Symmetry Detection in Building Footprints

Presentation of the Master's thesis

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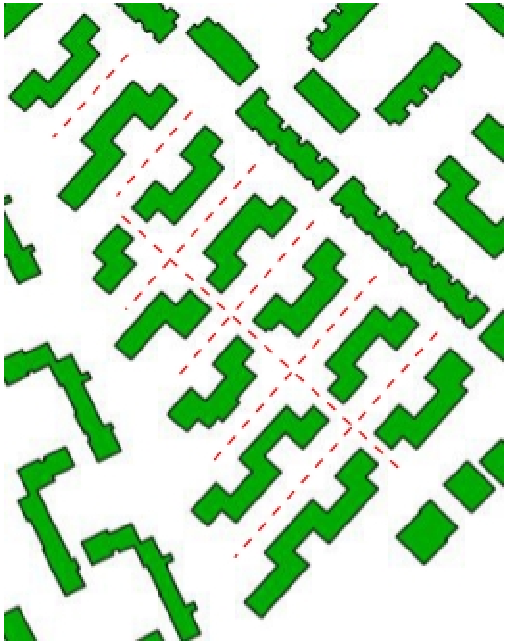
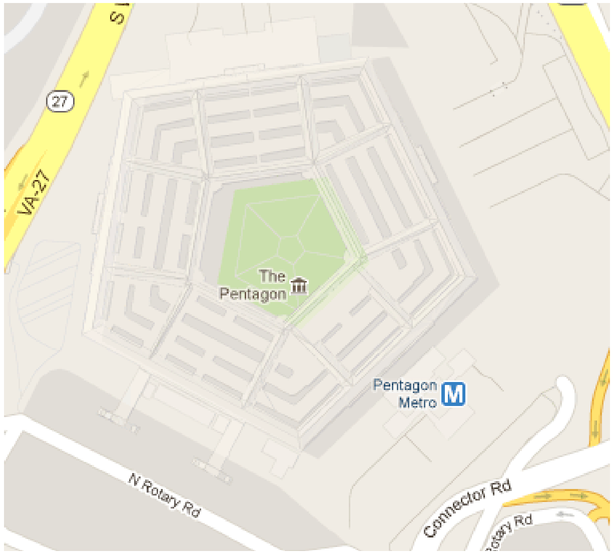
Introduction

- Symmetry is a fundamental element of design in architecture



Building group with reflectional symmetries (building dataset of Boston)



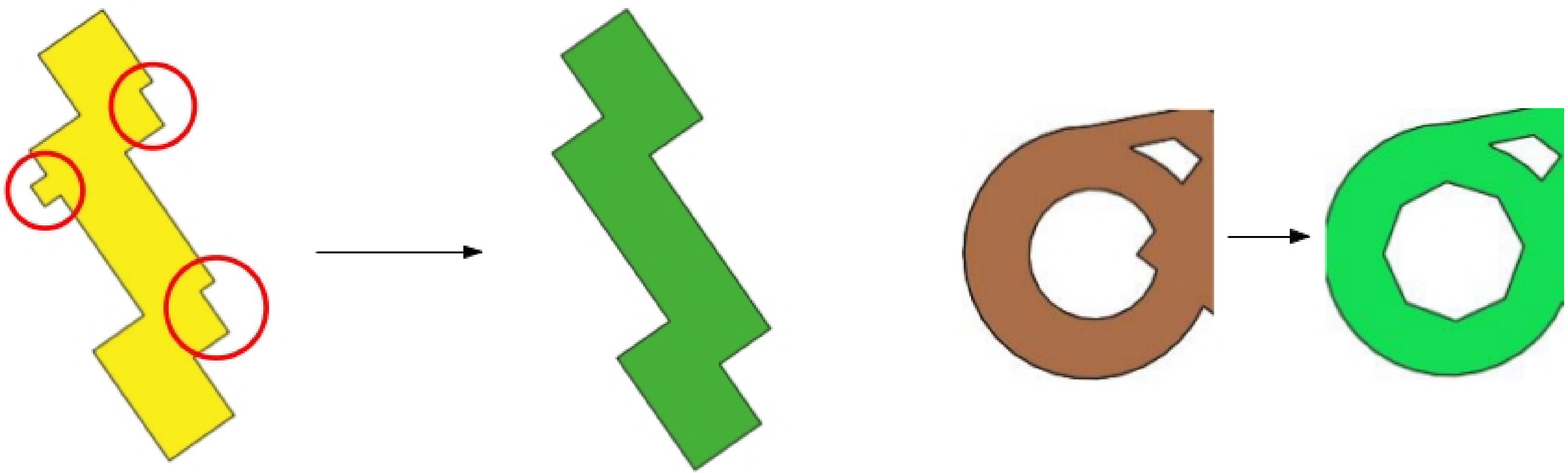
The rotational symmetric Pentagon (Google maps)

Introduction

- Applications
  - Symmetry aware building simplification
    - Preserving main characteristics
    - Recognizability
    - Aesthetic
  - Landmark selection
    - Humans are extremely good in detecting symmetries
  - Building classification according to functionality
    - Symmetry as a shape feature

Introduction

- Challenge
  - Simplification required



Simplifications obtained by „Pentagon“

Overview

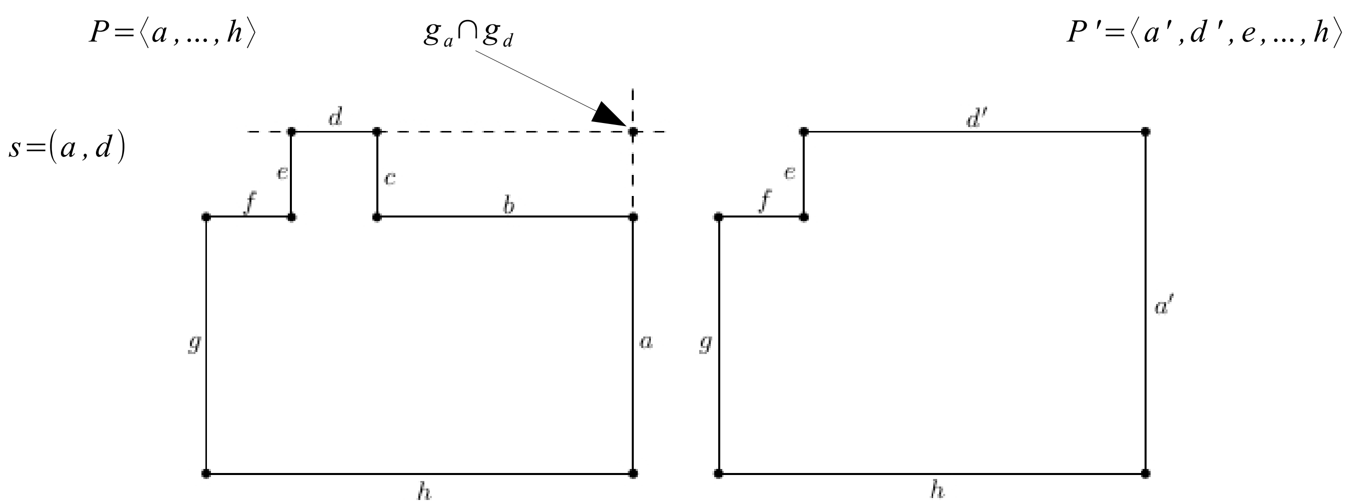
- Symmetry detection by Lladós et al.
- Simplification approach by Haunert and Wolff
- Comparison graph
- Symmetries between two different footprints
- Symmetries within one footprint
- Summery
- Open problems

Symmetry detection by Lladós et al.

- Polygons as sequences of edges in String-representation
- Comparision by dynamic programming detects symmetries
- New: operations for merging edges on the flow (simplification)
- We have a good example that will cause failing anyway
- Maybe working well for polygonally approximated shapes from image data

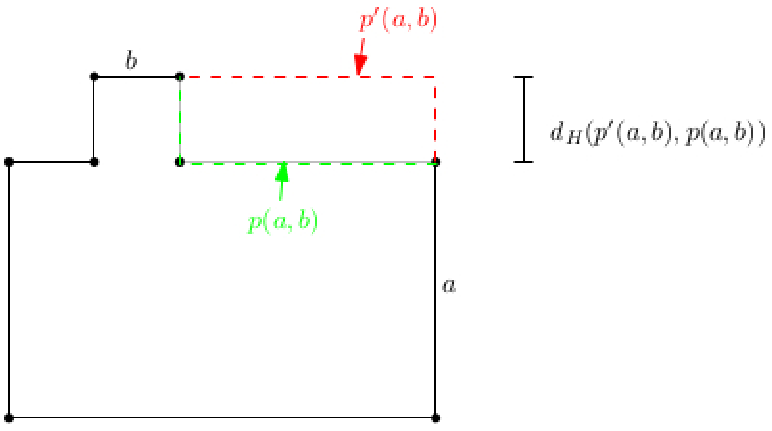
Simplification approach by Haunert and Wolff

- Polygons as sequences of edges
- Shortcuts as pairs of edges



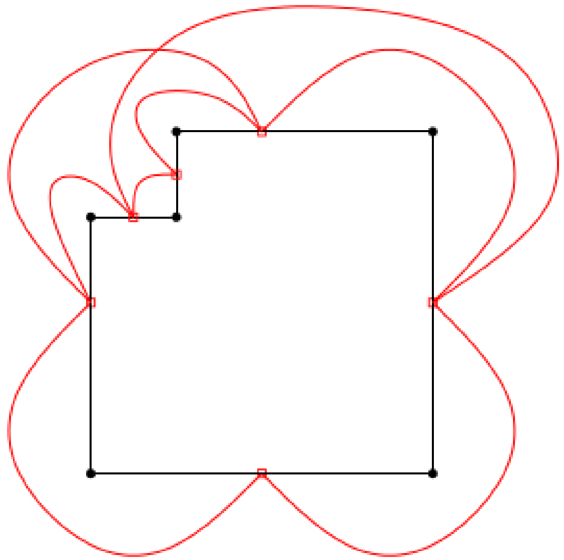
Shortcut selection

- Threshold for Hausdorff-distance between polygonal chains



Shortcut graph G

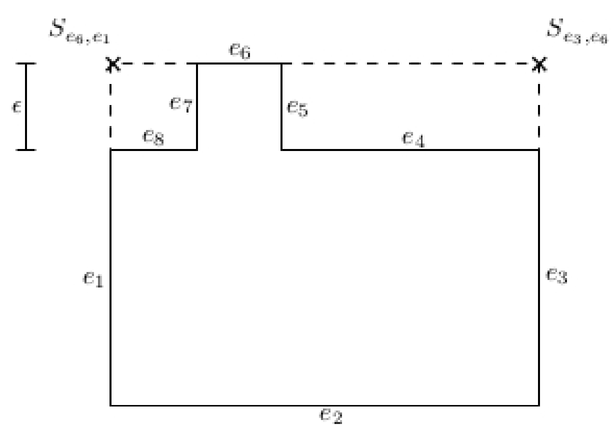
- Consider a shortcut as a graph edge
- G contains a Vertex for each polygon edge
- G contains an Edge for each shorcut
- A cycle is a simplification



Combining shortcuts

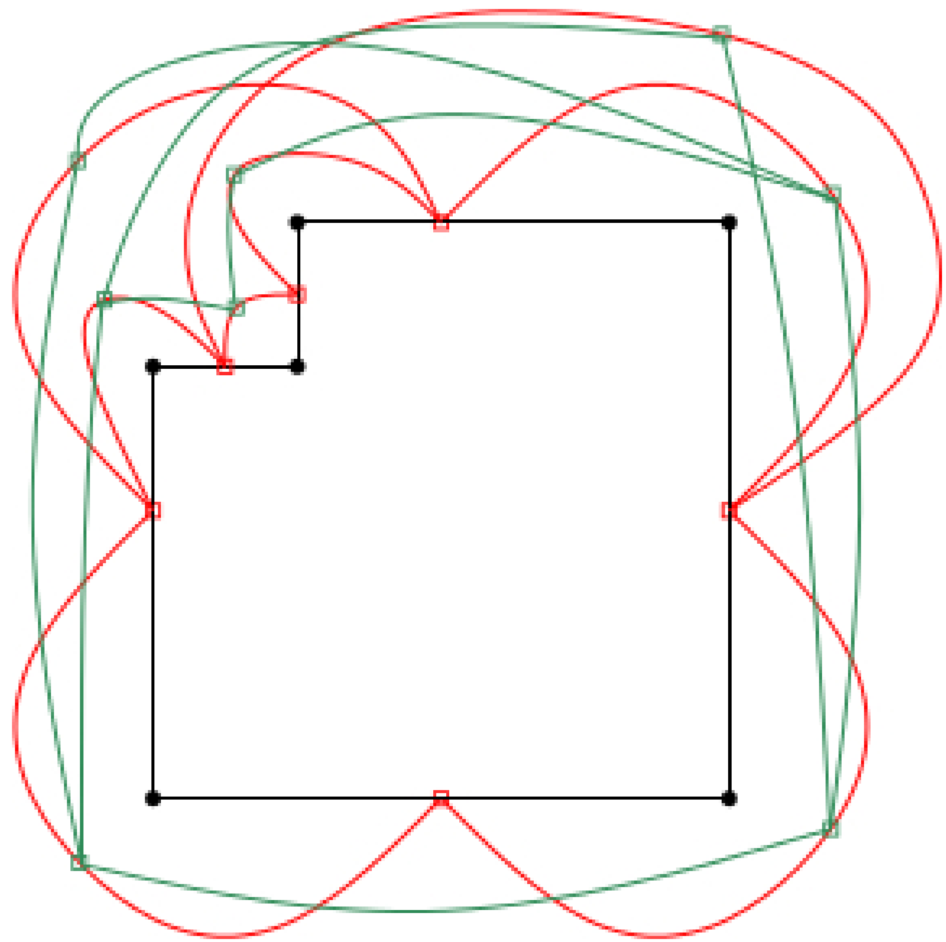
- A shortcut defines a vertex of the simplified polygon
- A combination of to consecutive shortcuts defines an edge of the simplified polygon

$$c=(s_1, s_2)=((e_3, e_6), (e_6, e_1))$$



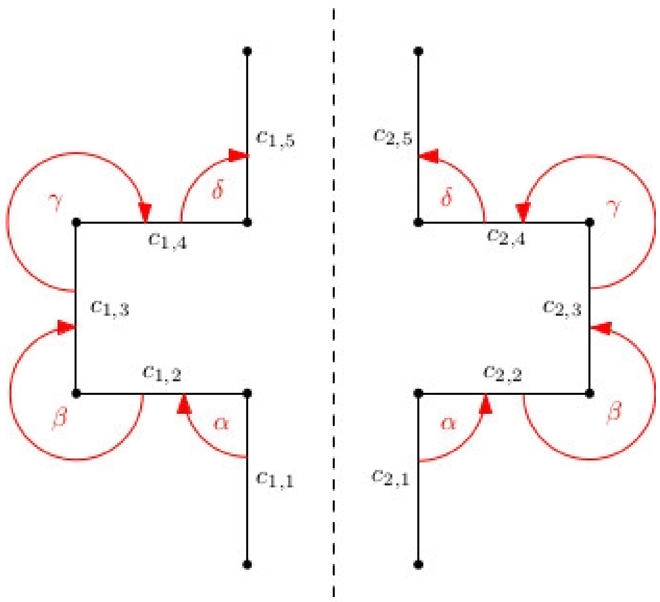
Combination graph

- Combining all consecutive shortcuts in the shortcut graph results in the combination graph
- A cycle in the combination graph is a cycle in the shortcut graph
- Consecutive combinations refer to consecutive polygon edges in the simplification



Comparing combinations

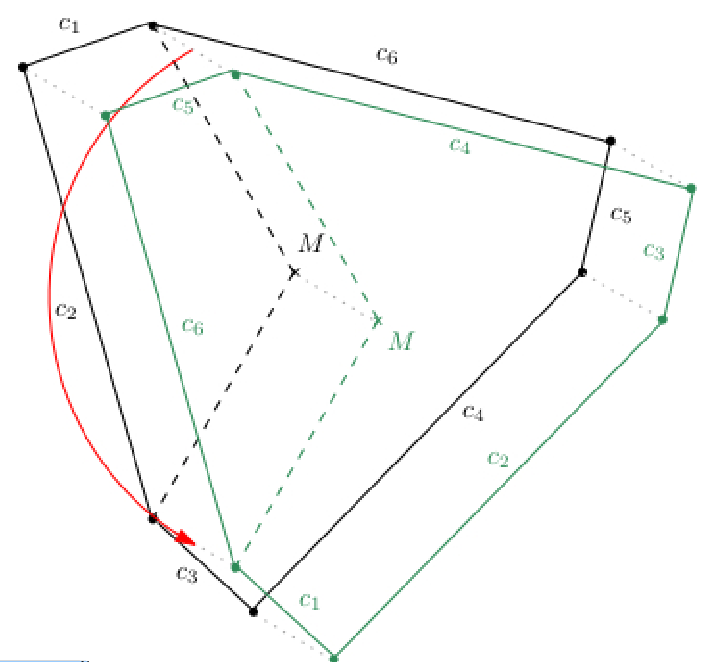
- Detecting symmetries by sequences of matching combinations
  - By length
  - By angle to predeccessor
- A comparison is a pair of combinations
- A comparison is selected if the combinations match

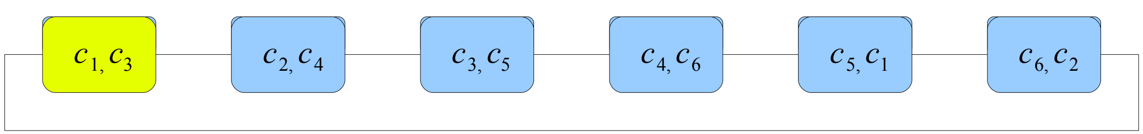


$$v_1 = (c_{1,1}, c_{2,1}), v_2 = (c_{1,2}, c_{2,2}), \ldots$$

Comparison graph

- Requires two combination graphs
- Starting with a combination from each combination graph
- Consecutive comparisons with consecutive combinations





Symmetries between two different footprints

- Comparison graph of two different footprints
  - Rotational direction
    - Identical: building matching
    - Contrary: reflections
- Searching the longest path
  - Length
    - Geometrically
    - Number of combinations
  - Minimum cost
- Any possible pair of start-combinations

Runtime

- Two different polygons of lengths n and m, n>m

	Less than	At least
Combination set	n^4, m^4	n, m
Comparison set	$O\left((n\cdot m)^4\right)$	$O(n\cdot m)$
Edges in comparison graph	$O\left((n\cdot m)^8\right)$	$O(n)$

- For any start-comparison compute the comparison graph and search the longest path

	Less than	At least
Compute comparison graph	$O\left((n\cdot m)^8\right)$	$O(n)$
Search longest path	$O\left((n\cdot m)^8\right)$	$O(n)$
Total amount	$O\left((n\cdot m)^{12}\right)$	$O(n^2\cdot m)$

Symmetries within one footprint

- Rotational
  - During the editing period of the thesis
    - Developement of a heuristical procedure
    - Discussion of exact approaches
  - Today
    - Completed an exact approach discussed to an polynomial time procedure
- Reflectional
  - Finding a simplification that is reflectional symmetric according to a single axis

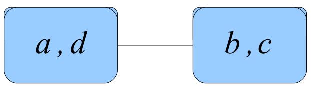
Rotational symmetries

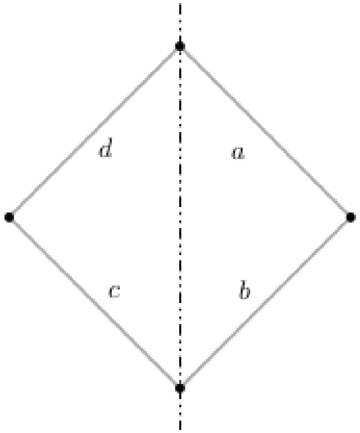
- A cycle in the comparison graph where the pre-image matches the image before and after the rotation
- Exact procedure in less than $O(n^{56})$ but at least $O(n^{2})$ time where n is the polygon length

Dataset	Boston urban area, about 4500 buildings
Runtime	About 20 seconds
Result	About 20.000 comparison graphs containin rotational symmetries
Shortcut threshold	5 meters
Length tolerance	15%
Angle tolerance	1%

Reflectional symmetries

- Finding a simplification that is reflectional symmetric according to a single axis
  - A path that starts and ends at a comparison of identical or consecutive combinations
  - Runtime less than $O(n^{24})$ but at least $O(n^3)$





Summery

- Discussed symmetry detection by Lladós et al. used with building footprints
- Introduced a procedure for building matching
- Introduced a procedure for finding reflectional symmetries between buildings
- Developed a procedure for finding rotational symmetries within a building footprint
- Introduced a procedure to find a simplification that is reflectional symmetric according to a single axis

Open problems

- Analogous to the detection of rotational symmetries find a procedure that can detect a simplification within a comparison graph that is reflectional symmetric to the most possible number of axes