Julius-Maximilians-Universität Würzburg
Chair for INFORMATICS I
Efficient Algorithms and Knowledge-Based Systems
Institute for Informatics

The Complexity of Finding Tangles

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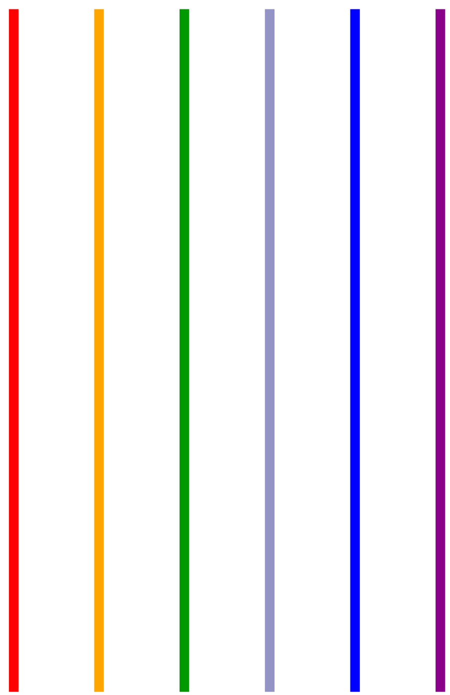
Lviv, Ukraine

Stefan Felsner

TU Berlin,
Germany

Introduction

Given an ordered set of n y-monotone wires



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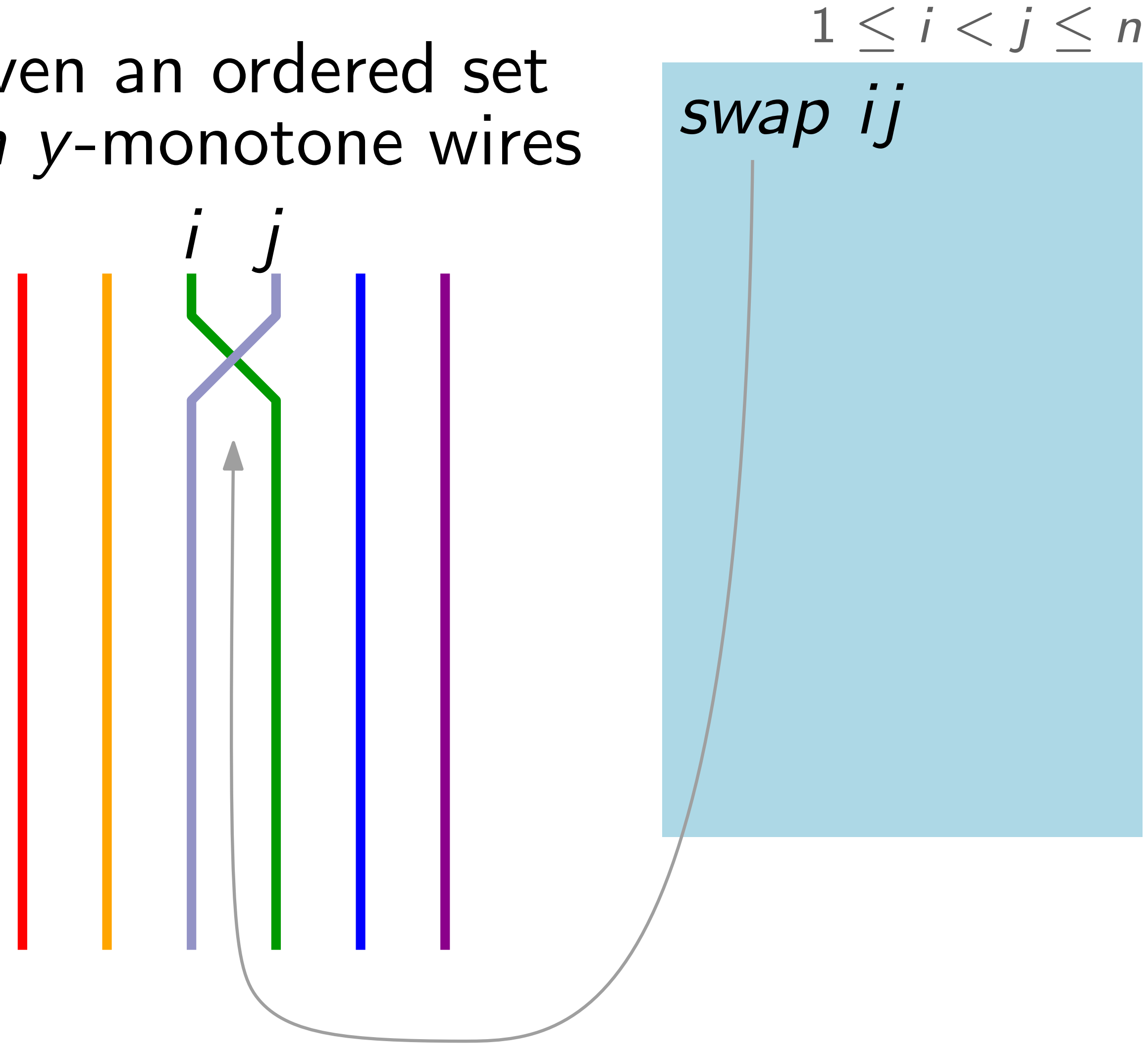

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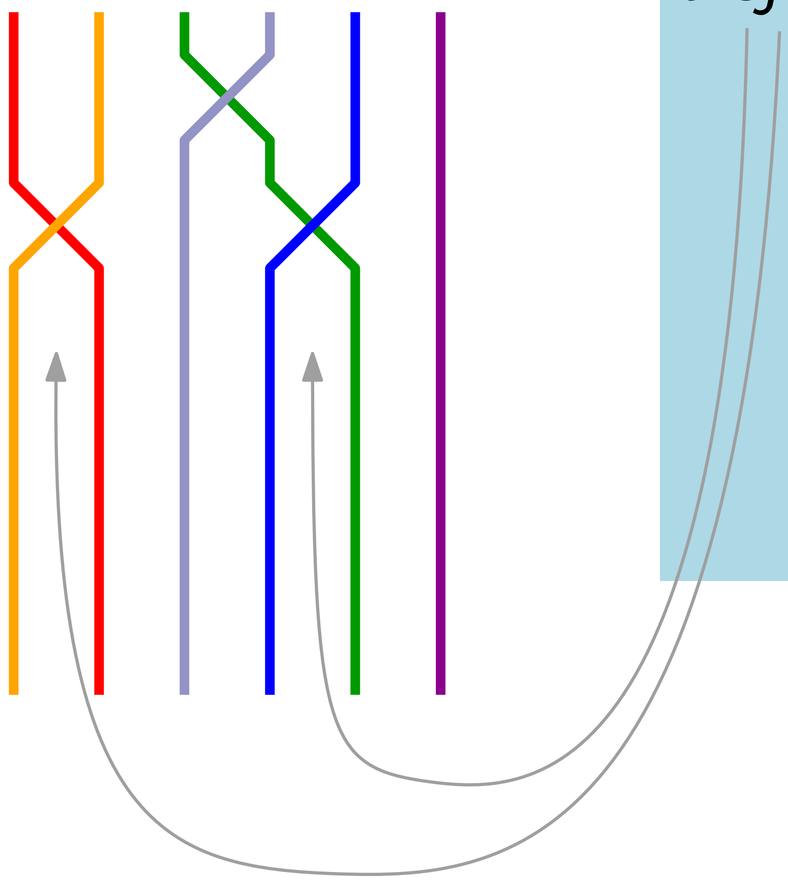
swap ij

disjoint swaps



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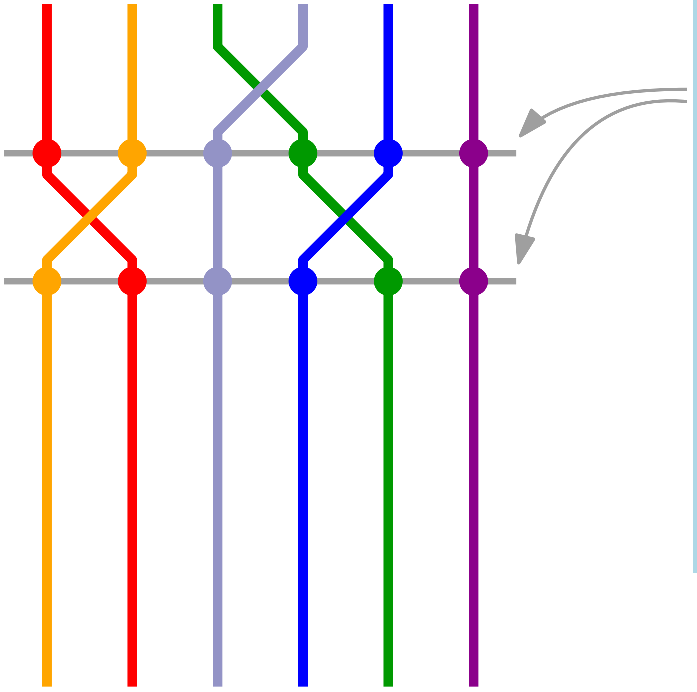

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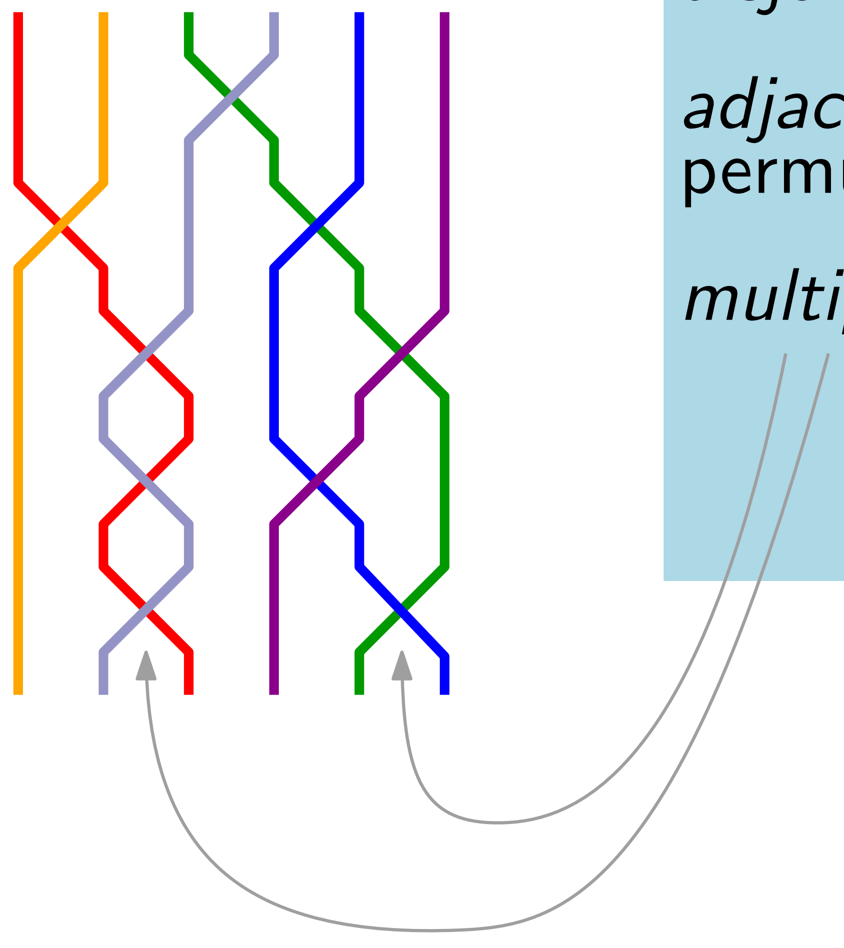

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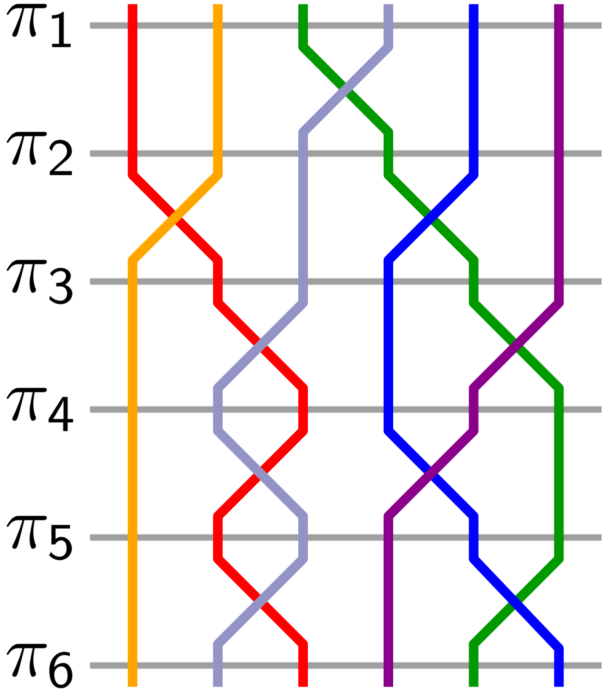

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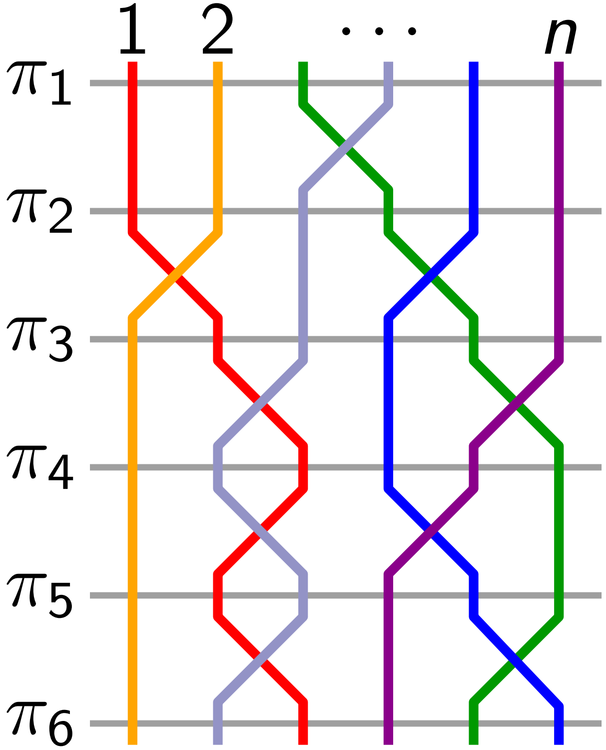

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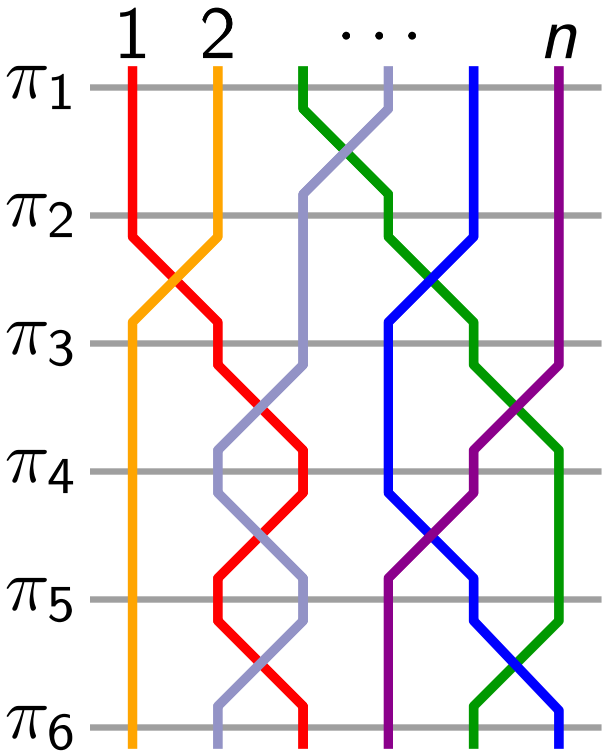

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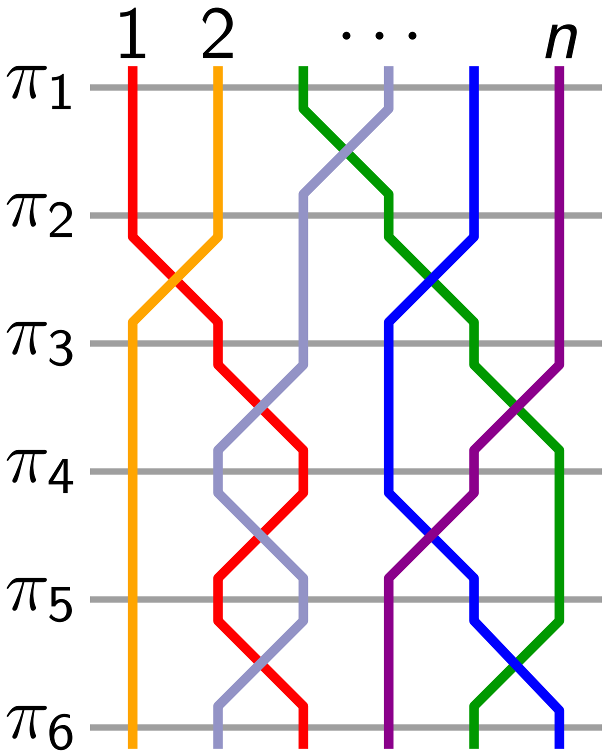

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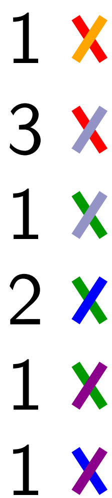

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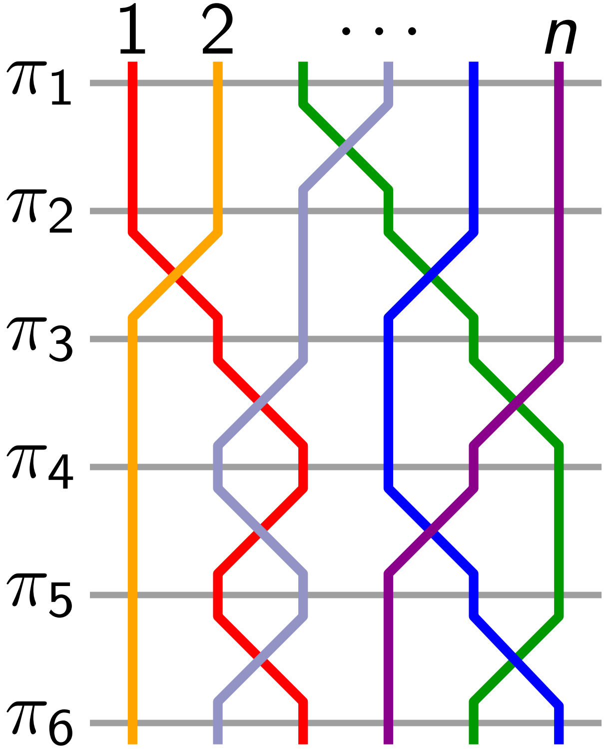

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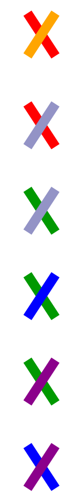
1
2
1
1



Tangle T *realizes* list L.

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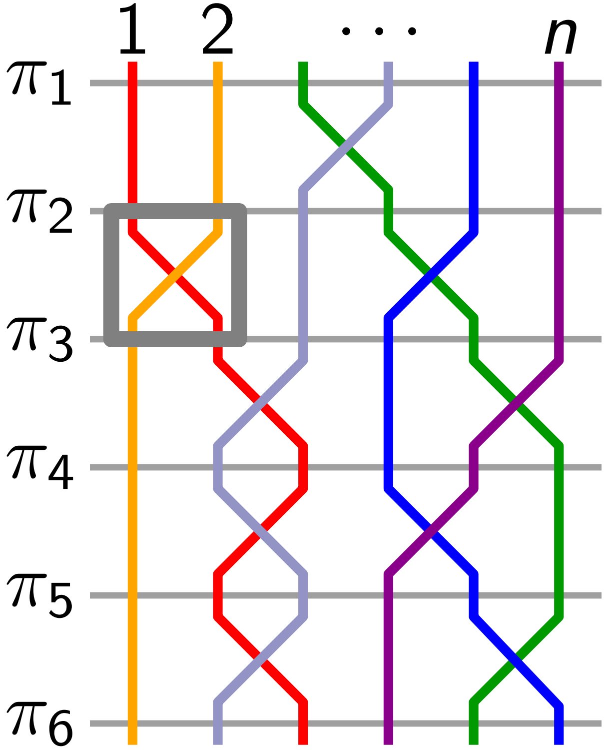

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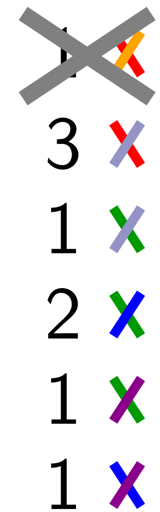

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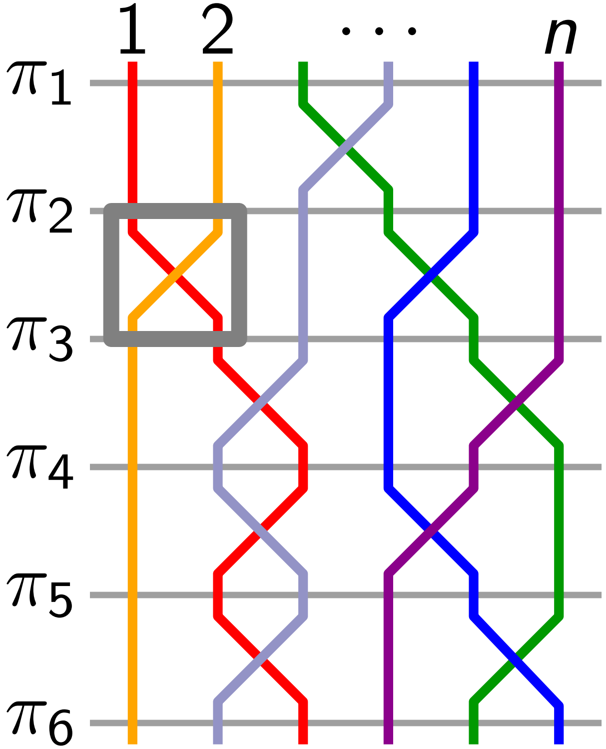

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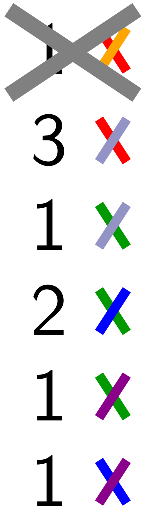

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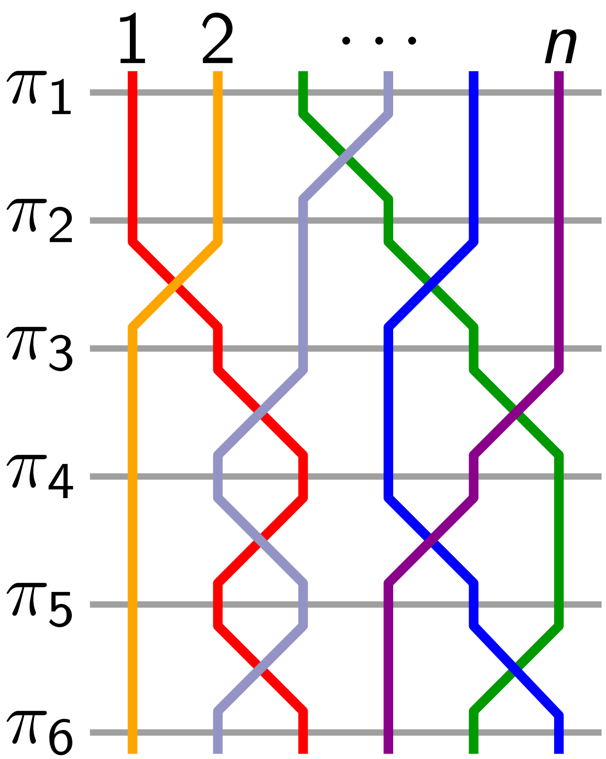

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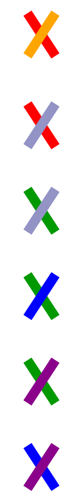

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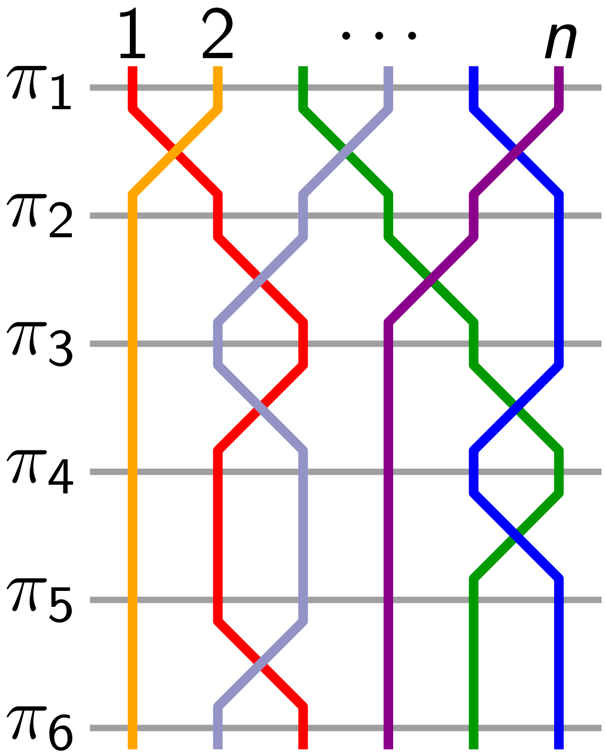

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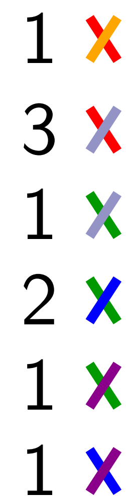

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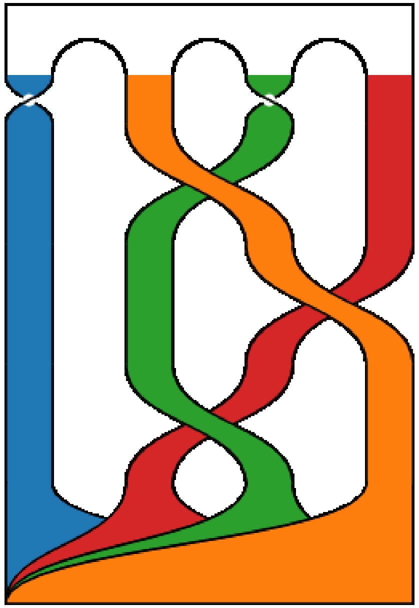

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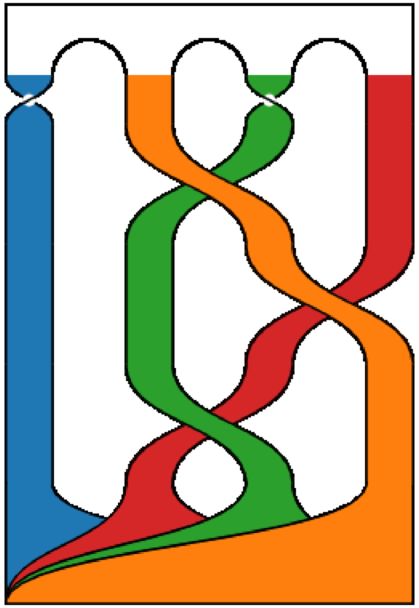

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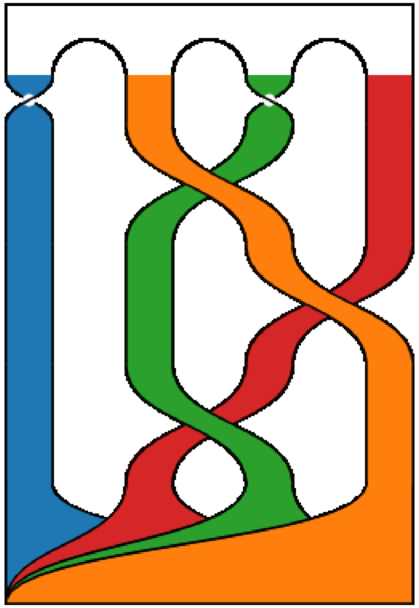
Exp.-time algorithm for finding optimal-height tangles



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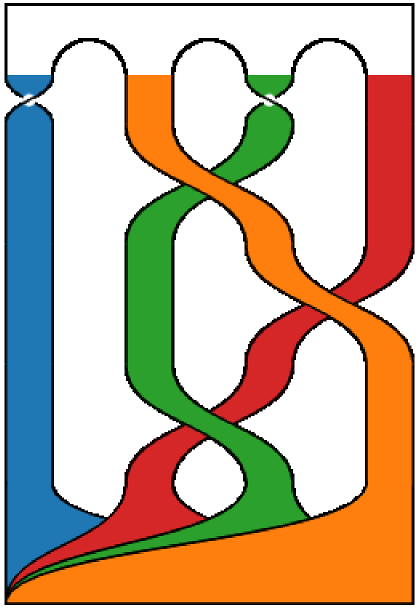

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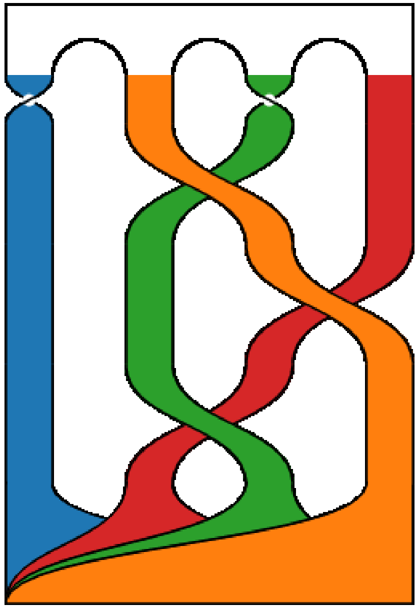

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Given: initial and final permutations

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list

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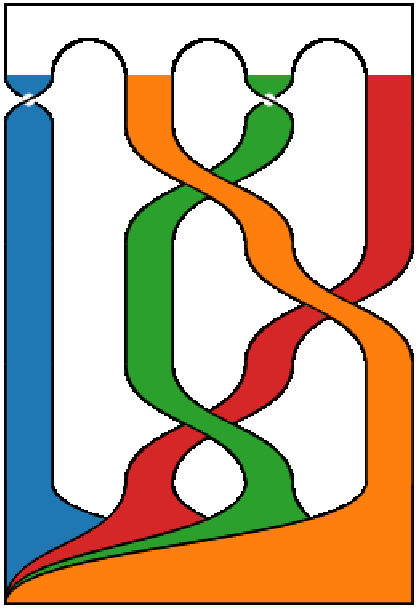

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Related Work

list
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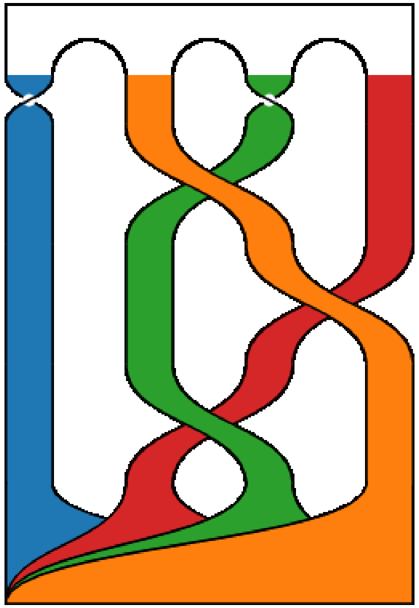

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list ⇕ template

Exp.-time algorithm for finding optimal-height tangles

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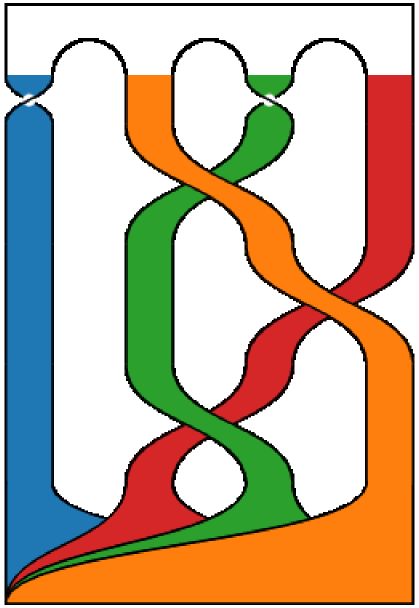

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Faster exp.-time algorithm for finding optimal-height tangles

Related Work

list
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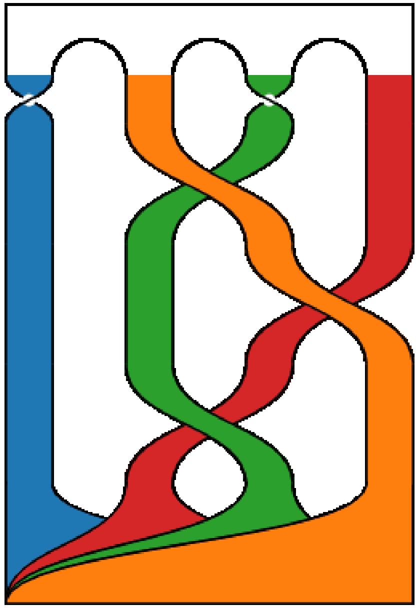

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Faster exp.-time algorithm for finding optimal-height tangles | Finding optimal-height tangles is NP-hard

Contribution

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Deciding whether a given list of swaps is feasible is NP-hard.

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~~negative literals~~

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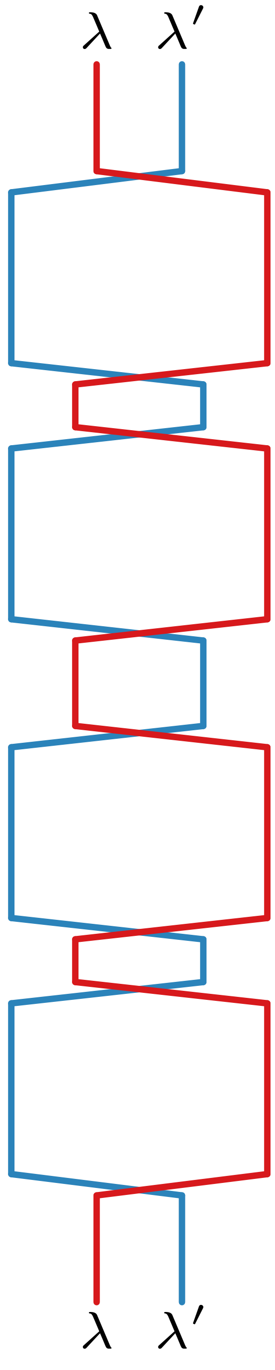

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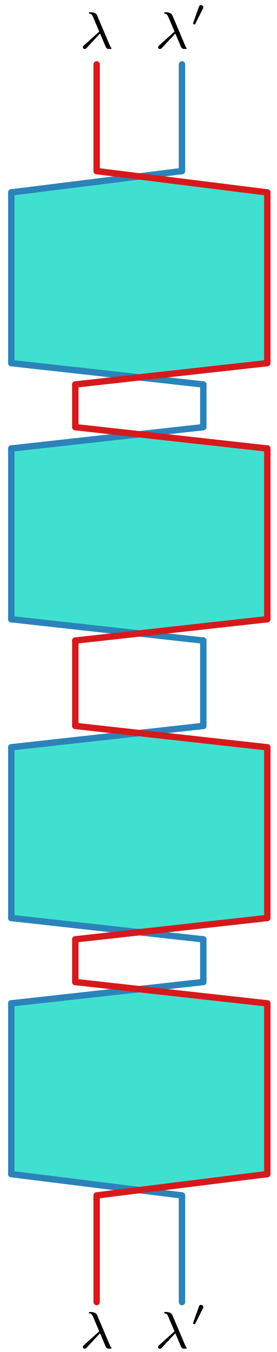

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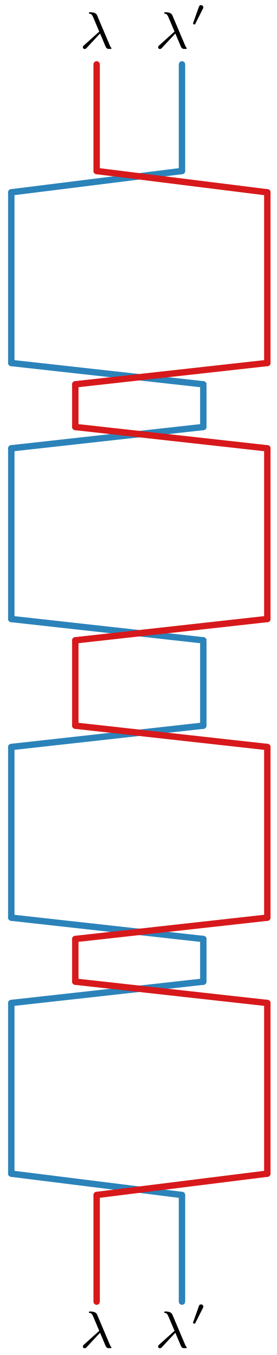

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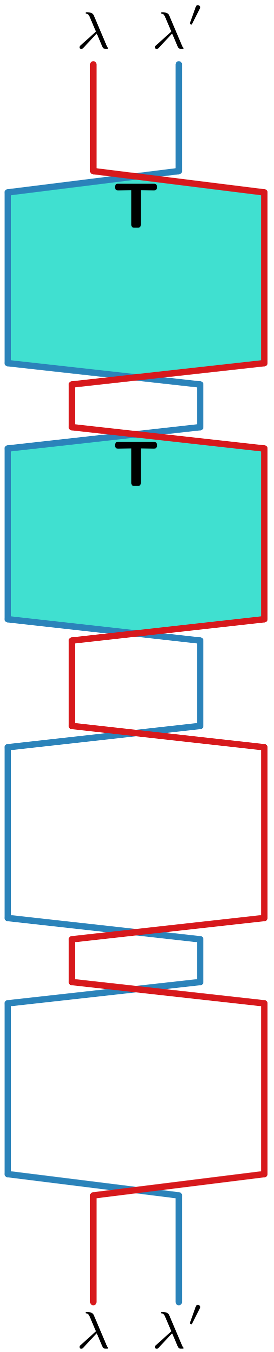

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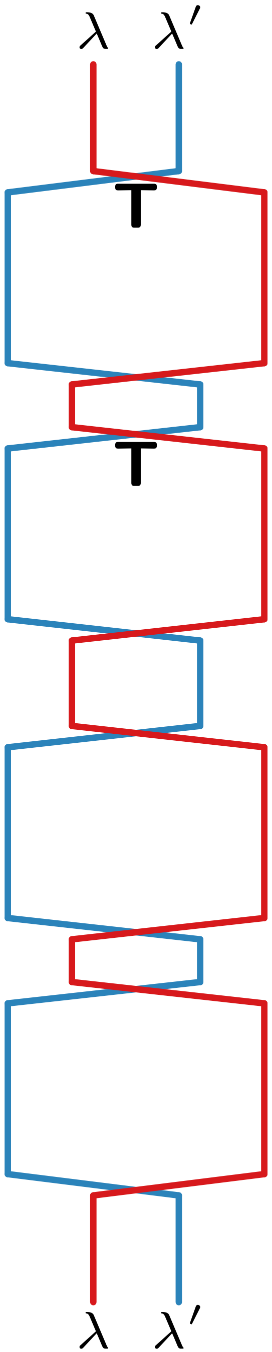

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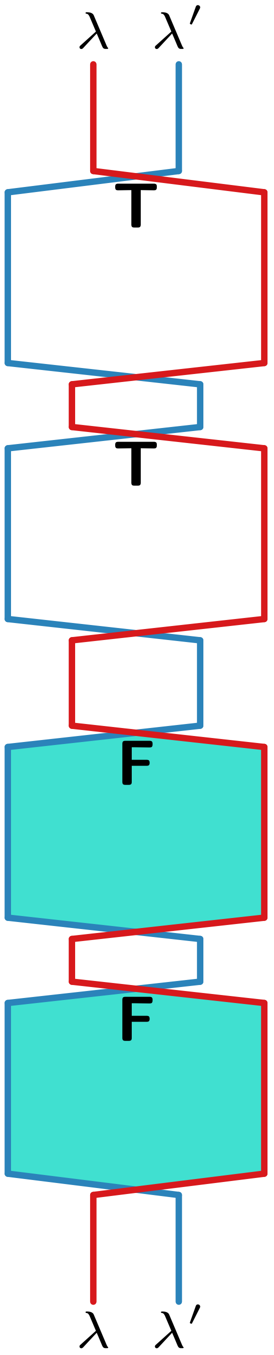

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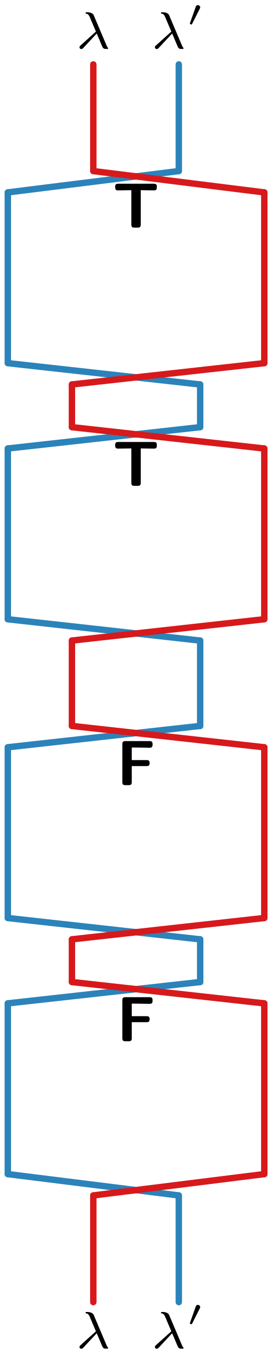

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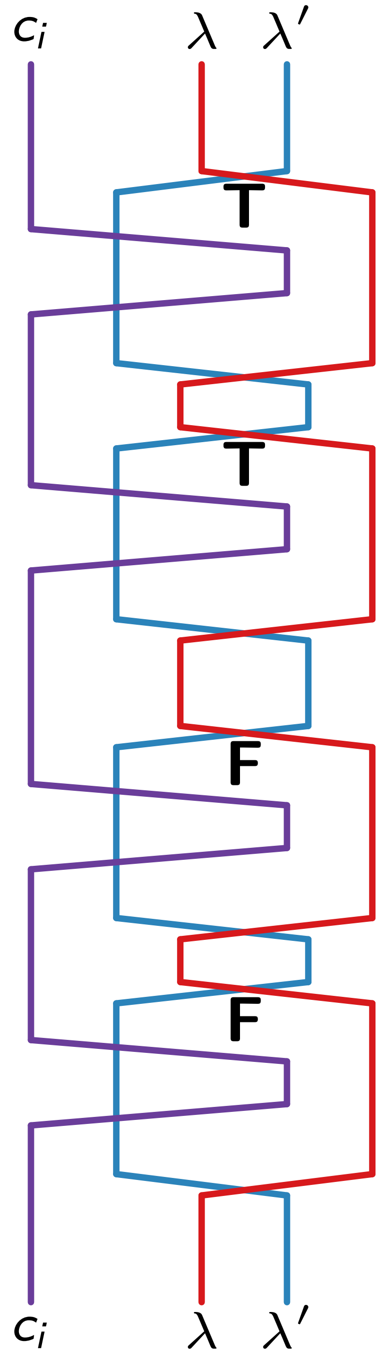

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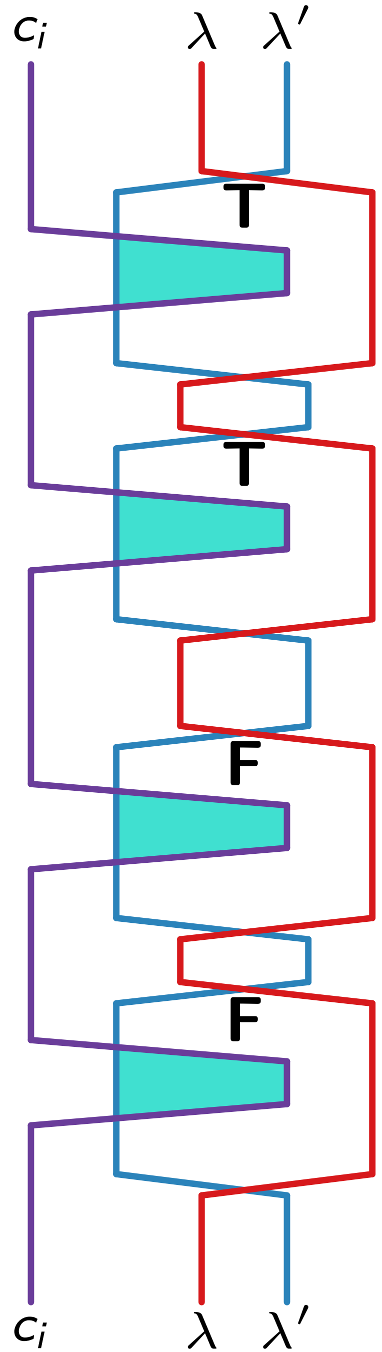

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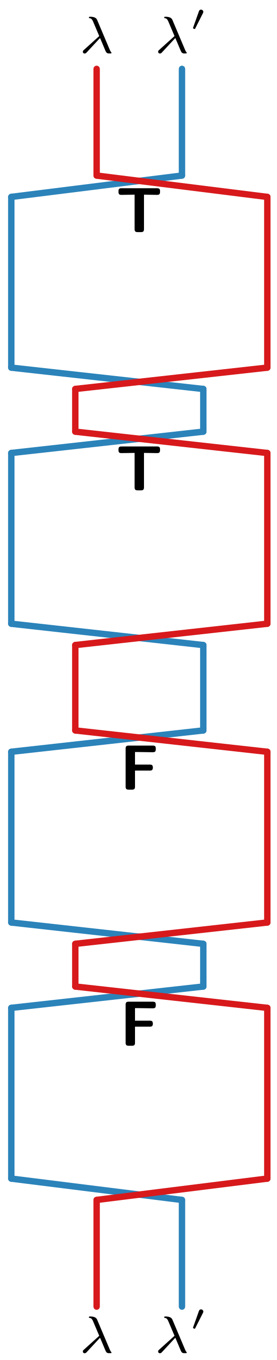

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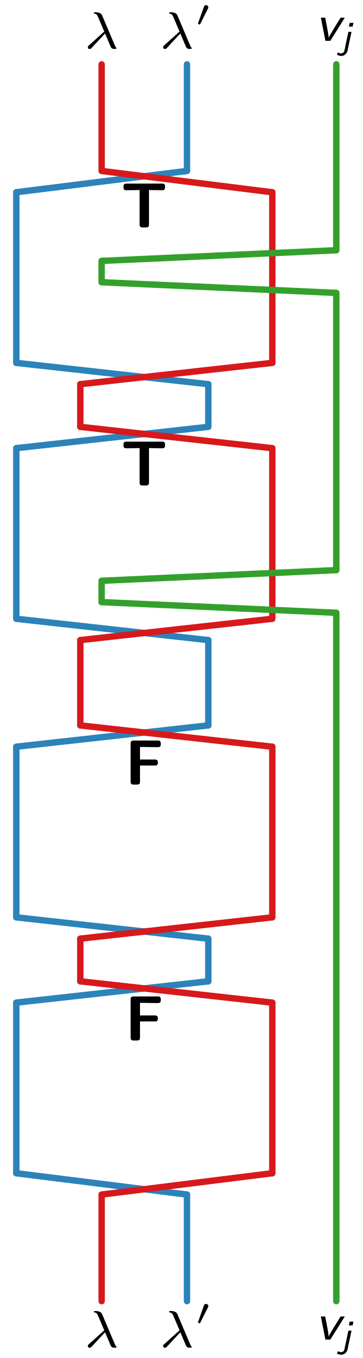

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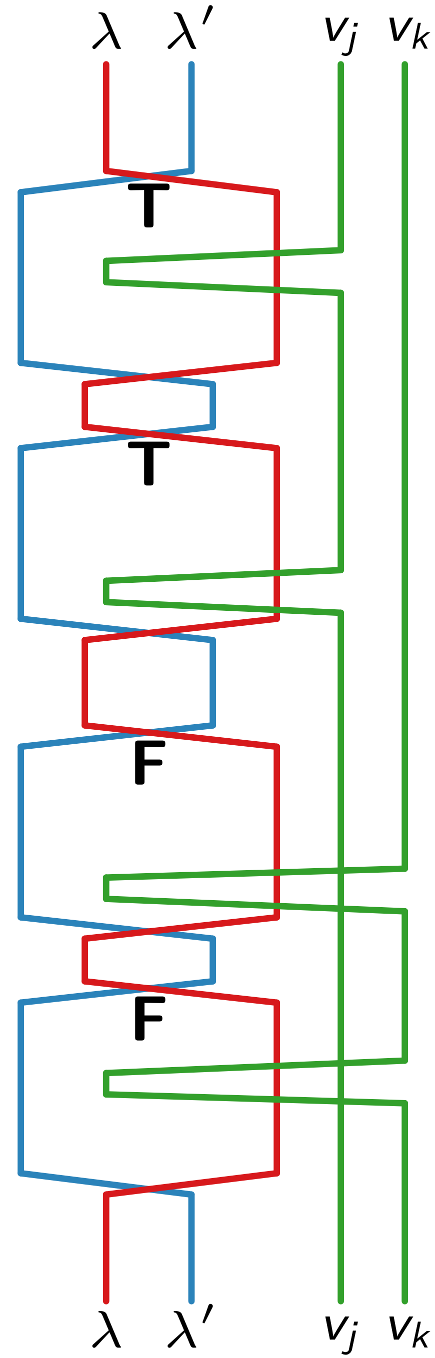

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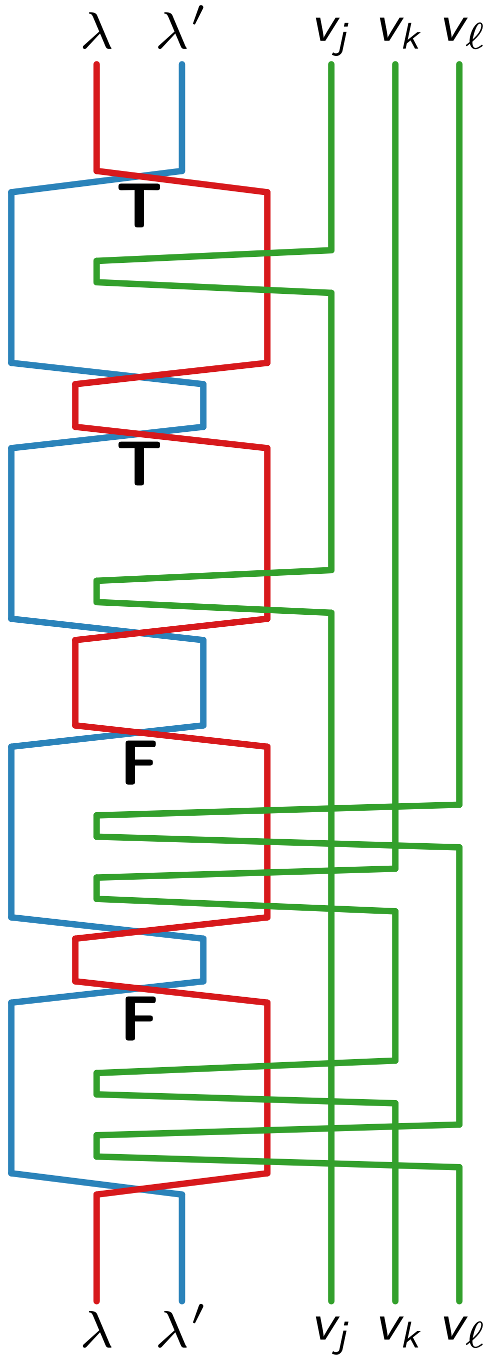

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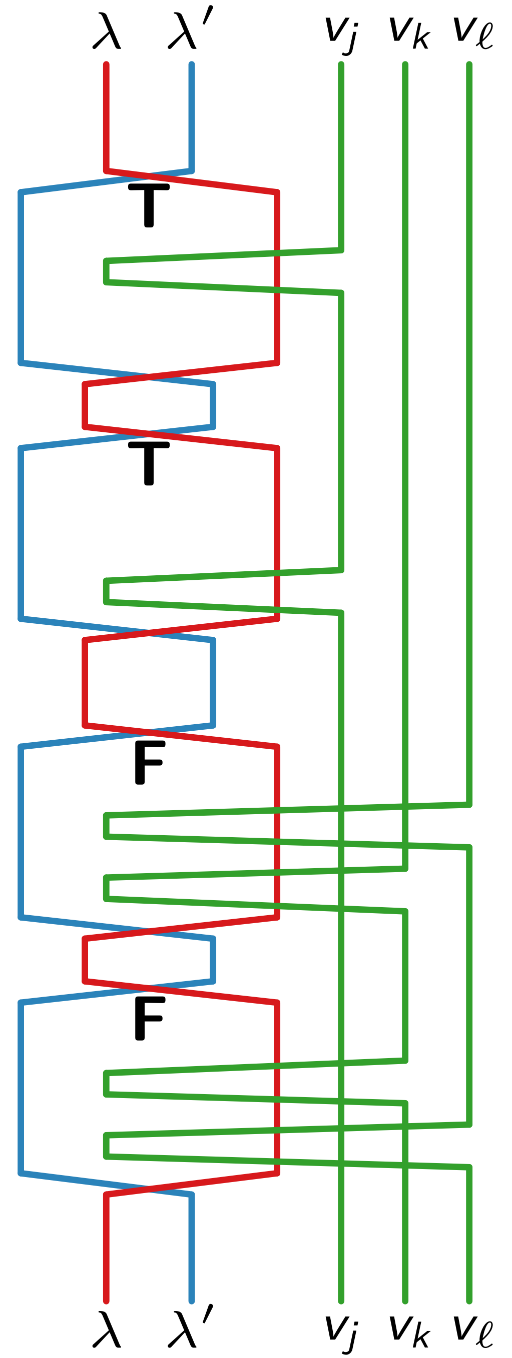

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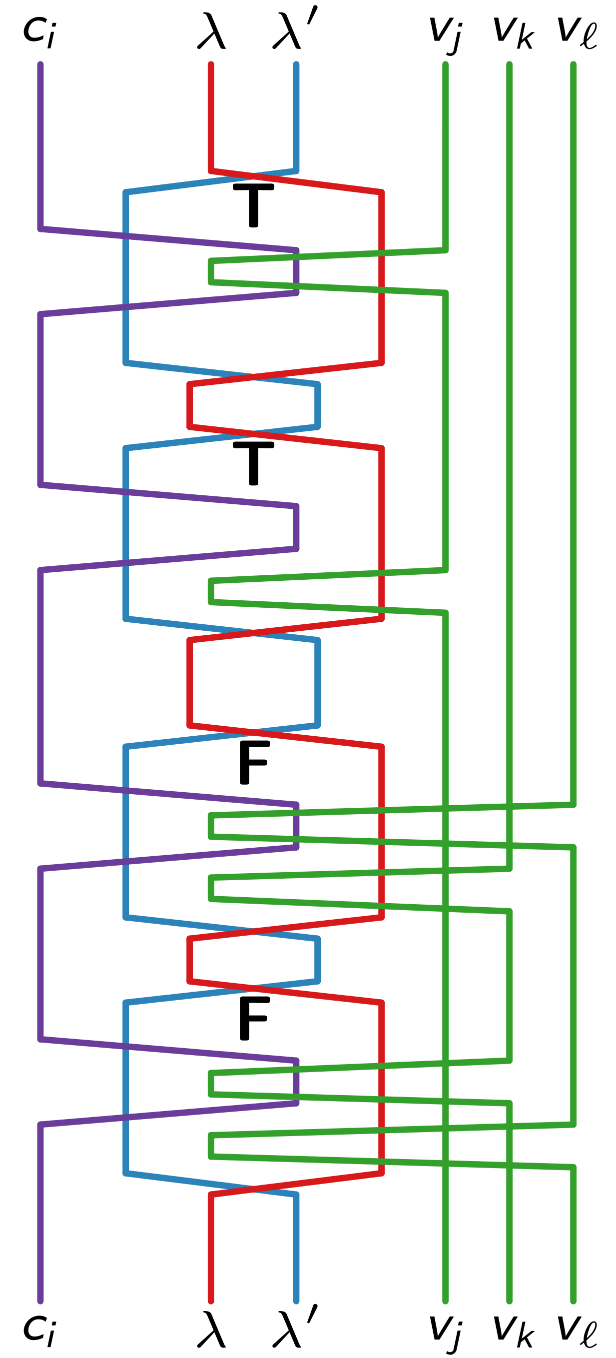

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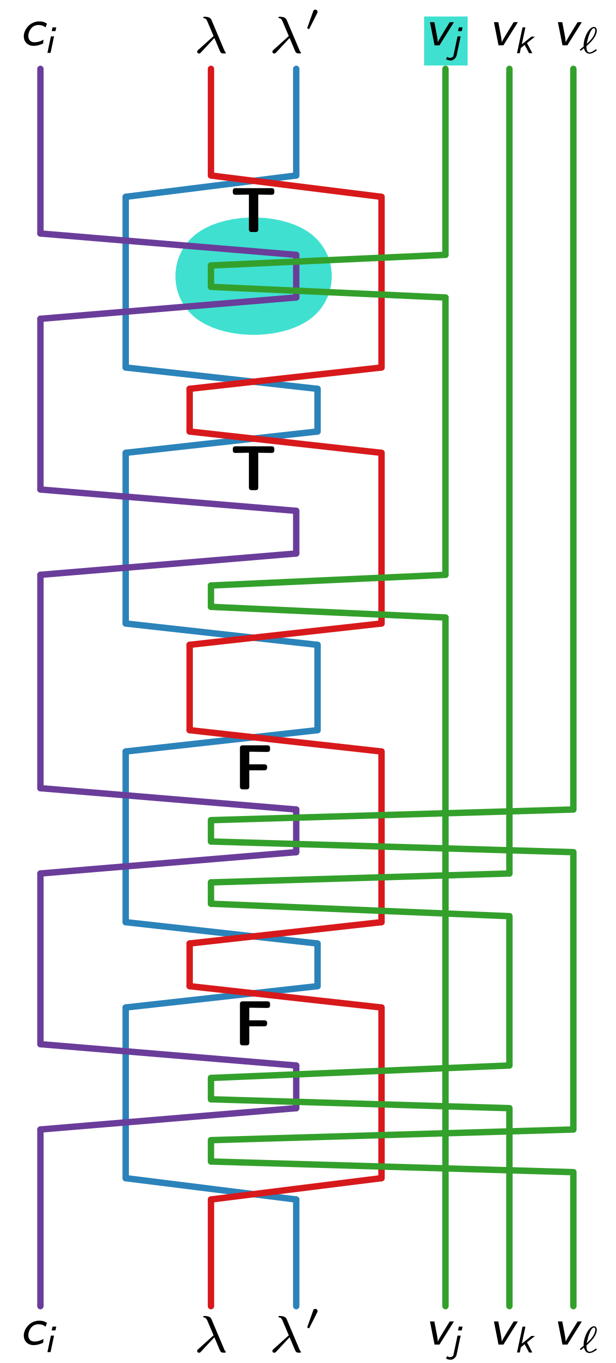

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- Two wires build 4 *loops* that we consider
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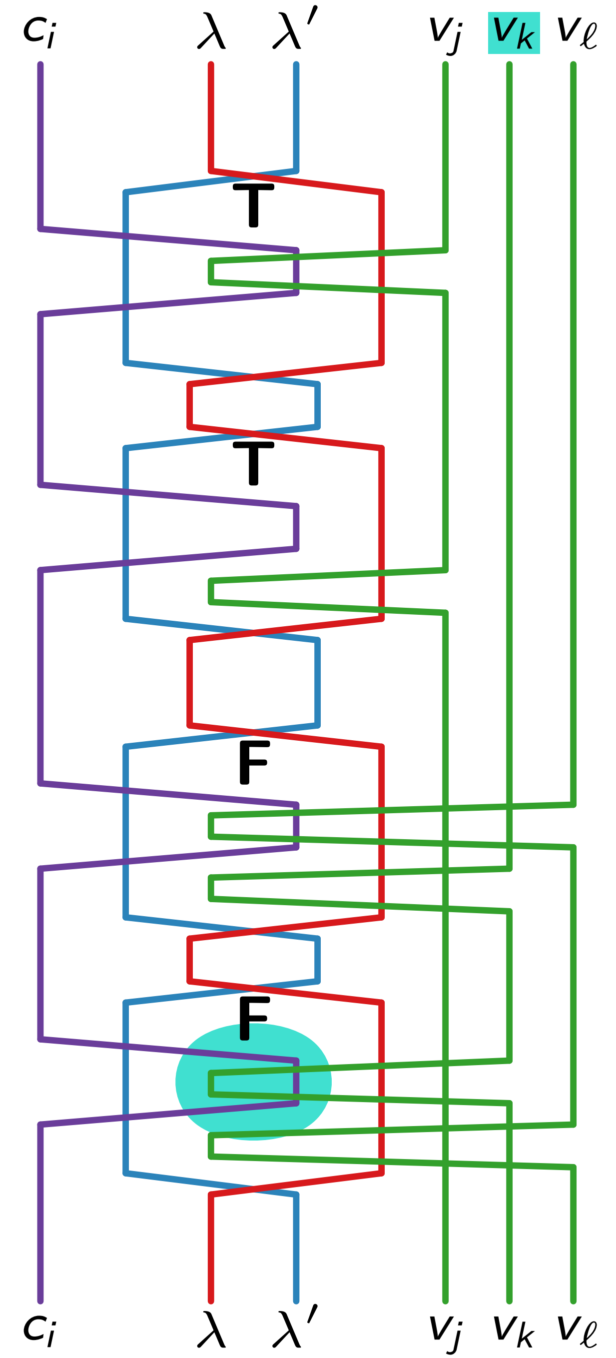

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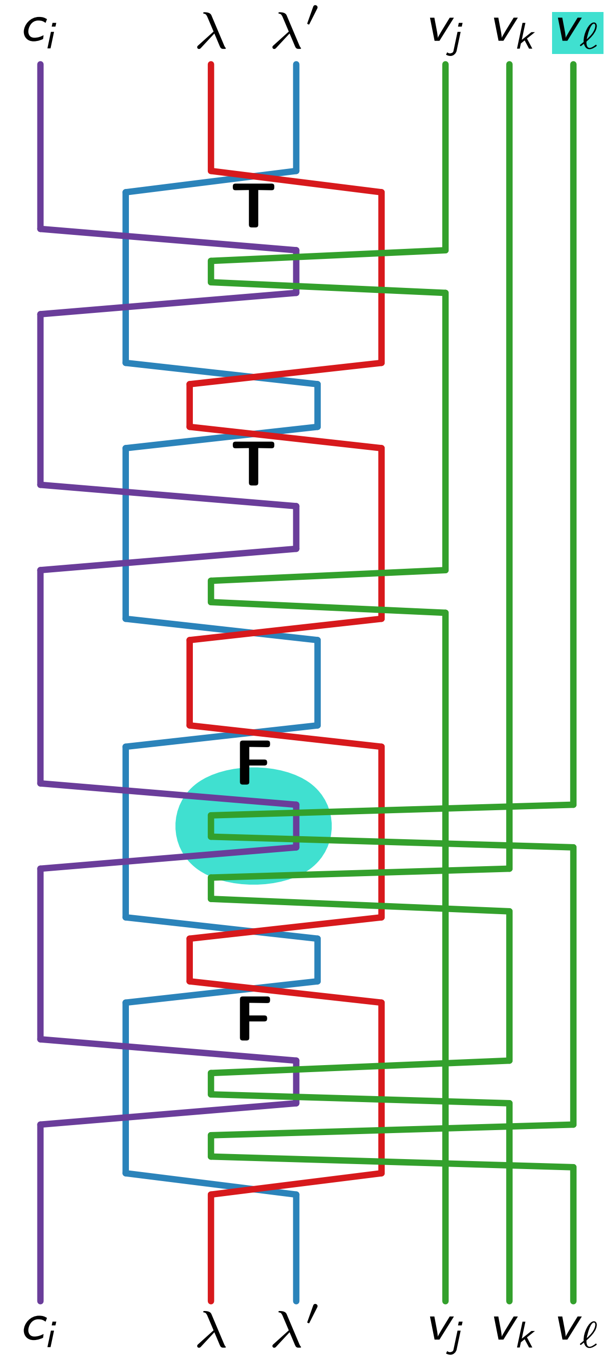

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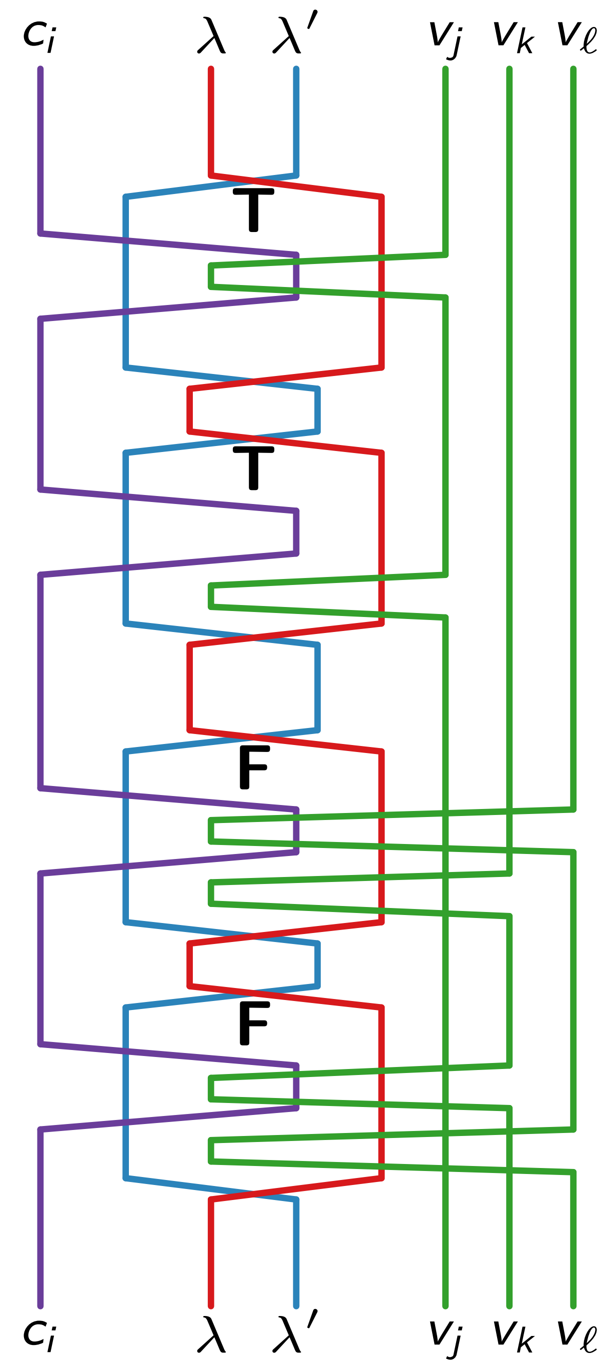
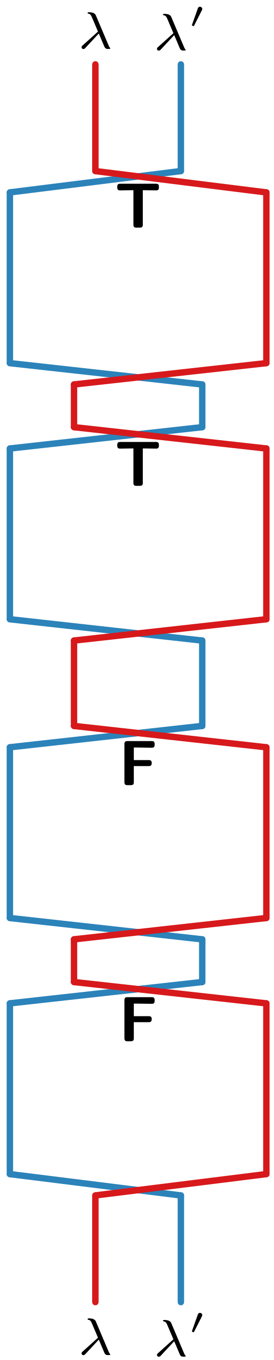
- Only 2 *true* loops and 2 *false* loops $\Rightarrow$ clause wires meet all their variable wires iff $\textsc{Positive Not-All-Equal}$ 3-SAT formula satisfiable



Variable Gadget



λ, λ': central loop structure

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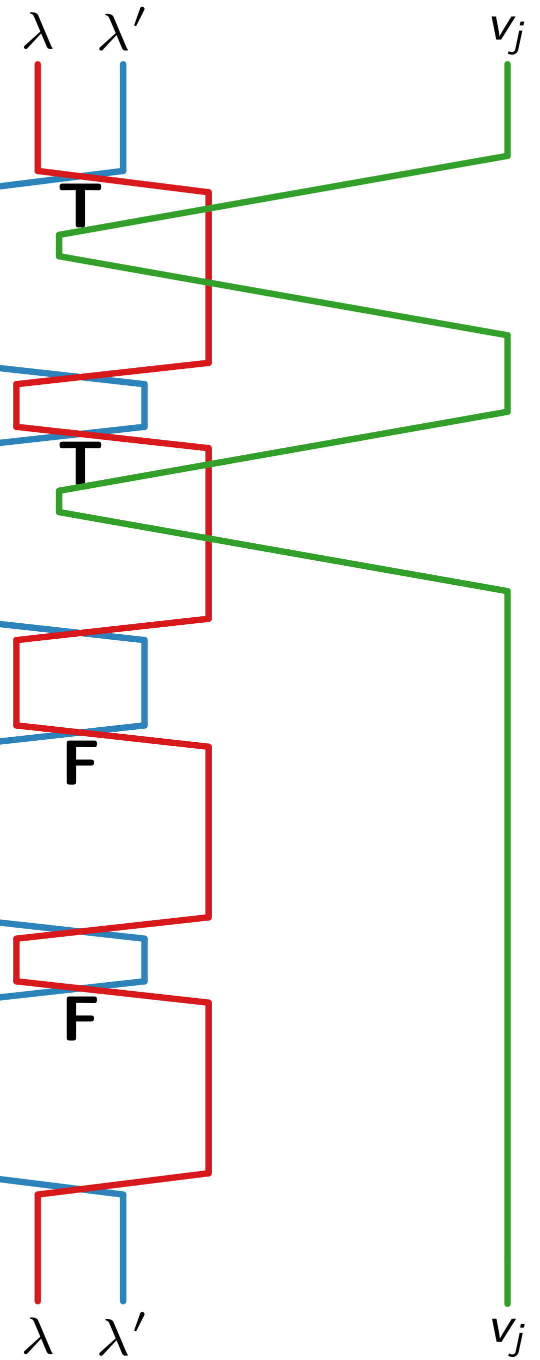

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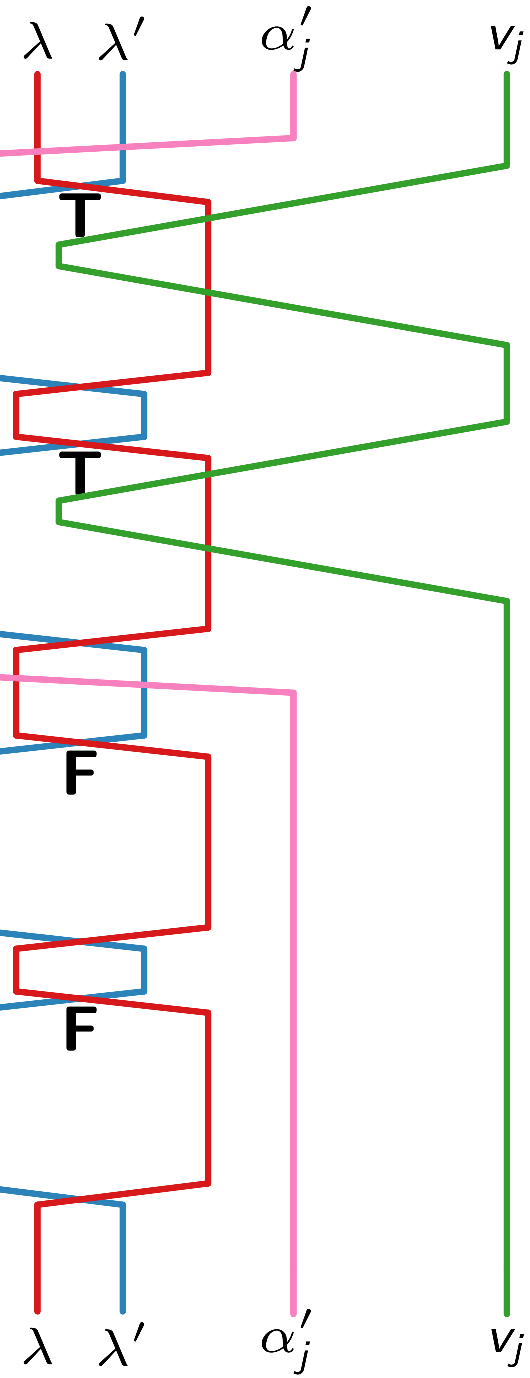

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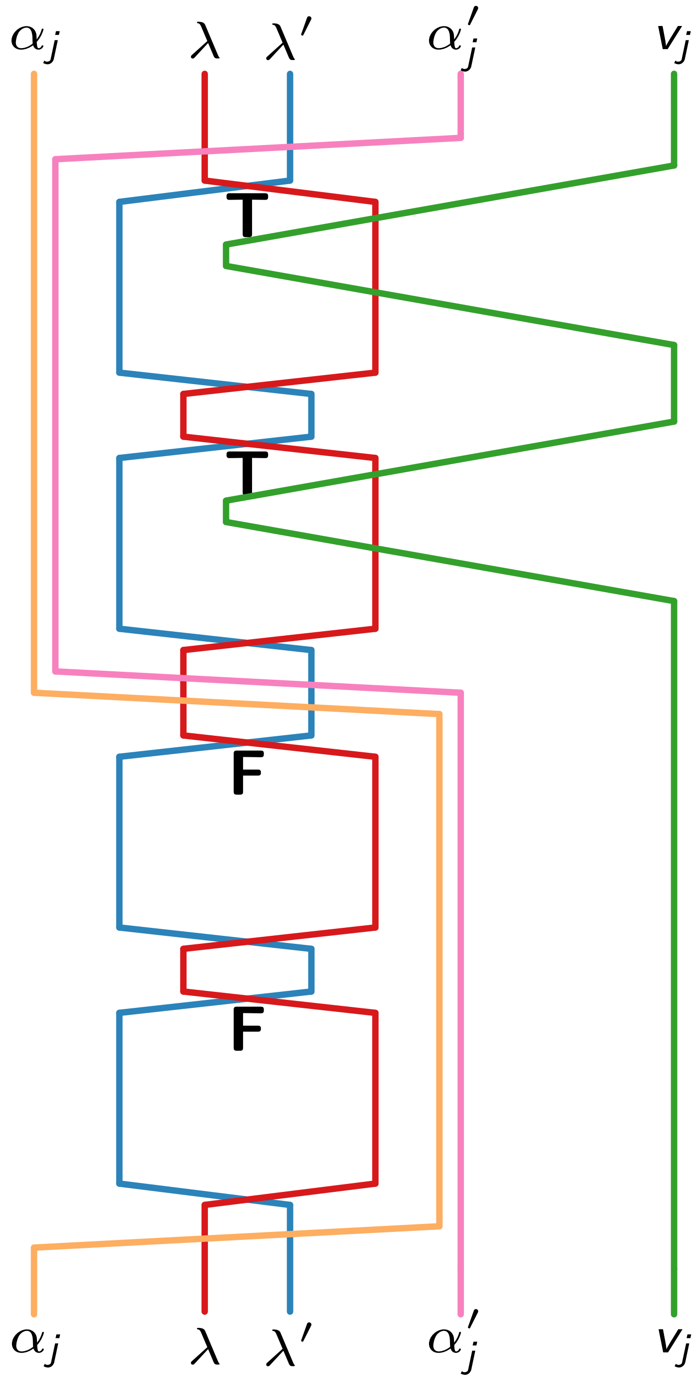

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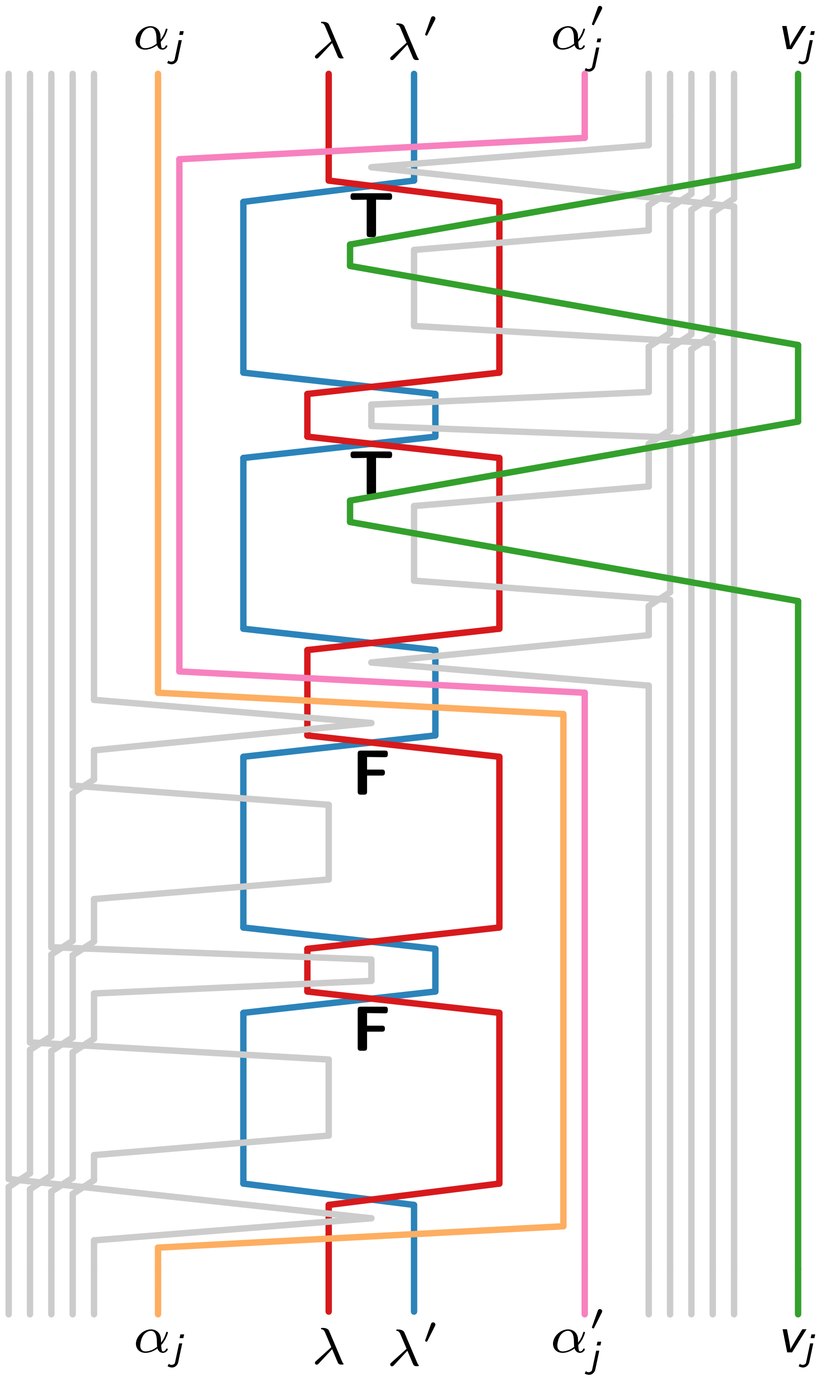

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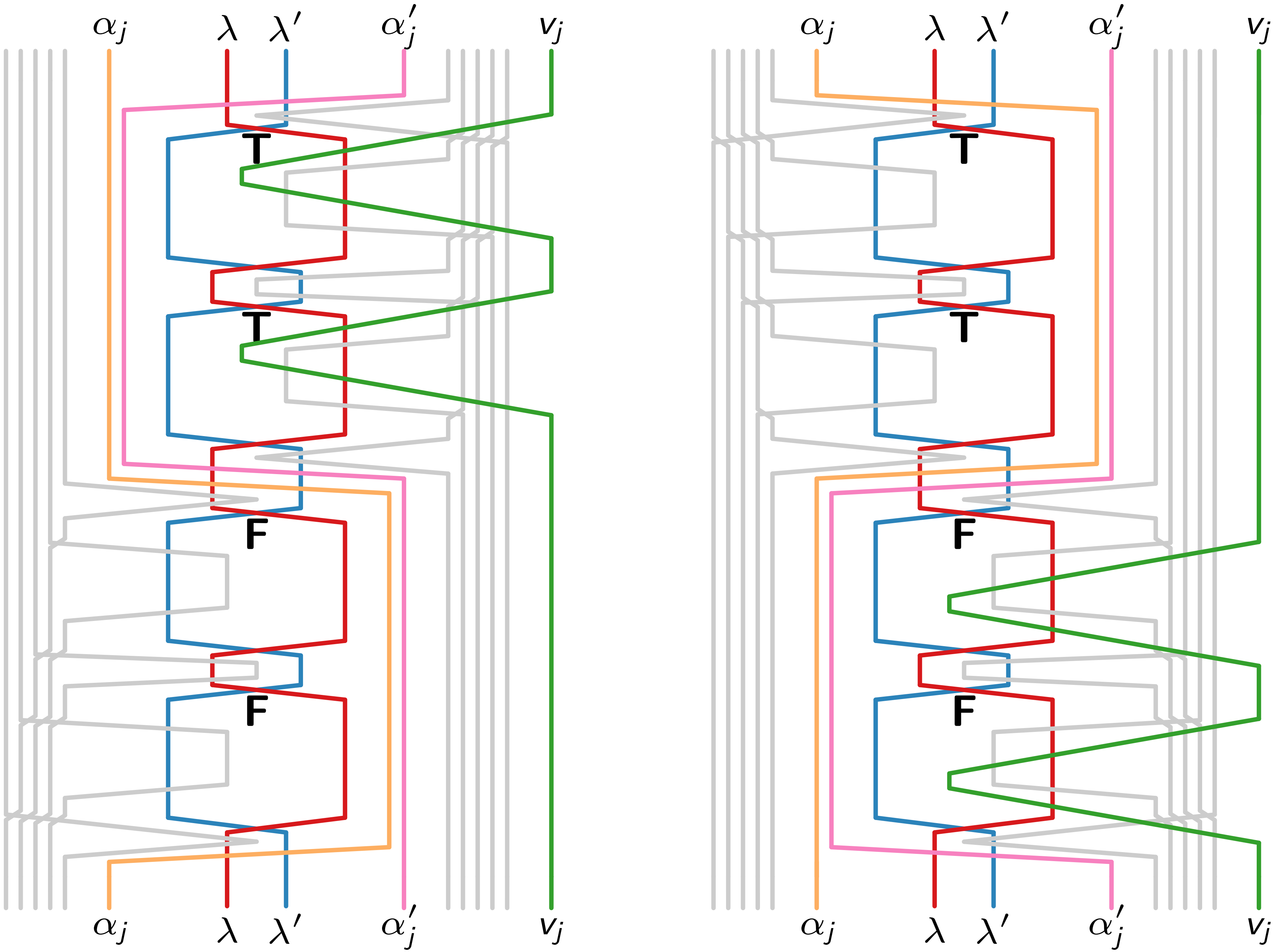

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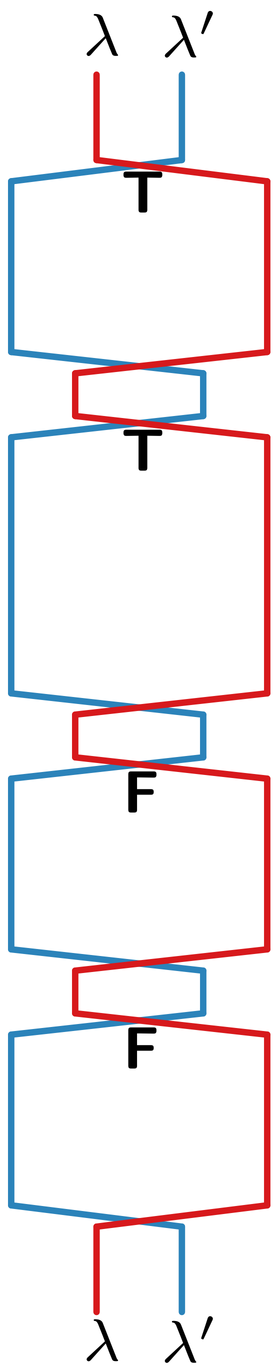

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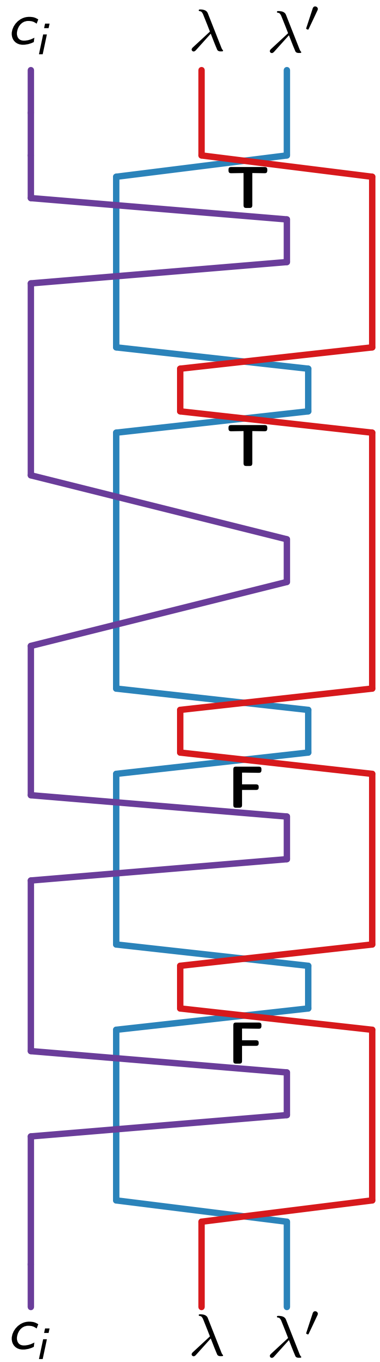

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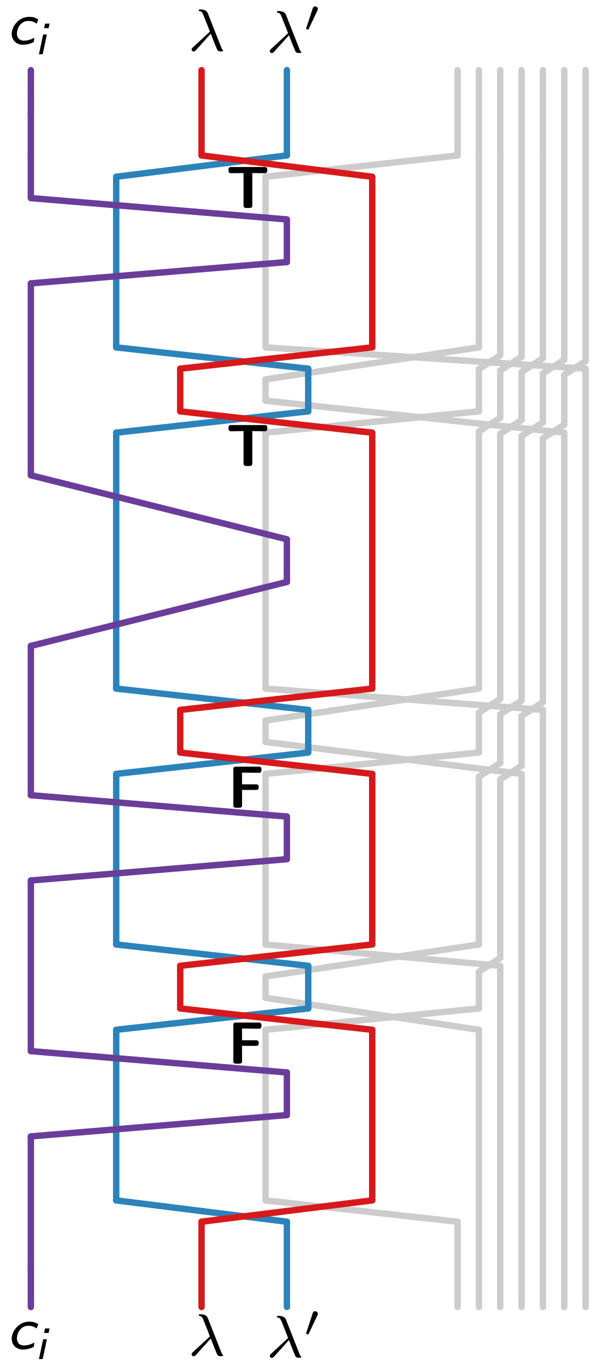

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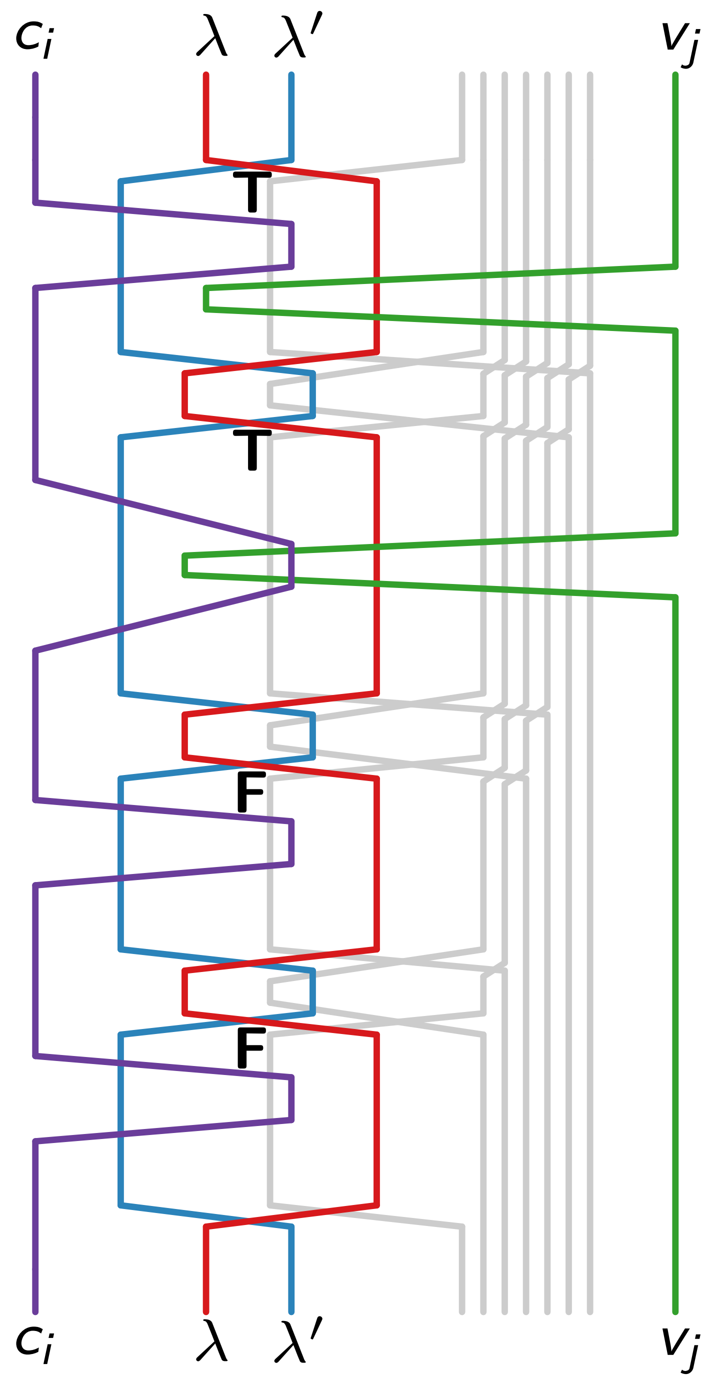

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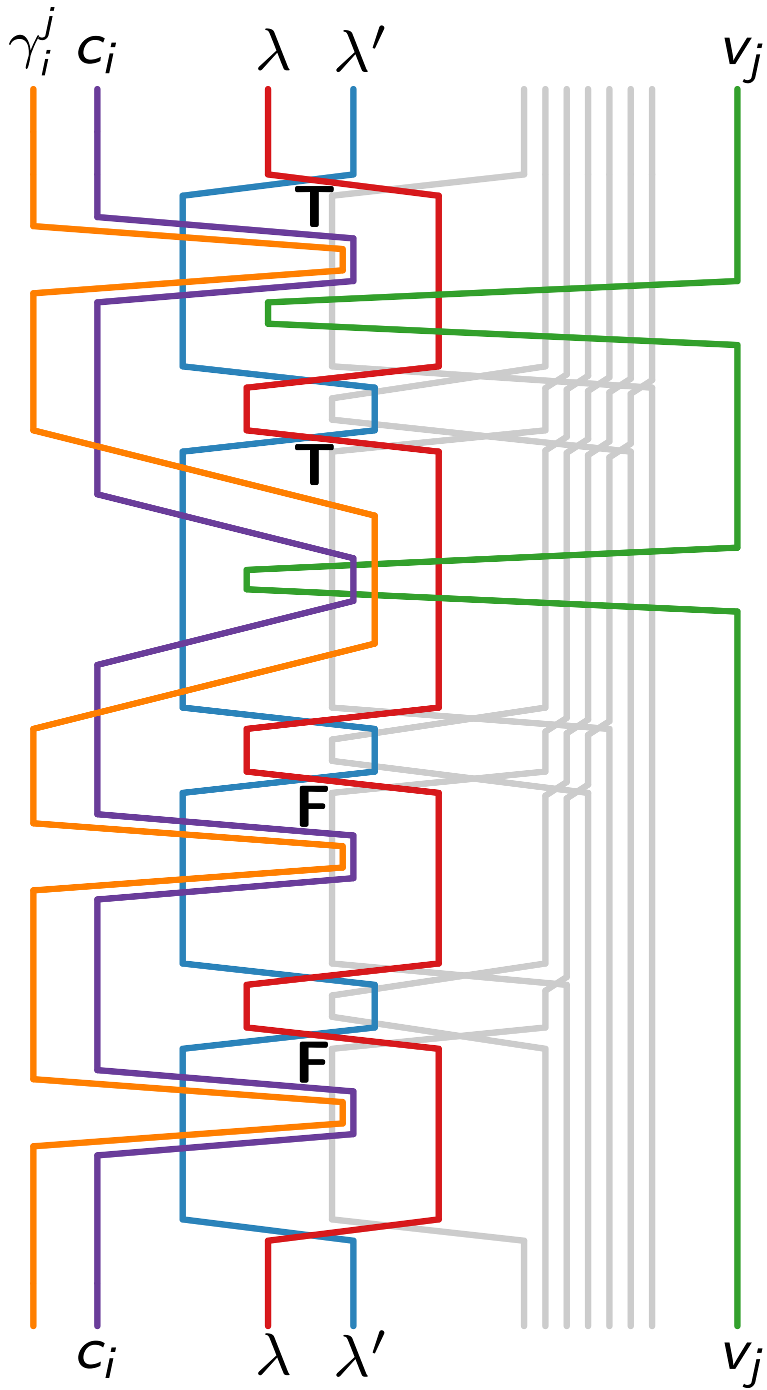

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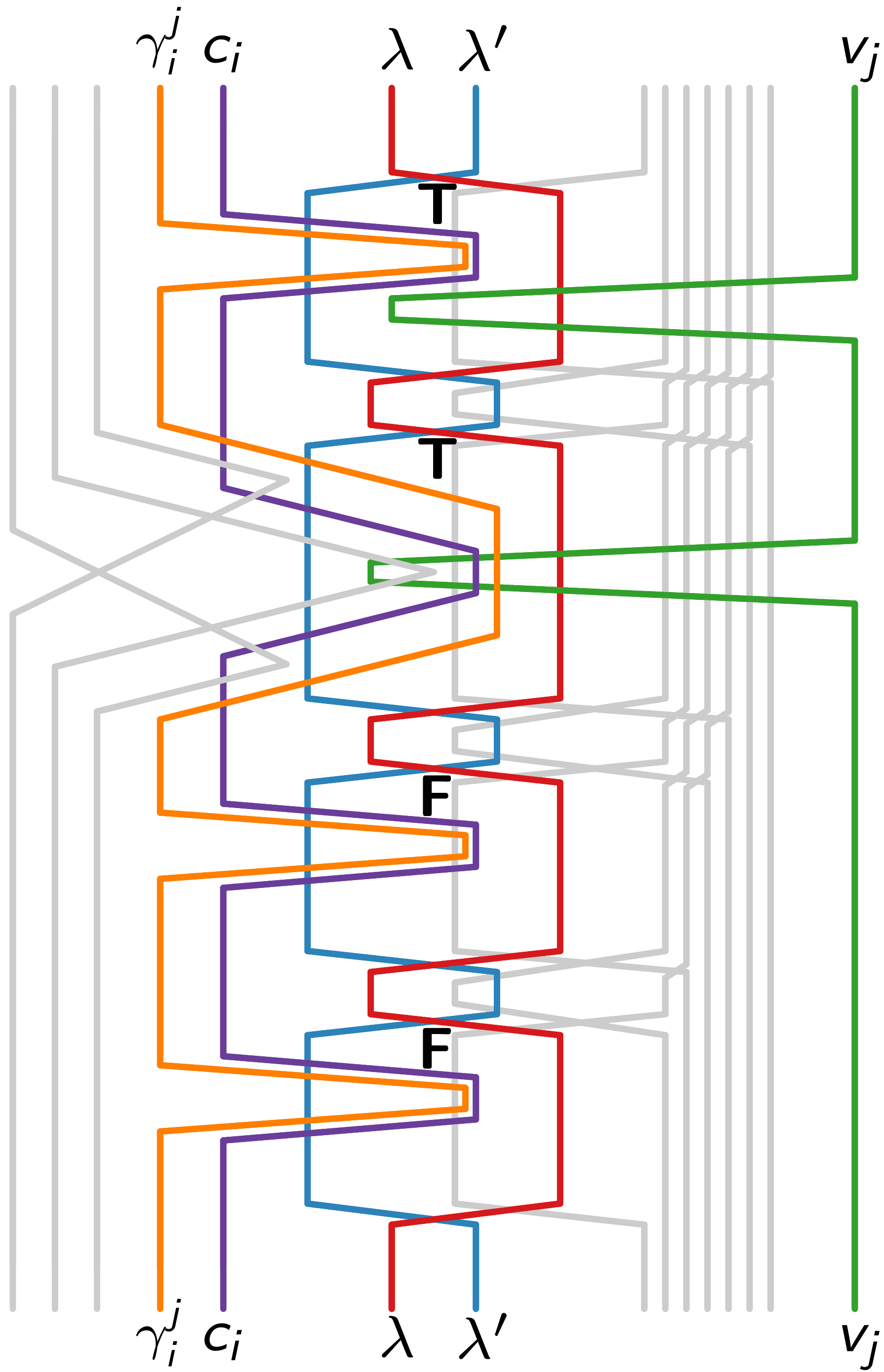

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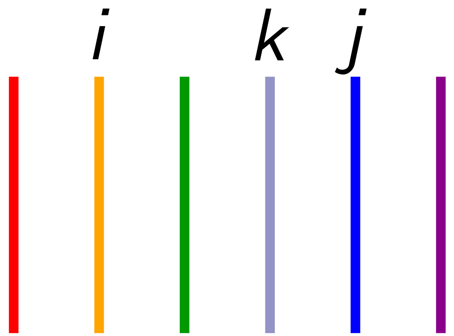

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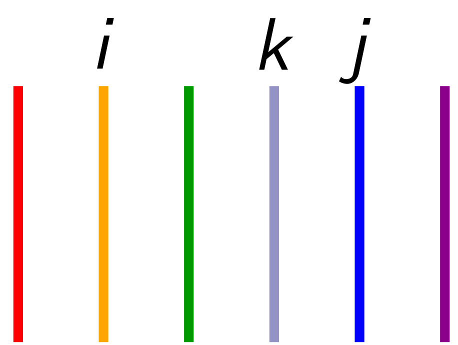

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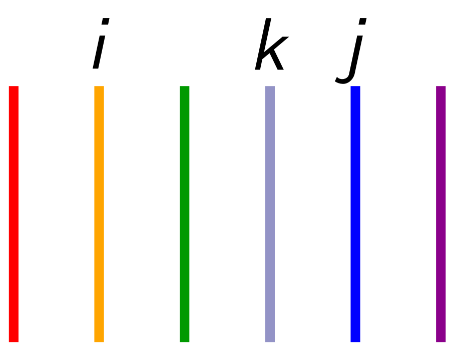

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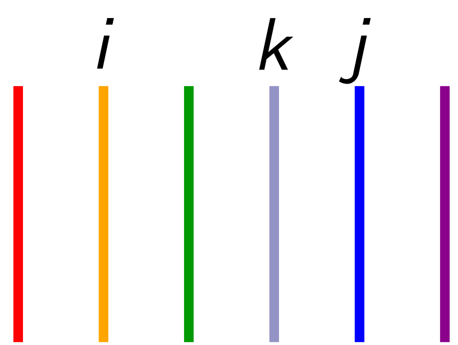

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