Polylogarithmic Approximation for Generalized Minimum Manhattan Networks

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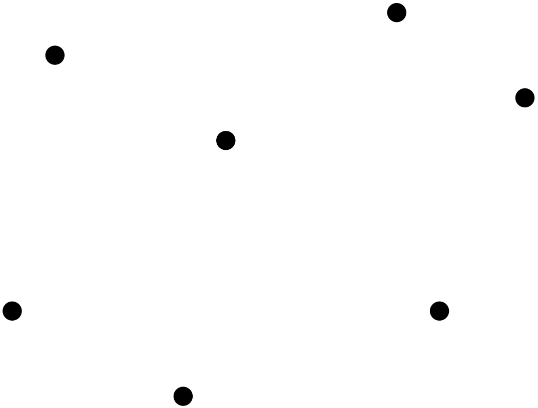
Universität Würzburg

14.06.2012
Mittagsseminar am Lehrstuhl 1

Minimum Manhattan Network Problem (MMN)

Problem Definition



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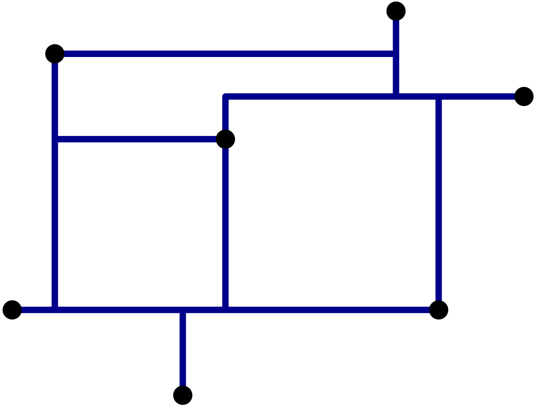

Rectilinear Steiner Arborescence Problem (RSA)

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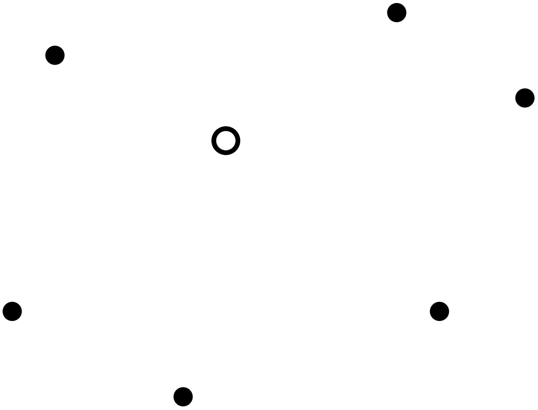

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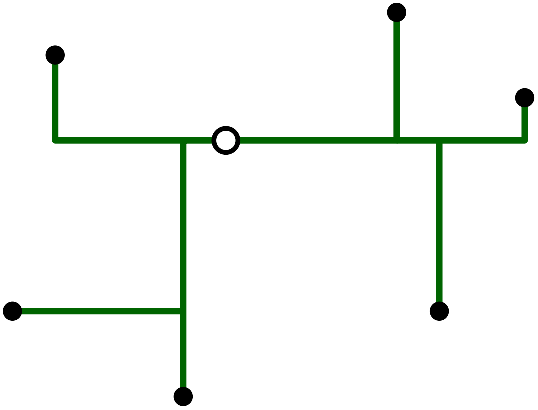

Generalized MMN Problem (GMMN)

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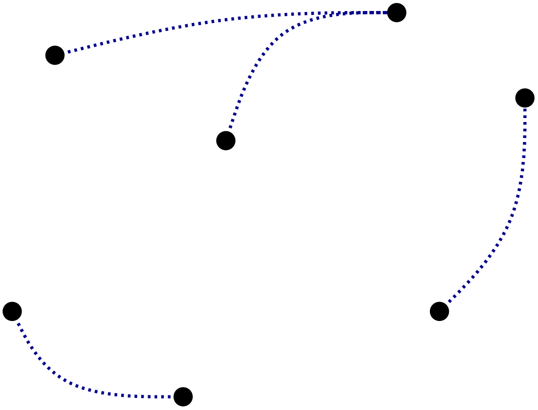

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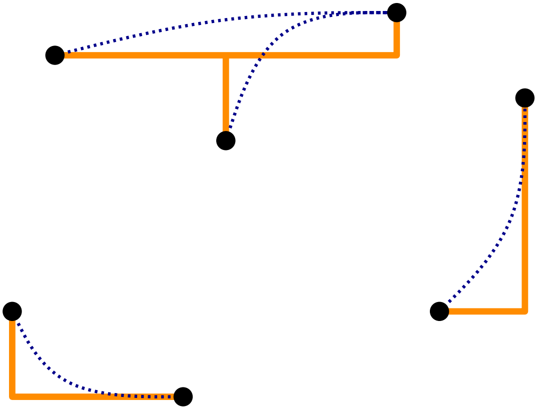

History
Introduction

- 1974: RSA
- 2000: RSA has PTAS
- 2000: RSA is NP-hard

- 1999: MMN
- 2004: MMN has 2-approximation
- 2009: MMN is NP-hard

- 2004: GMMN
- 2012: . . .

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Previous Results
Introduction

	2D	Dimension $d > 2$
RSA	$(1+\varepsilon)$	$O(n^{\varepsilon})$
MMN	2	$O(n^{\varepsilon})$
GMMN	?	?

Our Contribution
Introduction

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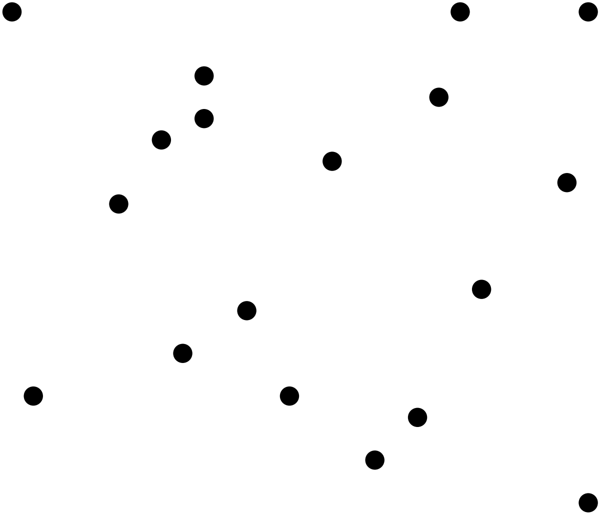
Problem Instance
Overview



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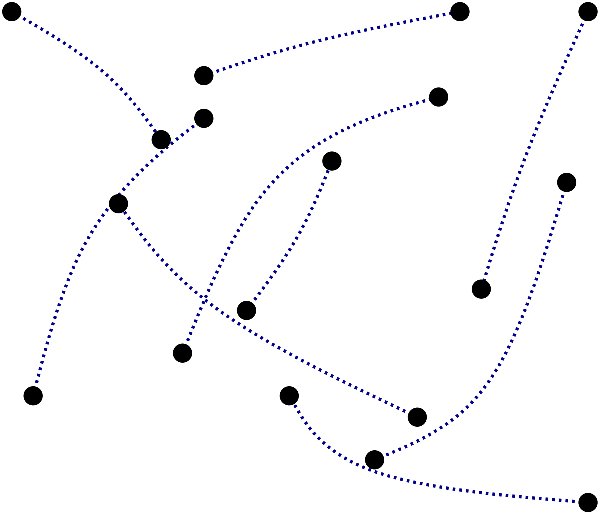

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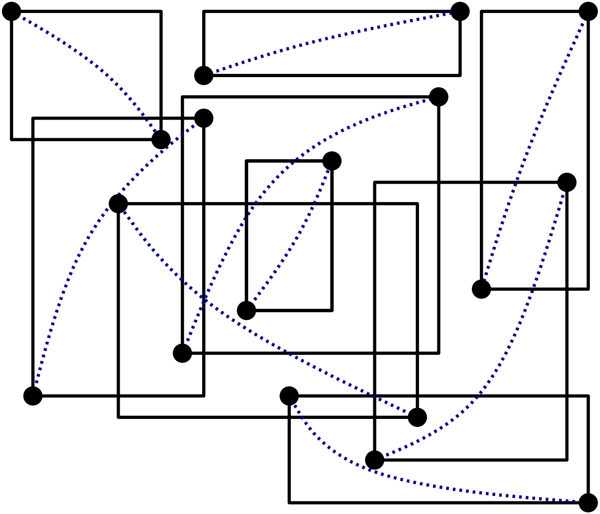
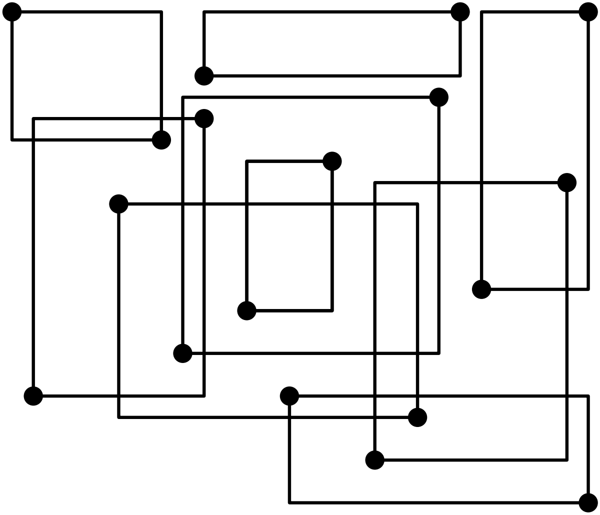


Partition . . .
Overview



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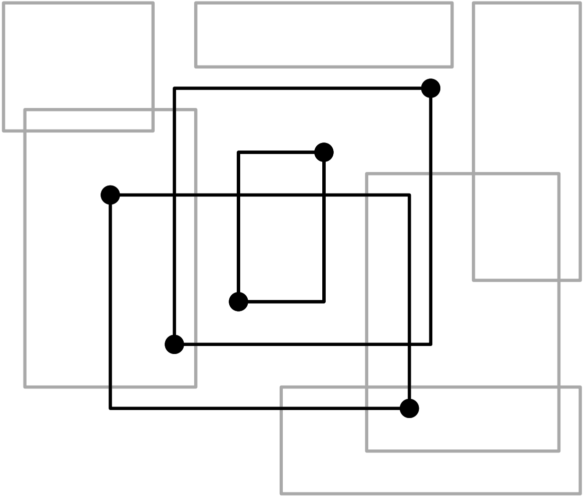

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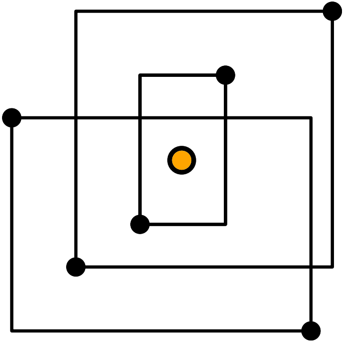
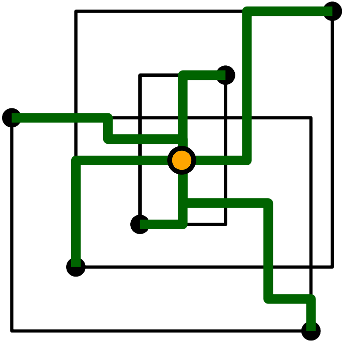


. . . And Reduce to RSA

Overview



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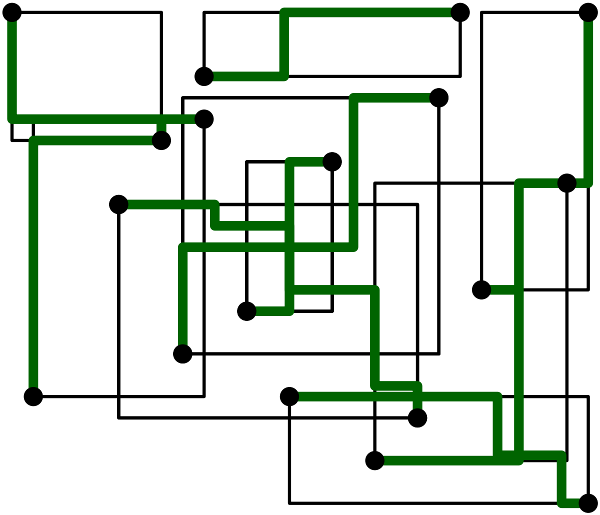
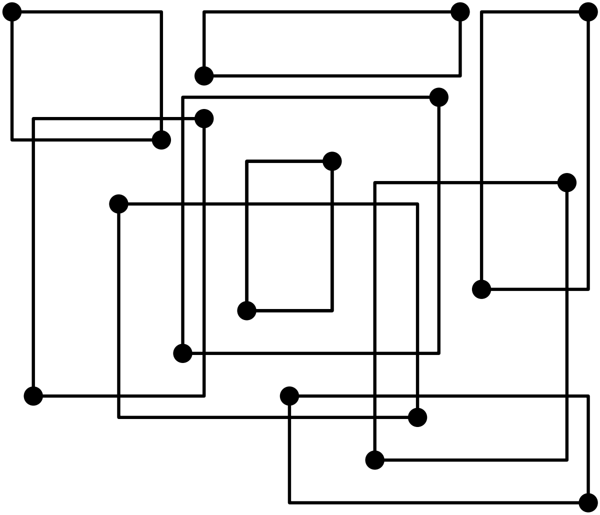


Partition

Construction



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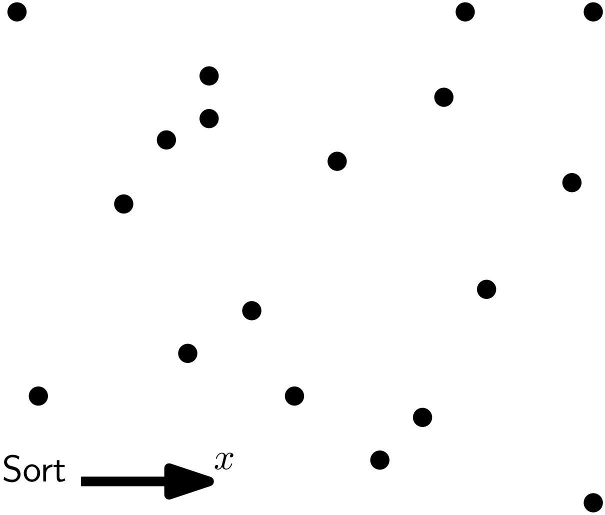

Partition
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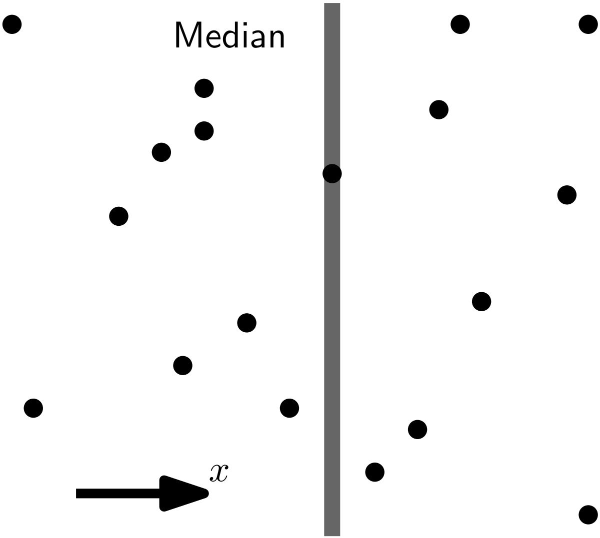

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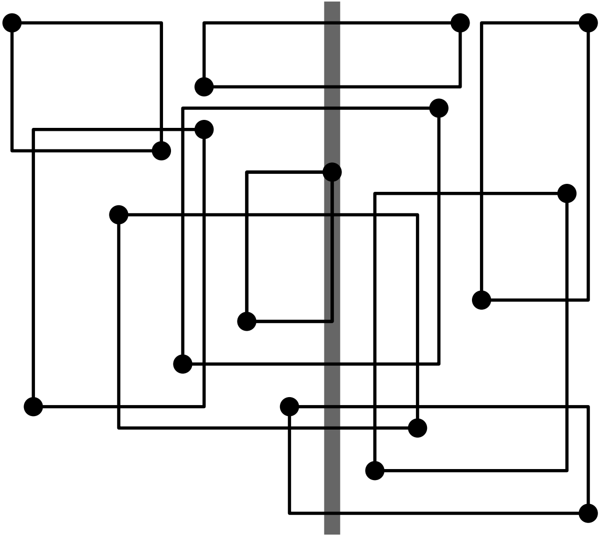
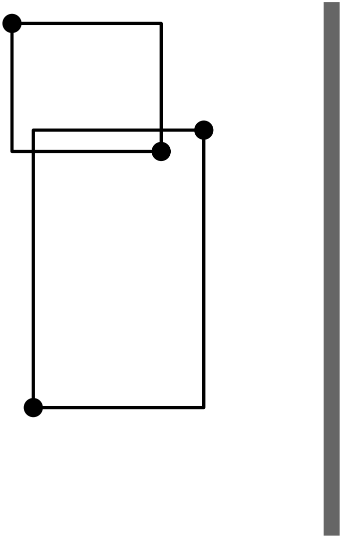


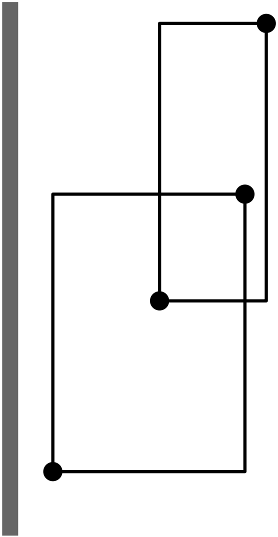
Left Sub-Instance

Construction



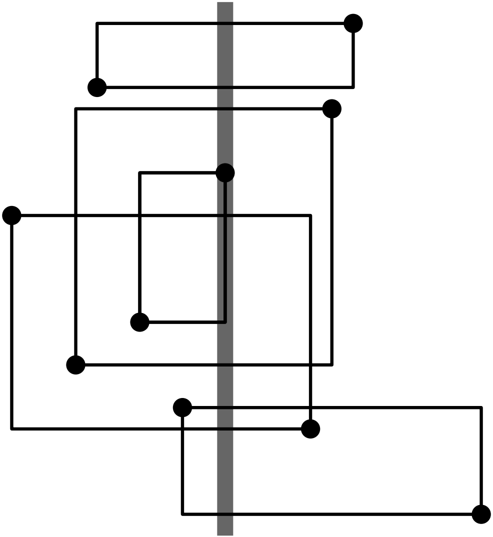
Right Sub-Instance

Construction



x-Separated Instance

Construction



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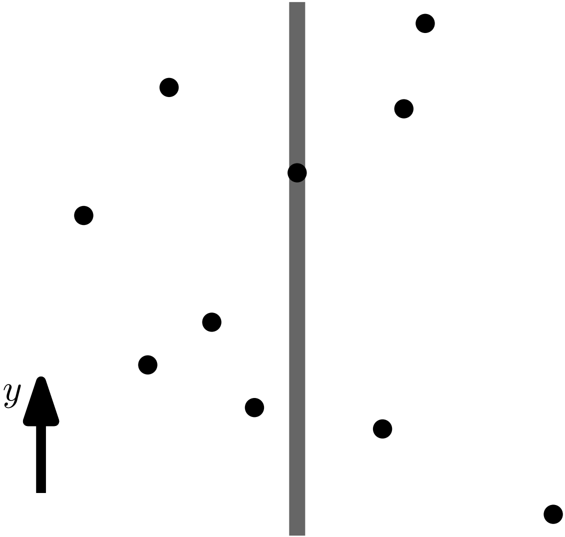

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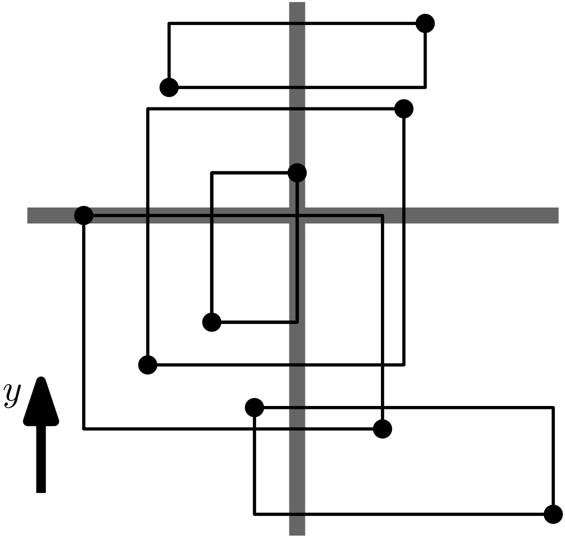

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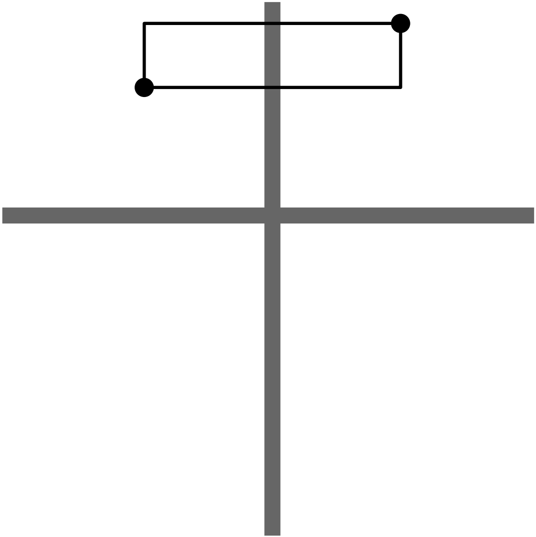

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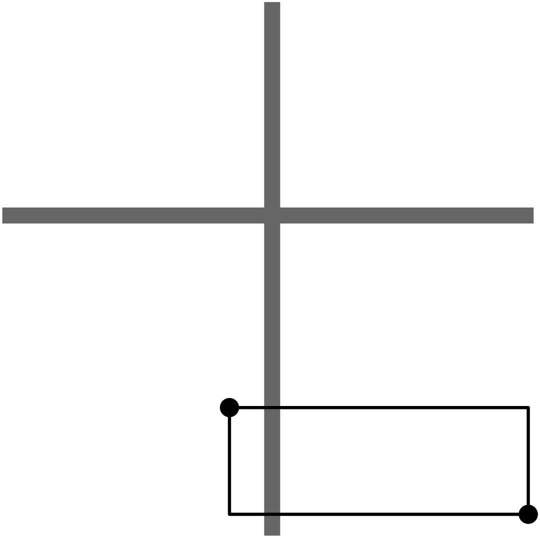

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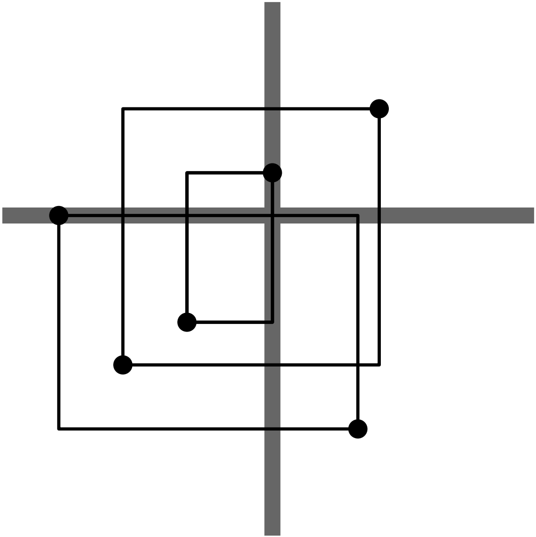
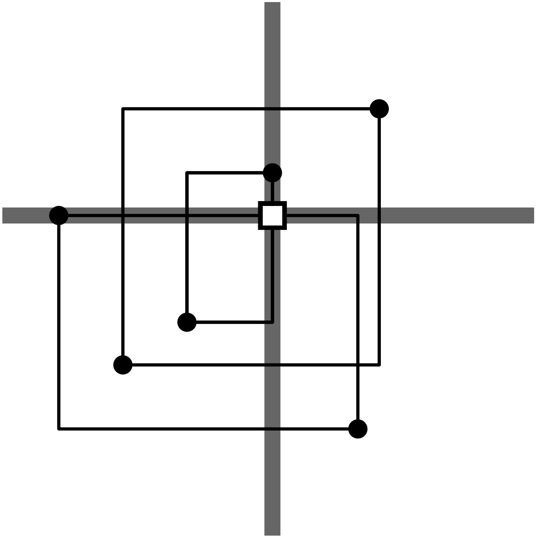


xy-Separated Instance

Construction



xy-Separated Instance

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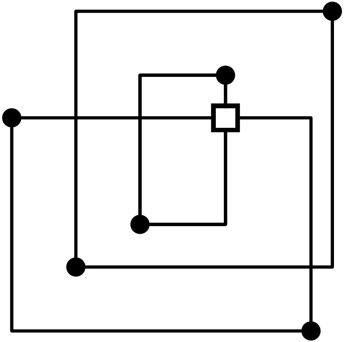
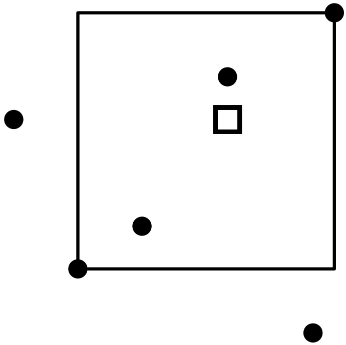


Observation

Construction



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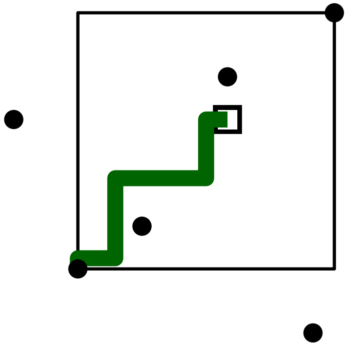
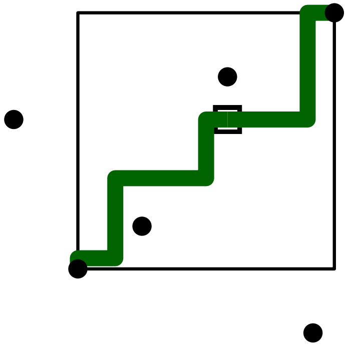


Observation
Construction

Manhattan-Path



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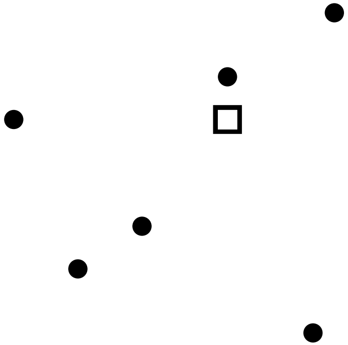

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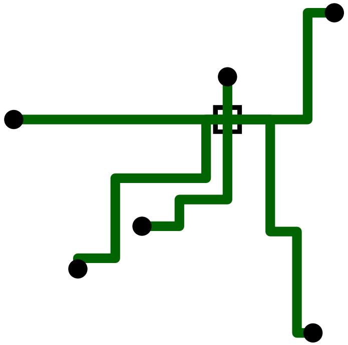
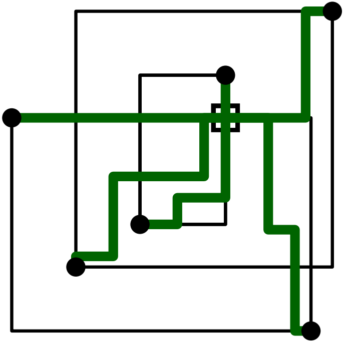


Reduce to RSA

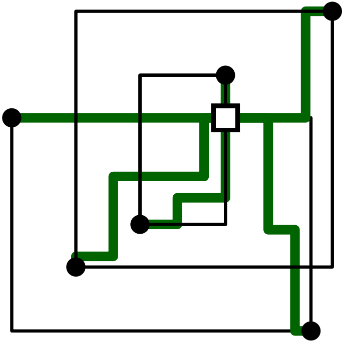
Construction



Cost of RSA

Cost Analysis

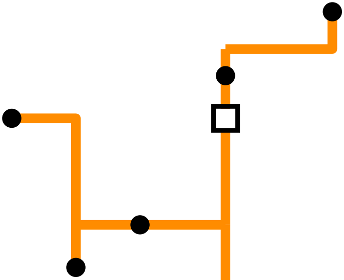
$$\text{cost} \leq (1+\varepsilon) \cdot \mathrm{OPT}_{\mathrm{RSA}}$$



$$\mathrm{OPT}_{\mathrm{RSA}} = ?$$

Cost of RSA
Cost Analysis

An optimum GMMN solution



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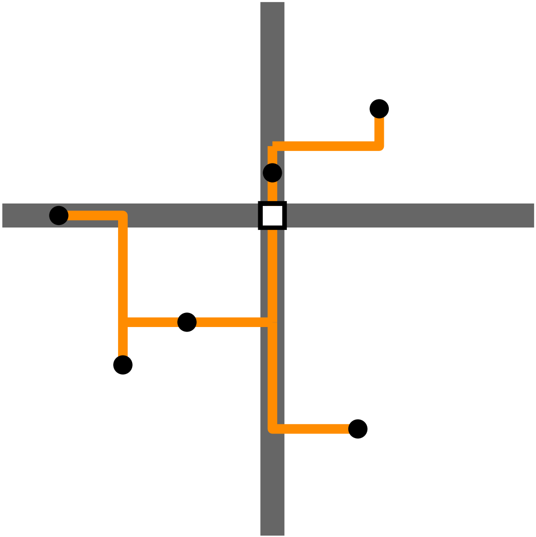

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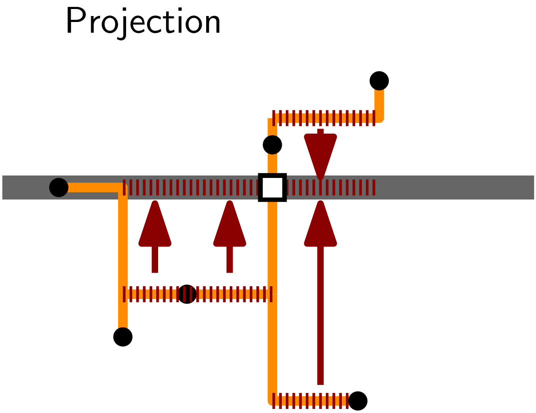

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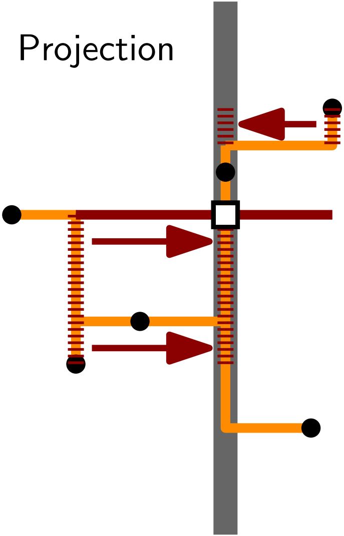

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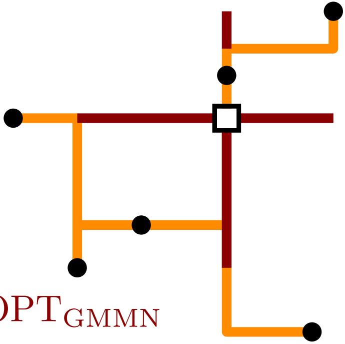

cost $\leq 2 \cdot \mathrm{OPT}_{\mathrm{GMMN}}$

Cost of RSA

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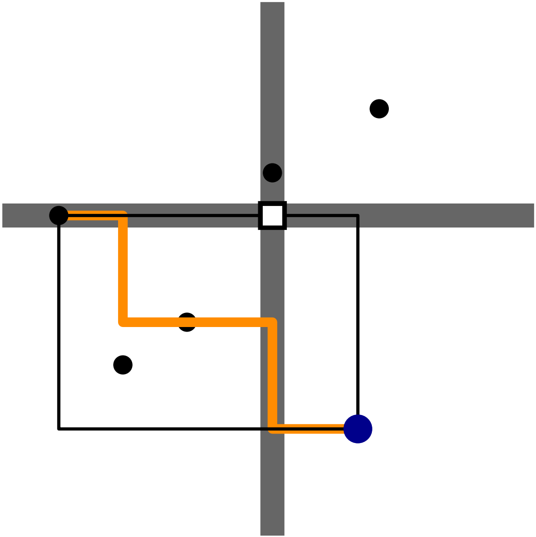

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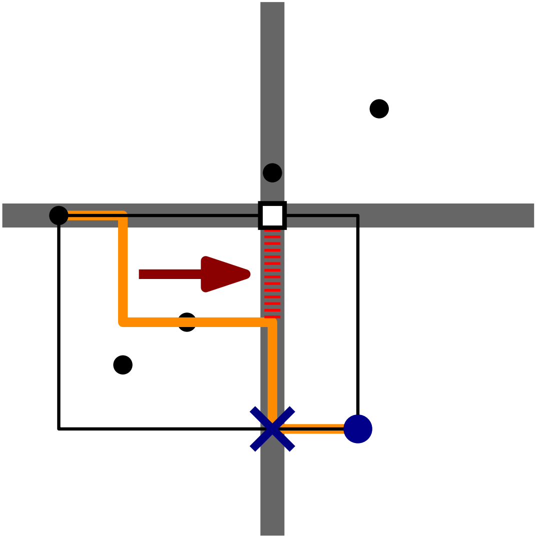

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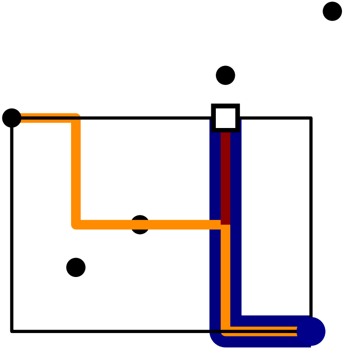
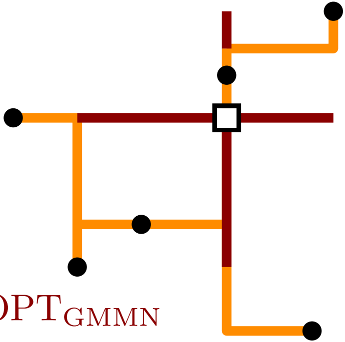


Cost of RSA
Cost Analysis

RSA solution



cost $\leq 2 \cdot \mathrm{OPT}_{\mathrm{GMMN}}$

$\Rightarrow \mathrm{OPT}_{\mathrm{RSA}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{GMMN}}$

Cost of RSA

Cost Analysis

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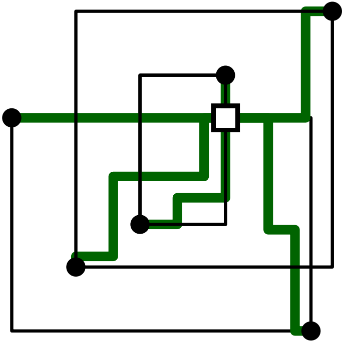
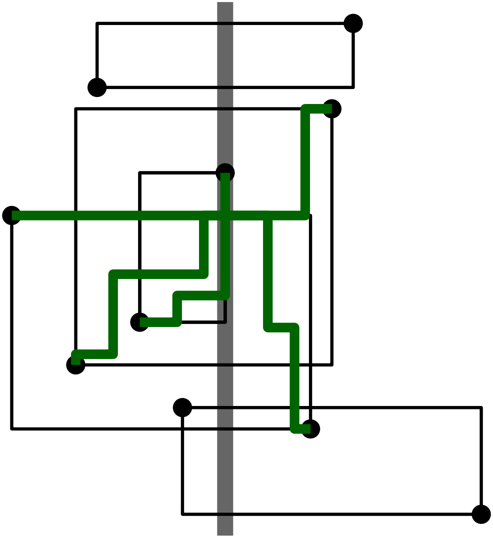


$$\Rightarrow \mathrm{OPT}_{\mathrm{RSA}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{GMMN}}$$

$$\Rightarrow \text{cost} \leq (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}_{\mathrm{GMMN}}$$

Cost of x-Separated Instance

Cost Analysis



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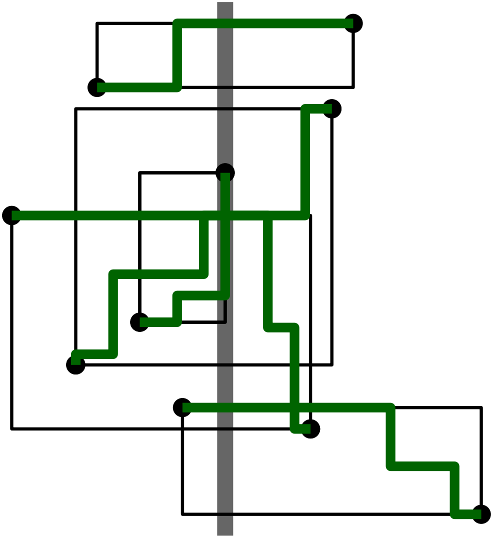

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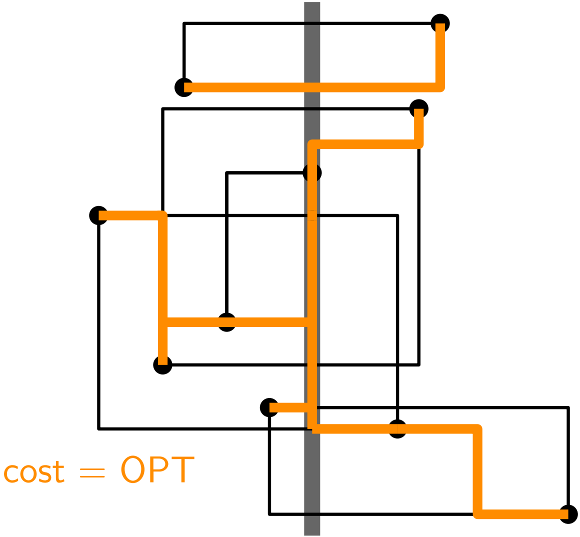
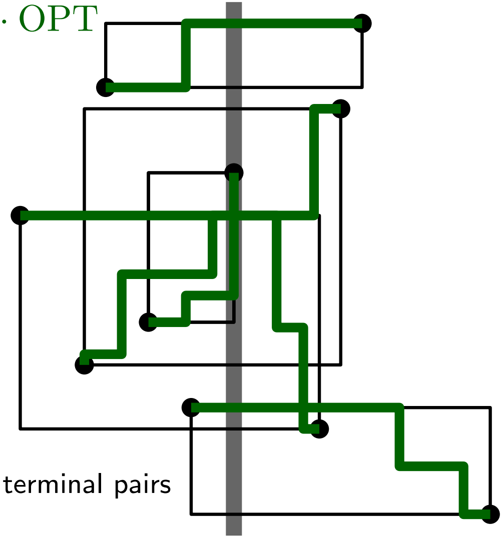


Cost of x-Separated Instance

Cost Analysis

$$\text{cost} \leq \rho_x(n) \cdot \mathrm{OPT}$$

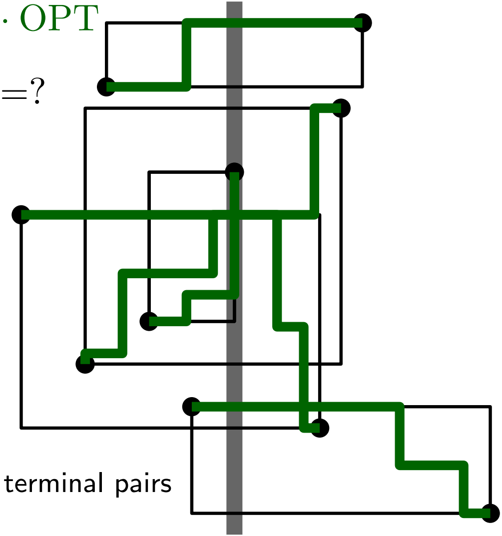


$n =$ number of terminal pairs

Cost of x-Separated Instance
Cost Analysis

$$\text{cost} \leq \rho_x(n) \cdot \text{OPT}$$

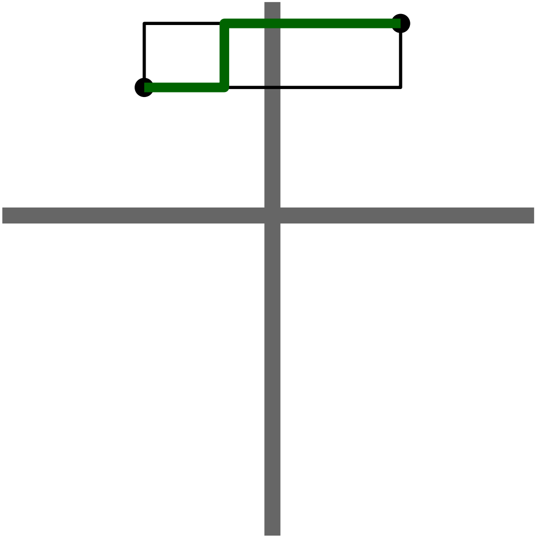
$$\rho_x(n) = ?$$



n = number of terminal pairs

Cost of x-Separated Instance

Cost Analysis



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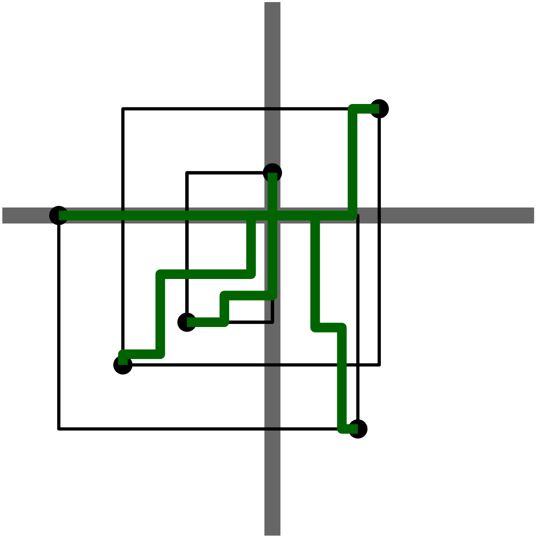

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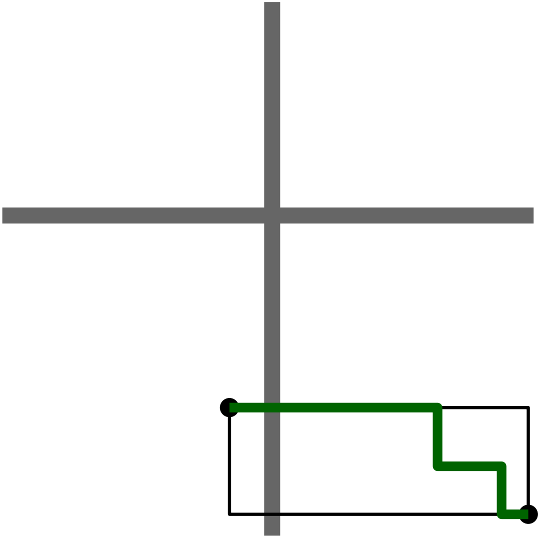
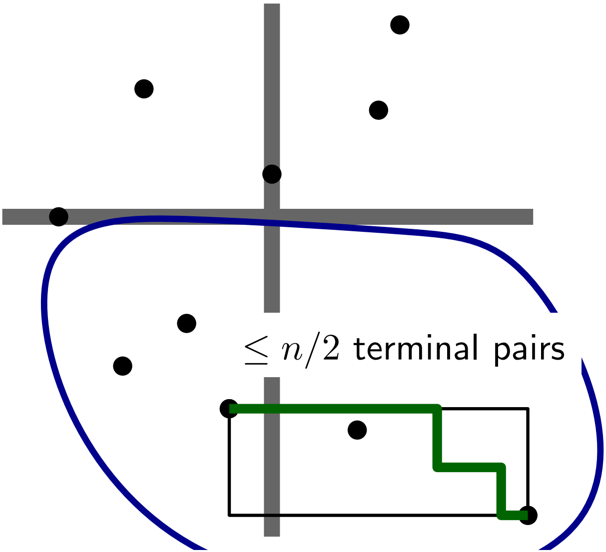


Cost of x-Separated Instance

Cost Analysis



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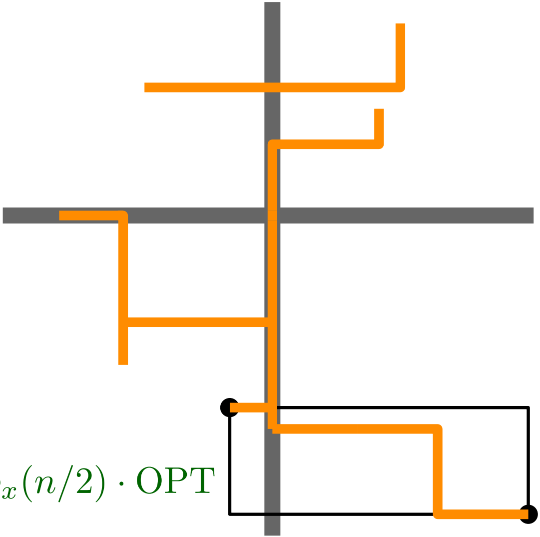
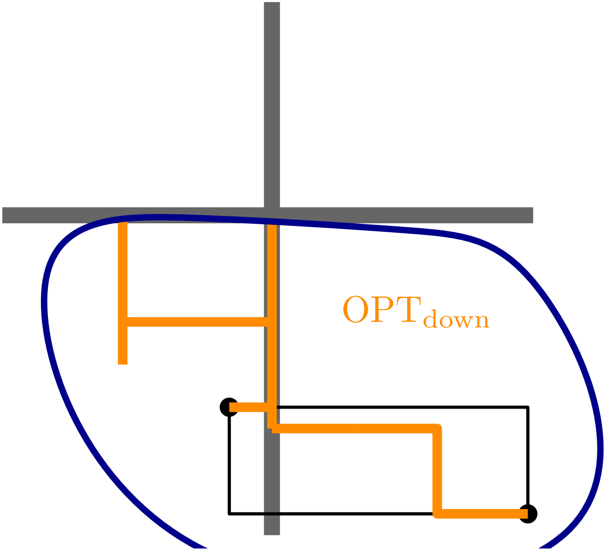


cost $\leq \rho_x(n/2) \cdot \mathrm{OPT}$

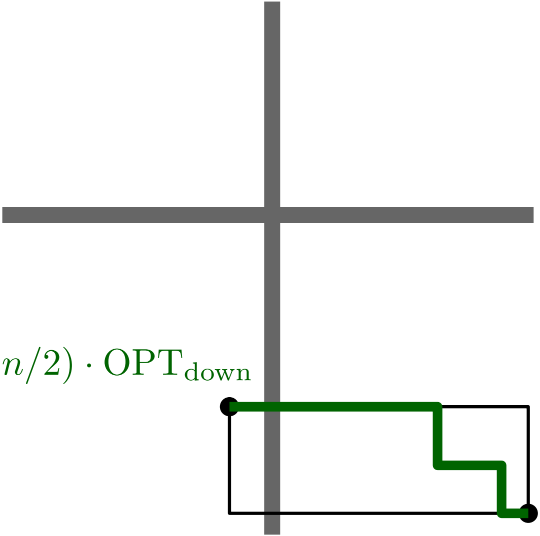
Cost of x-Separated Instance

Cost Analysis



Cost of x-Separated Instance

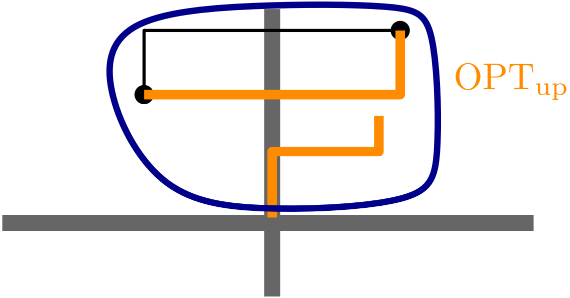
Cost Analysis



$$\text{cost} \leq \rho_x(n/2) \cdot \mathrm{OPT}_{\mathrm{down}}$$

Cost of x-Separated Instance

Cost Analysis

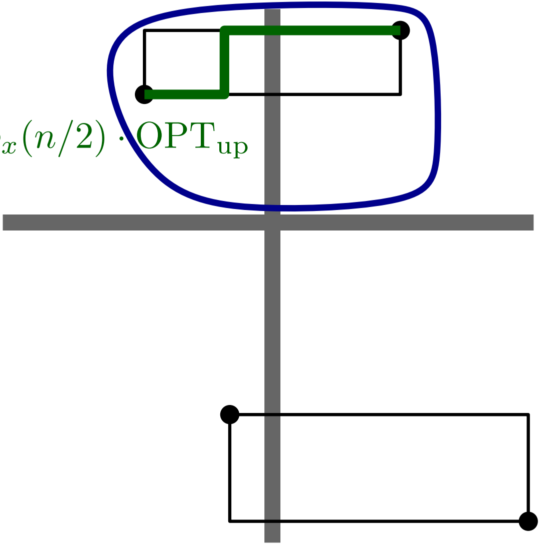


$$\text{OPT} \geq \text{OPT}_{\text{up}} + \text{OPT}_{\text{down}}$$

Cost of x-Separated Instance

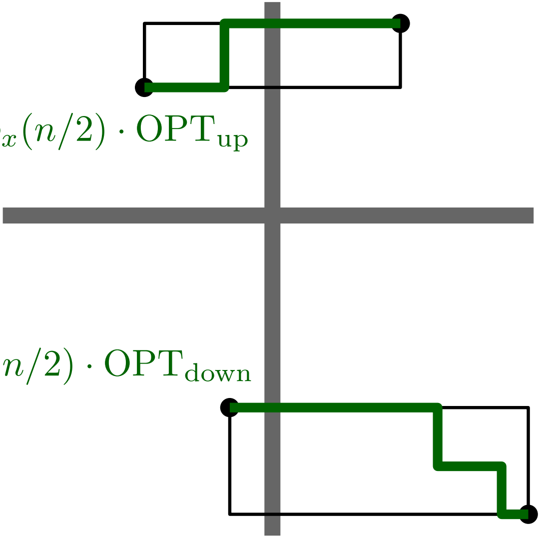
Cost Analysis

cost $\leq \rho_x(n/2) \cdot \mathrm{OPT}_{\mathrm{up}}$



Cost of x-Separated Instance

Cost Analysis

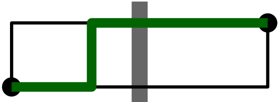


$$\text{cost} \leq \rho_x(n/2) \cdot \mathrm{OPT}_{\mathrm{up}}$$

$$\text{cost} \leq \rho_x(n/2) \cdot \mathrm{OPT}_{\mathrm{down}}$$

Cost of x-Separated Instance

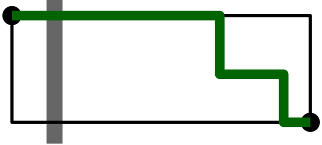
Cost Analysis



cost $\leq \rho_x(n/2) \cdot \mathrm{OPT}_{\mathrm{up}}$

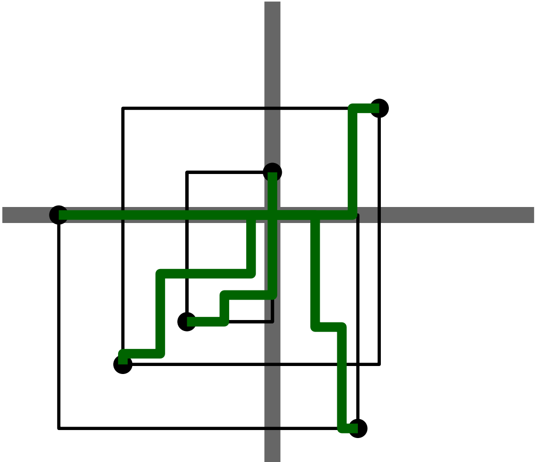
$$\leq \rho_x(n/2) \cdot (\mathrm{OPT}_{\mathrm{up}} + \mathrm{OPT}_{\mathrm{down}})$$
$$\leq \rho_x(n/2) \cdot \mathrm{OPT}$$

cost $\leq \rho_x(n/2) \cdot \mathrm{OPT}_{\mathrm{down}}$



Cost of x-Separated Instance

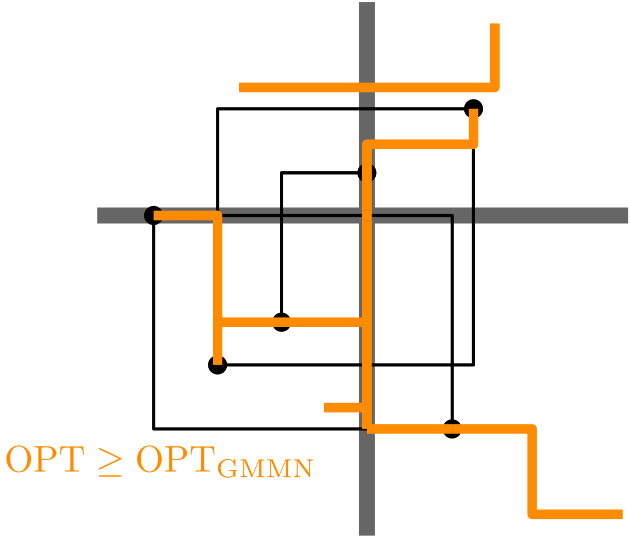
Cost Analysis



$$\text{cost} \leq (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}_{\mathrm{GMMN}}$$

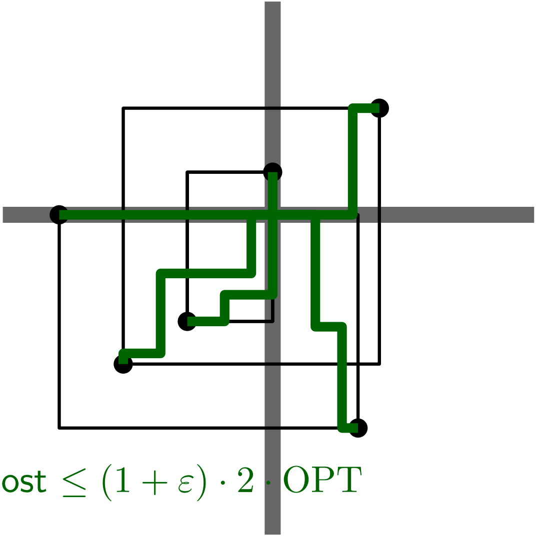
Cost of x-Separated Instance

Cost Analysis



Cost of x-Separated Instance

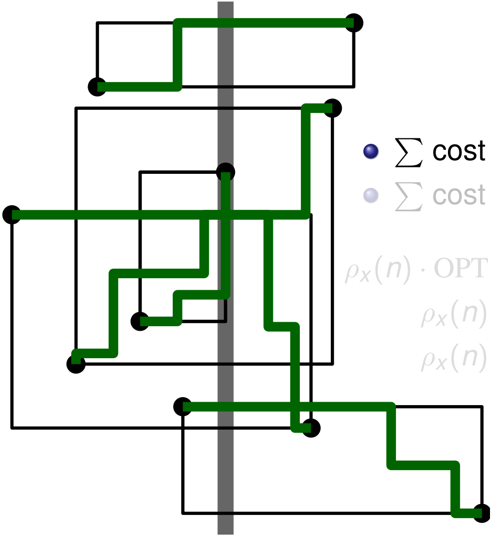
Cost Analysis



cost $\leq (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$

Cost of x-Separated Instance (Summed Up)

Cost Analysis



- $\sum \text{cost} \leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$
- $\sum \text{cost} \leq \rho_x(n) \cdot \mathrm{OPT}$

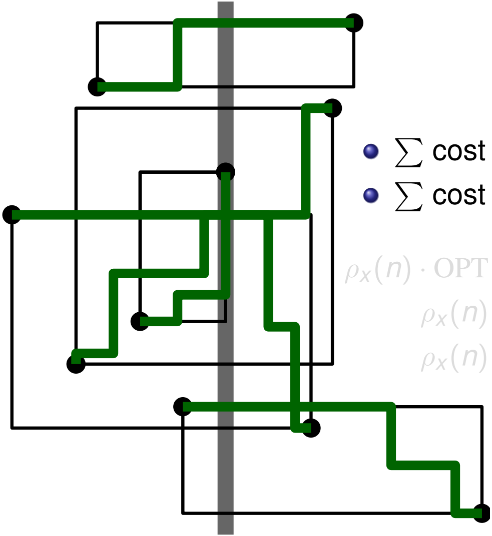
$$\rho_x(n) \cdot \mathrm{OPT} \leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$$

$$\rho_x(n) \leq \rho_x(n/2) + (1+\varepsilon) \cdot 2$$

$$\rho_x(n) \leq \log n \cdot (1+\varepsilon) \cdot 2$$

Cost of x-Separated Instance (Summed Up)

Cost Analysis



- $\sum$ cost $\leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$
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$$\rho_x(n) \cdot \mathrm{OPT} \leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$$
$$\rho_x(n) \leq \rho_x(n/2) + (1+\varepsilon) \cdot 2$$
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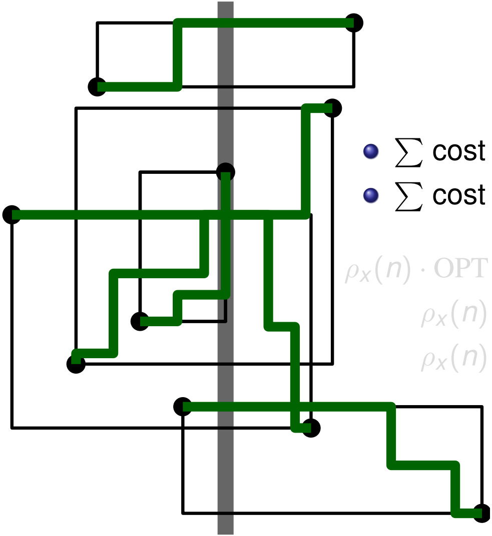
Cost of x-Separated Instance (Summed Up)

Cost Analysis



- $\sum \text{cost} \leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$
- $\sum \text{cost} = \rho_x(n) \cdot \mathrm{OPT}$

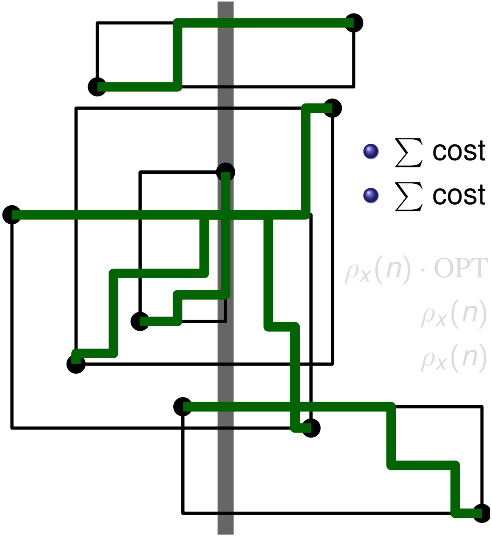
$$\rho_x(n) \cdot \mathrm{OPT} \leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$$

$$\rho_x(n) \leq \rho_x(n/2) + (1+\varepsilon) \cdot 2$$

$$\rho_x(n) \leq \log n \cdot (1+\varepsilon) \cdot 2$$

Cost of x-Separated Instance (Summed Up)

Cost Analysis



- $\sum$ cost $\leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$
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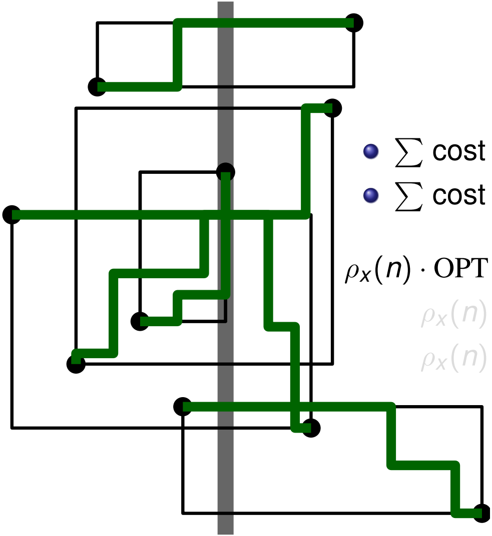
Cost of x-Separated Instance (Summed Up)

Cost Analysis



- $\sum$ cost $\leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$
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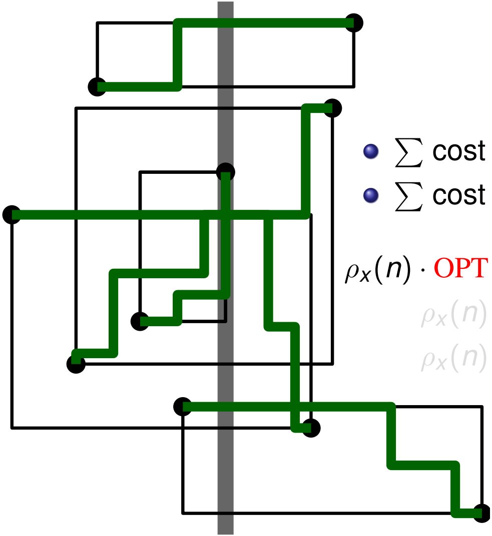
Cost of x-Separated Instance (Summed Up)

Cost Analysis



- $\sum$ cost $\leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$
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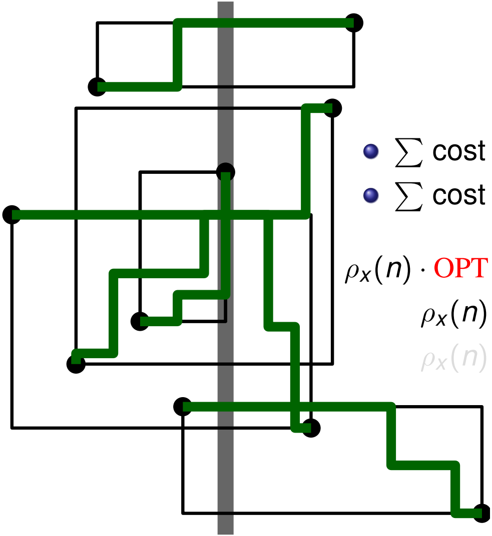

- $\sum \text{cost} \leq \rho_x(n/2) \cdot \mathrm{OPT} + (1+\varepsilon) \cdot 2 \cdot \mathrm{OPT}$
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Cost of x-Separated Instance (Summed Up)

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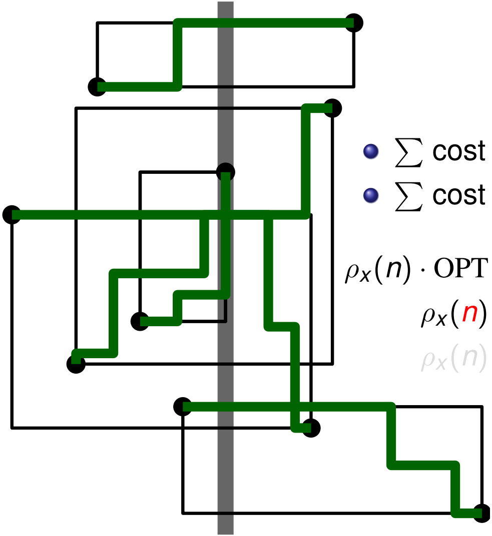

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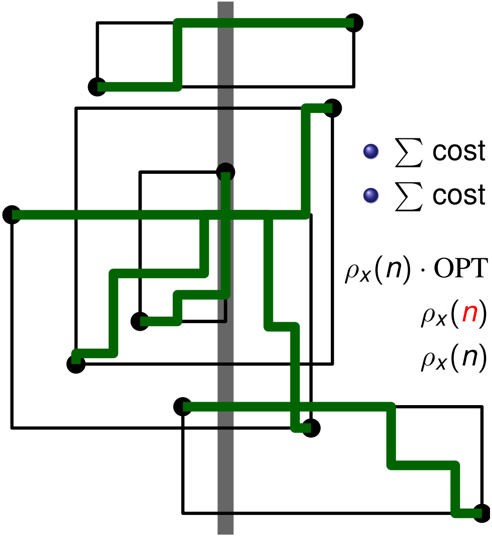
Cost Analysis



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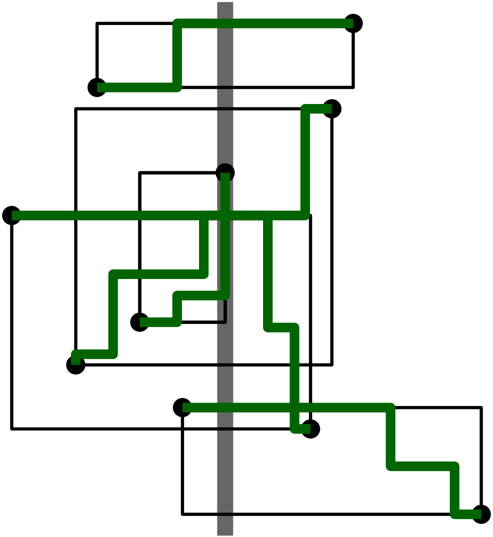
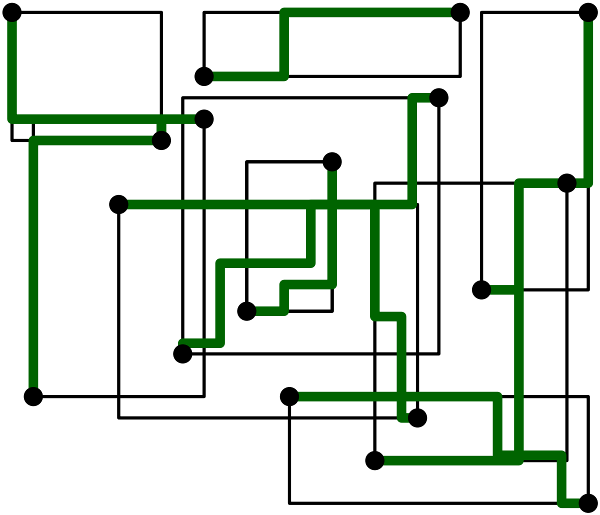
General Instance

Cost Analysis

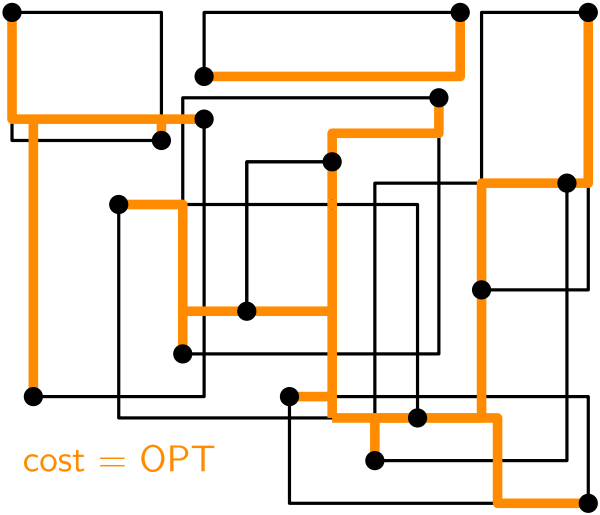


General Instance

Cost Analysis

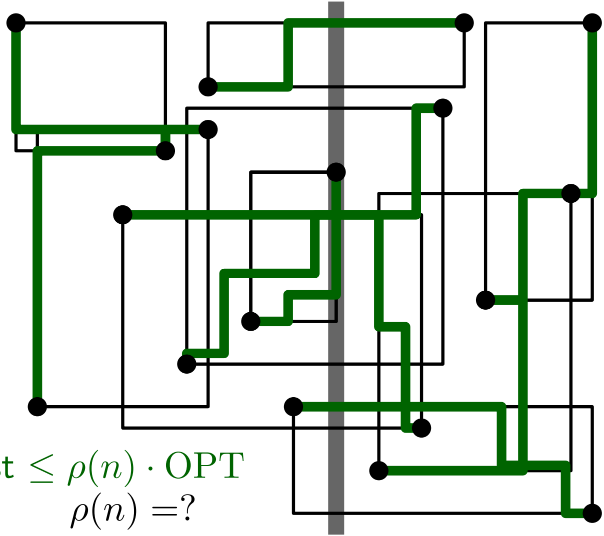


General Instance
Cost Analysis



General Instance

Cost Analysis



cost $\leq \rho(n) \cdot \mathrm{OPT}$

$\rho(n) = ?$

Total Cost
Cost Analysis

- $\sum \text{cost} \leq \rho(n/2) \cdot \mathrm{OPT} + \rho_x(n) \cdot \mathrm{OPT}$
- $\sum \text{cost} = \rho(n) \cdot \mathrm{OPT}$

$$\rho(n) \leq \log n \cdot \rho_x(n)$$

$$\rho(n) \leq \log n \cdot \log n \cdot 2 \in O(\log^2 n)$$

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Main Result in 2D

Theorem

2D-GMMN admits an $O(\log^2 n)$-approximation.

Main Result in Higher Dimension

Dimension	1.Step (Partition)	2.Step (RSA)	Altogether
2	$O(\log^2 n)$	$O(1)$	$O(\log^2 n)$
$d > 2$	$O(\log^d n)$	$O(\log n)$	$O(\log^{d+1} n)$

Theorem

GMMN in dimension d admits an $O(\log^{d+1} n)$-approximation.

Main Result in Higher Dimension

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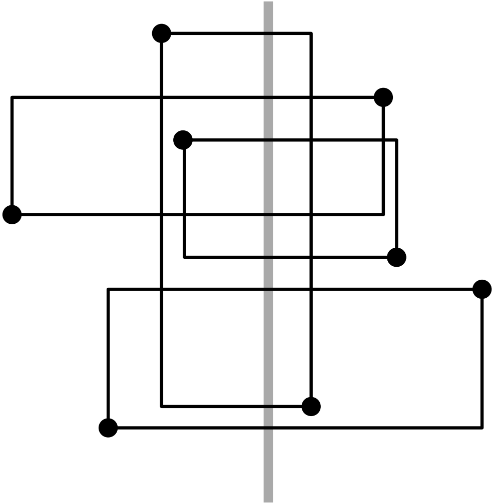
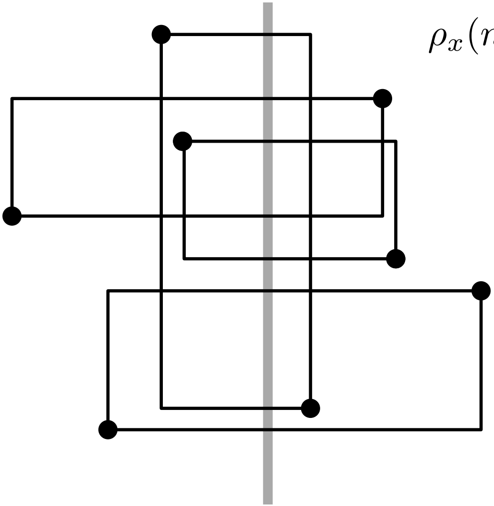
Optimization in 2D

Introduction



Optimization in 2D

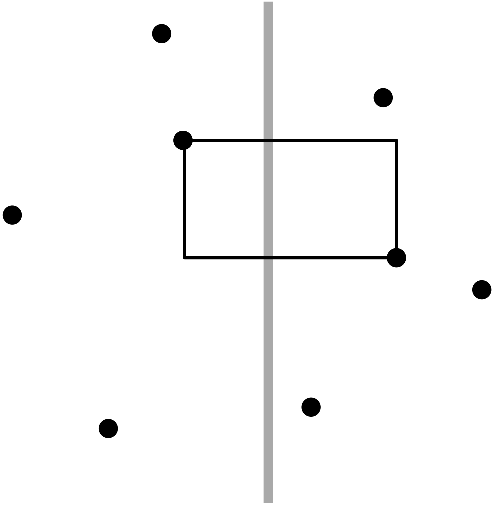
Introduction



$\rho_x(n) \in O(1)$?

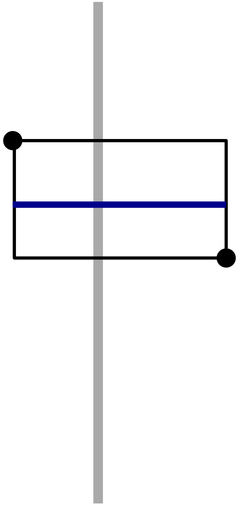
Idea

Introduction



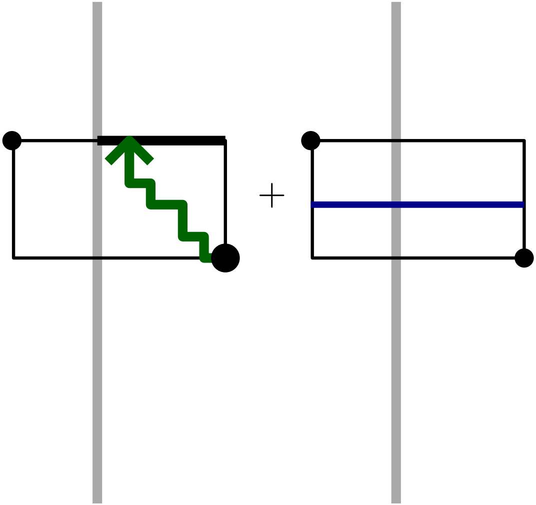
Idea

Introduction

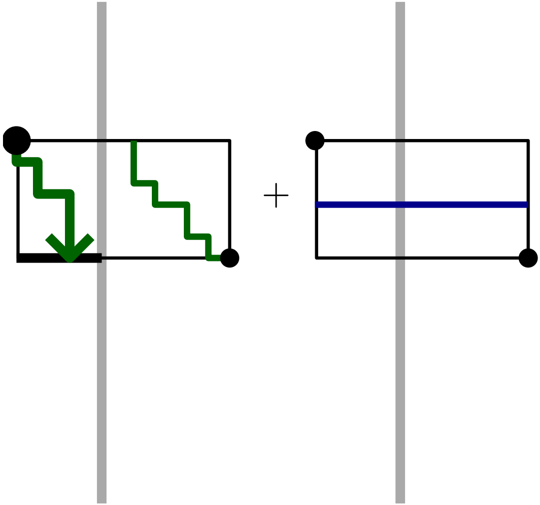


Idea

Introduction

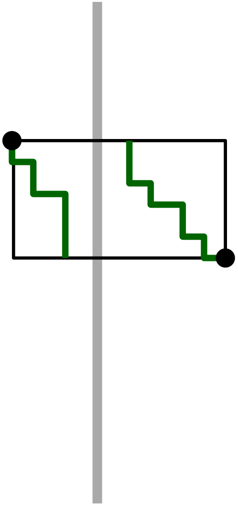


Idea
Introduction



Idea

Introduction



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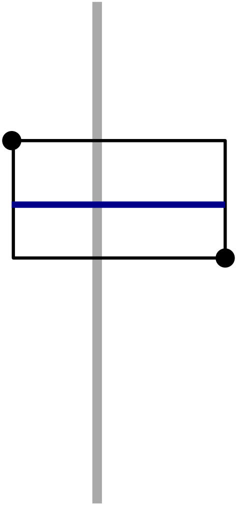

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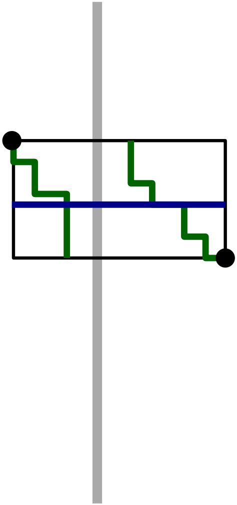

Idea

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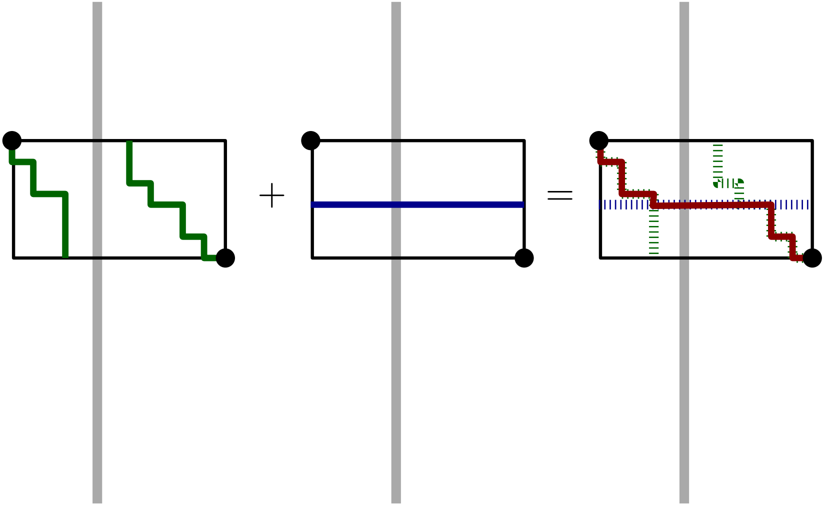

Idea

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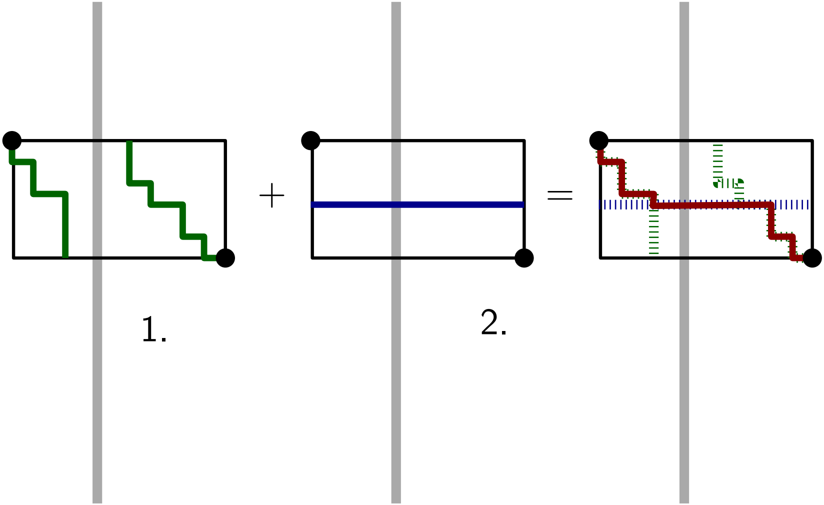
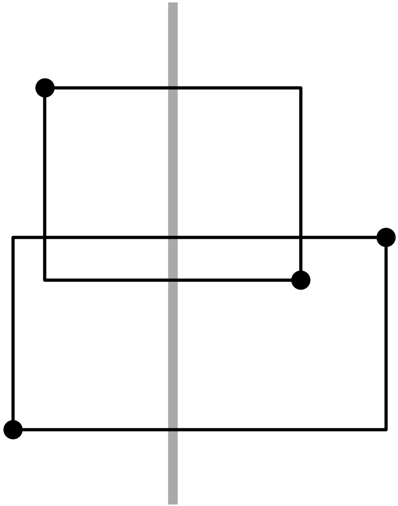


Construction

First Step



Construction

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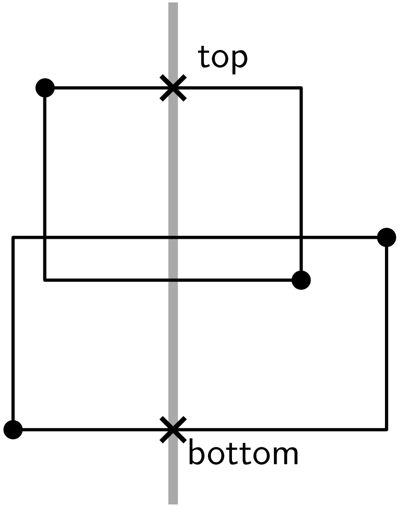
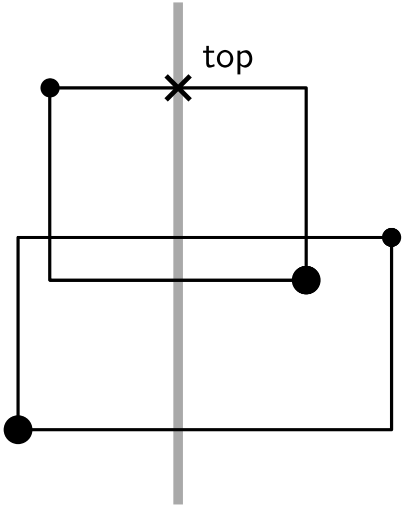


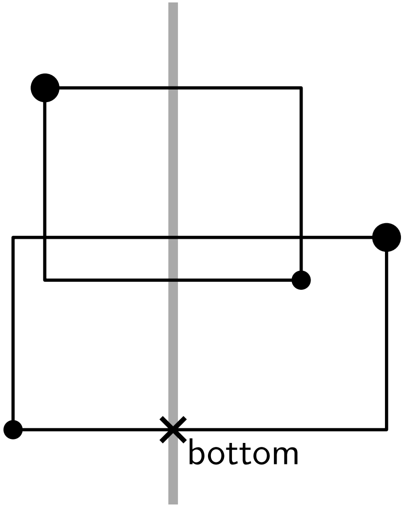
Construction

First Step



Construction

First Step



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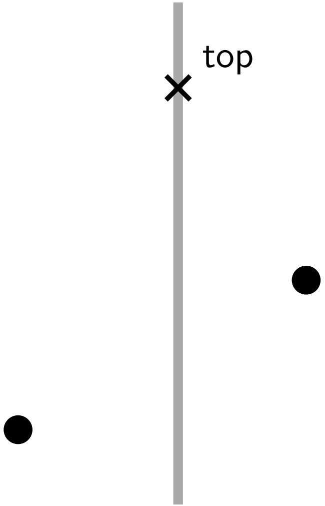
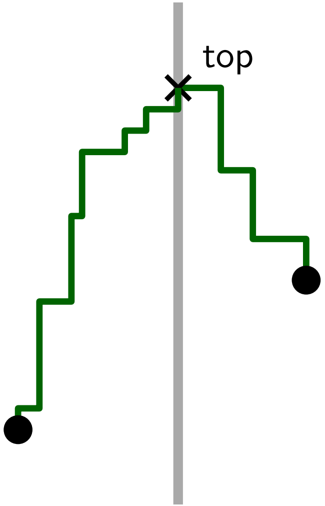
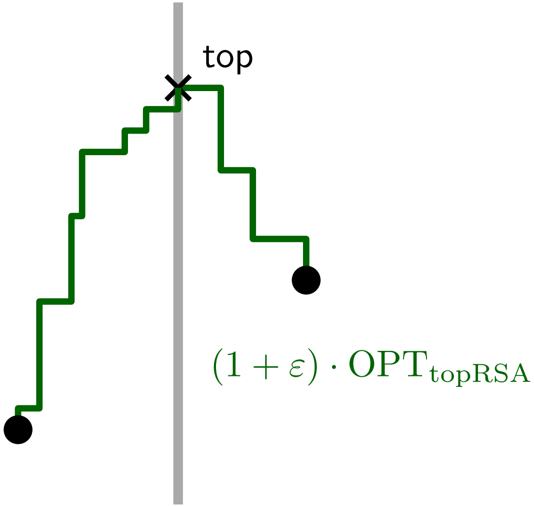


Construction

First Step



Construction
First Step



Construction

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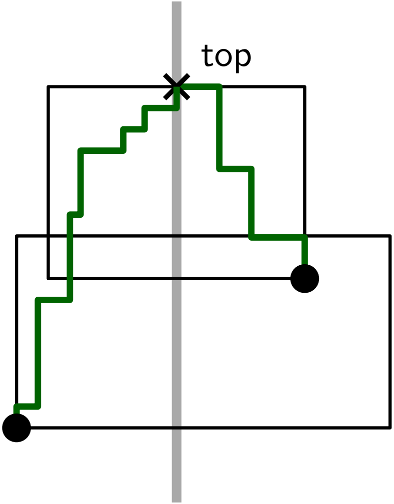
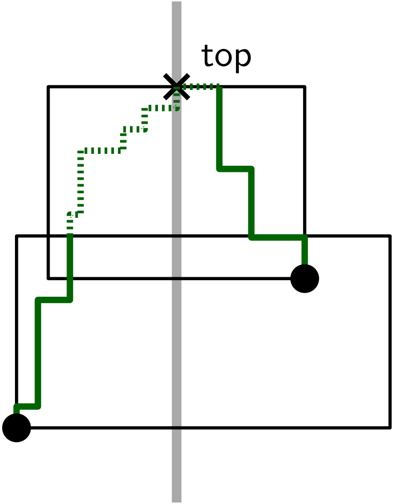


Construction

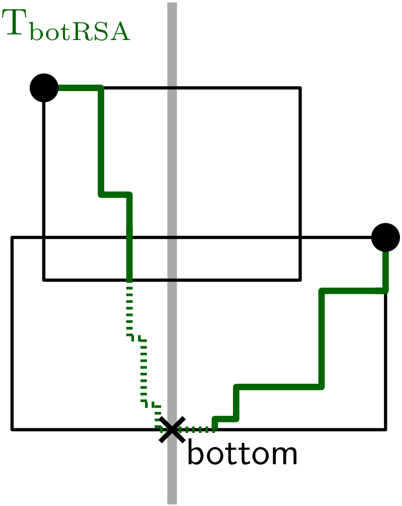
First Step



Construction

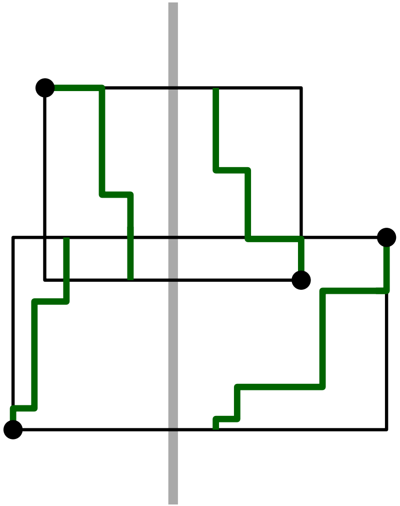
First Step

$$(1+\varepsilon) \cdot \mathrm{OPT}_{\mathrm{botRSA}}$$



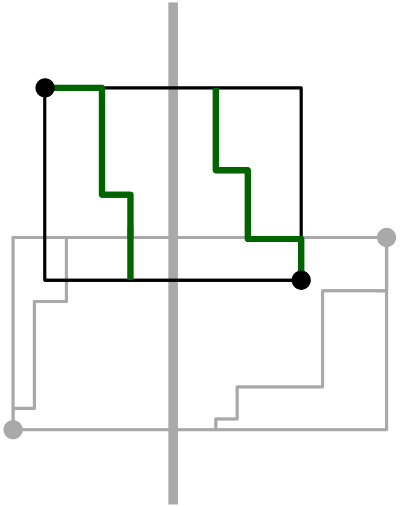
Construction

First Step



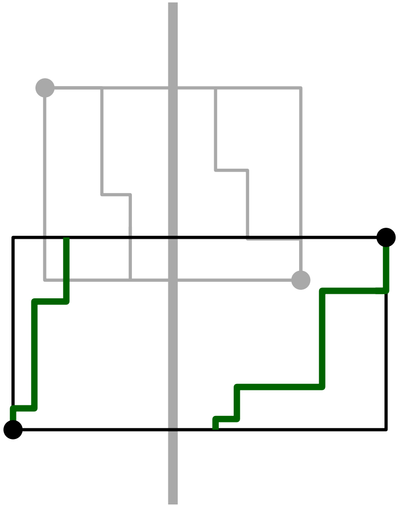
Construction

First Step



Construction

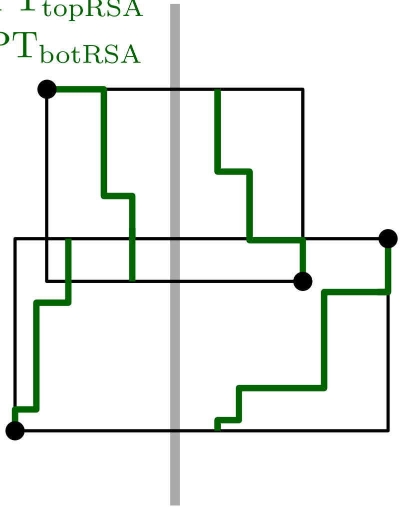
First Step



Cost Analysis

First Step

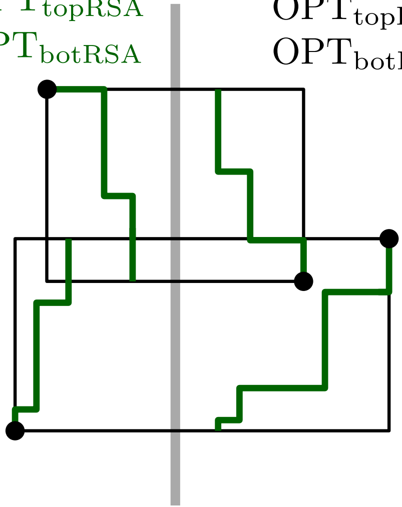
$$(1+\varepsilon)\cdot \mathrm{OPT}_{\mathrm{topRSA}}$$
$$+(1+\varepsilon)\cdot \mathrm{OPT}_{\mathrm{botRSA}}$$



Cost Analysis
First Step

$$(1+\varepsilon)\cdot \mathrm{OPT}_{\mathrm{topRSA}}$$
$$+(1+\varepsilon)\cdot \mathrm{OPT}_{\mathrm{botRSA}}$$

$$\mathrm{OPT}_{\mathrm{topRSA}}?$$
$$\mathrm{OPT}_{\mathrm{botRSA}}?$$



Cost Analysis

First Step

- $\mathrm{OPT}_{\mathrm{topRSA}}, \mathrm{OPT}_{\mathrm{botRSA}} \leq \mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}}$

Lemma

Step 1 costs $(2 + 2\varepsilon) \cdot (\mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}})$.

Cost Analysis

First Step

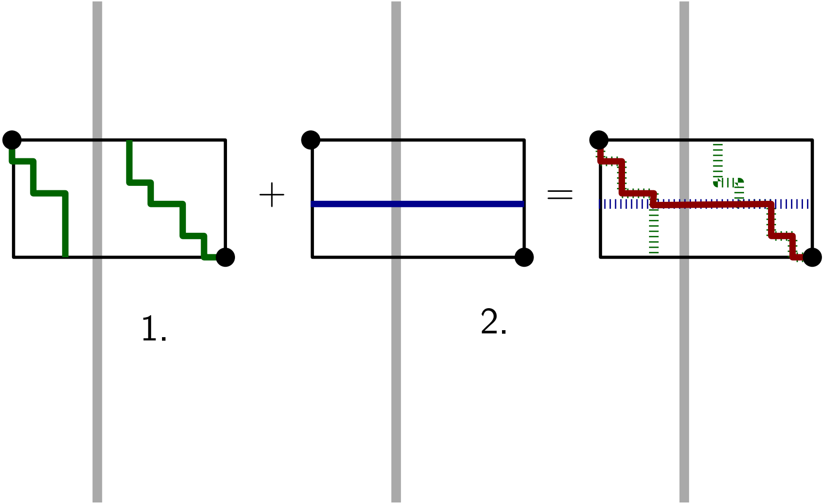
- $\mathrm{OPT}_{\mathrm{topRSA}}, \mathrm{OPT}_{\mathrm{botRSA}} \leq \mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}}$

Lemma

Step 1 costs $(2 + 2\varepsilon) \cdot (\mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}})$.

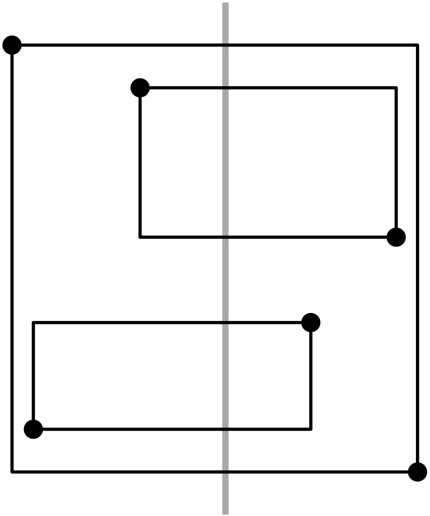
Second Step

Second Step



Observations

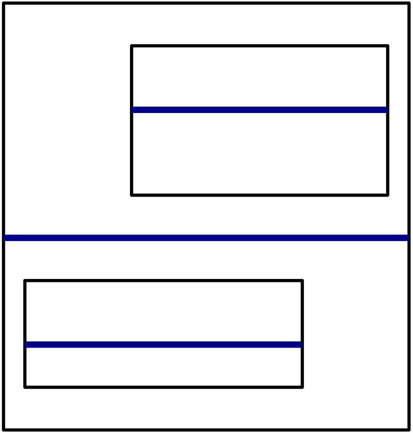
Second Step



Observations

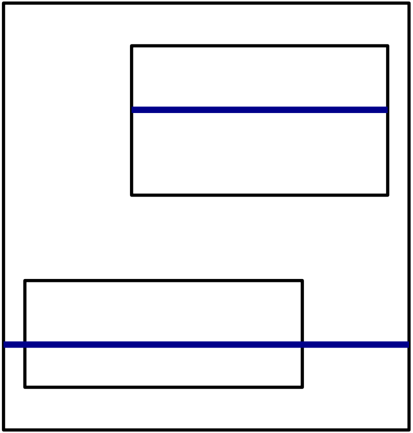
Second Step

Stabbing



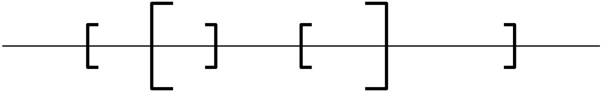
Observations
Second Step

Stabbing



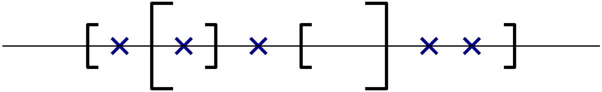
Excursion: Piercing

Second Step



Excursion: Piercing

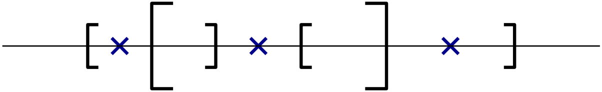
Second Step



Excursion: Piercing

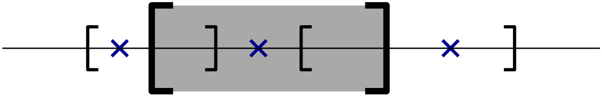
Second Step

Minimal Piercing



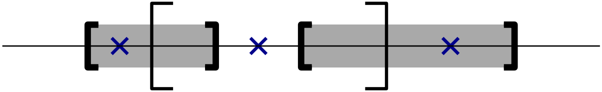
Excursion: Piercing
Second Step

Minimal Piercing



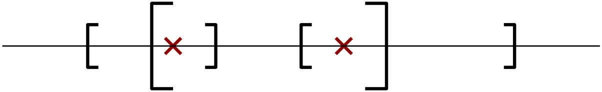
Excursion: Piercing
Second Step

Minimal Piercing



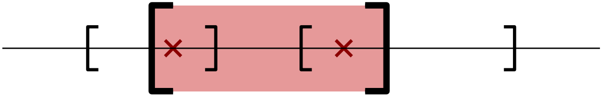
Excursion: Piercing
Second Step

Optimum Piercing



Excursion: Piercing
Second Step

Optimum Piercing



Excursion: Piercing

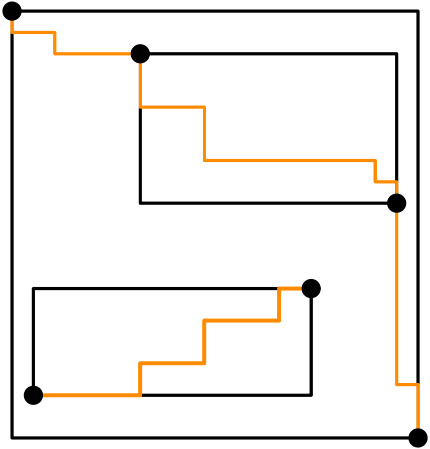
Second Step

Lemma

Minimal piercing $\leq 2\cdot$ optimum piercing.

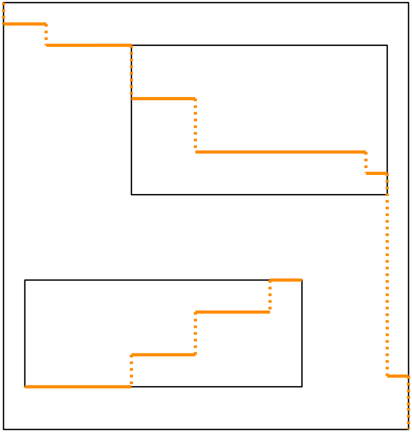
OPT is a Piercing

Second Step



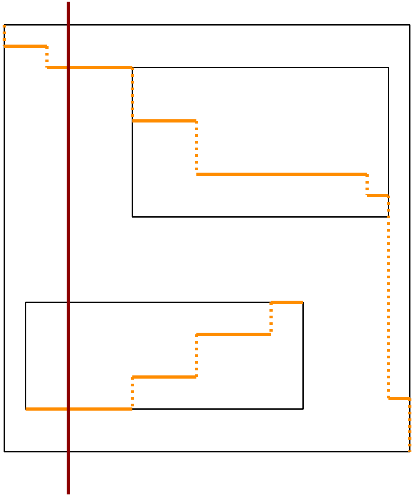
OPT is a Piercing

Second Step



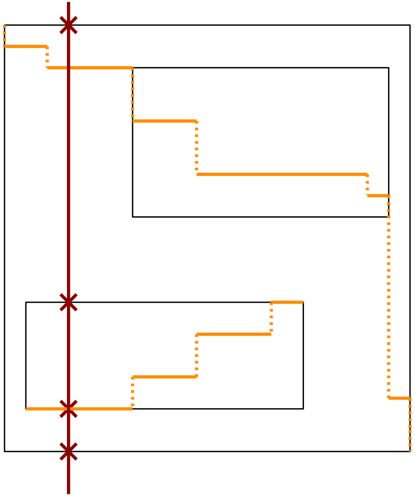
OPT is a Piercing

Second Step



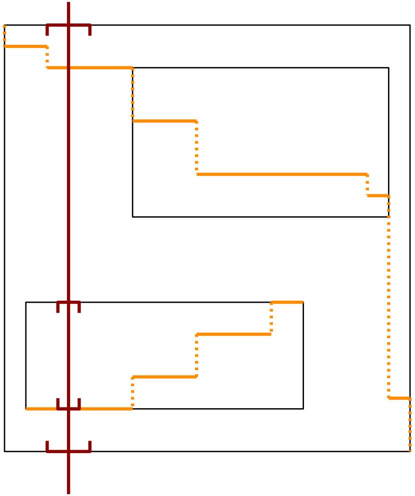
OPT is a Piercing

Second Step



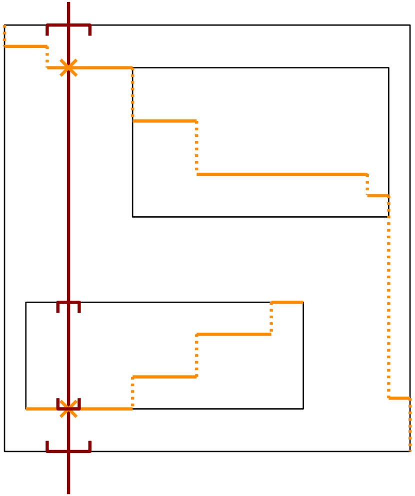
OPT is a Piercing

Second Step



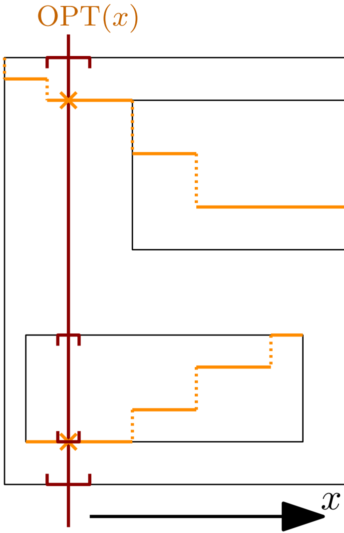
OPT is a Piercing

Second Step



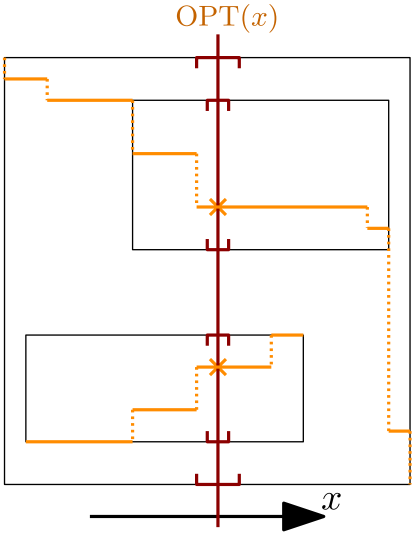
OPT is a Piercing

Second Step



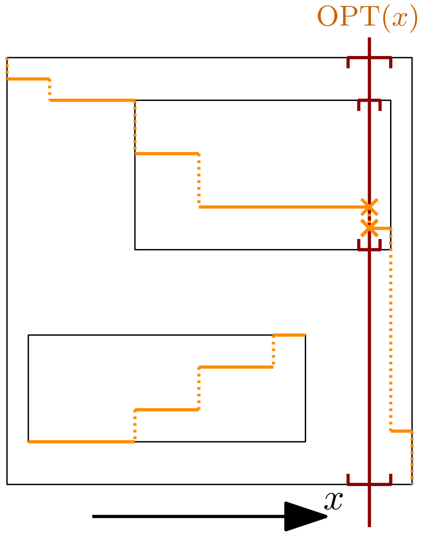
OPT is a Piercing

Second Step



OPT is a Piercing

Second Step



OPT is a Piercing

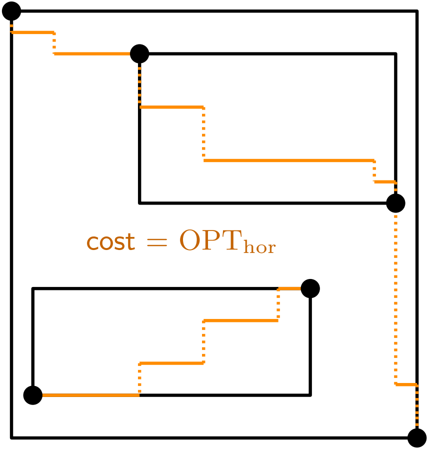
Second Step

Lemma

$\forall x$: *optimum piercing*$(x) \leq \mathrm{OPT}(x)$

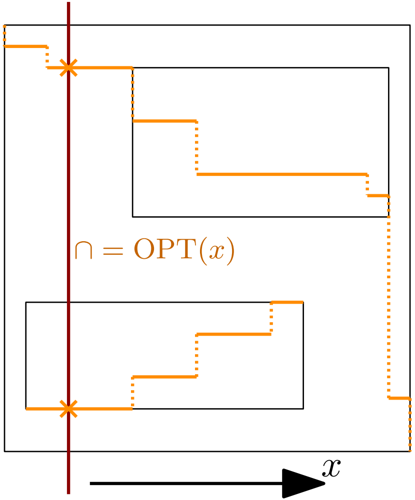
Piercing Summed Up

Second Step



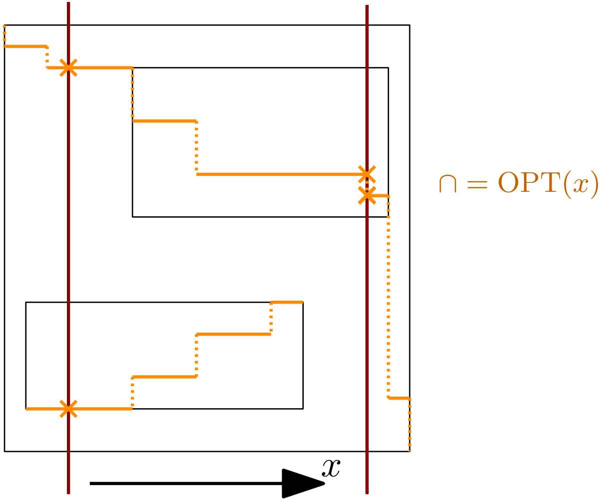
Piercing Summed Up

Second Step



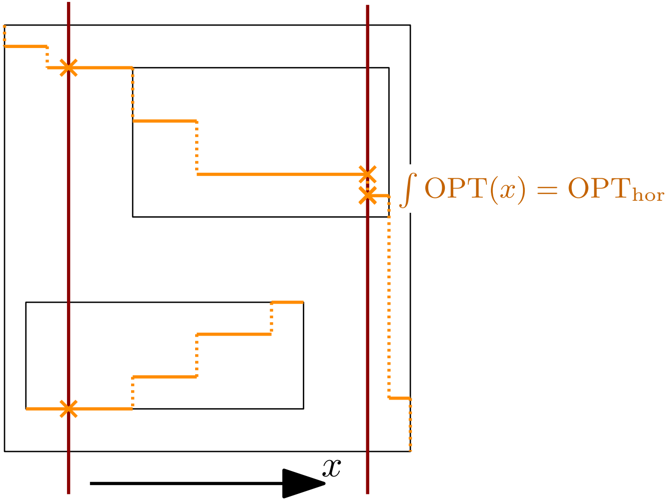
Piercing Summed Up

Second Step



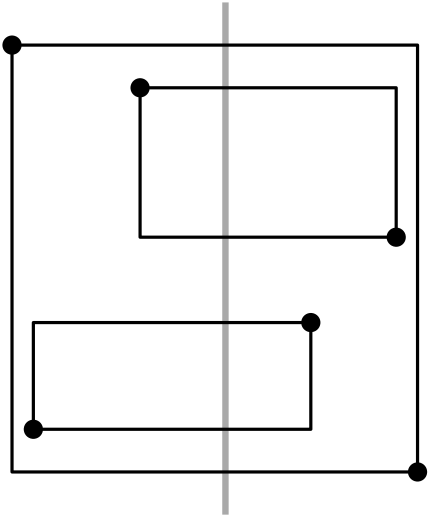
Piercing Summed Up

Second Step

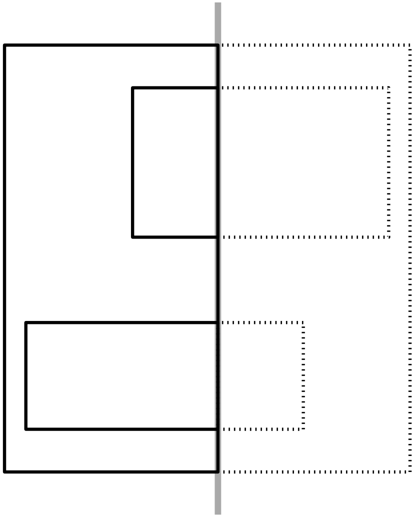


Stabbing the Left Side

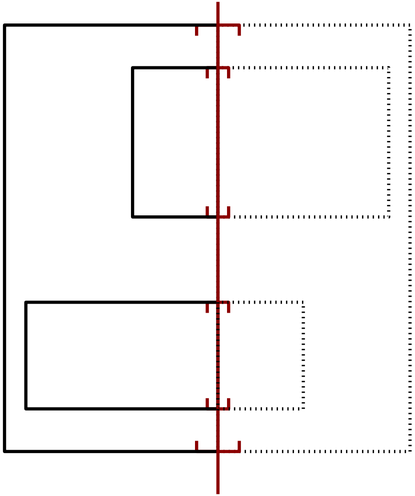
Second Step



Stabbing the Left Side
Second Step



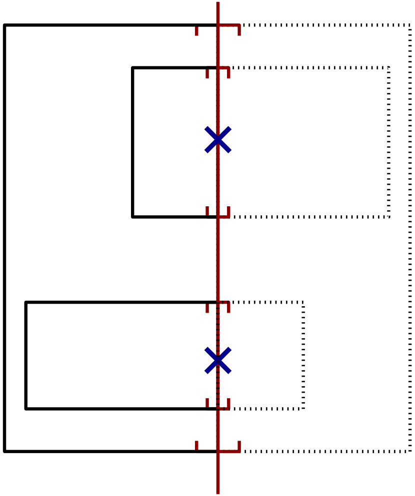
Stabbing the Left Side
Second Step



Stabbing the Left Side

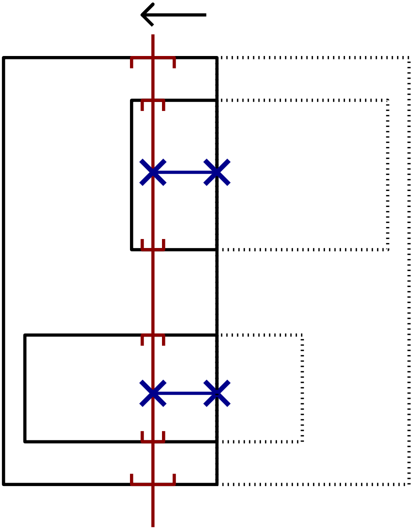
Second Step

Minimal Piercing



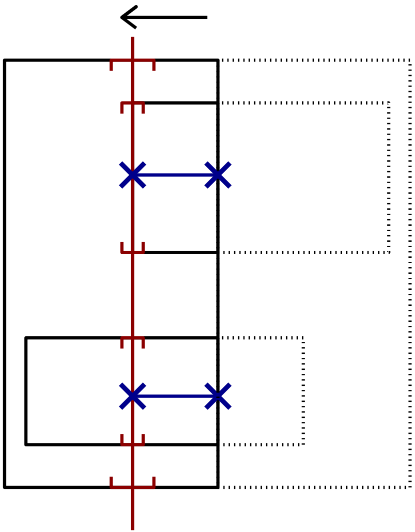
Stabbing the Left Side

Second Step



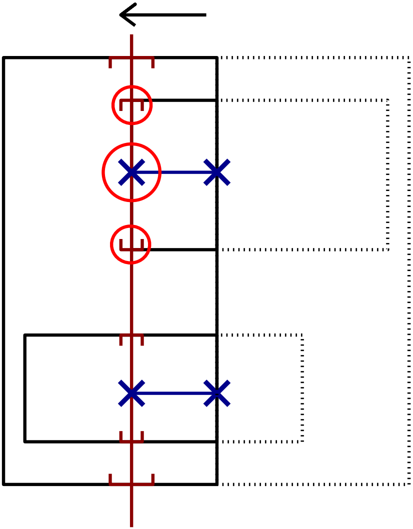
Stabbing the Left Side

Second Step



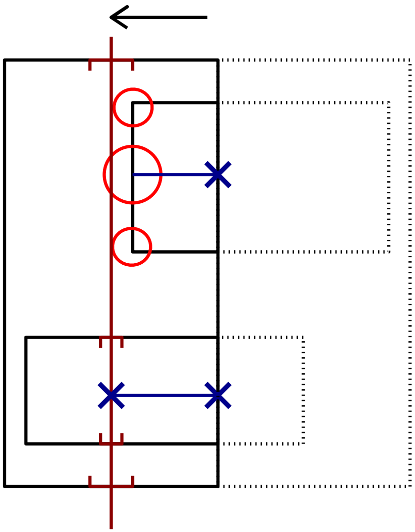
Stabbing the Left Side

Second Step



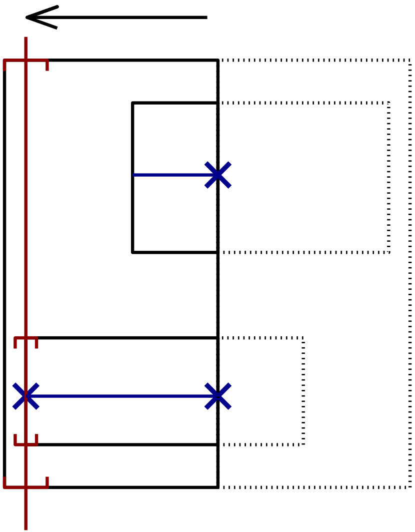
Stabbing the Left Side

Second Step



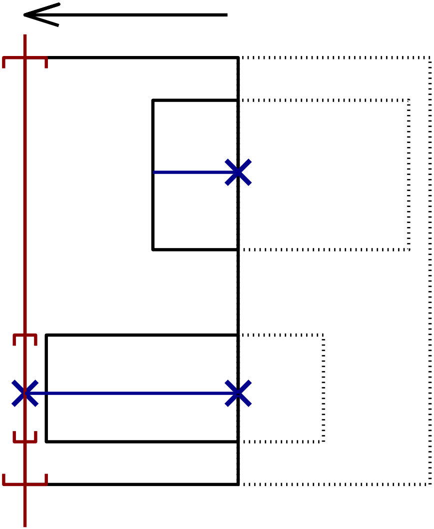
Stabbing the Left Side

Second Step



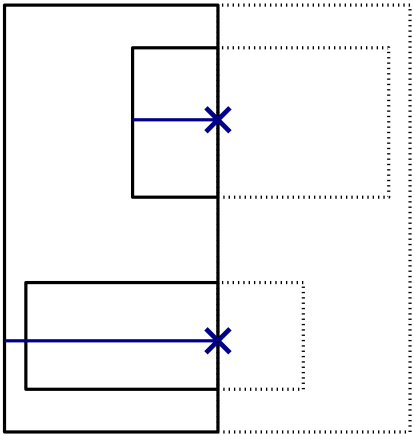
Stabbing the Left Side

Second Step



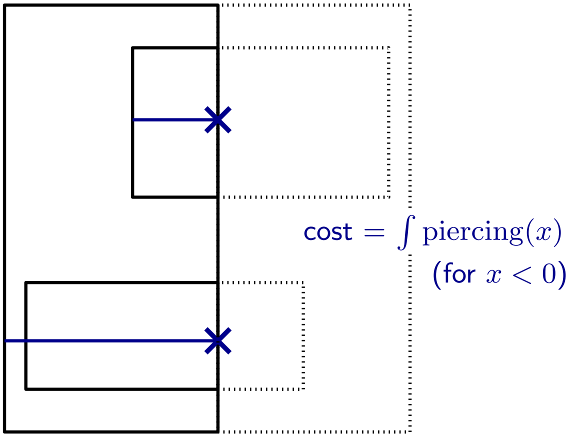
Stabbing the Left Side

Second Step



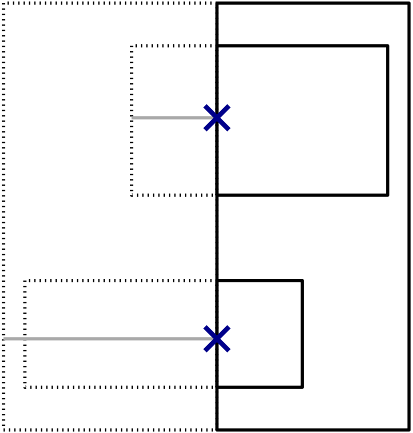
Stabbing the Left Side

Second Step



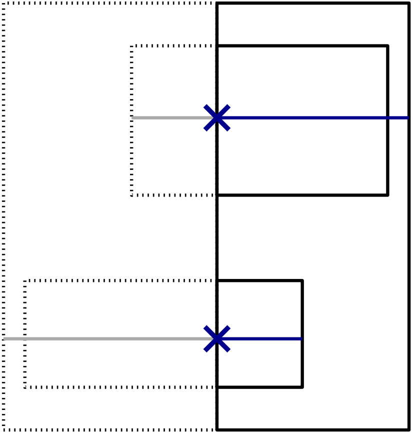
Stabbing Both Sides

Second Step



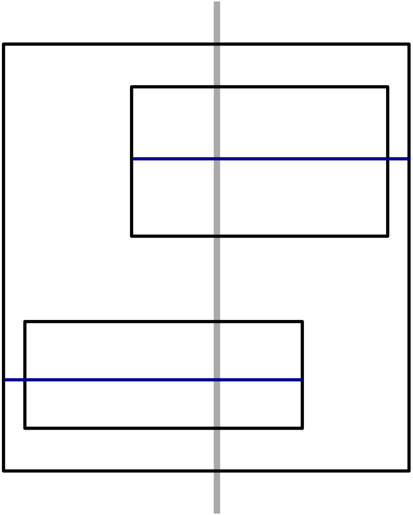
Stabbing Both Sides

Second Step



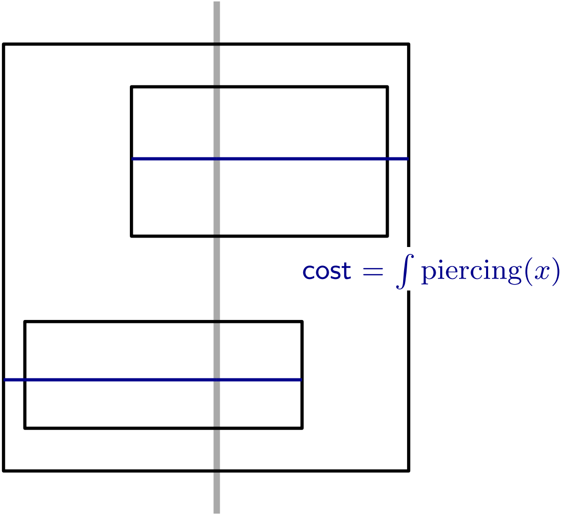
Stabbing Both Sides

Second Step



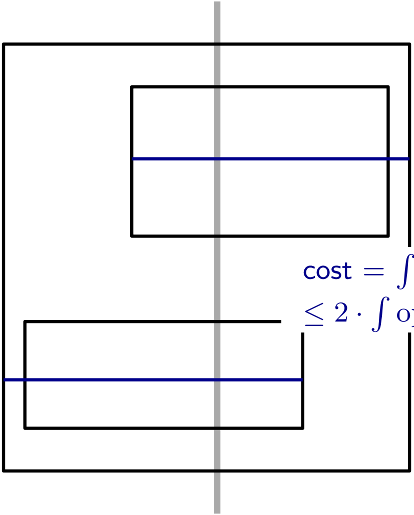
Stabbing Both Sides

Second Step



Stabbing Both Sides

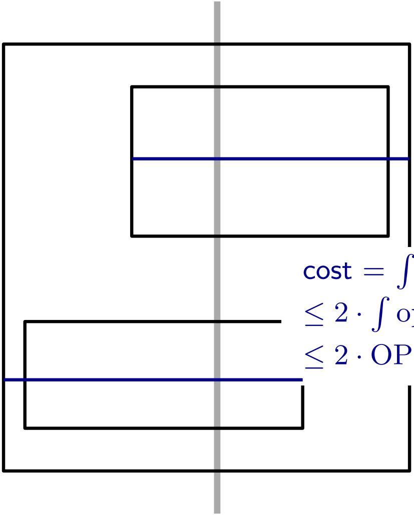
Second Step



$$\text{cost} = \int \text{piercing}(x) \leq 2 \cdot \int \text{optimum piercing}$$

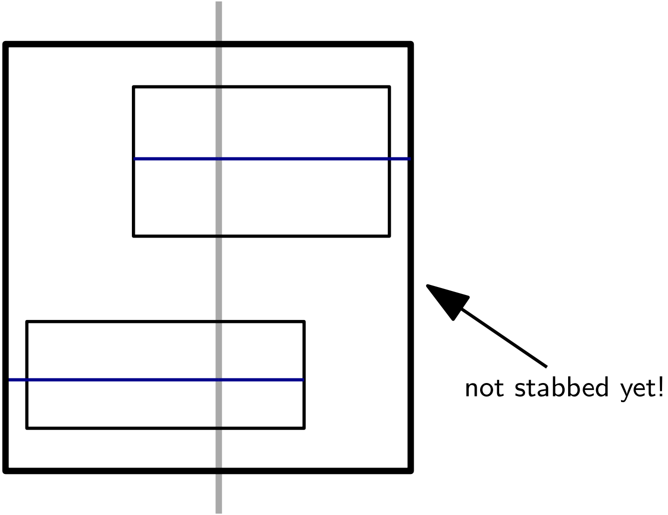
Stabbing Both Sides

Second Step



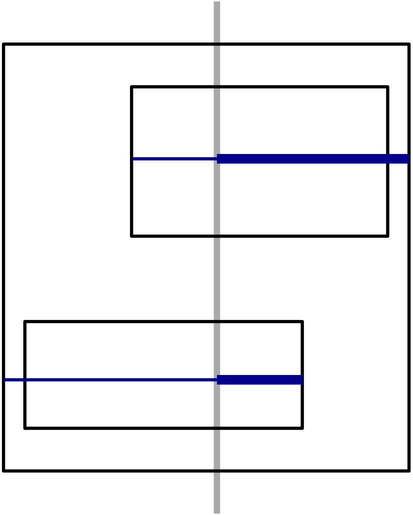
$$\text{cost} = \int \text{piercing}(x) \leq 2 \cdot \int \text{optimum piercing} \leq 2 \cdot \text{OPT}_{\text{hor}}$$

Mirror Step
Second Step



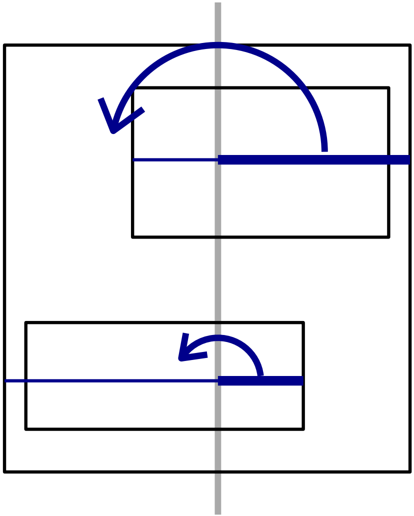
Mirror Step

Second Step



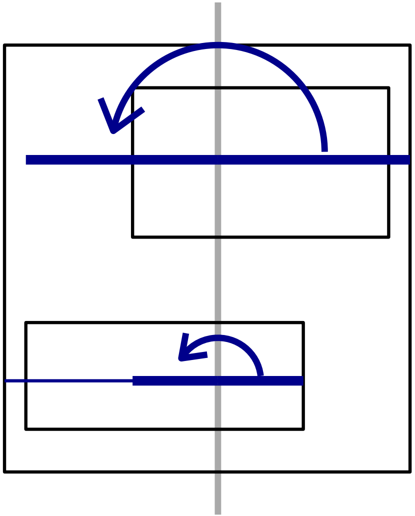
Mirror Step

Second Step



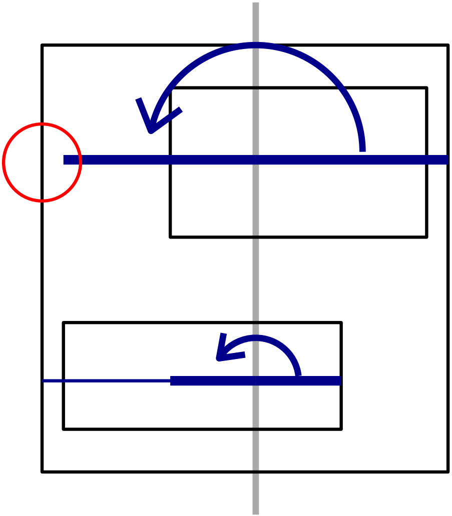
Mirror Step

Second Step



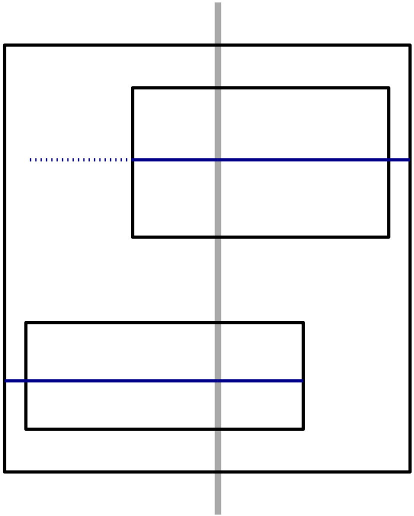
Mirror Step

Second Step

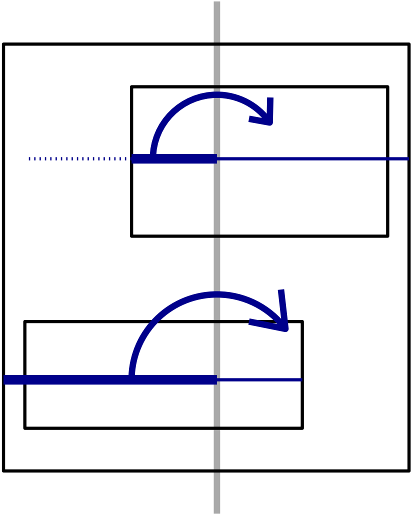


Mirror Step

Second Step

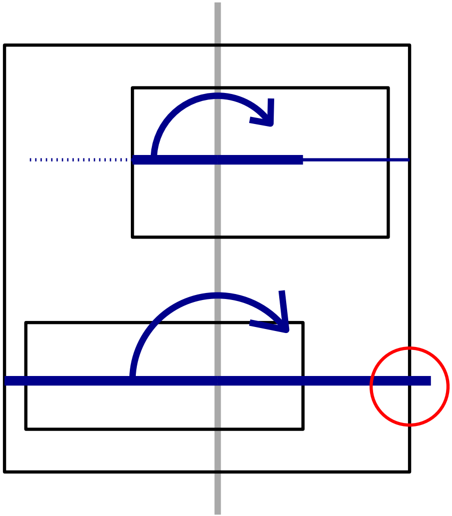


Mirror Step
Second Step



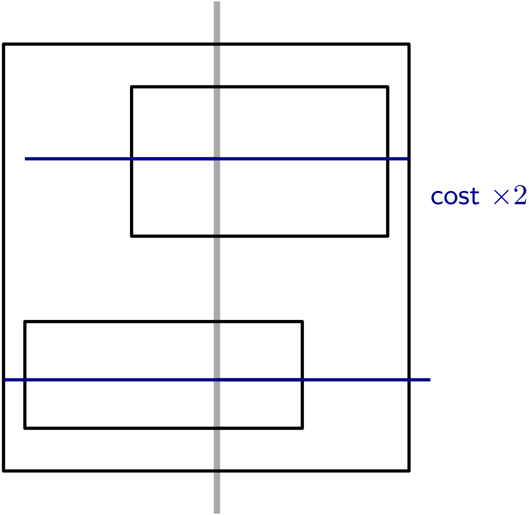
Mirror Step

Second Step

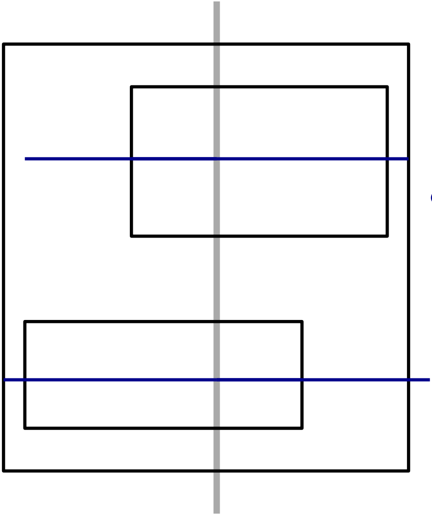


Mirror Step

Second Step



Mirror Step
Second Step

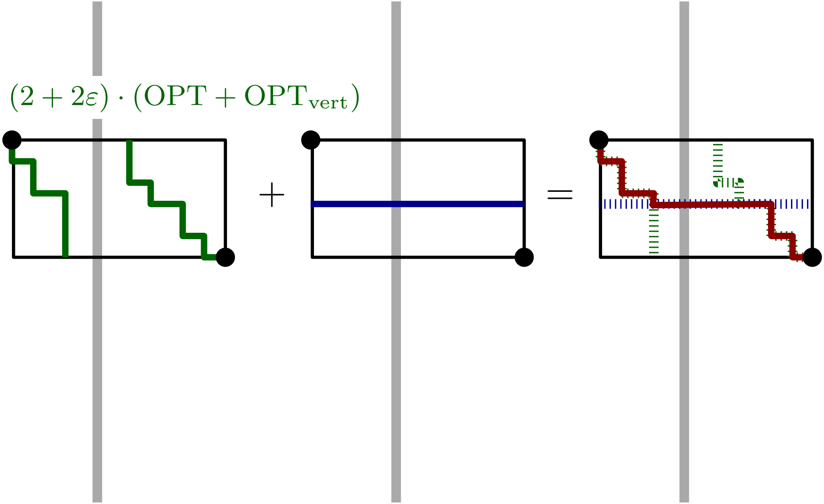


cost $\leq 4 \cdot \mathrm{OPT}_{\mathrm{hor}}$

Total Cost

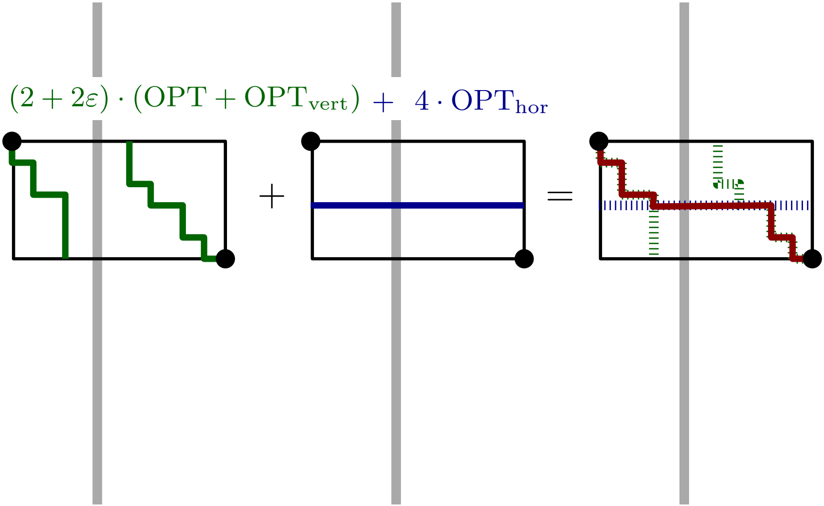
Both Steps Together

$$(2 + 2\varepsilon) \cdot (\mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}})$$



Total Cost
Both Steps Together

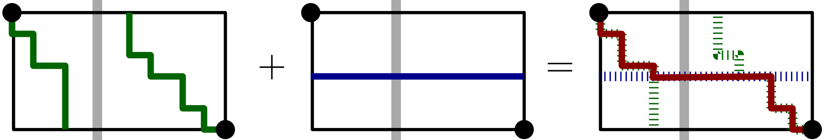
$$(2 + 2\varepsilon) \cdot (\mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}}) + 4 \cdot \mathrm{OPT}_{\mathrm{hor}}$$



Total Cost

Both Steps Together

$$(2+2\varepsilon)\cdot(\mathrm{OPT}+\mathrm{OPT}_{\mathrm{vert}}) + 4\cdot\mathrm{OPT}_{\mathrm{hor}}$$

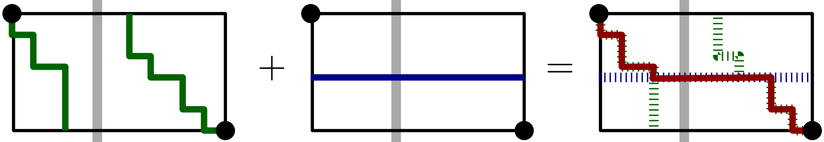


$$\le (6+\varepsilon')\cdot\mathrm{OPT} \quad \text{for } \varepsilon'=\varepsilon/2$$

Total Cost

Both Steps Together

$$(2 + 2\varepsilon) \cdot (\mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}}) + 4 \cdot \mathrm{OPT}_{\mathrm{hor}}$$



$$\leq \quad (6 + \varepsilon') \cdot \mathrm{OPT} \qquad \text{for } \varepsilon' = \varepsilon/2$$

$$\Rightarrow \rho_x(n) = (6 + \varepsilon) \in O(1)$$

Optimized Result in 2D

Both Steps Together

Theorem (New)

2D-GMMN admits an $((6+\varepsilon)\cdot\log n)$-approximation.

Theorem (Before)

2D-GMMN admits an $O(\log^2 n)$-approximation.

Optimized Result in 2D

Both Steps Together

Theorem (New)

2D-GMMN admits an $O(\log n)$-approximation.

Theorem (Before)

2D-GMMN admits an $O(\log^2 n)$-approximation.

Optimized Result in 2D

Both Steps Together

Theorem (New)

2D-GMMN admits an $O(\log n)$-approximation.

Theorem (Before)

2D-GMMN admits an $O(\log^2 n)$-approximation.

Summary
Current Results

	2D	Dimension $d > 2$
RSA	$(1 + \varepsilon)$	$O(\log n)$
MMN	2	$O(\log^{d+1} n)$
GMMN	$O(\log n)$	$O(\log^{d+1} n)$

Open Problems

Future Results?

- O(1)-Approximation for 2D-GMMN?
- O(1)-Approximation for 3D-RSA?
- APX-Hardness of 2D-GMMN?

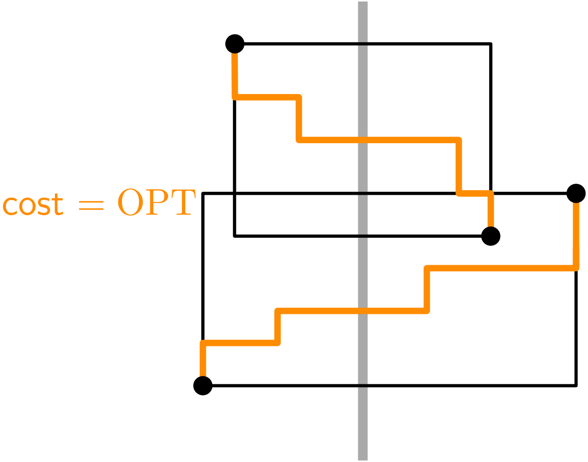
Outline

Appendix
Detailed Analysis

(RSA Cost Analysis)
(Minimal Piercing Analysis)

Cost Analysis
Detailed Analysis

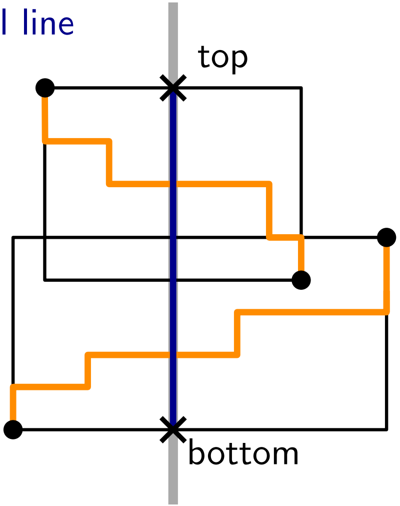
An optimum GMMN solution



Cost Analysis

Detailed Analysis

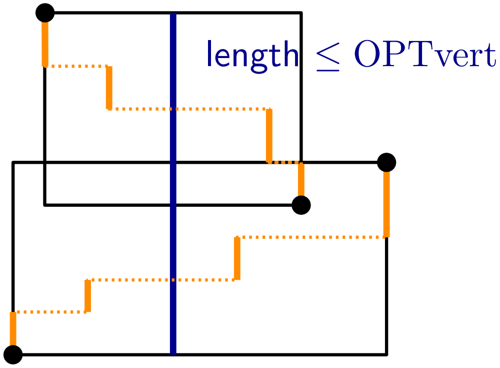
Add vertical line



Cost Analysis

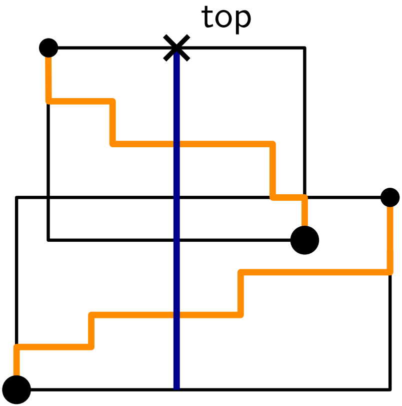
Detailed Analysis

Add vertical line



Cost Analysis

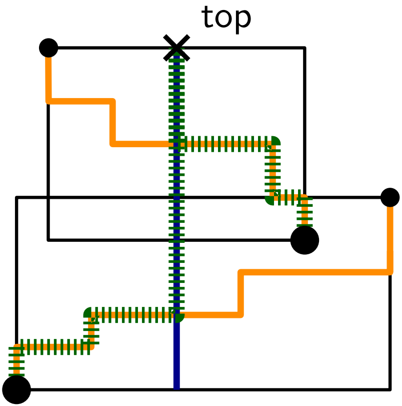
Detailed Analysis



Cost Analysis

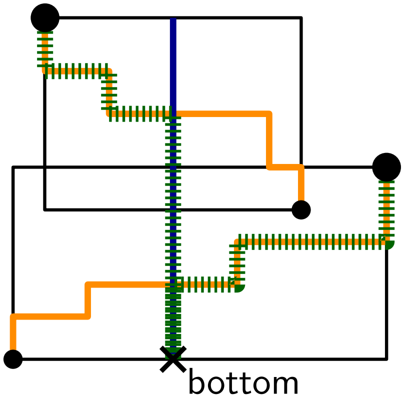
Detailed Analysis

Optimal solution + vertical line = top RSA



Cost Analysis
Detailed Analysis

Optimal solution + vertical line = bottom RSA



Cost Analysis
Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$\begin{aligned}(1+\varepsilon) \cdot \text{OPT}_{\text{topRSA}} + (1+\varepsilon) \cdot \text{OPT}_{\text{botRSA}} \\ \leq \quad & (1+\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + (1+\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) \\ \leq \quad & 2 \cdot (1+\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})\end{aligned}$$

(Go To Short Analysis)
(Go To Open Problems)

Cost Analysis
Detailed Analysis

- $\mathrm{OPT}_{\mathrm{topRSA}} \leq \mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}}$
- $\mathrm{OPT}_{\mathrm{botRSA}} \leq \mathrm{OPT} + \mathrm{OPT}_{\mathrm{vert}}$
- Our solution cost:

$$
\begin{aligned}
&(1+\varepsilon)\cdot \mathrm{OPT}_{\mathrm{topRSA}} + (1+\varepsilon)\cdot \mathrm{OPT}_{\mathrm{botRSA}} \\
&\quad \leq \quad (1+\varepsilon)\cdot(\mathrm{OPT}+\mathrm{OPT}_{\mathrm{vert}}) + (1+\varepsilon)\cdot(\mathrm{OPT}+\mathrm{OPT}_{\mathrm{vert}}) \\
&\quad \leq \quad 2\cdot(1+\varepsilon)\cdot(\mathrm{OPT}+\mathrm{OPT}_{\mathrm{vert}})
\end{aligned}
$$

(Go To Short Analysis)
(Go To Open Problems)

Cost Analysis
Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$(1+\varepsilon) \cdot \text{OPT}_{\text{topRSA}} + (1+\varepsilon) \cdot \text{OPT}_{\text{botRSA}}$$
$$\leq (1+\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + (1+\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})$$
$$\leq 2 \cdot (1+\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})$$

(Go To Short Analysis)
(Go To Open Problems)

Cost Analysis
Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$
\begin{aligned}
&(1+\varepsilon)\cdot \text{OPT}_{\text{topRSA}} + (1+\varepsilon)\cdot \text{OPT}_{\text{botRSA}} \\
&\quad \leq \quad (1+\varepsilon)\cdot(\text{OPT} + \text{OPT}_{\text{vert}}) + (1+\varepsilon)\cdot(\text{OPT} + \text{OPT}_{\text{vert}}) \\
&\quad \leq \quad 2\cdot(1+\varepsilon)\cdot(\text{OPT} + \text{OPT}_{\text{vert}})
\end{aligned}
$$

(Go To Short Analysis)
(Go To Open Problems)

Cost Analysis
Detailed Analysis

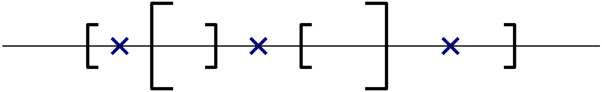
- $\text{OPT}_{\text{topRSA}} \le \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \le \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$
\begin{aligned}
&(1+\varepsilon)\cdot \text{OPT}_{\text{topRSA}} + (1+\varepsilon)\cdot \text{OPT}_{\text{botRSA}} \\
&\quad \le \quad (1+\varepsilon)\cdot(\text{OPT}+\text{OPT}_{\text{vert}}) + (1+\varepsilon)\cdot(\text{OPT}+\text{OPT}_{\text{vert}}) \\
&\quad \le \quad 2\cdot(1+\varepsilon)\cdot(\text{OPT}+\text{OPT}_{\text{vert}})
\end{aligned}
$$

(Go To Short Analysis)
(Go To Open Problems)

Excursion: Piercing
Detailed Analysis

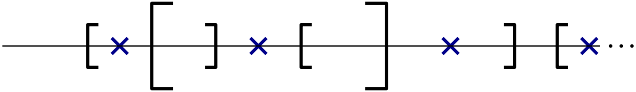
Minimal Piercing $\leq 2\cdot$ Optimum Piercing



Excursion: Piercing

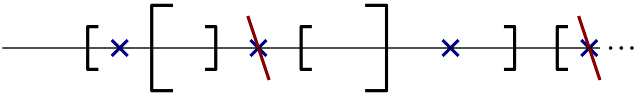
Detailed Analysis

Minimal Piercing



Excursion: Piercing
Detailed Analysis

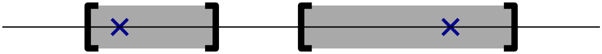
Minimal Piercing



Excursion: Piercing

Detailed Analysis

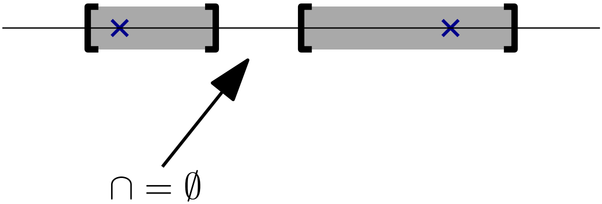
Minimal Piercing



Excursion: Piercing

Detailed Analysis

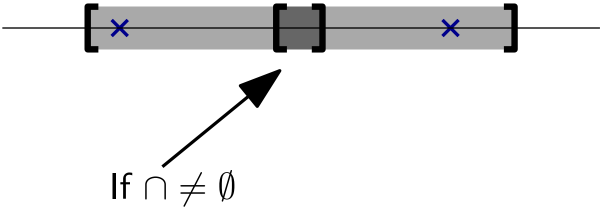
Minimal Piercing



Excursion: Piercing

Detailed Analysis

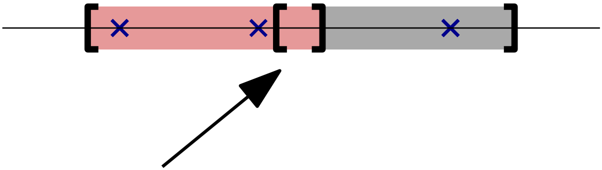
Minimal Piercing



Excursion: Piercing

Detailed Analysis

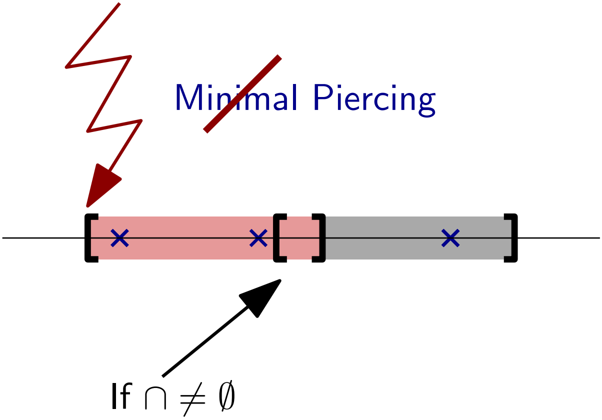
Minimal Piercing



If $\cap \neq \emptyset$

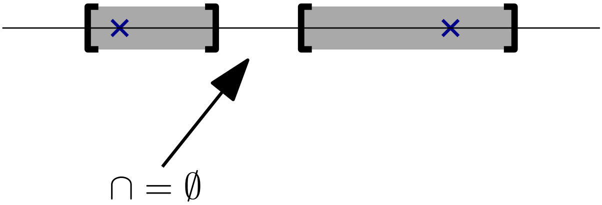
Excursion: Piercing

Detailed Analysis



Excursion: Piercing
Detailed Analysis

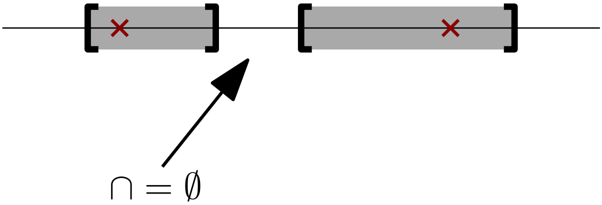
Minimal Piercing



Excursion: Piercing

Detailed Analysis

Optimum Piercing



Excursion: Piercing

Detailed Analysis

Lemma

Minimal piercing $\leq 2\cdot$ optimum piercing.

(Continue With Piercing)
(Go To Open Problems)