

Kreuzungen, Kurven und Constraints beim Zeichnen von Graphen

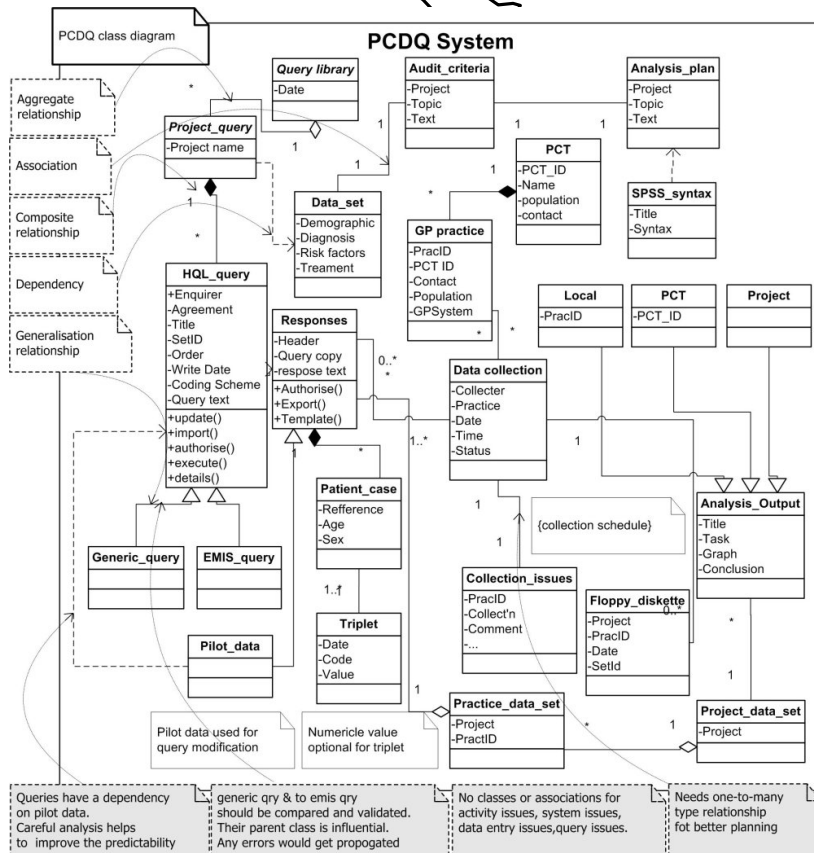
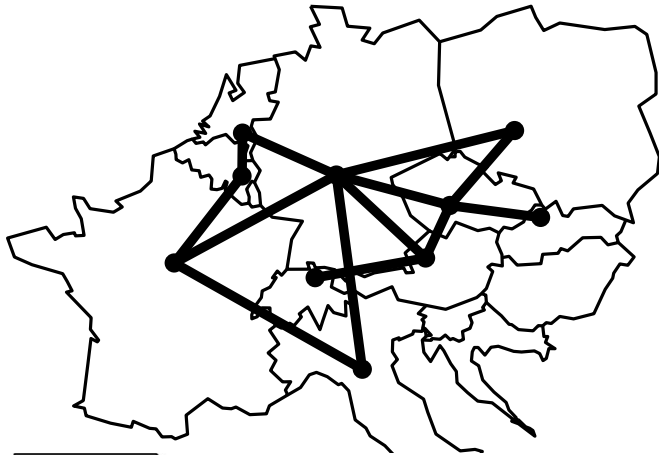
Crossings, Curves, and Constraints
in Graph Drawing

Martin Fink
Lehrstuhl für Informatik I
Universität Würzburg

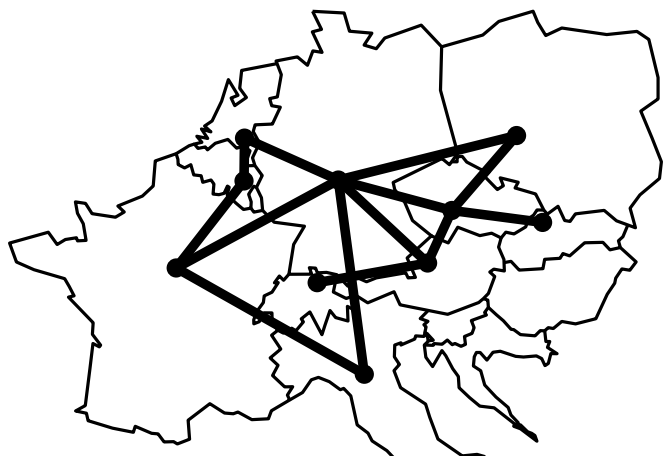
Warum Graphen zeichnen?



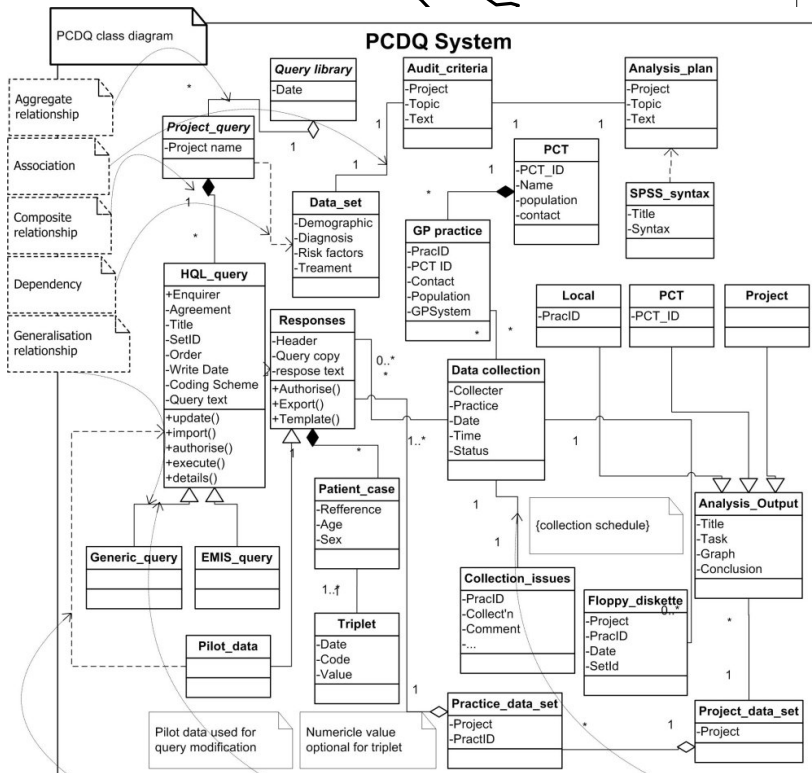
Warum Graphen zeichnen?



Warum Graphen zeichnen?



U-Bahn-Netz
Wien
Stand: 2012



Vieles als Graph modellierbar

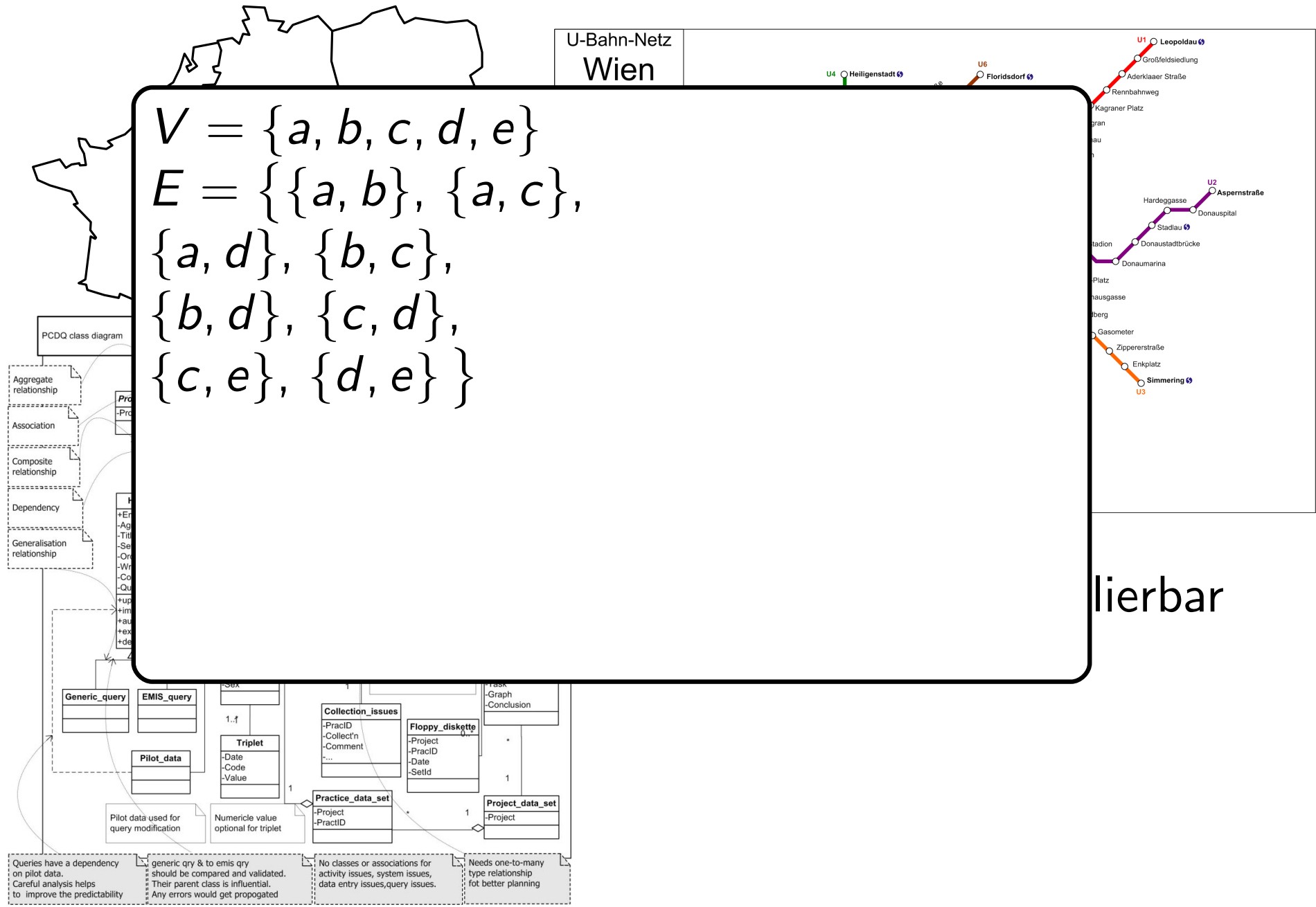
Queries have a dependency on pilot data. Careful analysis helps to improve the predictability

generic qry & to emis qry should be compared and validated. Their parent class is influential. Any errors would get propagated

No classes or associations for activity issues, system issues, data entry issues, query issues.

Needs one-to-many type relationship for better planning

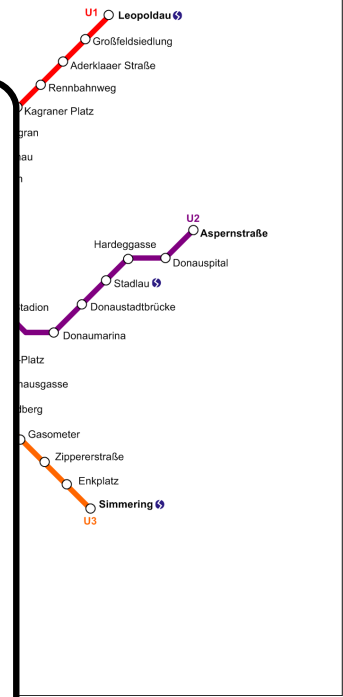
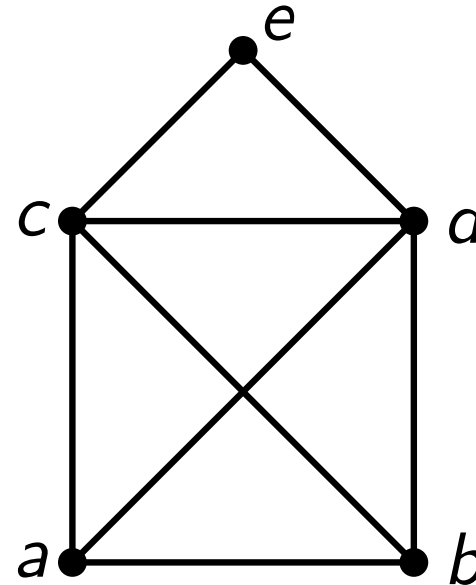
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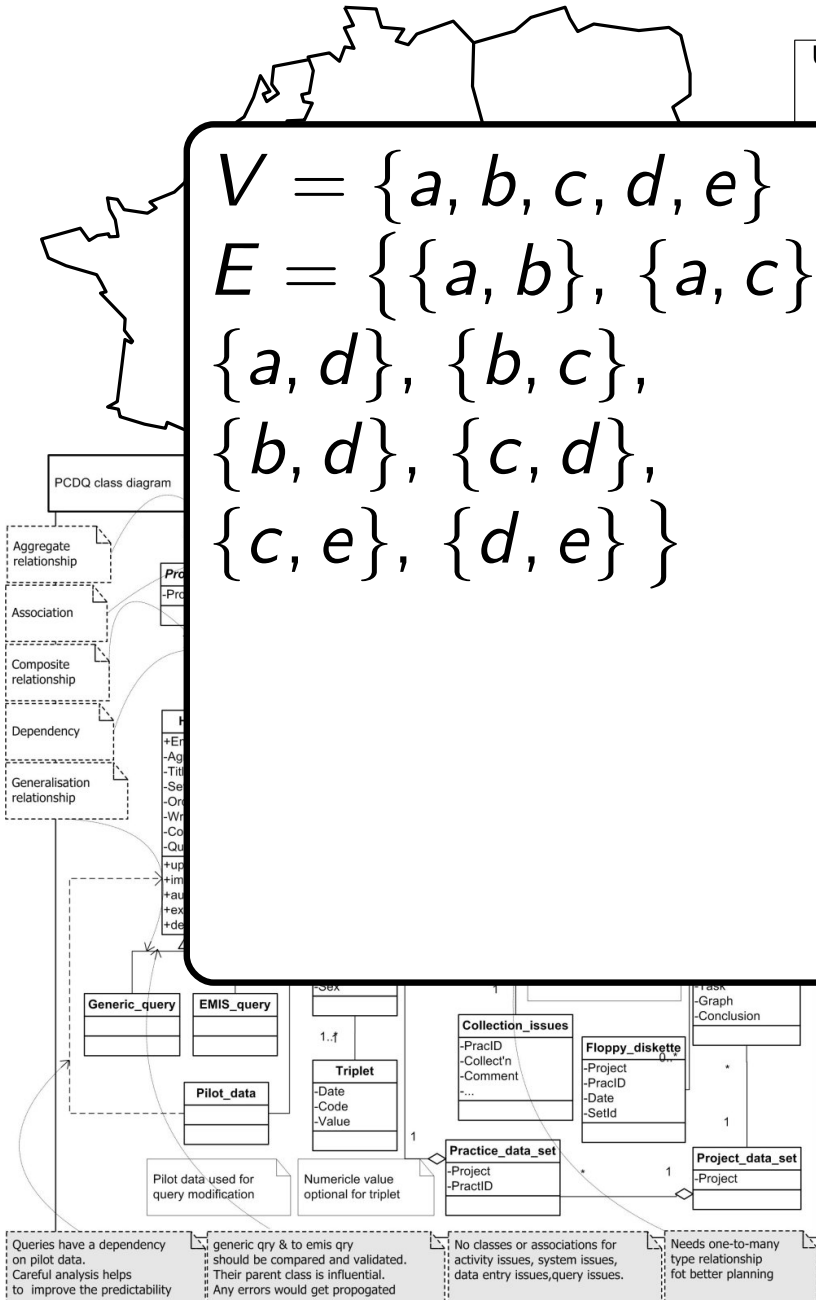
Warum Graphen zeichnen?

U-Bahn-Netz
Wien

$$V = \{a, b, c, d, e\}$$

$$E = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{c, e\}, \{d, e\}\}$$


lierbar



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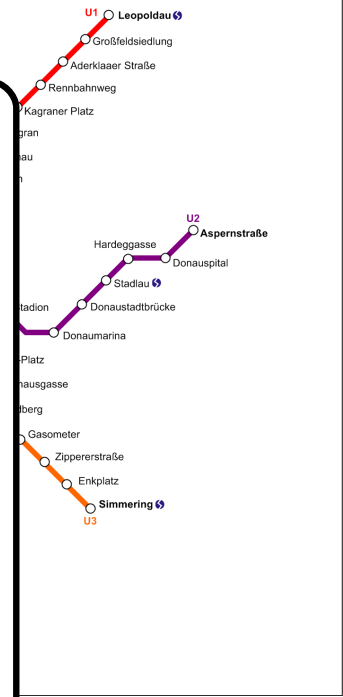
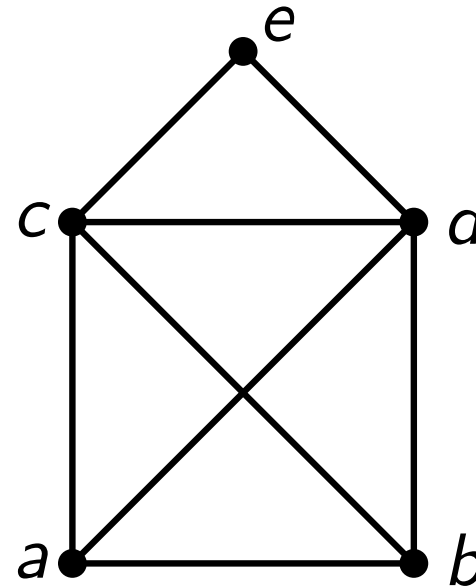
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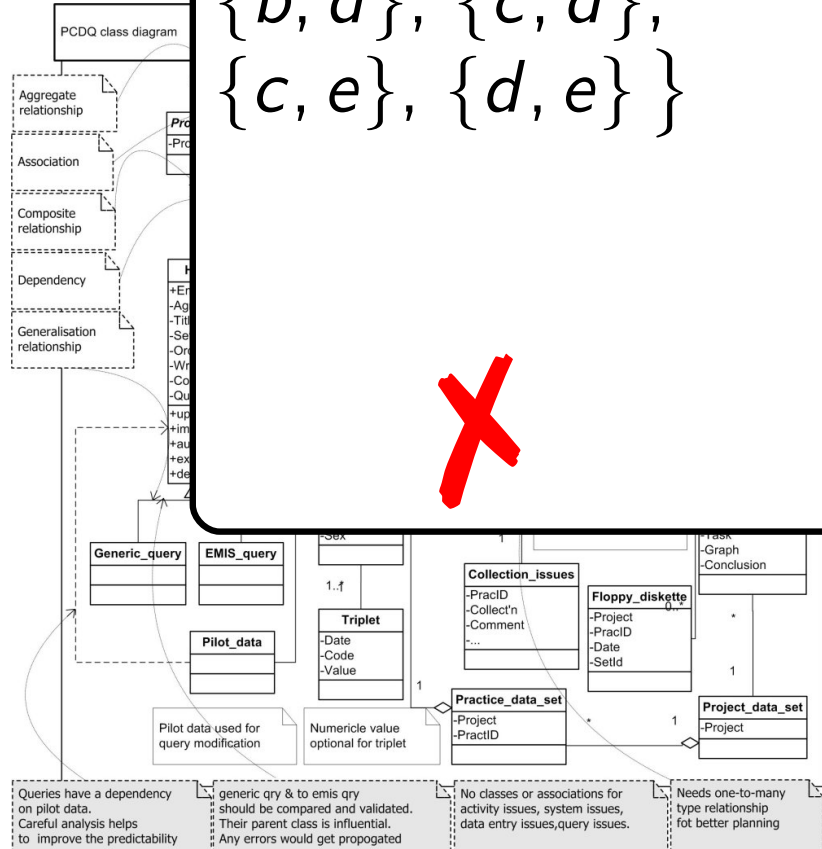
Warum Graphen zeichnen?

U-Bahn-Netz
Wien

$V = \{a, b, c, d, e\}$
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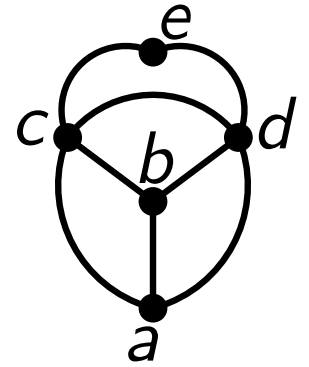
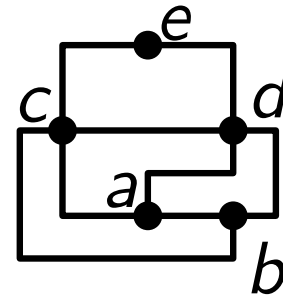
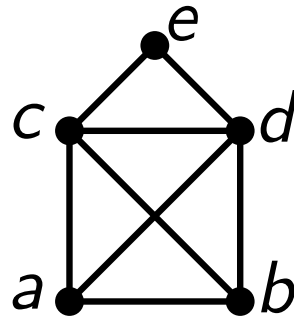


...lierbar



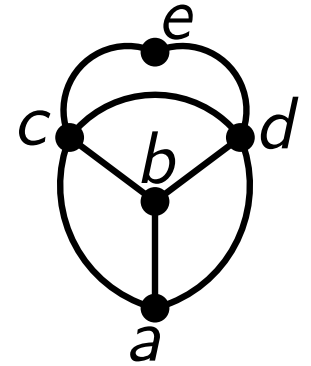
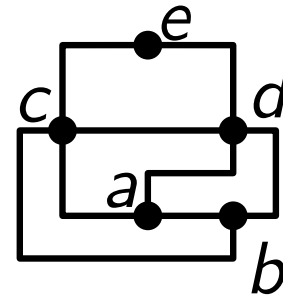
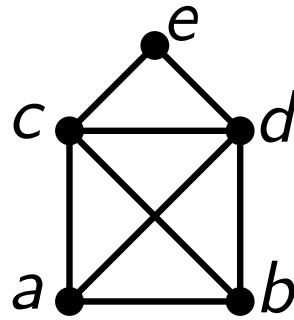
Constraints

- Kantendarstellung



Constraints

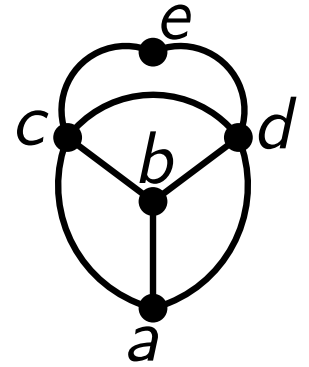
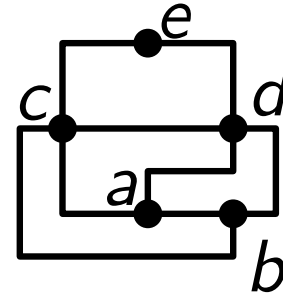
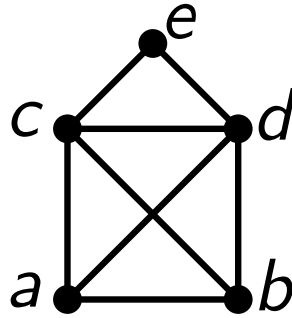
- Kantendarstellung



- Planarität

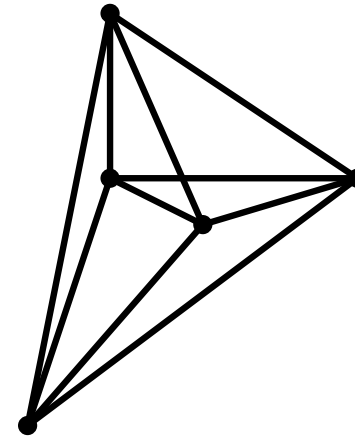
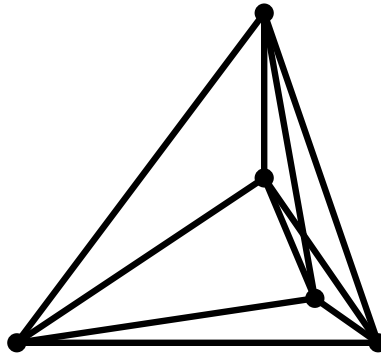
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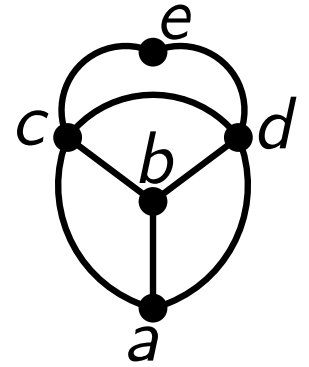
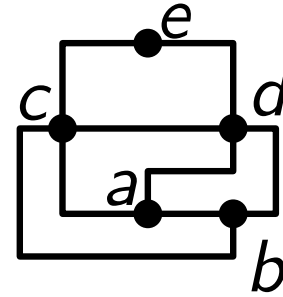
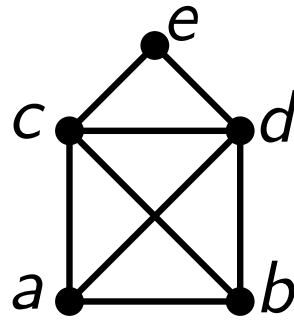
- Planarität

- Kreuzungswinkel



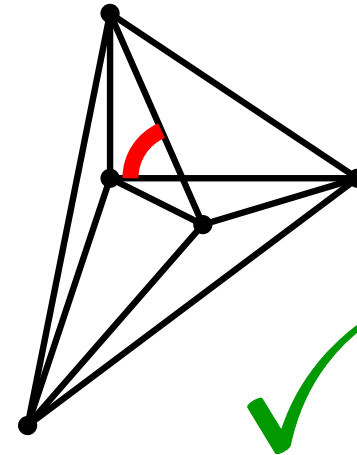
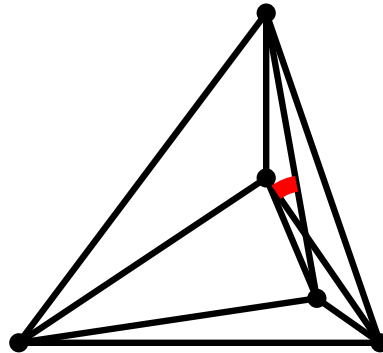
Constraints

- Kantendarstellung



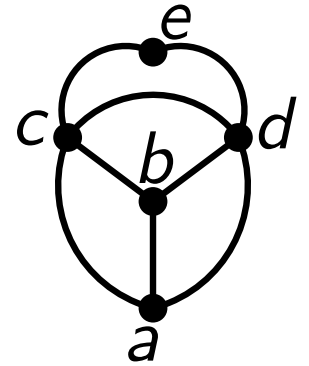
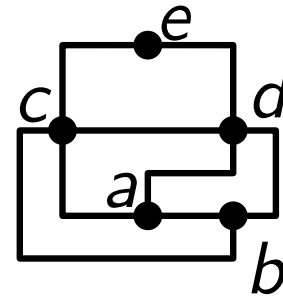
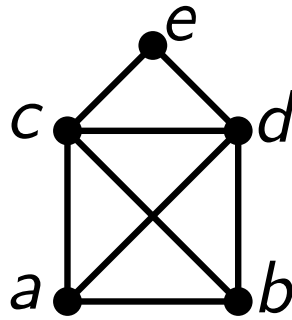
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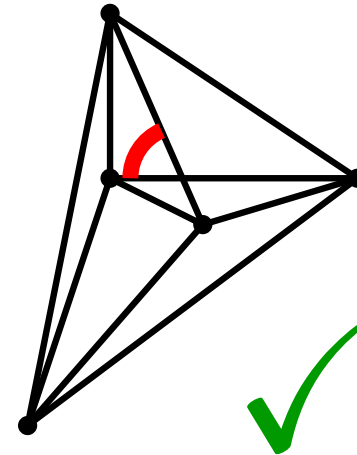
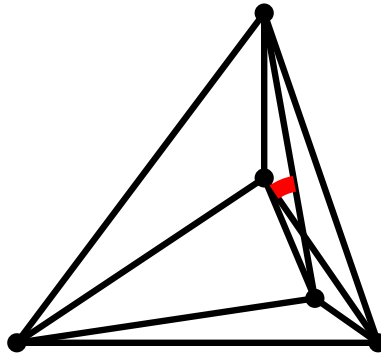
Constraints

- Kantendarstellung

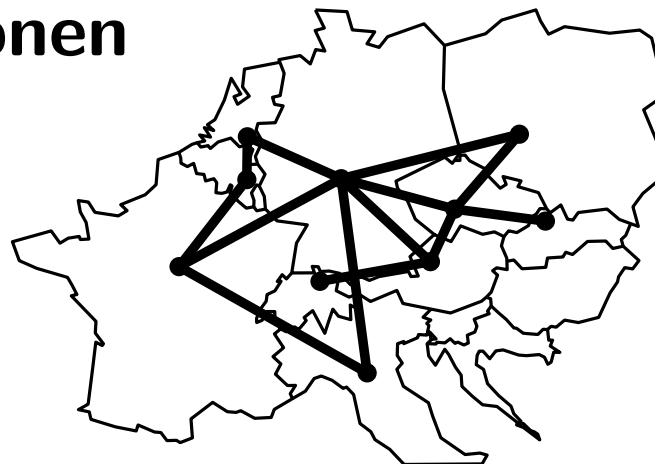


- Planarität

- Kreuzungswinkel



- Knotenpositionen



Gliederung

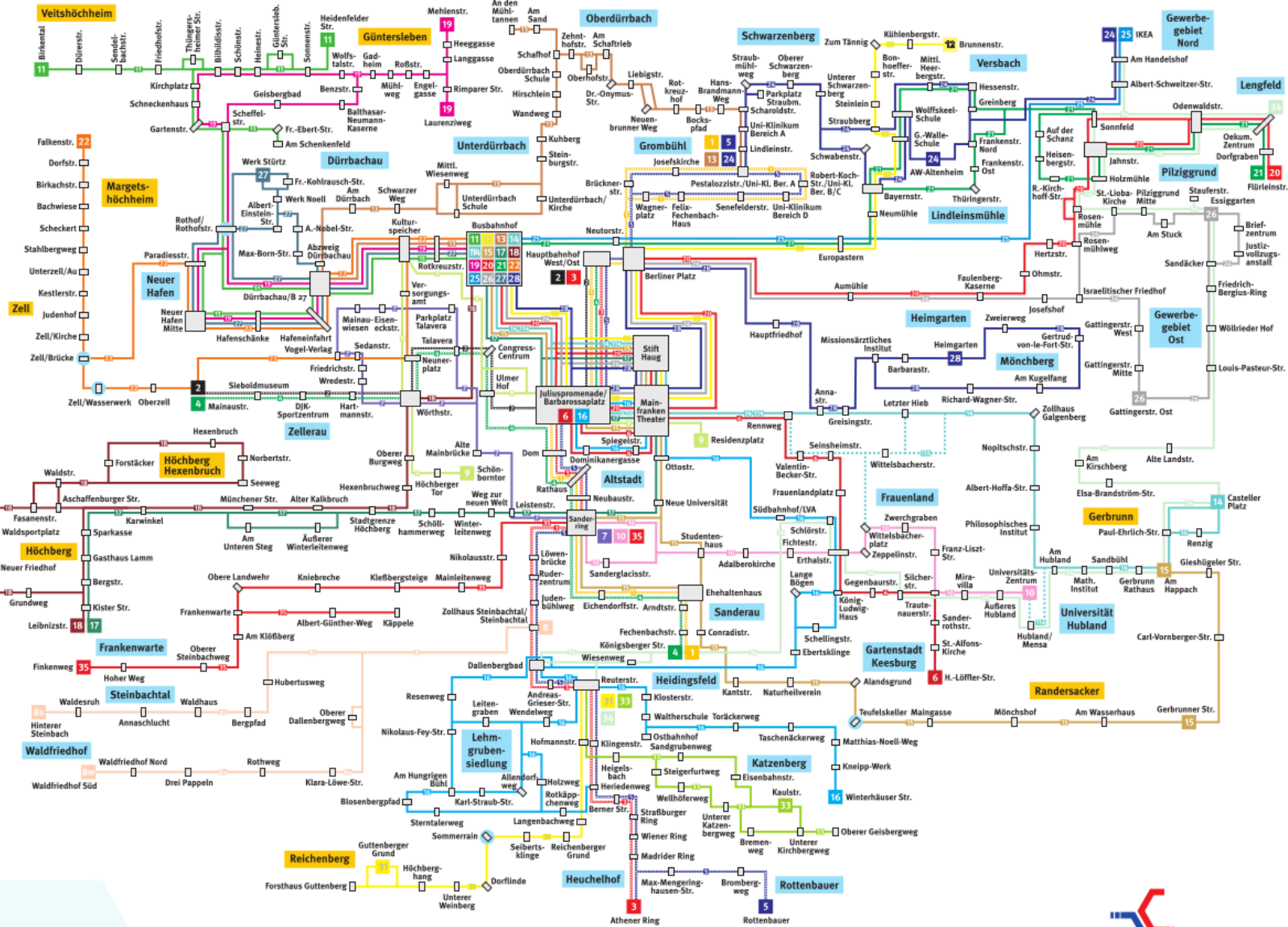
- **Teil I: U-Bahn-Linienpläne**
- **Teil II: Punktmengen-Einbettbarkeit**
- **Teil III: Randbeschriftungen**

Teil I: U-Bahn-Linienpläne

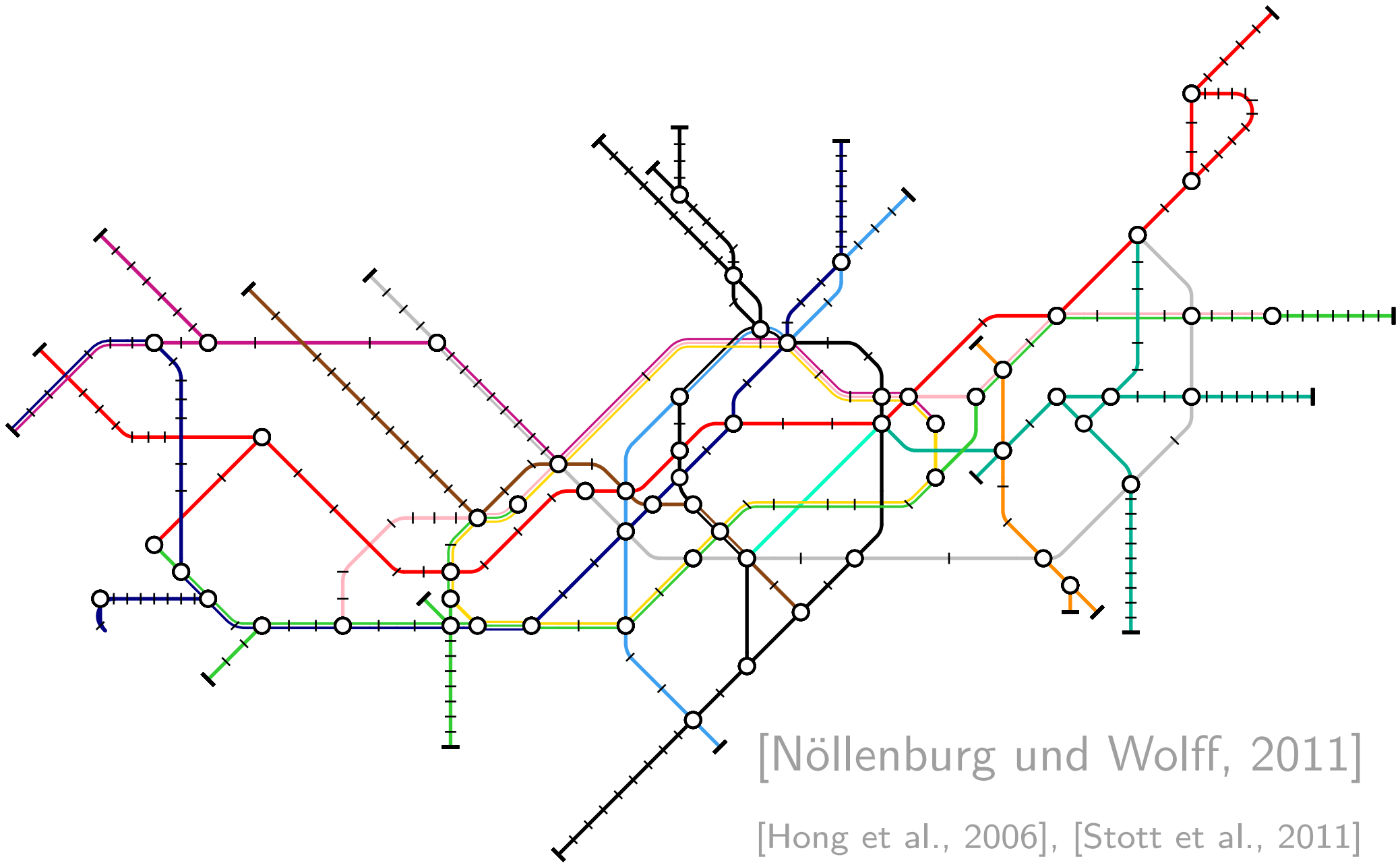
Métro Paris



Würzburg



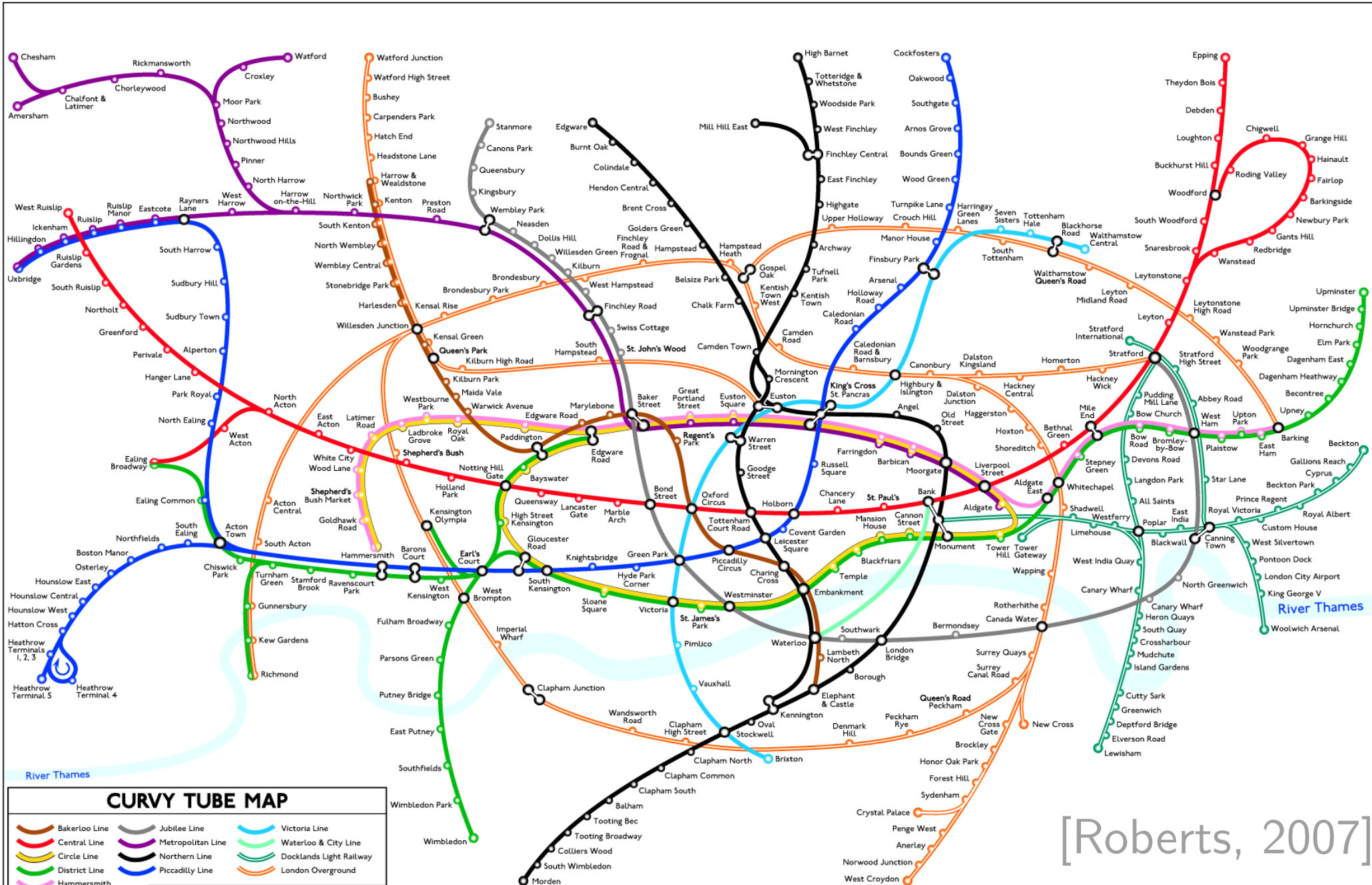
Bisher: schematische Pläne



[Nöllenburg und Wolff, 2011]

[Hong et al., 2006], [Stott et al., 2011]

Wunsch: kurviger Plan



CURVY TUBE MAP

Station
 Interchange between lines
 Overground

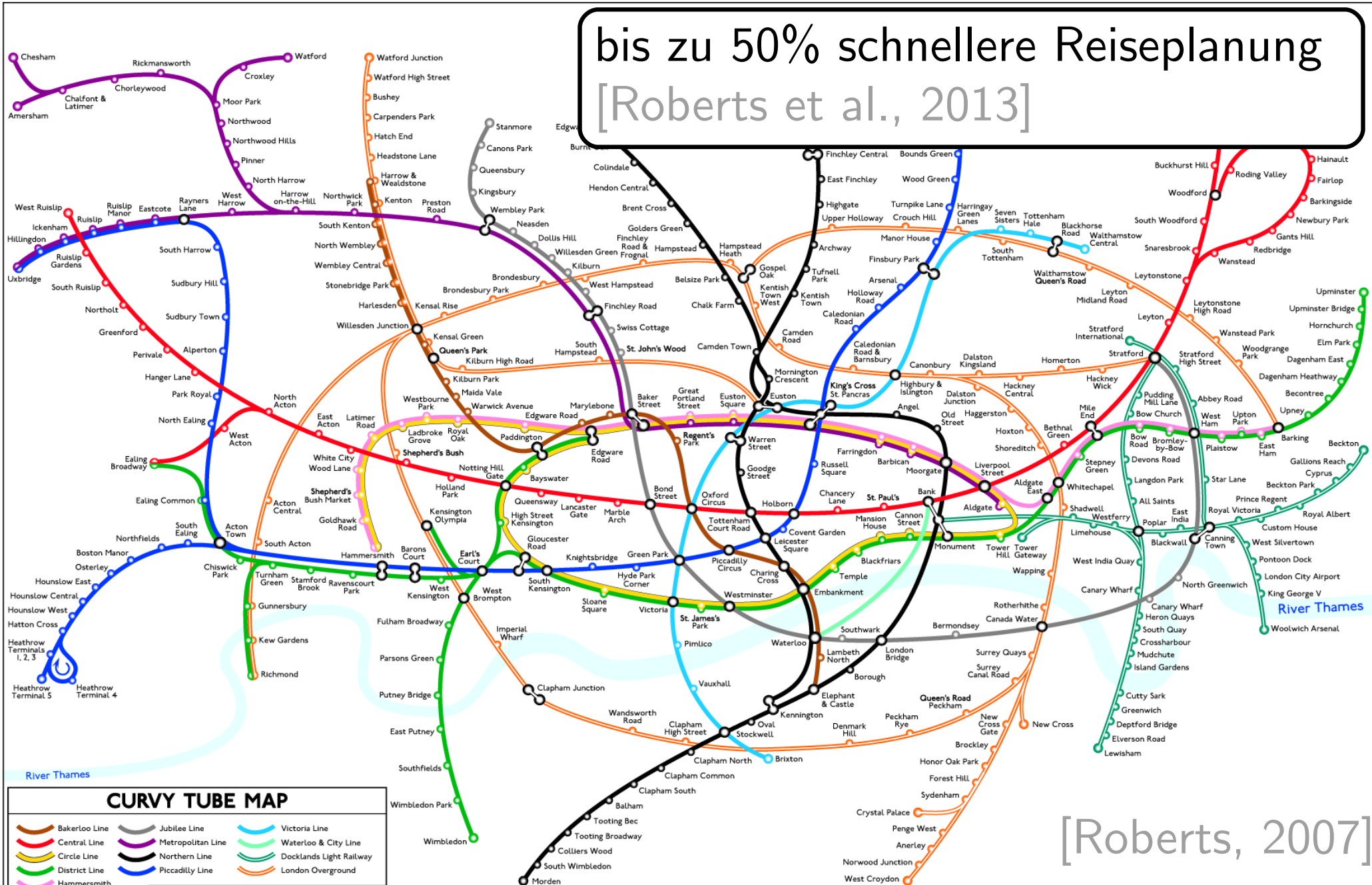
A curves-only diagrammatic map of TfL supported railways, © Maxwell J. Roberts, 30/01/2008

[Roberts, 2007]

THIS MAP IS DERIVED FROM REGISTERED DESIGN NO. 400180: A CURVES-ONLY DIAGRAMMATIC MAP OF THE LONDON UNDERGROUND © MAXWELL J. ROBERTS, 26/02/2007

Wunsch: kurviger Plan

bis zu 50% schnellere Reiseplanung
[Roberts et al., 2013]



[Roberts, 2007]

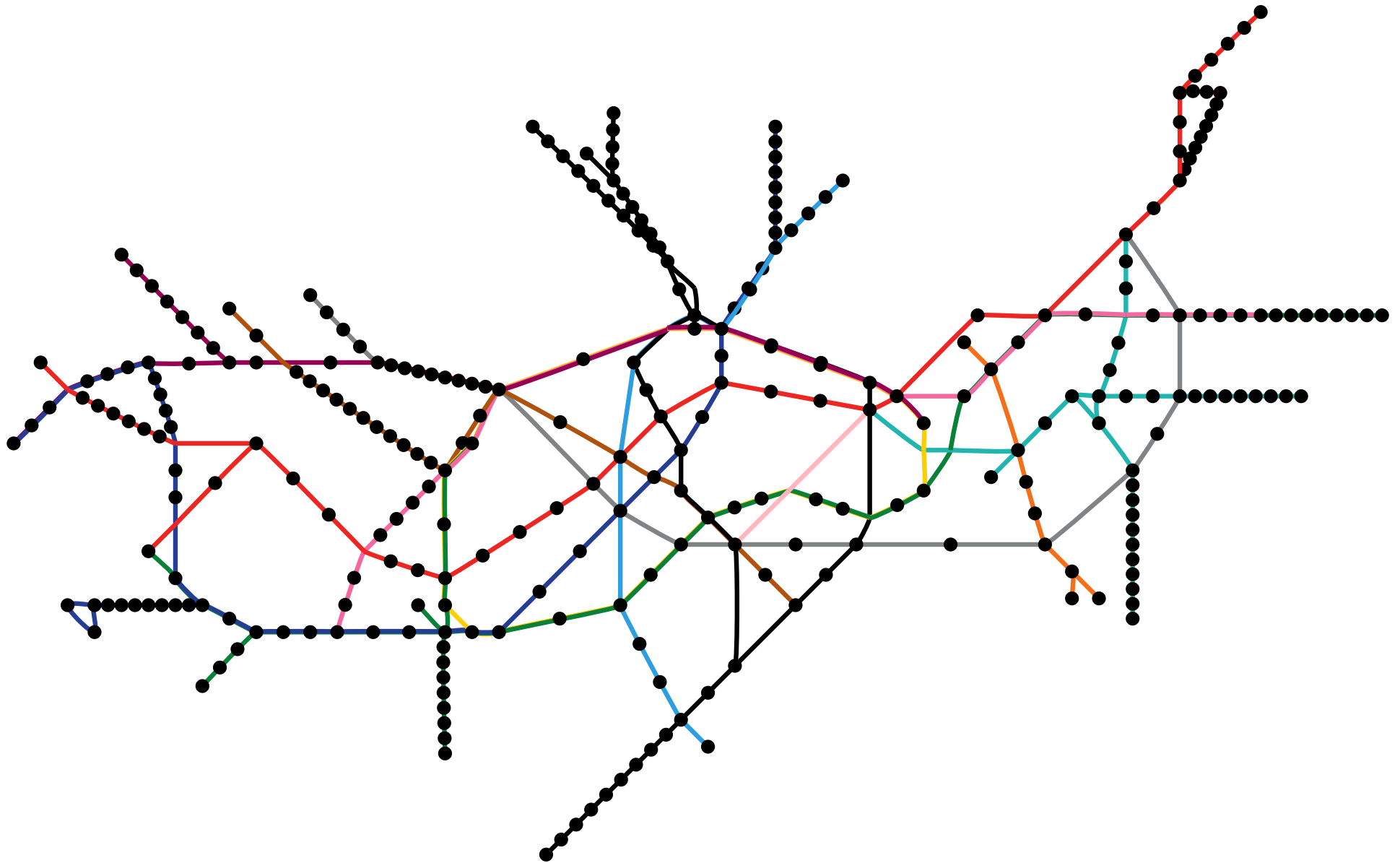
CURVY TUBE MAP

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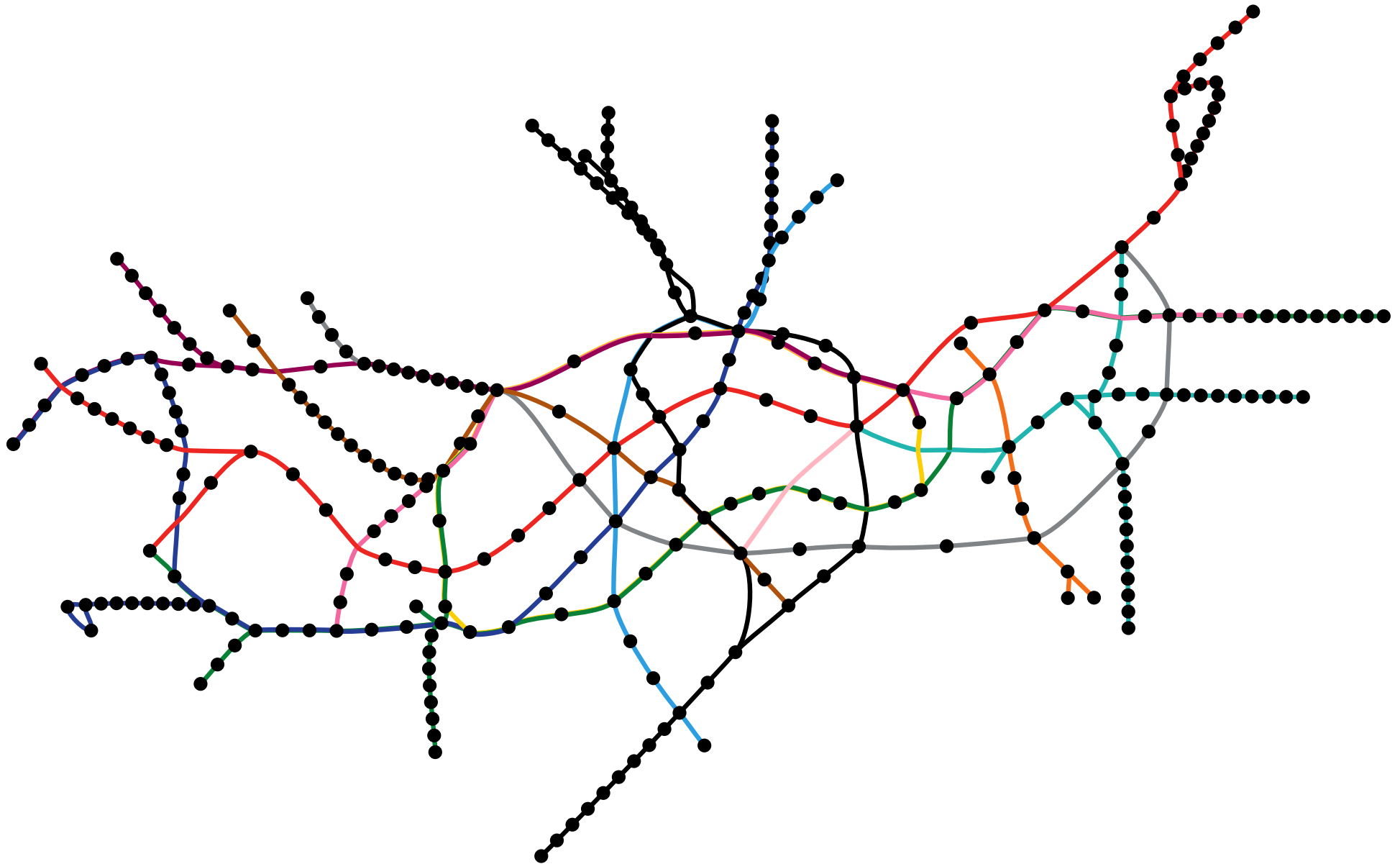
A curves-only diagrammatic map of TfL supported railways, © Maxwell J. Roberts, 30/01/2008

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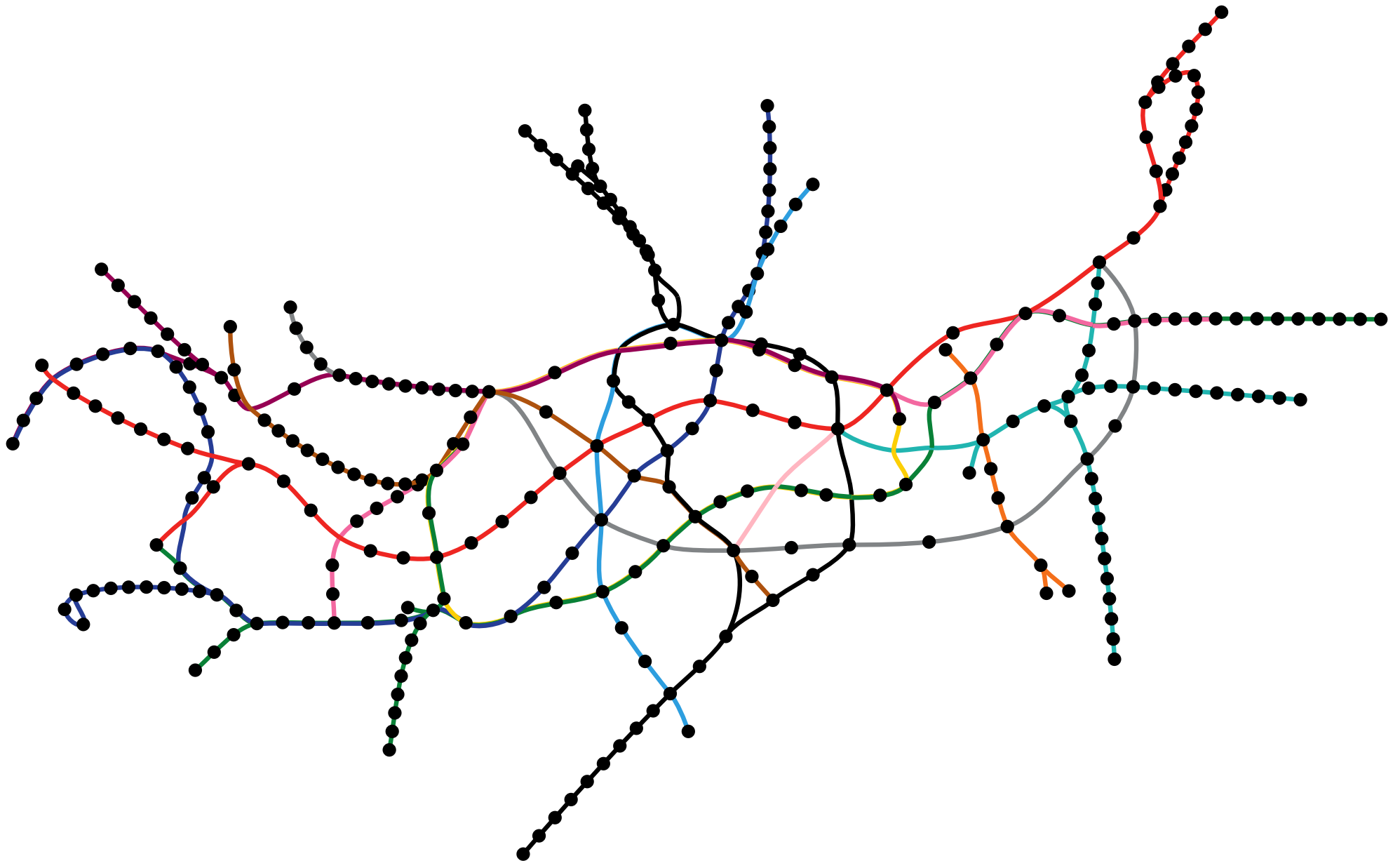
Unser Verfahren mit Bézierkurven



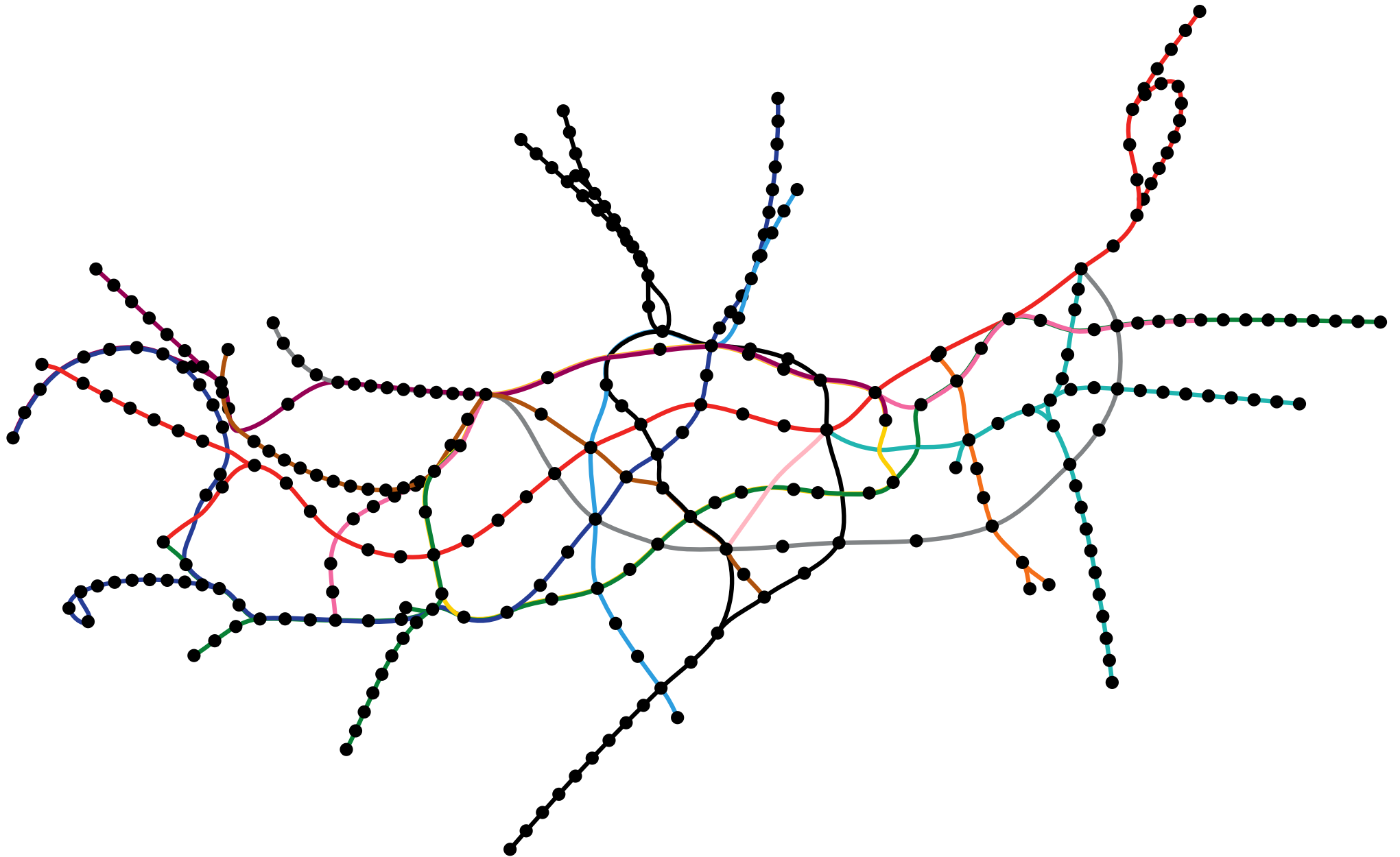
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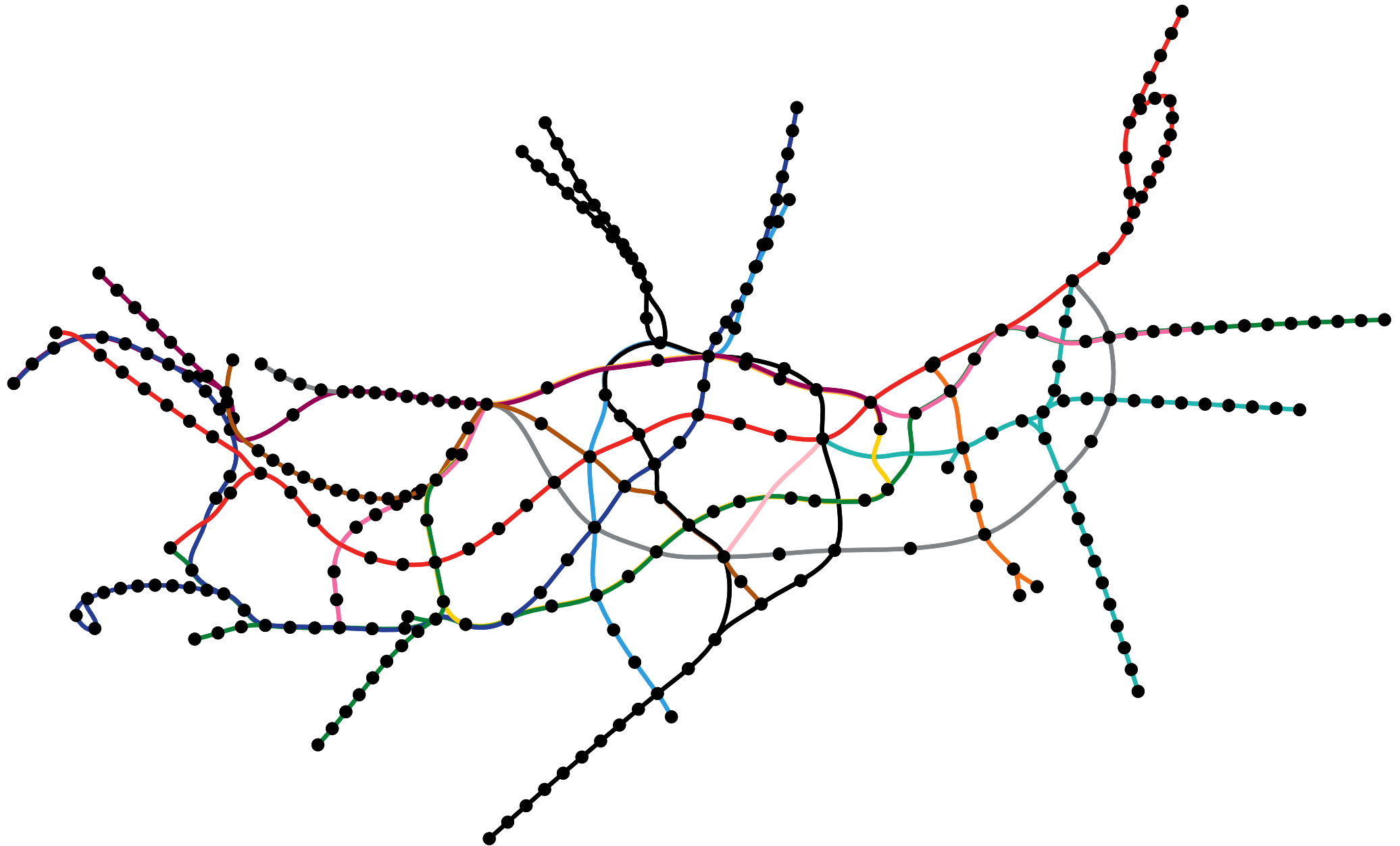
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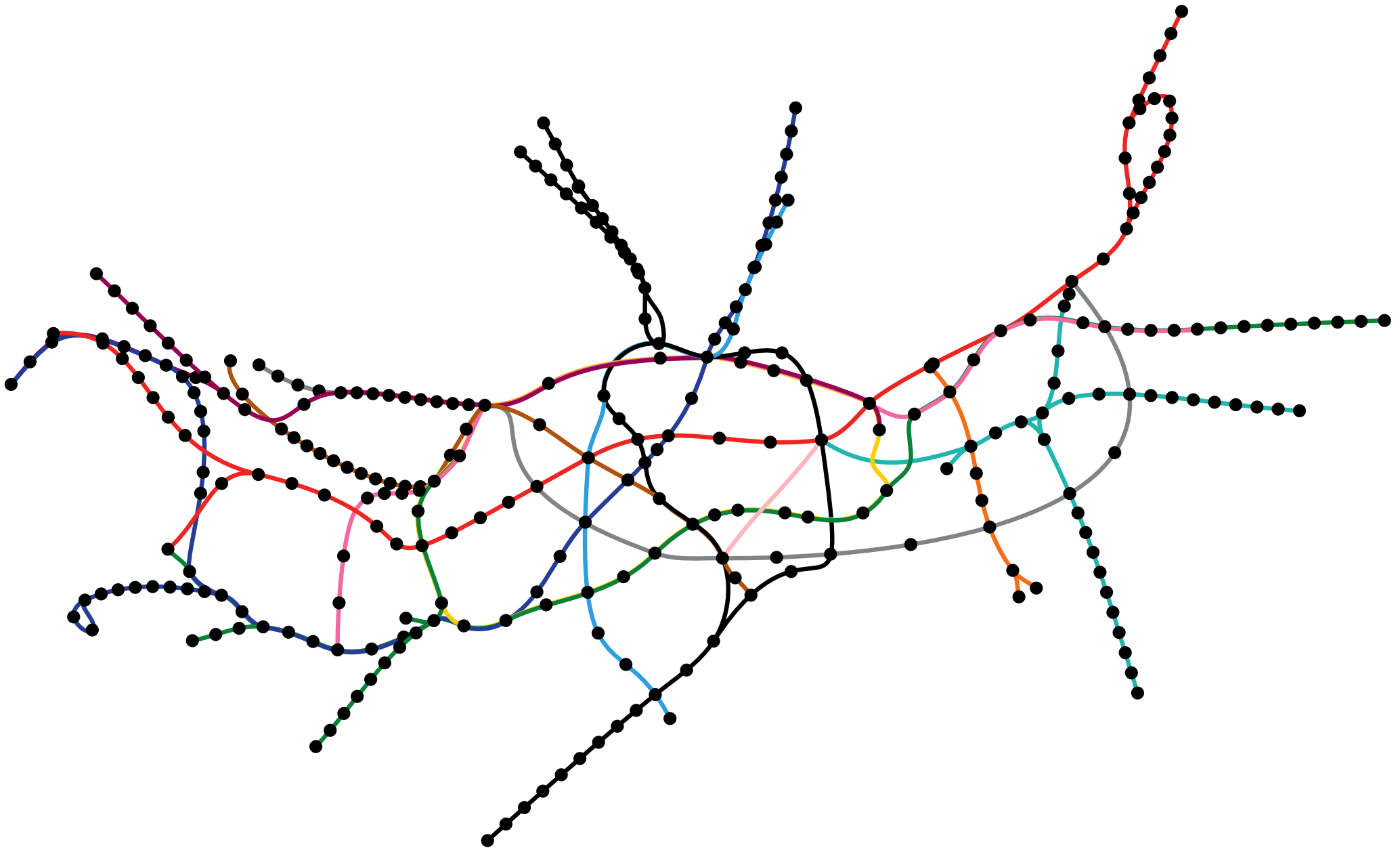
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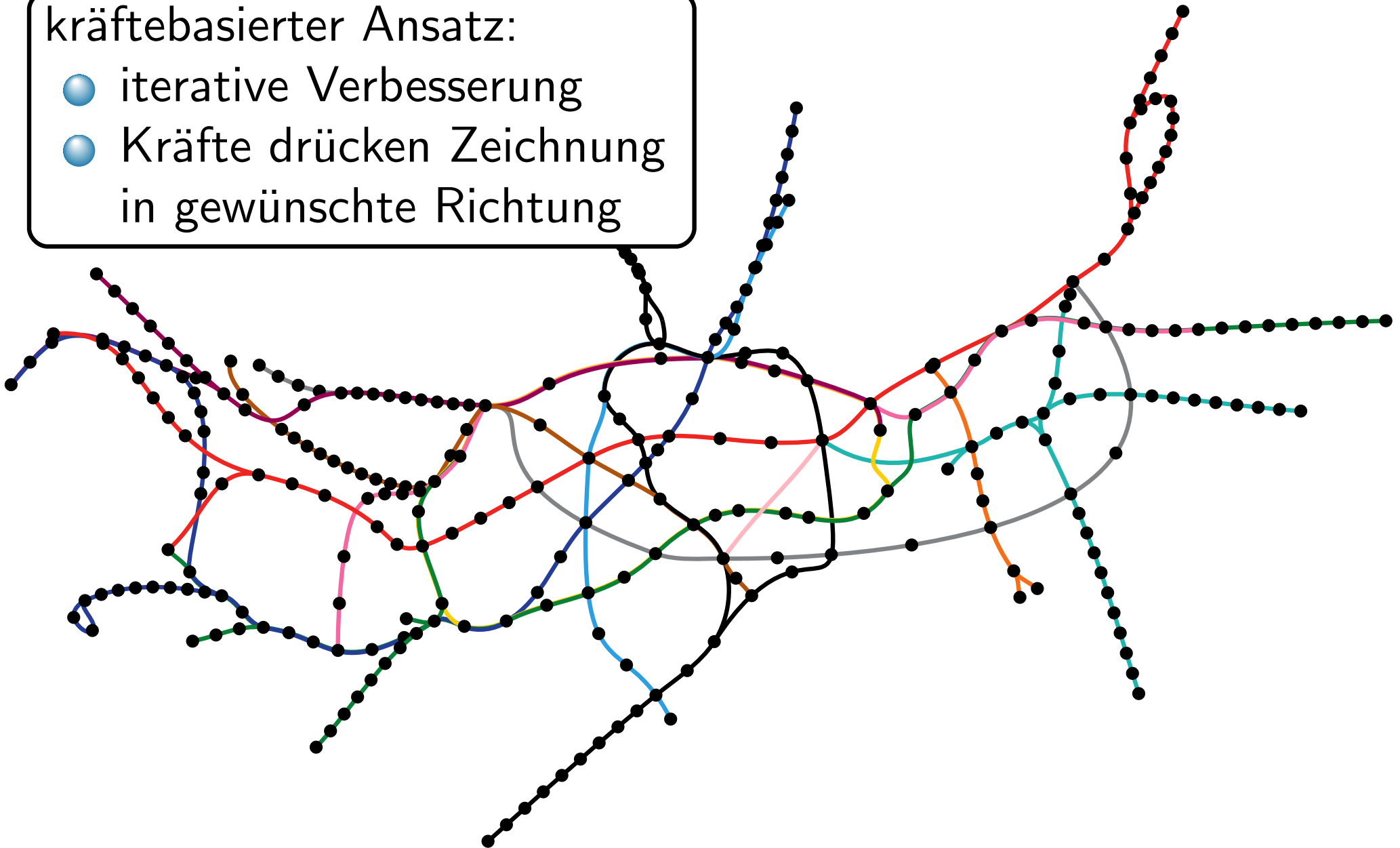
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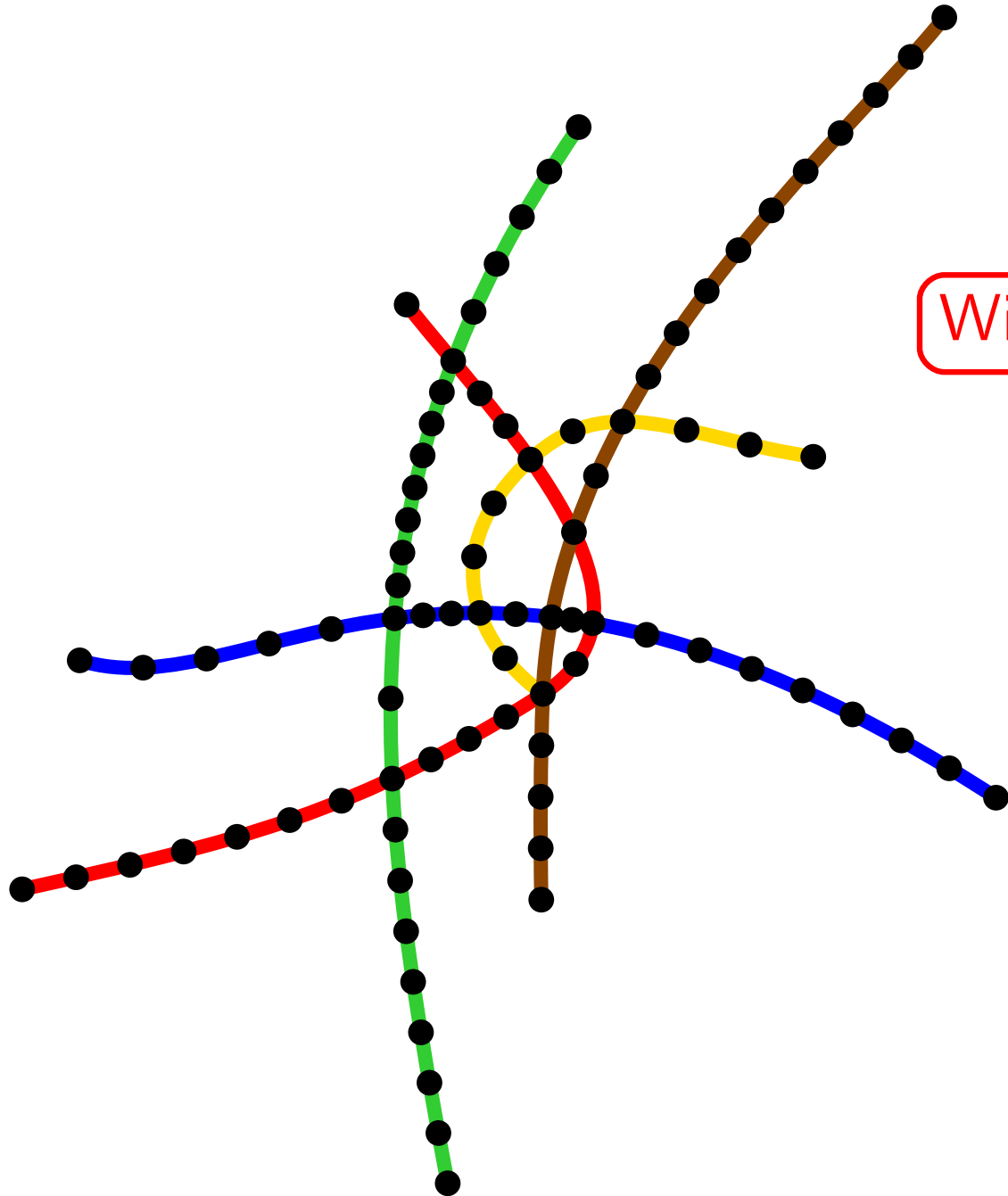
Unser Verfahren mit Bézierkurven

kräftebasierterer Ansatz:

- iterative Verbesserung
- Kräfte drücken Zeichnung in gewünschte Richtung



Wien



Wichtig: Verschmelze Kurven!

U-Bahn-Linien



U-Bahn-Linien



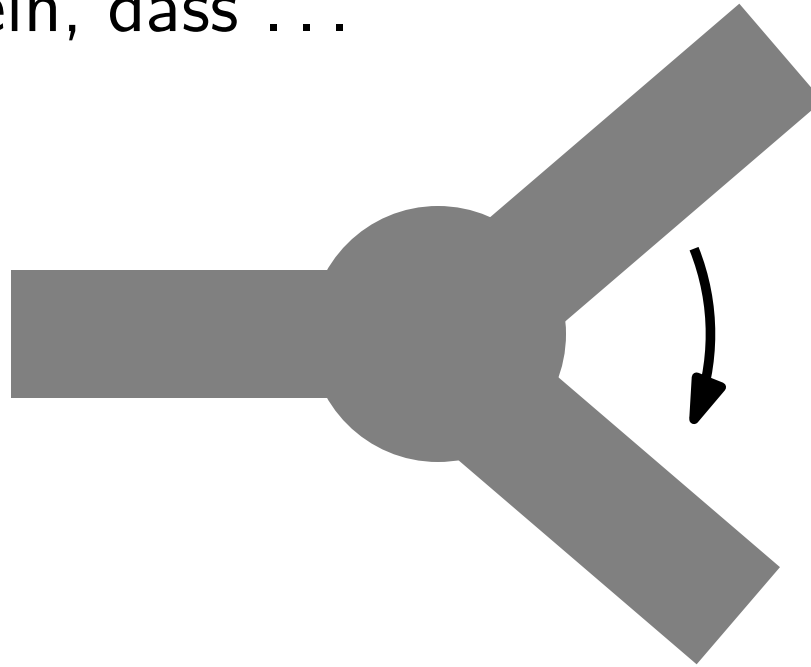
eine Kante – viele
U-Bahn-Linien

U-Bahn-Linien



Kreuzungsminimierung

Füge (U-Bahn-)Linien L so in *eingebetteten Graphen* $G = (V, E)$ ein, dass ...



Kreuzungsminimierung

Füge (U-Bahn-)Linien L so in *eingebetteten Graphen* $G = (V, E)$ ein, dass ...

- #Kreuzungen minimal.
- Kreuzungen auf Kanten platziert.

Annahmen:

- Linien sind einfache Pfade.
- Linien schneiden sich in Pfaden.

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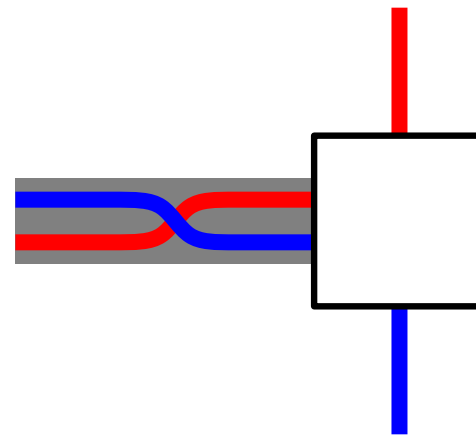
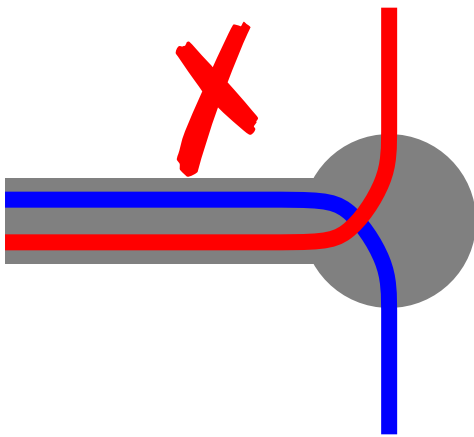
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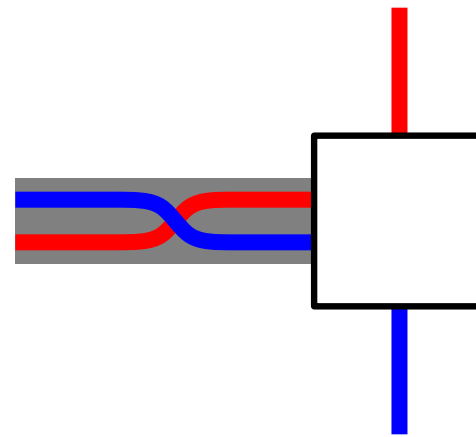
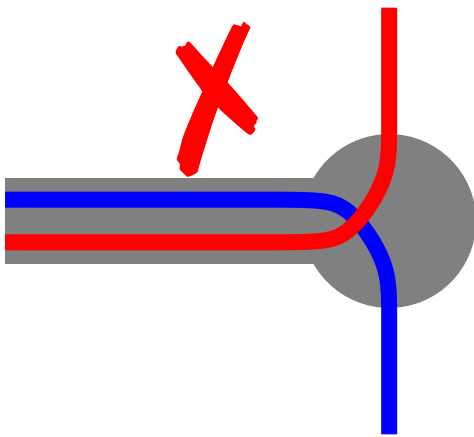
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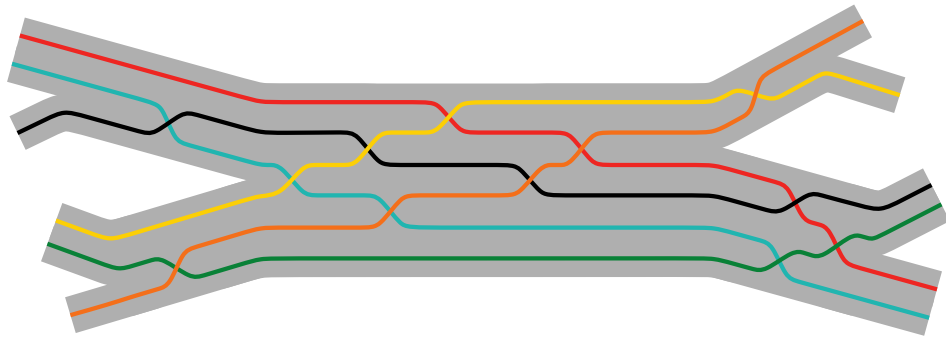
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Hauptergebnisse:

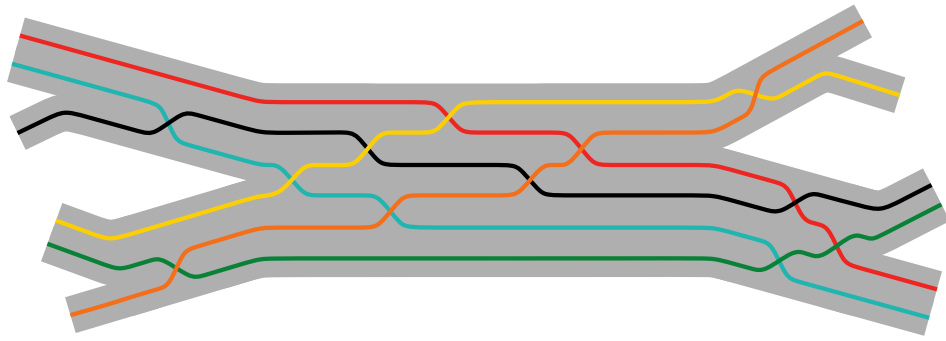
- Allgemeines Problem NP-schwer.
- $O(\sqrt{\log |L|})$ -Approximation für (NP-schwere) Variante

Neues Modell: Blockkreuzungen

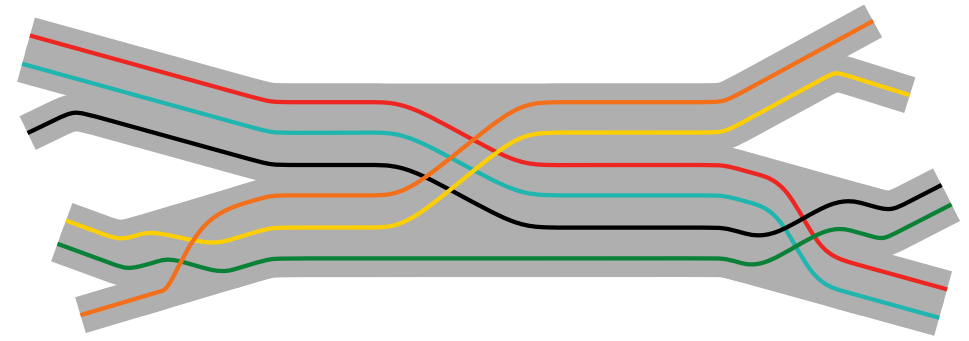


12 Kreuzungen

Neues Modell: Blockkreuzungen

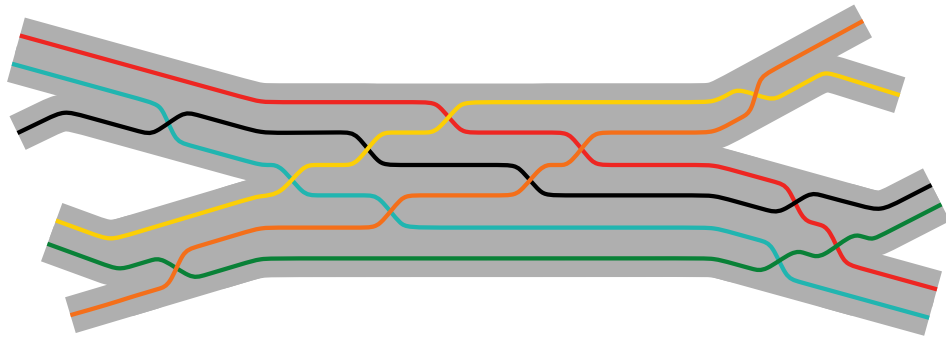


12 Kreuzungen



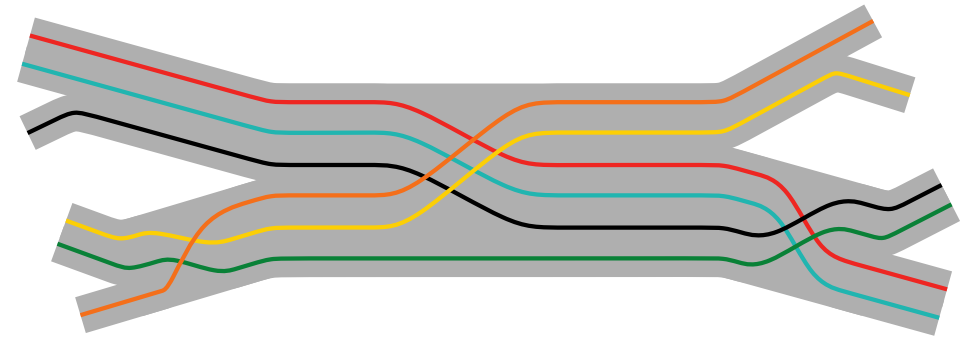
12 Kreuzungen

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12 Kreuzungen

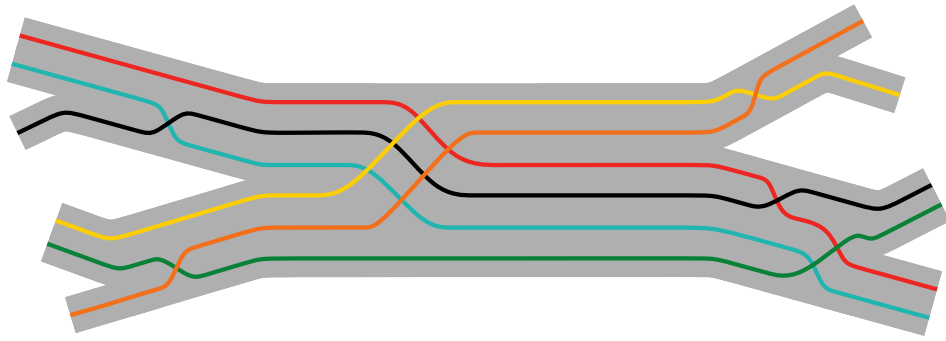
12 Blockkreuzungen



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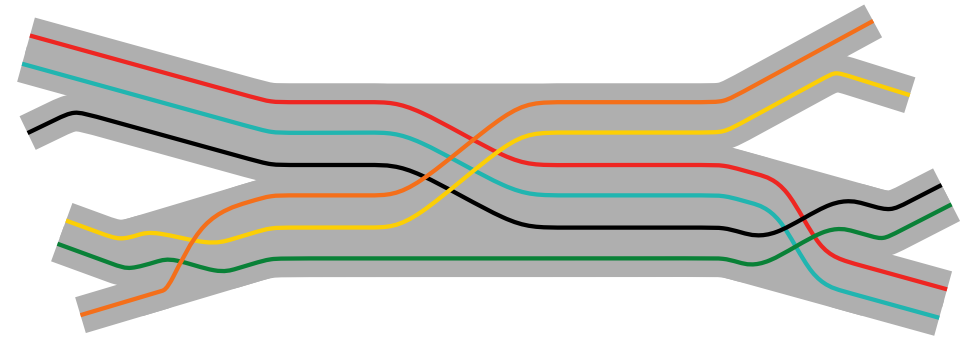
3 Blockkreuzungen

Neues Modell: Blockkreuzungen



12 Kreuzungen

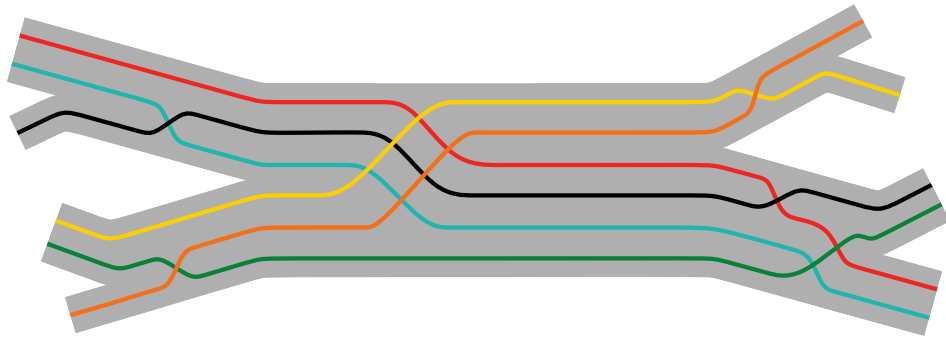
6 ~~12~~ Blockkreuzungen



12 Kreuzungen

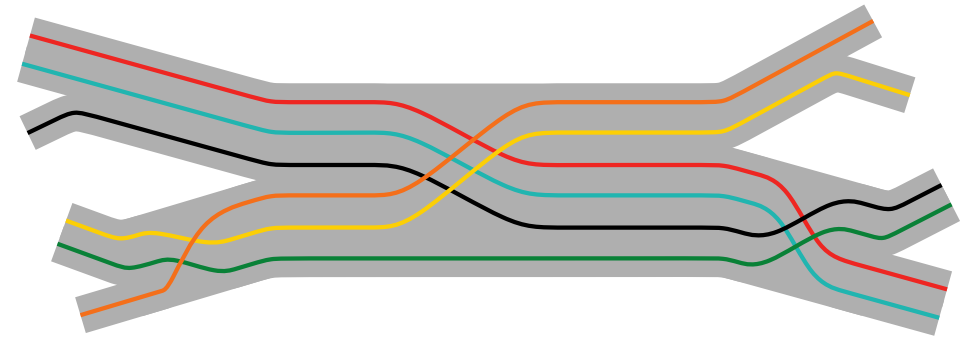
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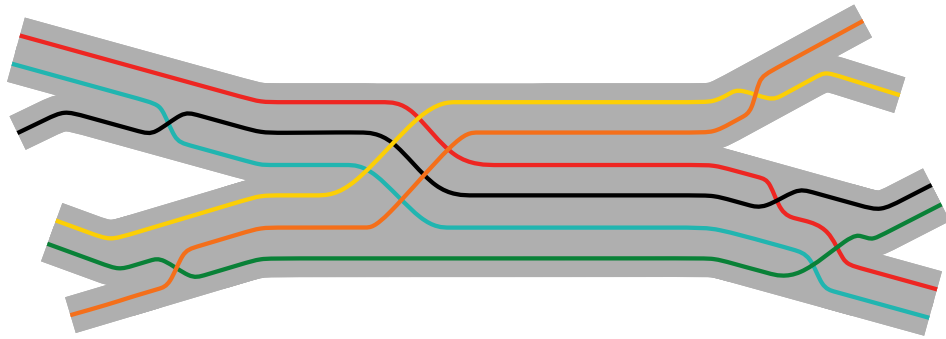


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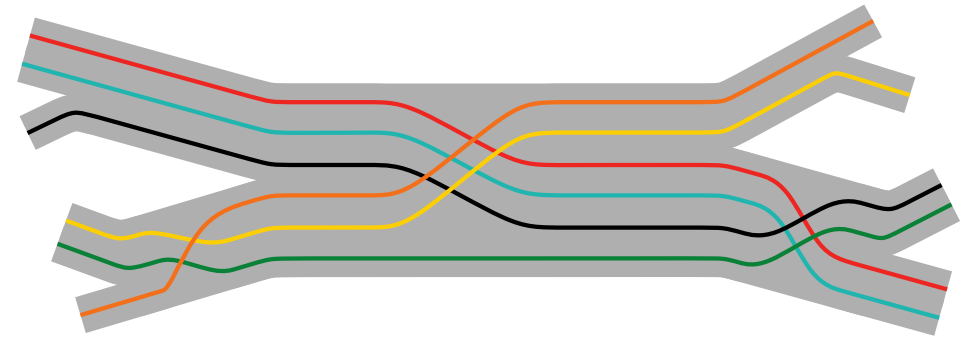
#Blockkreuzungen besseres Qualitätsmaß

Neues Modell: Blockkreuzungen



12 Kreuzungen

6 ~~12~~ Blockkreuzungen

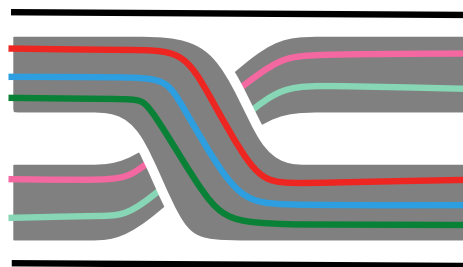


12 Kreuzungen

3 Blockkreuzungen

➔ #Blockkreuzungen besseres Qualitätsmaß

Blockkreuzung:



Linien in selbem Block parallel

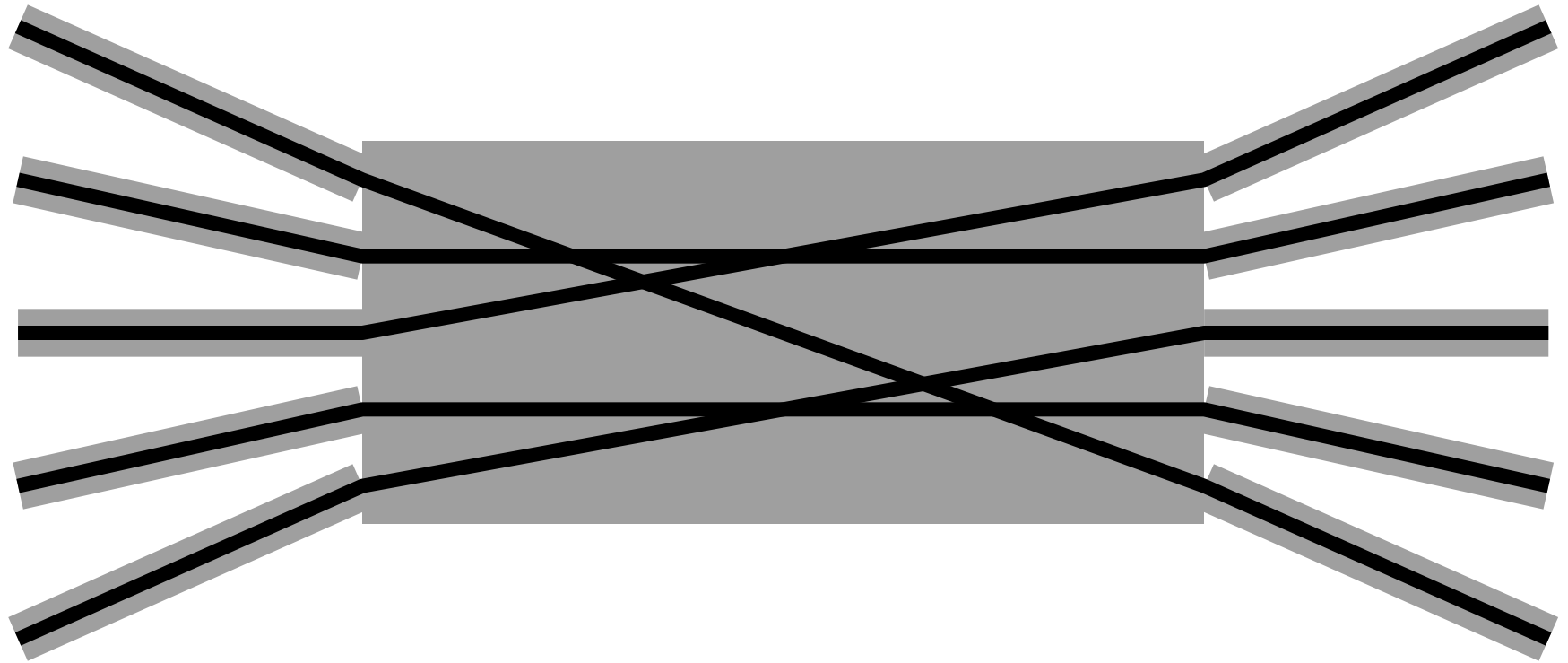
Blockkreuzungen auf einer Kante



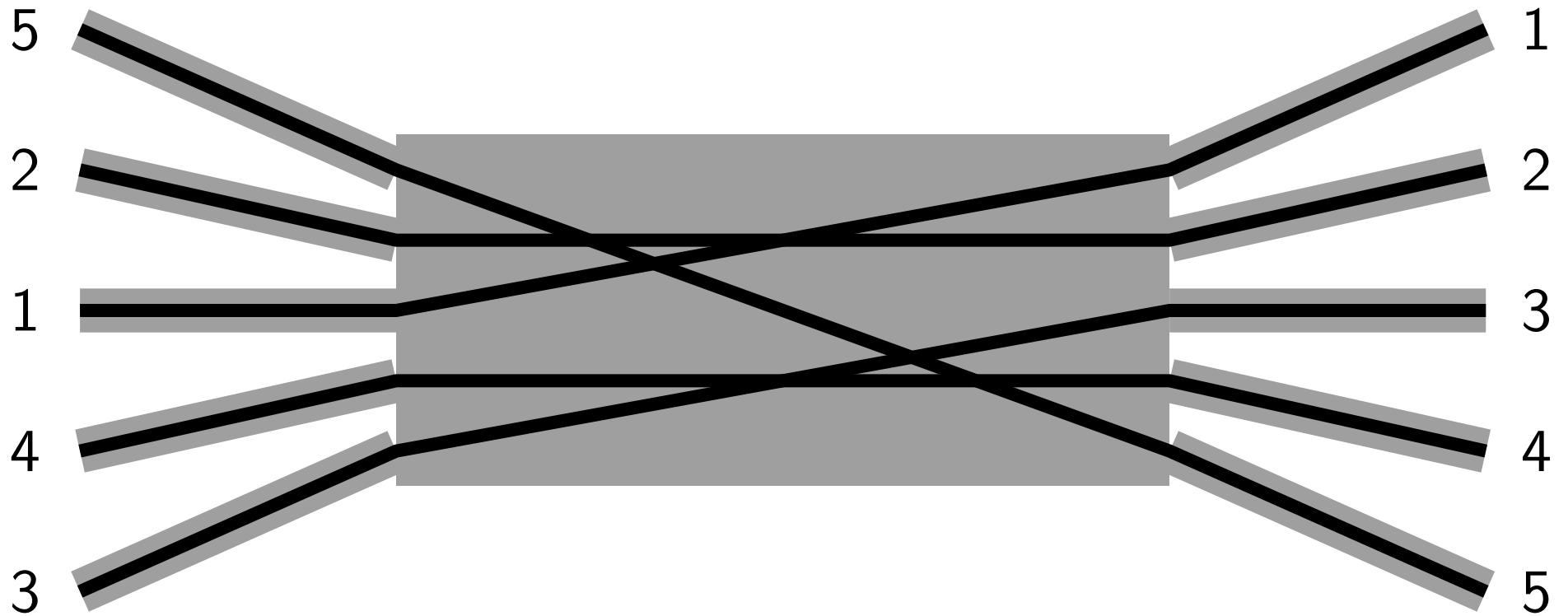
Blockkreuzungen auf einer Kante



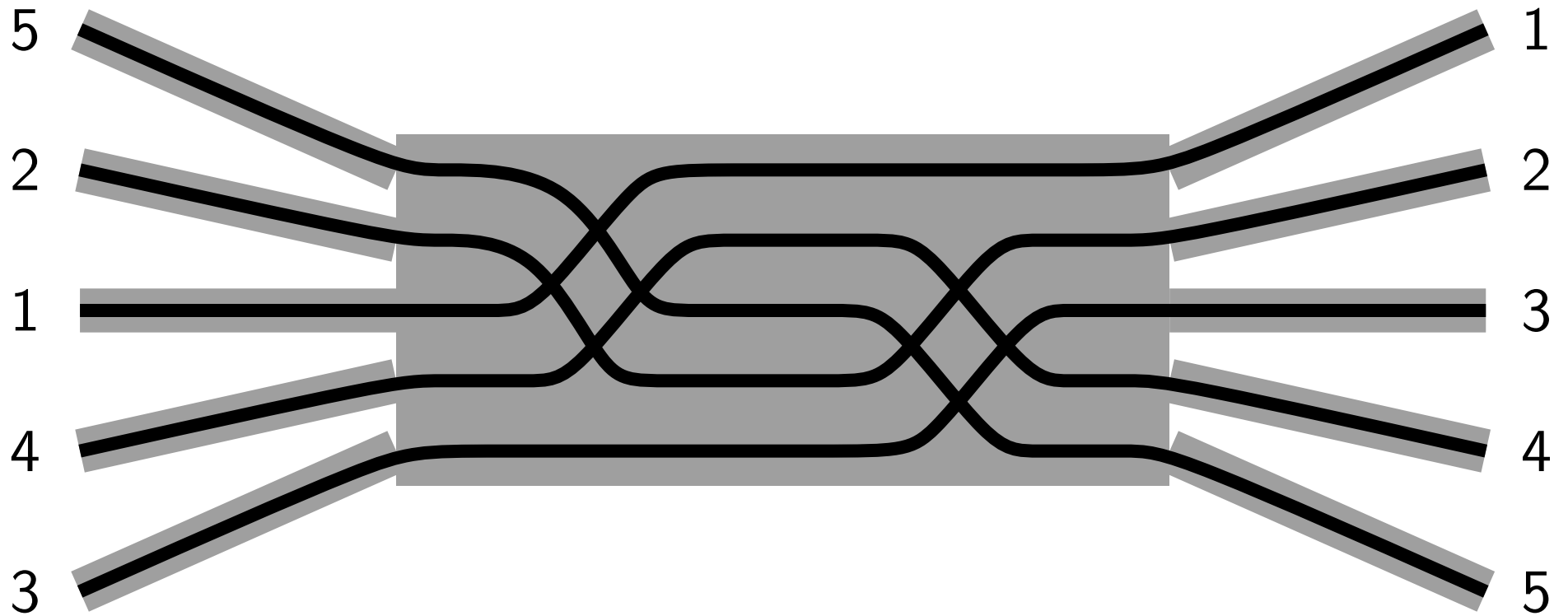
Blockkreuzungen auf einer Kante



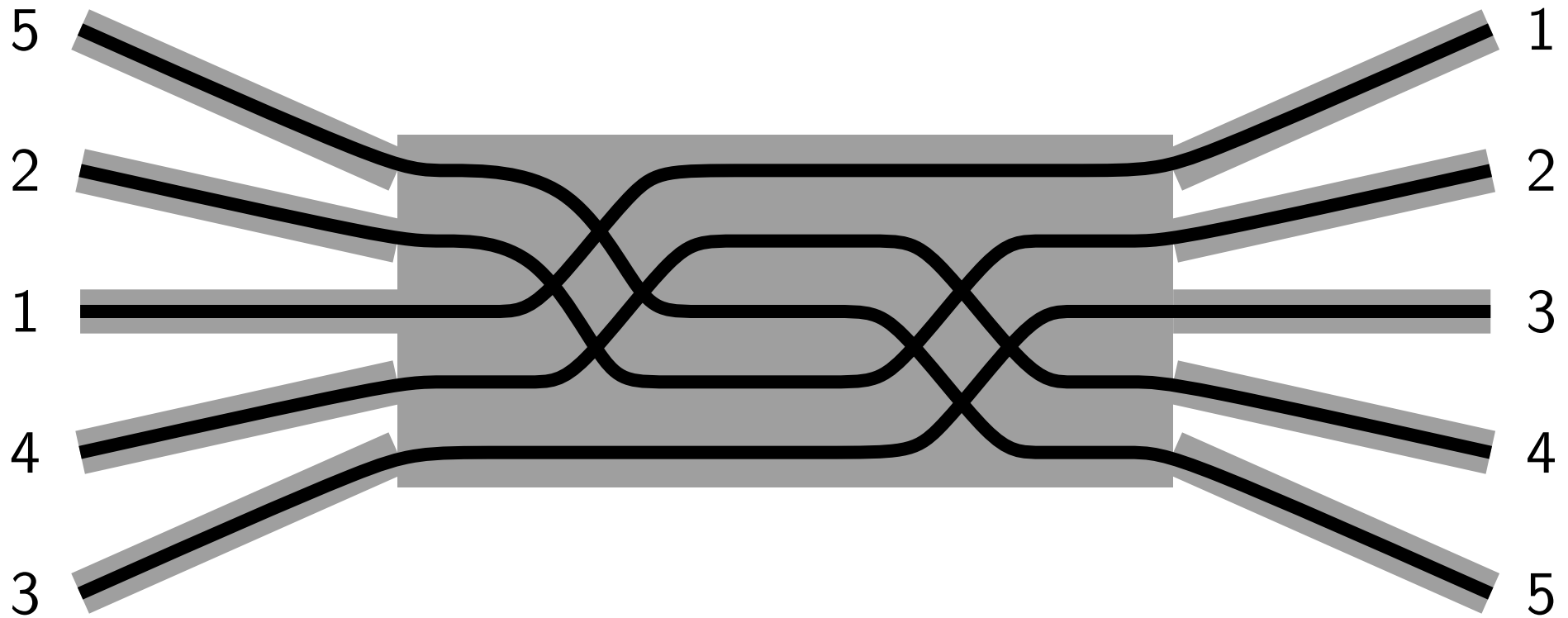
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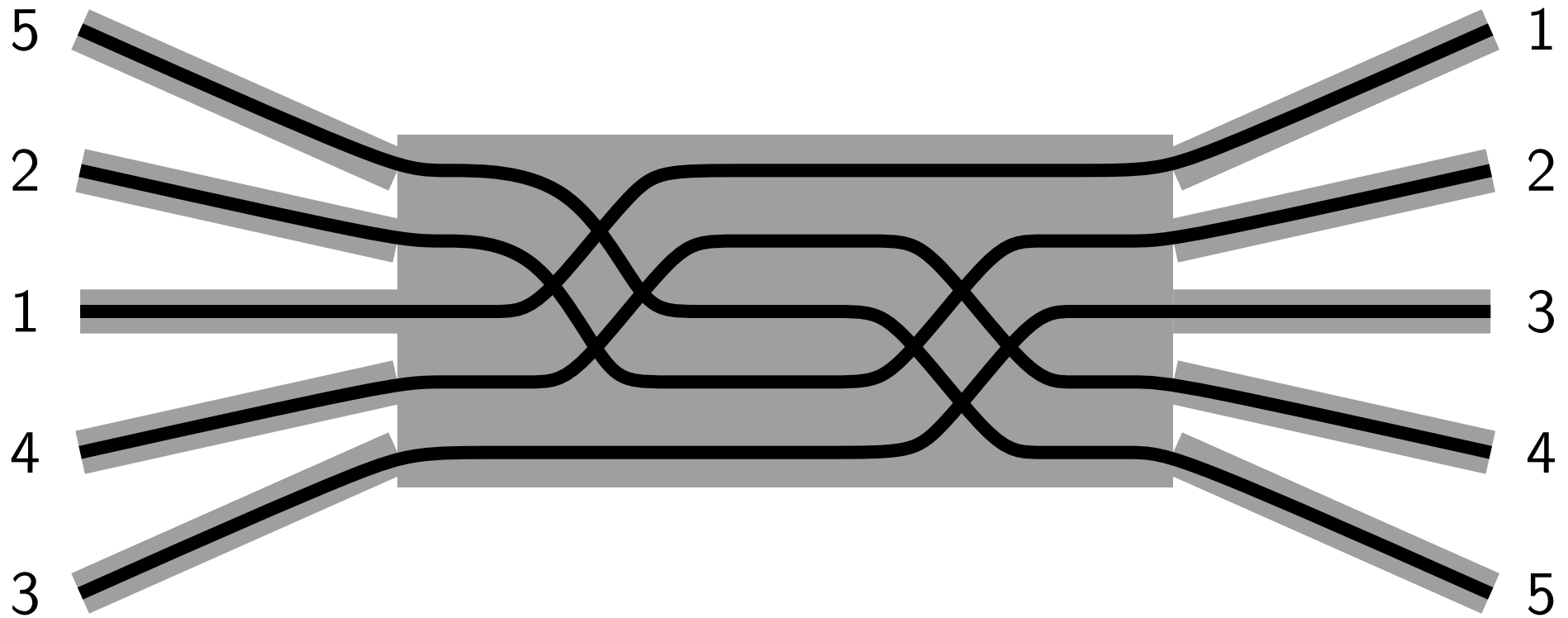


Blockkreuzungen auf einer Kante



- Entspricht Sortierproblem *Sorting by Transpositions*.

Blockkreuzungen auf einer Kante

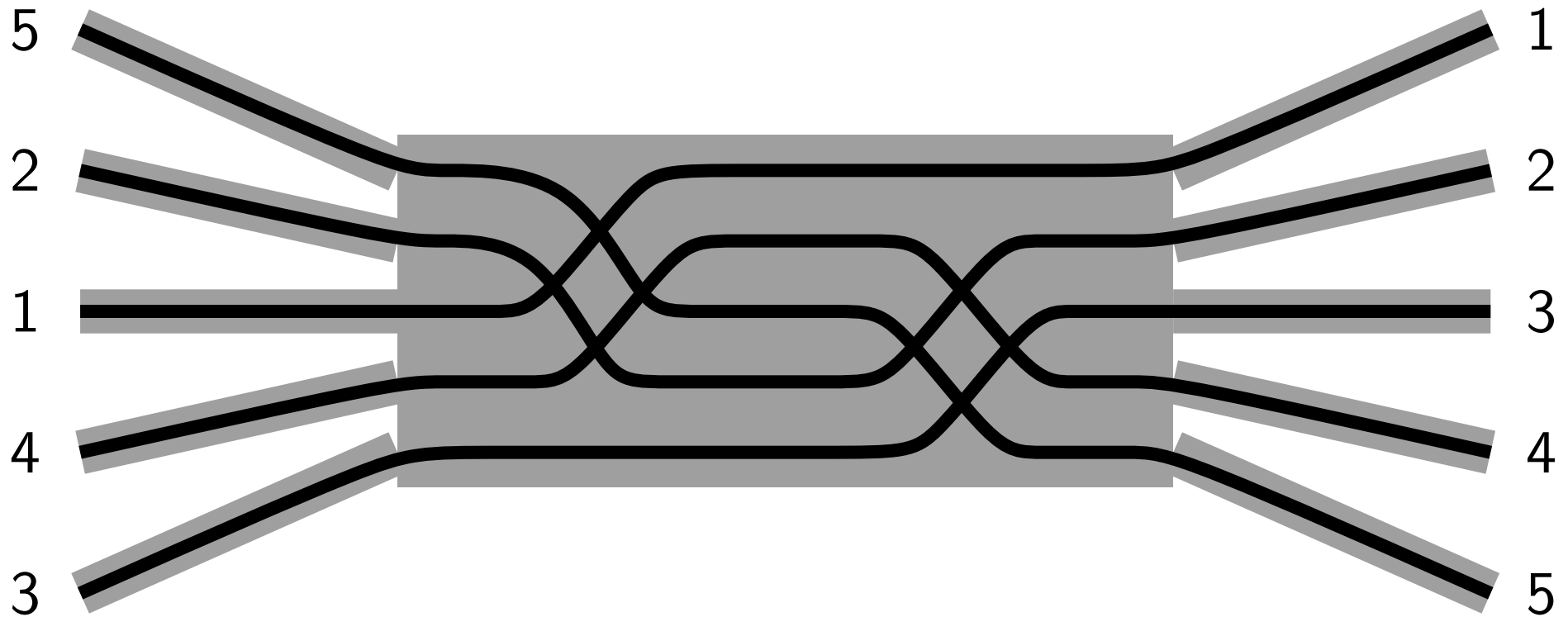


● Entspricht Sortierproblem *Sorting by Transpositions*.

● NP-schwer

[Bulteau et al., 2012]

Blockkreuzungen auf einer Kante



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● einfache 3-Approximation

[Bafna, Pevzner, 1998]

Blockkreuzungen auf einer Kante



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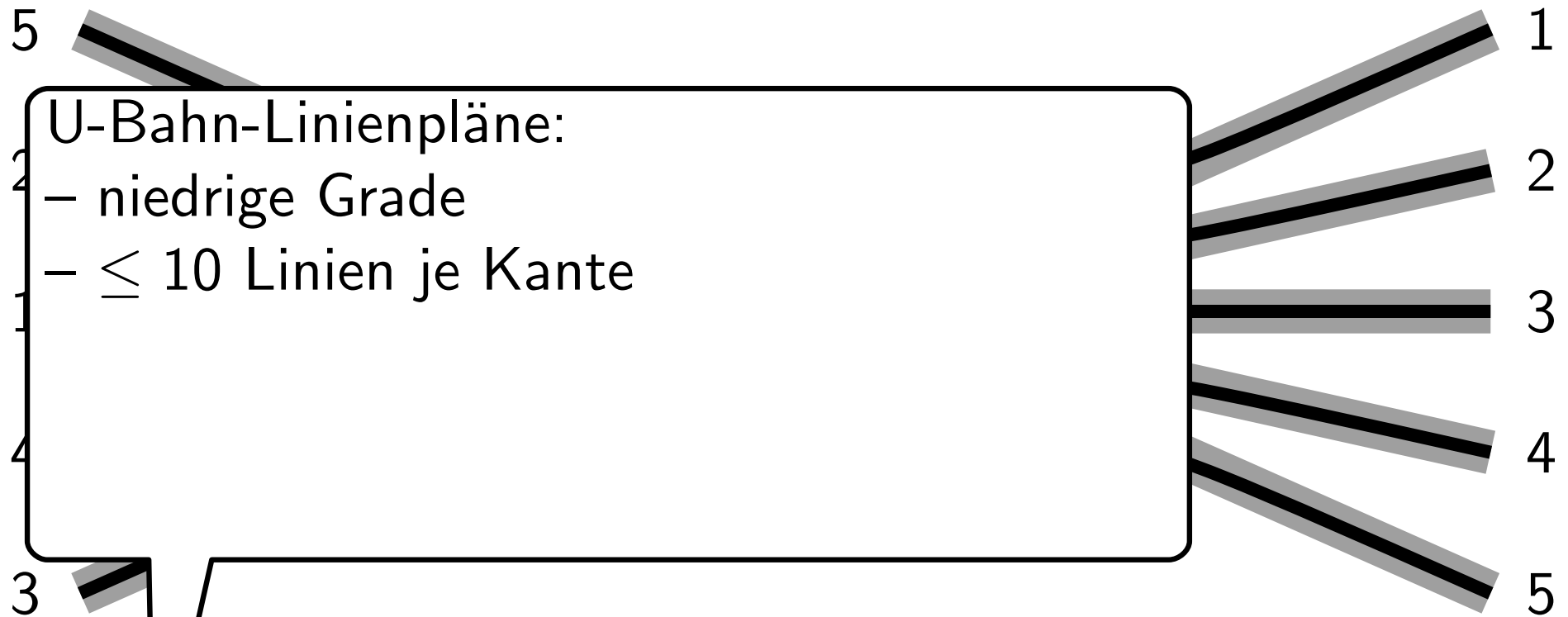
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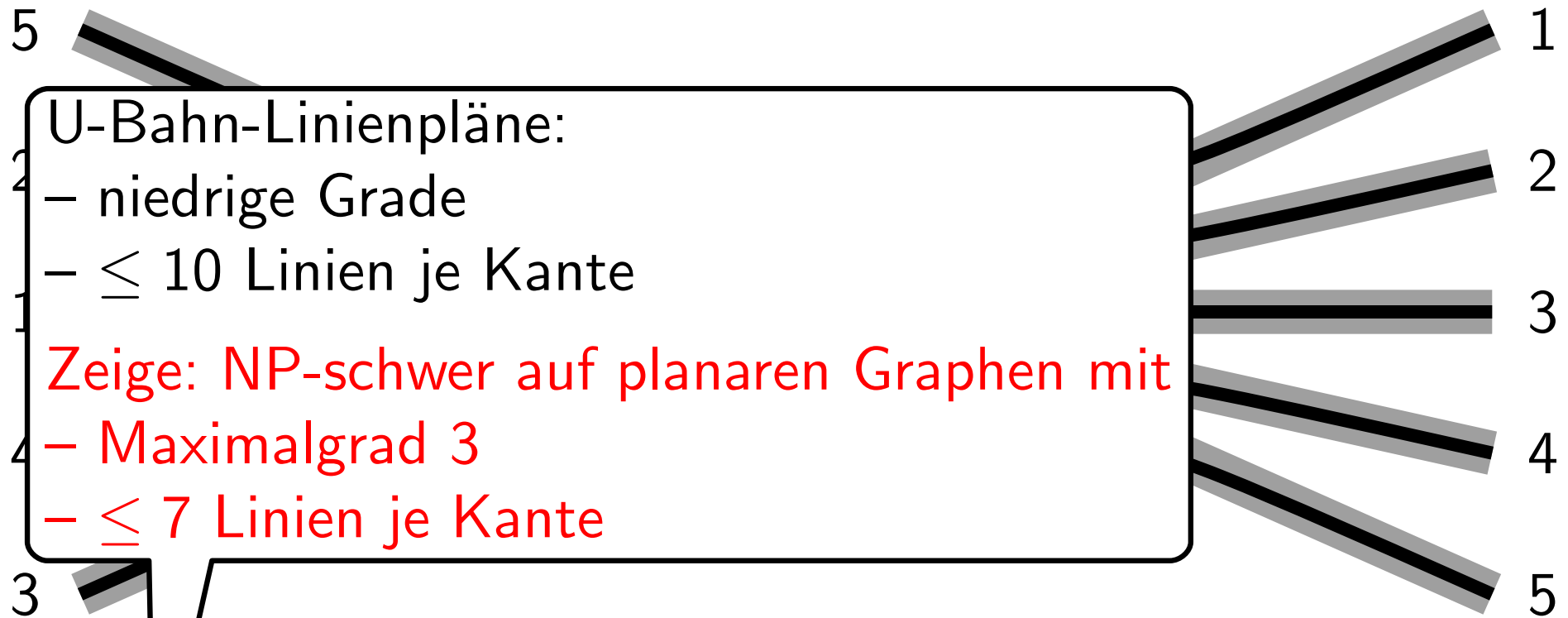
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[Bulteau et al., 2012]

● einfache 3-Approximation

[Bafna, Pevzner, 1998]

Blockkreuzungen auf einer Kante



● Entspricht Sortierproblem *Sorting by Transpositions*.

● NP-schwer

[Bulteau et al., 2012]

● einfache 3-Approximation

[Bafna, Pevzner, 1998]

Blockkreuzungen auf allgemeinen Graphen

Idee:

- Löse Kanten iterativ.
- Platziere jeweils alle *nötigen* Kreuzungen.
- Rechtfertige Blockkreuzungen durch *Informationsgewinn*.

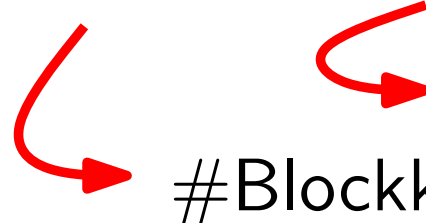
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Genauer:

- $(bc(e))^2 \leq I(e)$

 $bc(e)$ → #Blockkreuzungen auf Kante e

$I(e)$ → Informationsgewinn = #neue Paare von Linien

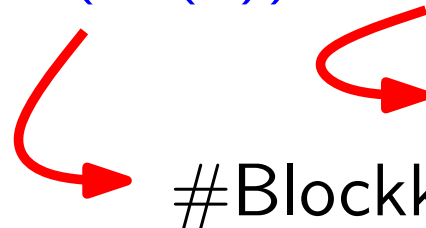
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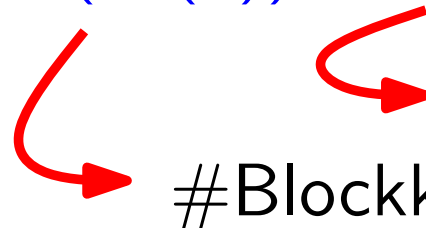
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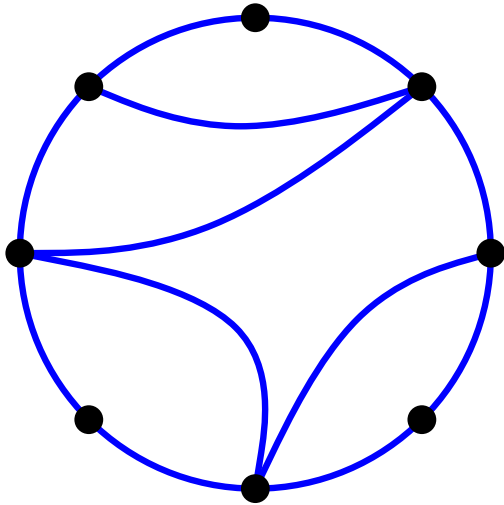
- es ex. Instanzen: $\Omega(|L| \sqrt{|E|})$ Blockkreuzungen nötig

Teil II:

Punkt mengen-Einbettbarkeit

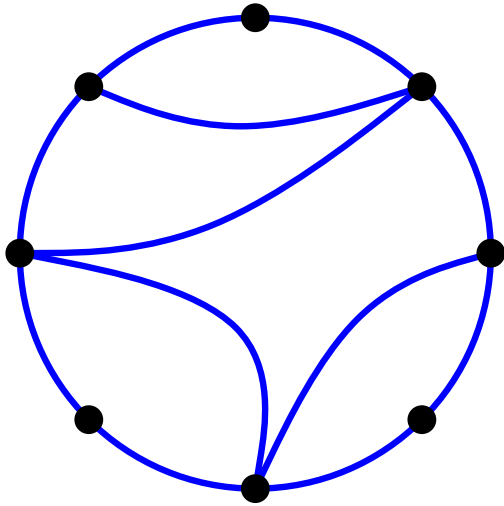
Punkt mengen-Einbettbarkeit

Graph

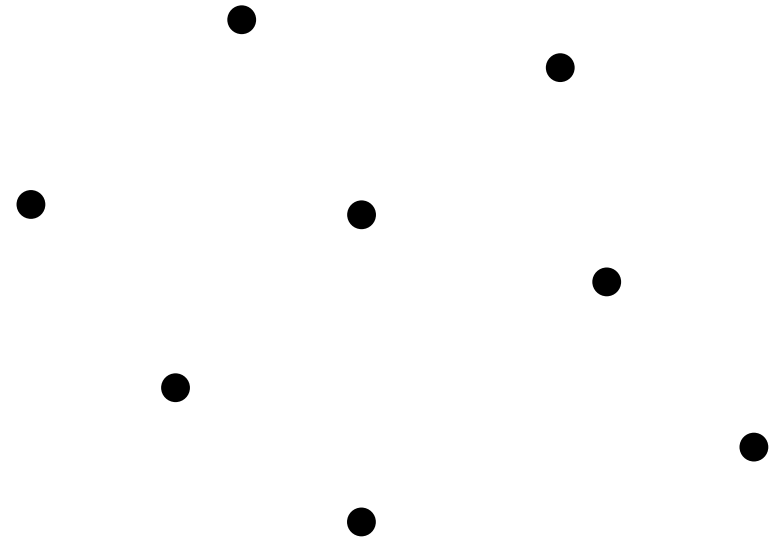


Punktmenge-Einbettbarkeit

Graph

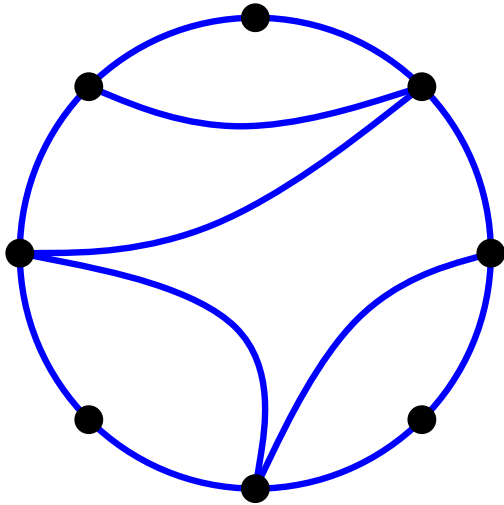


Punktmenge

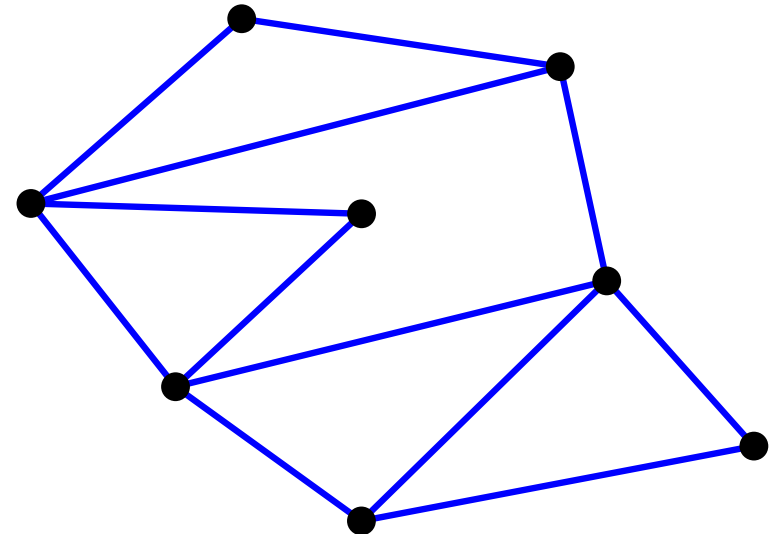


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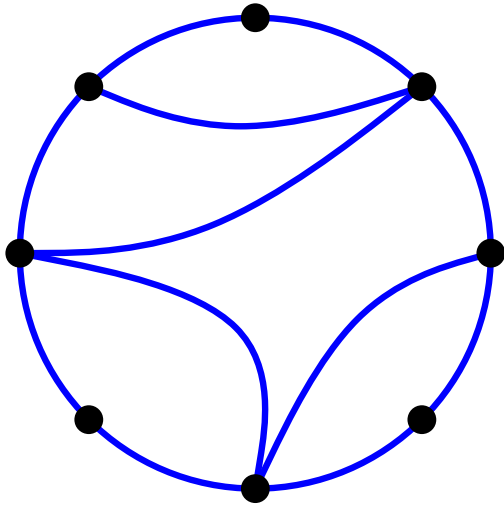


Punktmenge



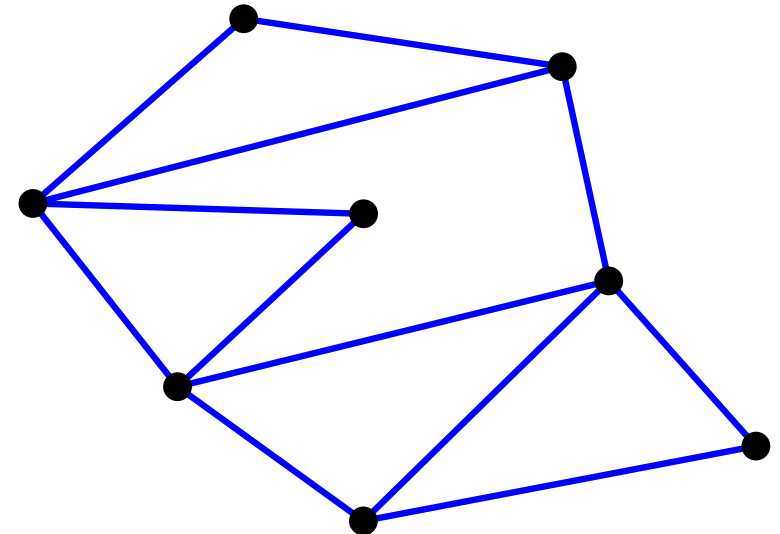
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Graph



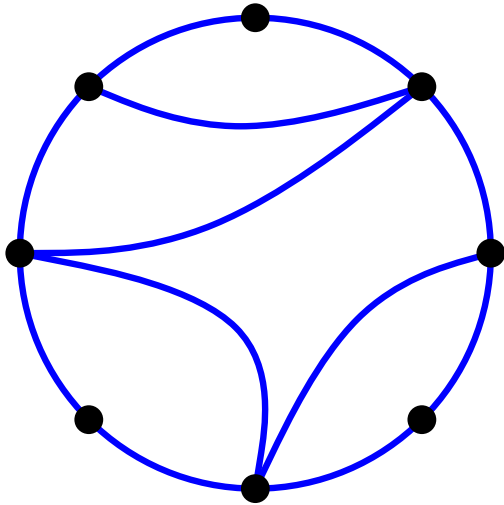
finde
→
Einbettung

Punktmenge



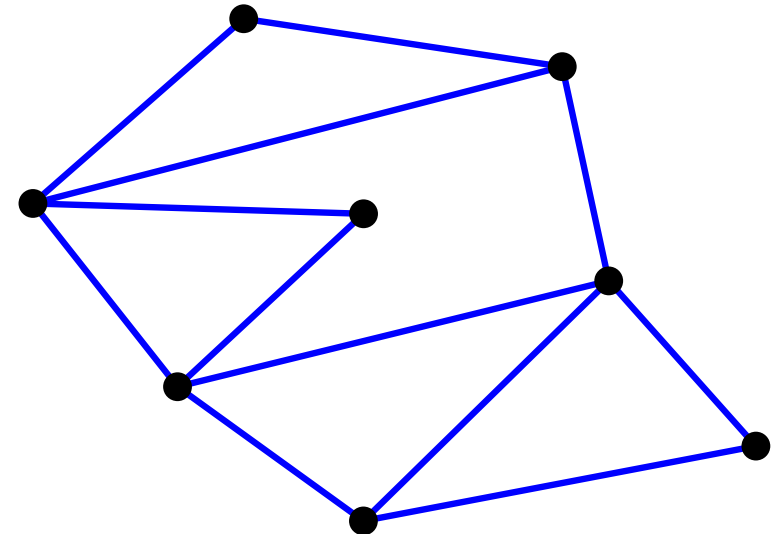
Punktmenge-Einbettbarkeit

Graph



finde
→
Einbettung

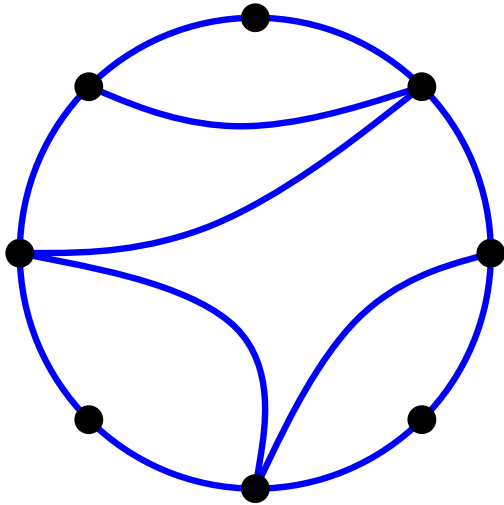
Punktmenge



planar, geradlinig: NP-schwer [Cabello, 2006]

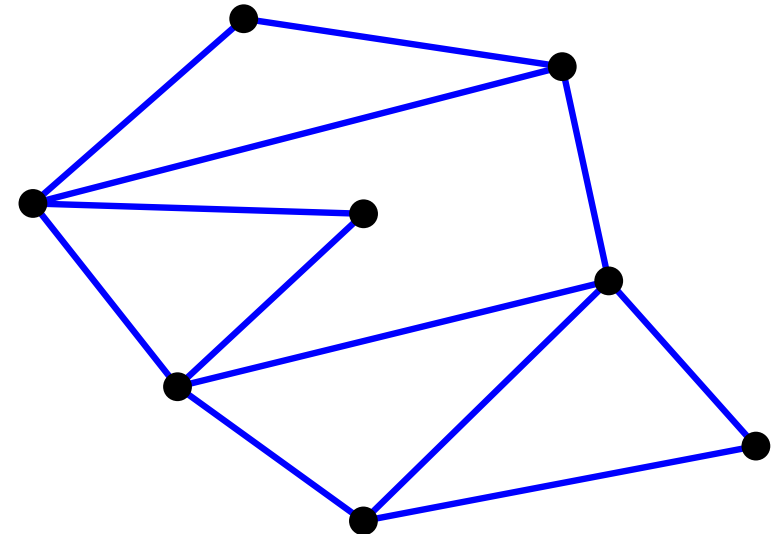
Punktmengen-Einbettbarkeit

Graph



finde
→
Einbettung

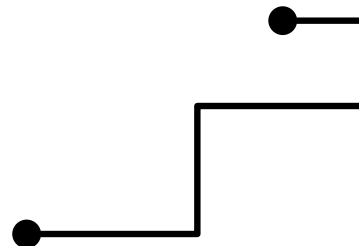
Punktemenge



planar, geradlinig: NP-schwer [Cabello, 2006]

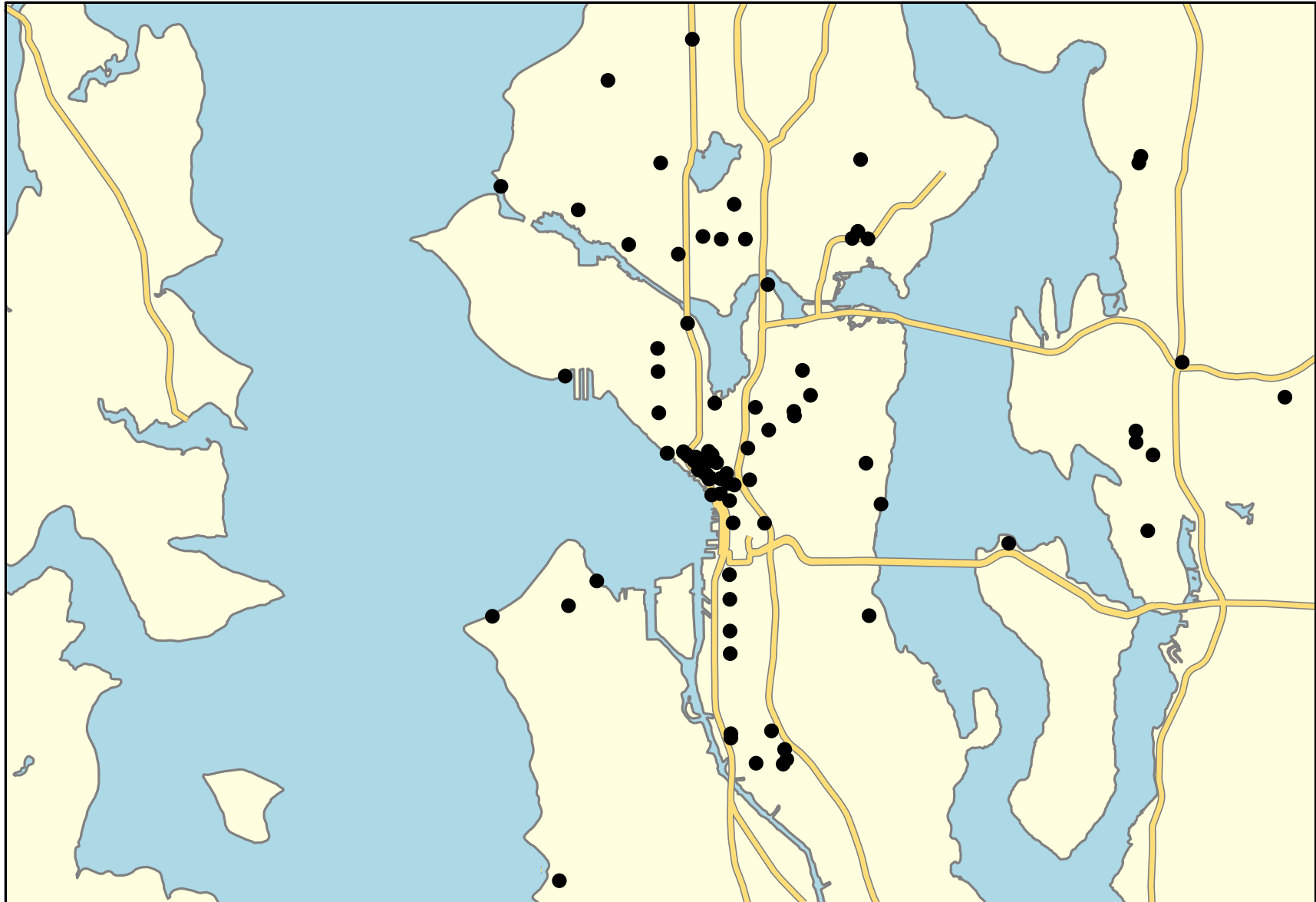
untersucht:

- große Kreuzungswinkel
- orthogonaler Stil

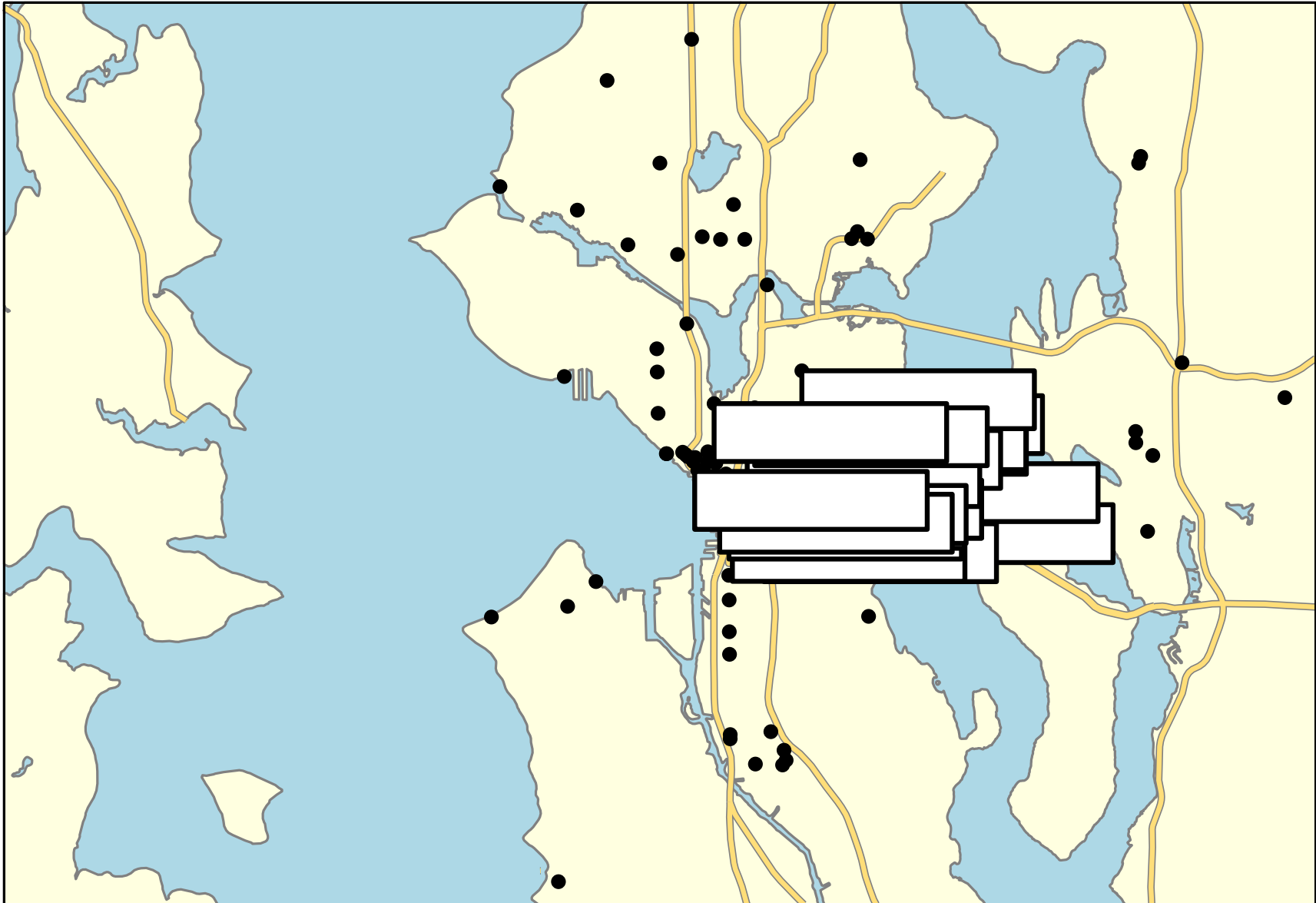


Teil III: Randbeschriftungen

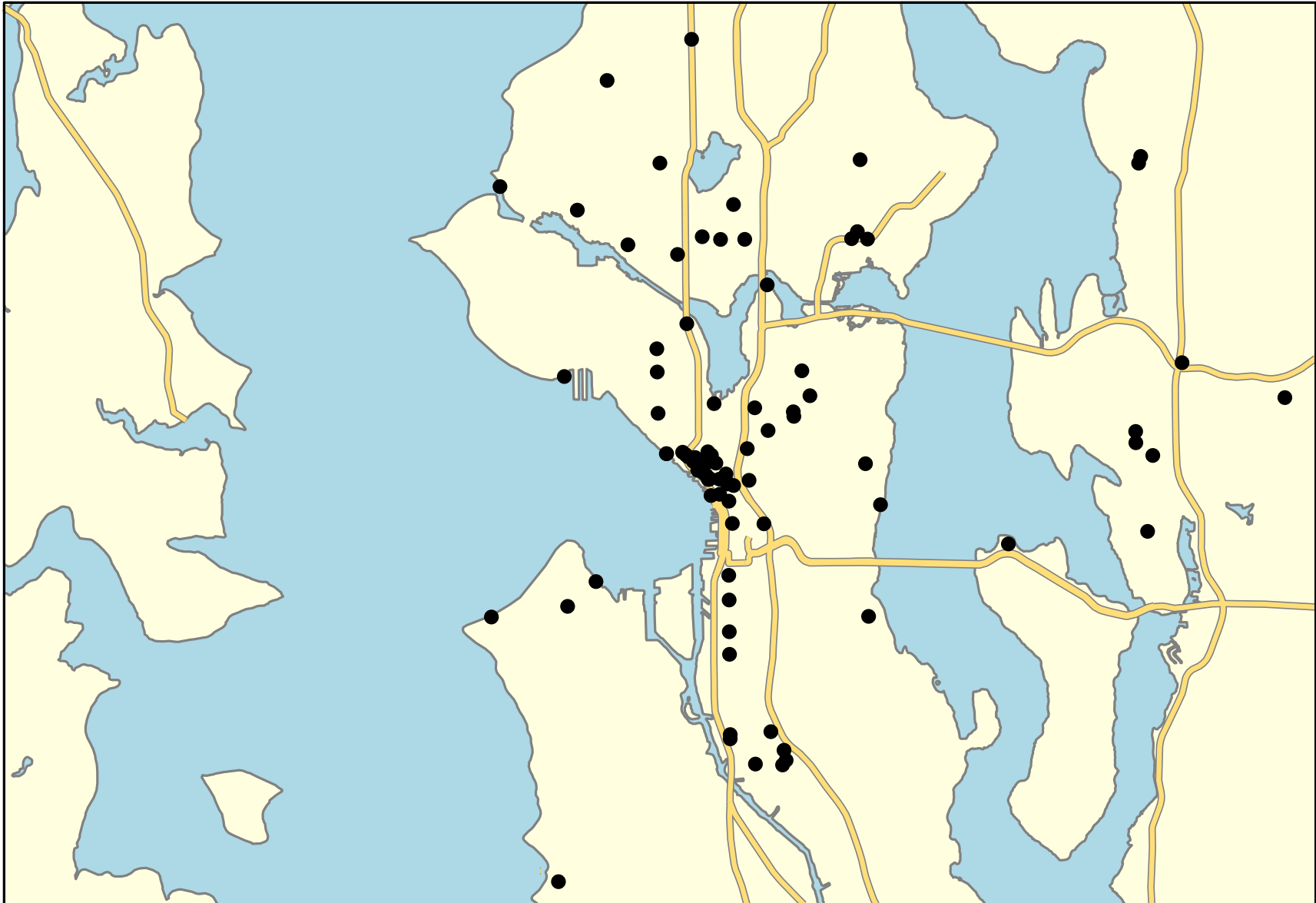
Aufgabe: Beschriftung von Landkarten



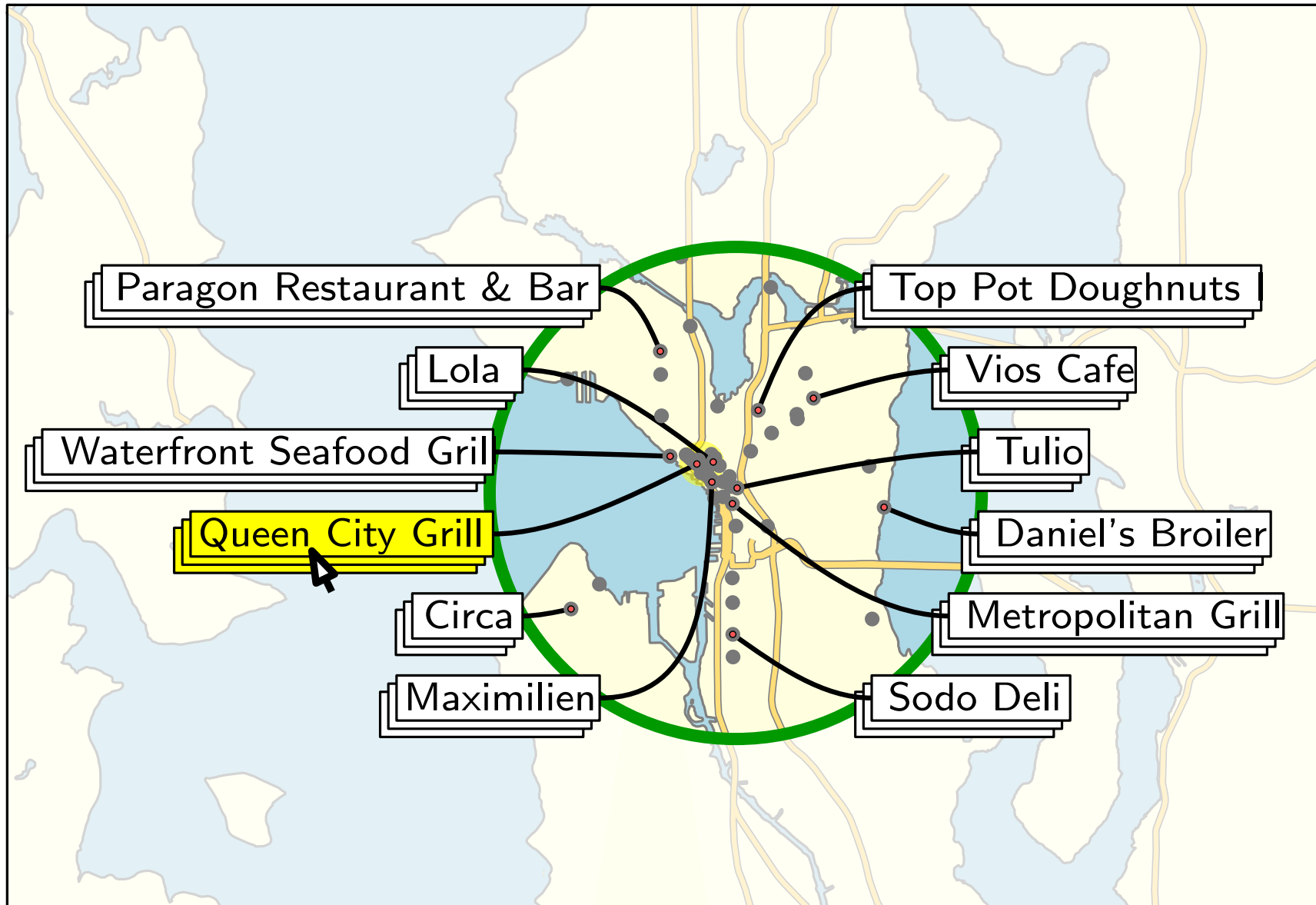
Aufgabe: Beschriftung von Landkarten



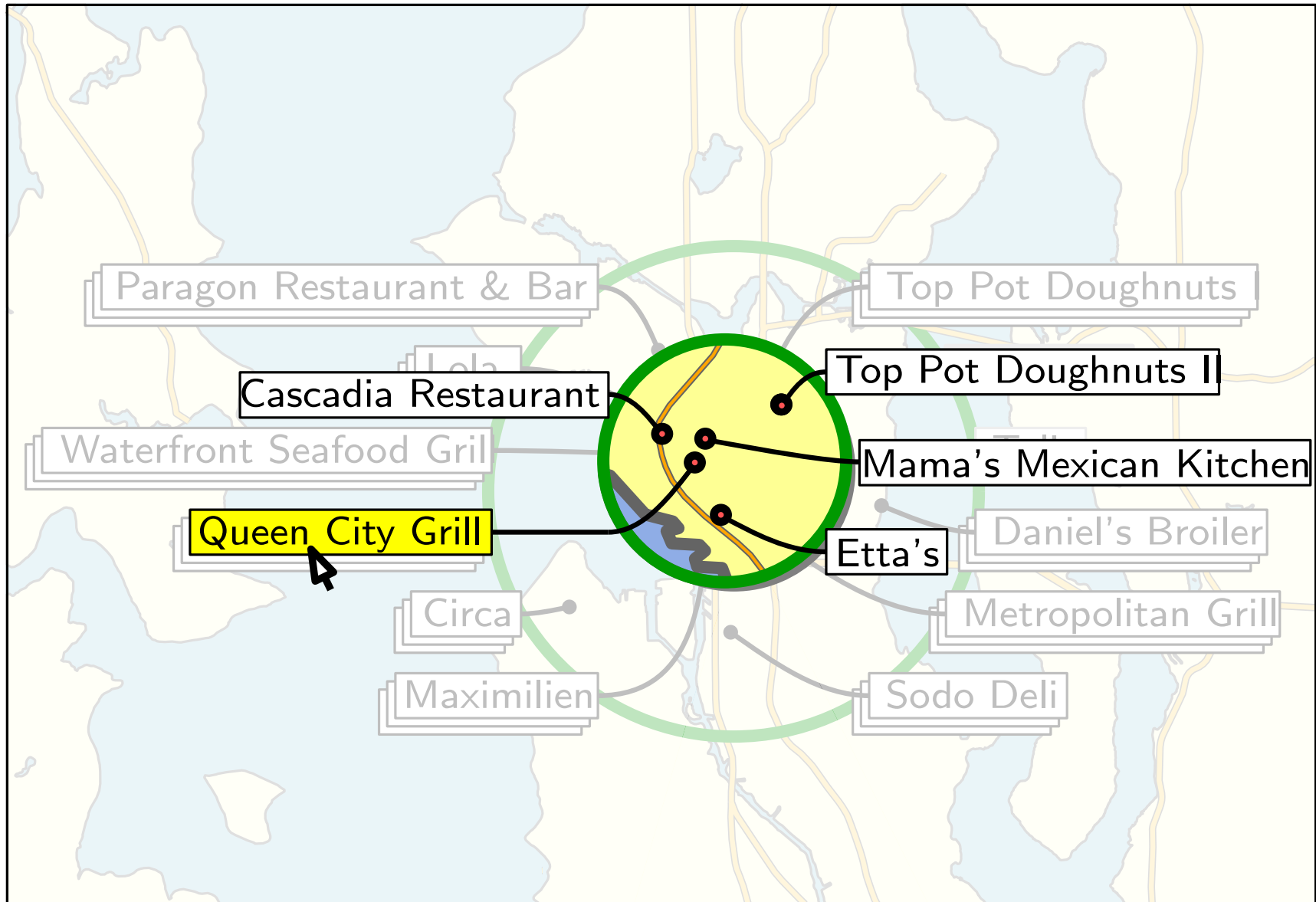
Aufgabe: Beschriftung von Landkarten



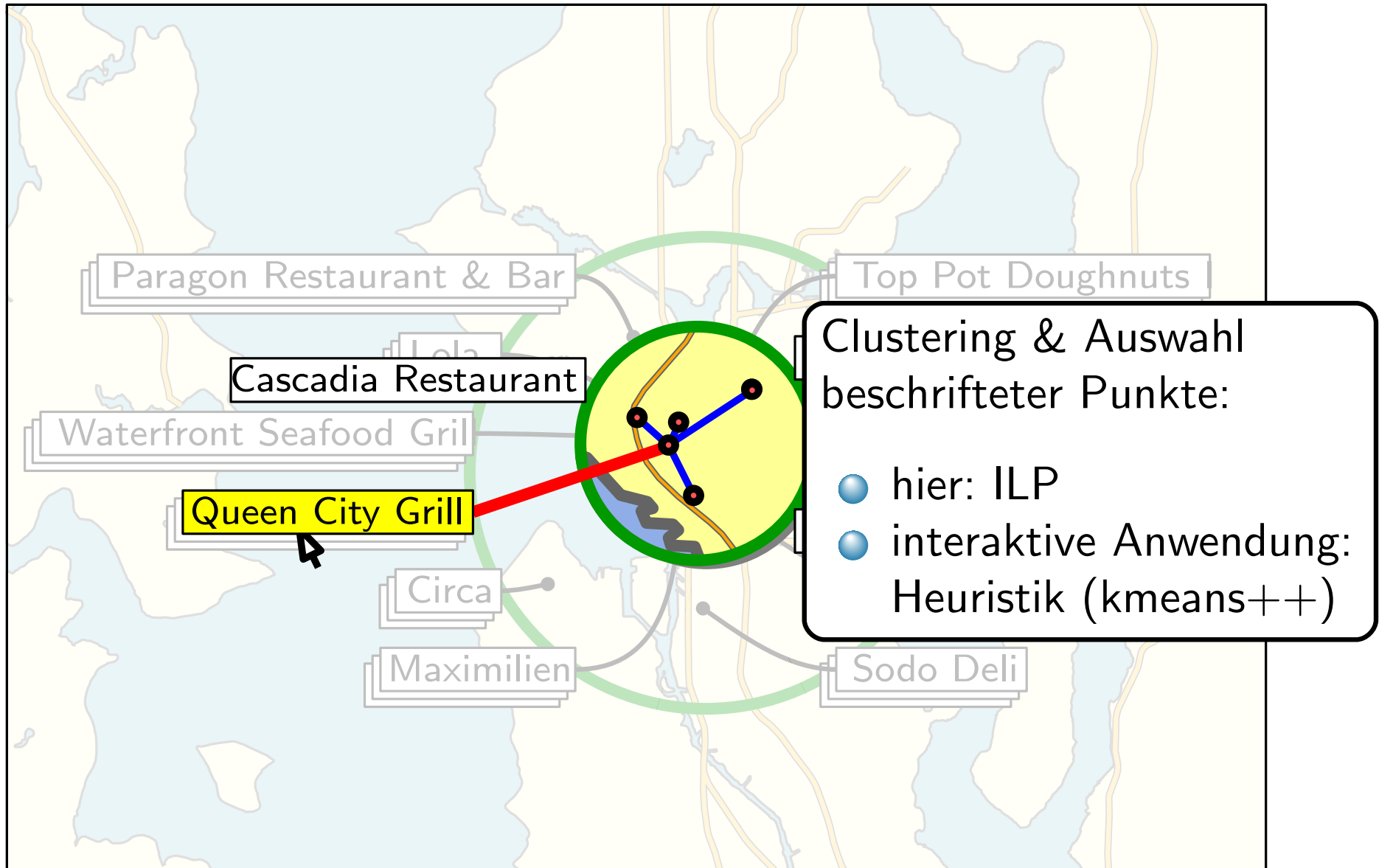
Aufgabe: Beschriftung von Landkarten



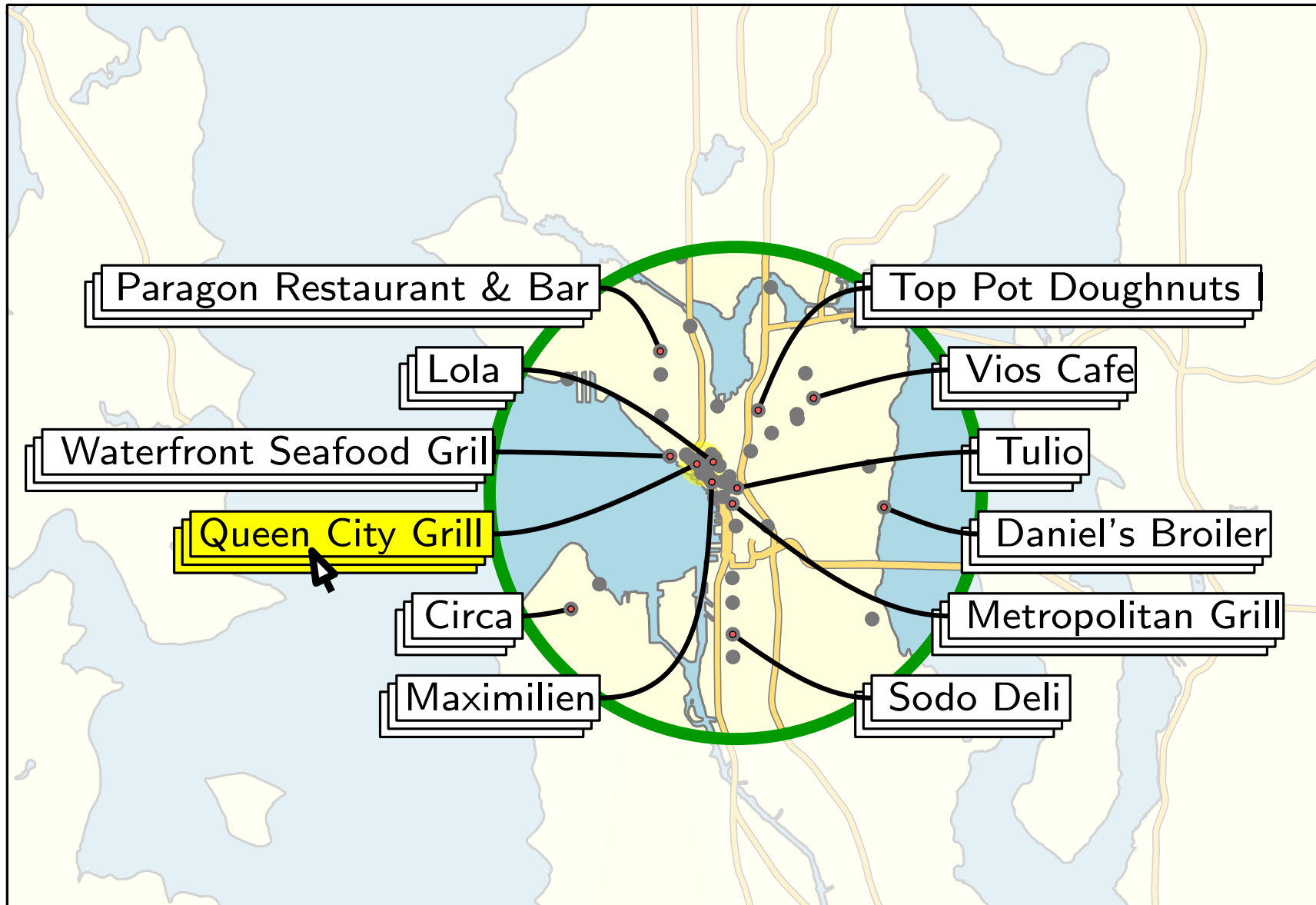
Aufgabe: Beschriftung von Landkarten



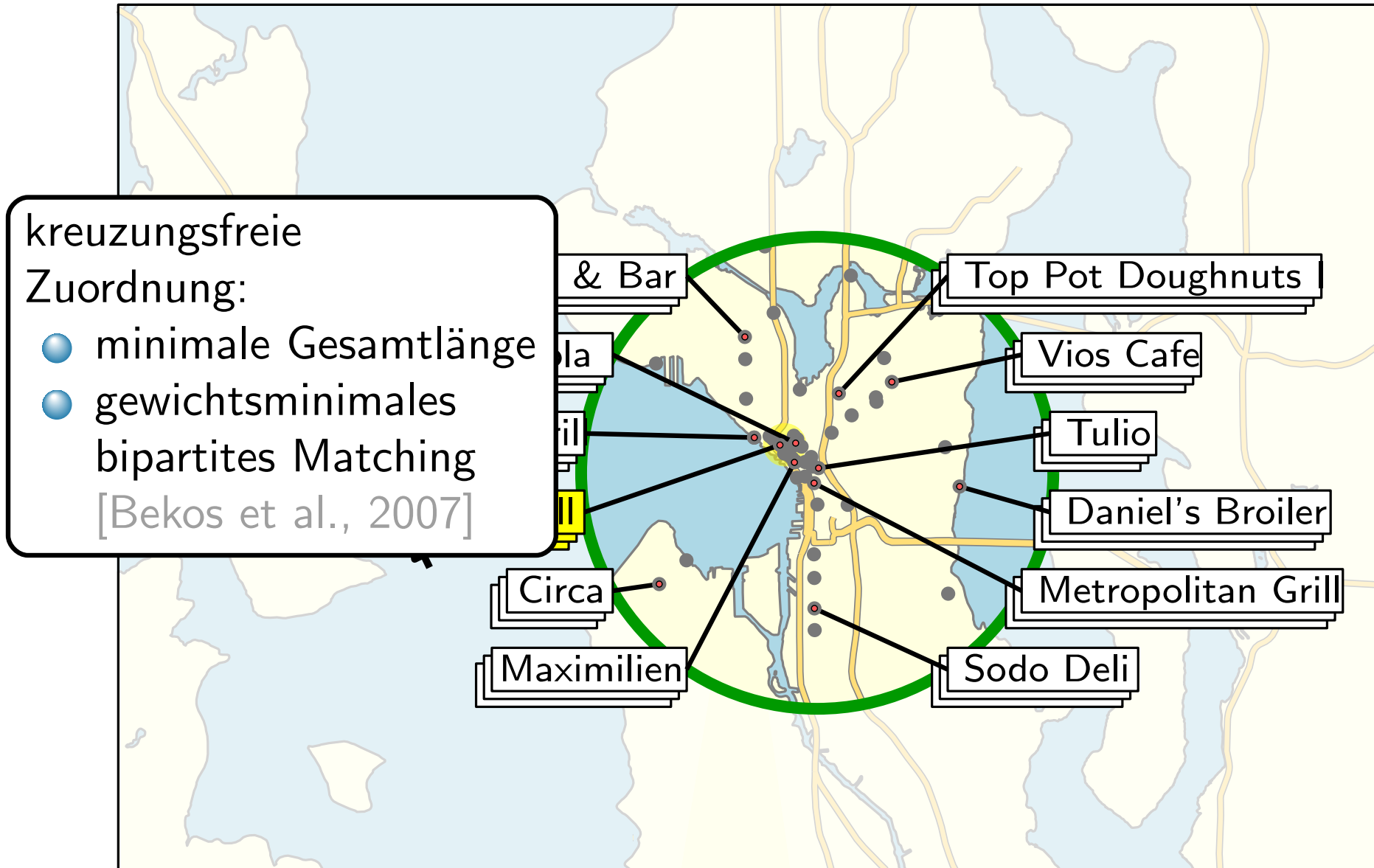
Aufgabe: Beschriftung von Landkarten



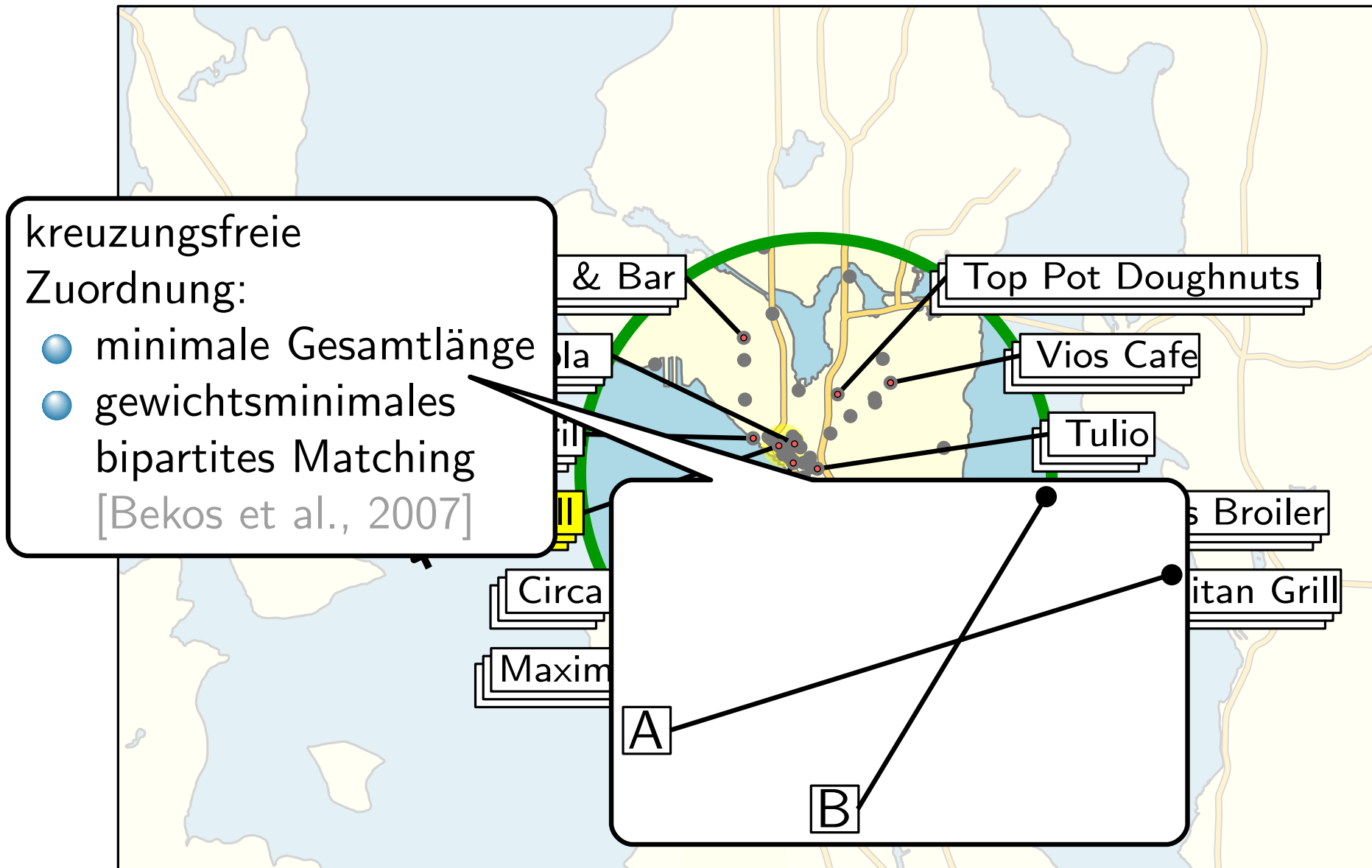
Aufgabe: Beschriftung von Landkarten



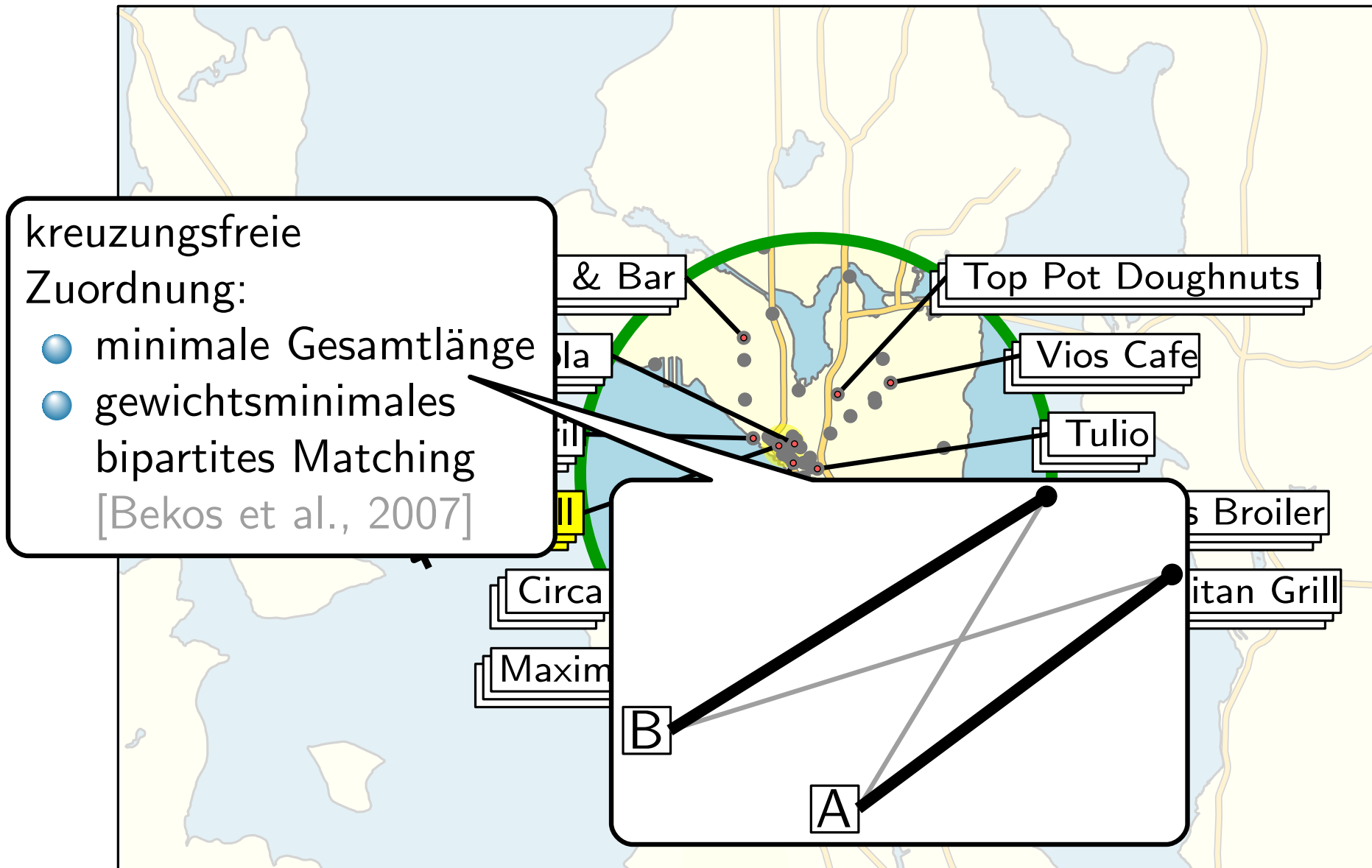
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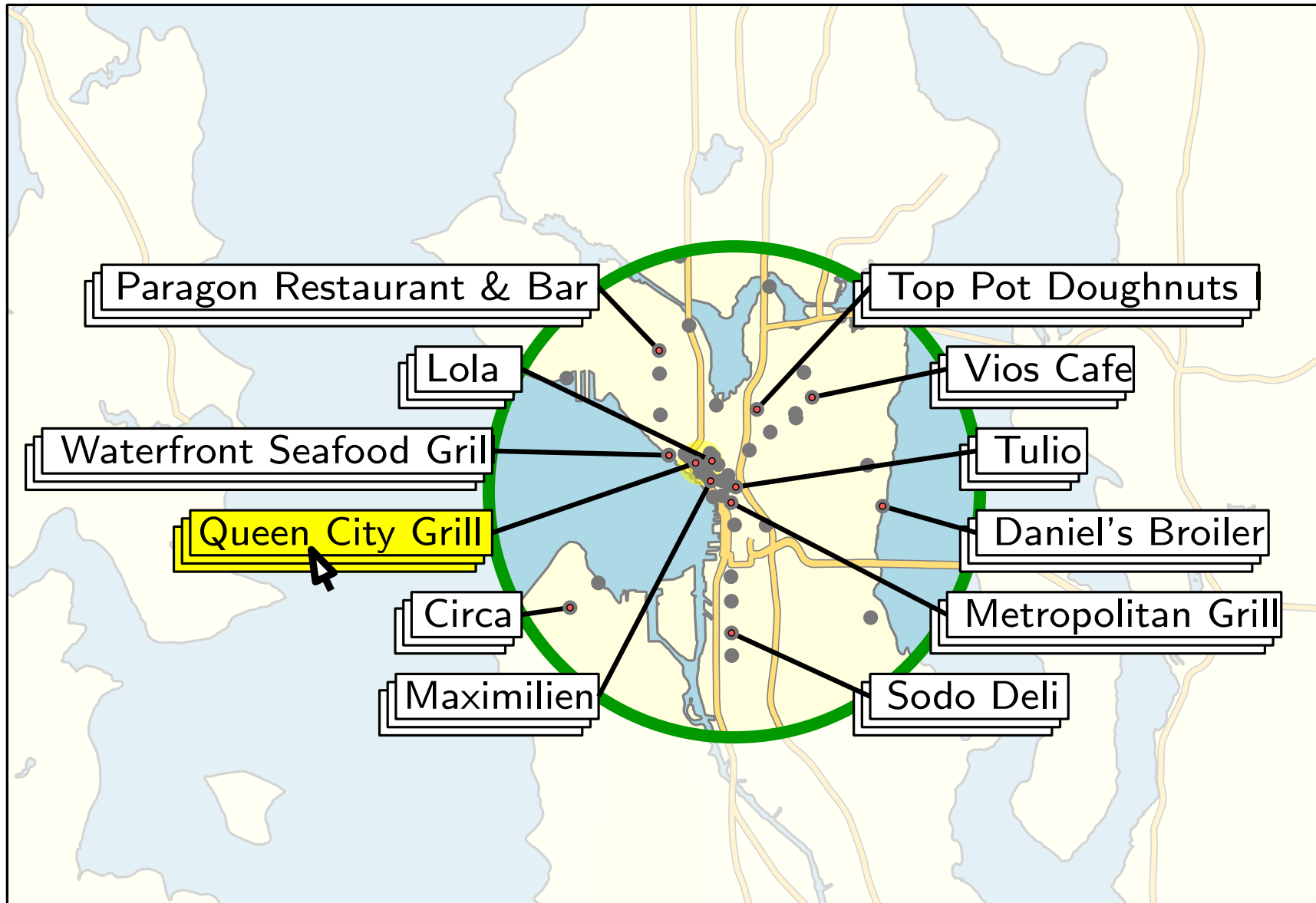
Aufgabe: Beschriftung von Landkarten



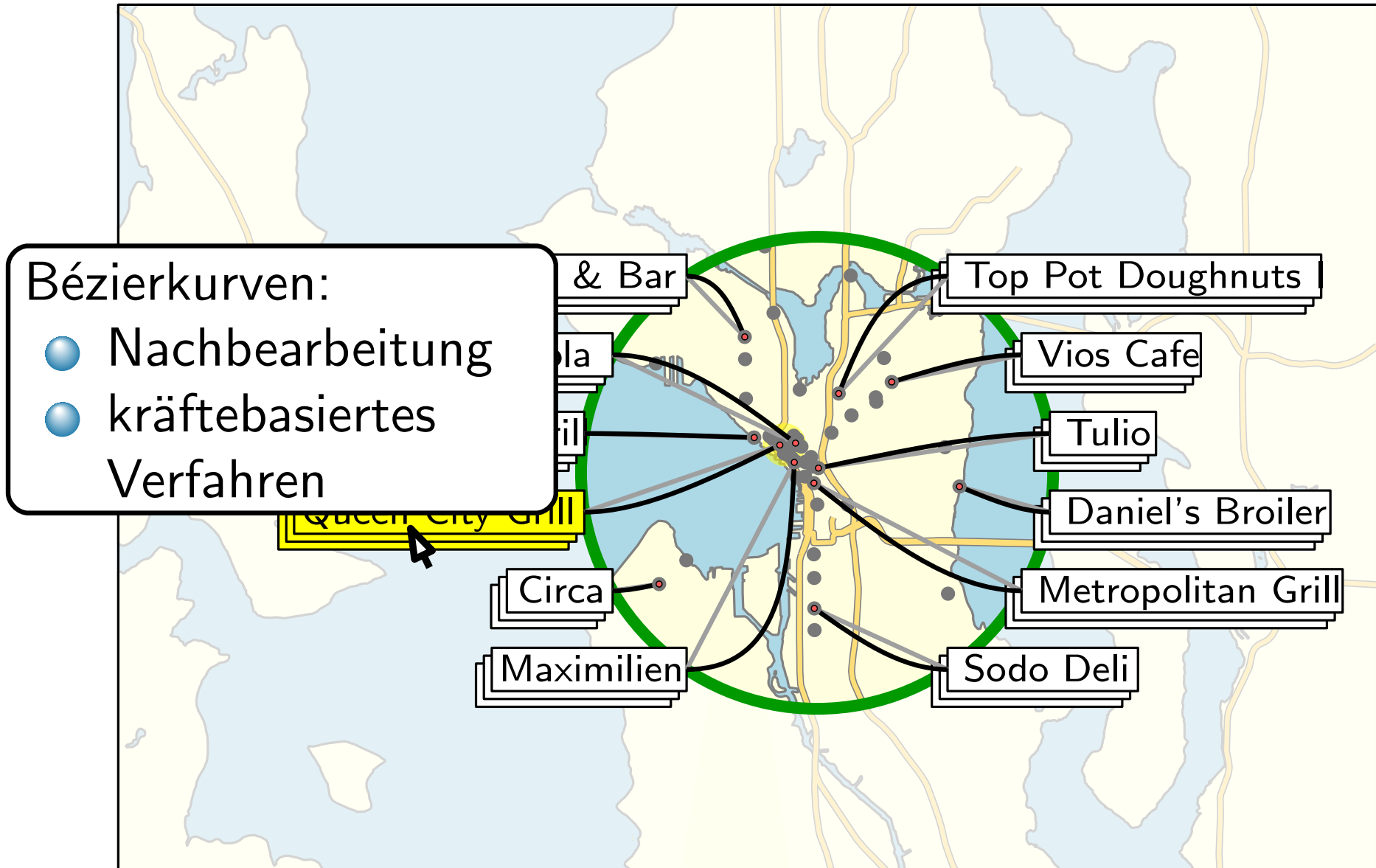
Aufgabe: Beschriftung von Landkarten



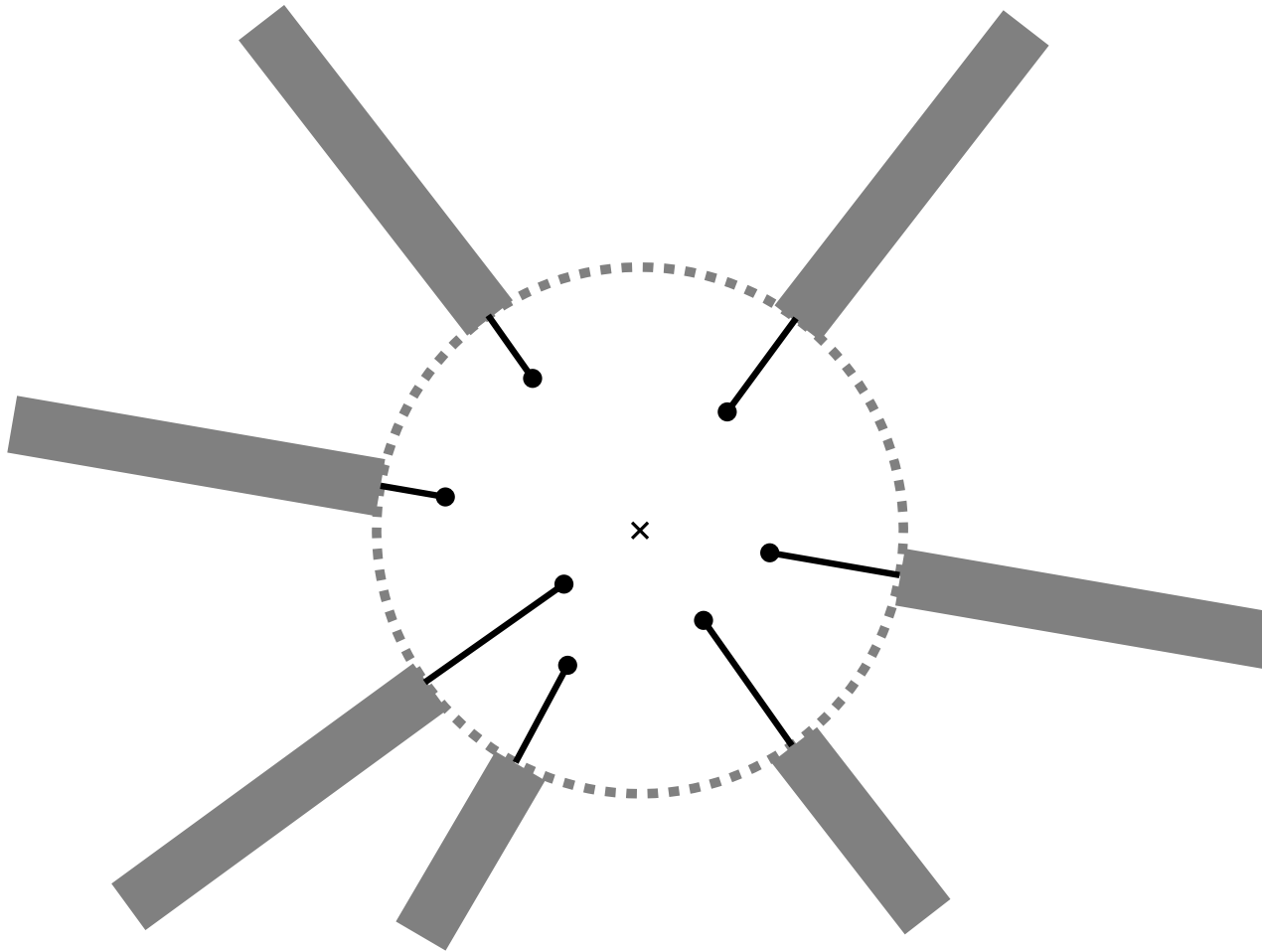
Aufgabe: Beschriftung von Landkarten



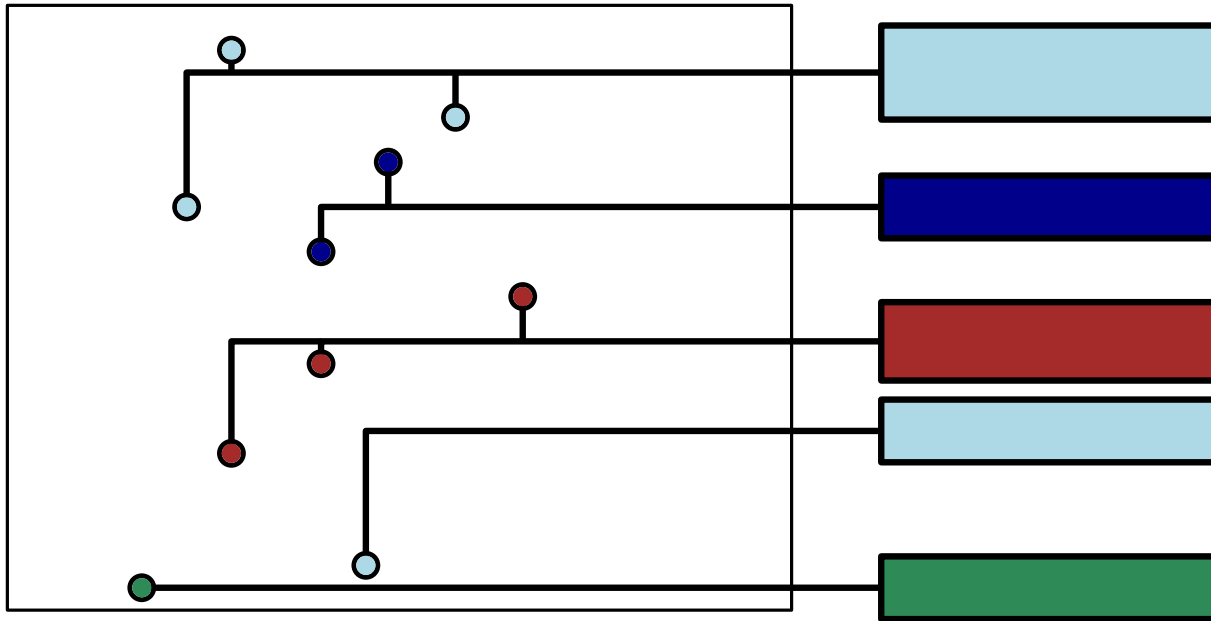
Aufgabe: Beschriftung von Landkarten



Radiale Beschriftungen

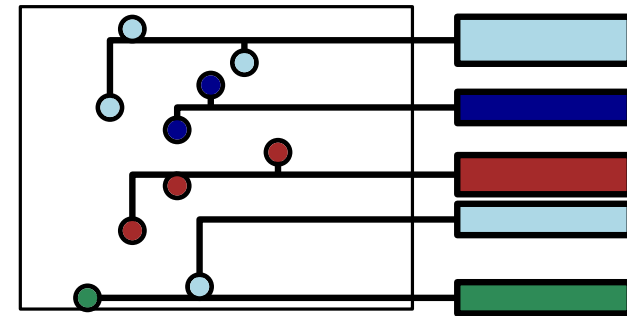
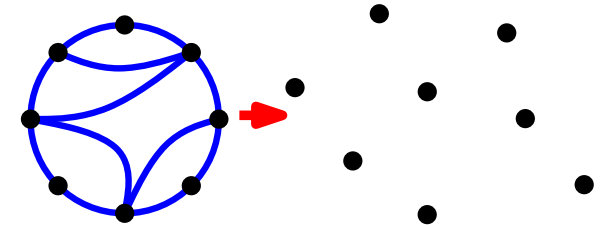
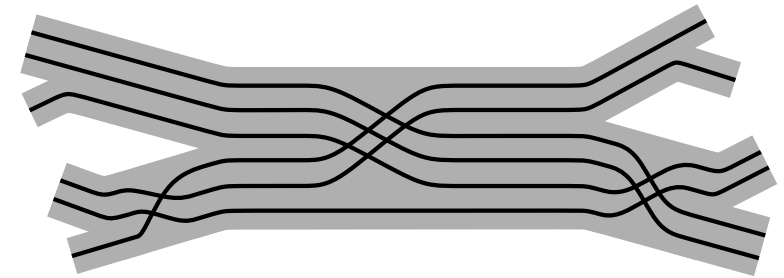
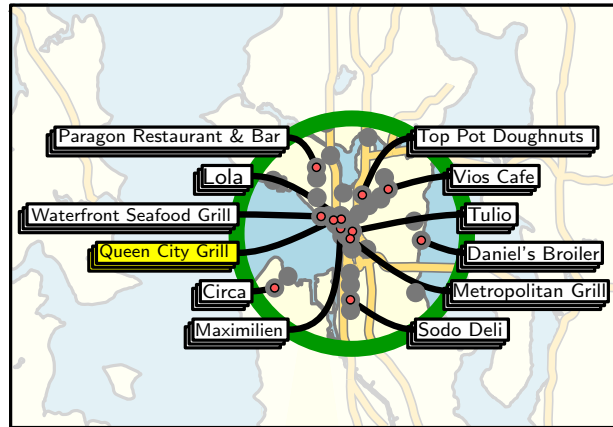
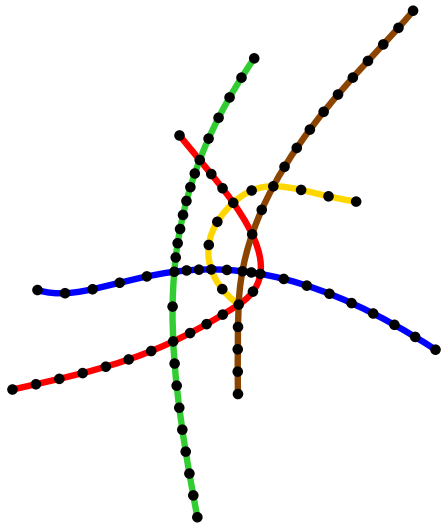


Mehrfachbeschriftungen



Beschriftung nach
Kategorien

Zusammenfassung



← experimentell

→ theoretisch