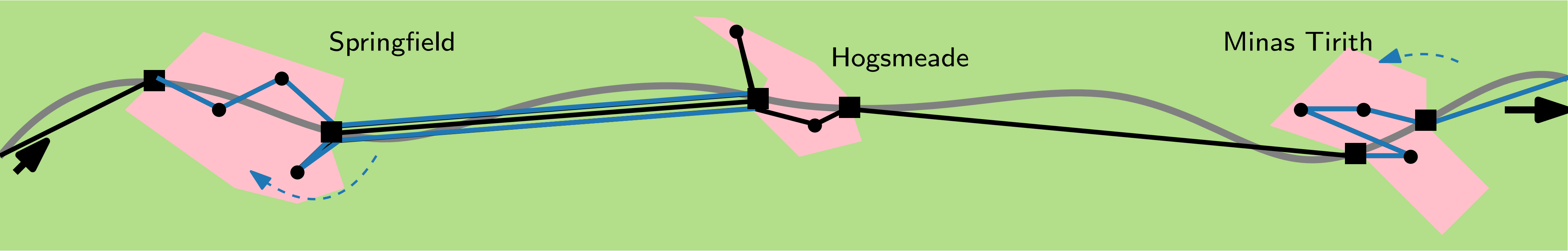

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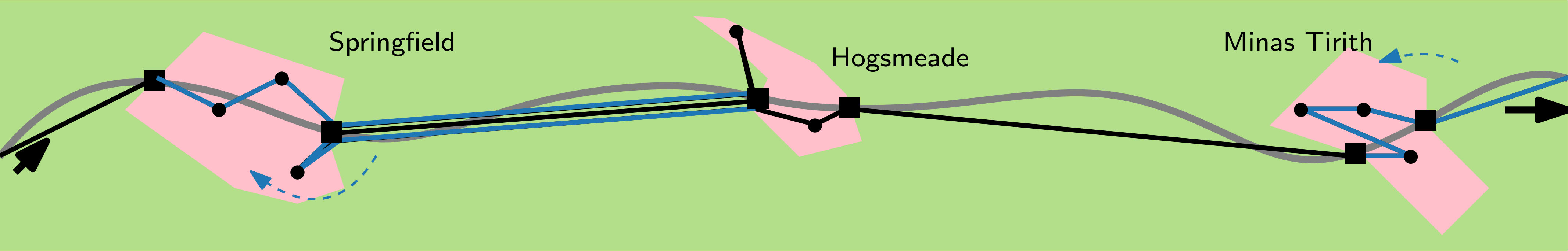


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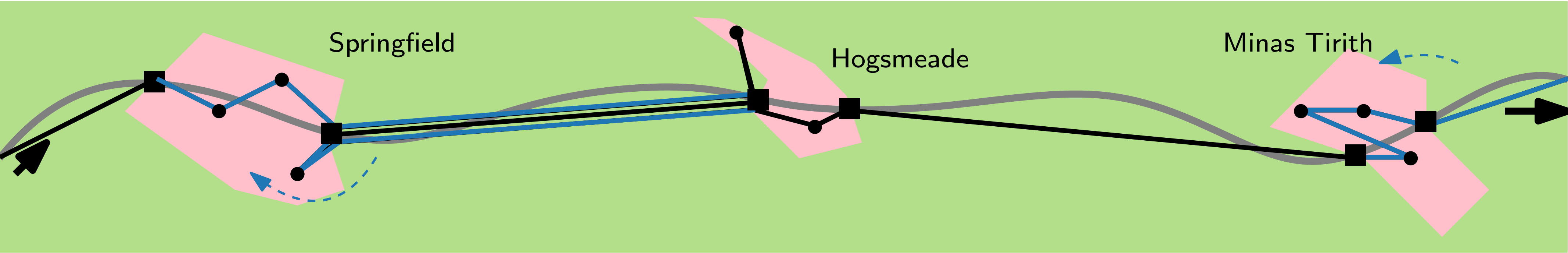


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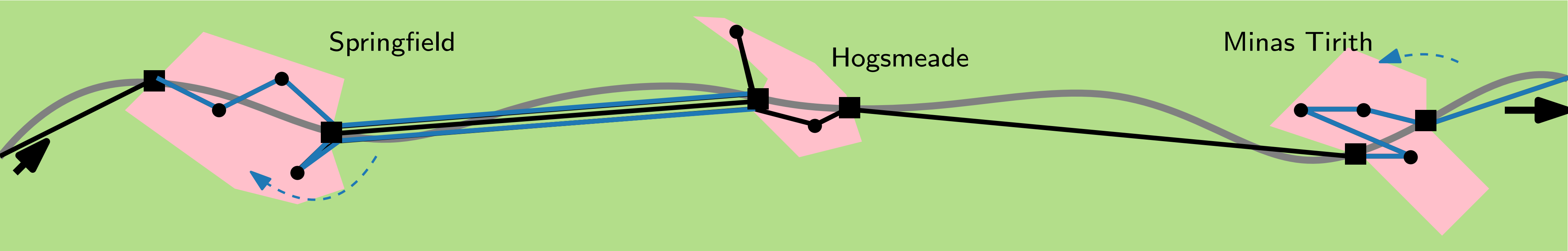


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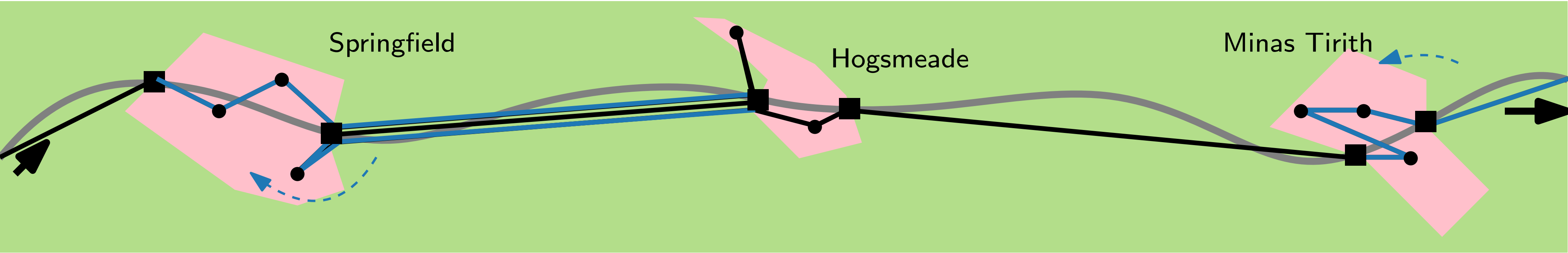


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Let $\Phi(C_i)$ be a lower bound on $\Upsilon(T^*, C_i)$.

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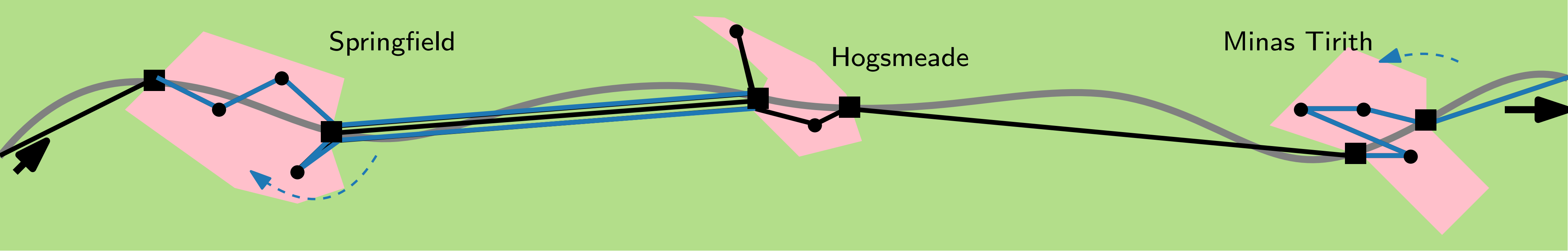


Let $C_1, \ldots C_q$ be the clusters.

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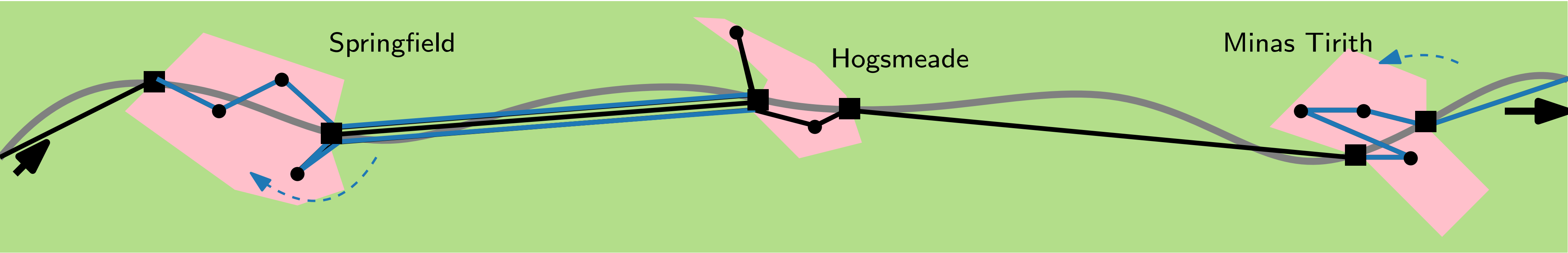


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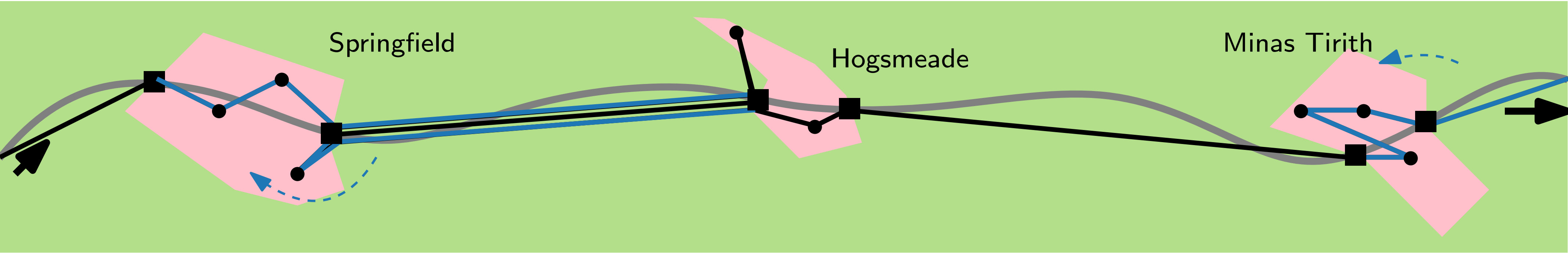


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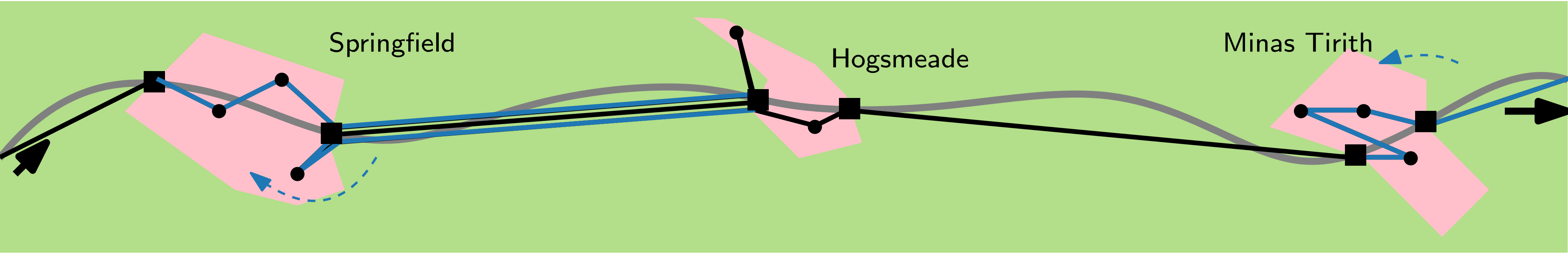


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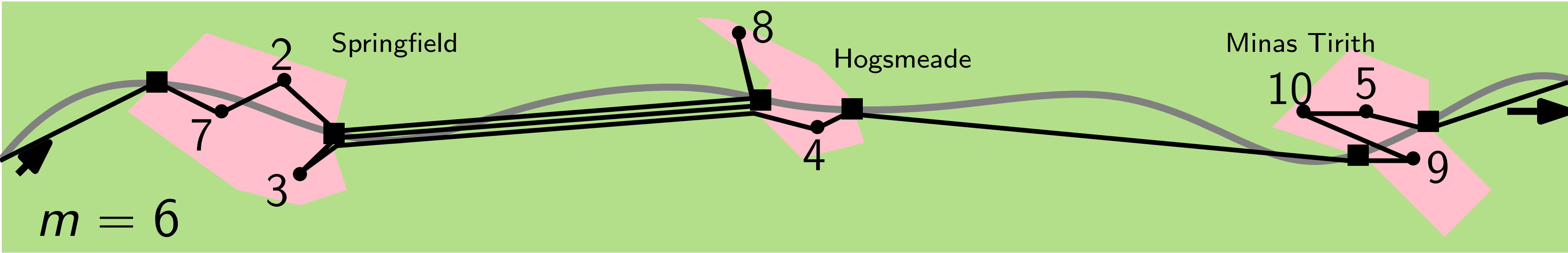

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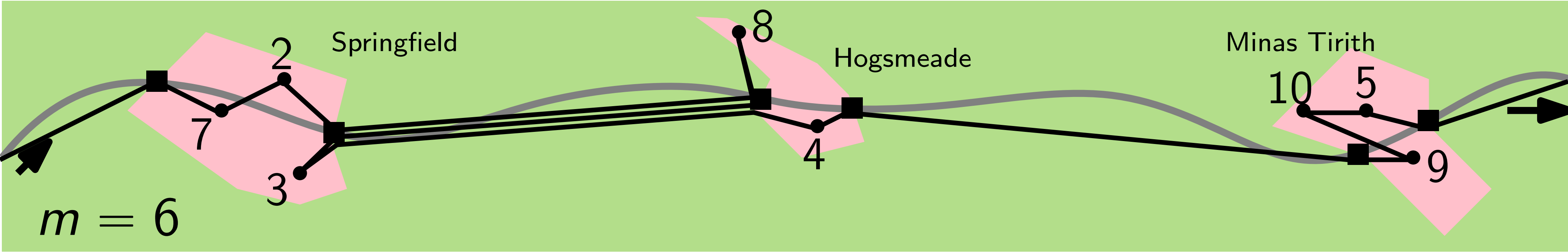

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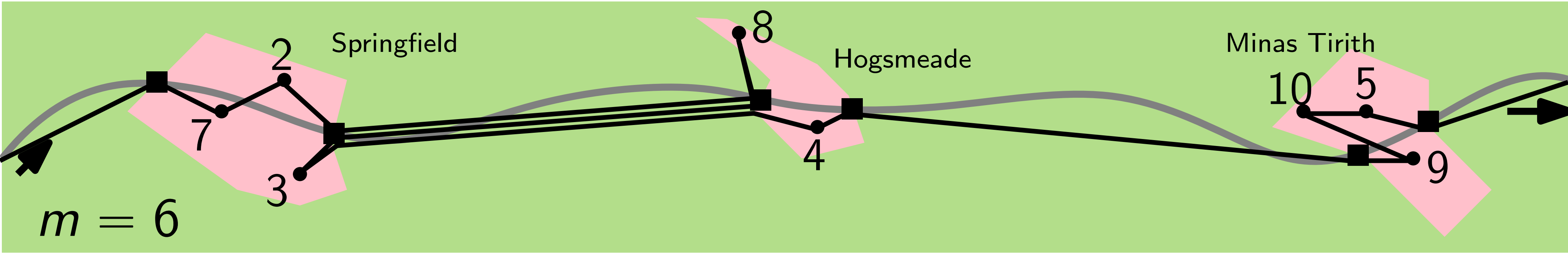


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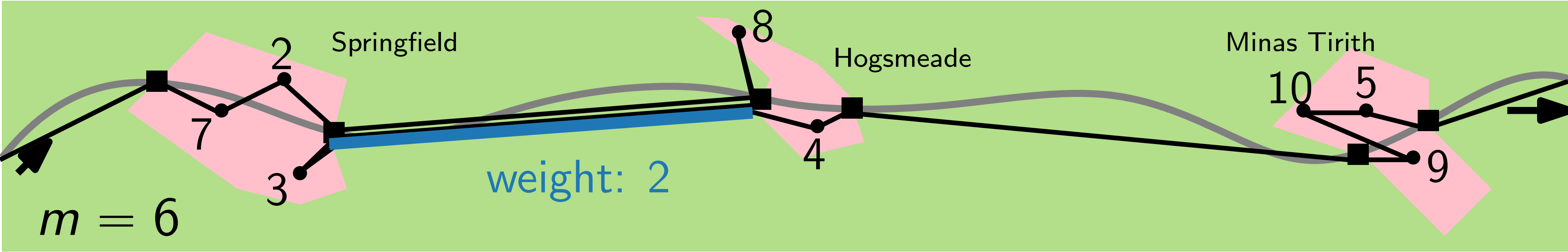


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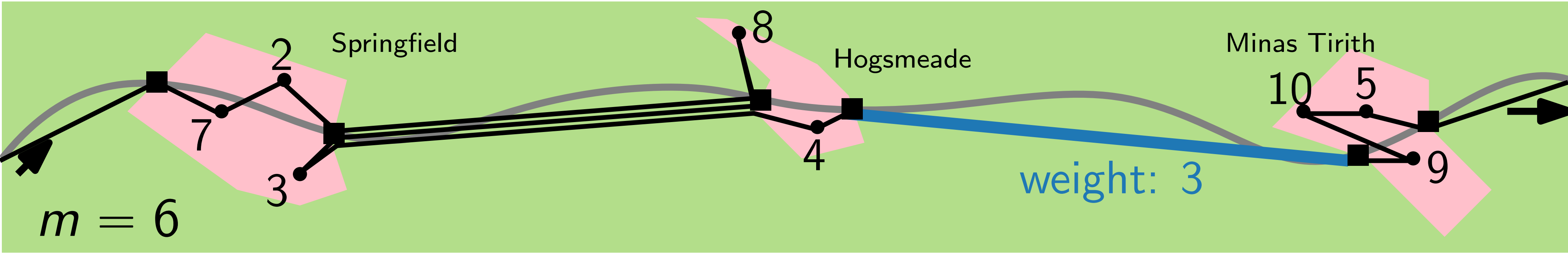


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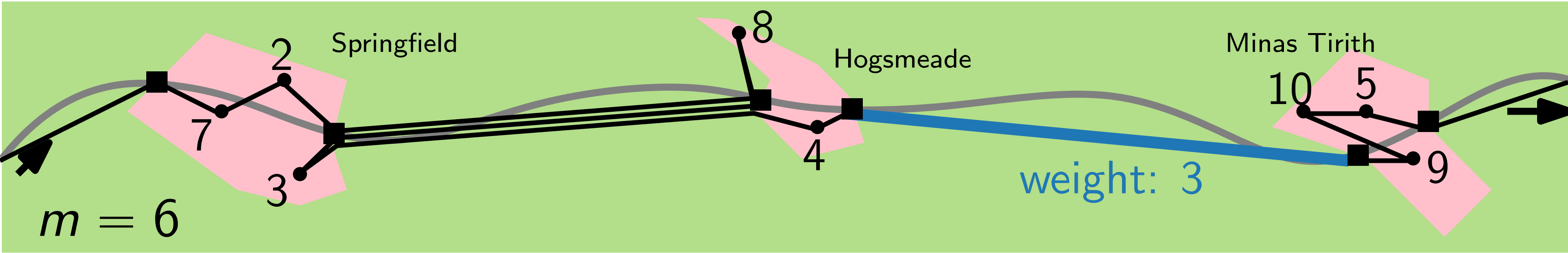


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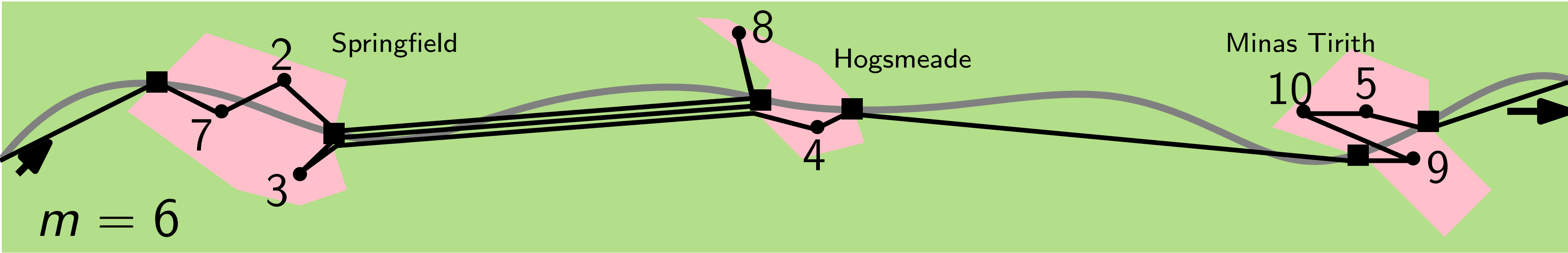


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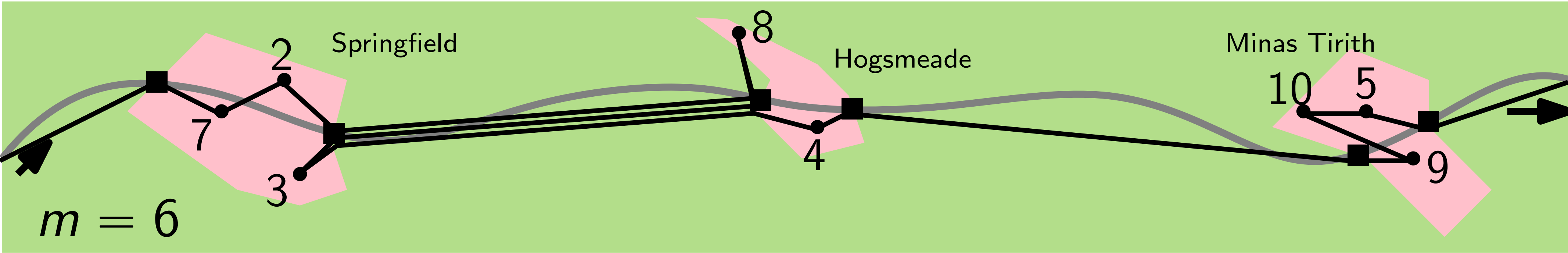


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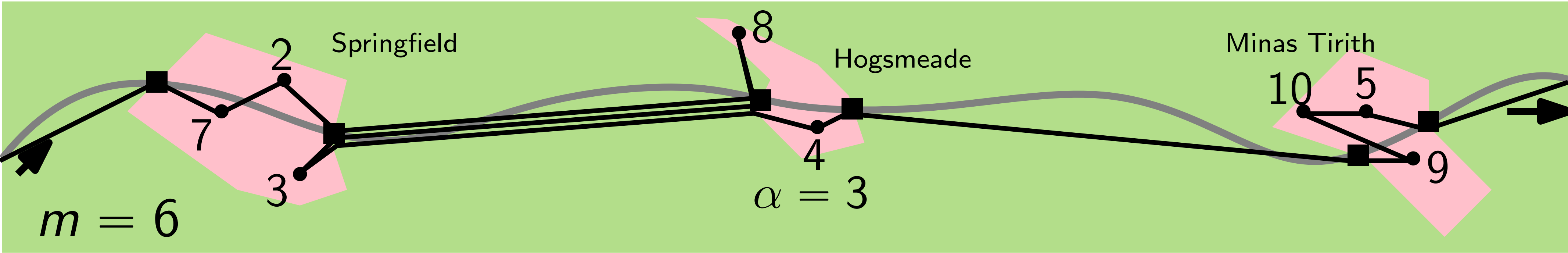


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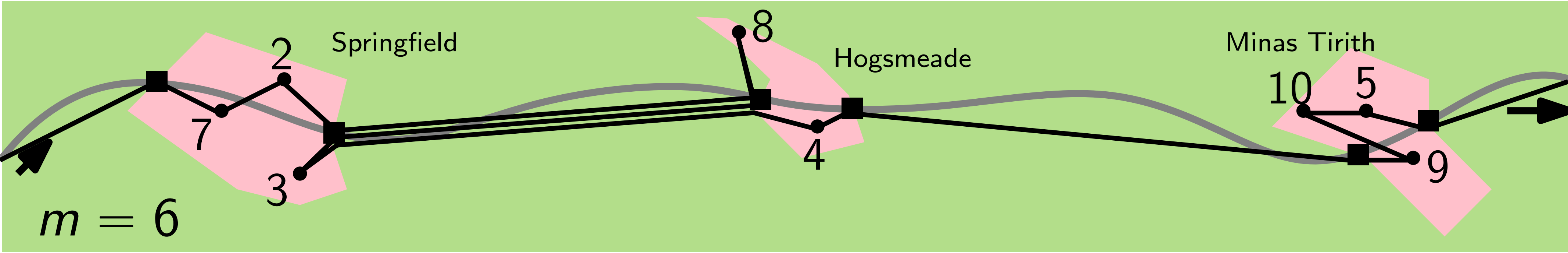


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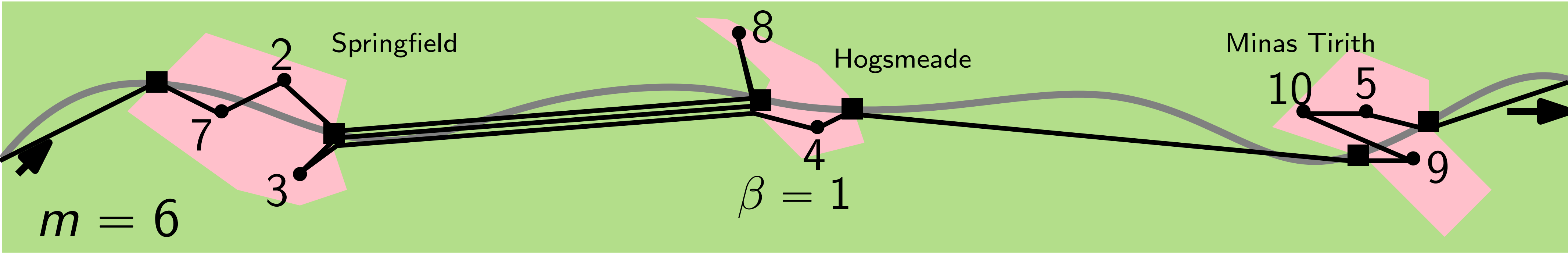


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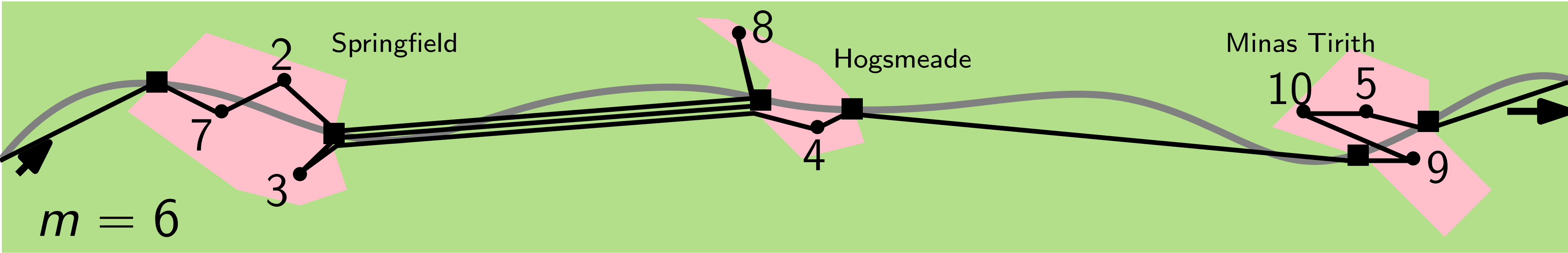


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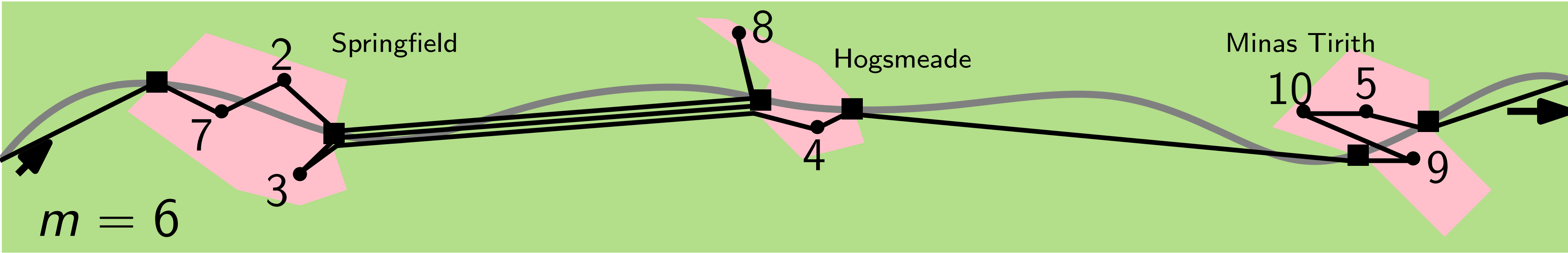


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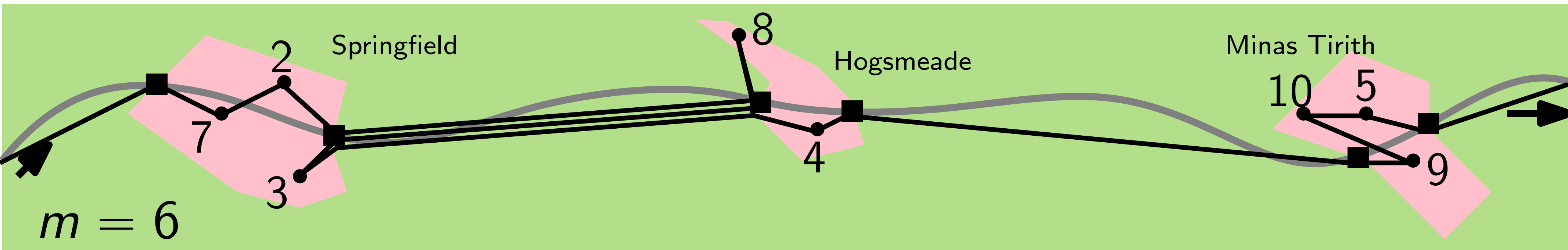


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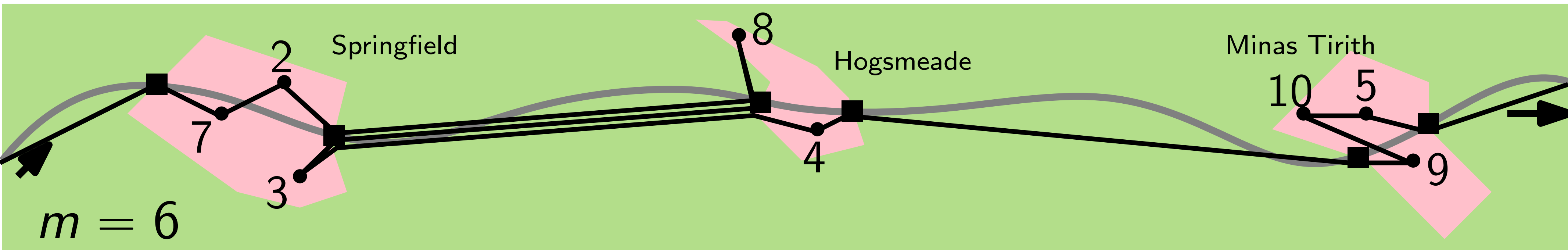


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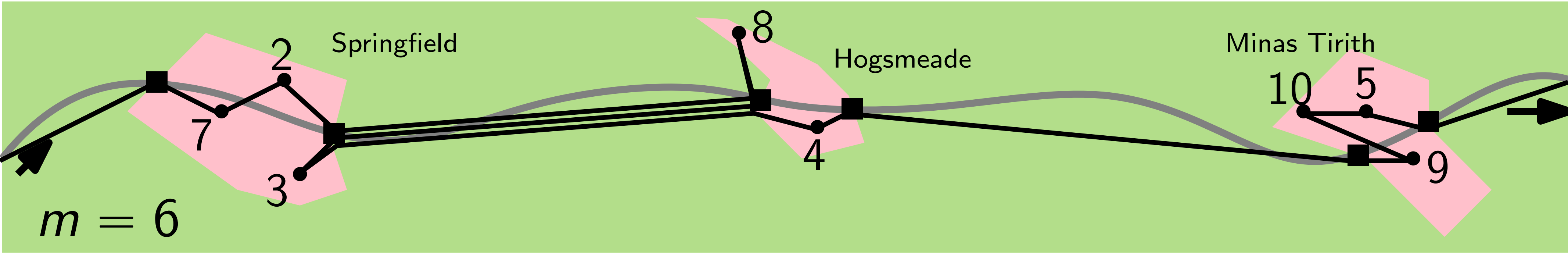


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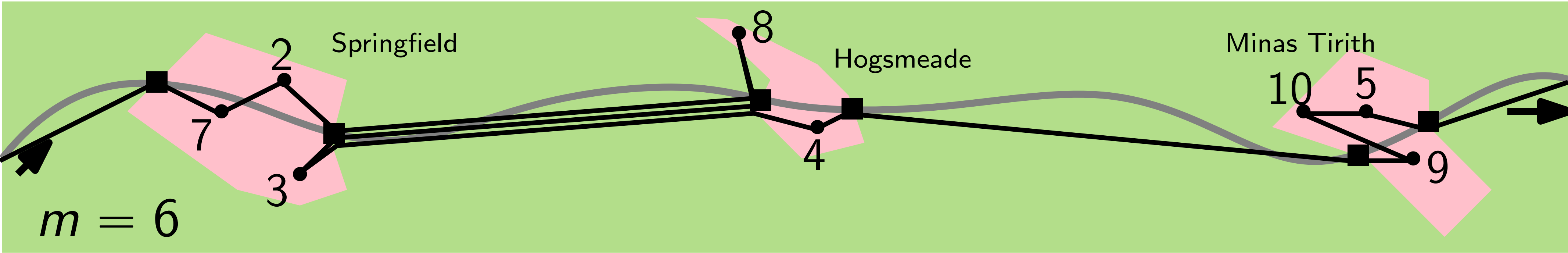


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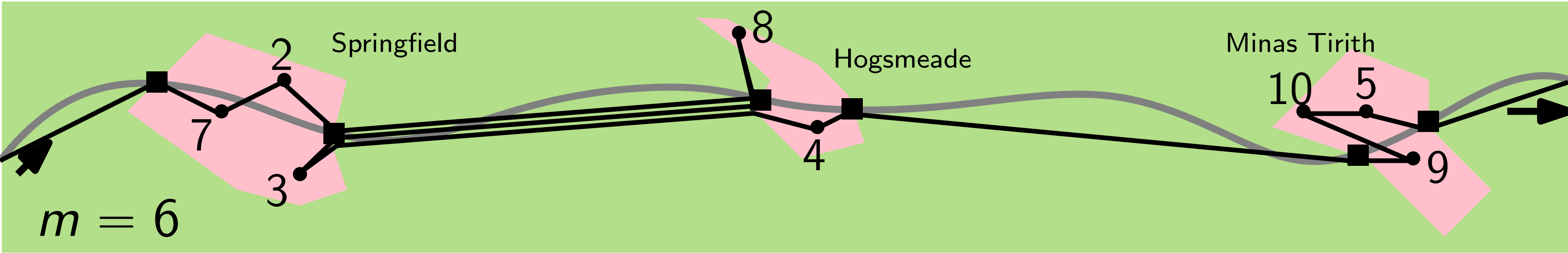


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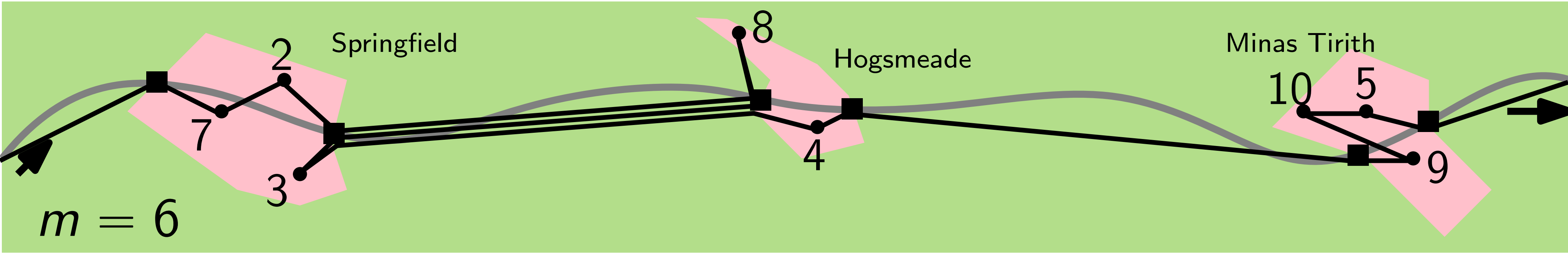


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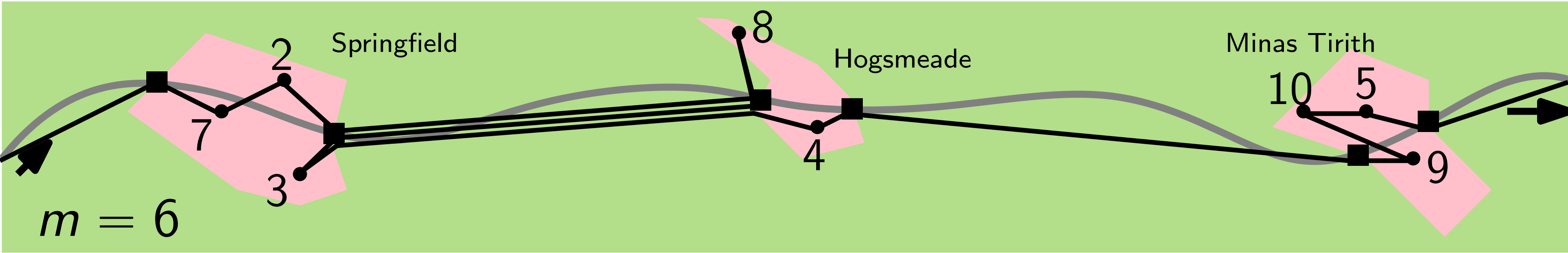
Todo: $\Phi(C_i) \leqslant \Upsilon(T^*, C_i)$

# Lower Bound on $\Upsilon(T^*, C_i)$ (Sketch)



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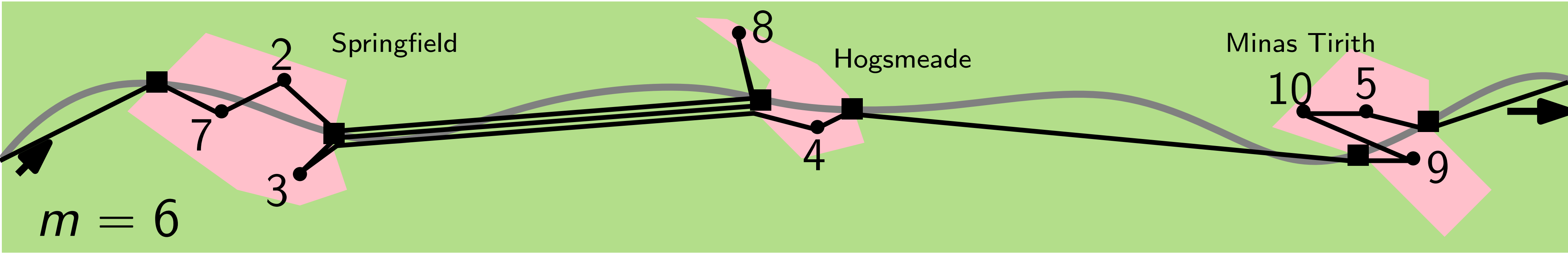

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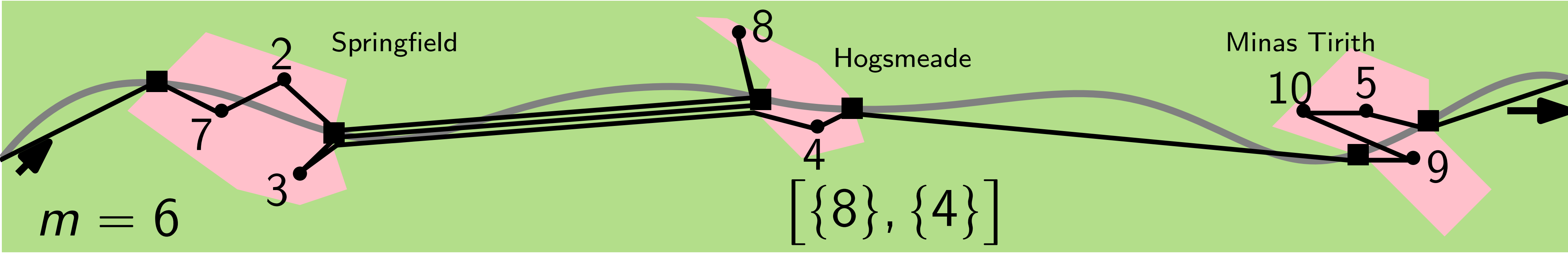

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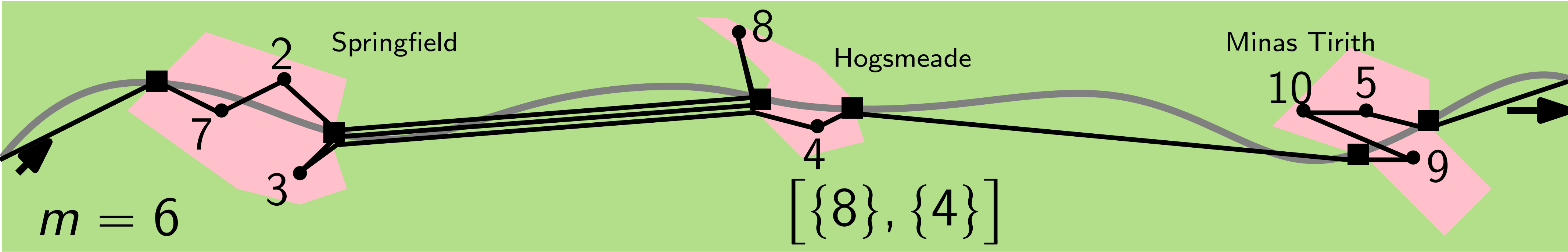


Other Possibilities?

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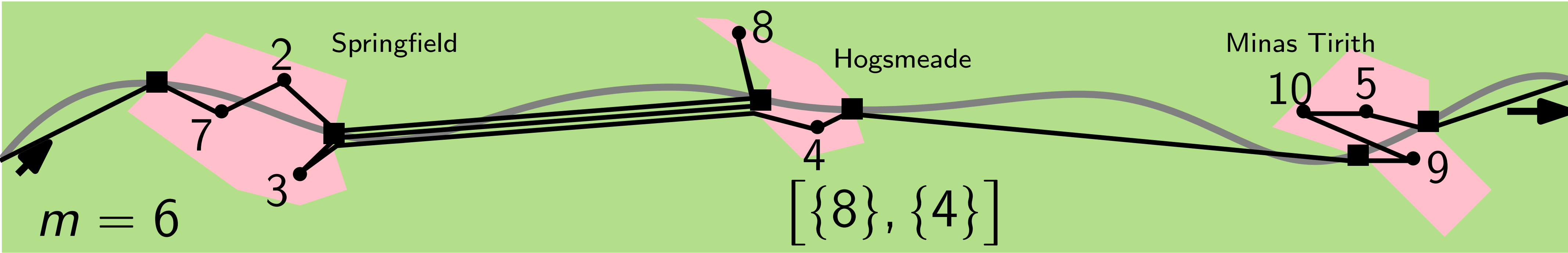


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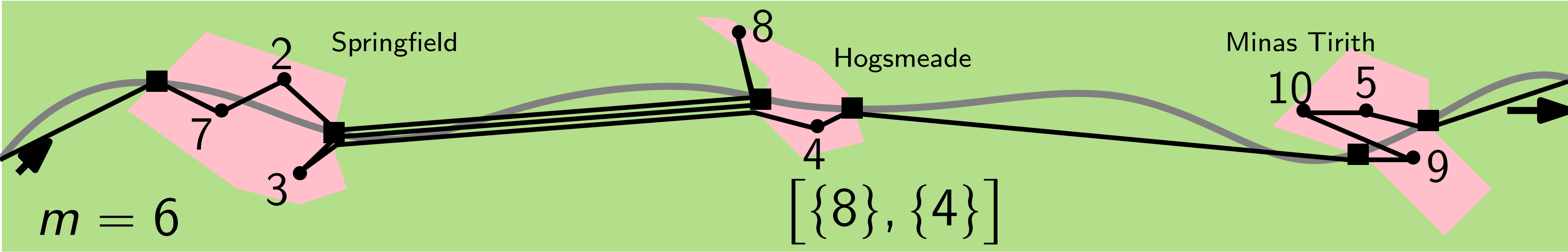


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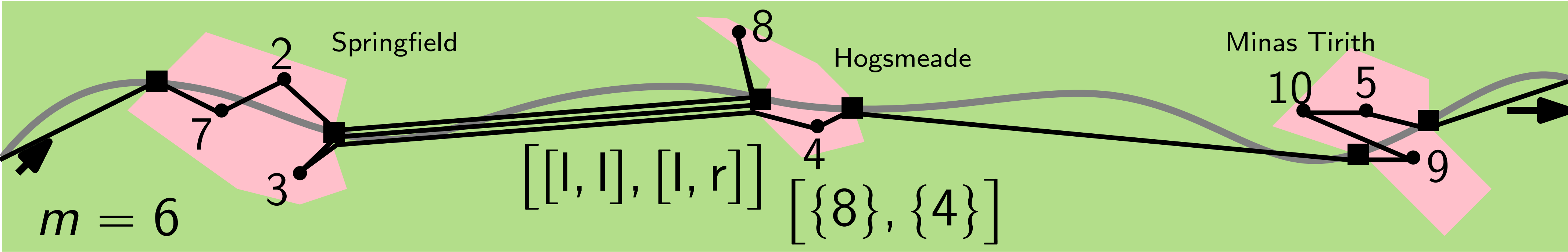


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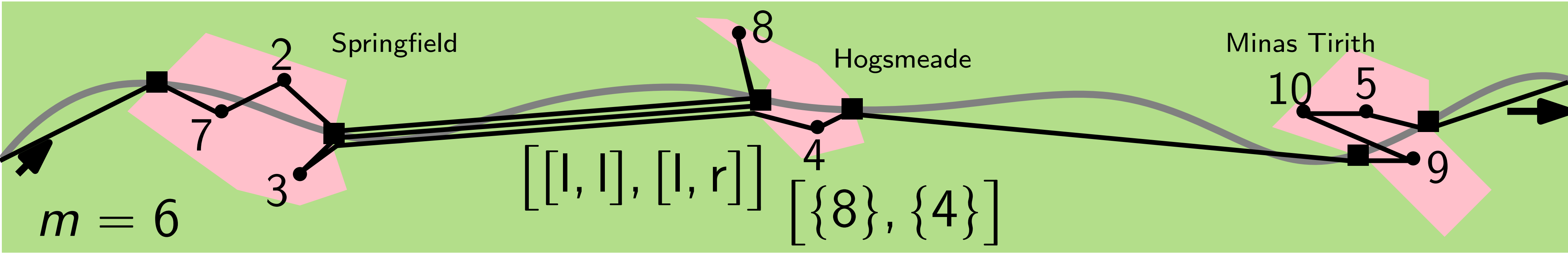


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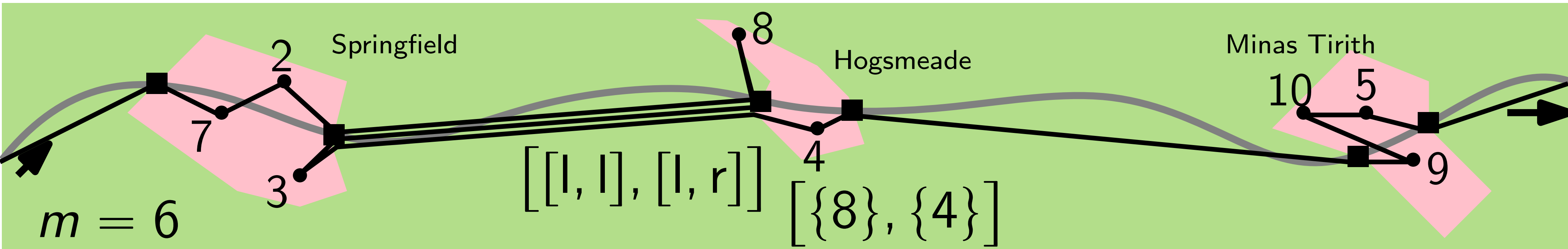


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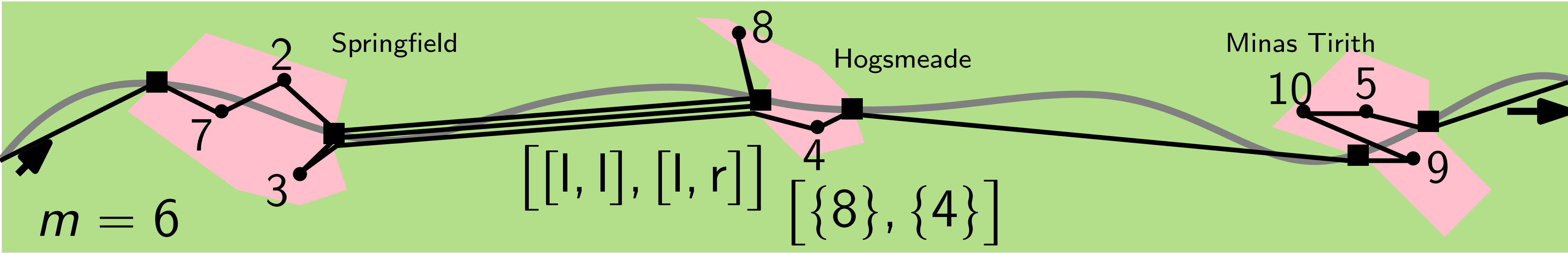


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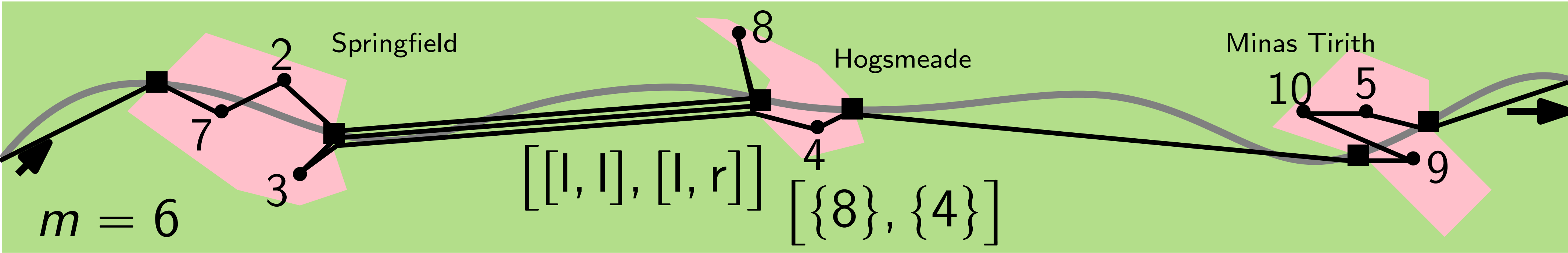


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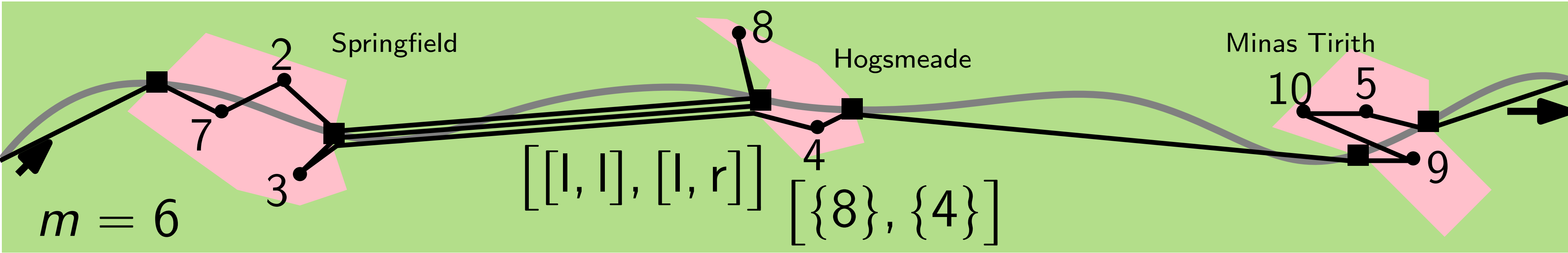


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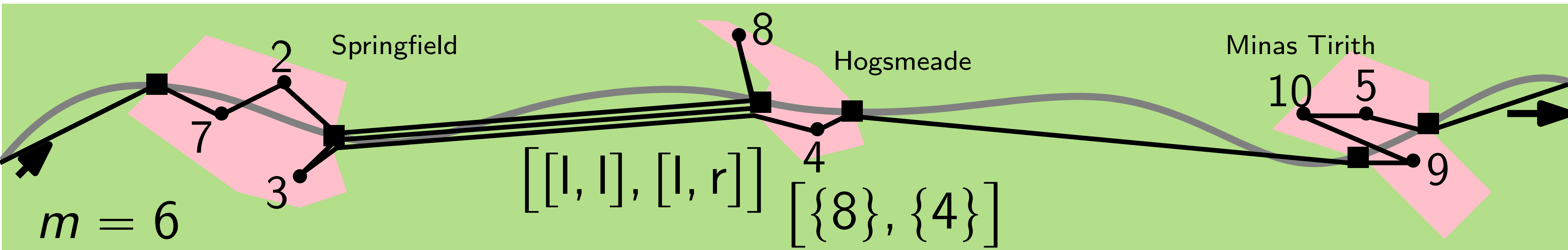


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$|C_i| = 6$: 5 227 236 choices

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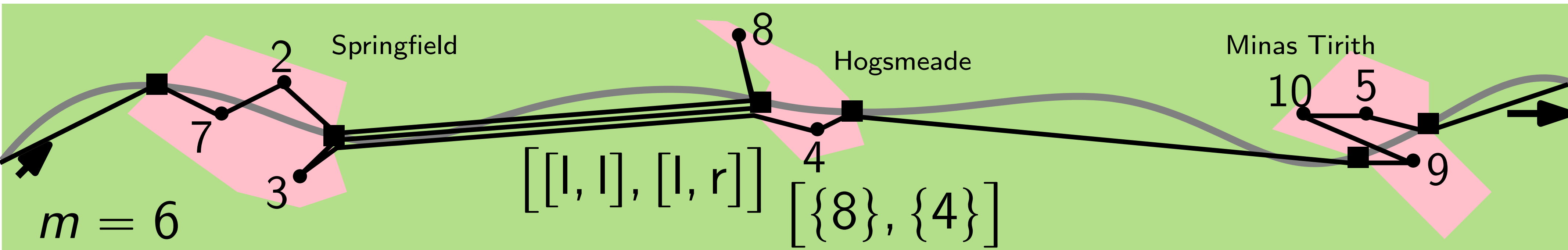


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Practical Limit!

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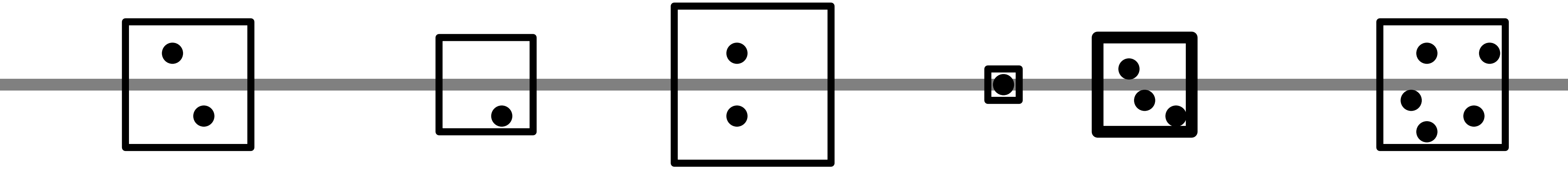

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for $n = 12$

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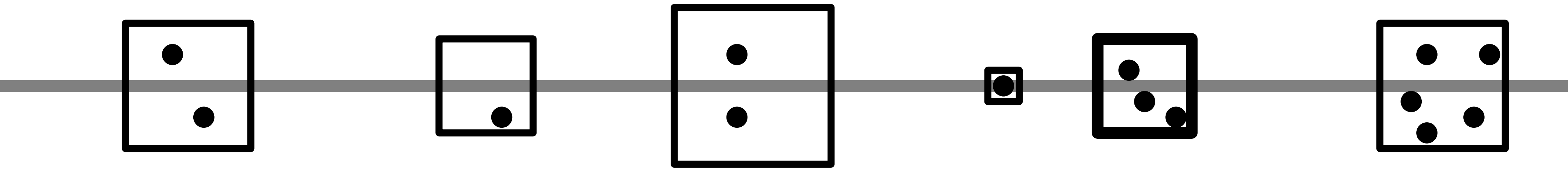

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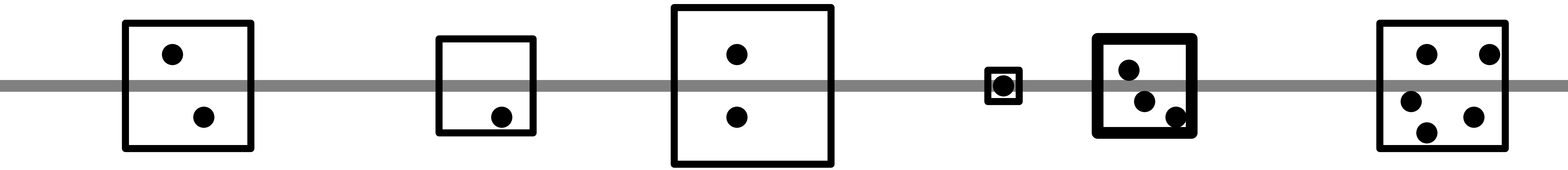

**Runtimes:**

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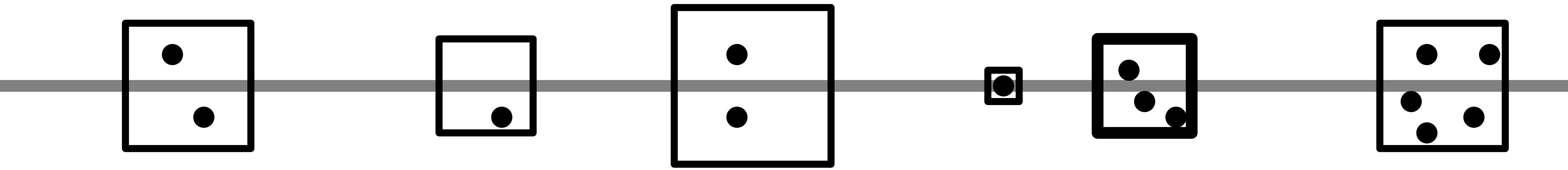

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**Exact:** 120 s  $\overrightarrow{T^*}$-**Algorithm**: 3 ms  **Classifier:** 4 s

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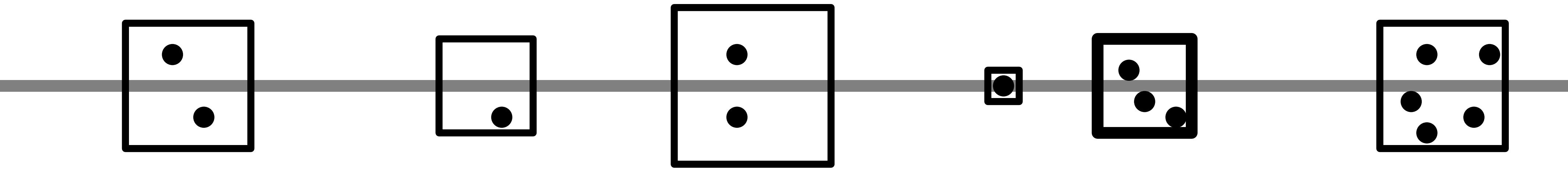

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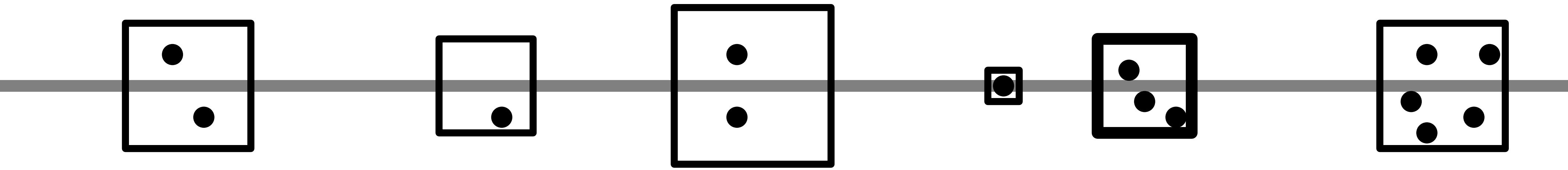

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$$\text{Ratio } T^* = \overrightarrow{T^*}$$

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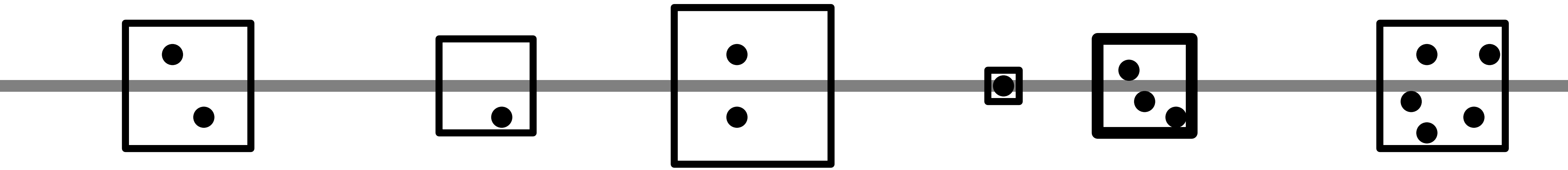

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| Clusters close together ($\sim$ 6km): | 59 % |

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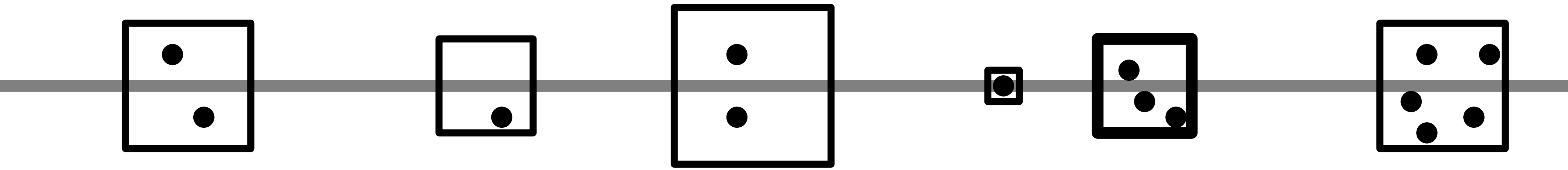

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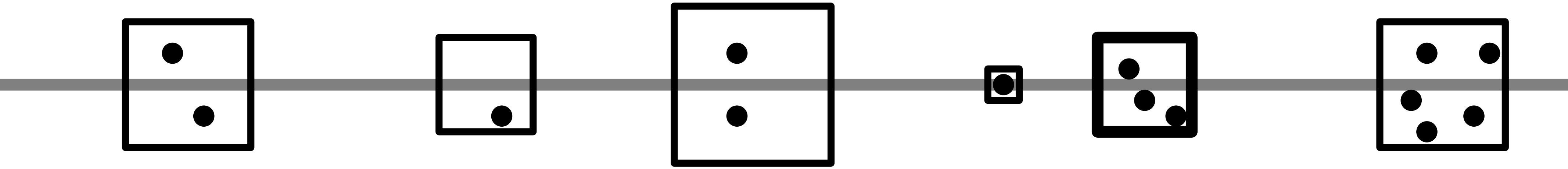

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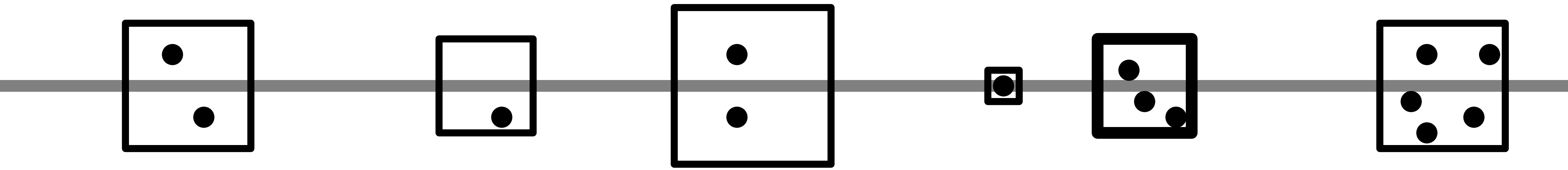

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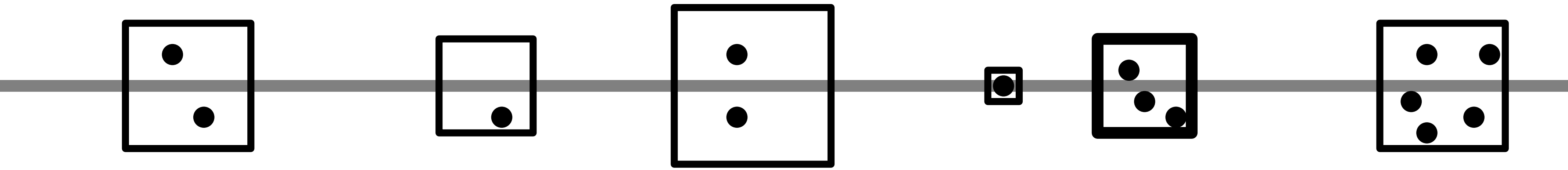

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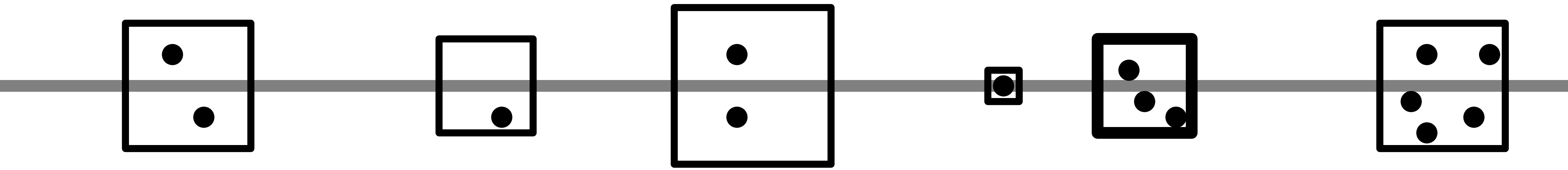

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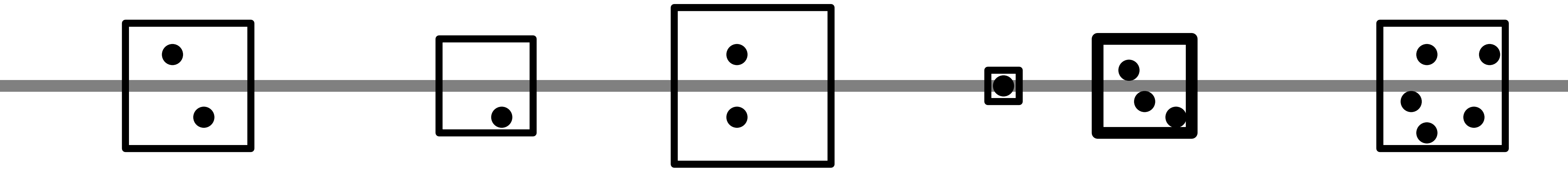

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$\overrightarrow{T^*}$-**Algorithm as Heuristic:**

Approximation Quality (empiric): $\leqslant 1.1$

# Topology of Street Networks

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Street Networks often do not meet the assumptions.

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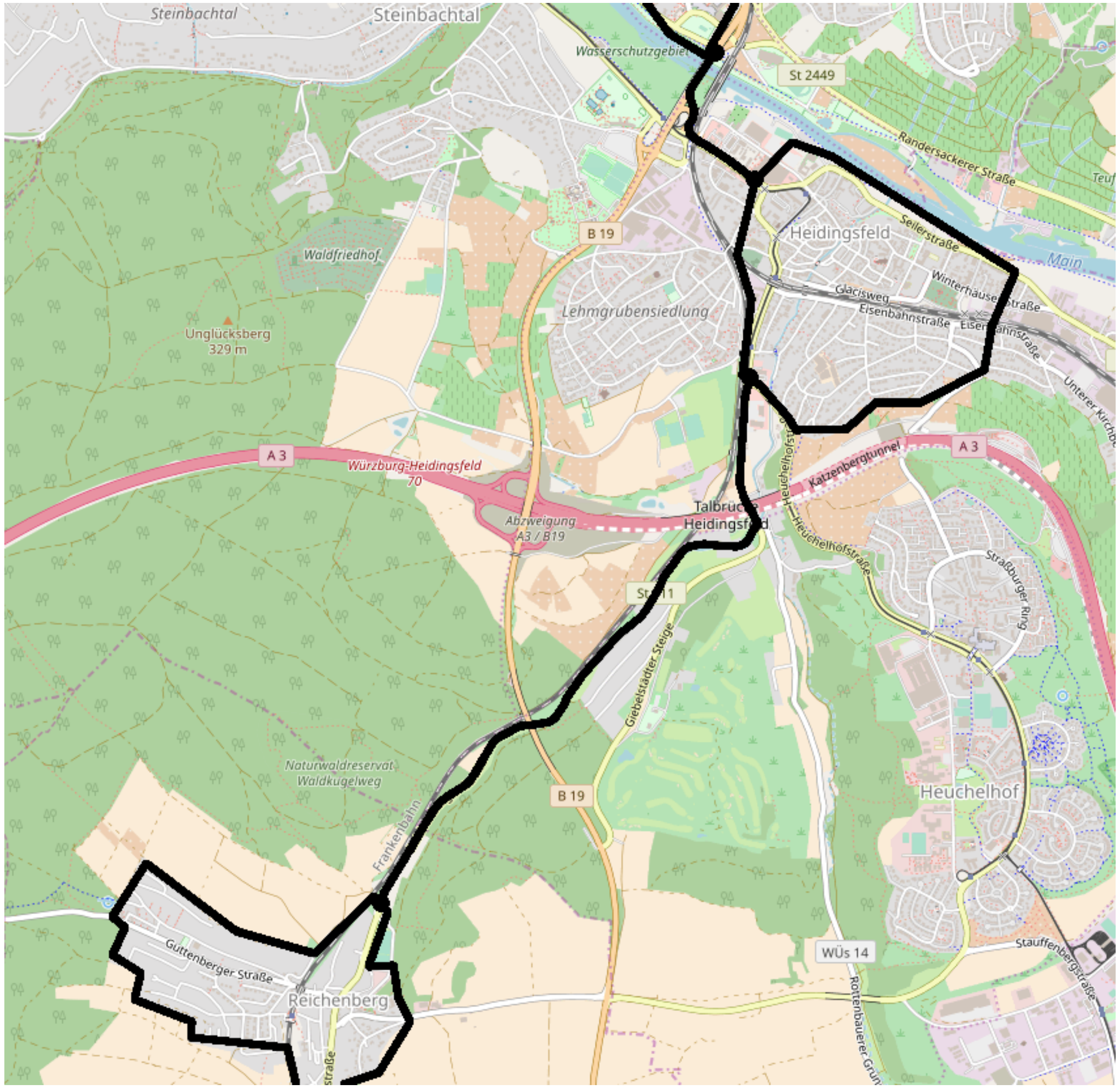


Example #1:
Rural Instance

# Topology of Street Networks

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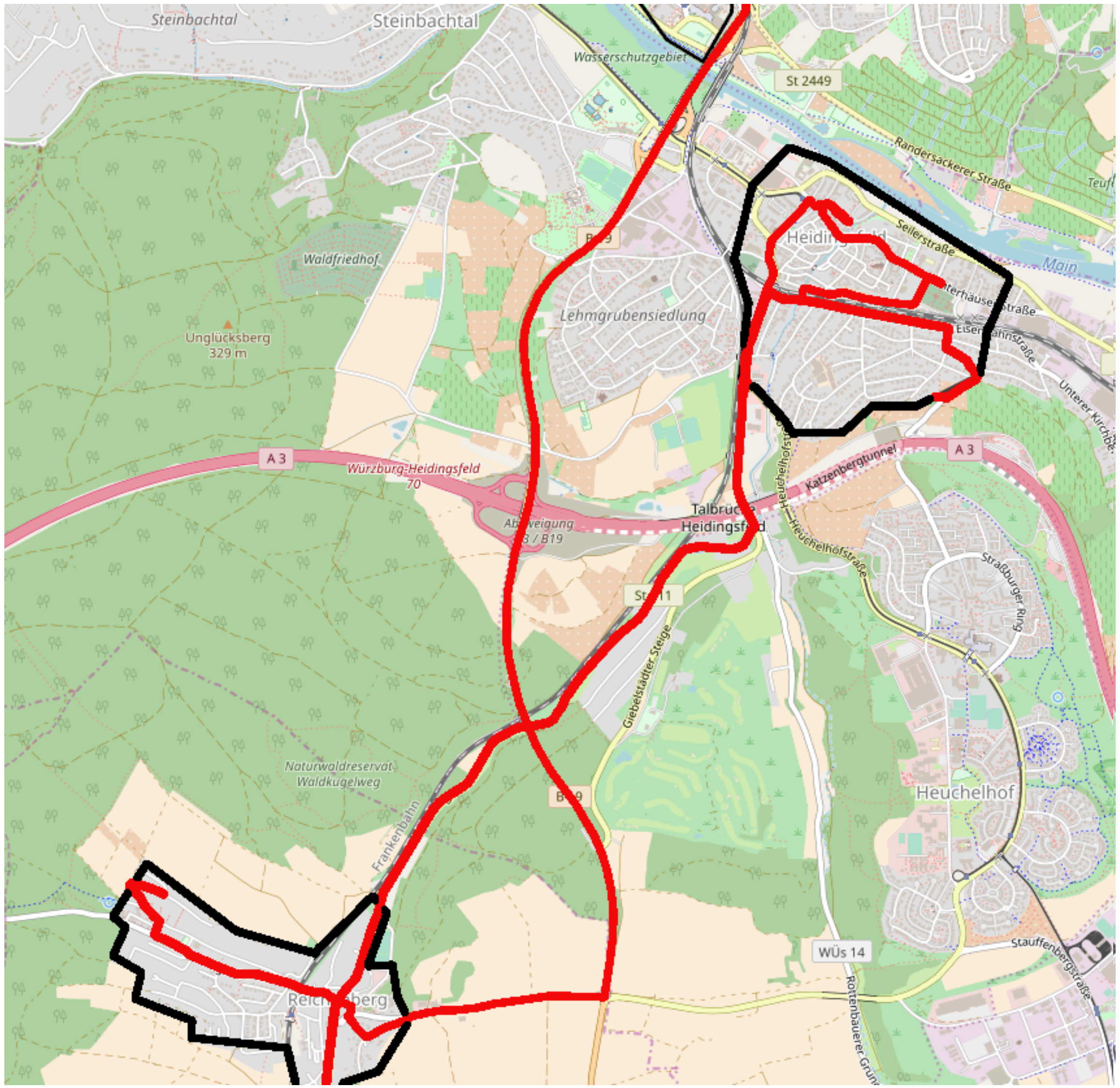


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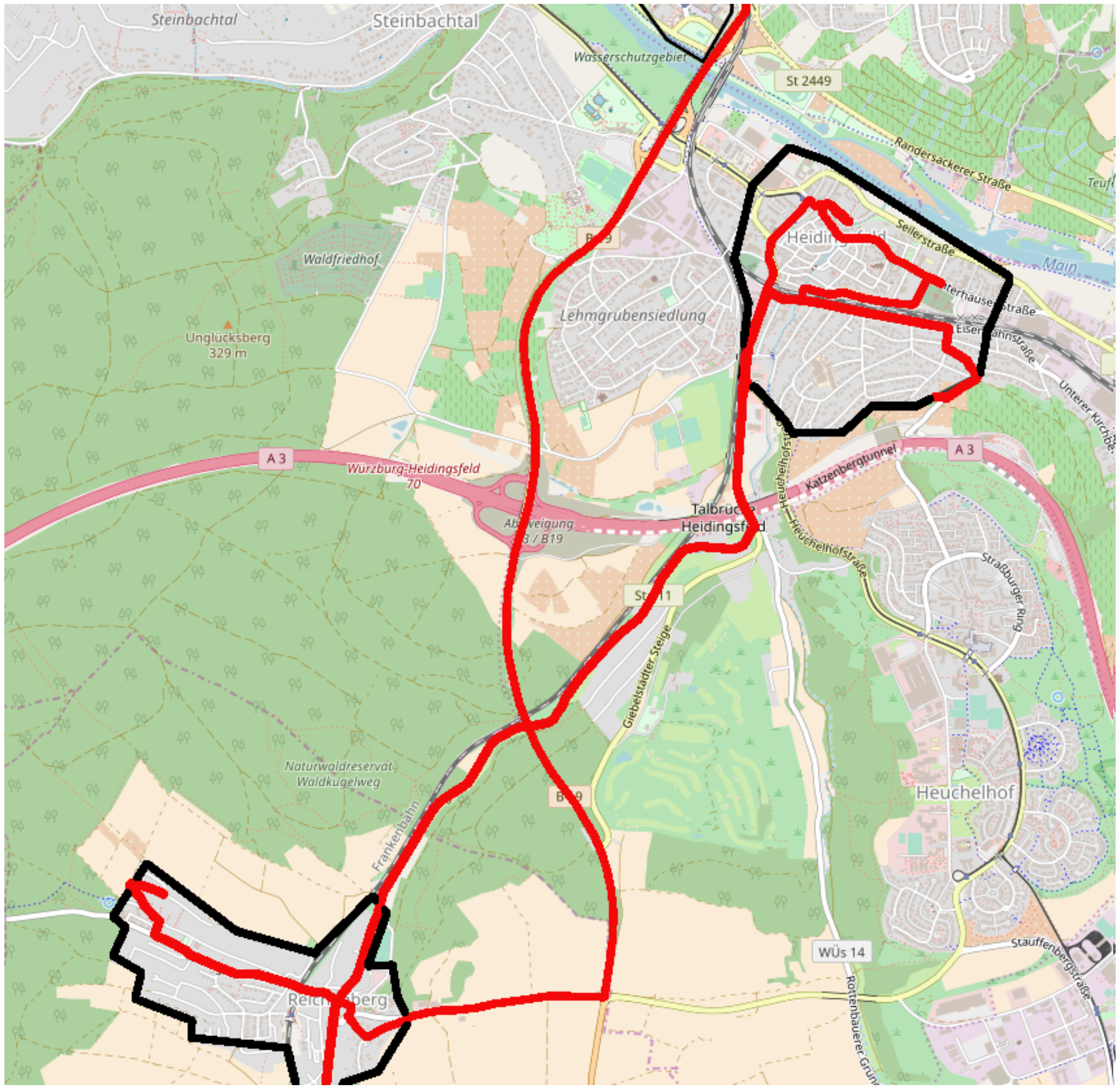


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$T^*$ bypasses a cluster!

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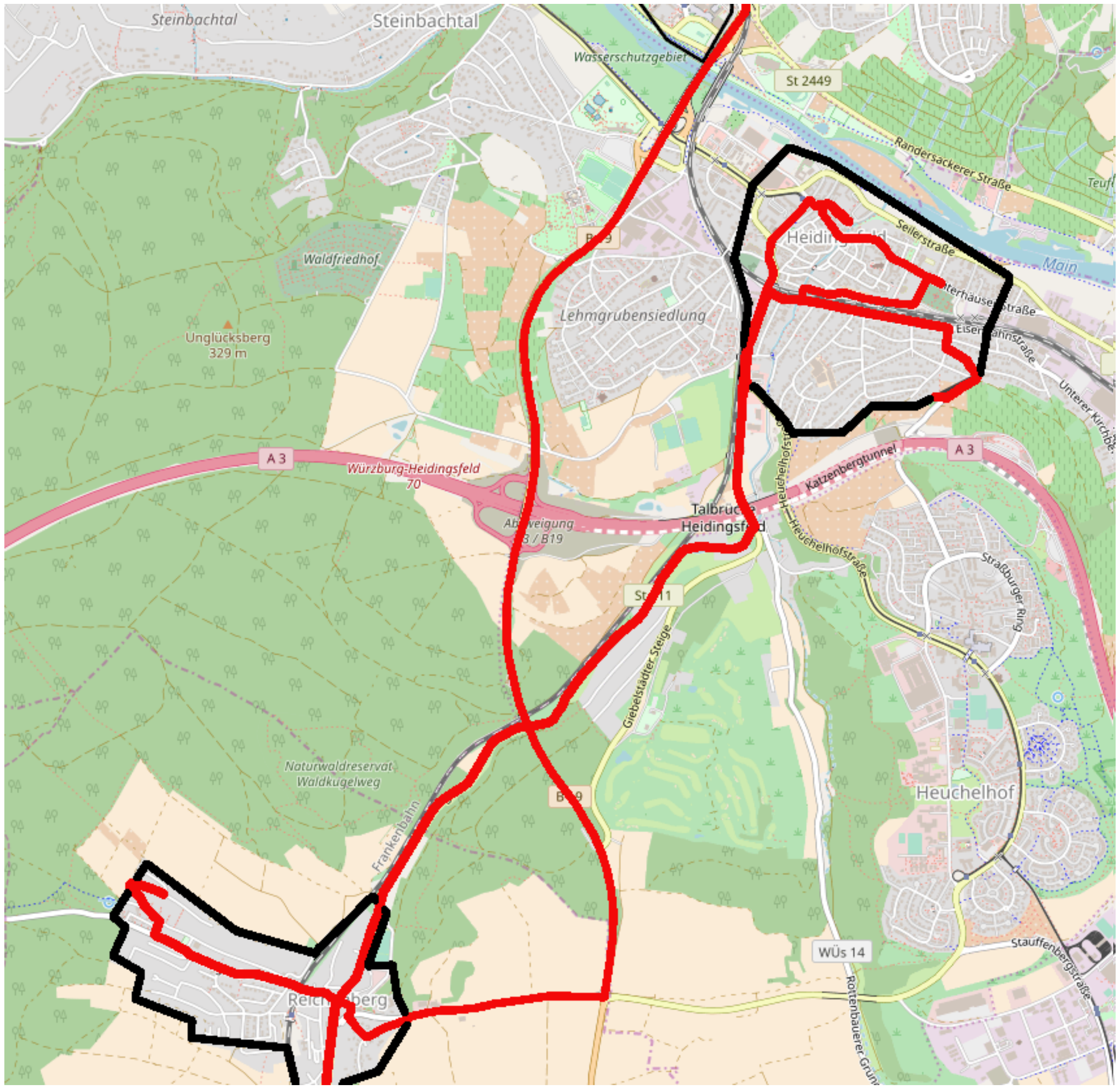


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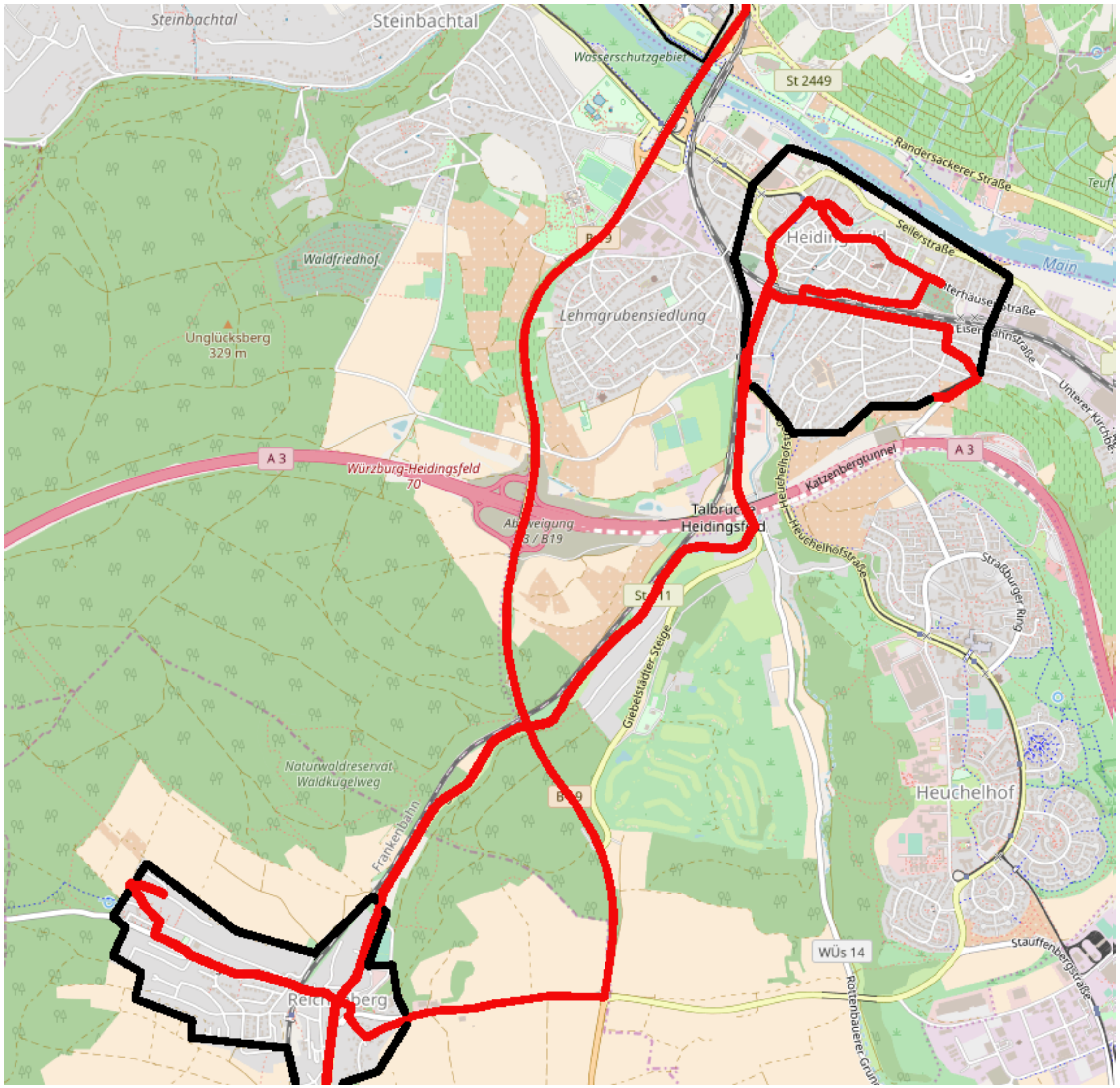


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$\Rightarrow$ Classifier is robust to some extent.

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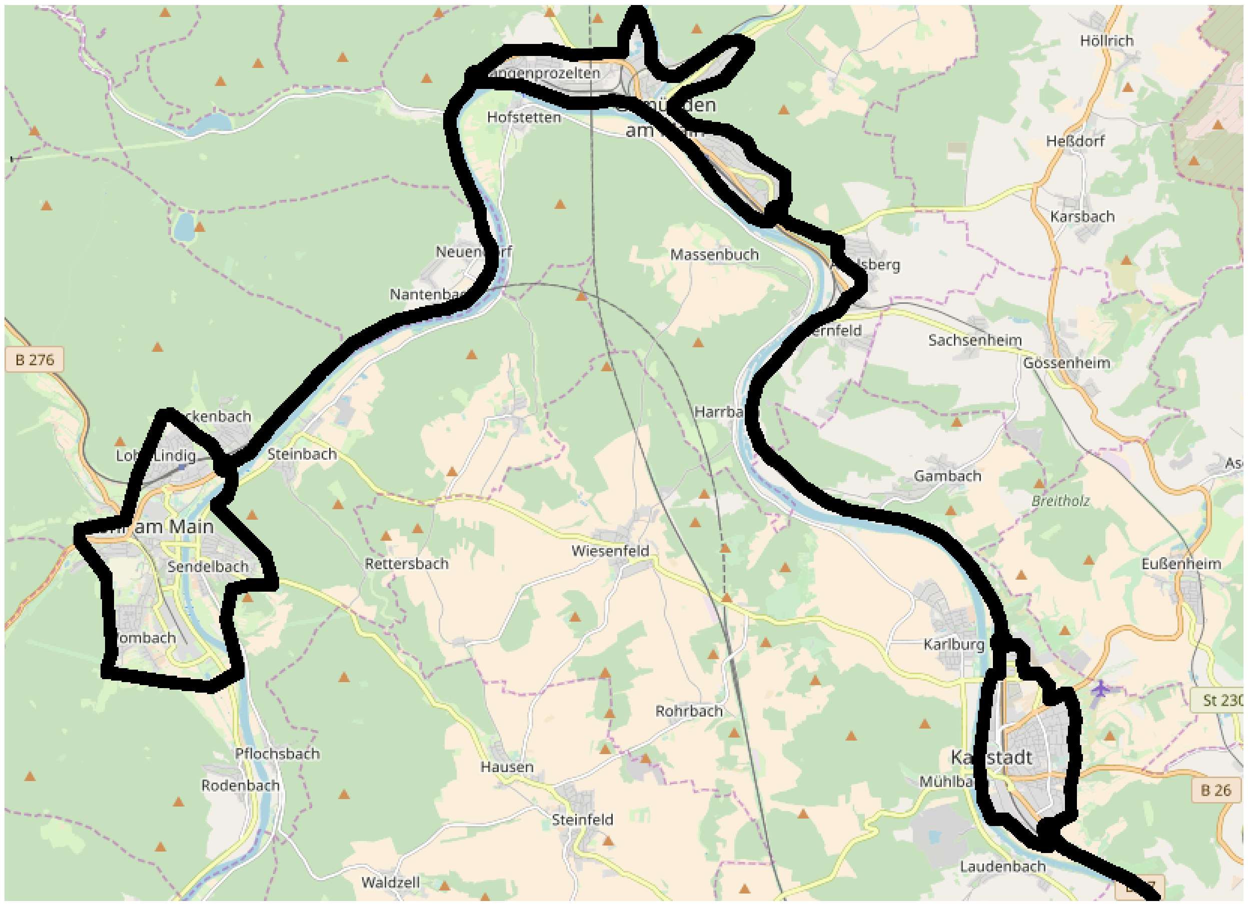


Example #2:
Regional Instance

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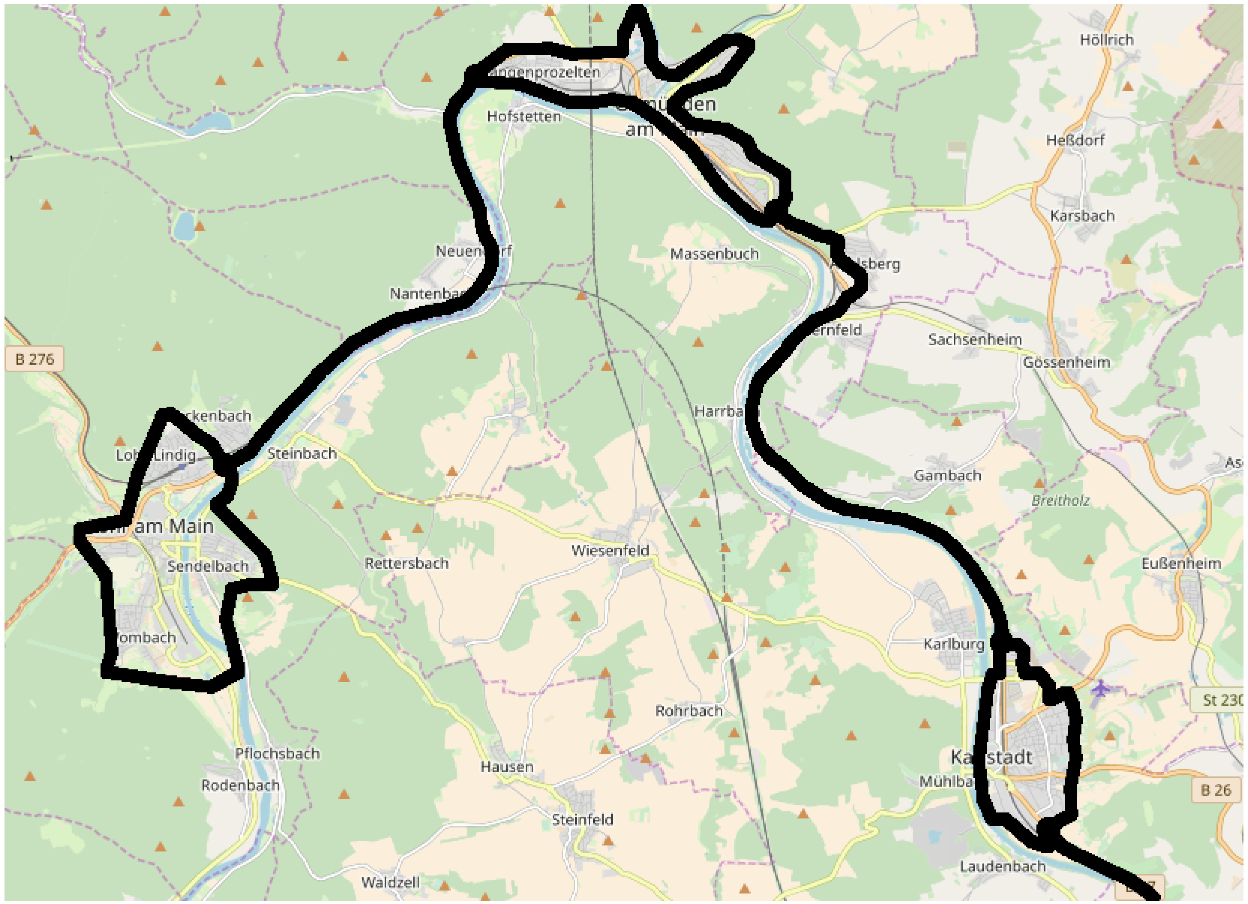


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Really hard scenario . . .

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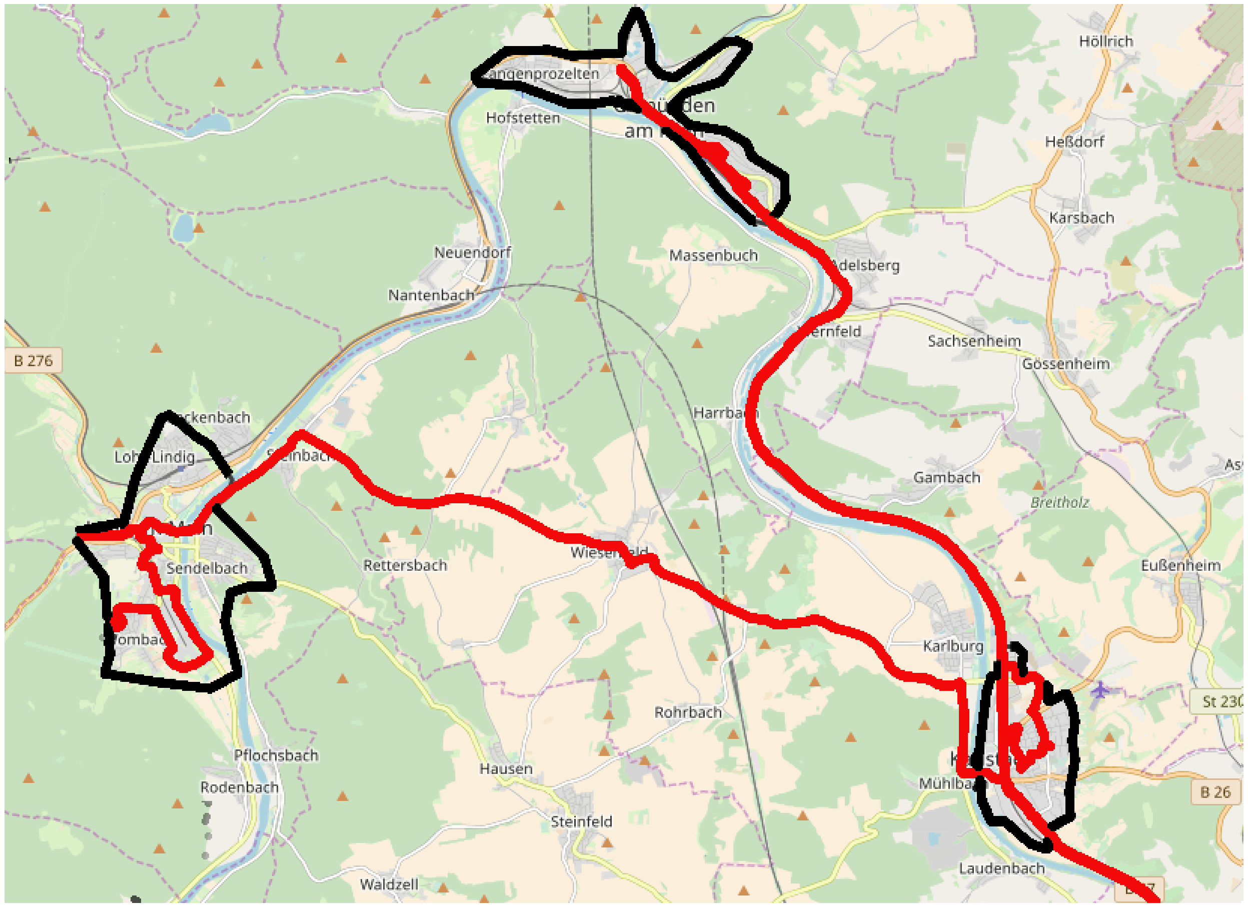


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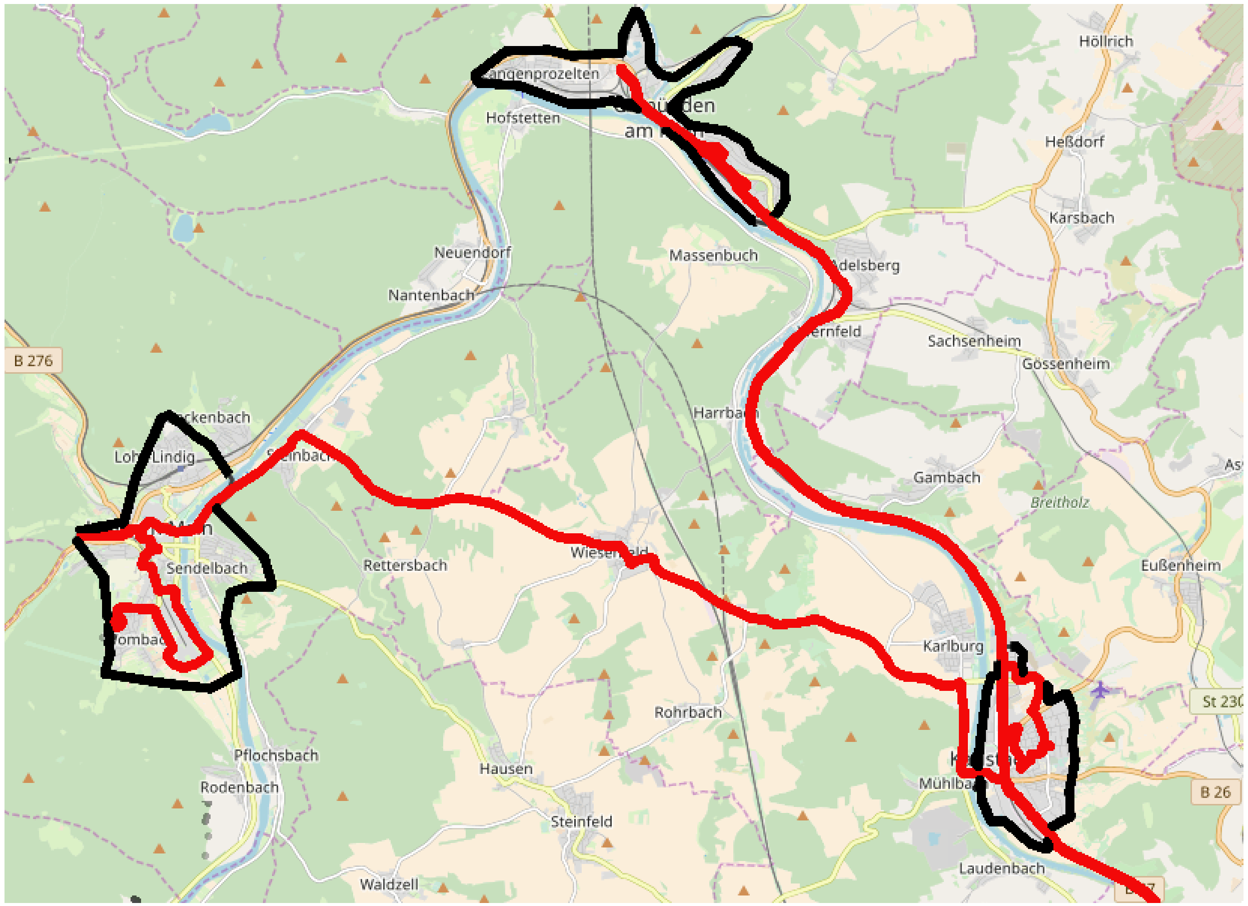


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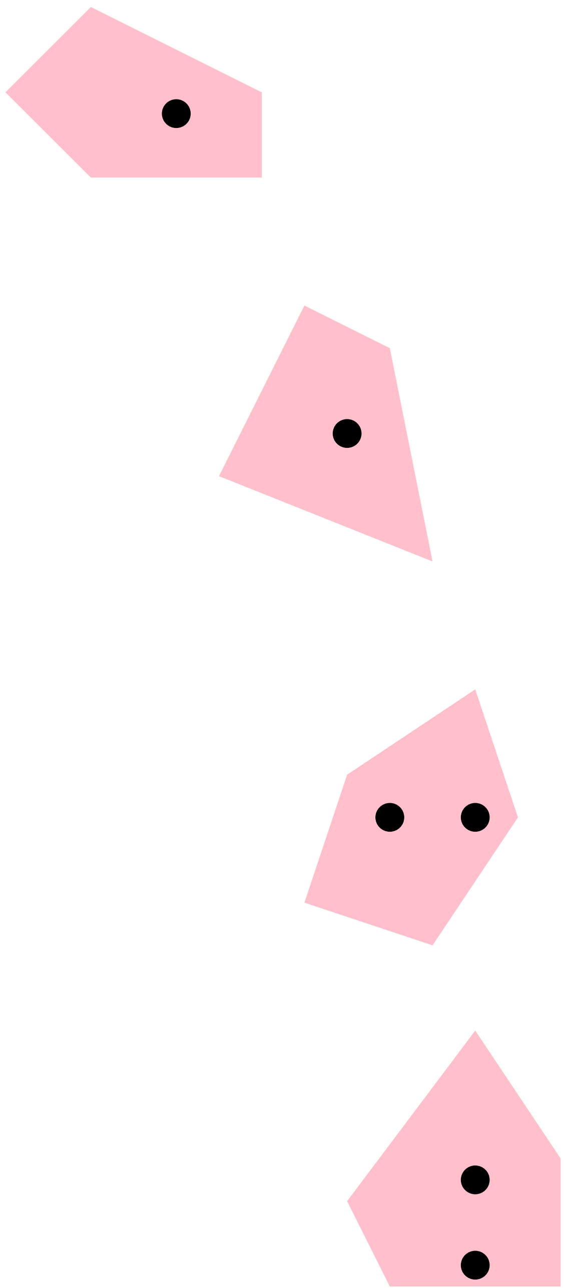
Really hard scenario . . .

False positives are to be expected in this case.

# Conclusion



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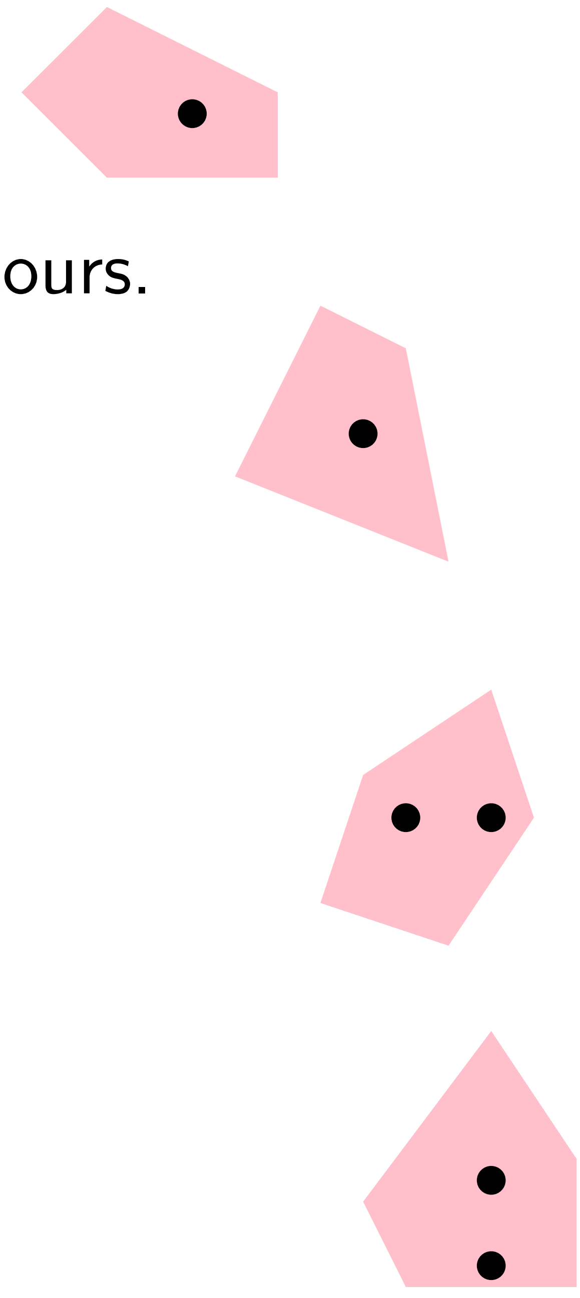

The Exact Algorithm considers unsensible tours.

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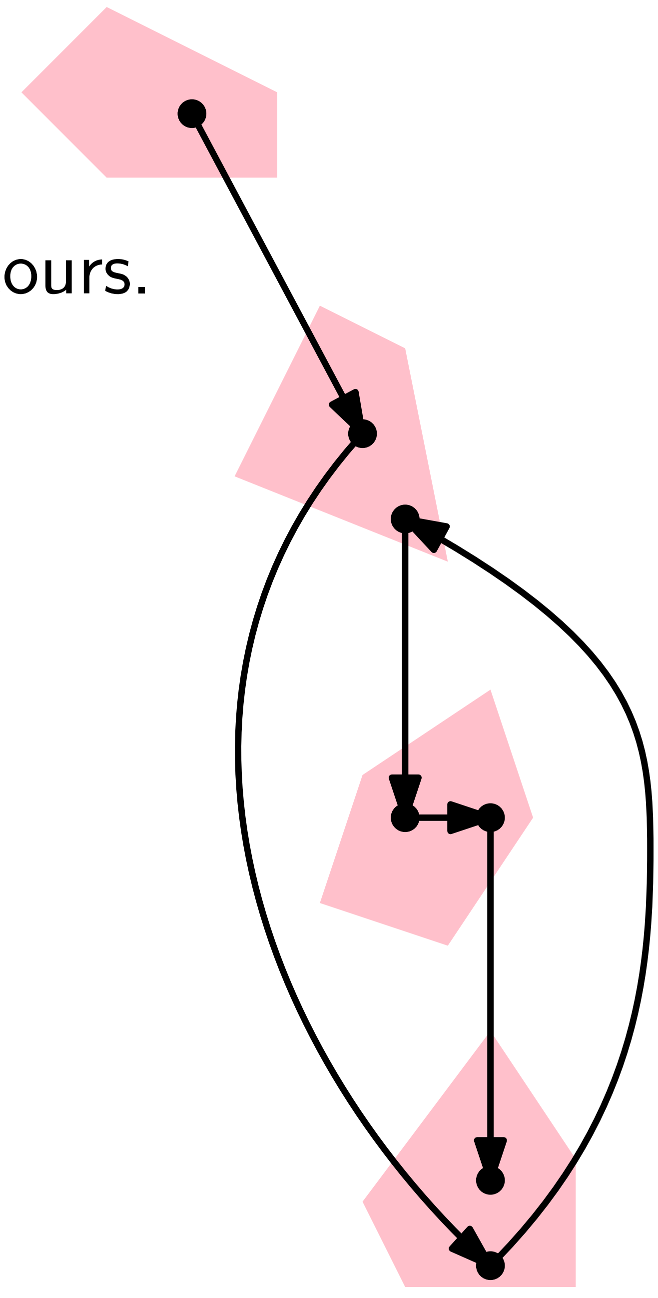

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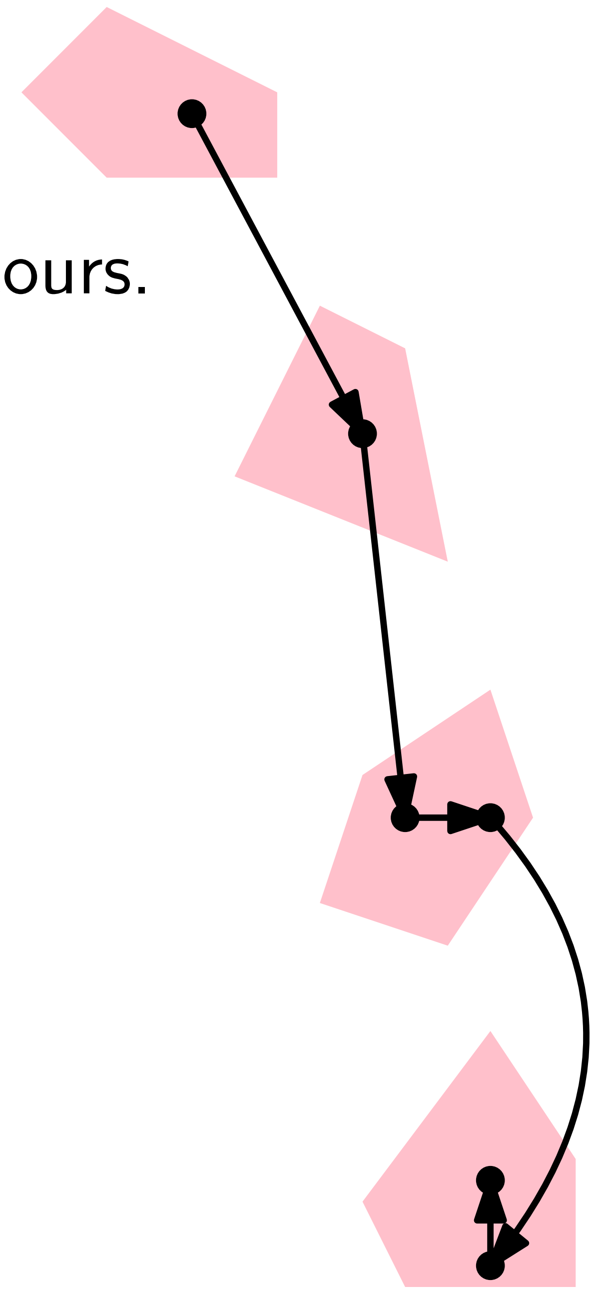

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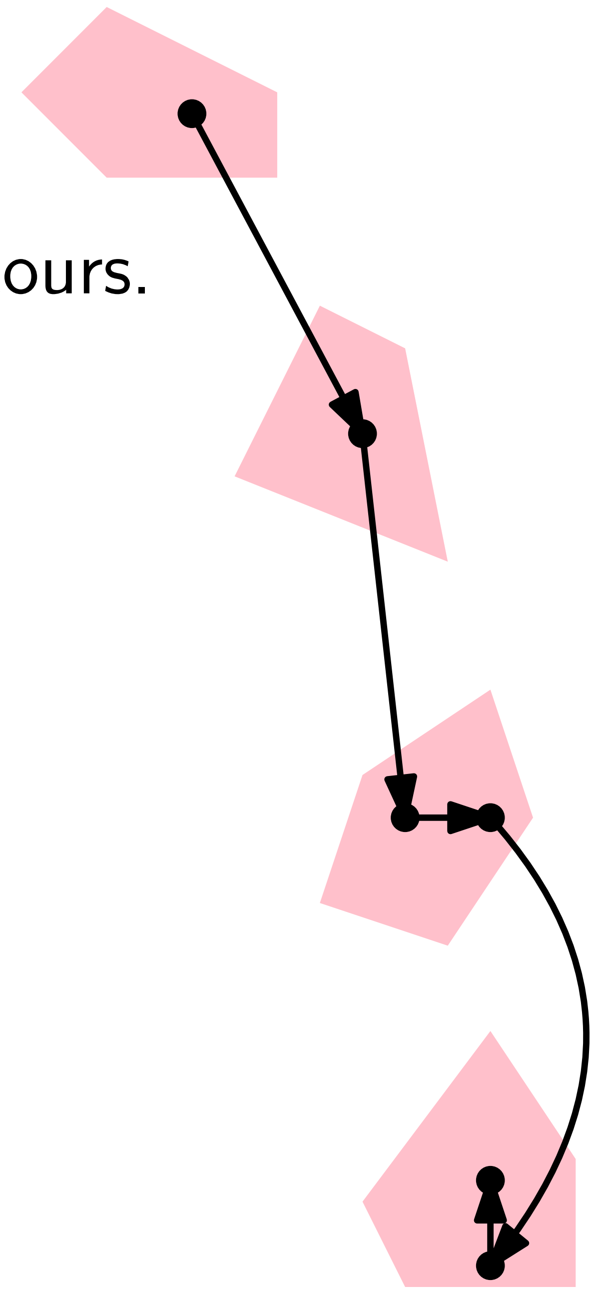

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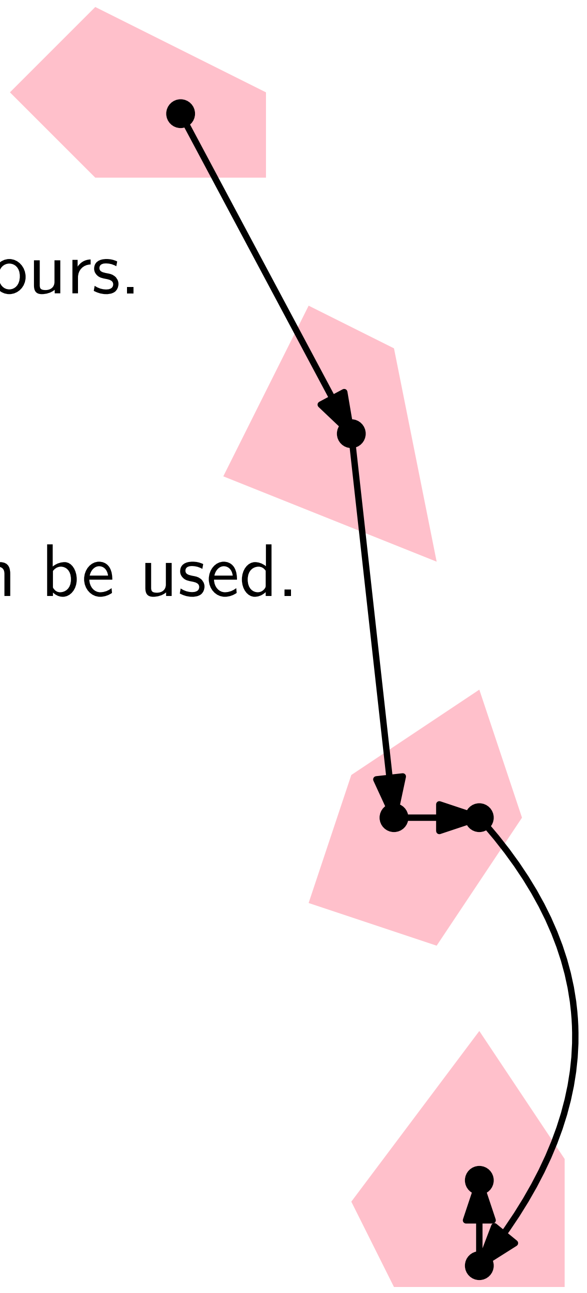

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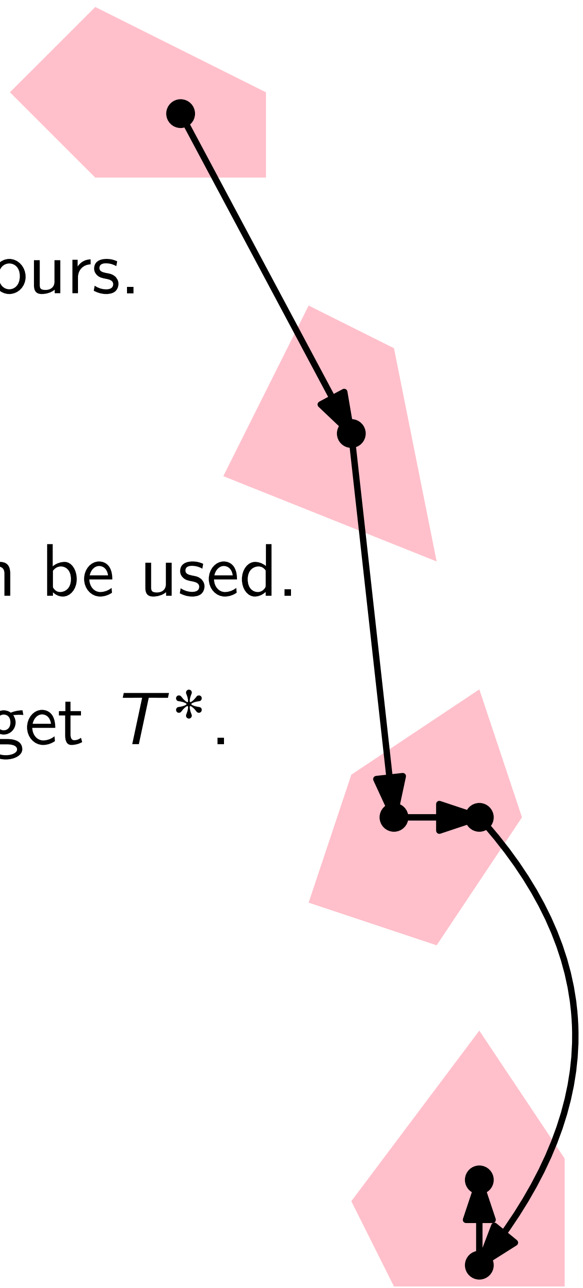

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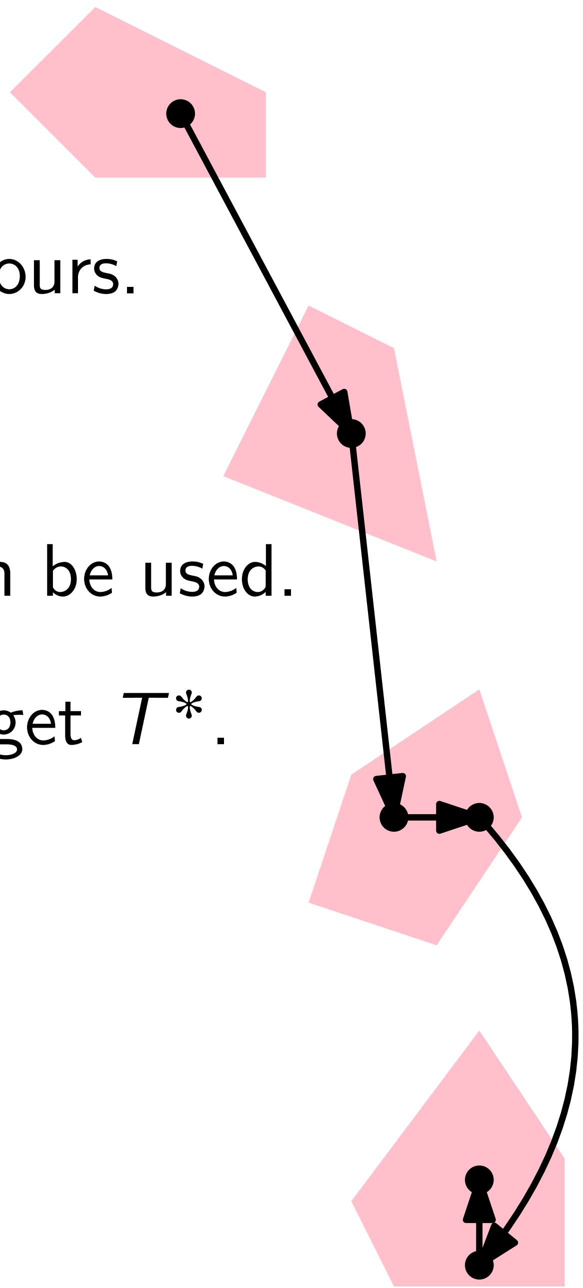

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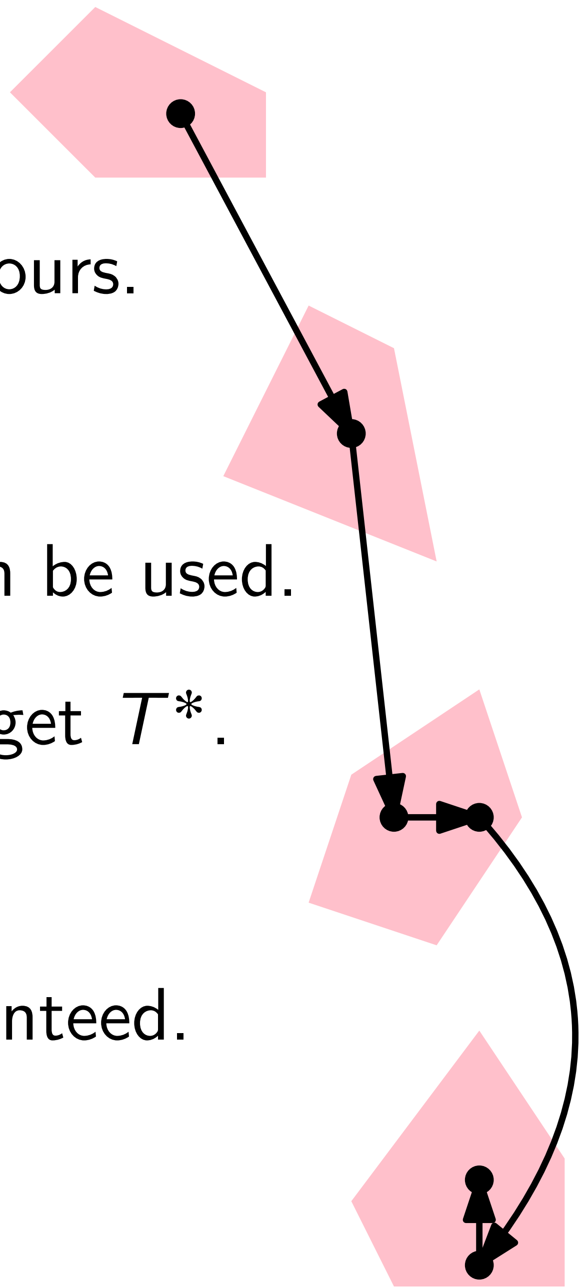

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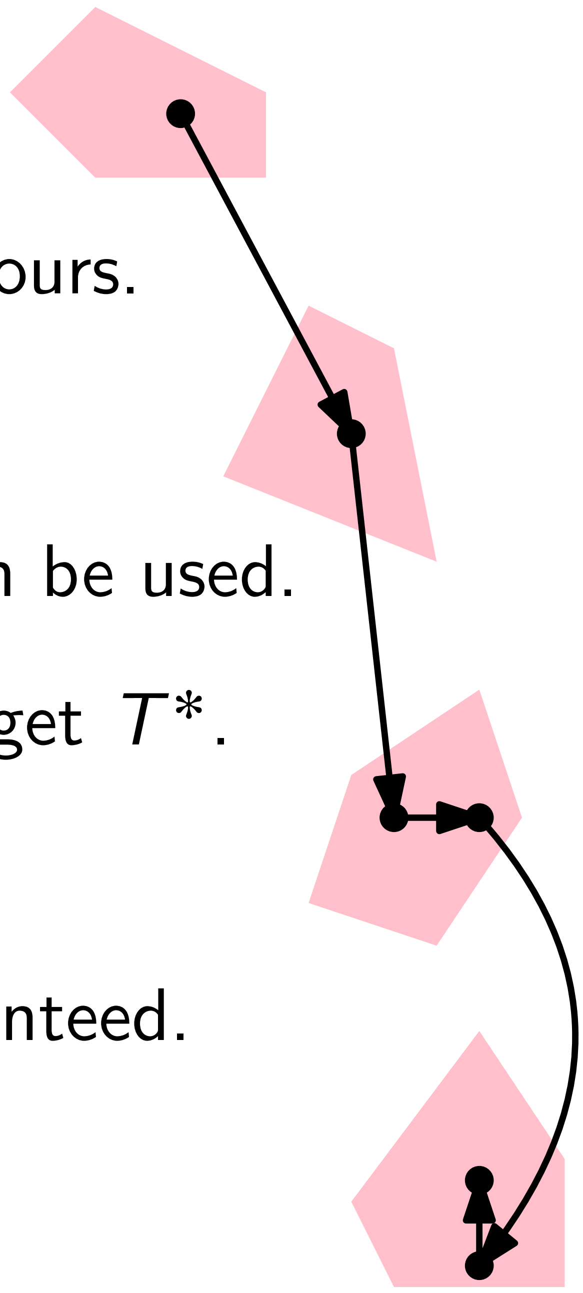

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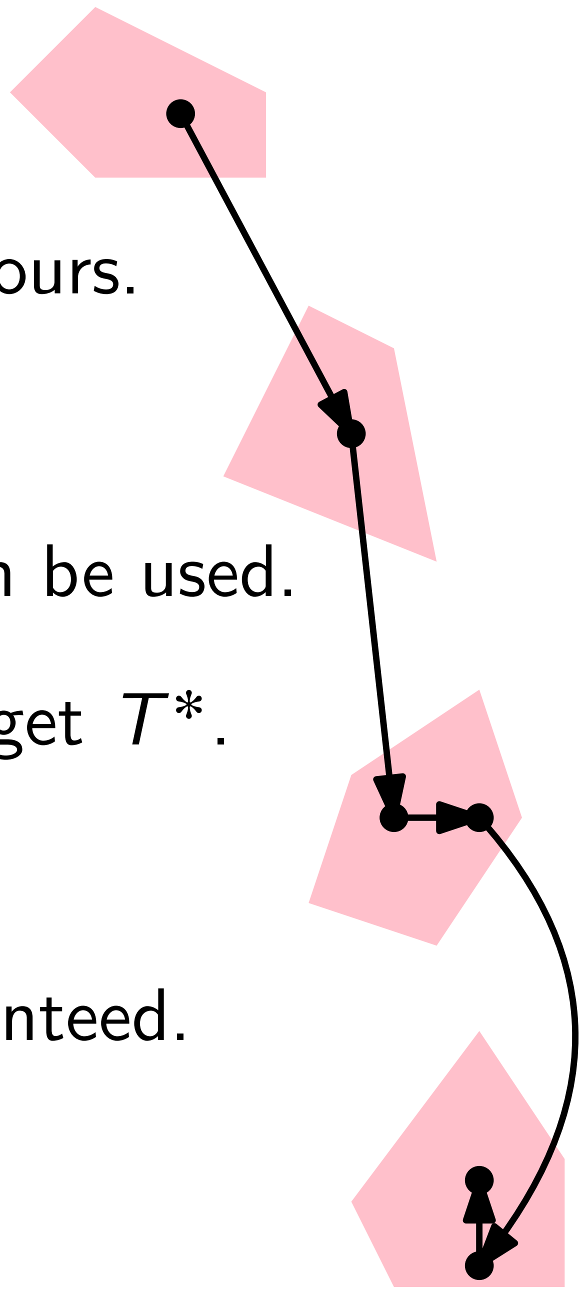

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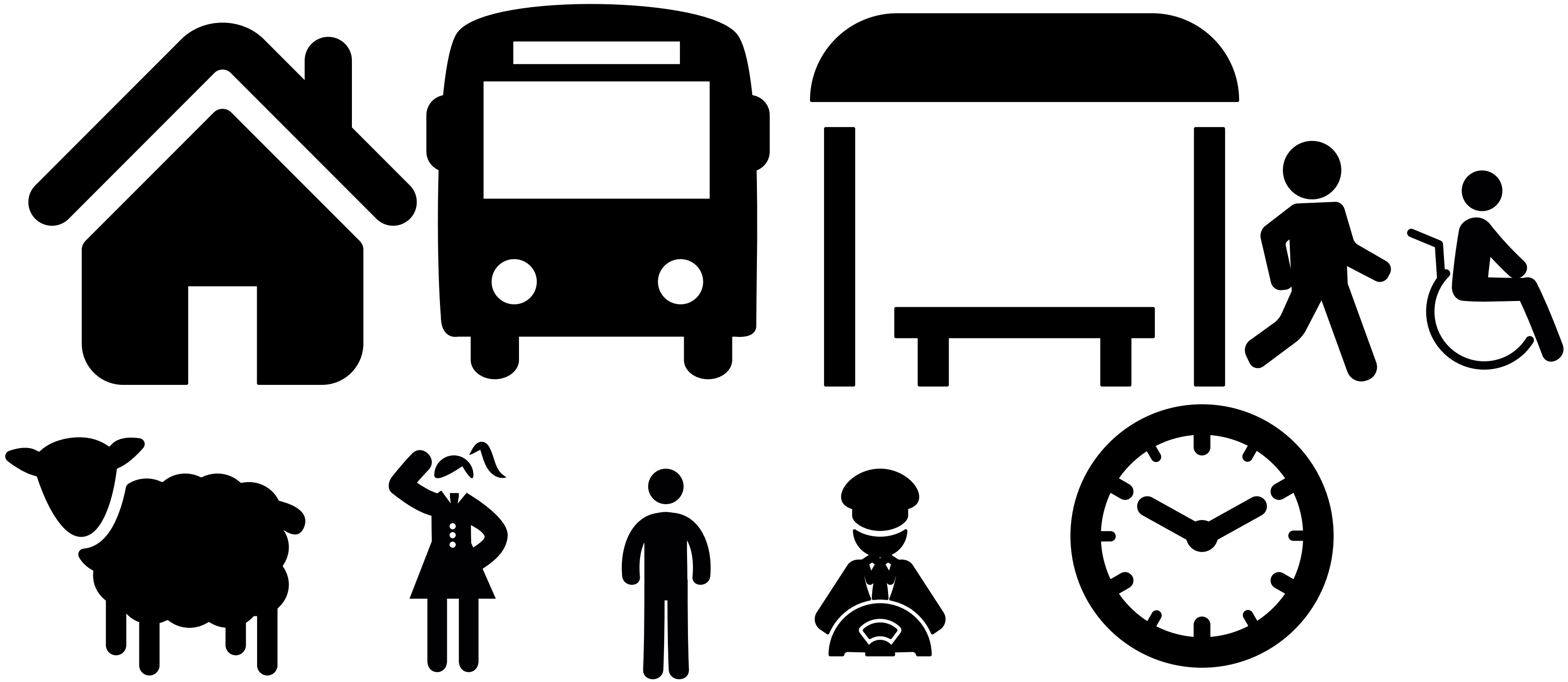
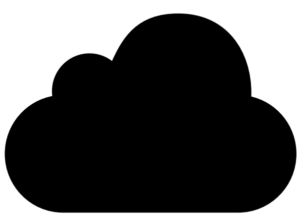
# Attributions



The above icons are made by Freepik from flaticon.com

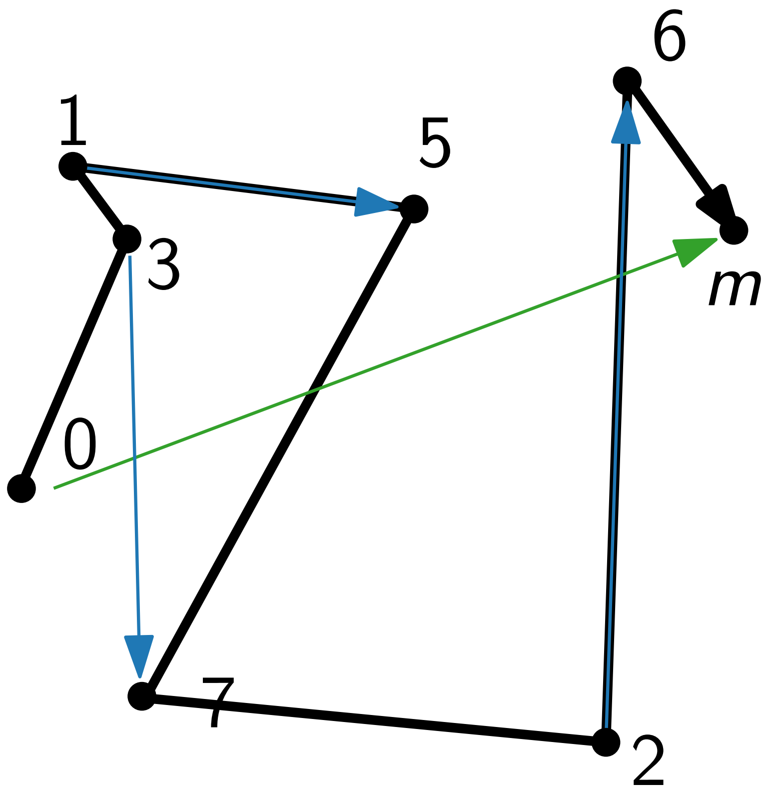


← CC 3.0 BY by SimpleIcon from flaticon.com

(c) Map Images from OpenStreetMap (osm.org)

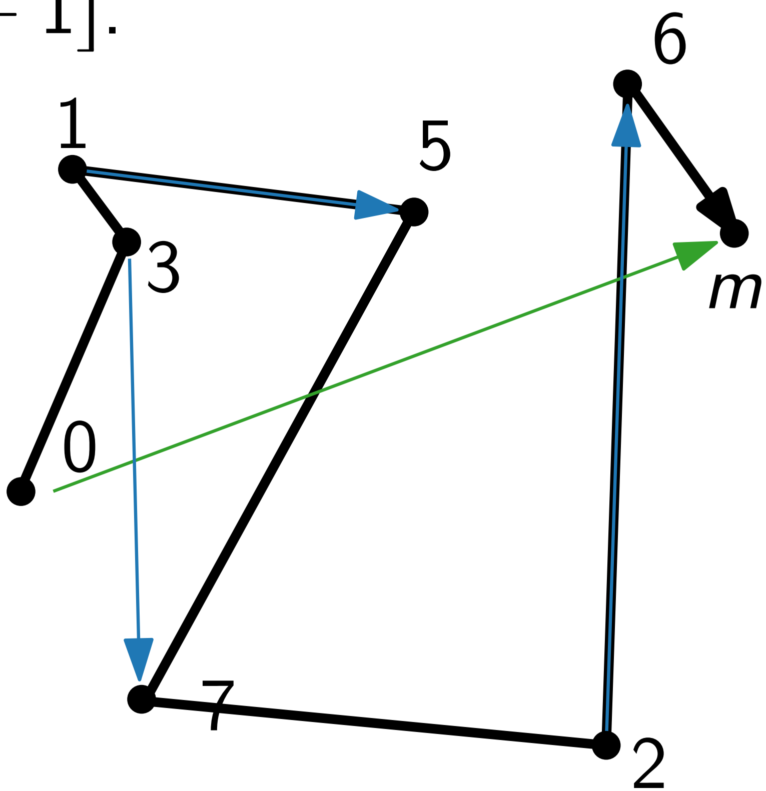
The following slides were abandoned at some point and not officially shown at the presentation. They may contain errors or are incomplete. Maybe they help you nonetheless.

# The Objective Function



# The Objective Function

A *tour* $T$ is a permutation of $[0, 2m - 1]$.



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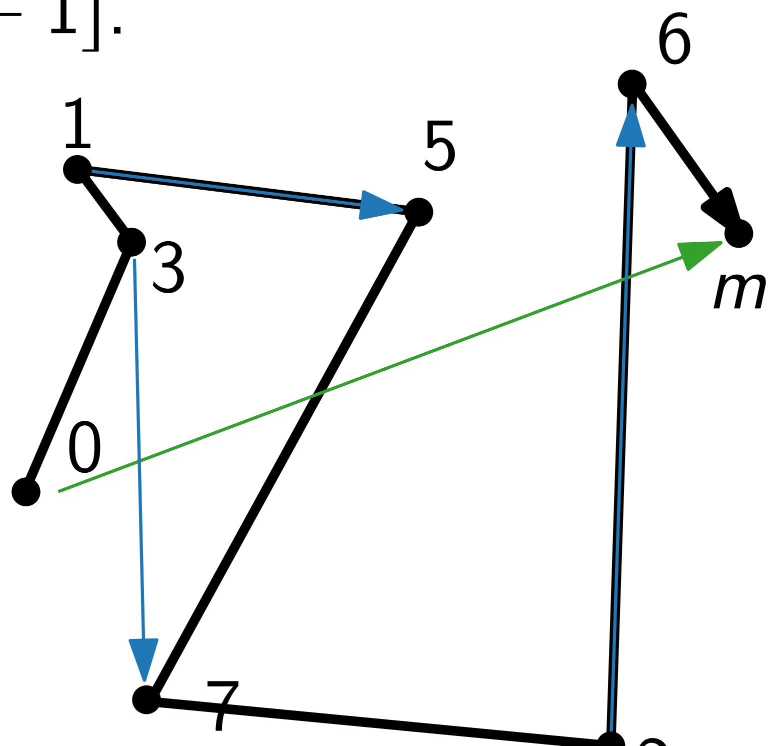


$T = [0, 3, 1, 5, 7, 2, 6, 4]$

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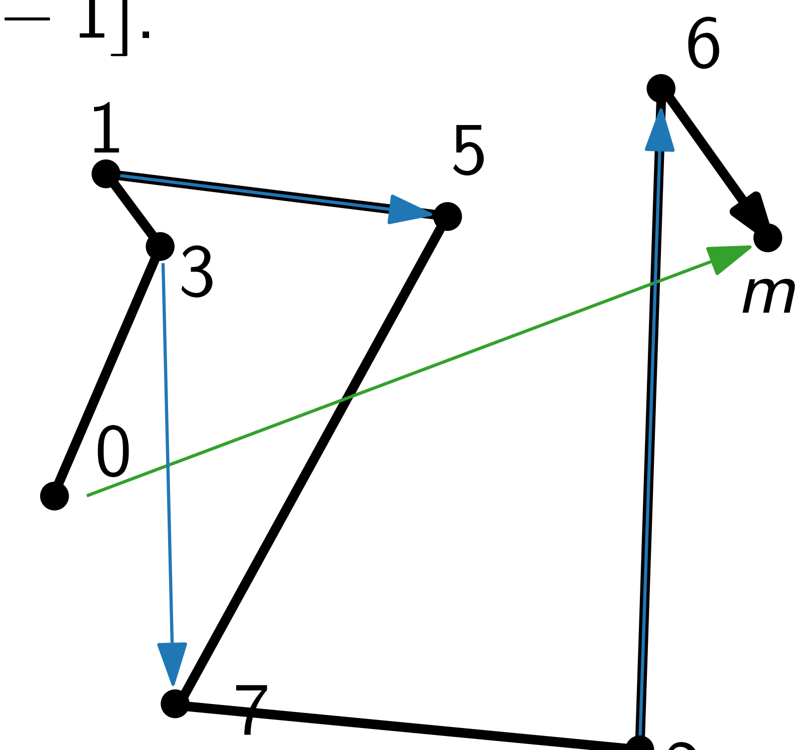


$T$ feasible $\Leftrightarrow$

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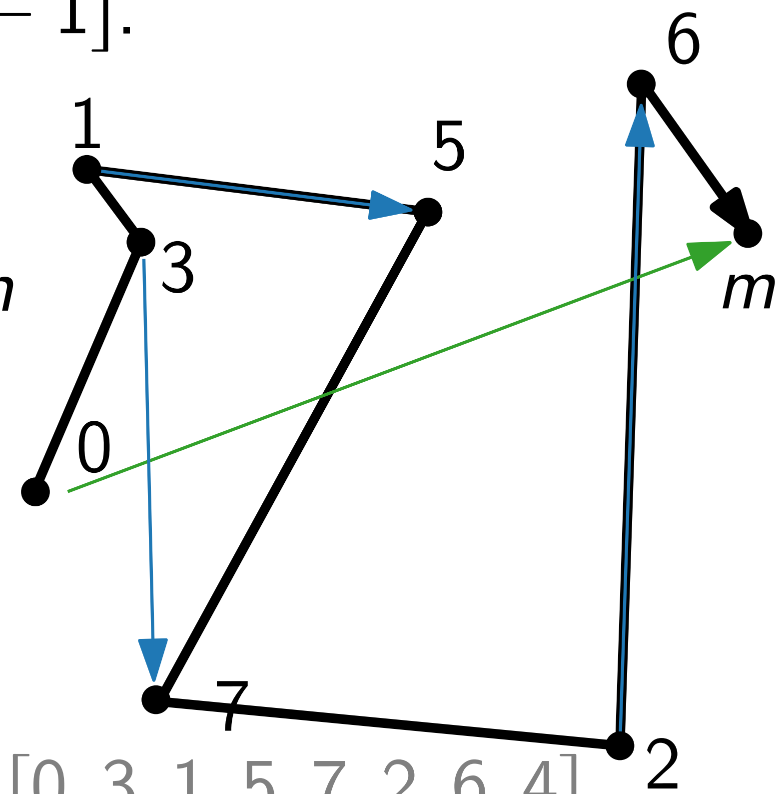
$T$ feasible $\Leftrightarrow T[1] = 0$ & $T[2m] = m$

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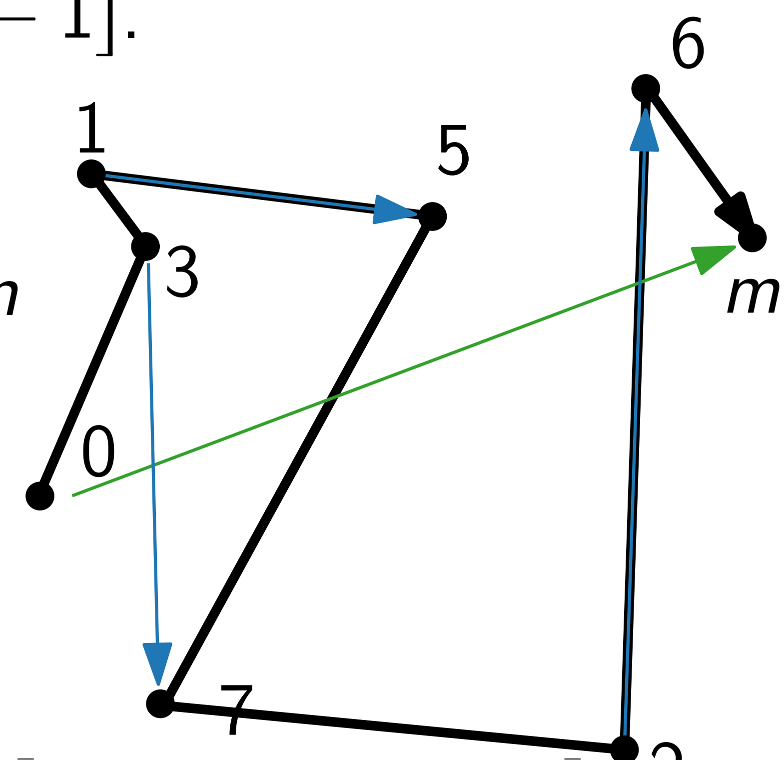


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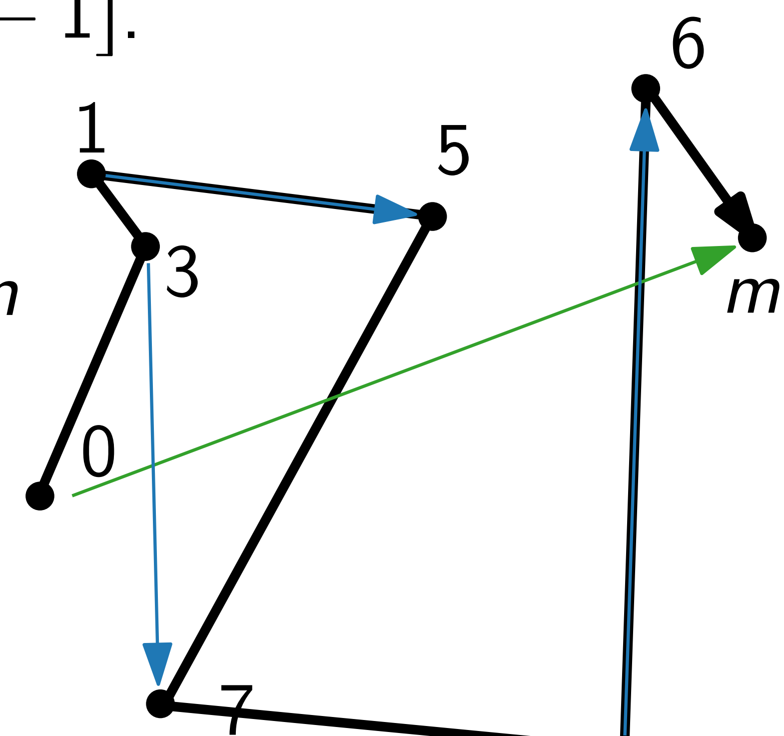


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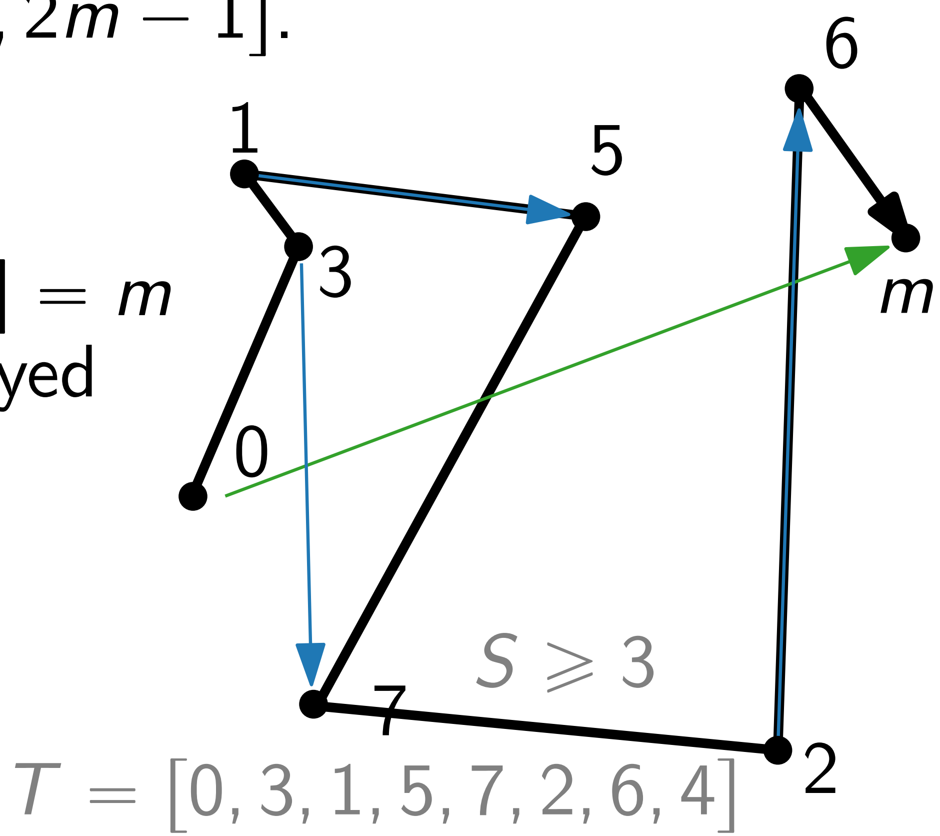

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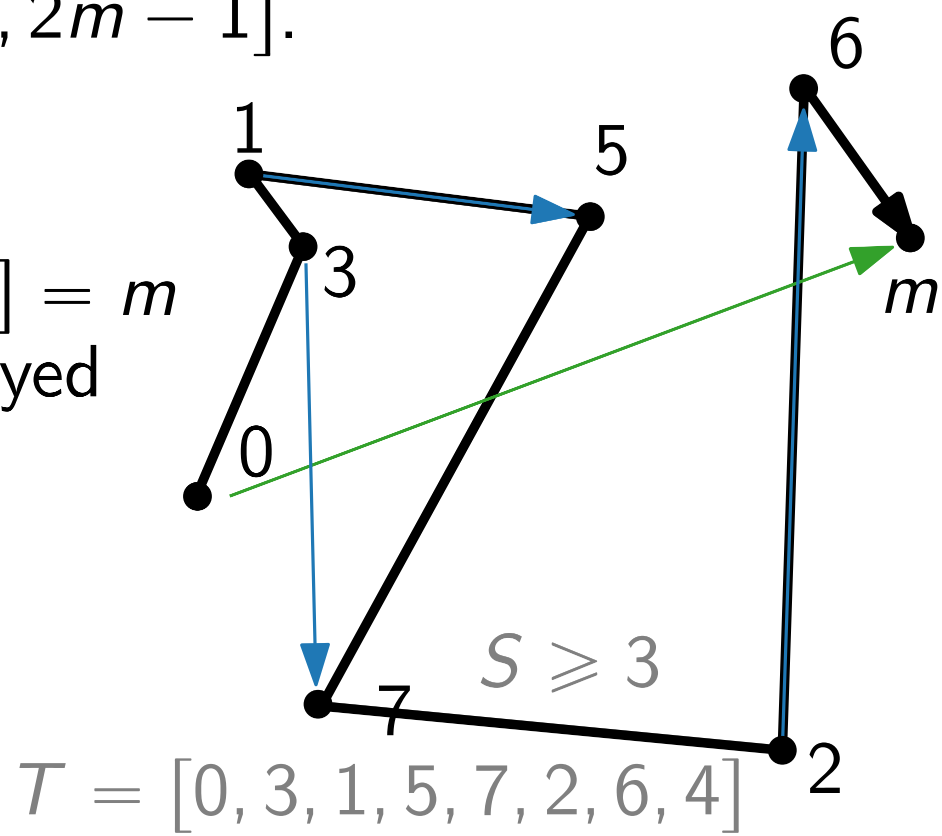


Objective:

$$\min_{T \text{ feasible}} \sum_{i=2}^{2m} k(i-1) \cdot d\Big[T[i-1], T[i]\Big]$$

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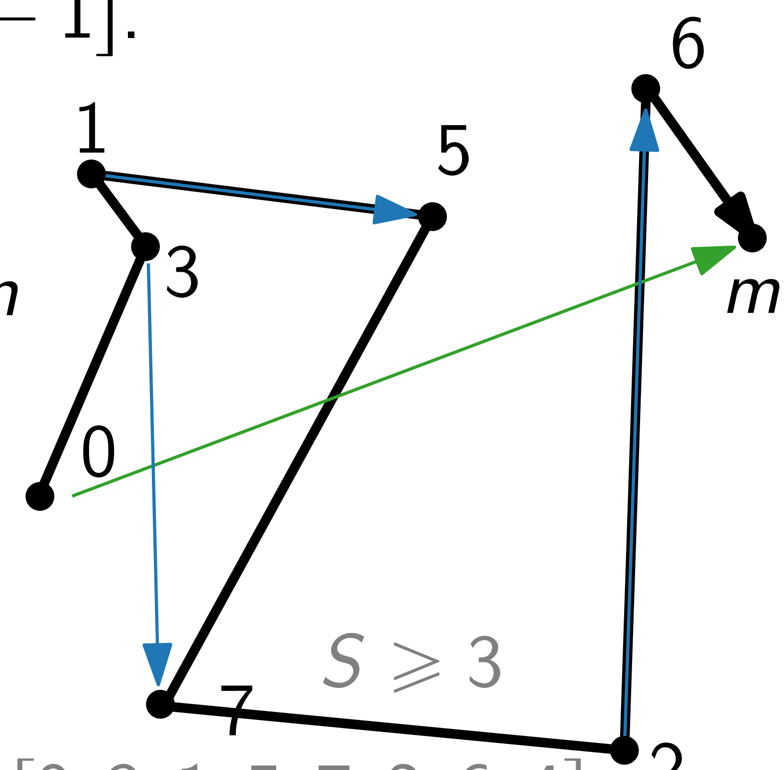


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$k(j)$ is the number of persons after step $j$ of $T$.

# An Exact Algorithm

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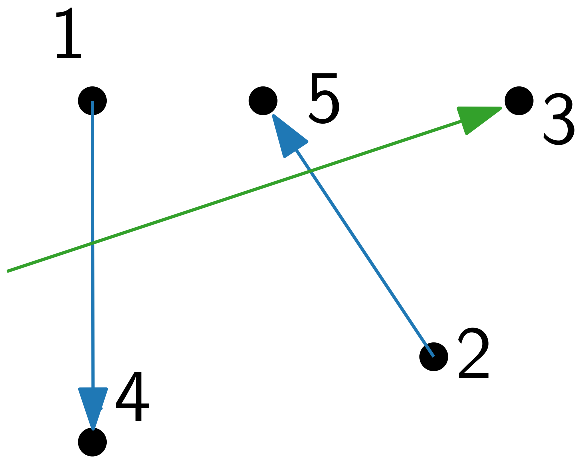
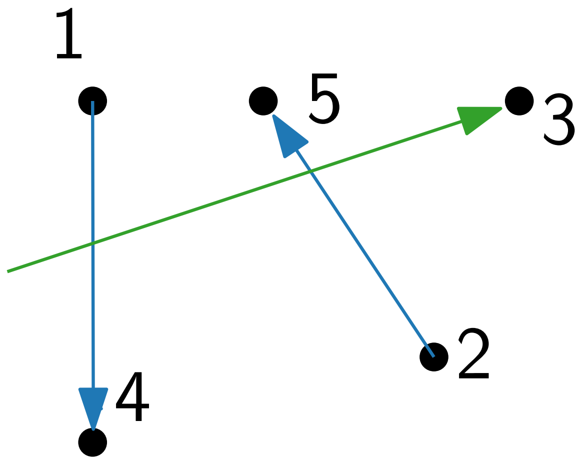


# An Exact Algorithm

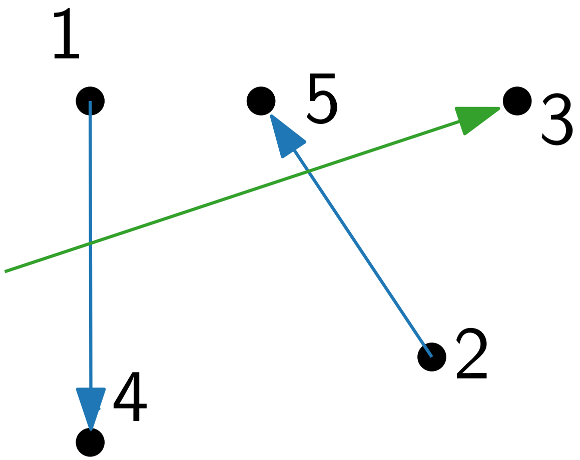
Find a tour with 6 steps:



# An Exact Algorithm

Find a tour with 6 steps:

1 2 3 4 5



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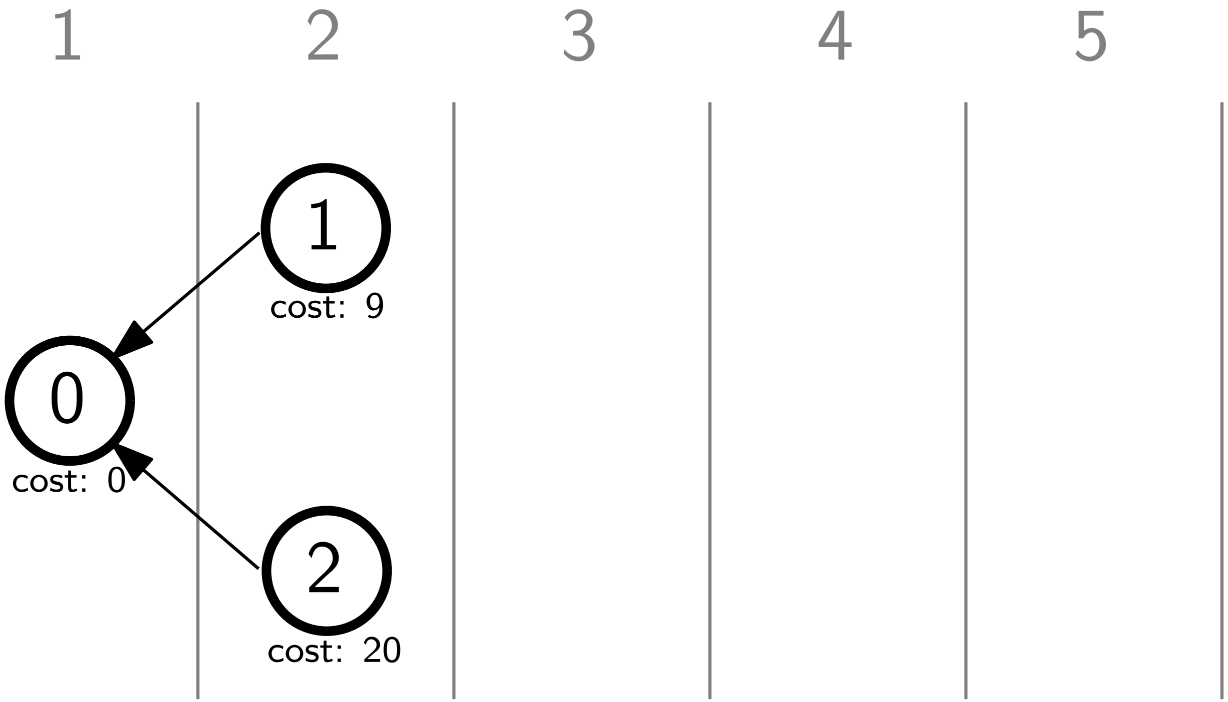


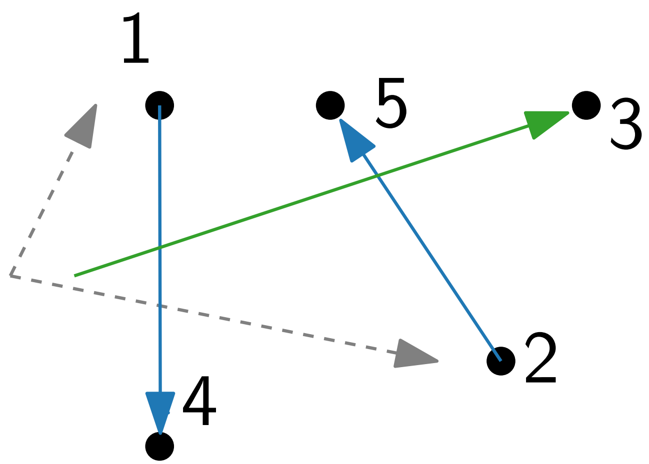

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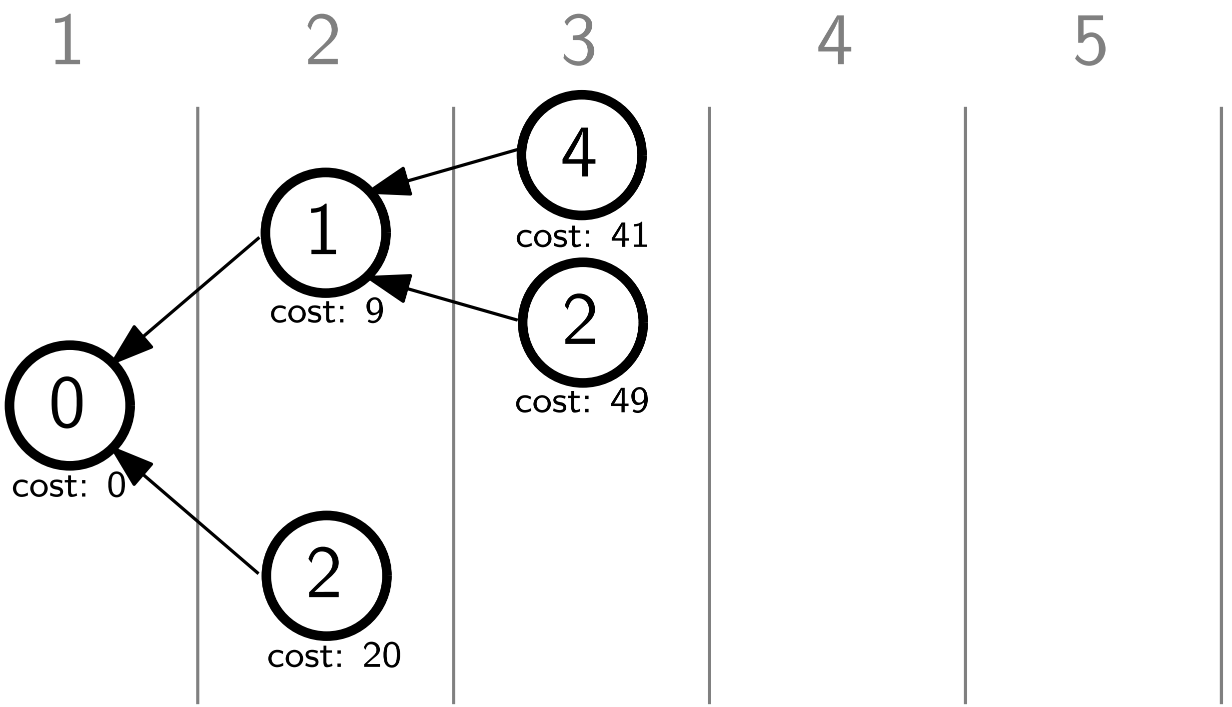


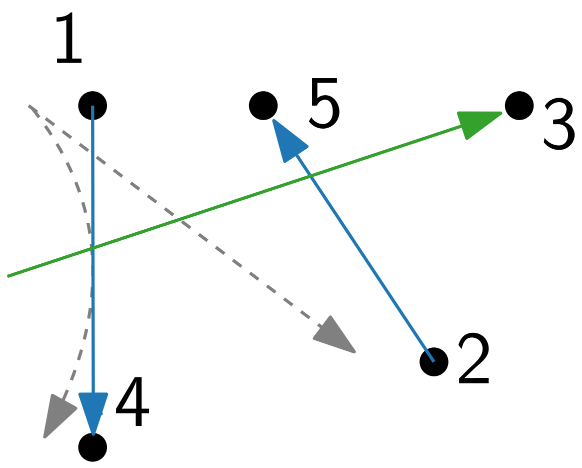

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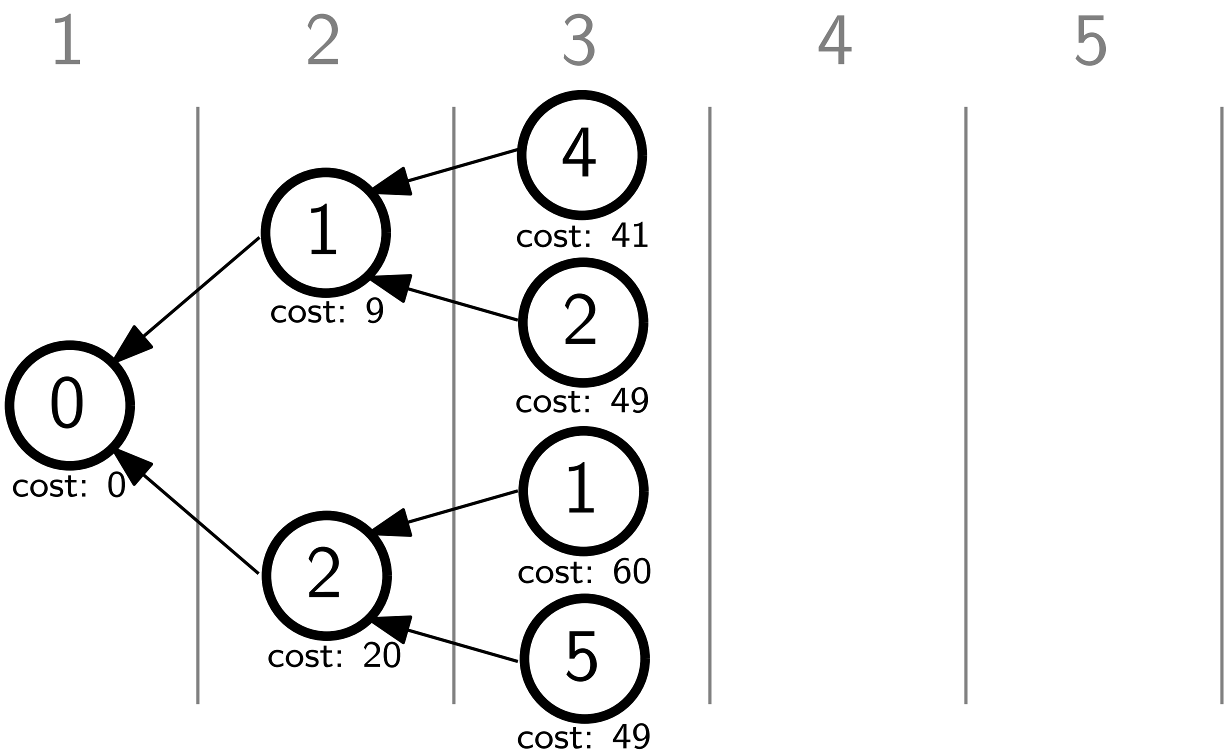


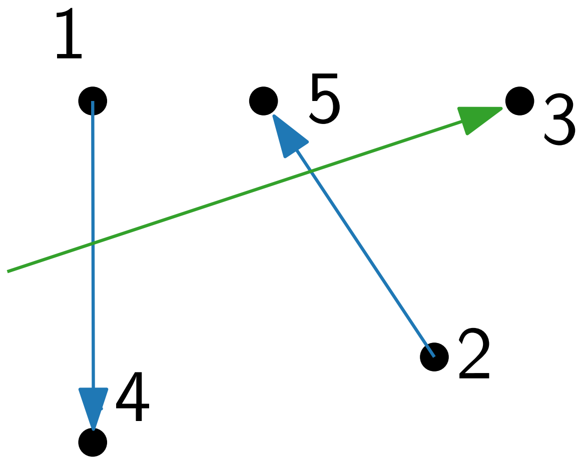

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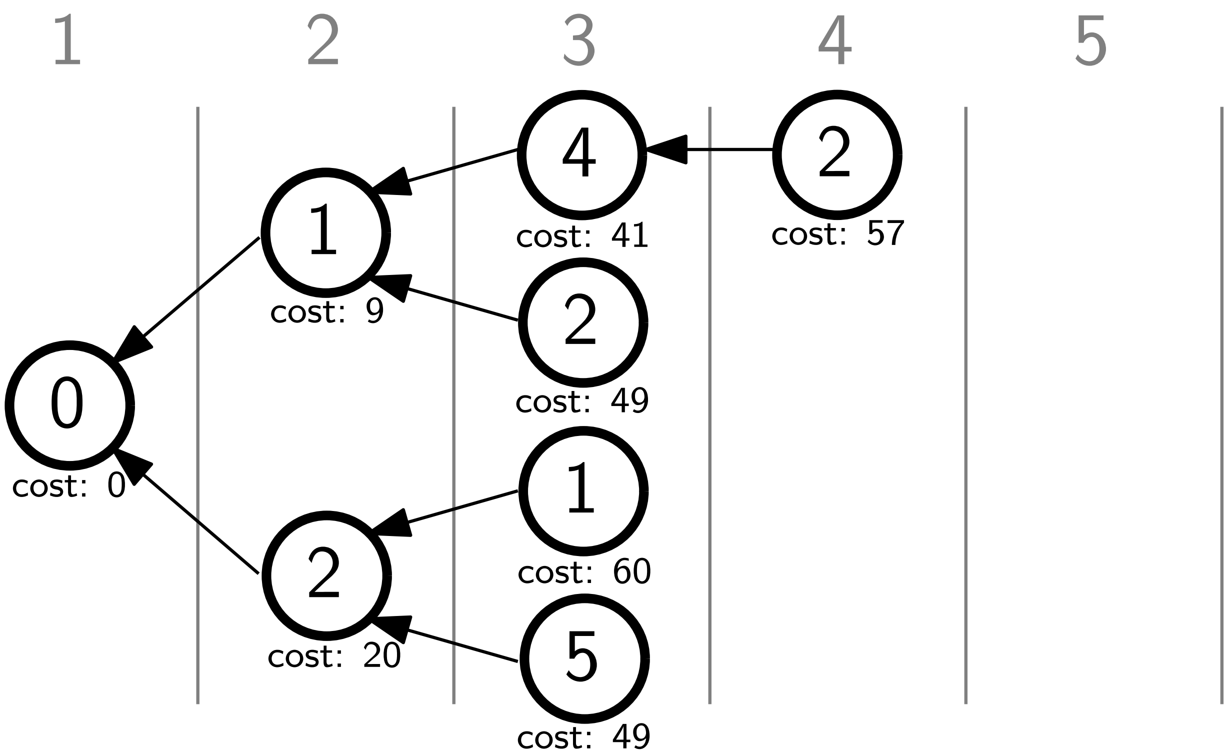


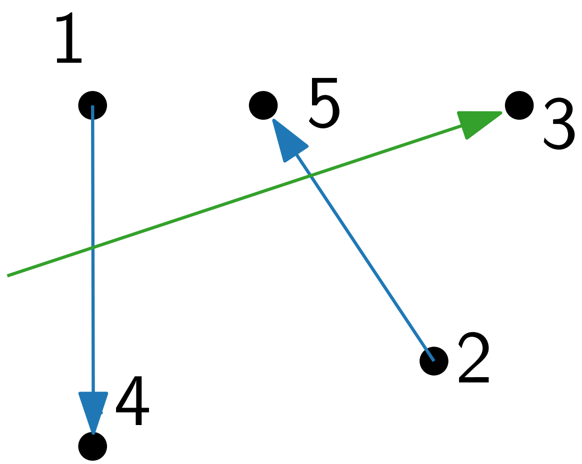

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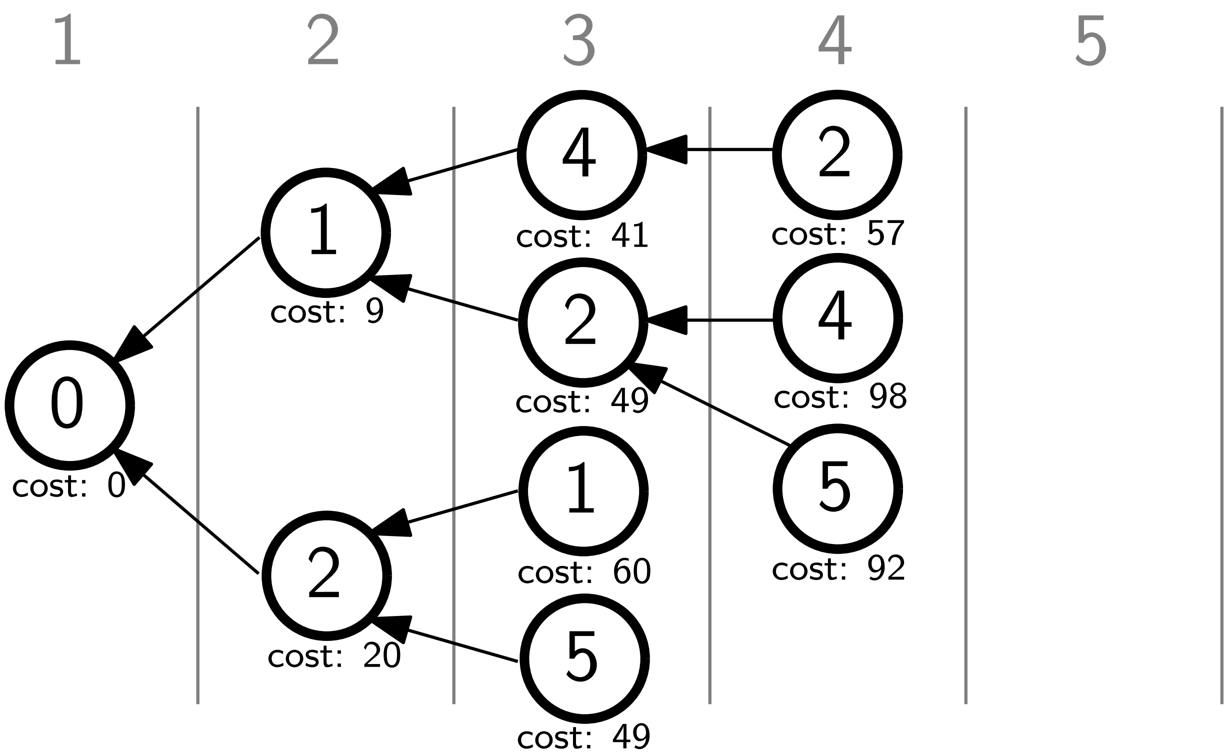


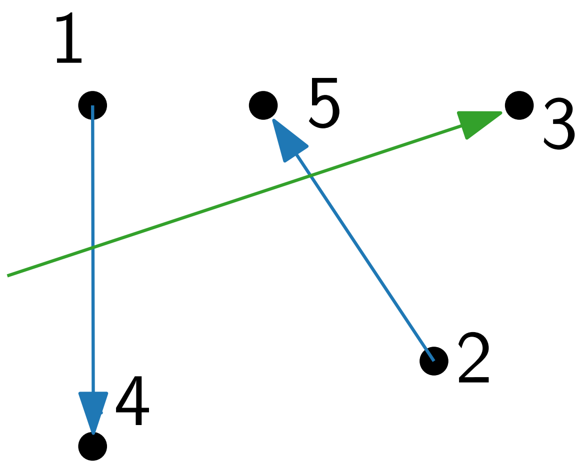

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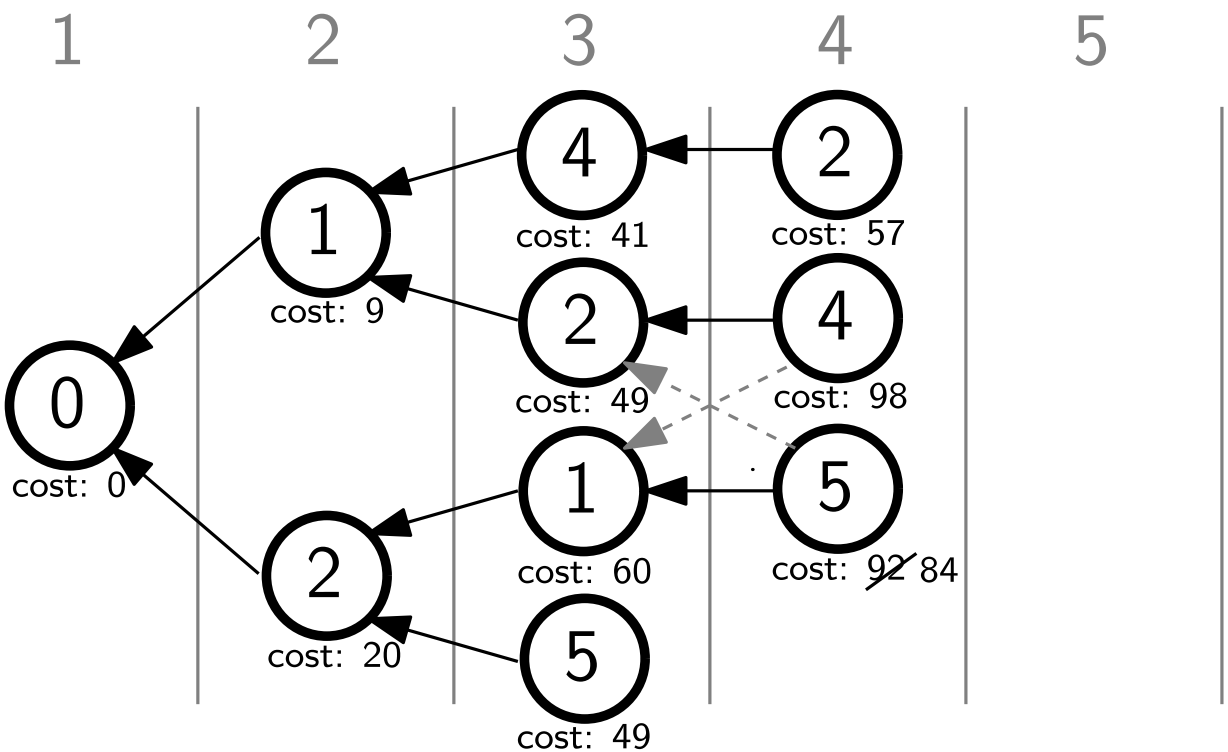


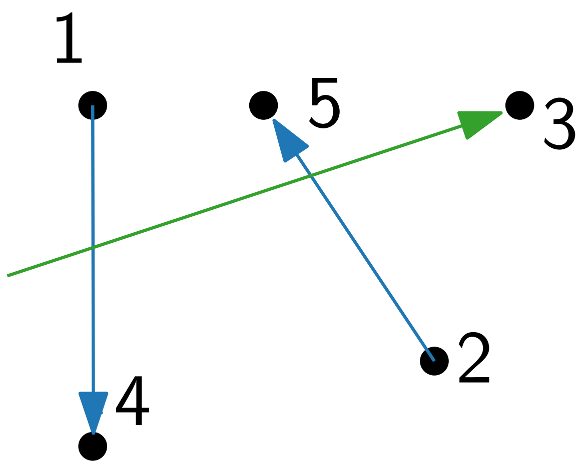

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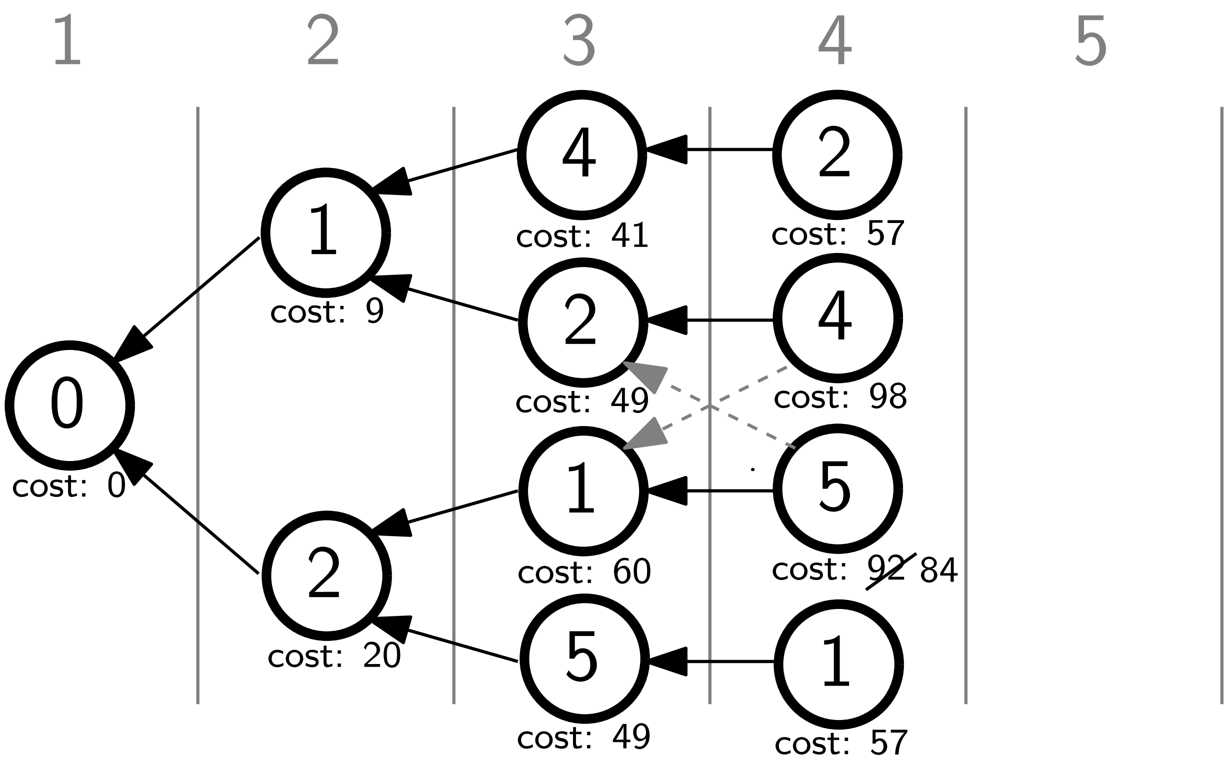


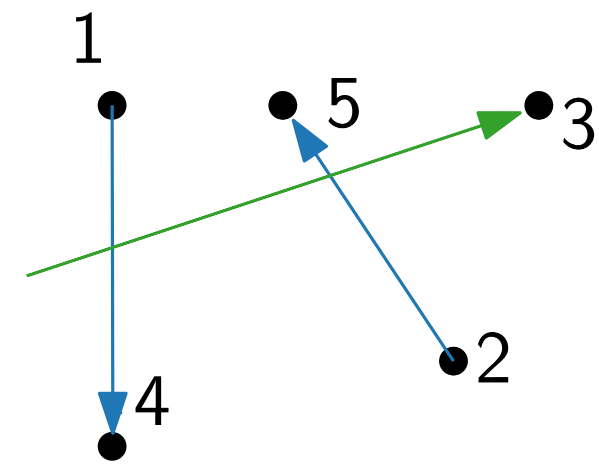

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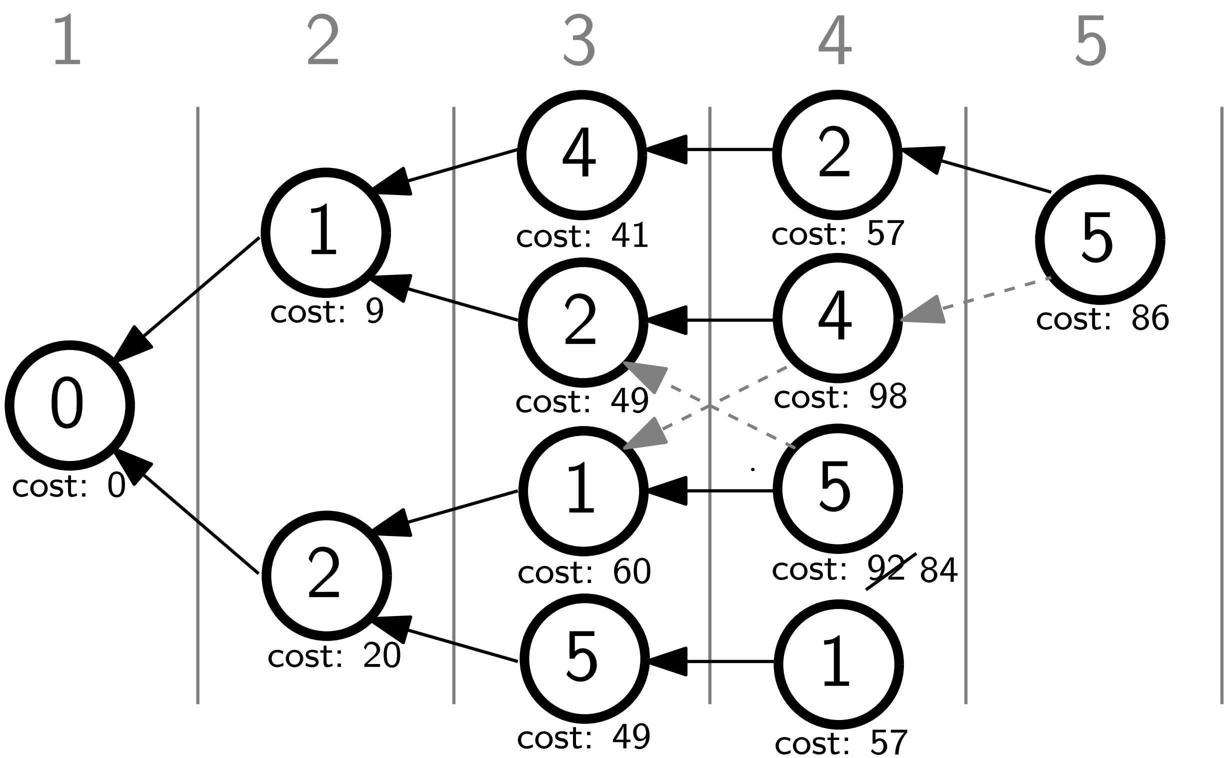


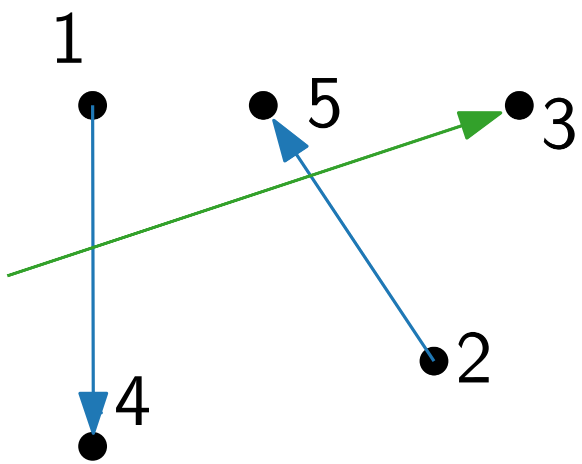

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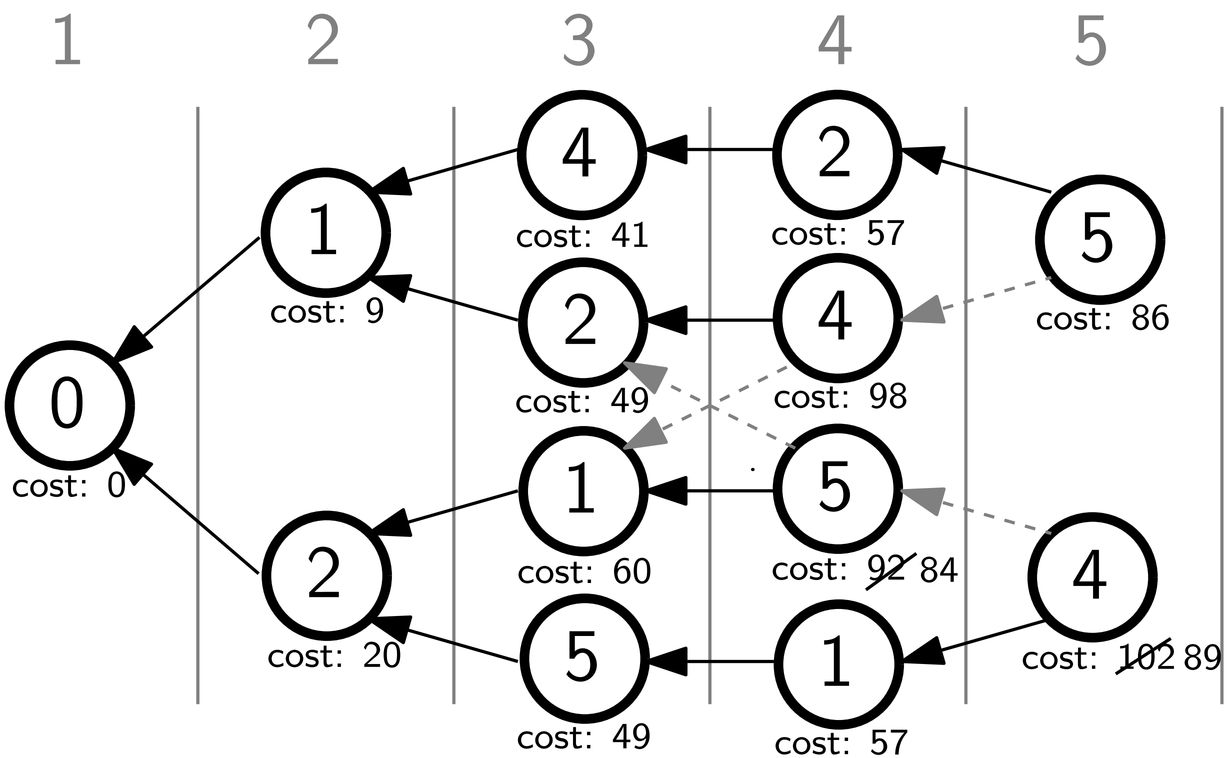


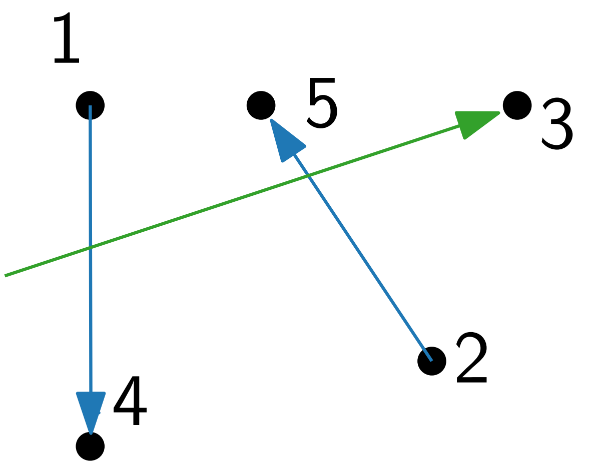

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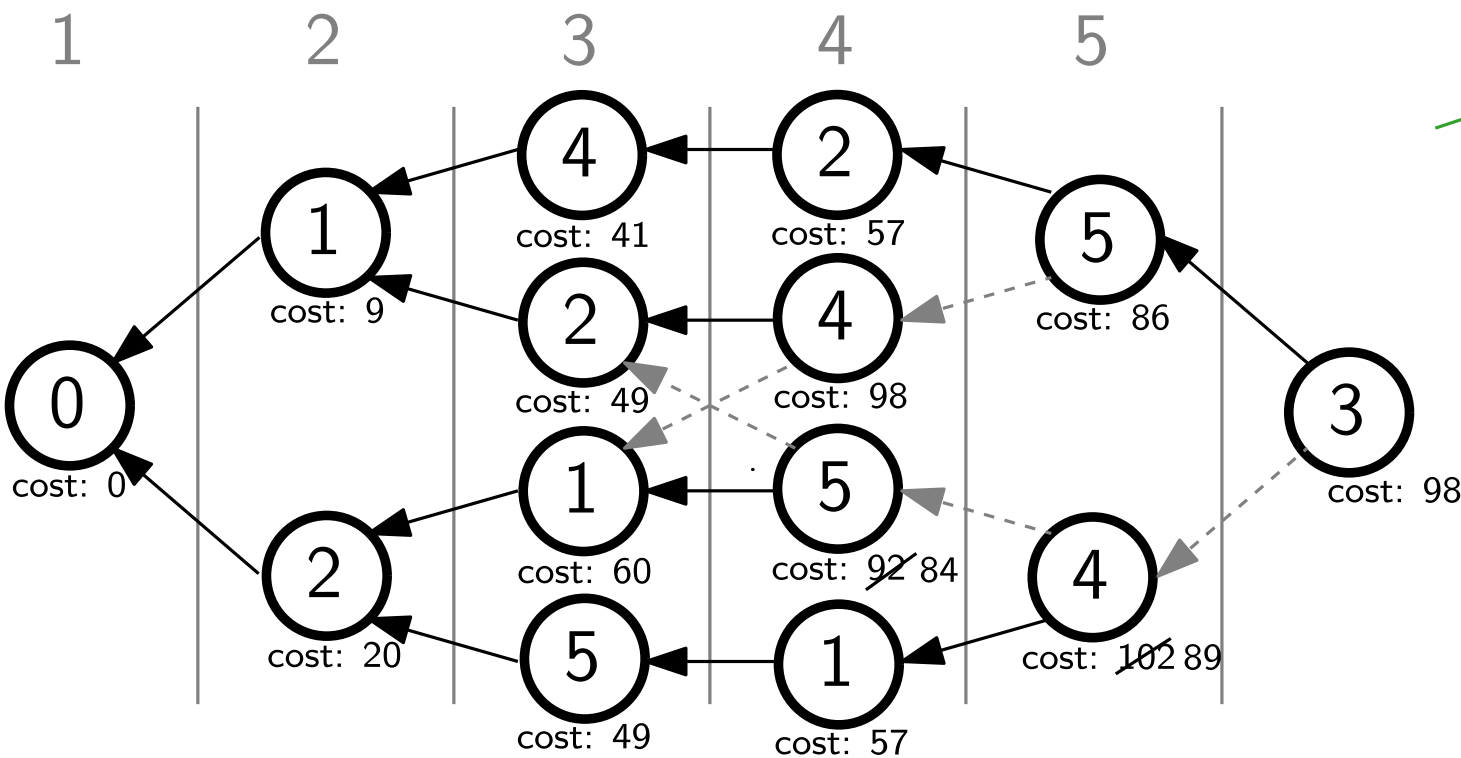


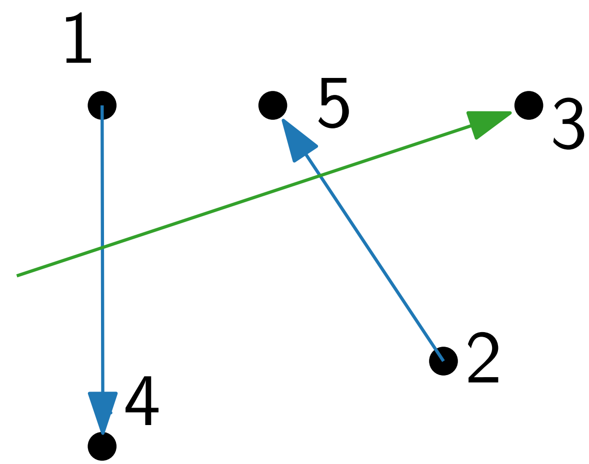

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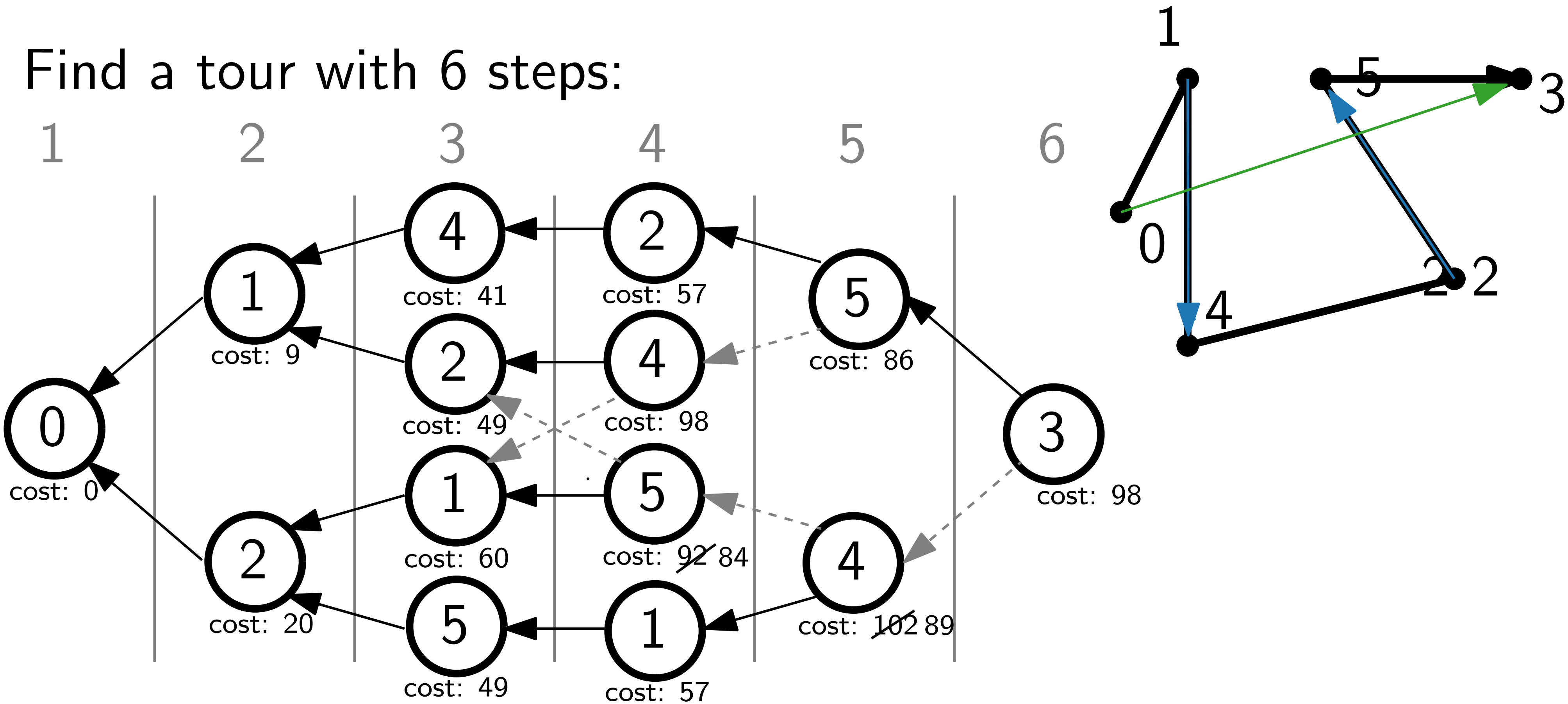

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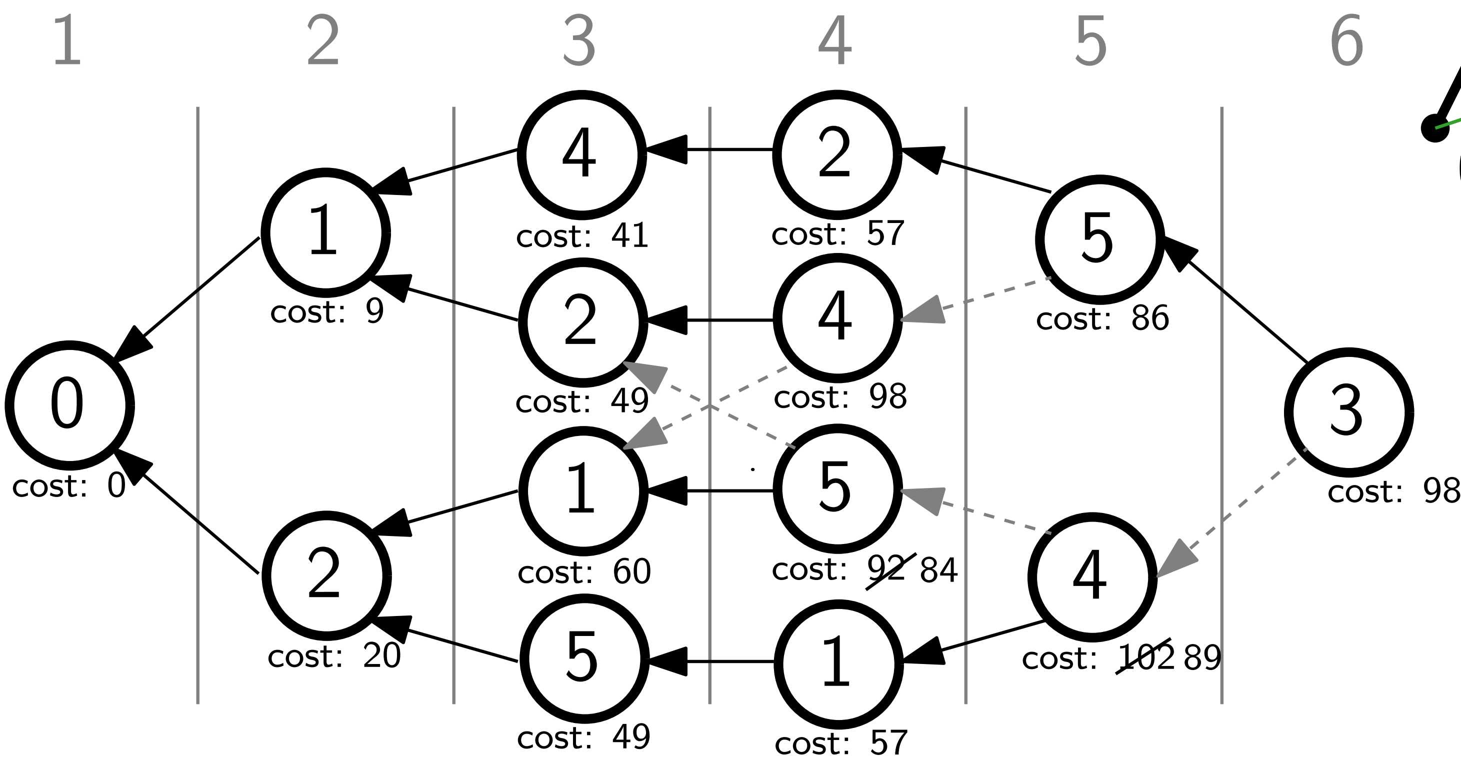


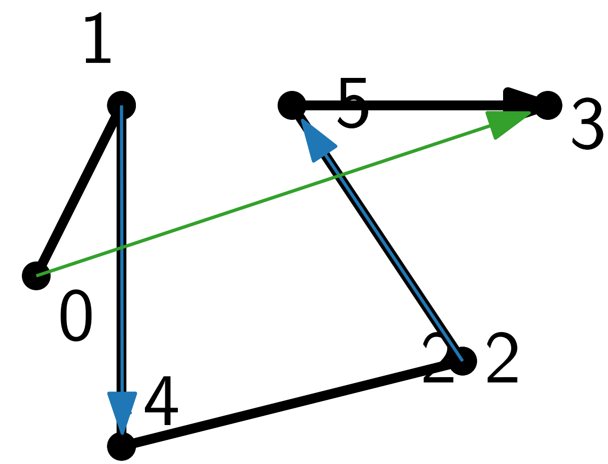


→ Generalizes to an algorithm with exchangeable objective

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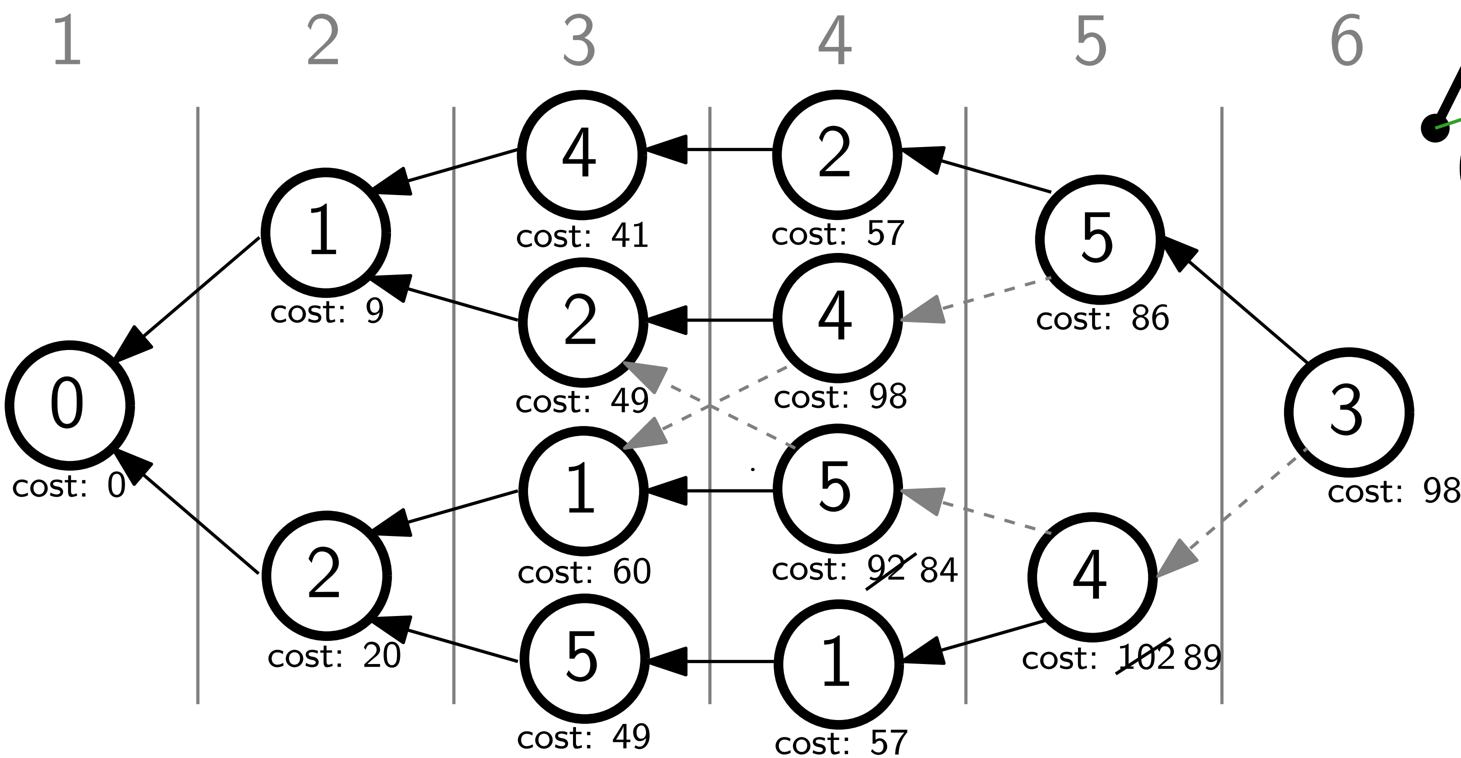


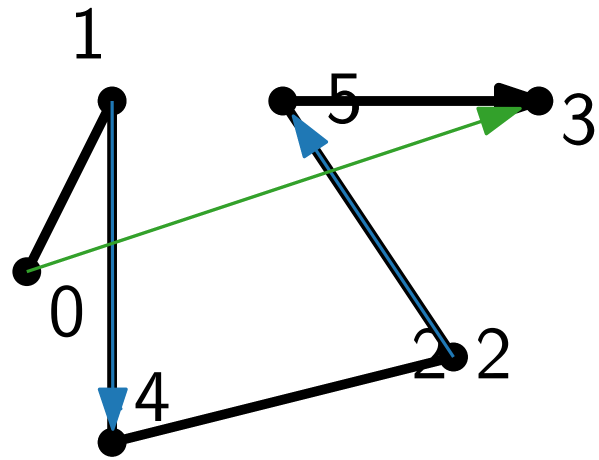


→ Generalizes to an algorithm with exchangeable objective

→ DFS-like traversal also possible [Psaraftis 1980]

# An Exact Algorithm

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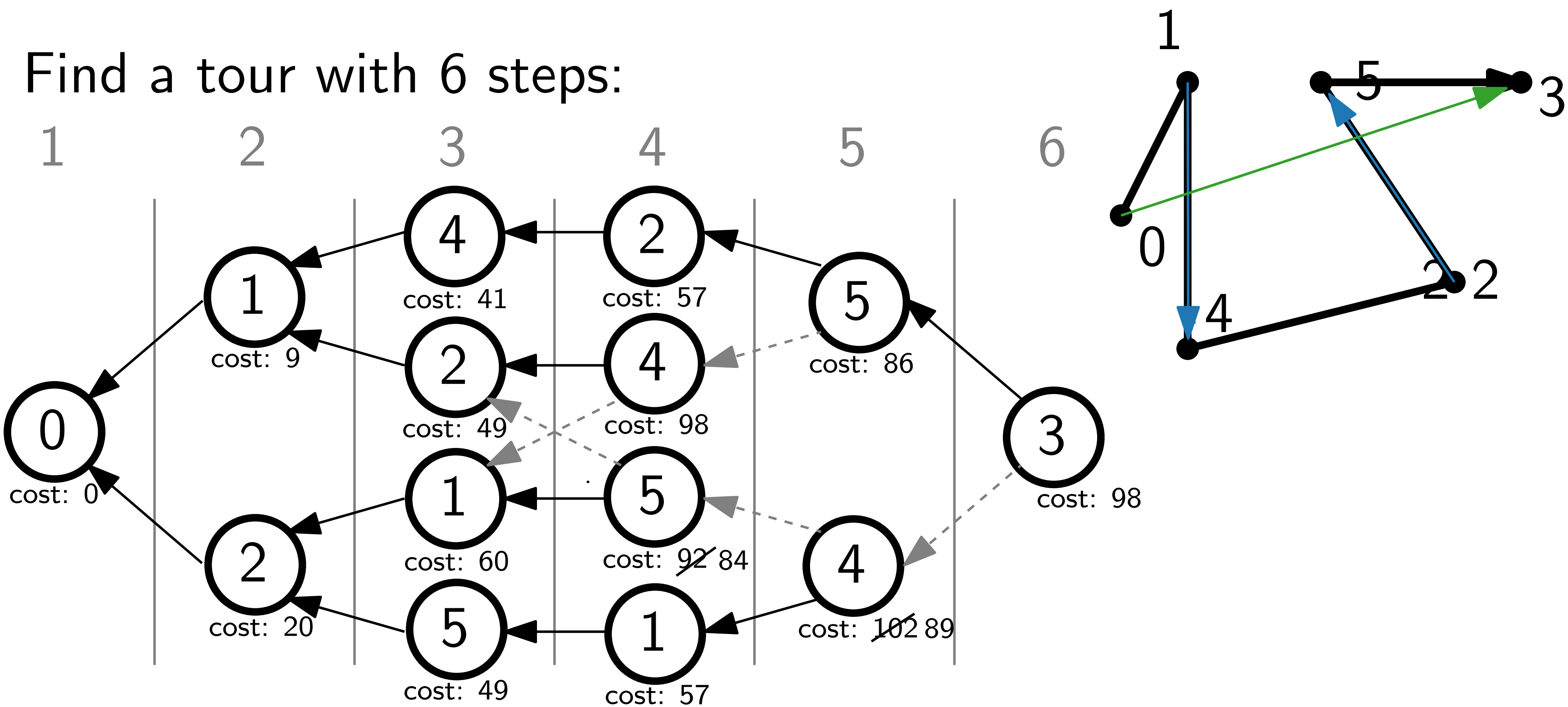
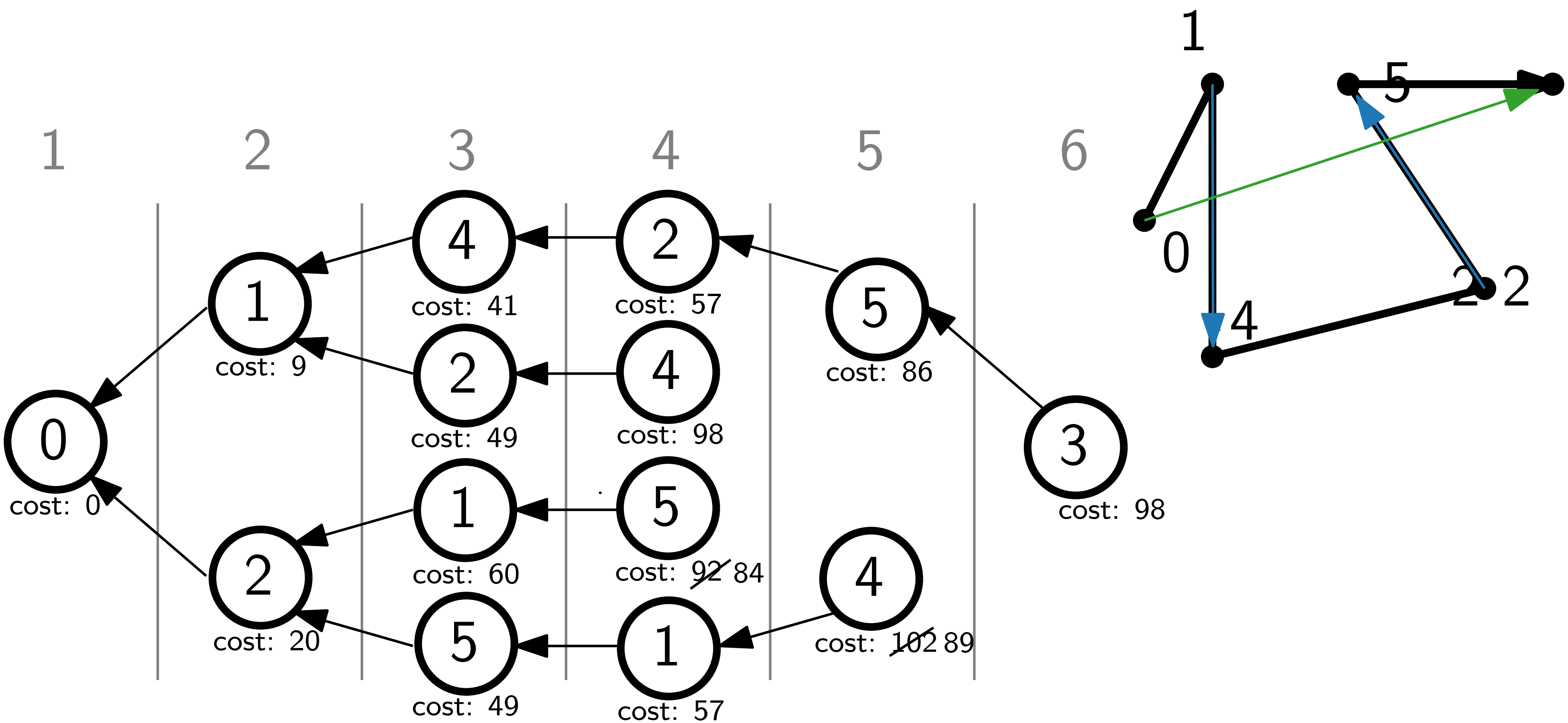



→ Generalizes to an algorithm with exchangeable objective
→ DFS-like traversal also possible [Psaraftis 1980]
→ BFS-like traversal can save storage

# Running Time and Partial Execution



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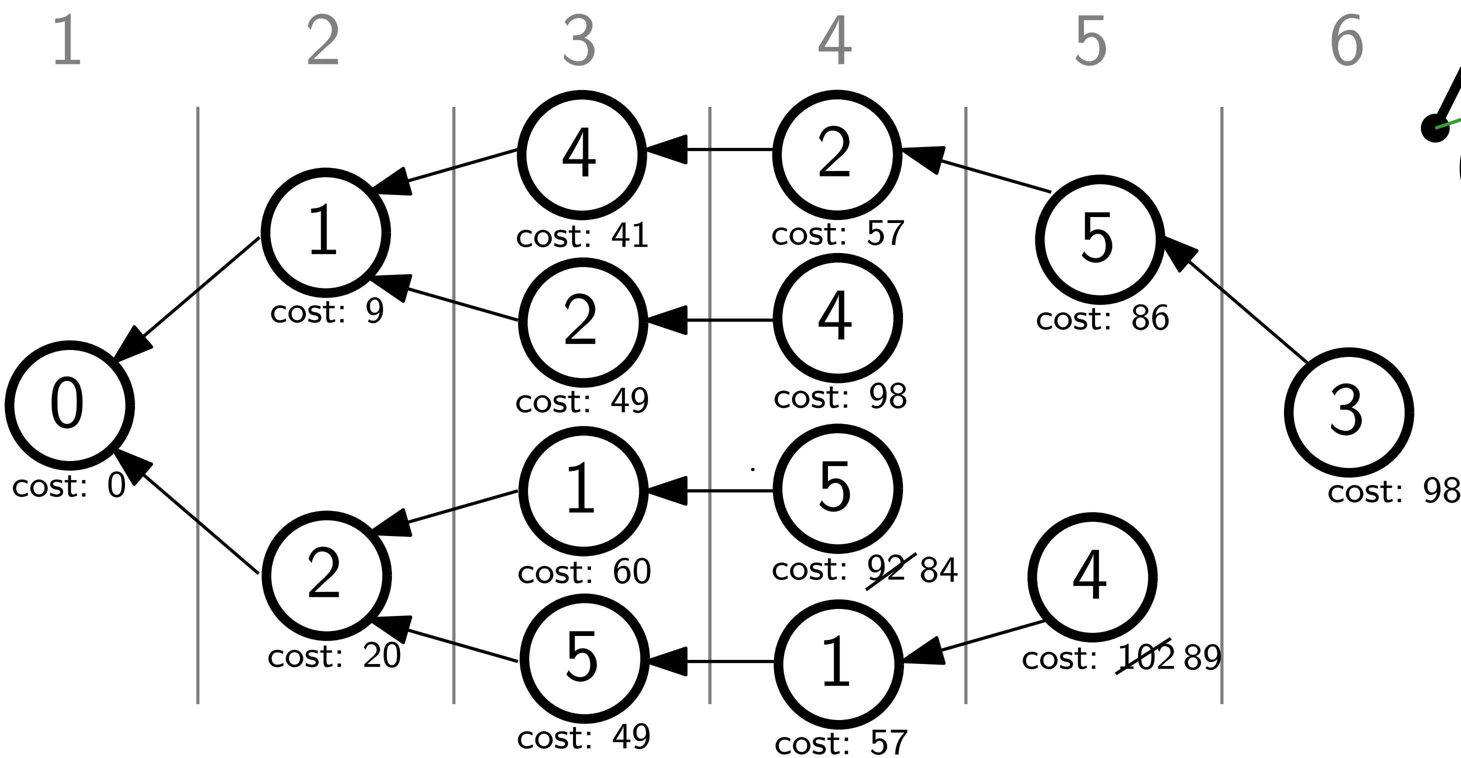


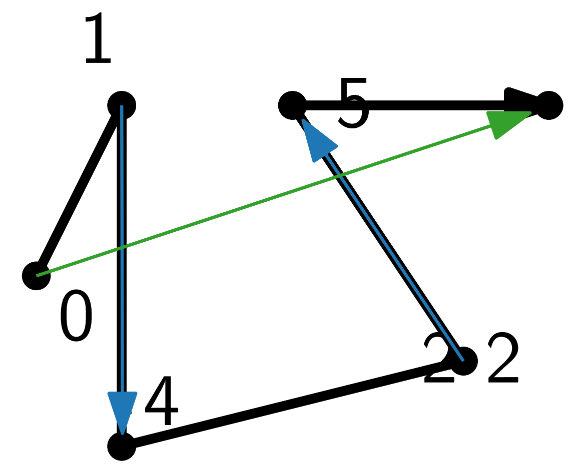


At every step, a rider can have three steps: *wait*, *travel*, *finish*.

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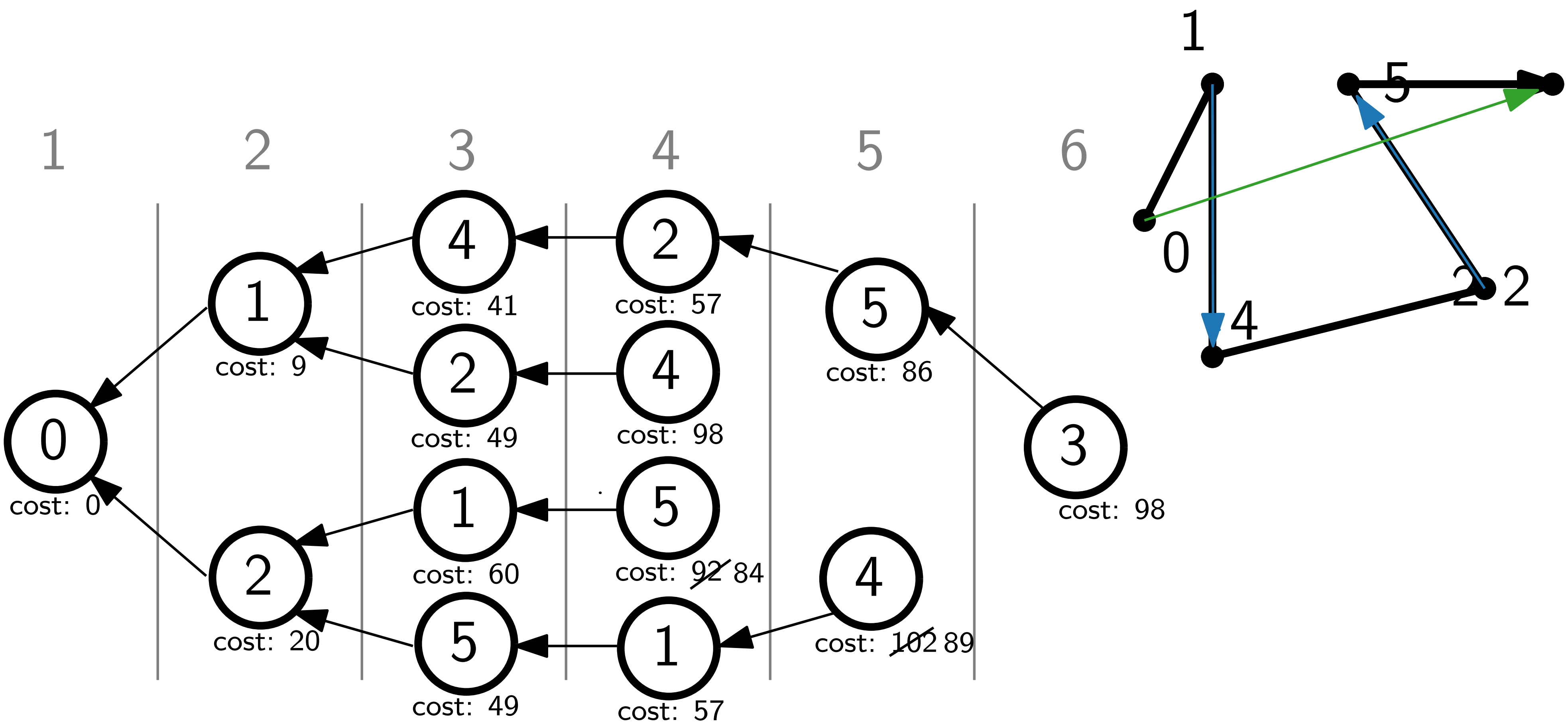


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$\Rightarrow$ for a fixed location $\notin \{0, m\}$ there are $3^{n-1}$ states.

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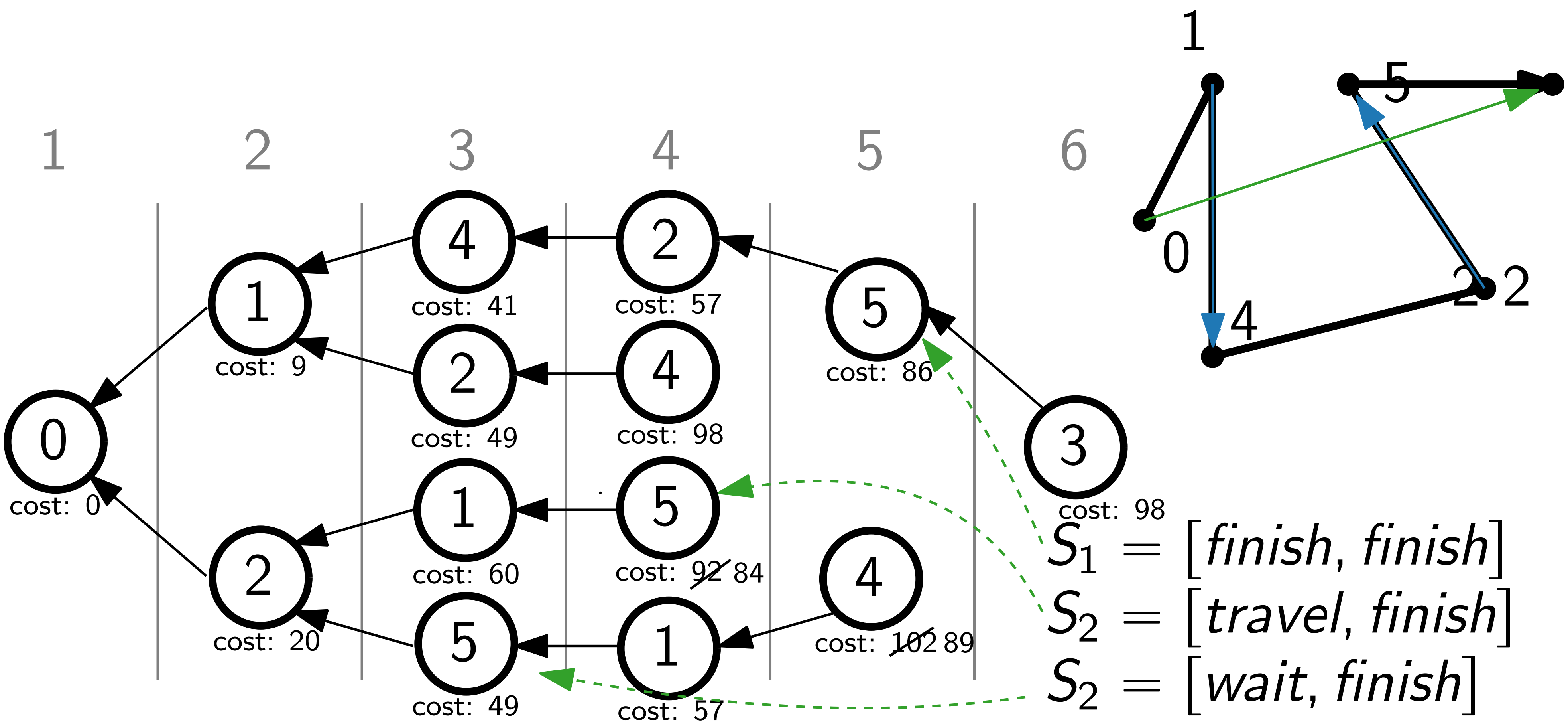


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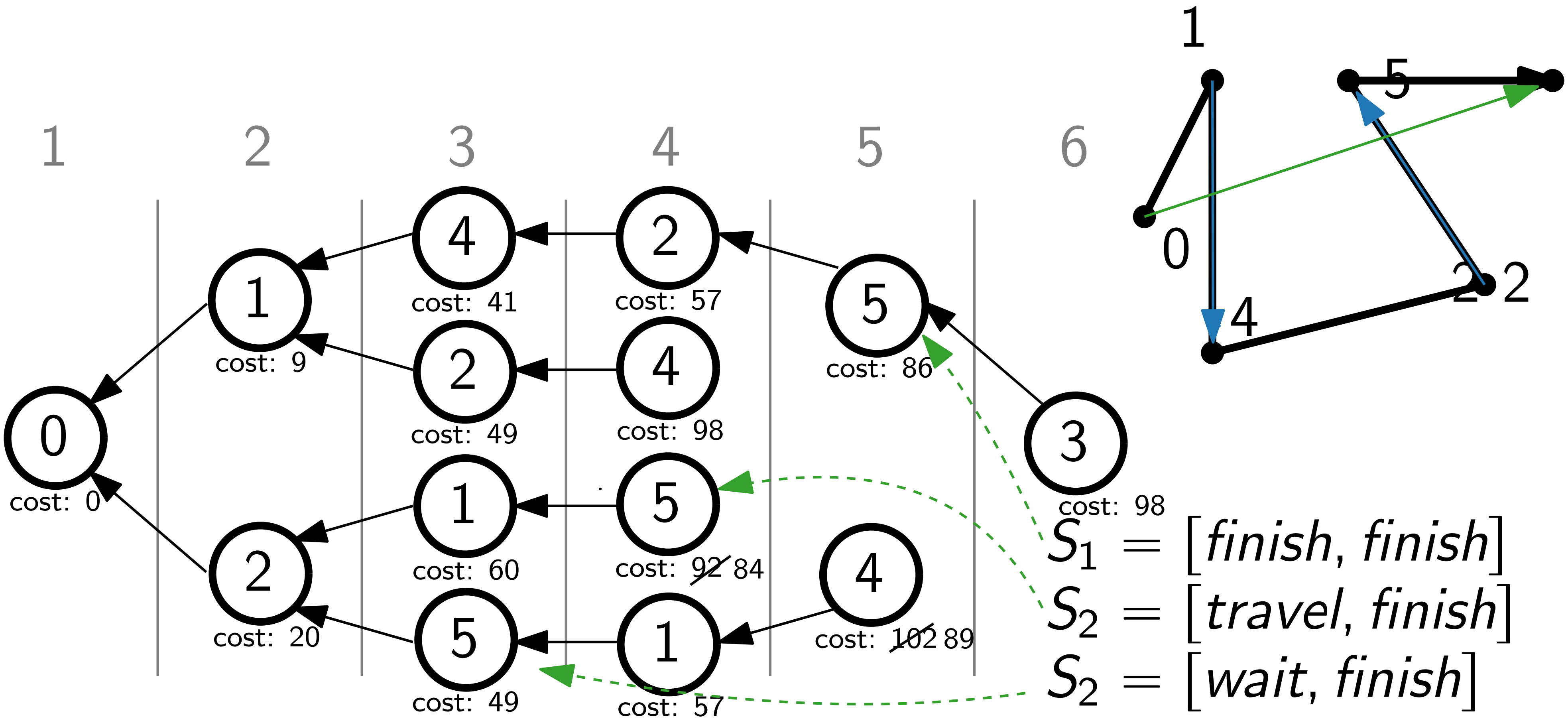


At every step, a rider can have three steps: *wait*, *travel*, *finish*.

$\Rightarrow$ for a fixed location $\notin \{0, m\}$ there are $3^{n-1}$ states.

$\Rightarrow 2n3^{n-1} + 2$ vertices, which yields $O^*(3^{n-1})$ running time.

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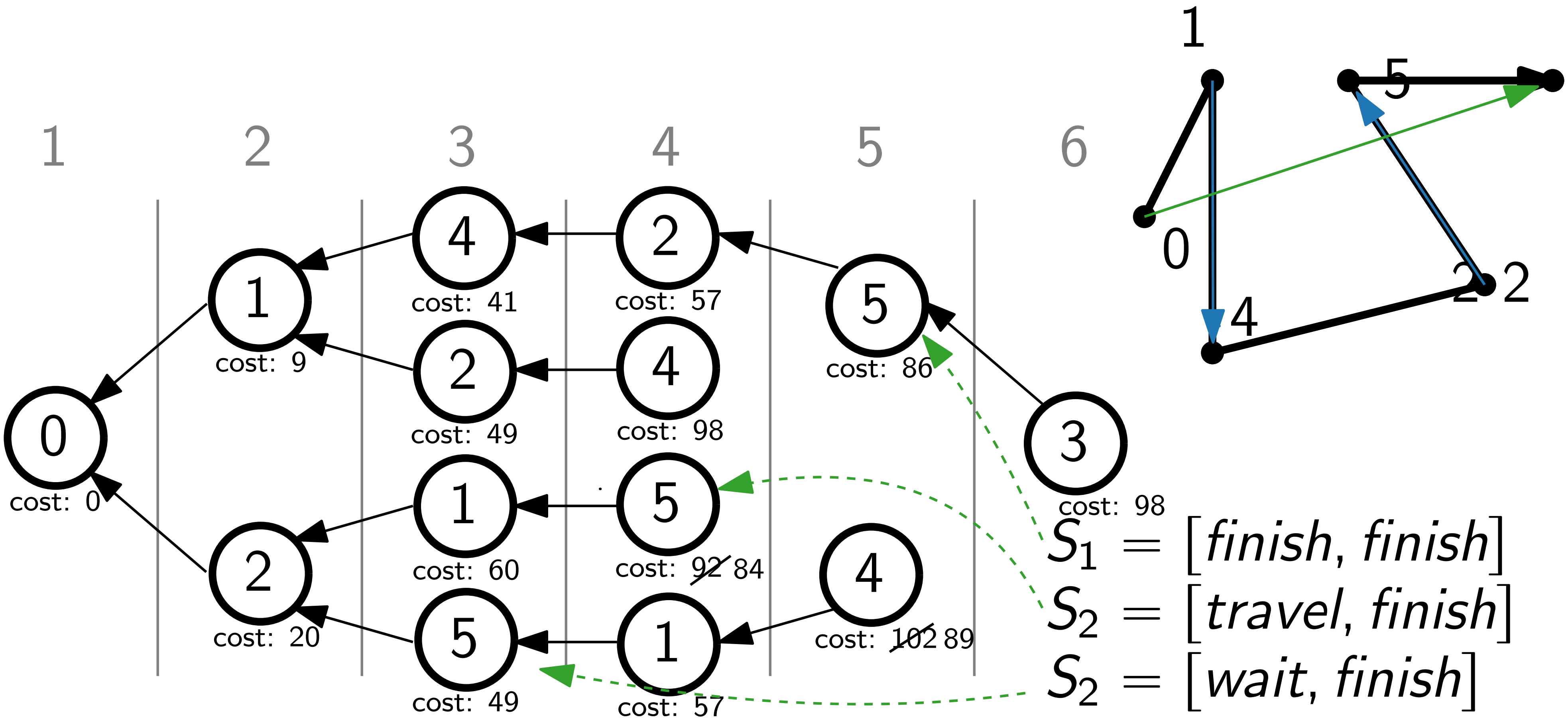


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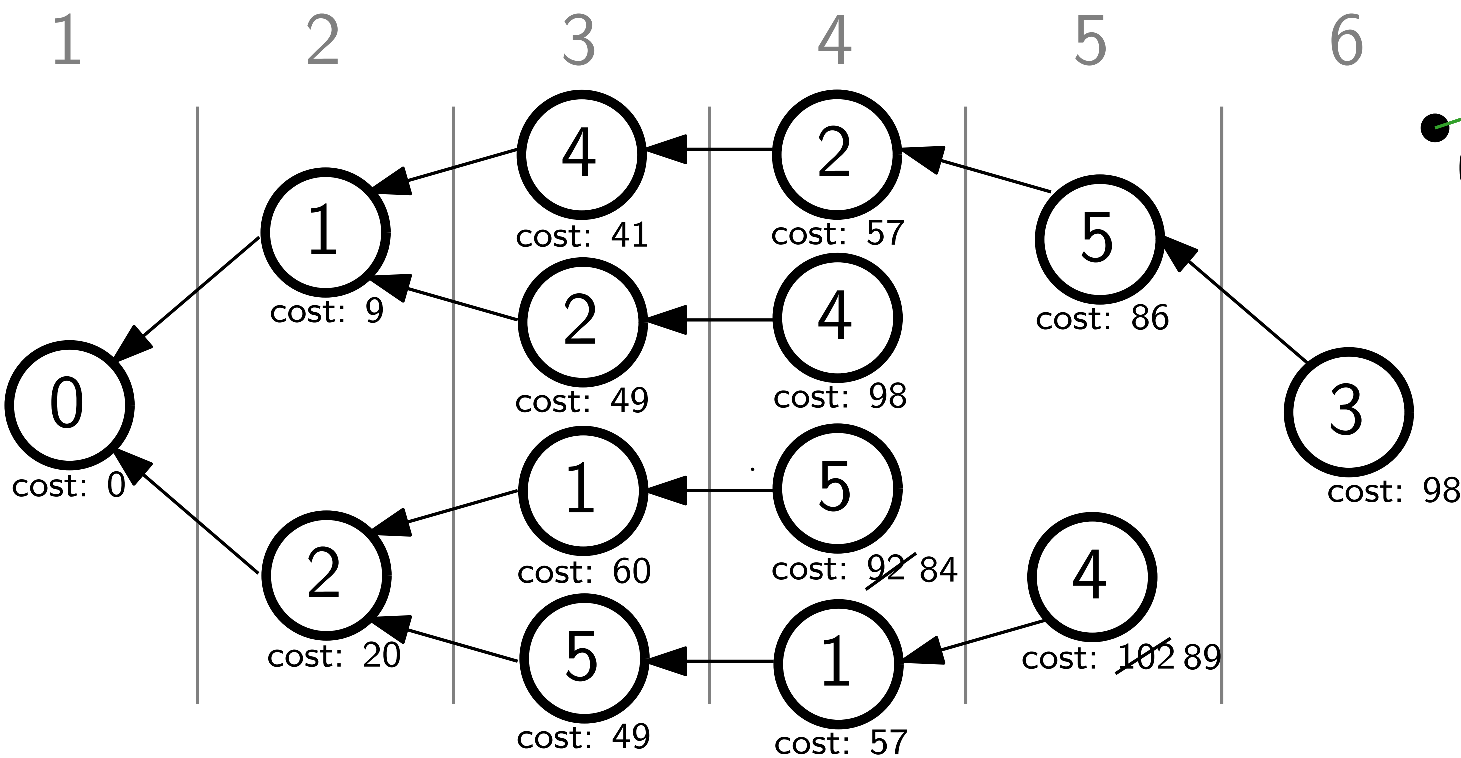


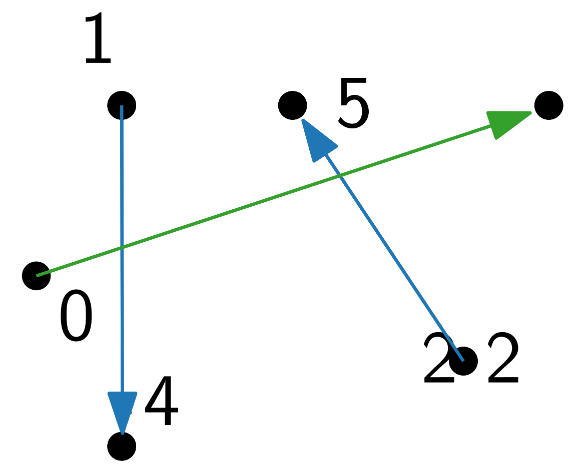


At every step, a rider can have three steps: *wait*, *travel*, *finish*.

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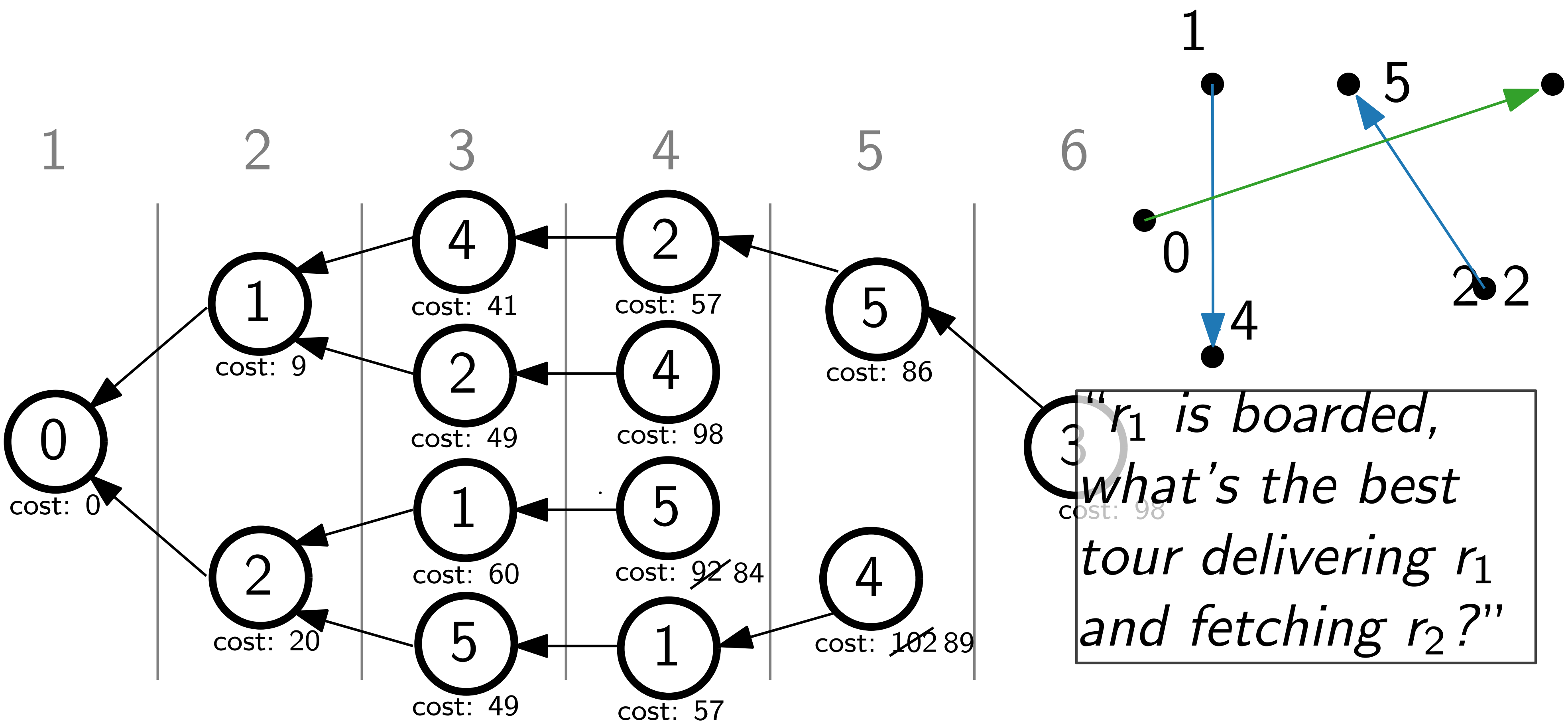


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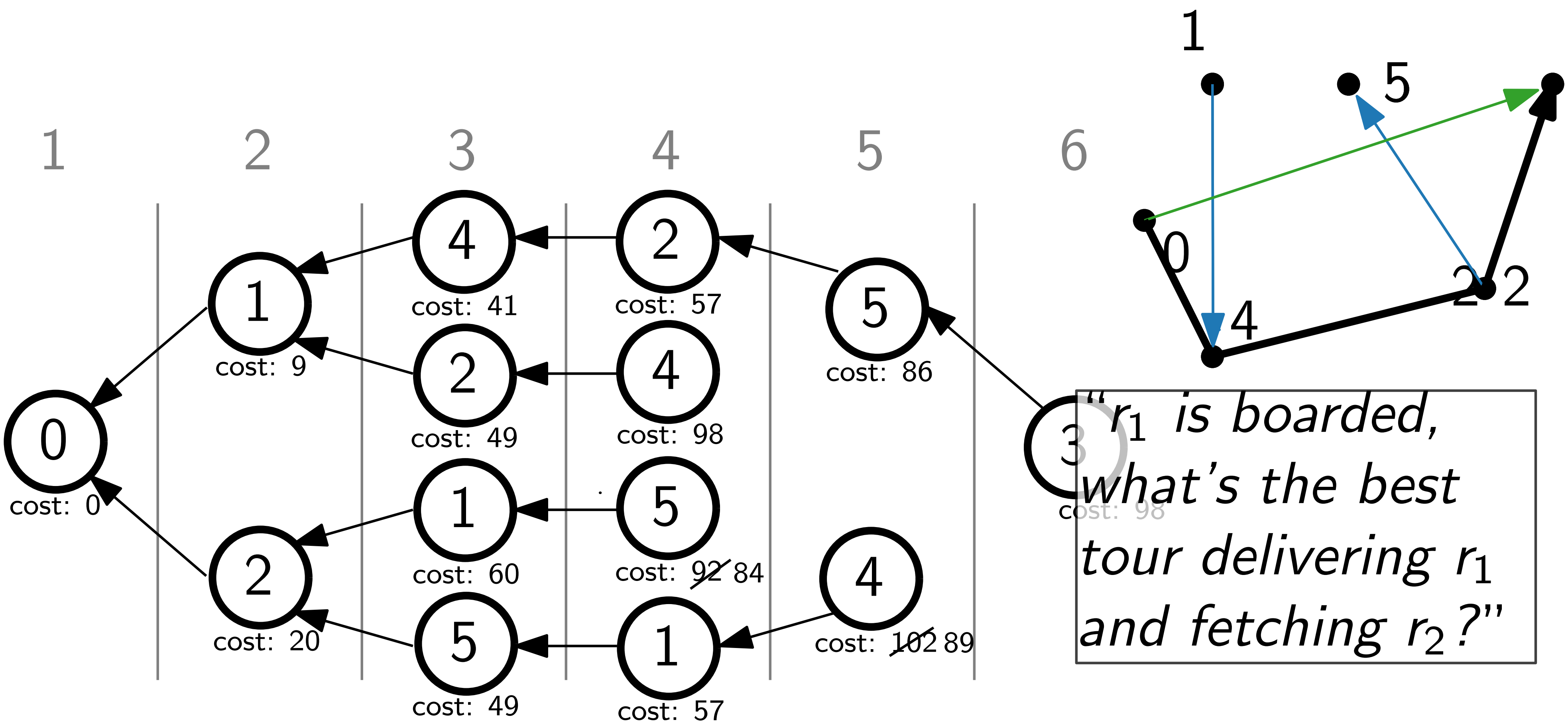


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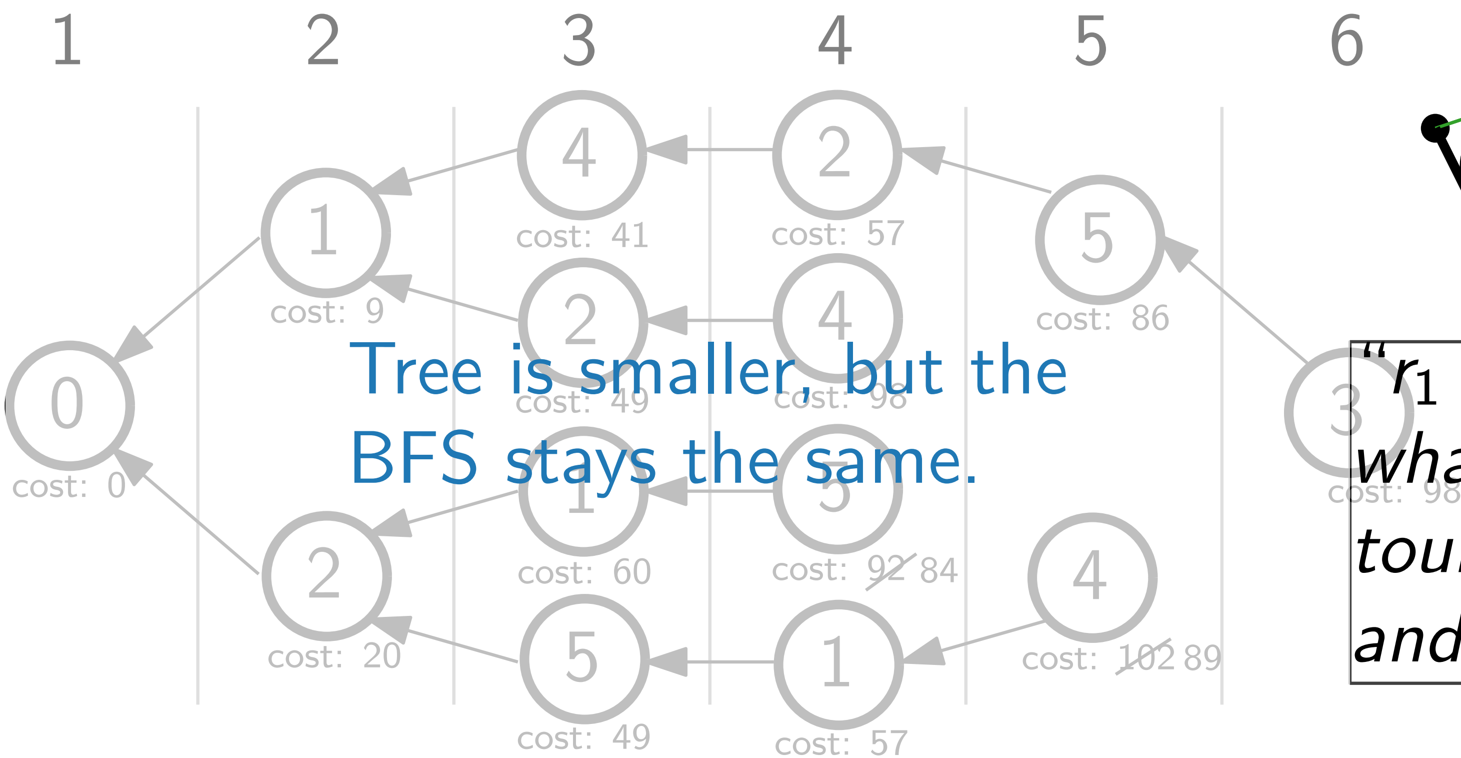
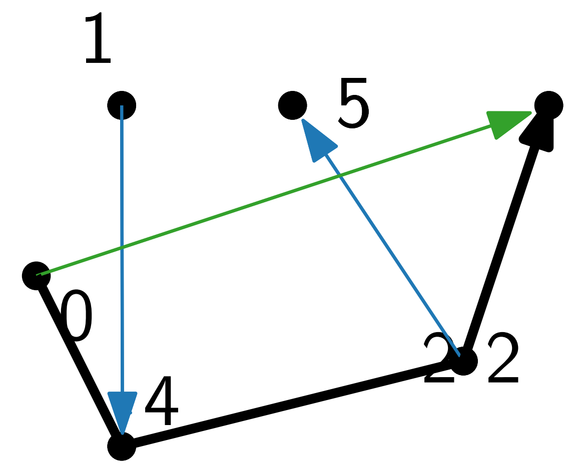



Tree is smaller, but the BFS stays the same.




"*$r_1$ is boarded, what's the best tour delivering $r_1$ and fetching $r_2$?*"

At every step, a rider can have three steps: *wait*, *travel*, *finish*.

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# Evalution of Realistic Examples

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**Rural Scenario**

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**Intercity Scenario**

# Evalution of Realistic Examples

**Rural Scenario**

Six small villages with $\varnothing$1.2 km distance.

**Regional Scenario**

**Intercity Scenario**

# Evalution of Realistic Examples

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Six small villages with ∅1.2 km distance.

**Regional Scenario**

Six small towns with ∅7.2 km distance.

**Intercity Scenario**

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Six major german cities with ∅129 km distance.

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All optimal tours are unidirectional, recall $> 0.9$.

# Evalution of Realistic Examples

**Rural Scenario**

Six small villages with ∅1.2 km distance.

$>$ 70% optimal tours unidirectional, recall $<$ 0.1.

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Six small towns with ∅7.2 km distance.

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Six small villages with ∅1.2 km distance.
$>$ 70% optimal tours unidirectional, recall $<$ 0.1.

Bad, distances are too small.

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Six small towns with ∅7.2 km distance.

$>$ 50% optimal tours unidir., recall $>$ 0.55, precision 0.61.

**Intercity Scenario**

Six major german cities with ∅129 km distance.

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Wait ... What?!

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Six major german cities with ∅129 km distance.

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