Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

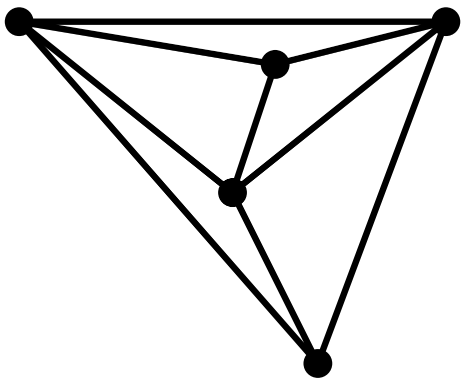
Institute for Informatics

1-Bend RAC Drawings of NIC-Planar Graphs in Quadratic Area

Steven Chaplick, Fabian Lipp,
Alexander Wolff, and **Johannes Zink**

Beyond-Planar Graphs

Types of Drawings:

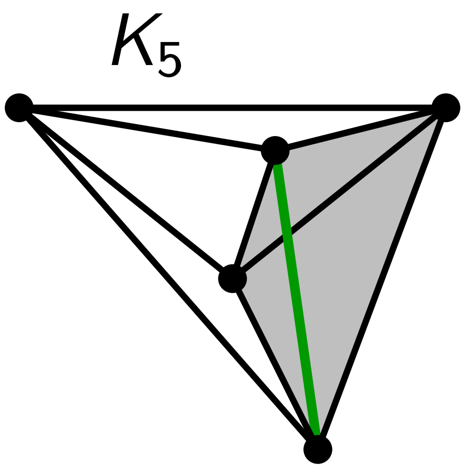


Planar: No crossings

Beyond-Planar Graphs

Types of Drawings:

1-Planar: ≤ 1 crossings per edge



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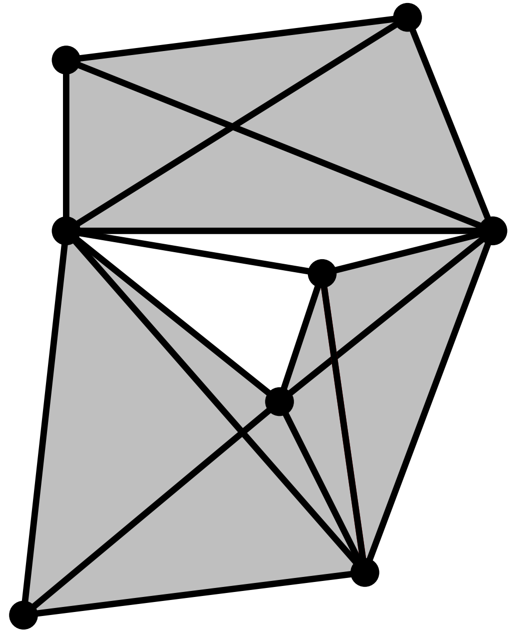

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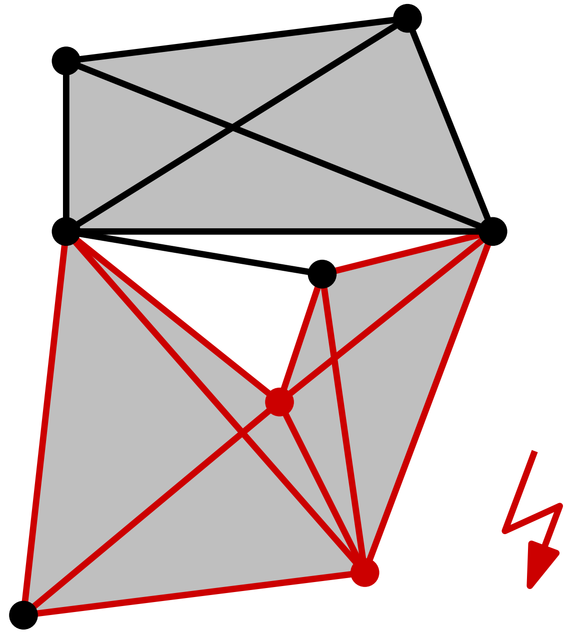

1-Planar: ≤ 1 crossings per edge

NIC-Planar: Two crossings share ≤ 1 vertices

Planar: No crossings

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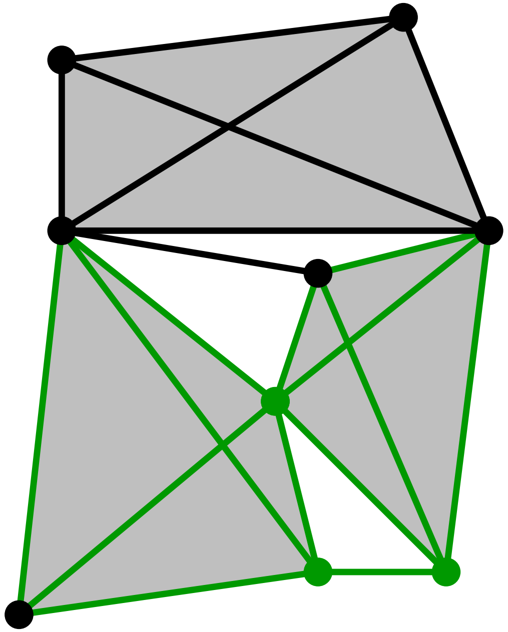

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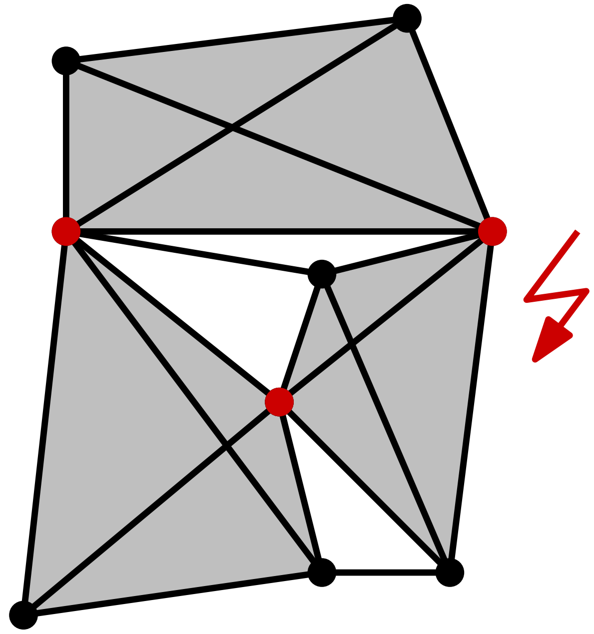

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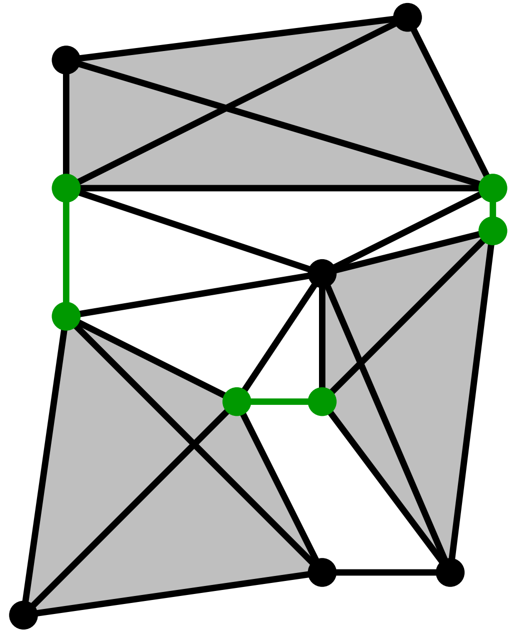

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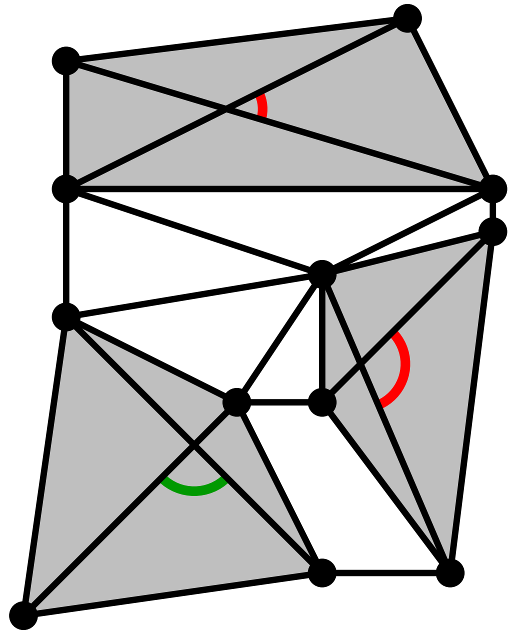

Types of Drawings:

1-Planar:	≤ 1 crossings per edge
NIC-Planar:	Two crossings share ≤ 1 vertices
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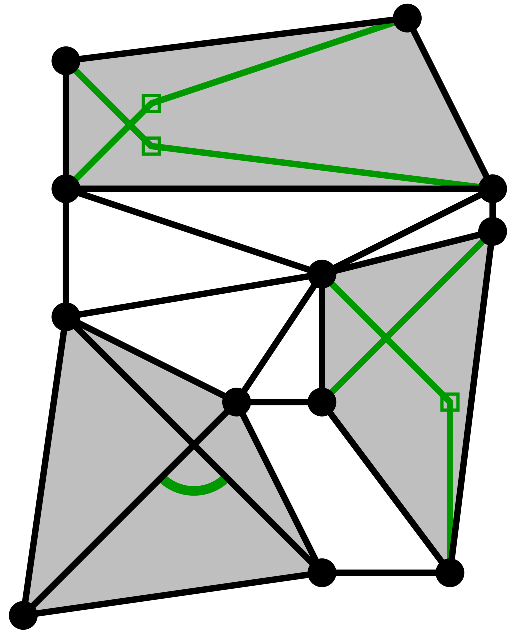
Planar:	No crossings
RAC:	Right angle crossings

Beyond-Planar Graphs



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RAC: Right angle crossings

RAC_1: with ≤ 1 bends per edge
RAC_0: with straight-line edges

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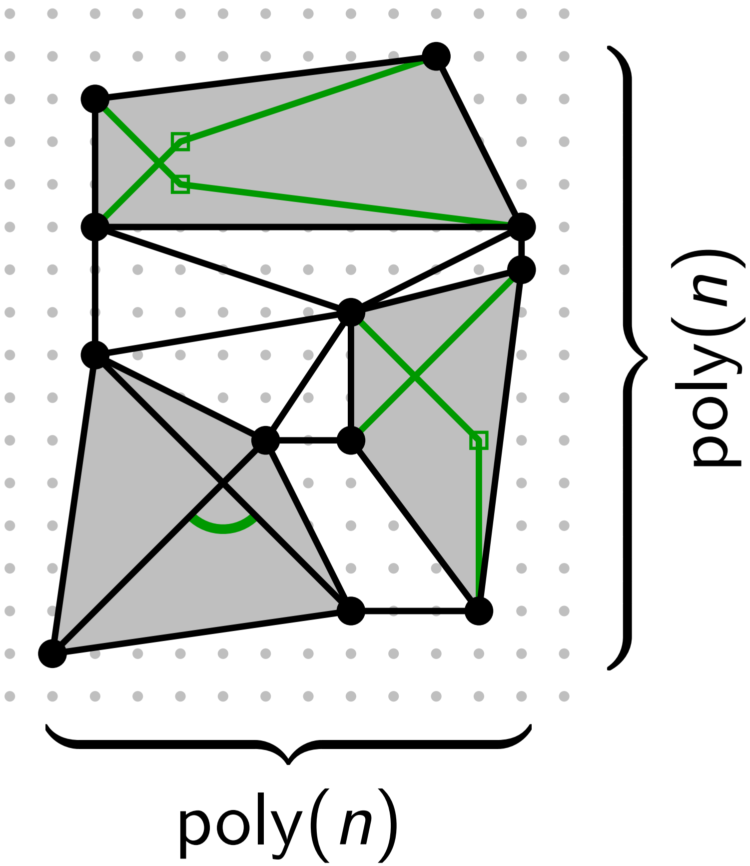

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- RAC^{poly}: in polynomial area

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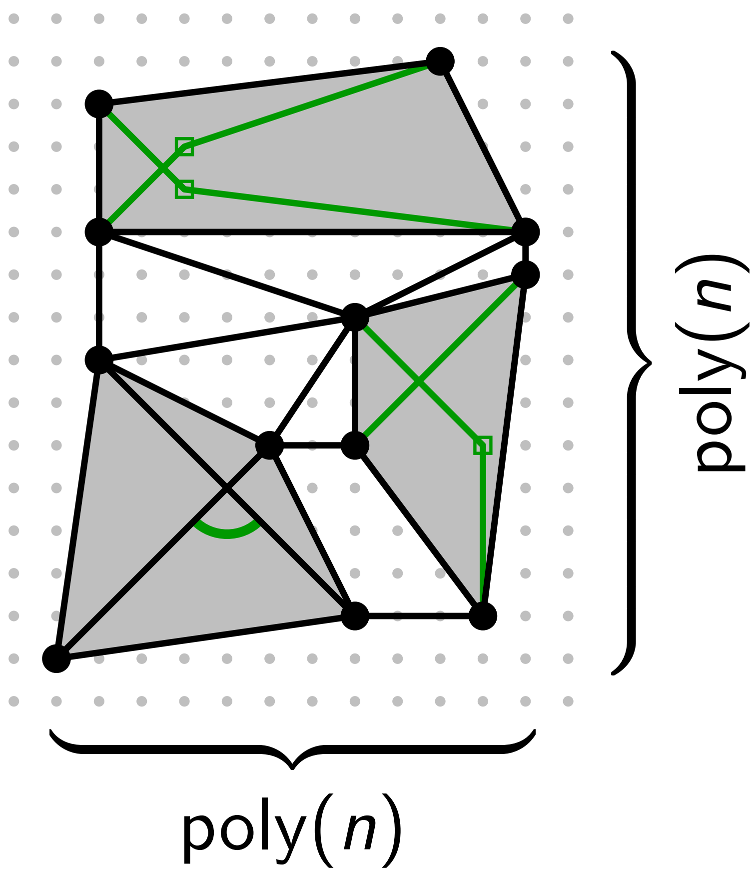

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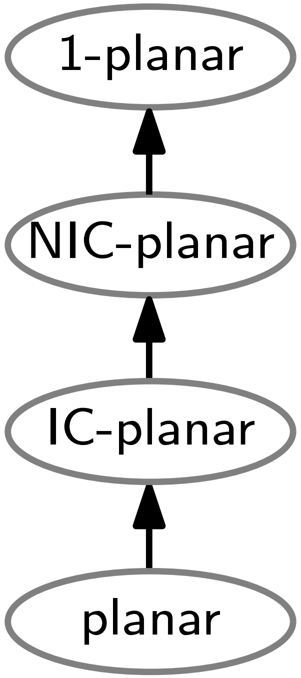
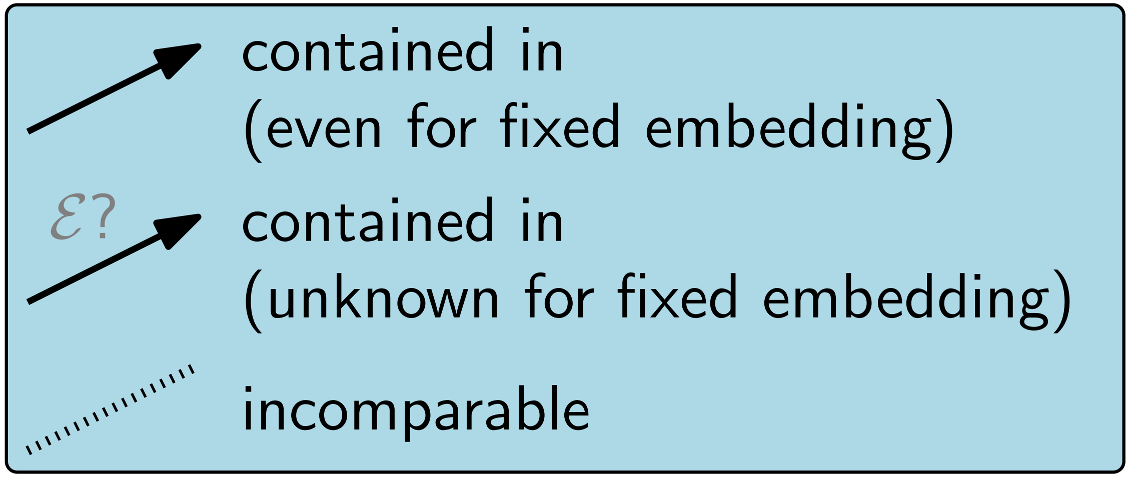
Related Work



$= \{\text{graph } G \mid G \text{ has a NIC-planar drawing}\}$



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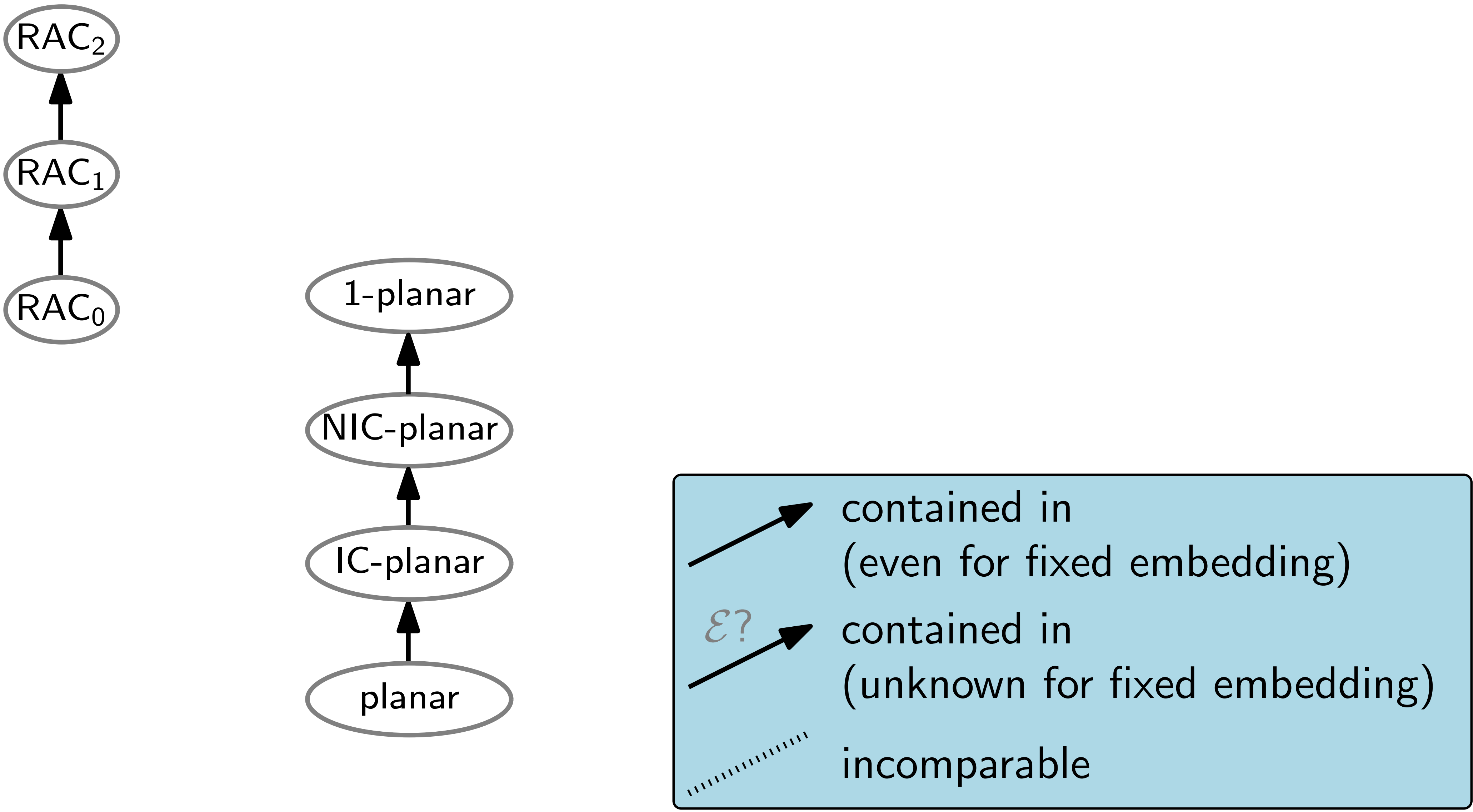

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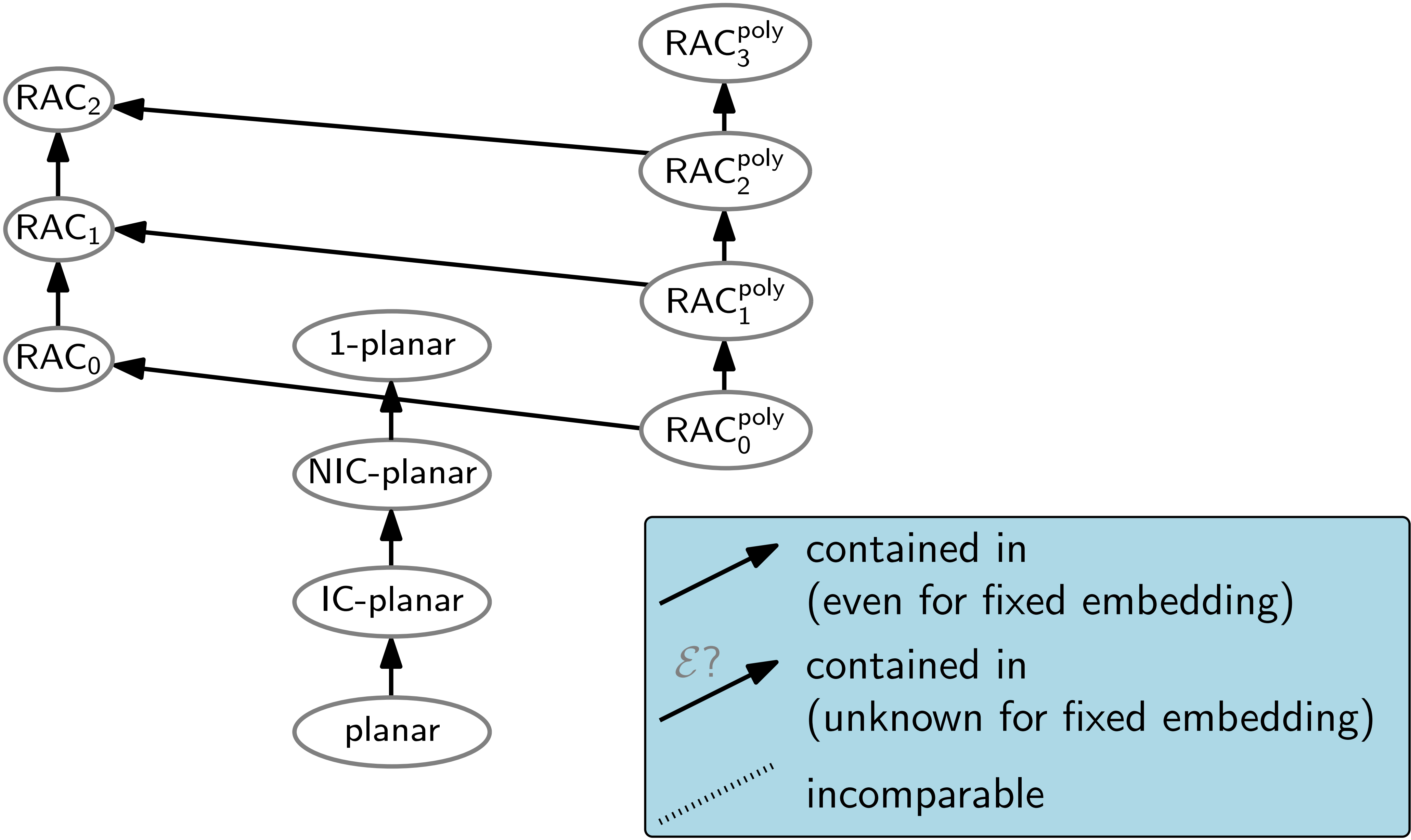

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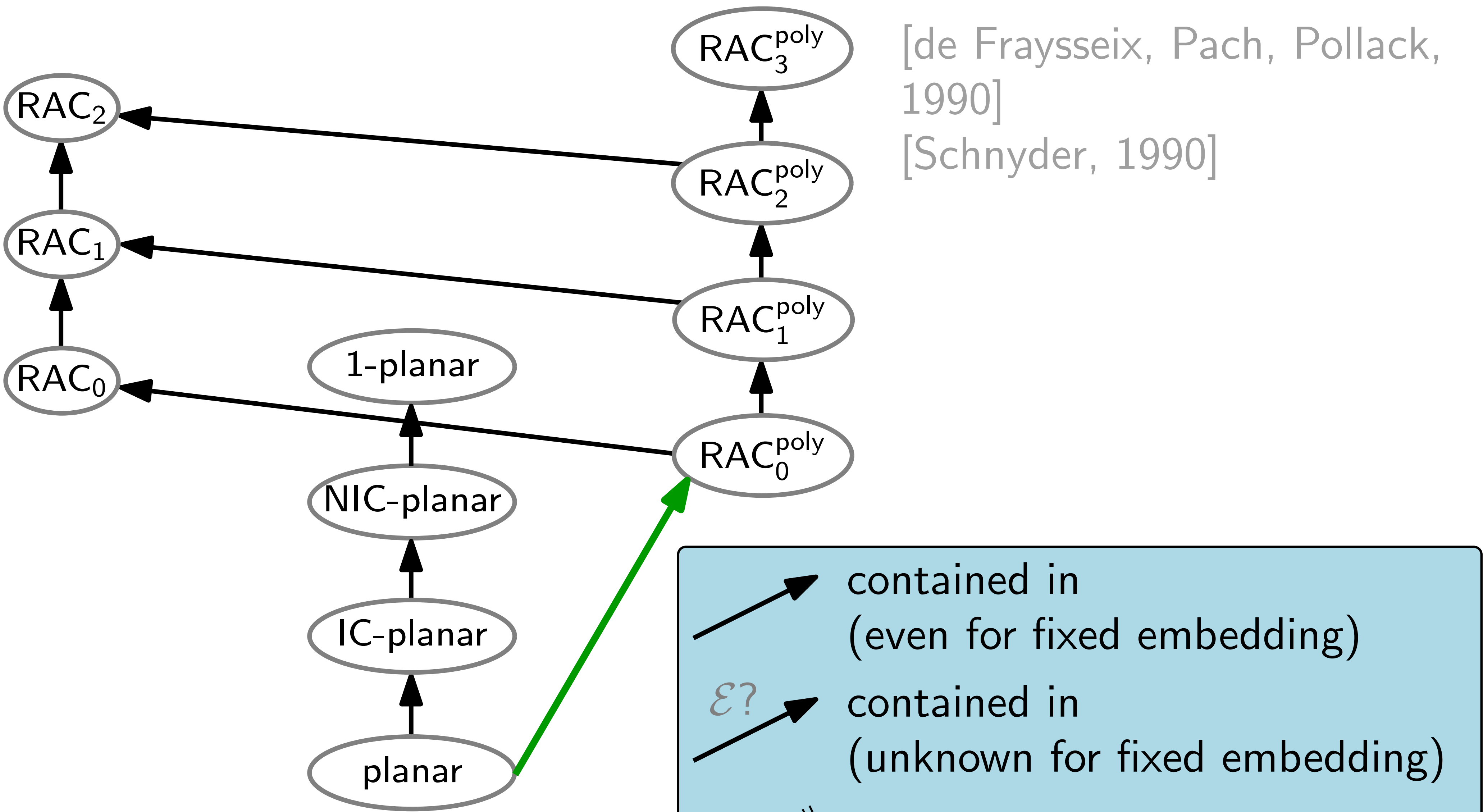
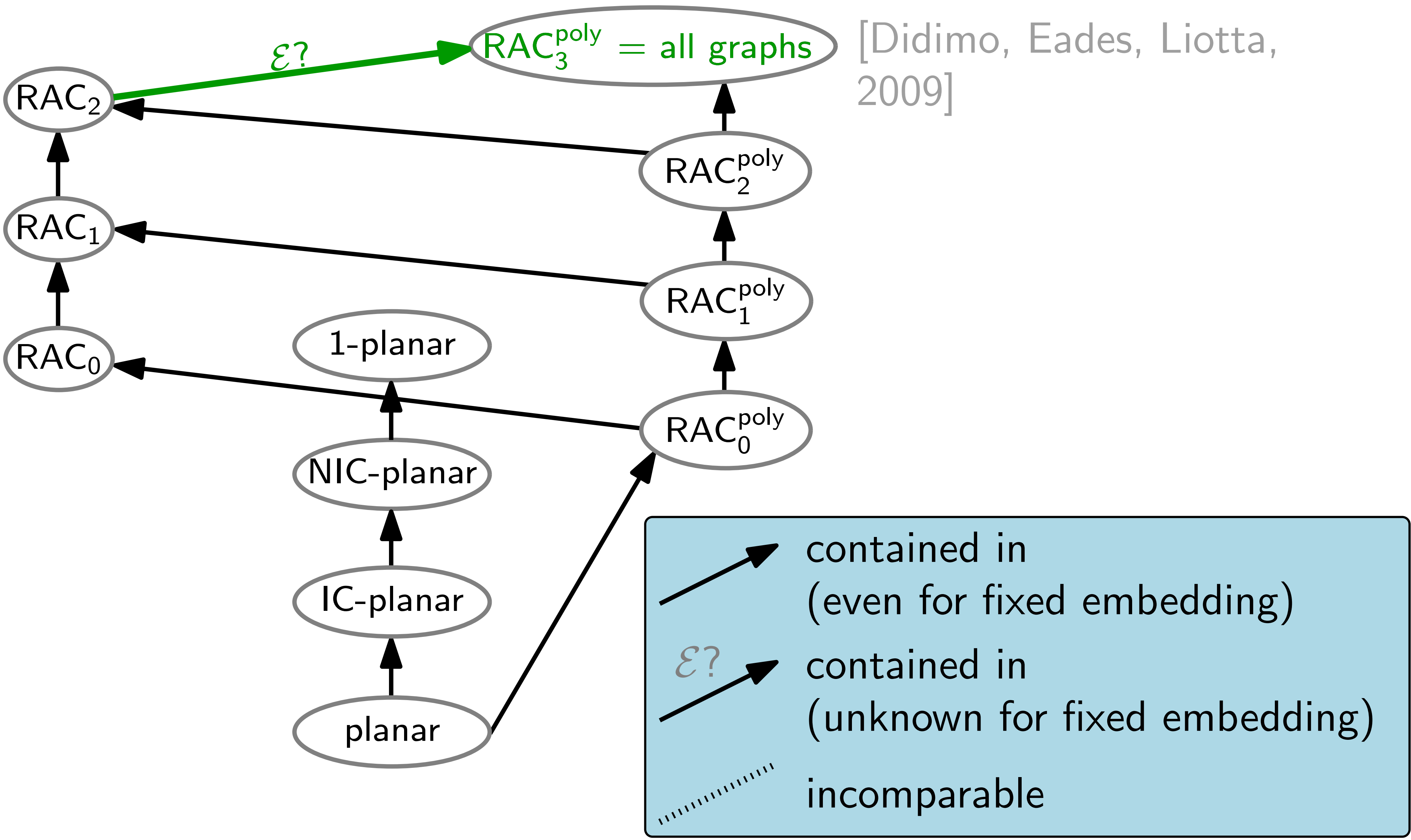


[de Fraysseix, Pach, Pollack, 1990]
[Schnyder, 1990]

Related Work



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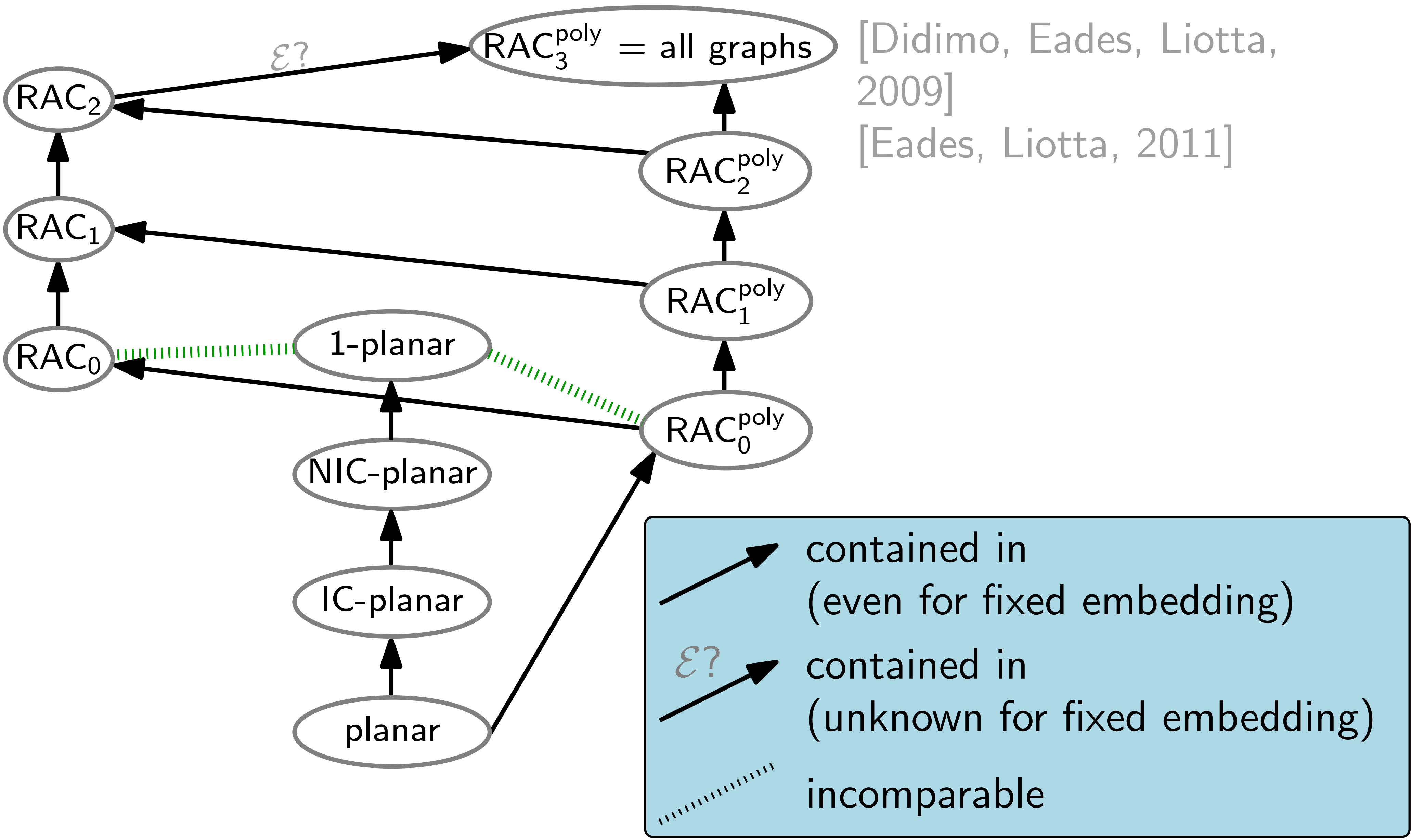

[Didimo, Eades, Liotta, 2009]
[Eades, Liotta, 2011]

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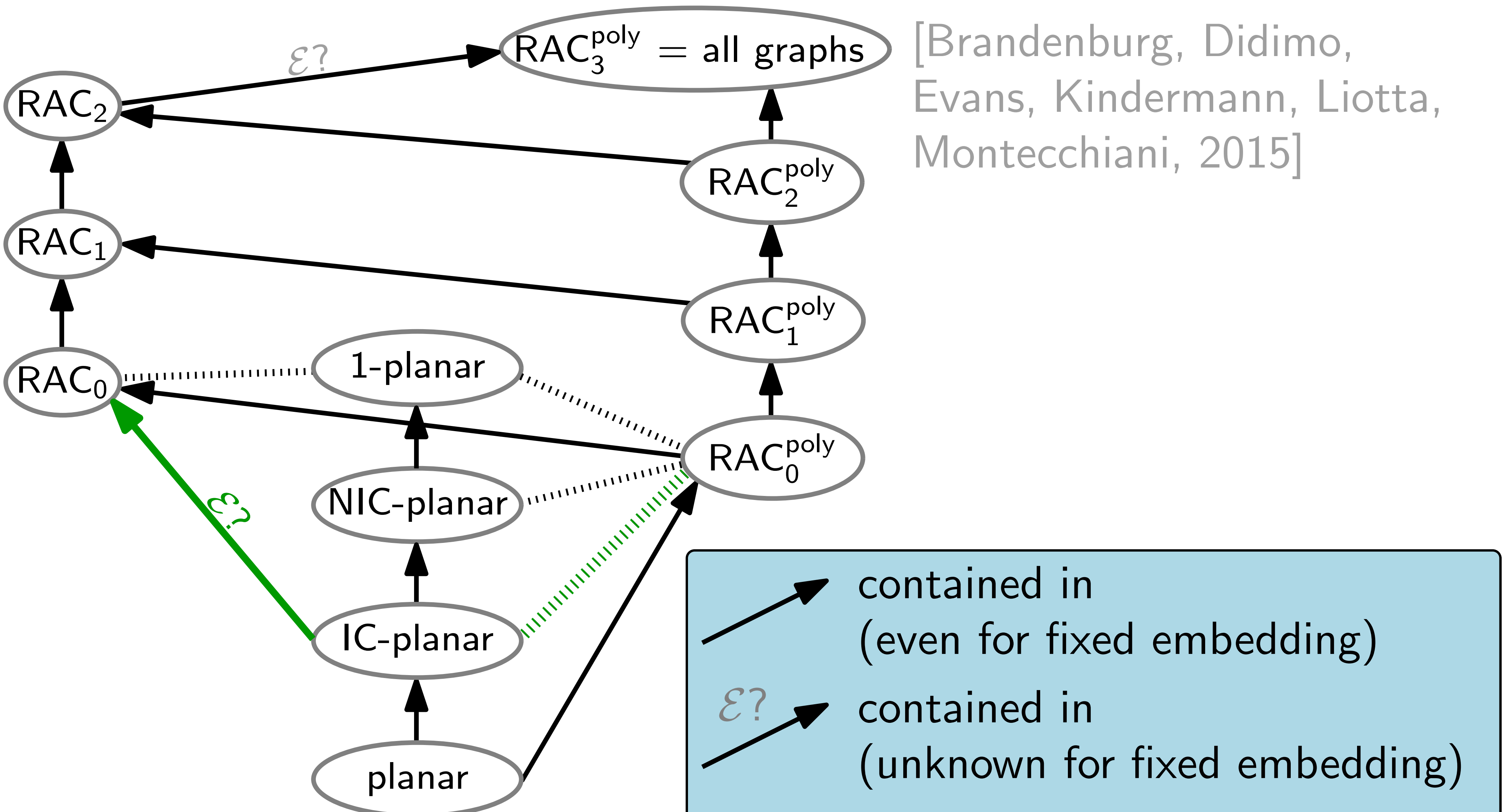

[Brandenburg, Didimo, Evans, Kindermann, Liotta, Montecchiani, 2015]

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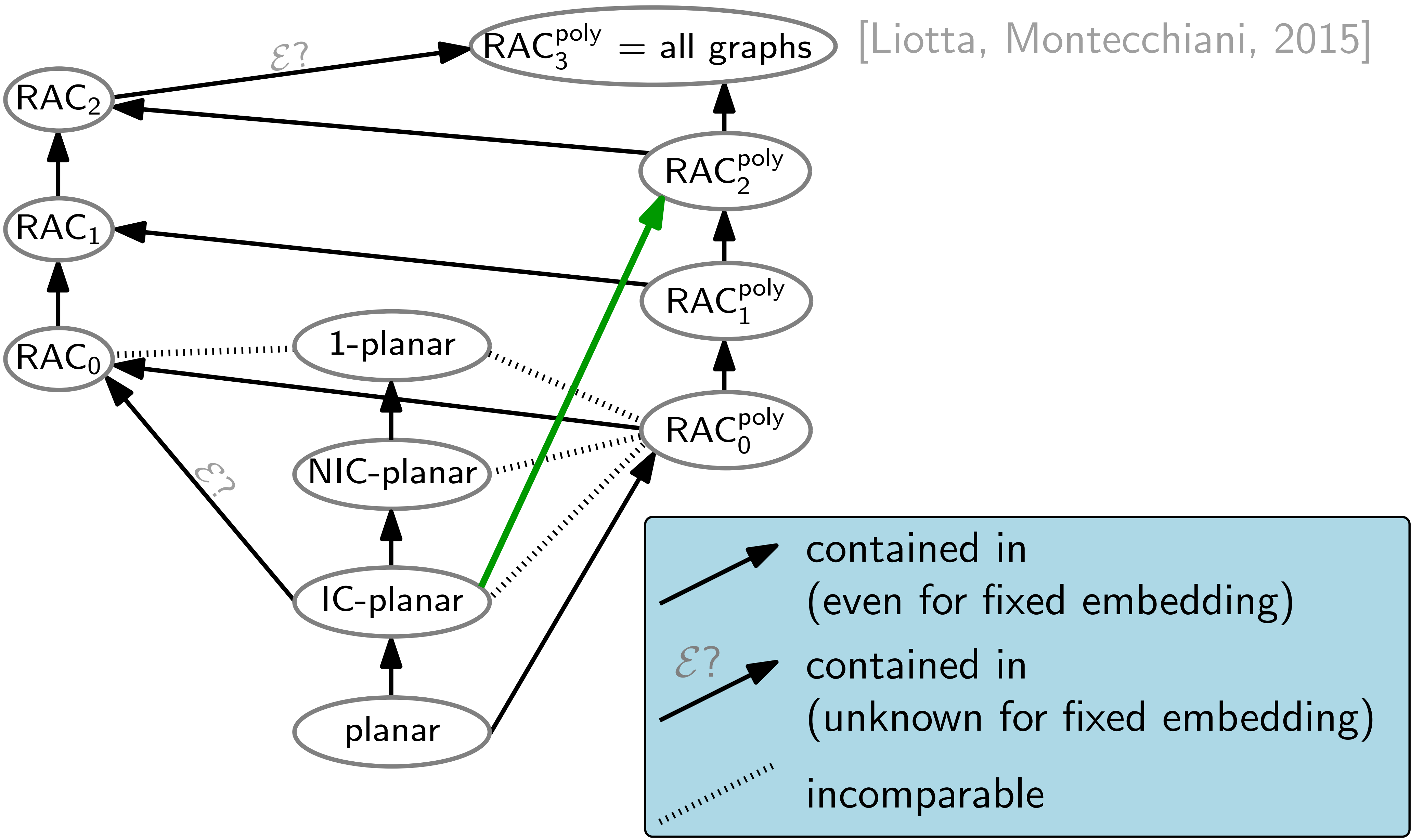

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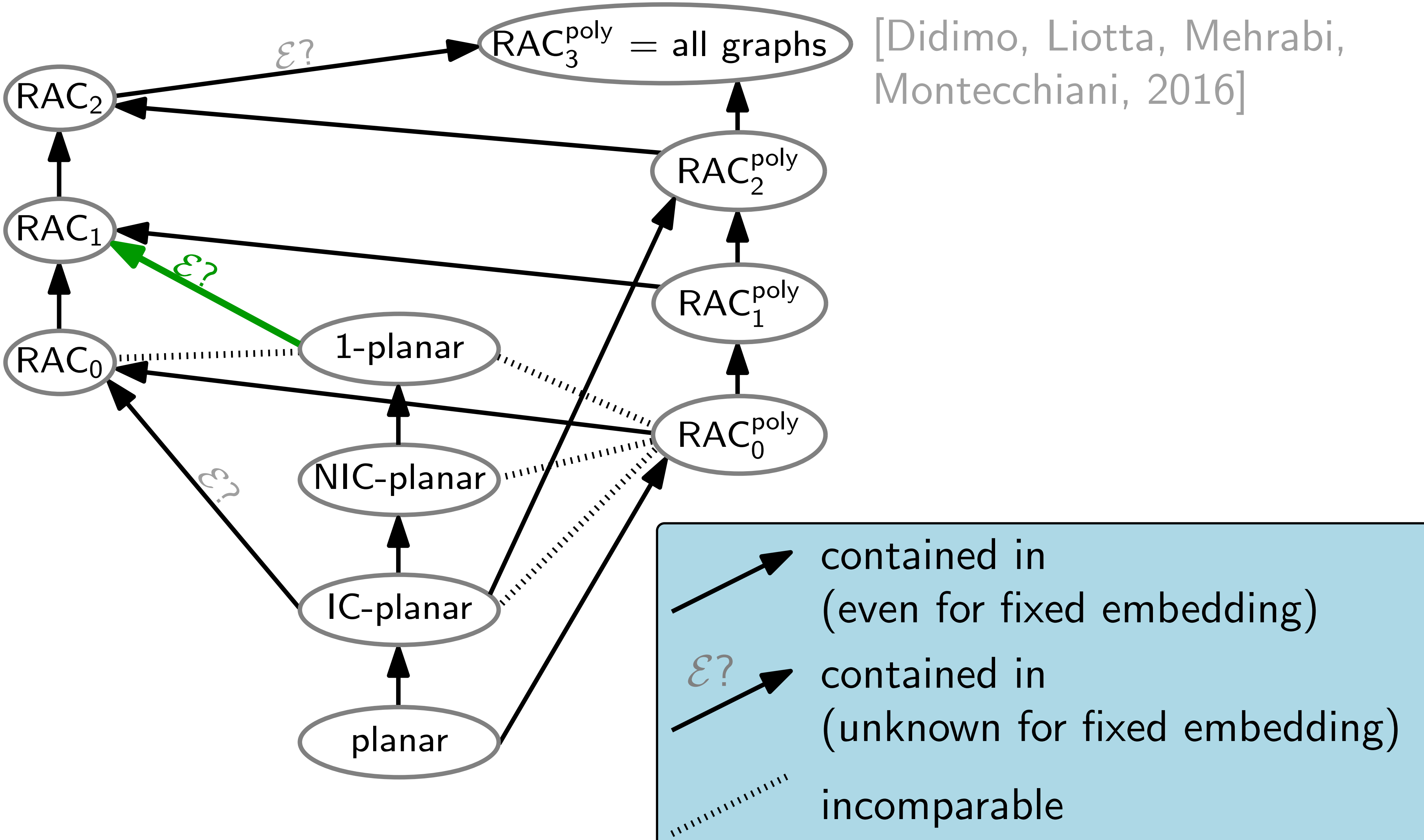

[Didimo, Liotta, Mehrabi, Montecchiani, 2016]

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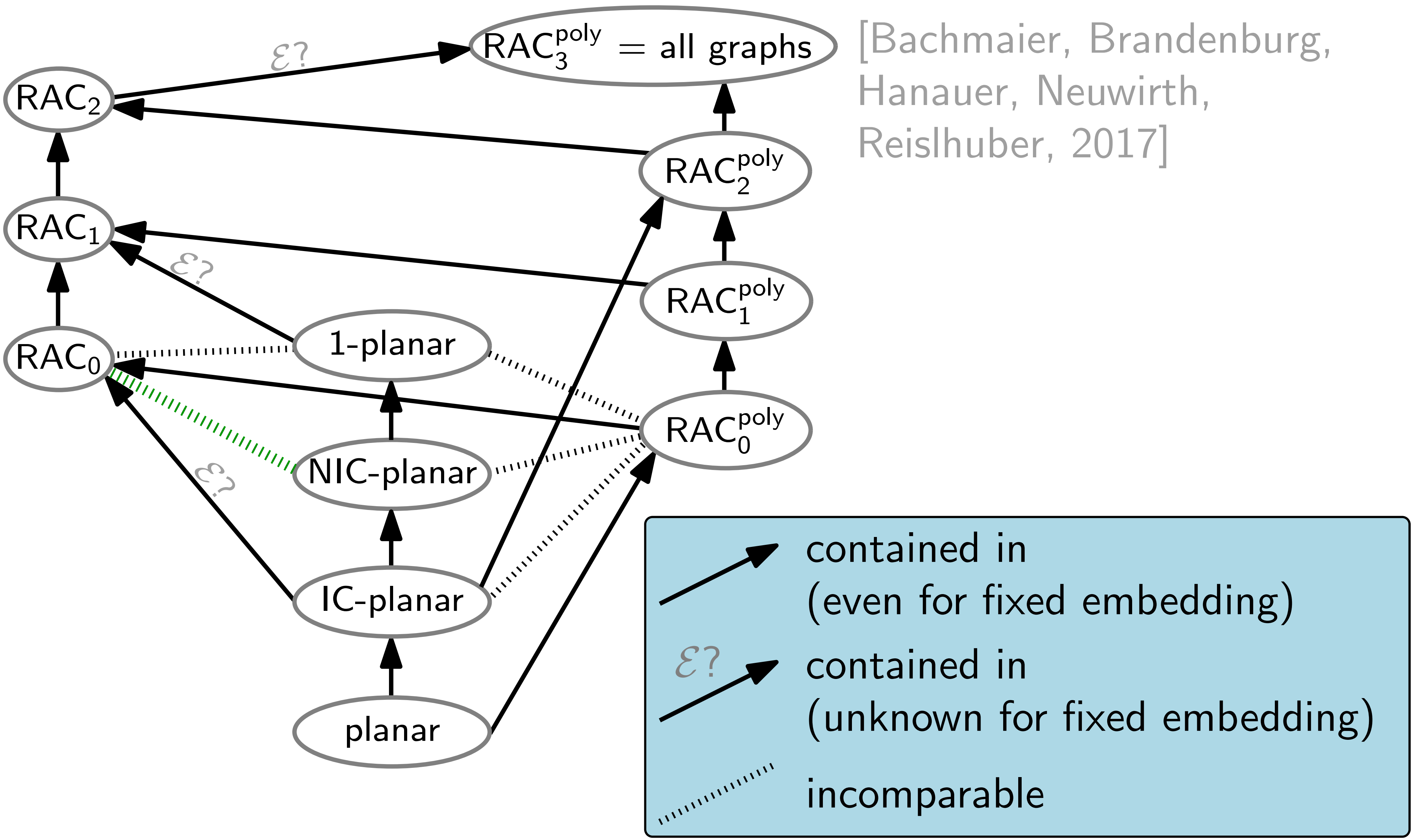

[Bachmaier, Brandenburg, Hanauer, Neuwirth, Reislhuber, 2017]

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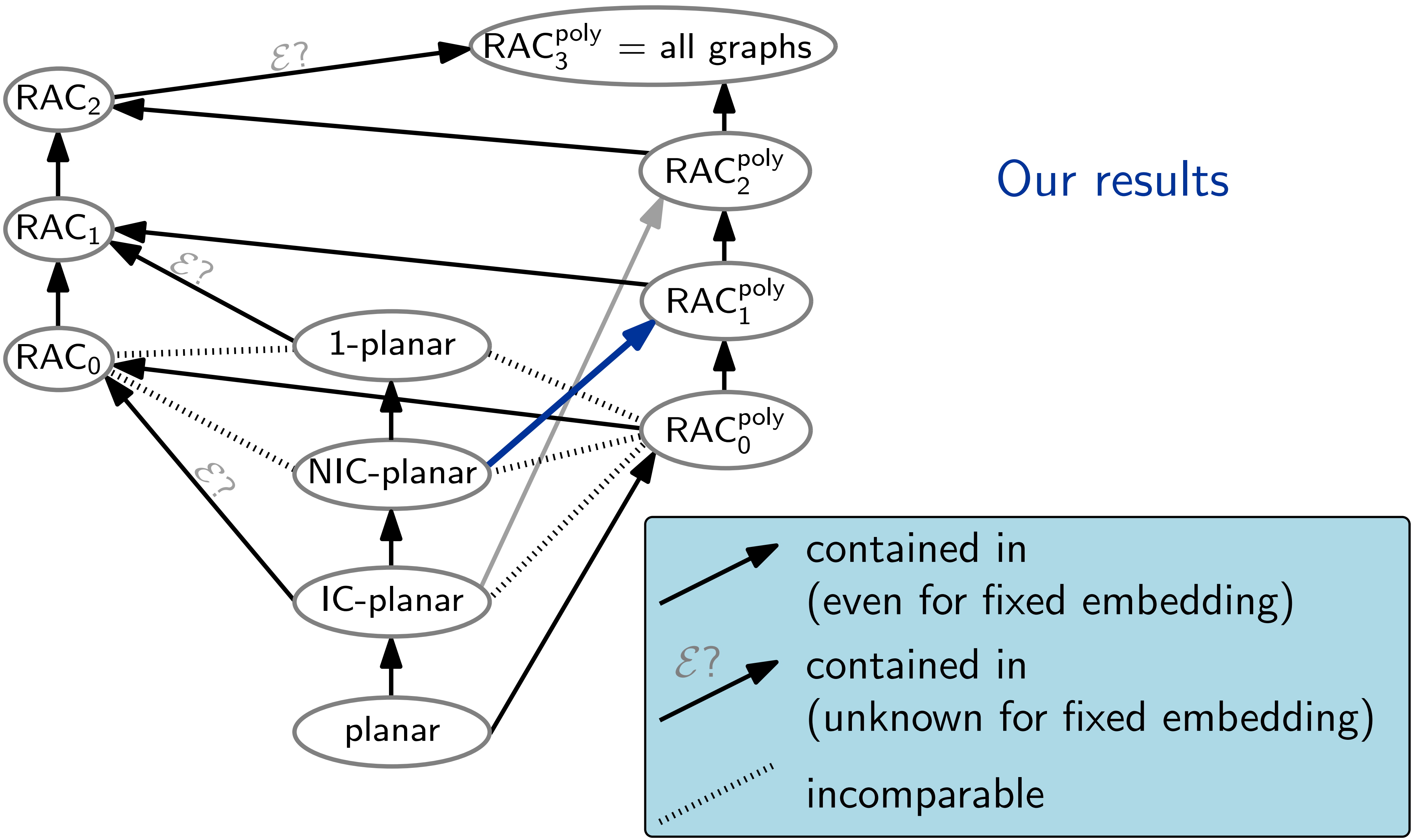

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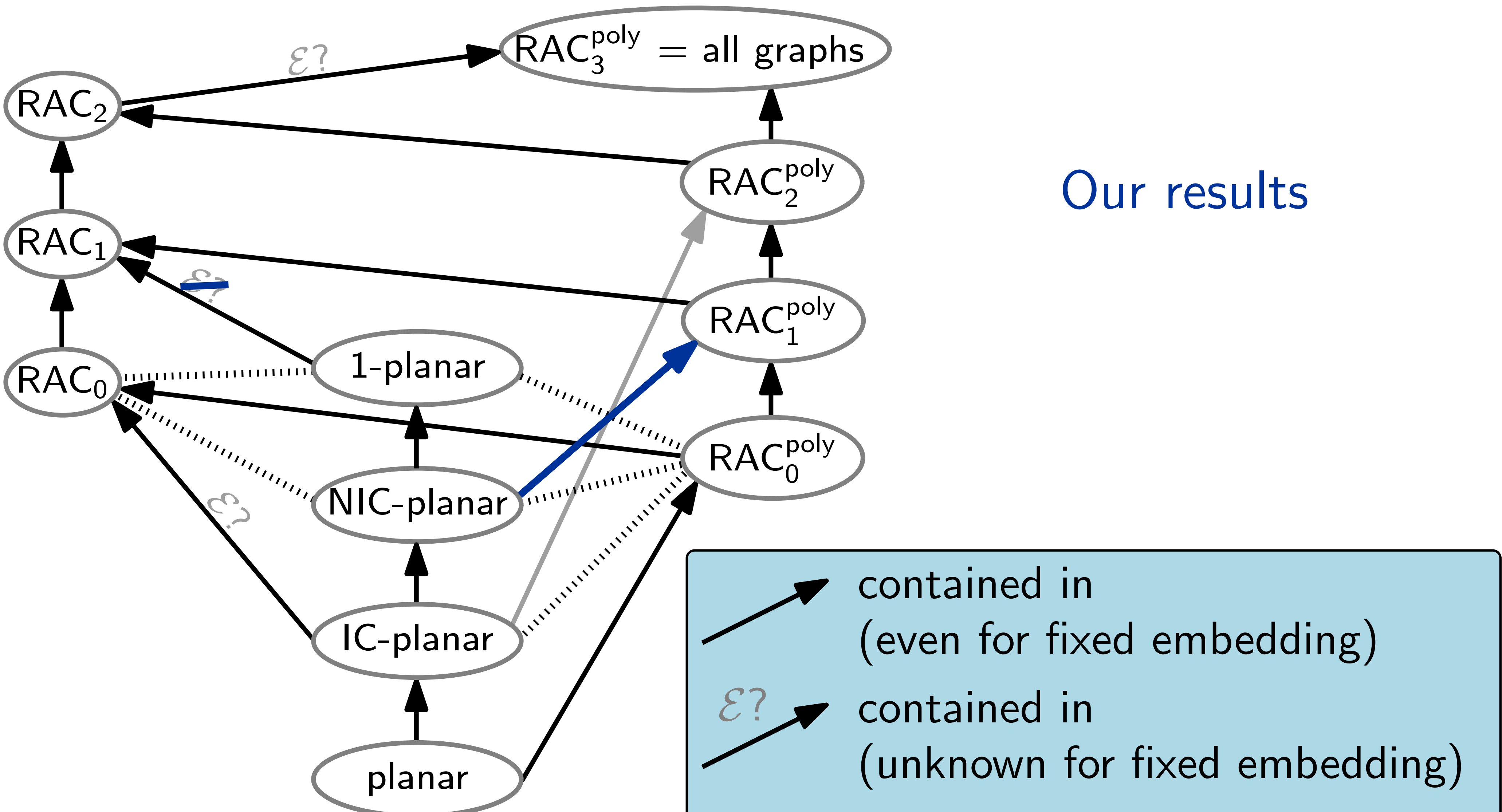
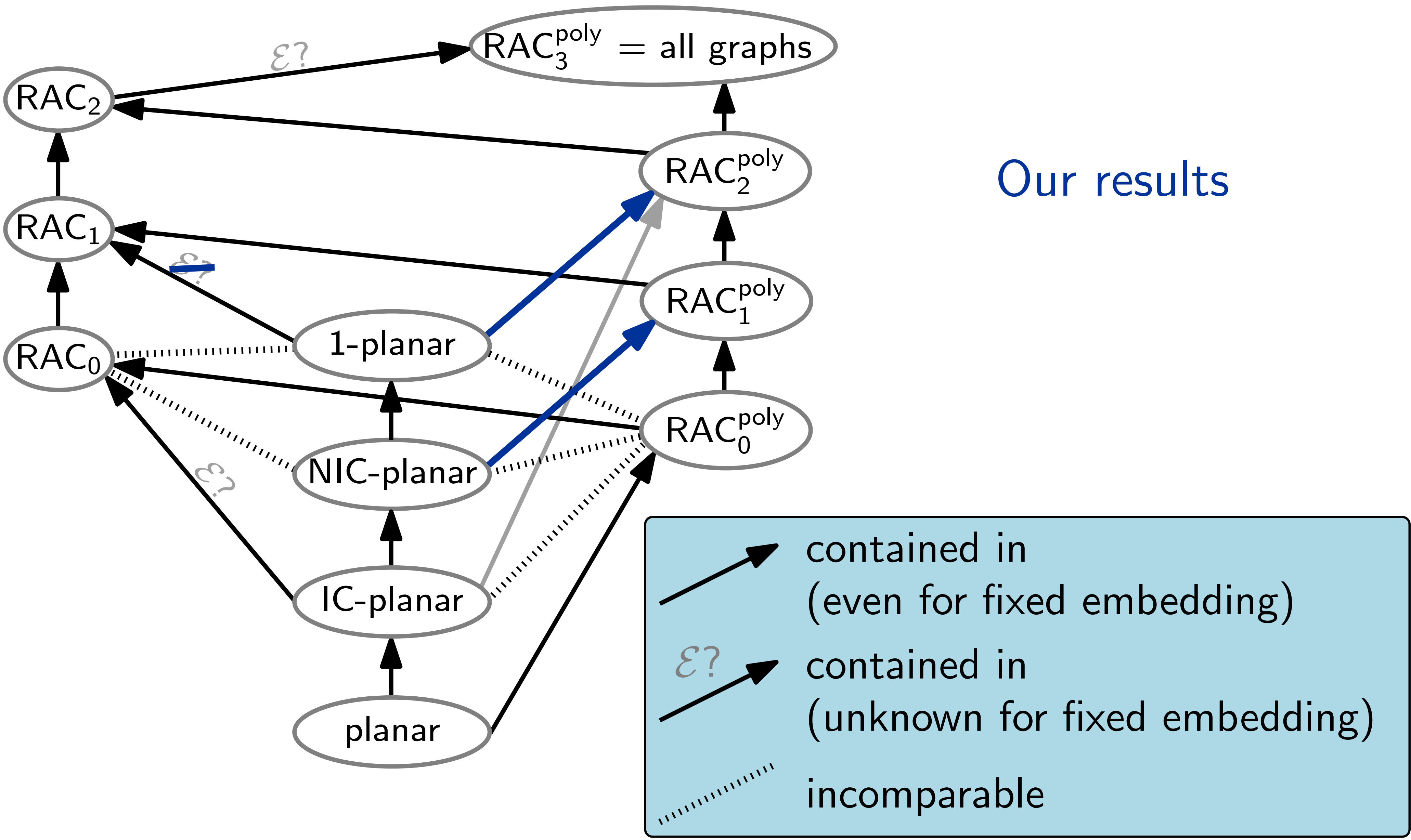


Our results

Related Work



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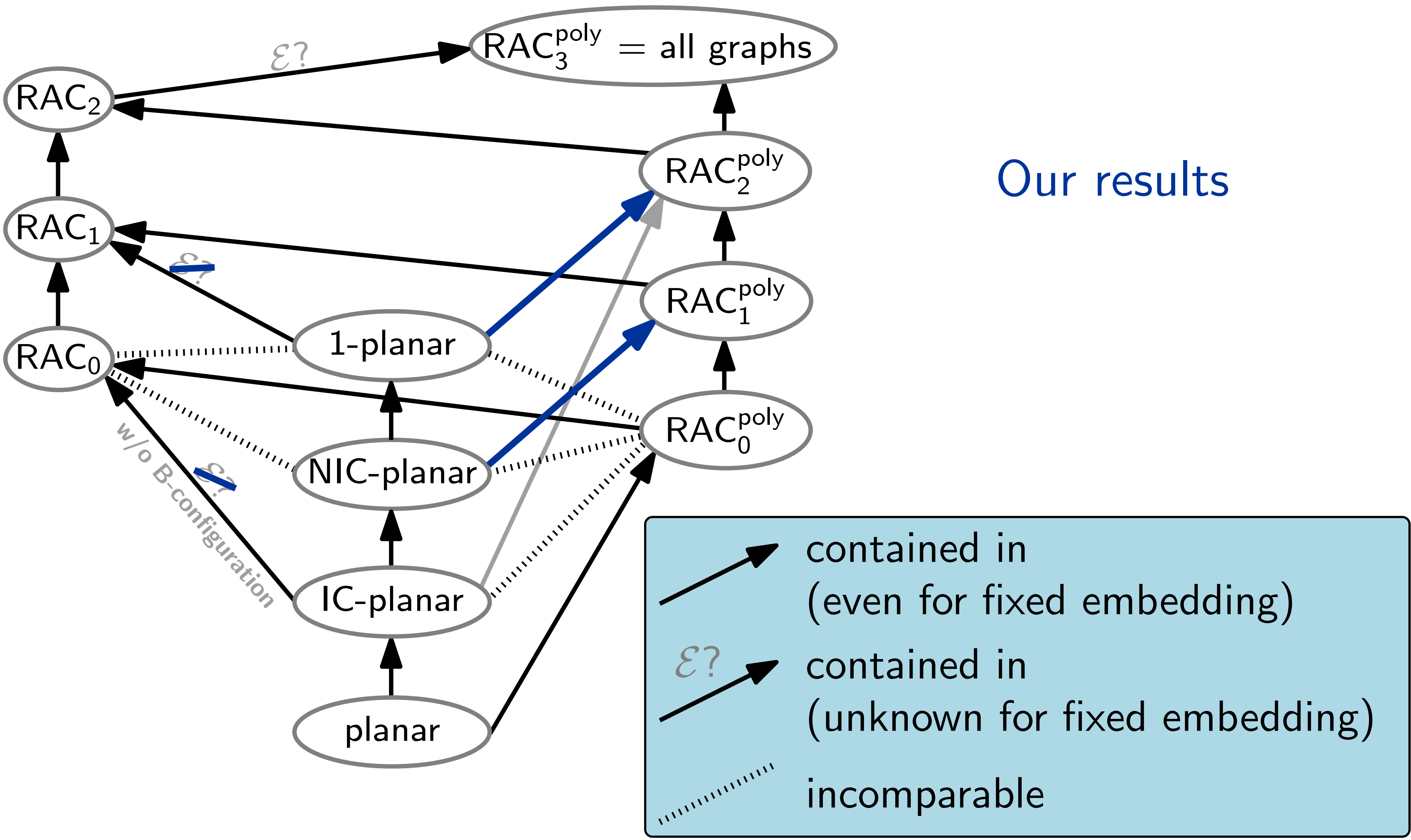

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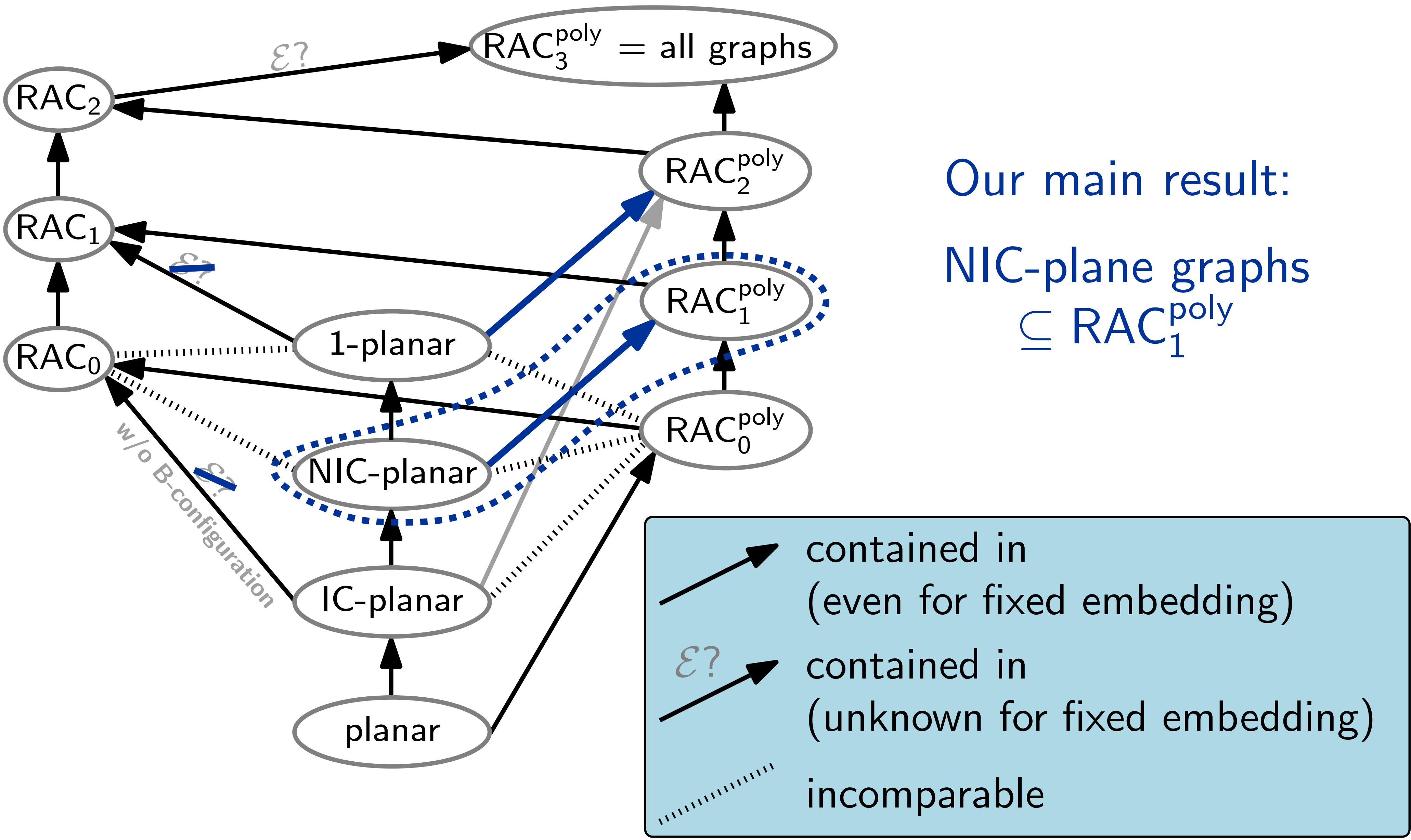

Our main result:

NIC-plane graphs $\subseteq$ RAC$_1^{poly}$

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Algorithm
in $O(n)$ **time:**

NIC-plane graphs $\subseteq$ $\mathrm{RAC}_1^{\mathrm{poly}}$

Algorithm
in $O(n)$ time:

Input:
NIC-plane graph $(G, \mathcal{E})$ with n vertices

NIC-plane graphs $\subseteq \mathrm{RAC}_1^{\mathrm{poly}}$

Algorithm
in $O(n)$ time:

Graph G with a NIC-planar embedding $\mathcal{E}$

Input:
NIC-plane graph $(G, \mathcal{E})$ with n vertices

NIC-plane graphs $\subseteq \text{RAC}_1^{\text{poly}}$

Algorithm
in $O(n)$ time:

Graph G with a NIC-planar embedding $\mathcal{E}$

Input:
NIC-plane graph $(G, \mathcal{E})$ with n vertices

Output:
1-bend RAC drawing Γ of G according to $\mathcal{E}$
Every vertex, bend point, and crossing point of Γ lies on a grid of size $O(n) \times O(n)$

The Shift Algorithm

[de Fraysseix, Pach, and Pollack, 1990]
[Chrobak and Payne, 1995]

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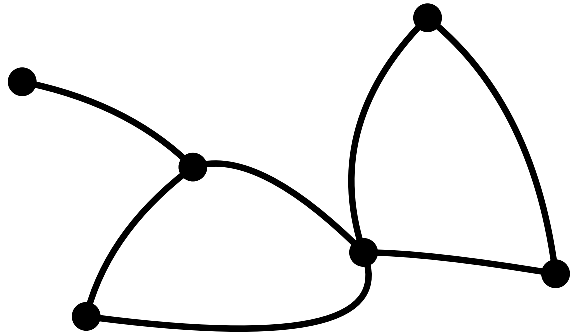
Idea:



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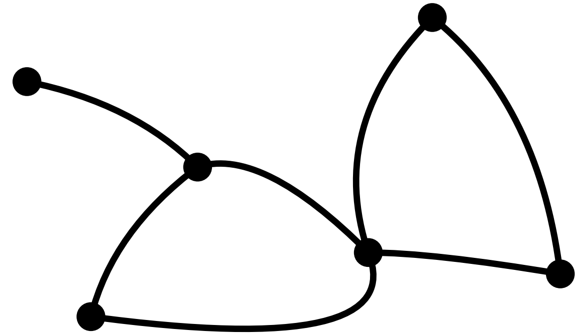
Idea:

- Triangulate given plane graph.



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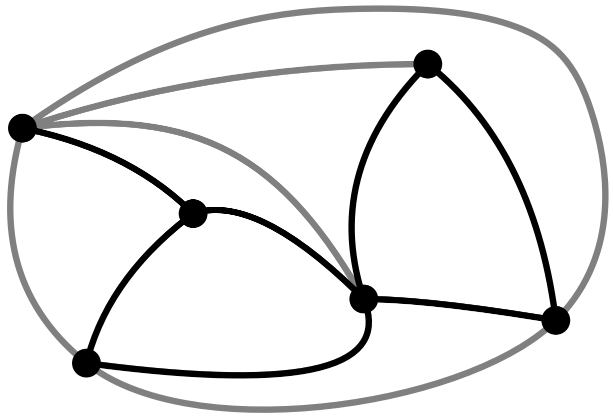

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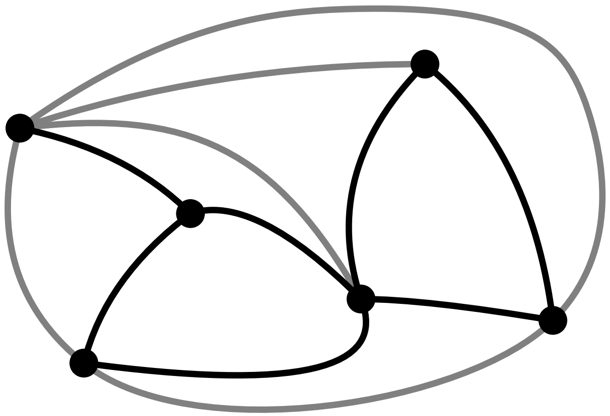

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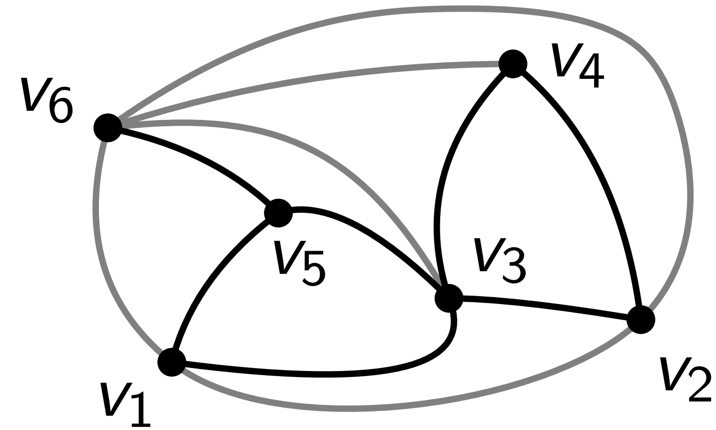

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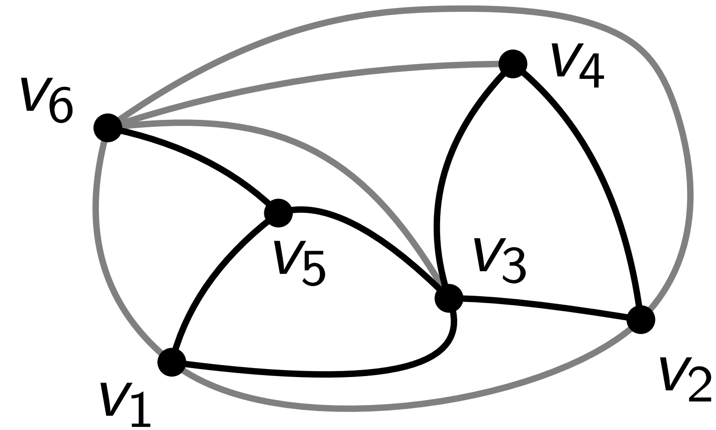

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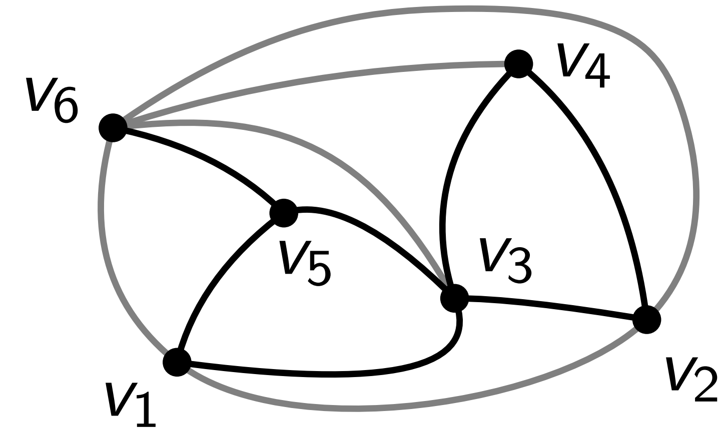

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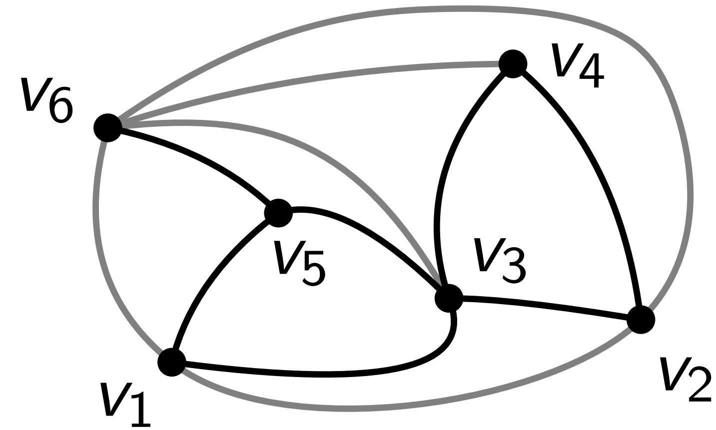

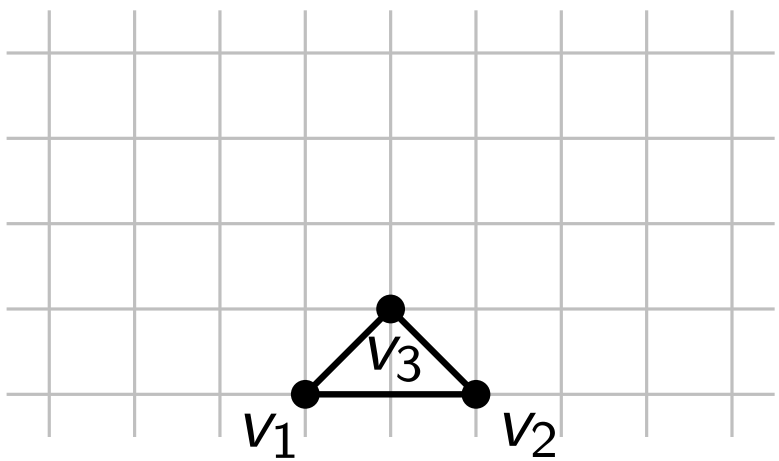

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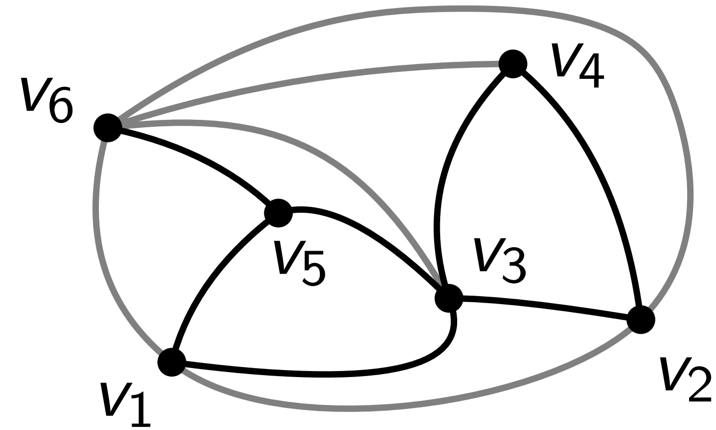

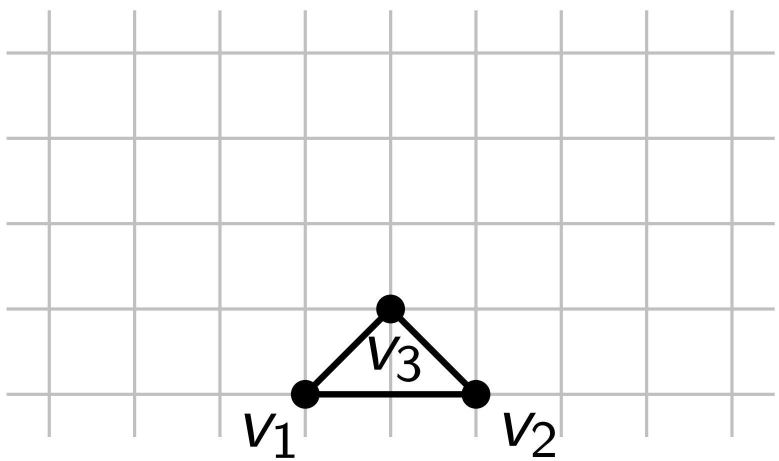

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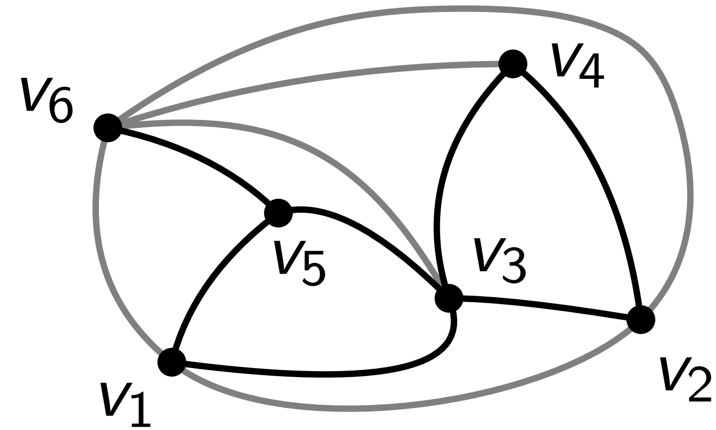

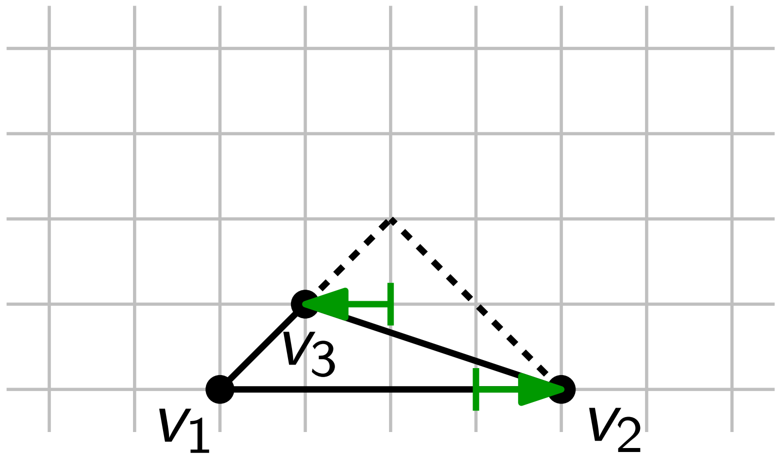

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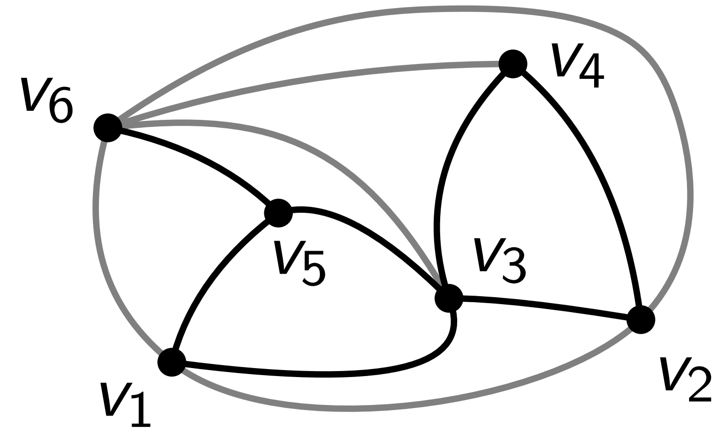

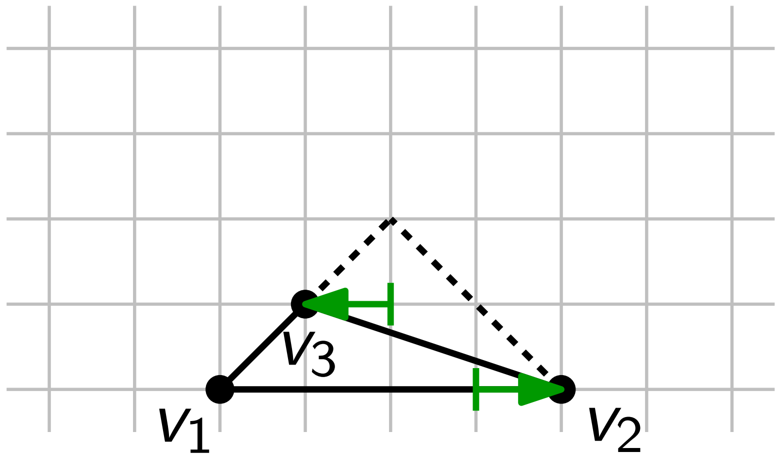

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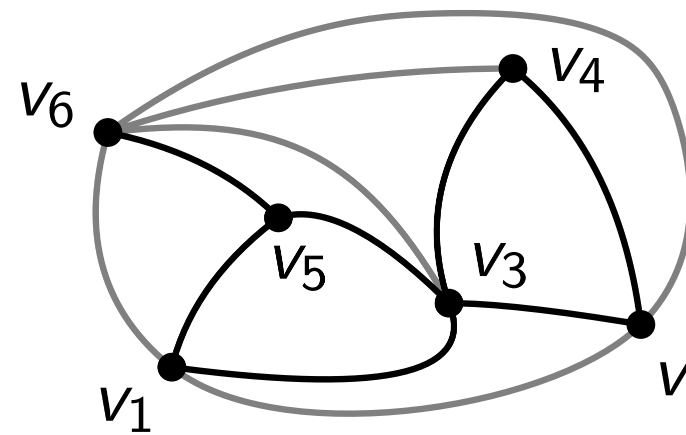

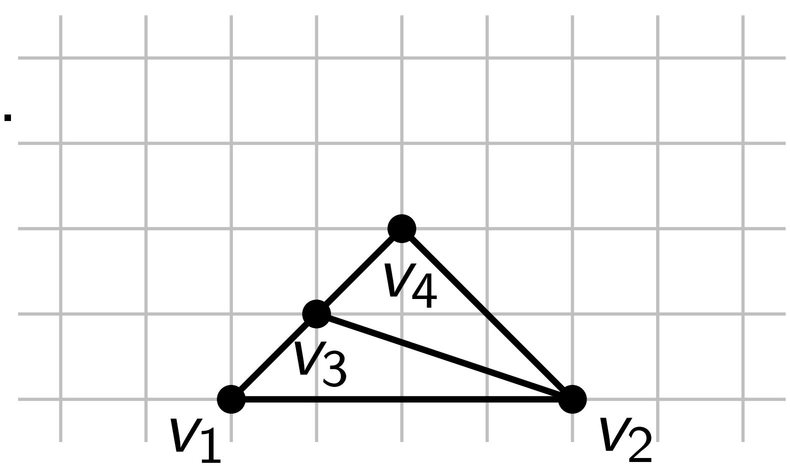

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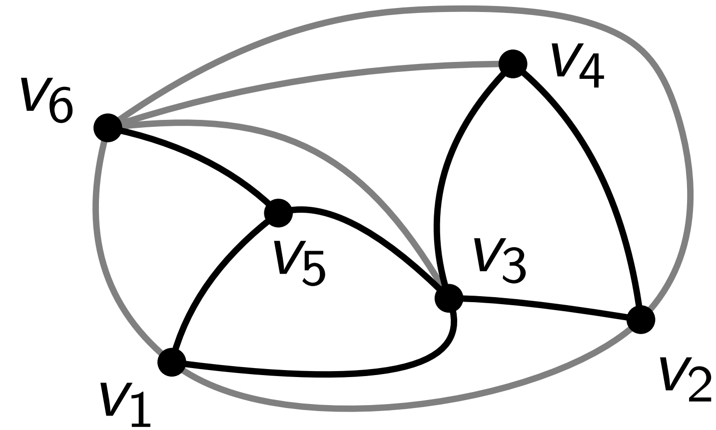
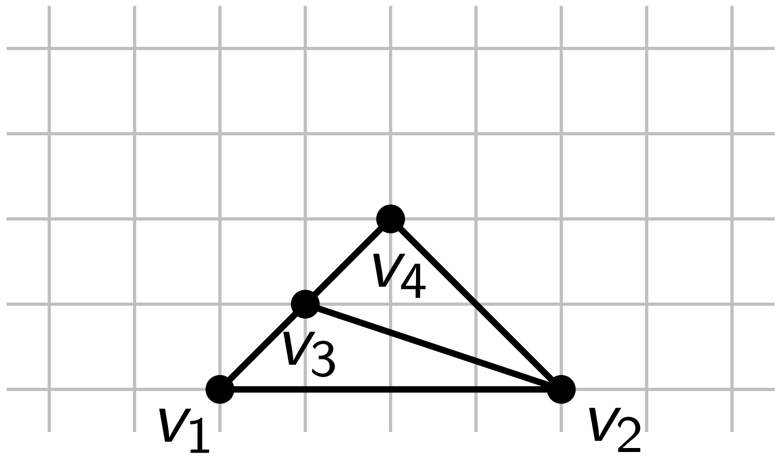
  $\Rightarrow$ all slopes on outer face ± 1 (except for $v_1 v_2$)





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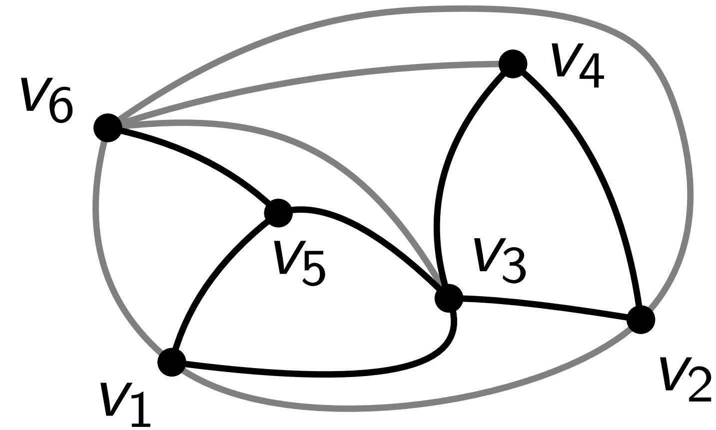

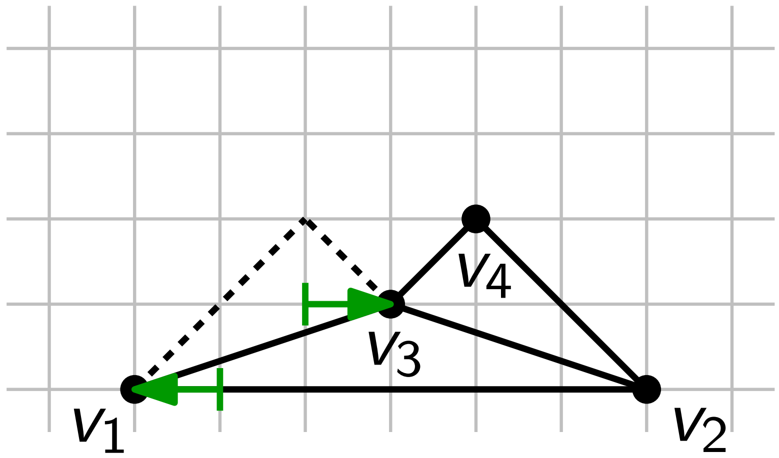

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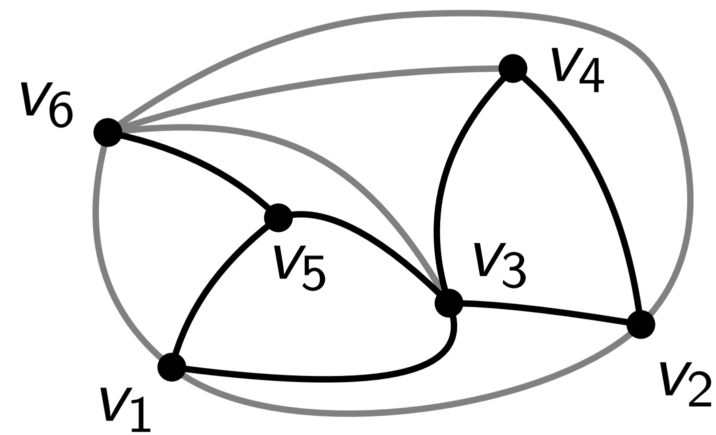

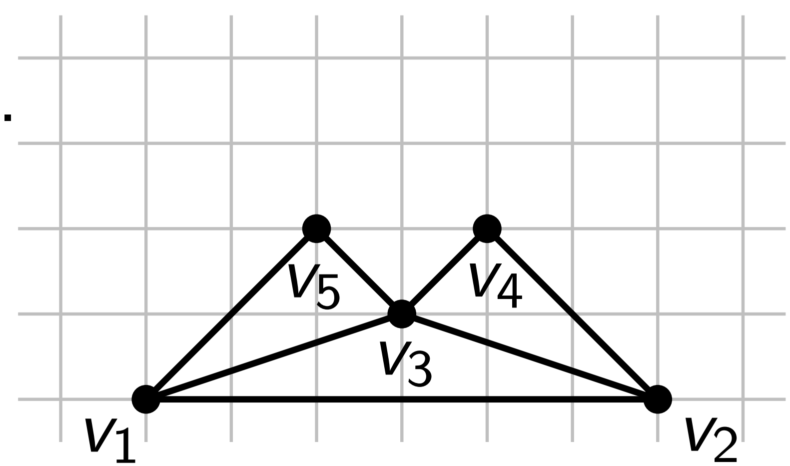

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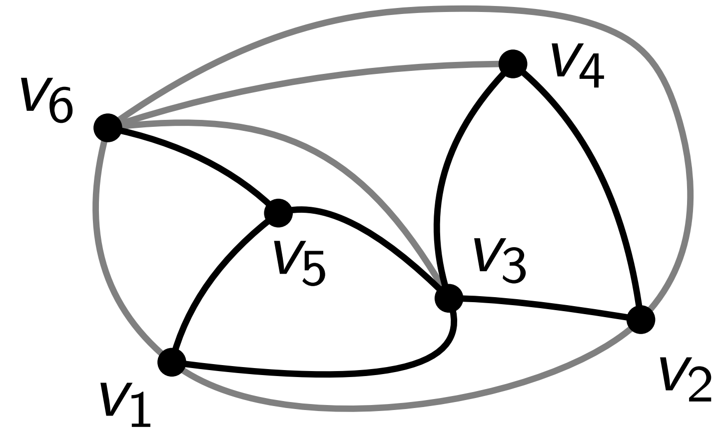

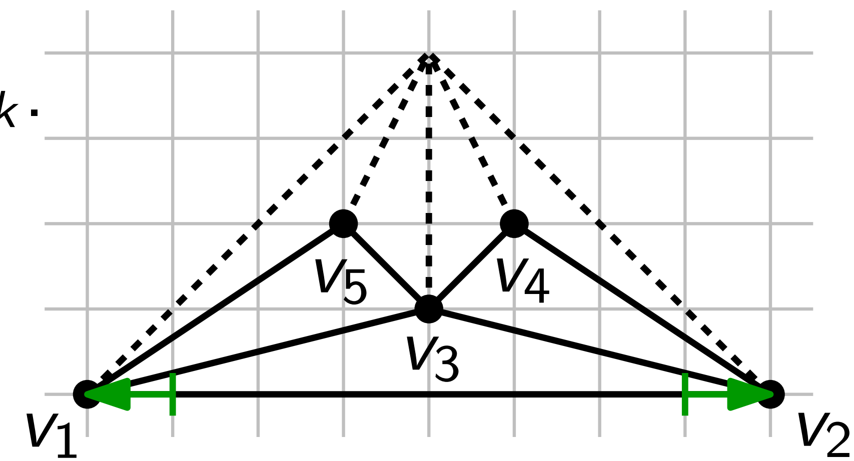

The Shift Algorithm

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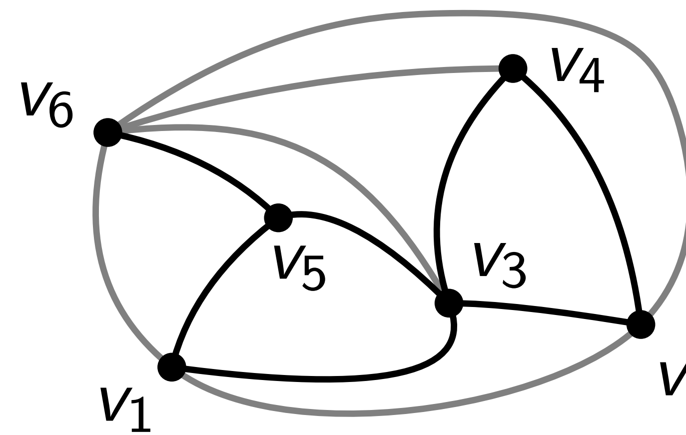

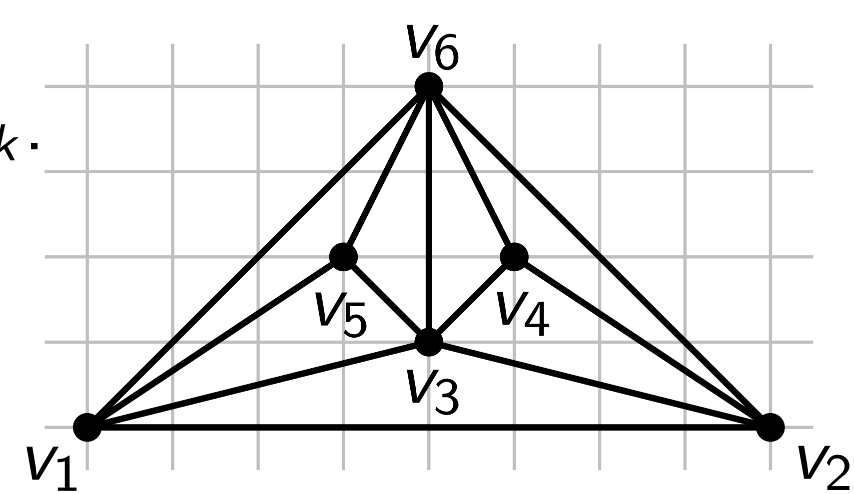

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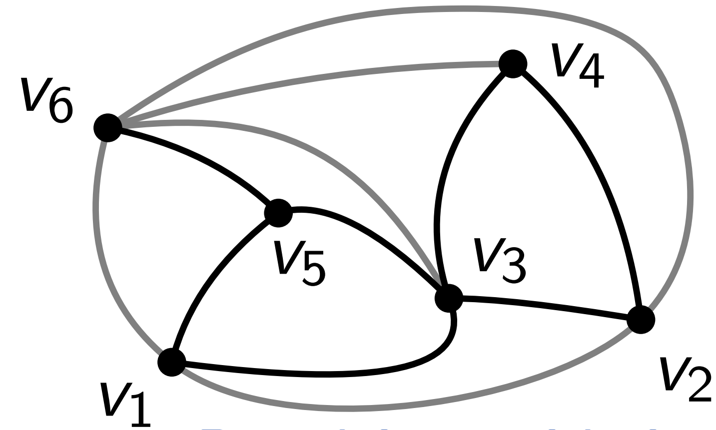

Resulting grid size:
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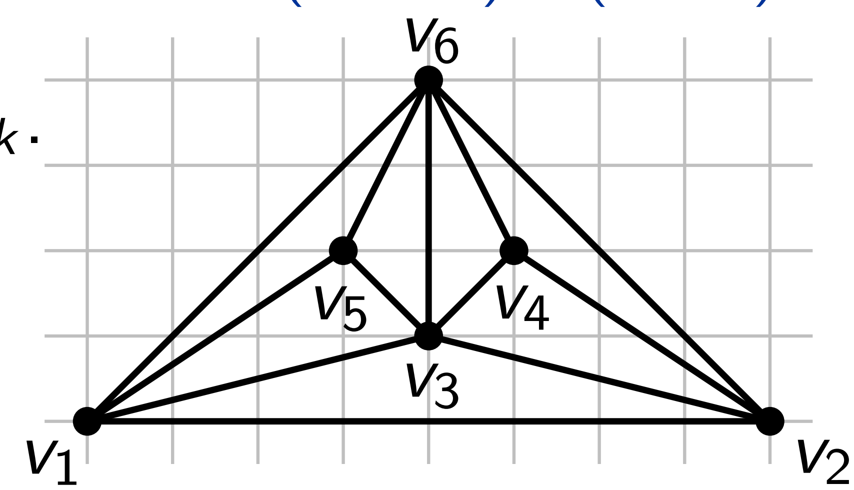

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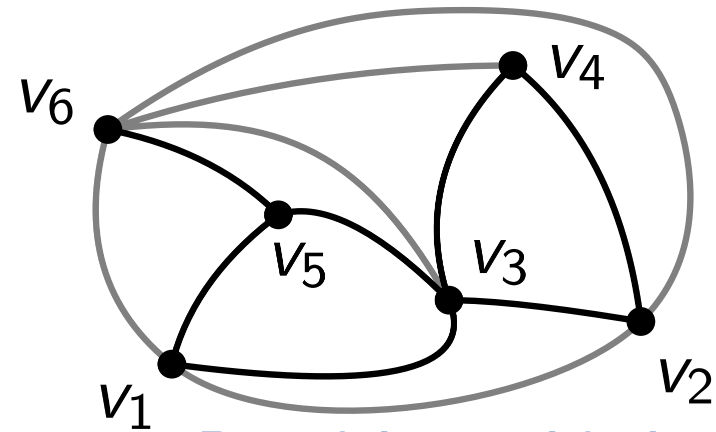

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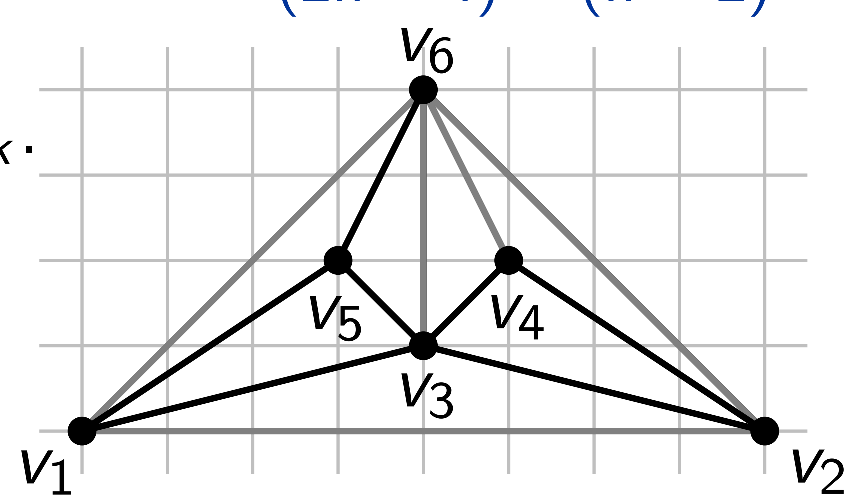

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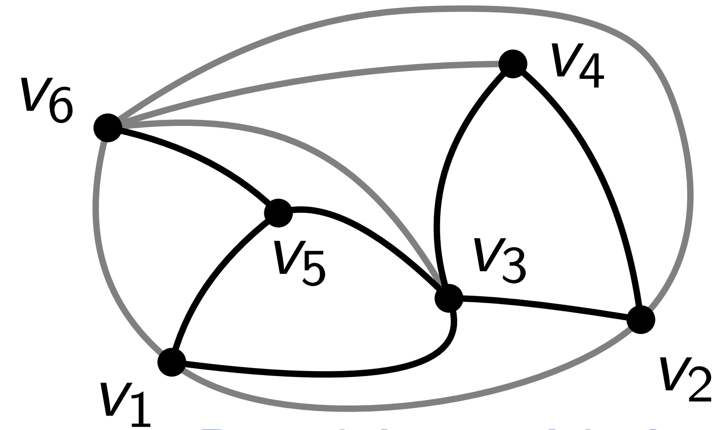

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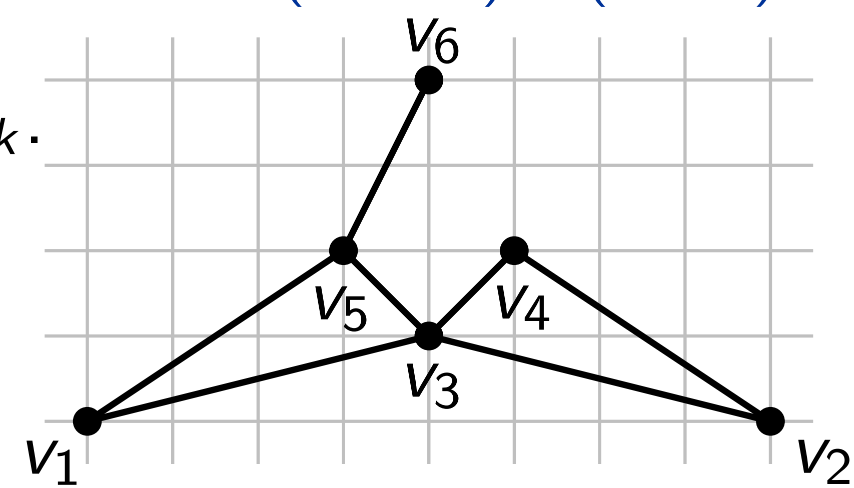

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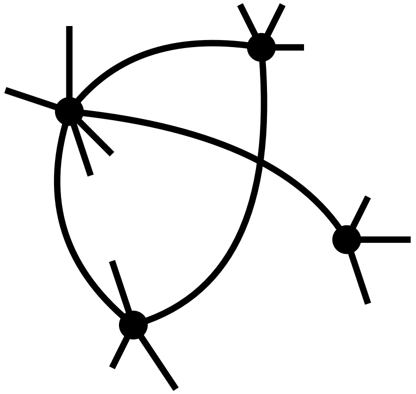

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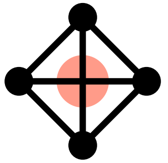
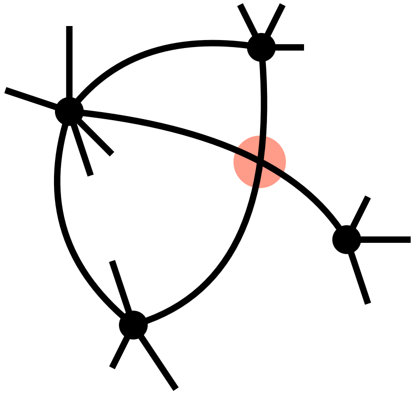
- Enclose each crossing by a so called *empty kite* ()



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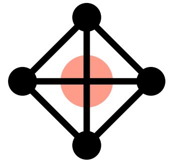
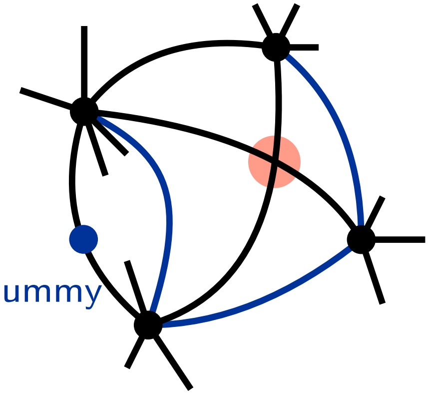
- Enclose each crossing by a so called *empty kite* ()



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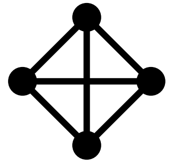
- Enclose each crossing by a so called *empty kite* ()
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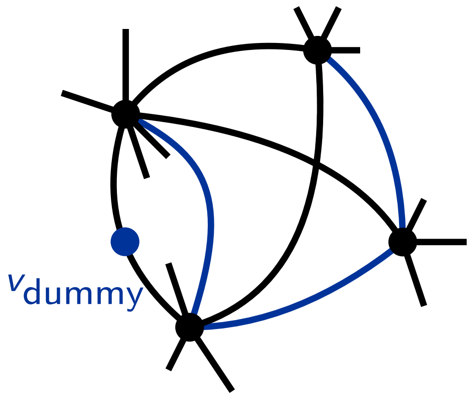

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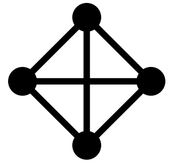
- Enclose each crossing by a so called *empty kite* ()
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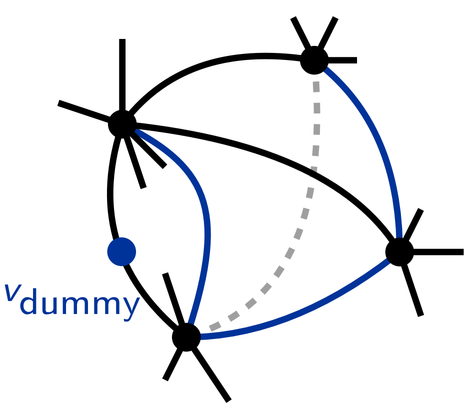

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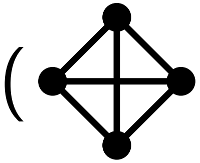
- Enclose each crossing by a so called *empty kite* ()
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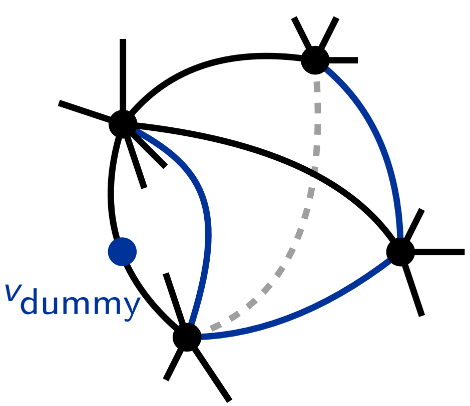

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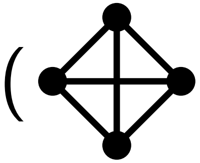
- Enclose each crossing by a so called *empty kite* ()
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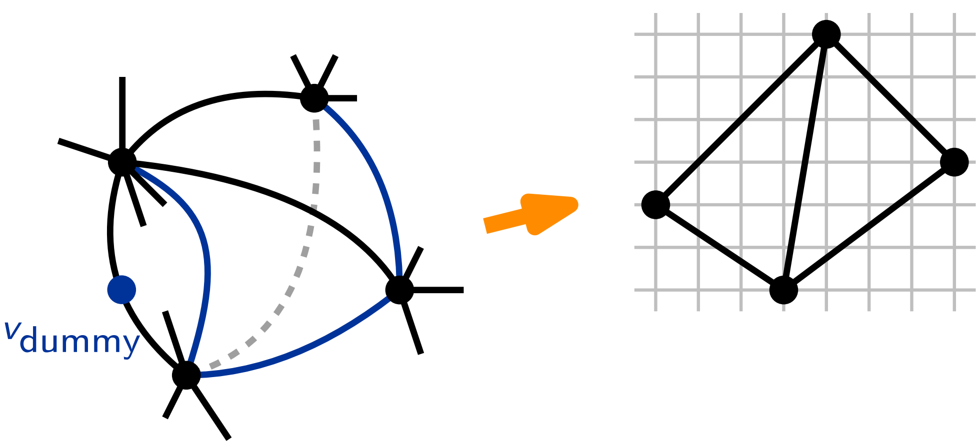

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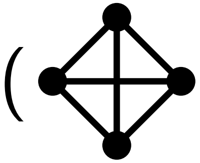
- Enclose each crossing by a so called *empty kite* ()
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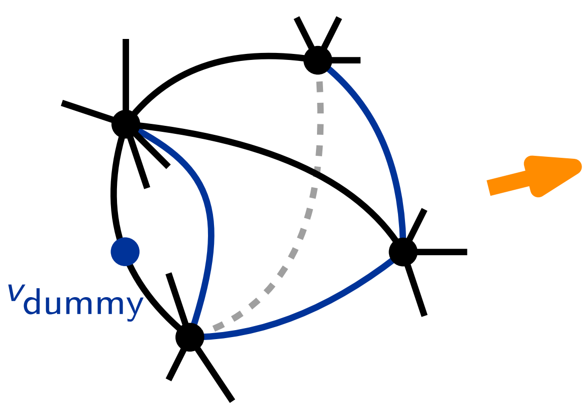

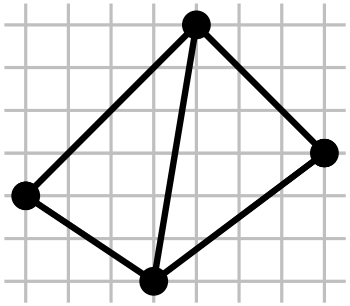

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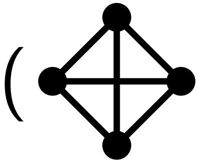
- Enclose each crossing by a so called *empty kite* ()
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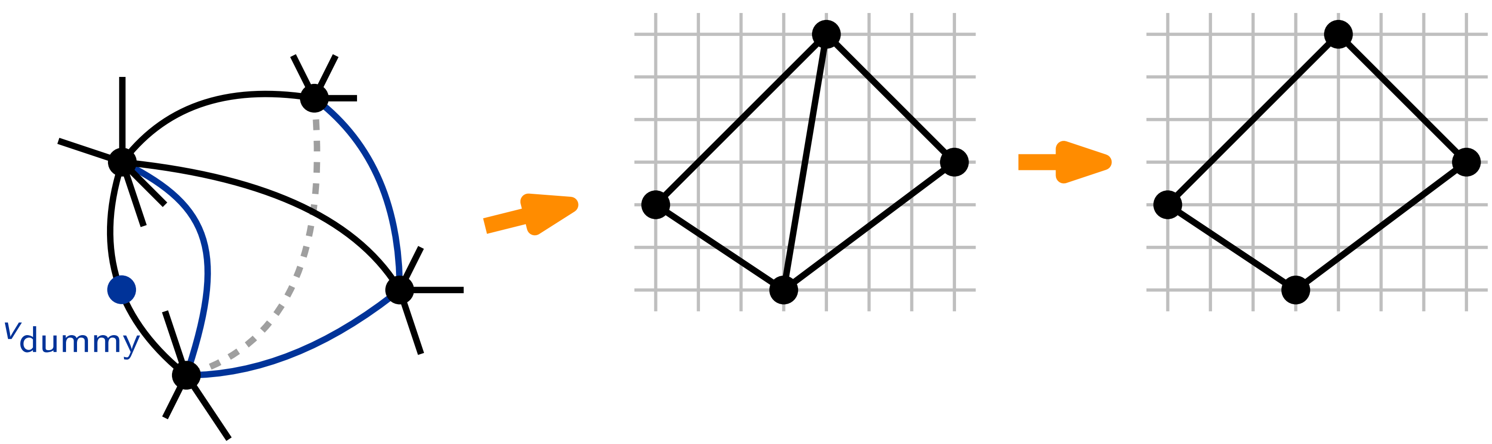

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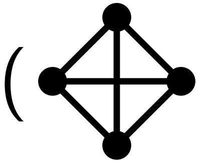
- Enclose each crossing by a so called *empty kite* ()
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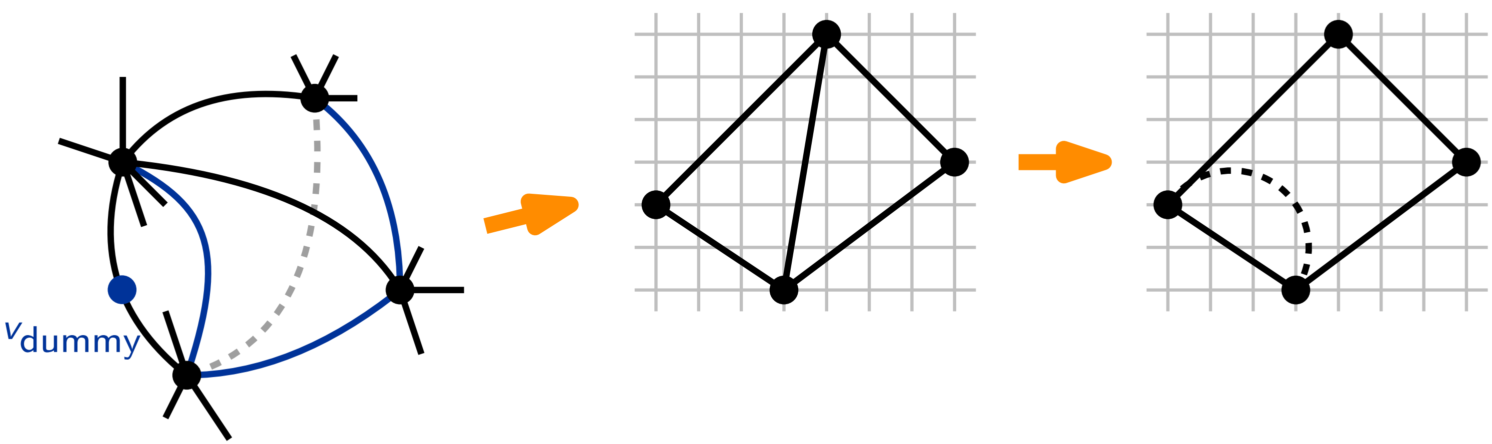

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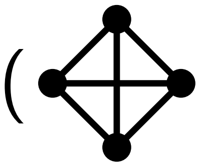
- Enclose each crossing by a so called *empty kite* ()
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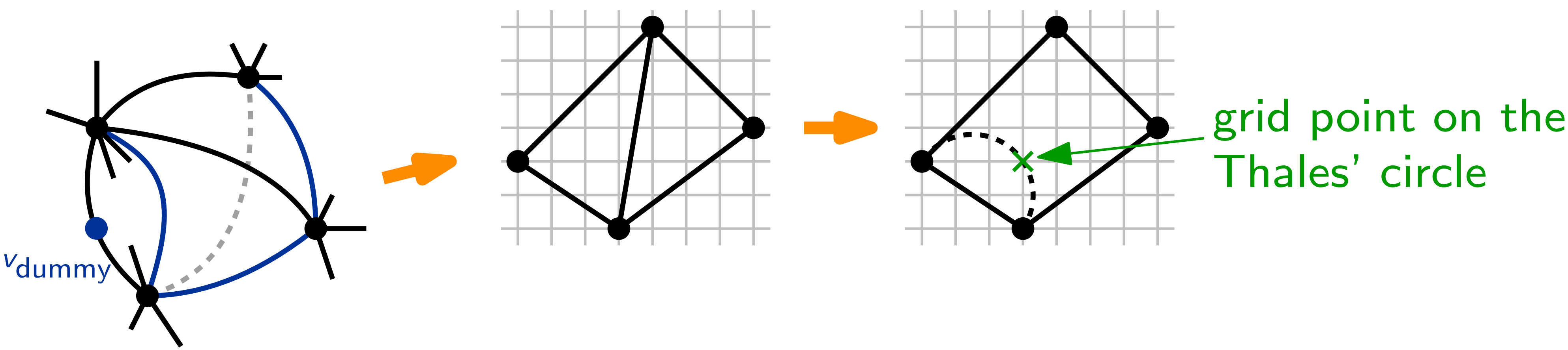

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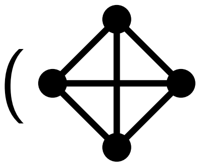
- Enclose each crossing by a so called *empty kite* ()
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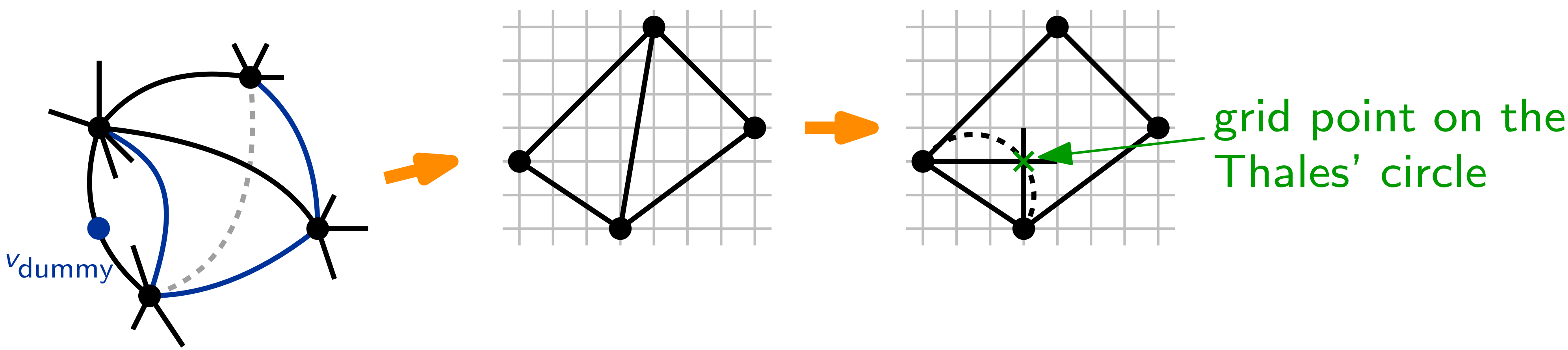

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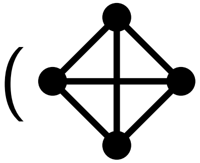
- Enclose each crossing by a so called *empty kite* ()
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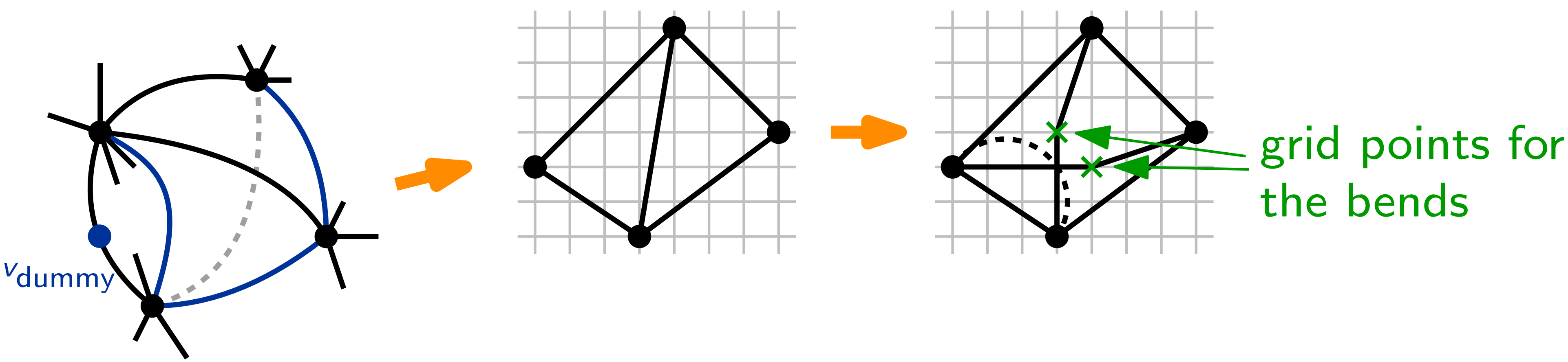

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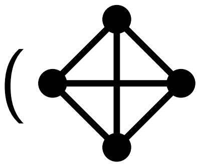
- Enclose each crossing by a so called *empty kite* ()
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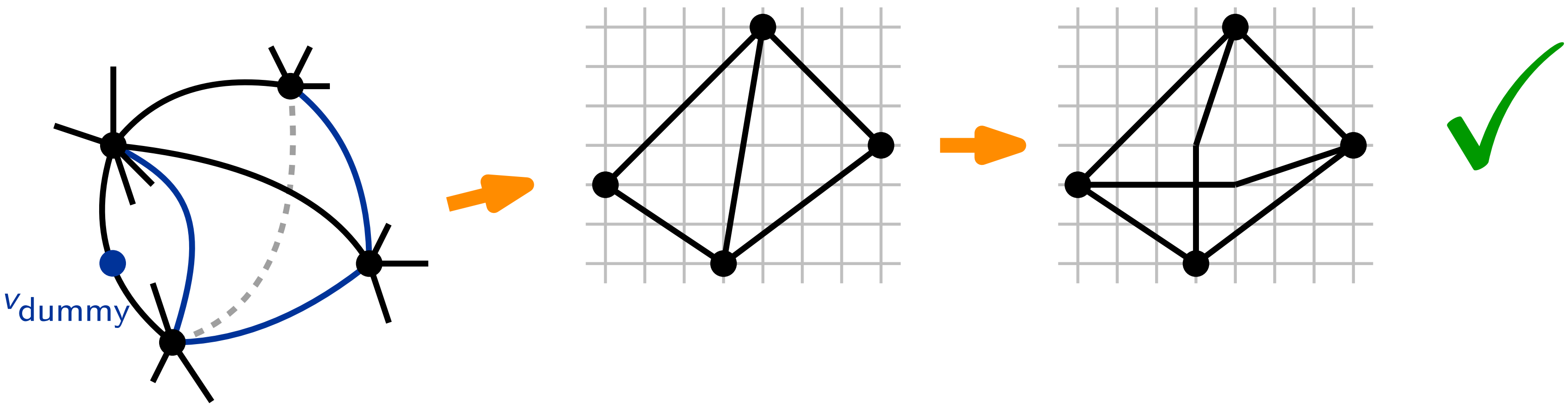

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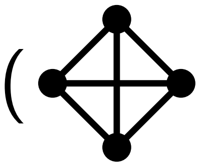
- Enclose each crossing by a so called *empty kite* ()
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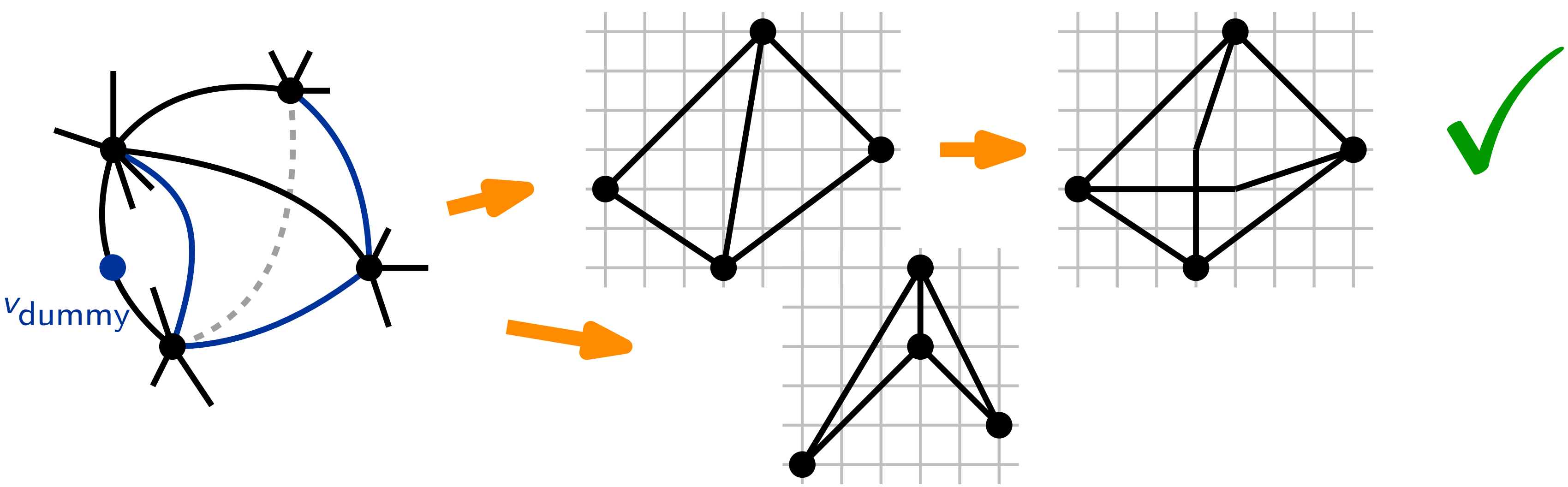

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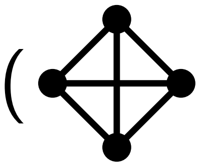
- Enclose each crossing by a so called *empty kite* ()
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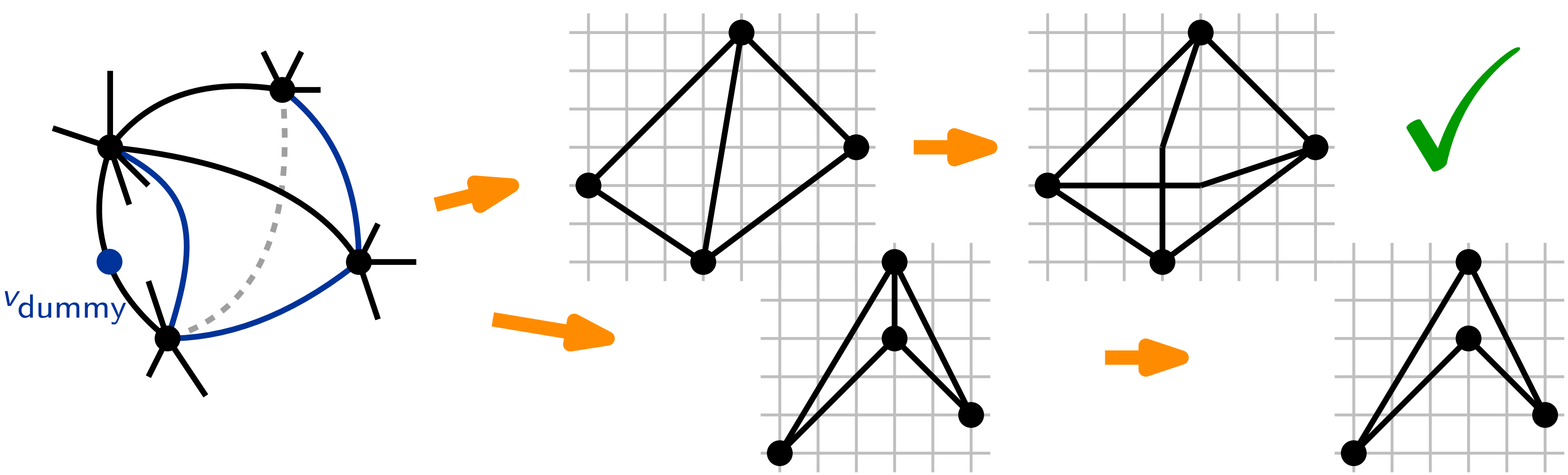

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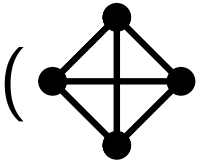
- Enclose each crossing by a so called *empty kite* ()
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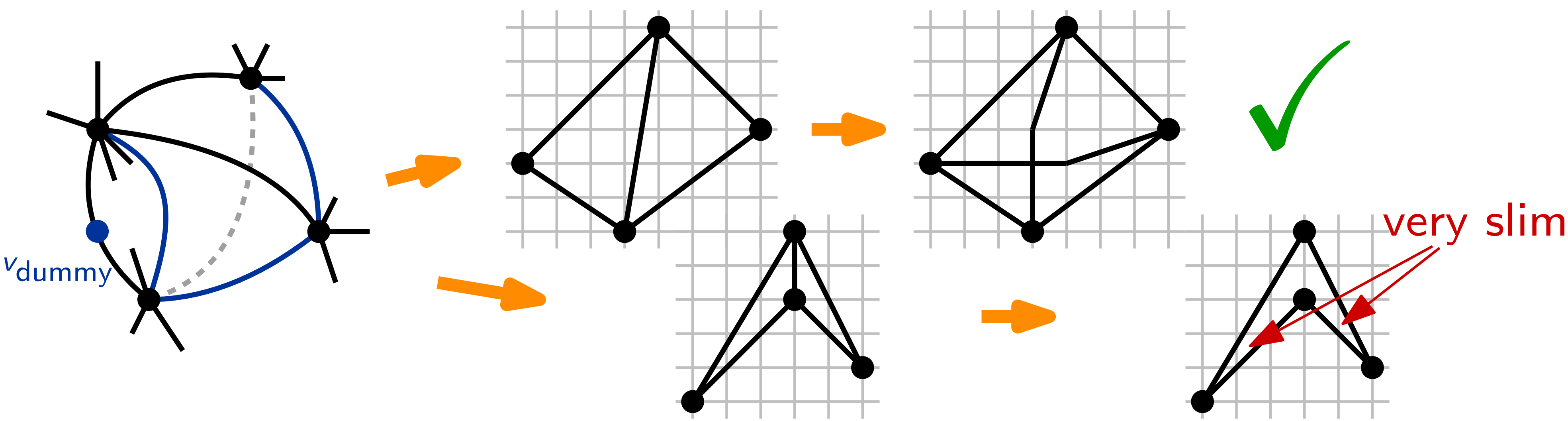

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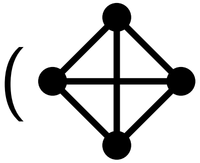
- Enclose each crossing by a so called *empty kite* ()
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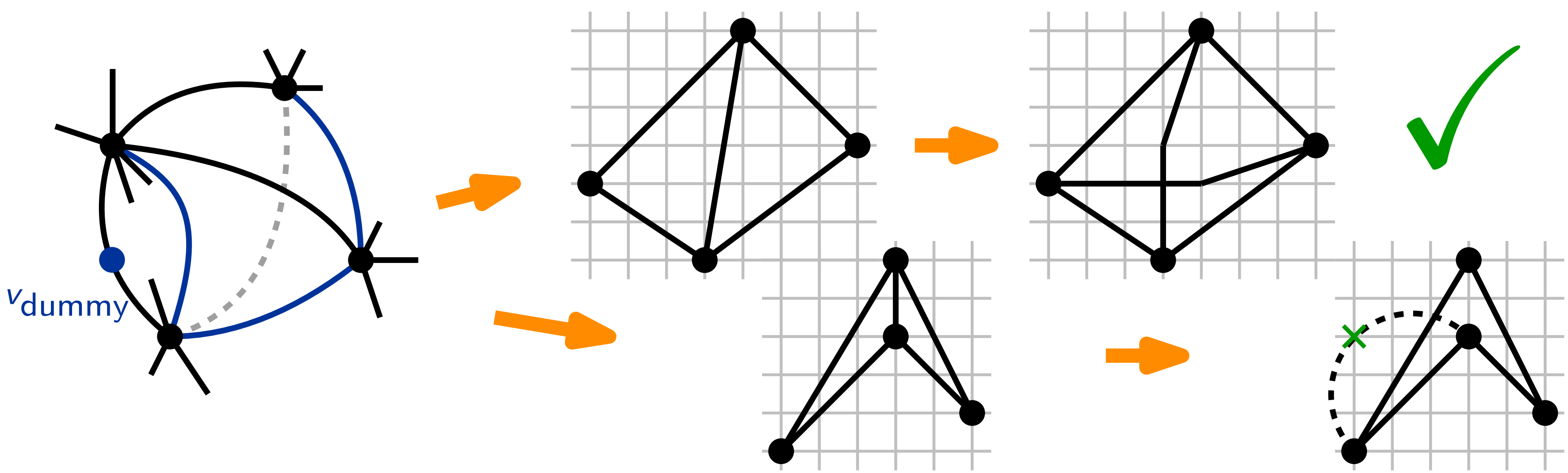

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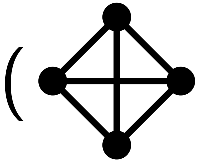
- Enclose each crossing by a so called *empty kite* ()
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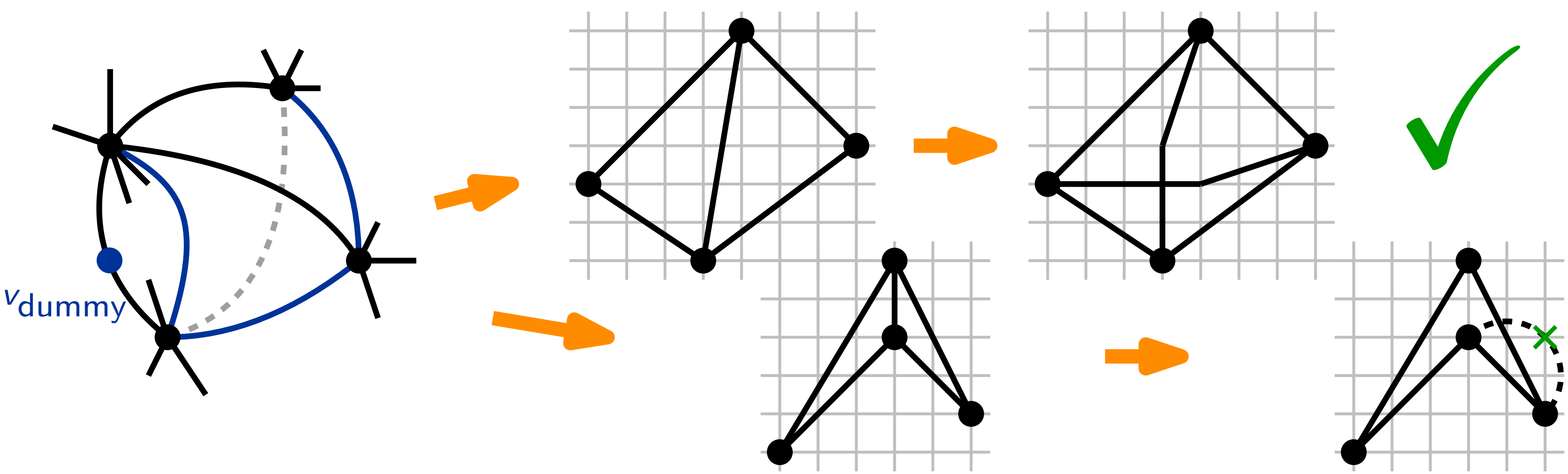

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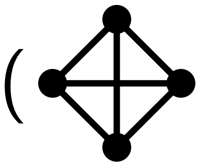
- Enclose each crossing by a so called *empty kite* ()
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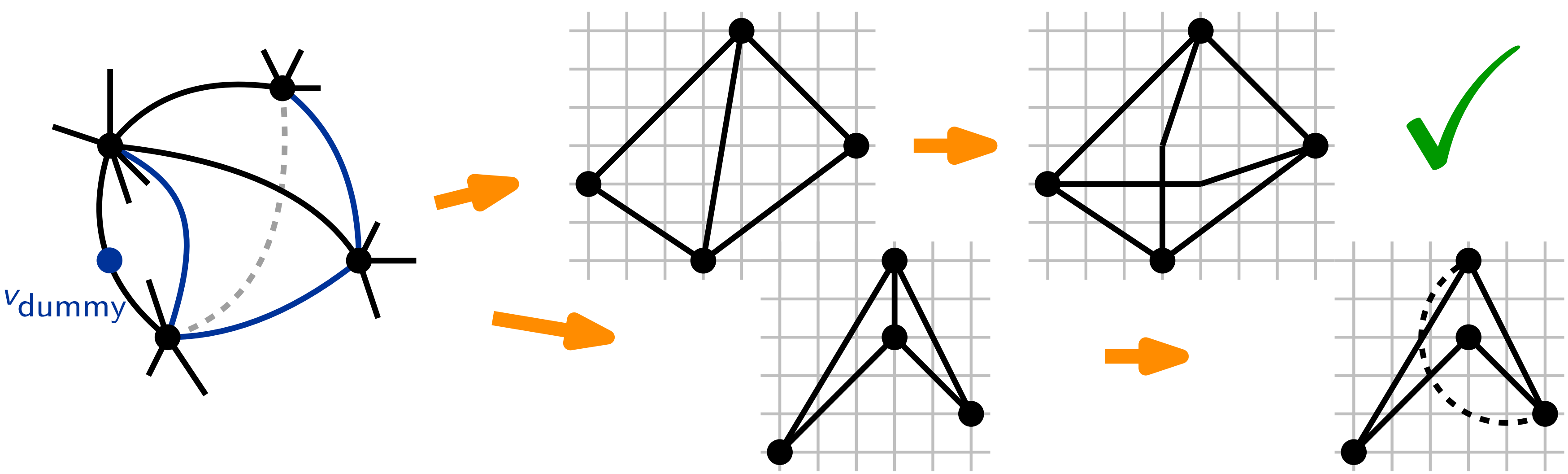

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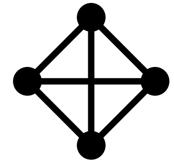
- Enclose each crossing by a so called *empty kite* ()
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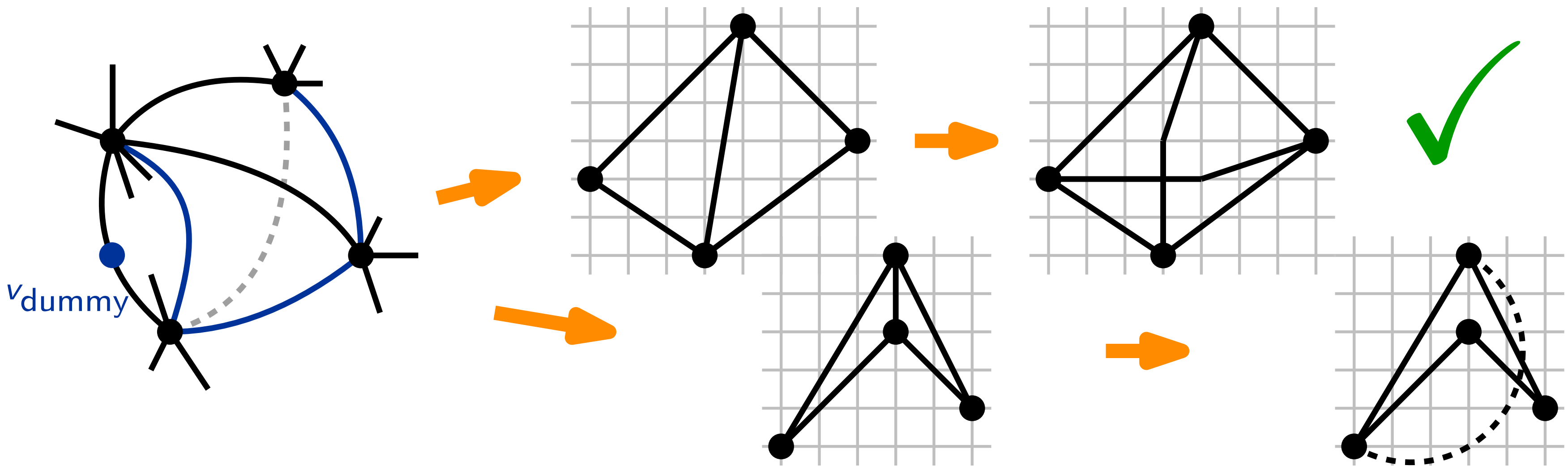

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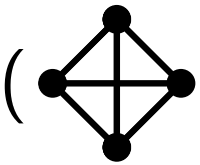
- Enclose each crossing by a so called *empty kite* ()
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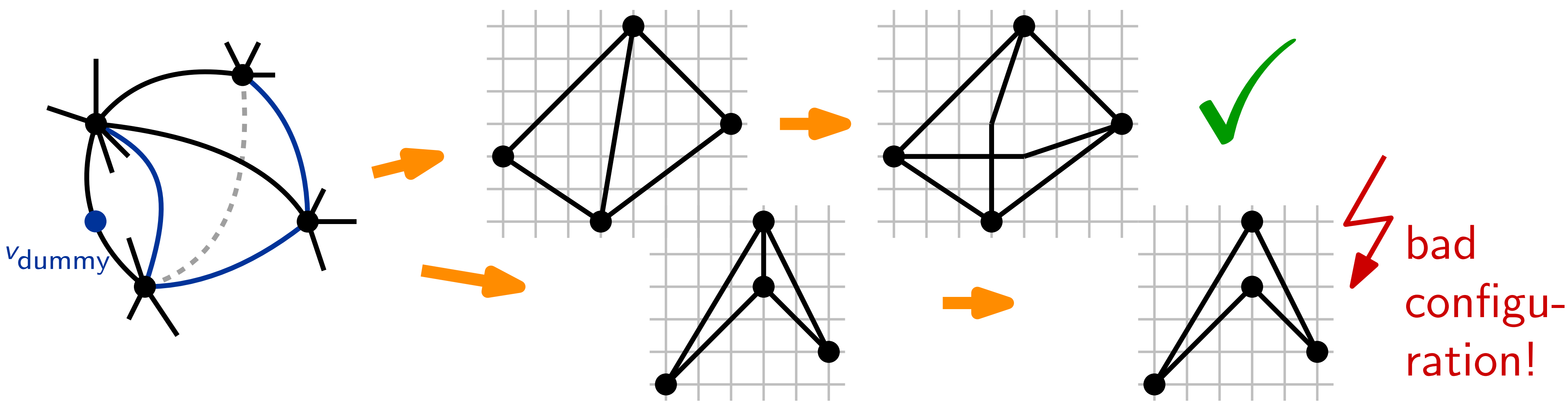
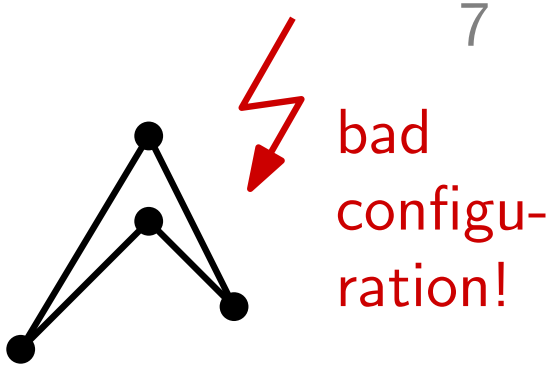


Our Algorithm



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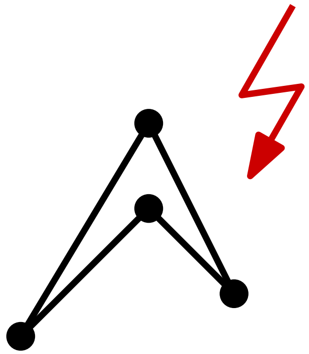

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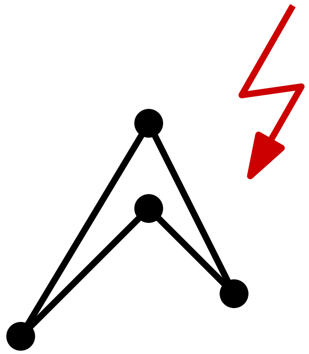

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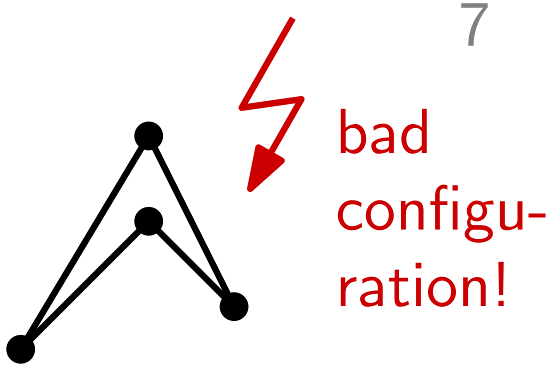

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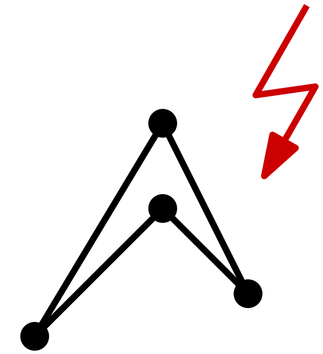

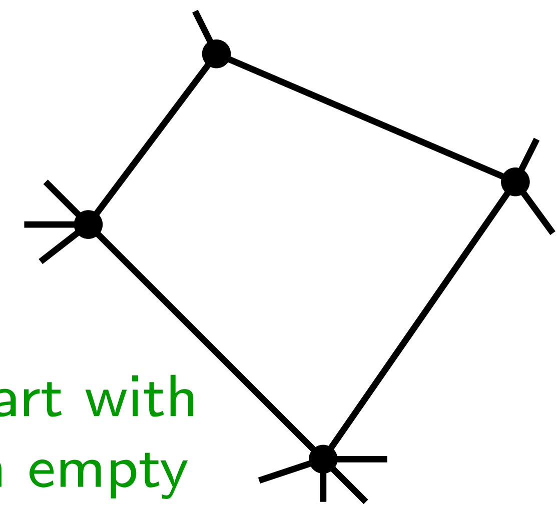

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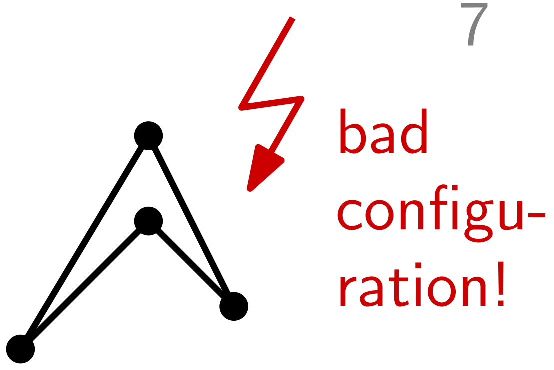

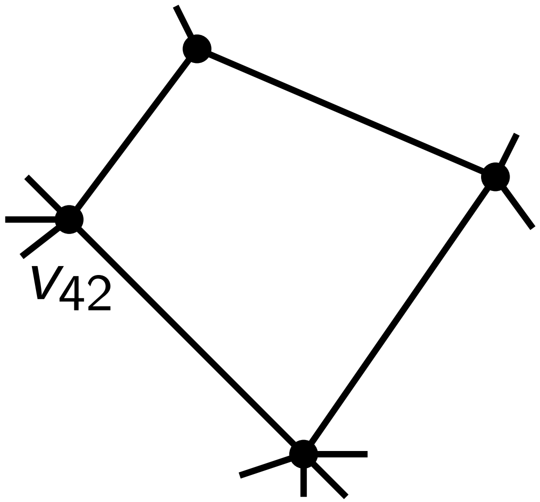

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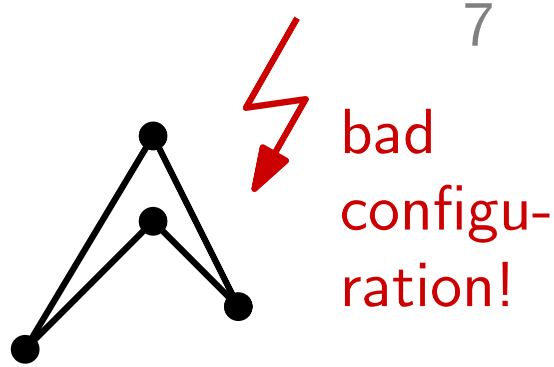

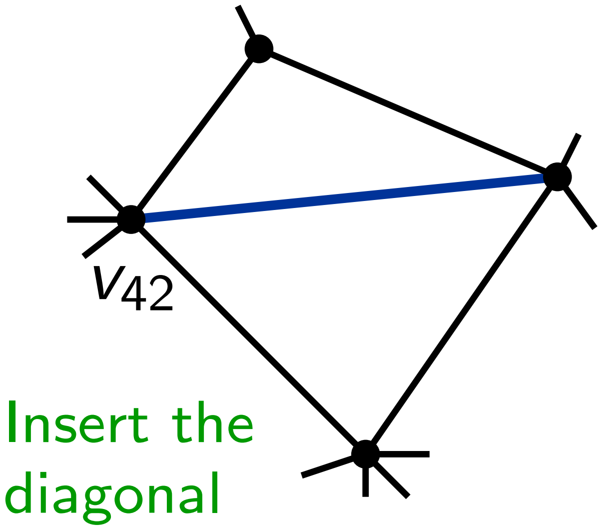

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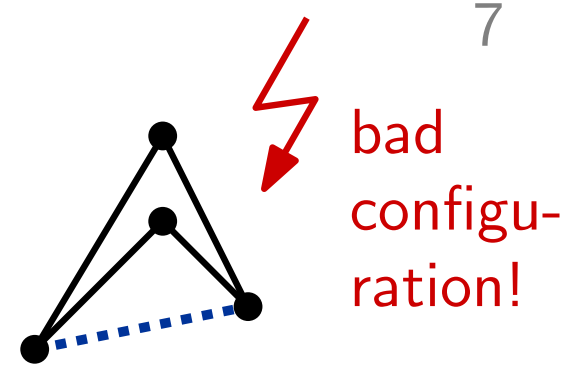

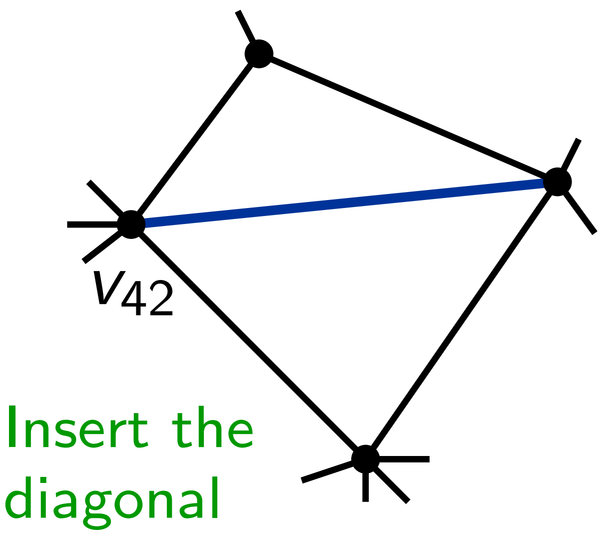

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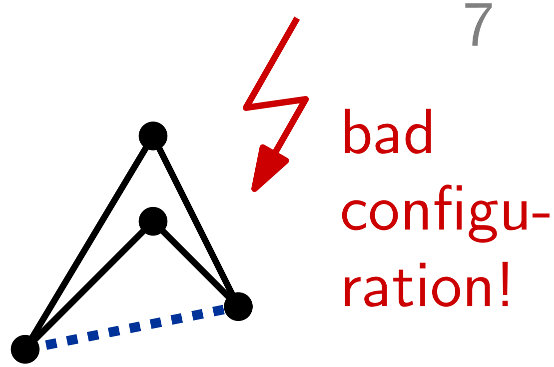

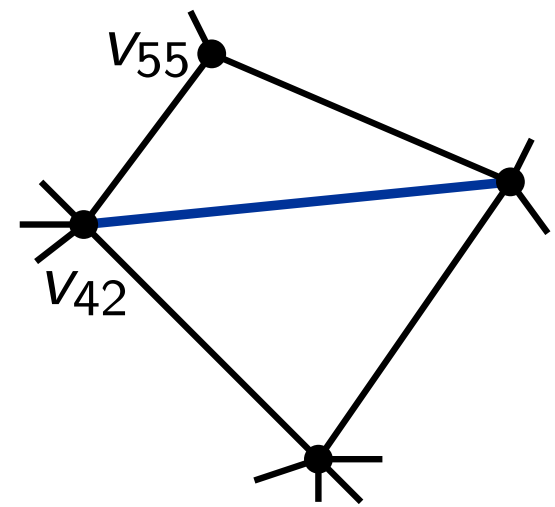

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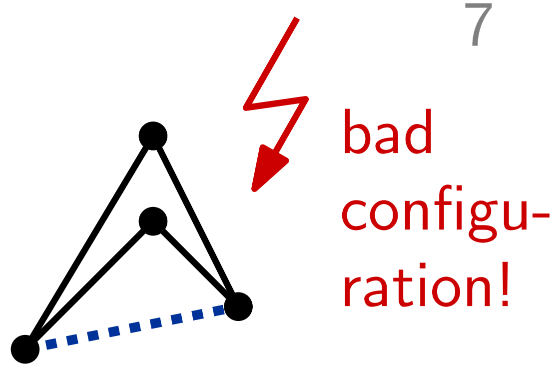

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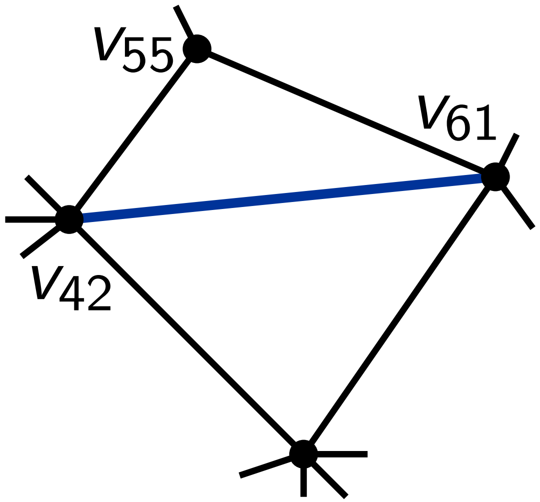

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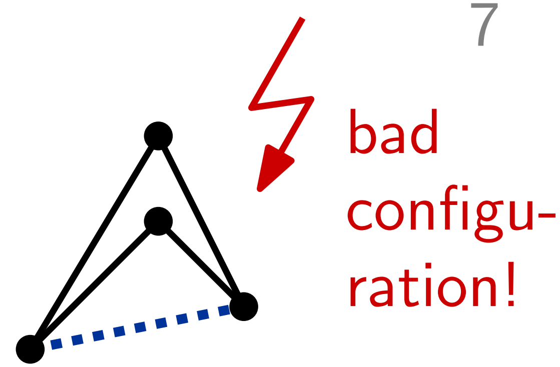

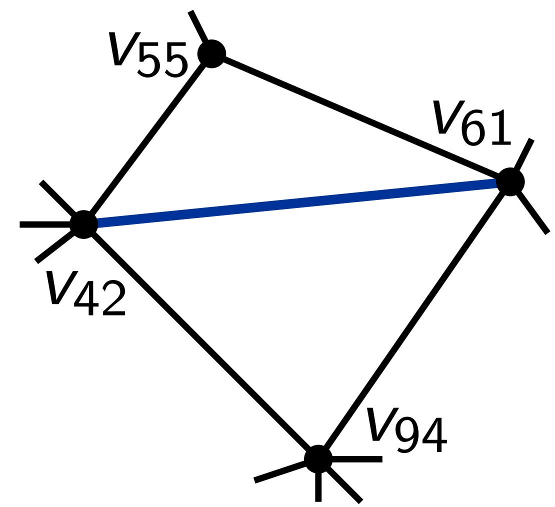

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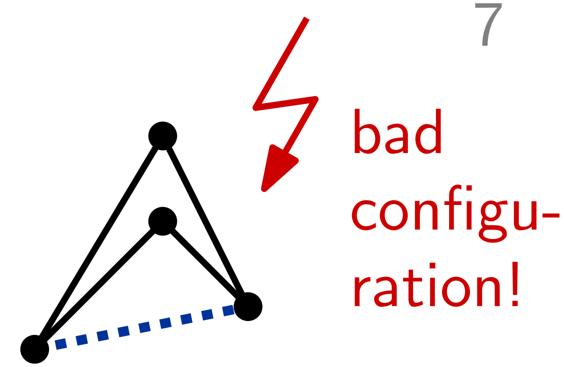

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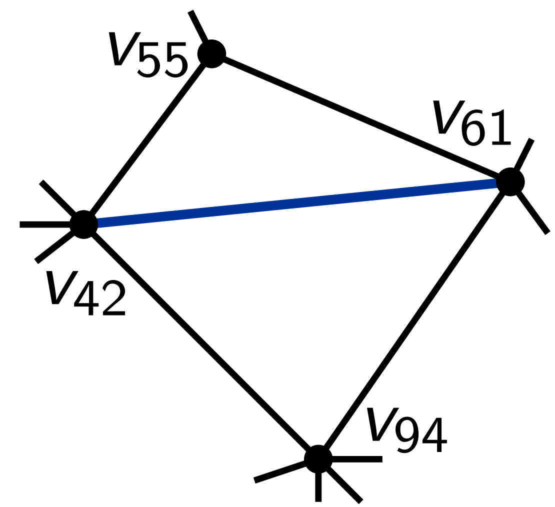

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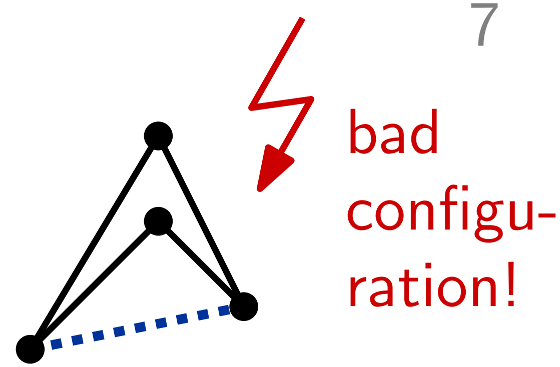

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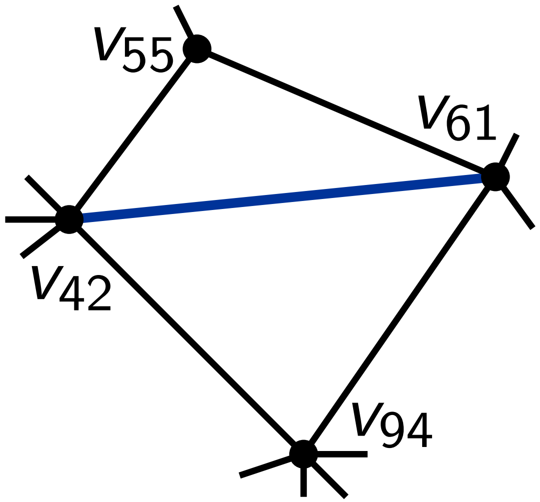

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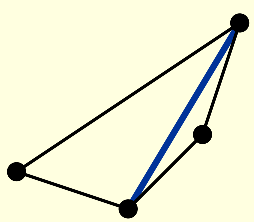

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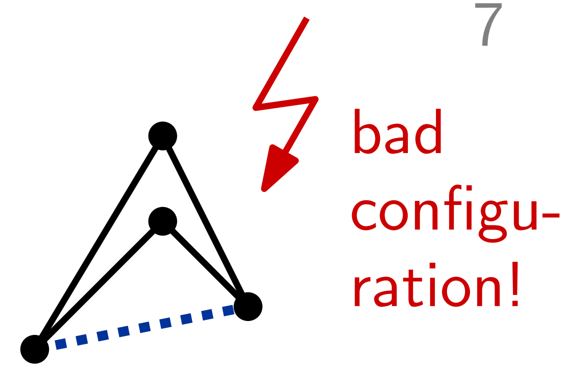

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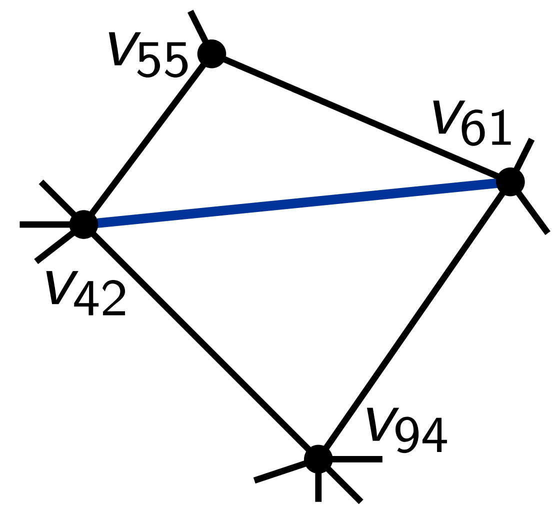

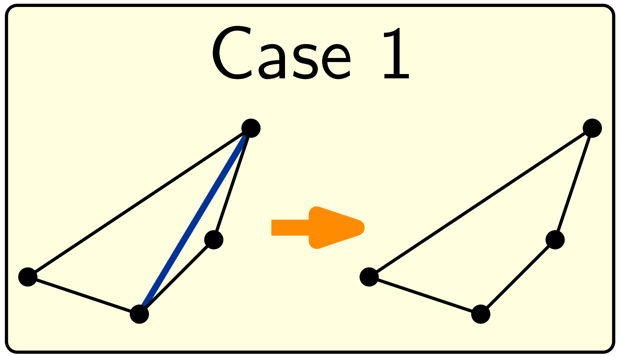

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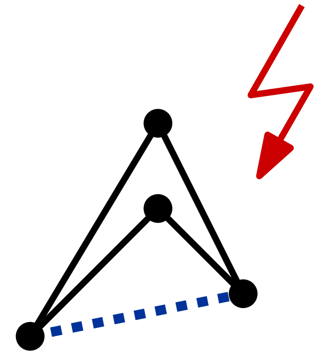

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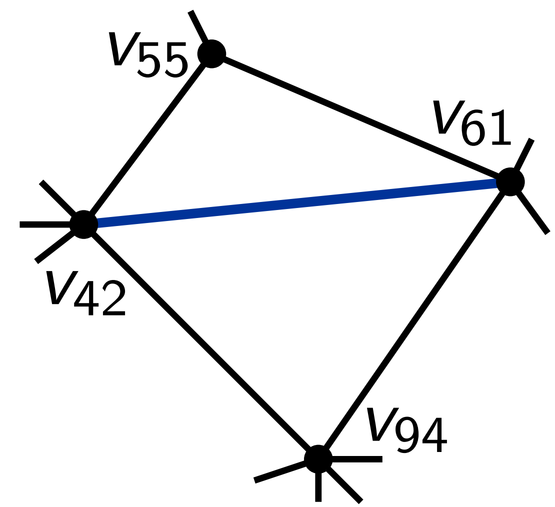

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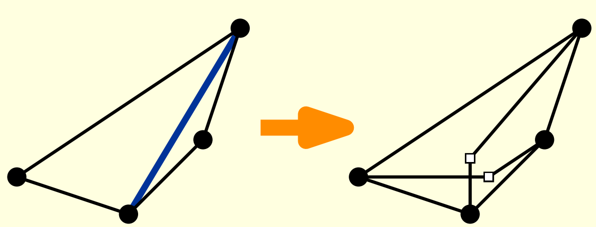

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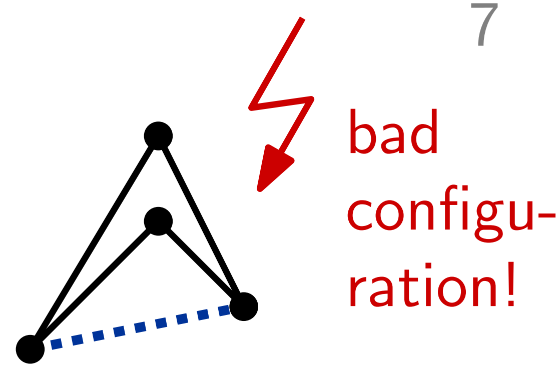

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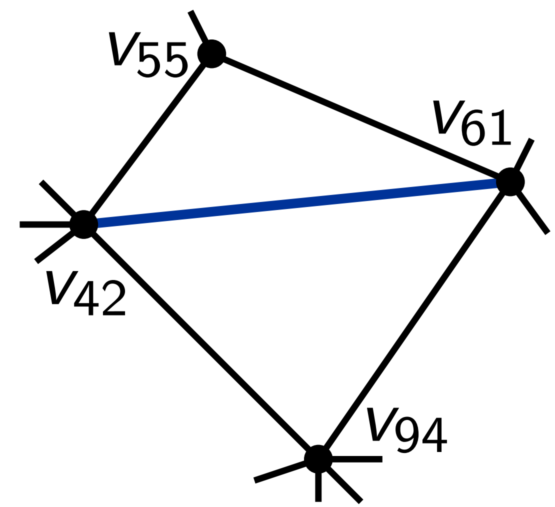

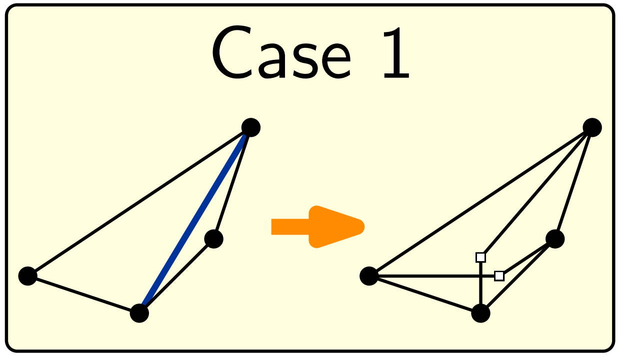

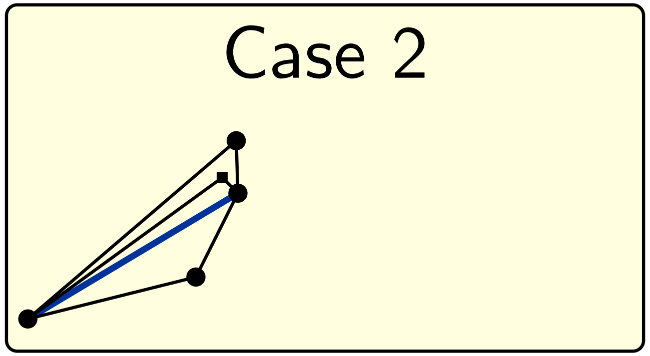

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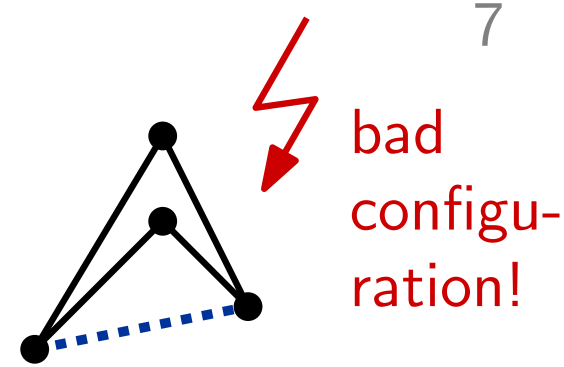

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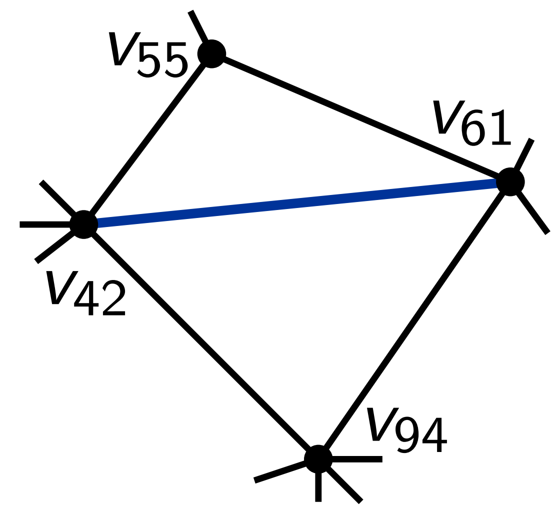

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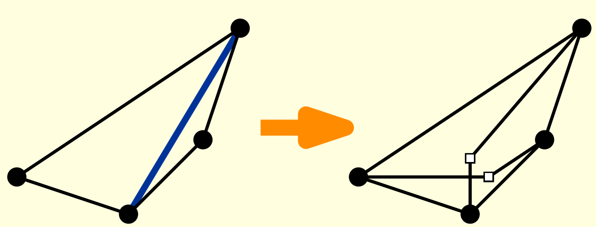

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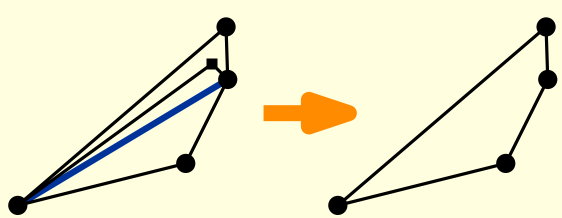

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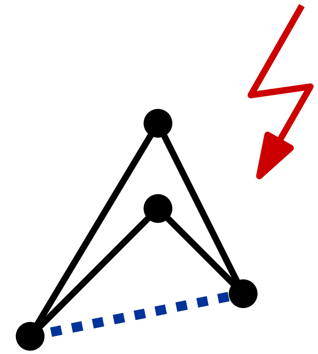

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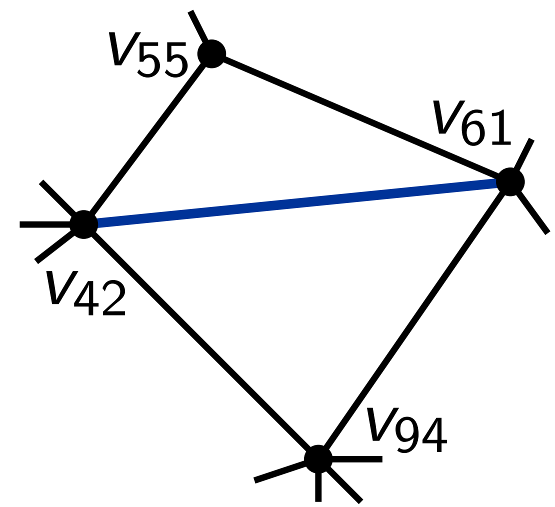

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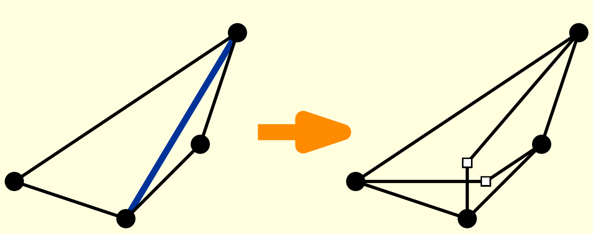

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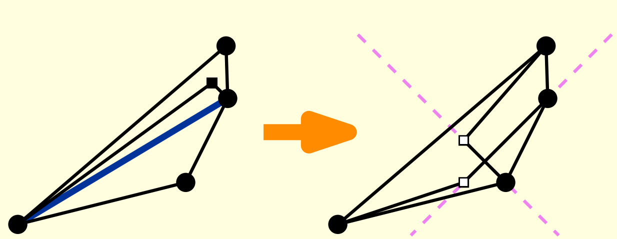

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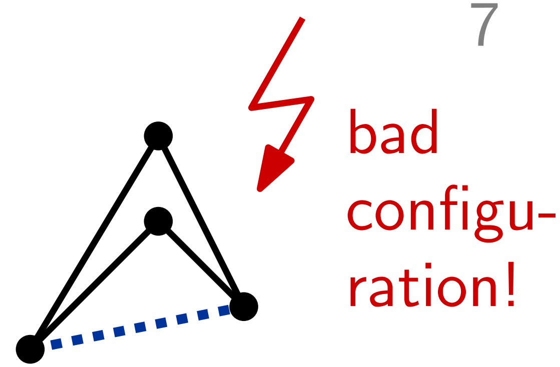

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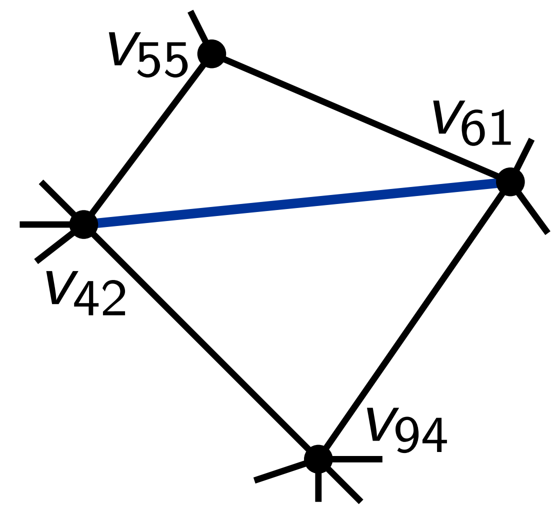

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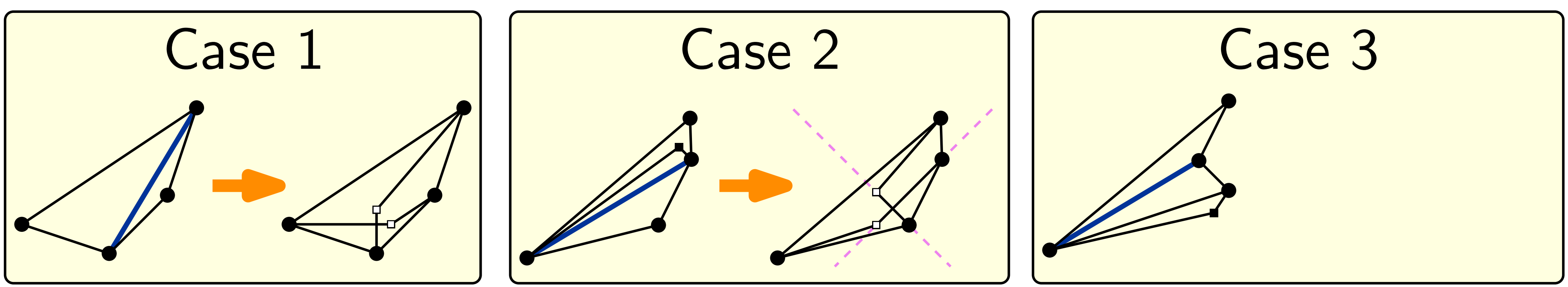

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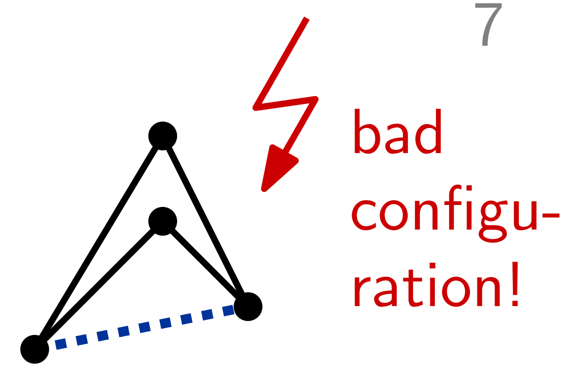

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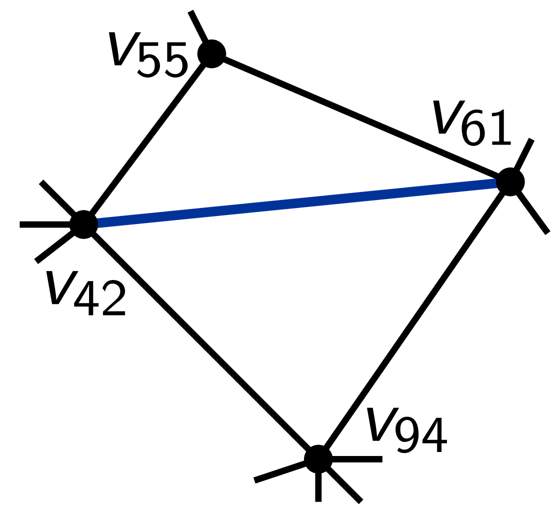

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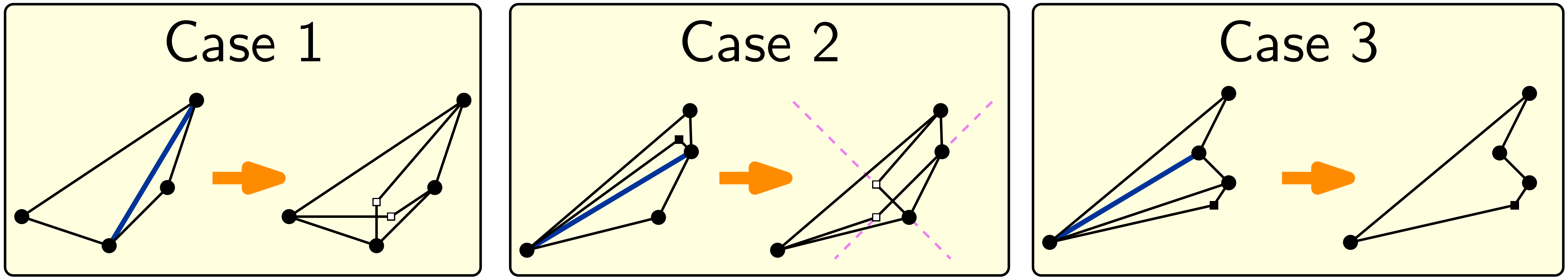

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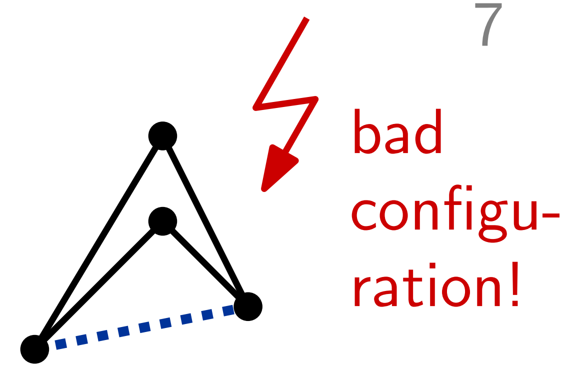

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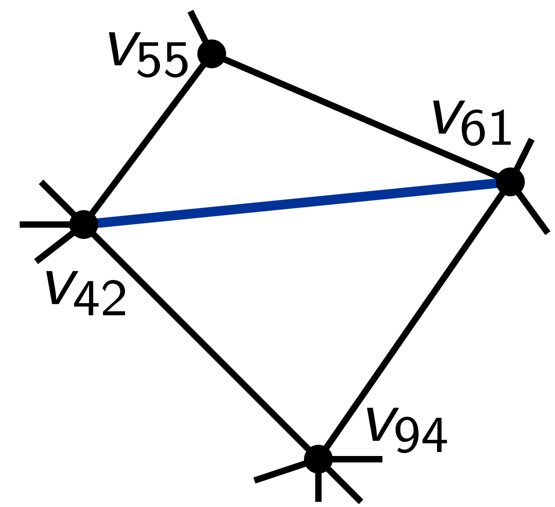

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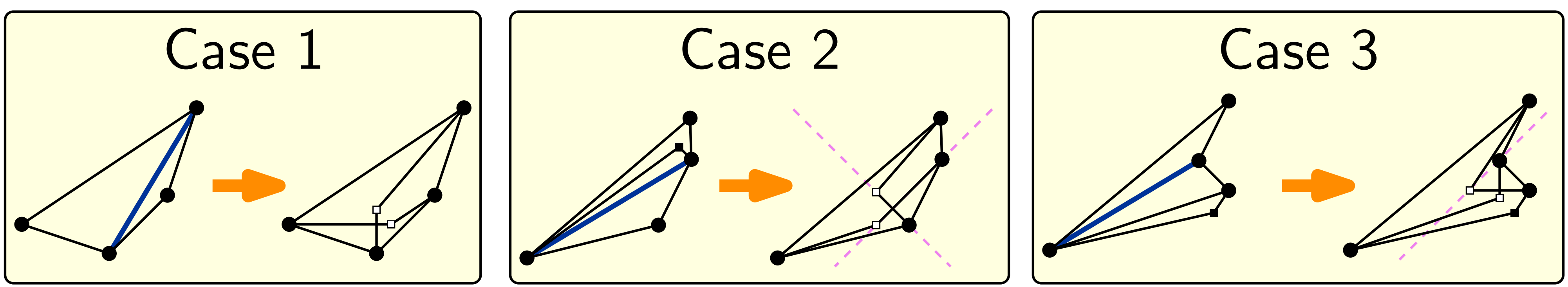

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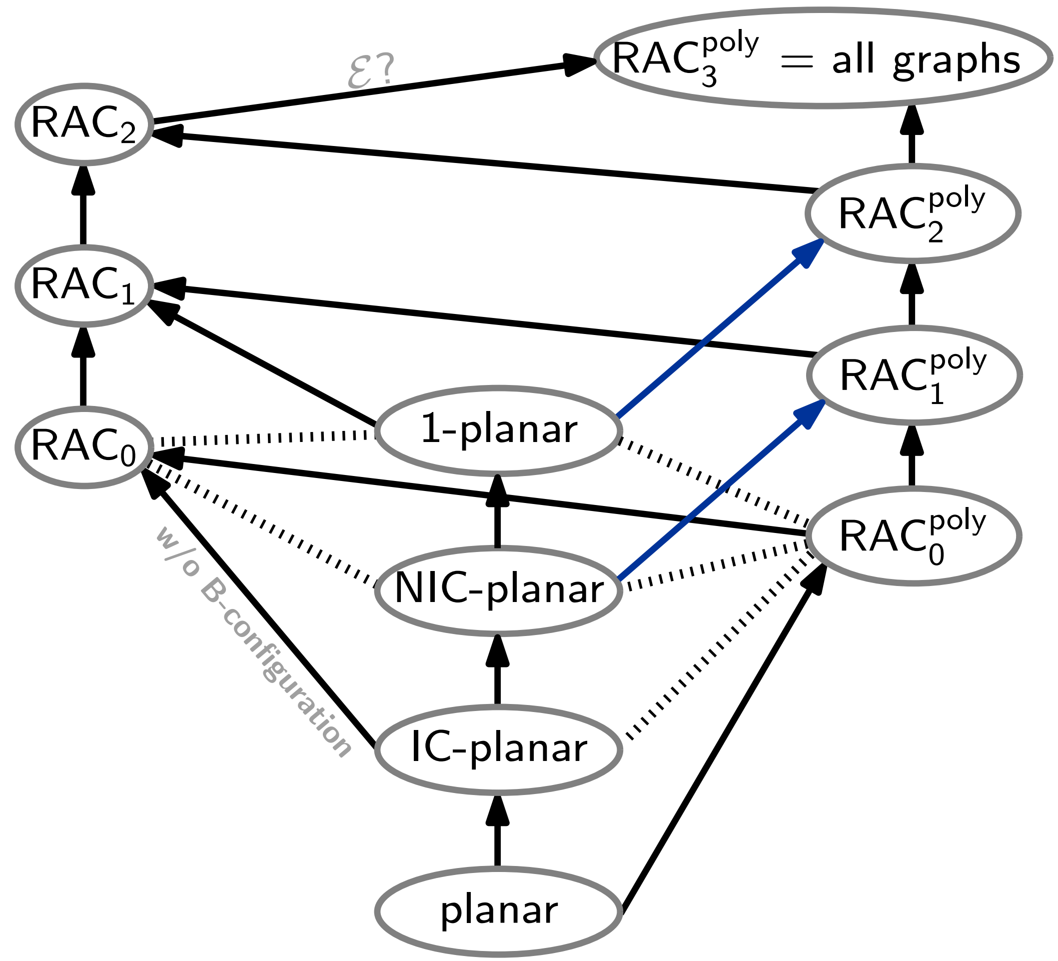

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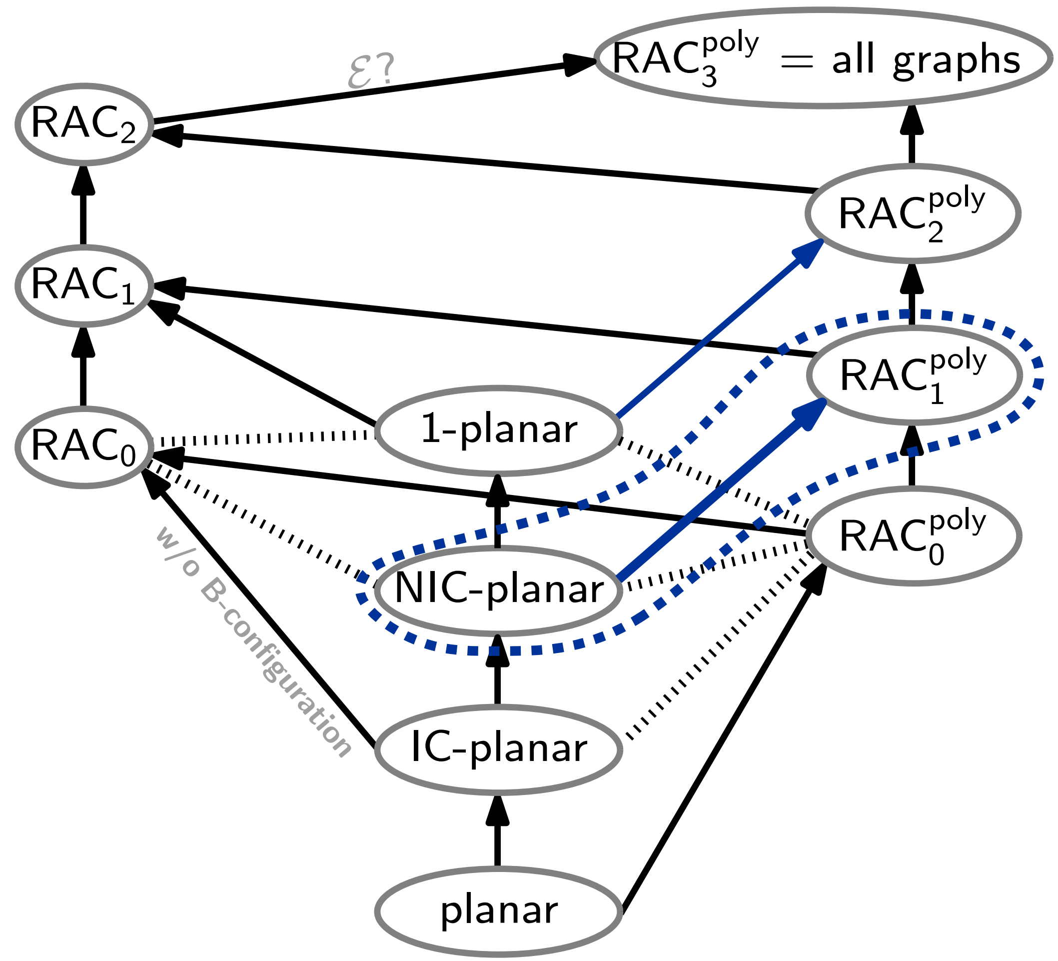
Our main result:
NIC-plane graphs $\subseteq$ RAC_1^{poly}



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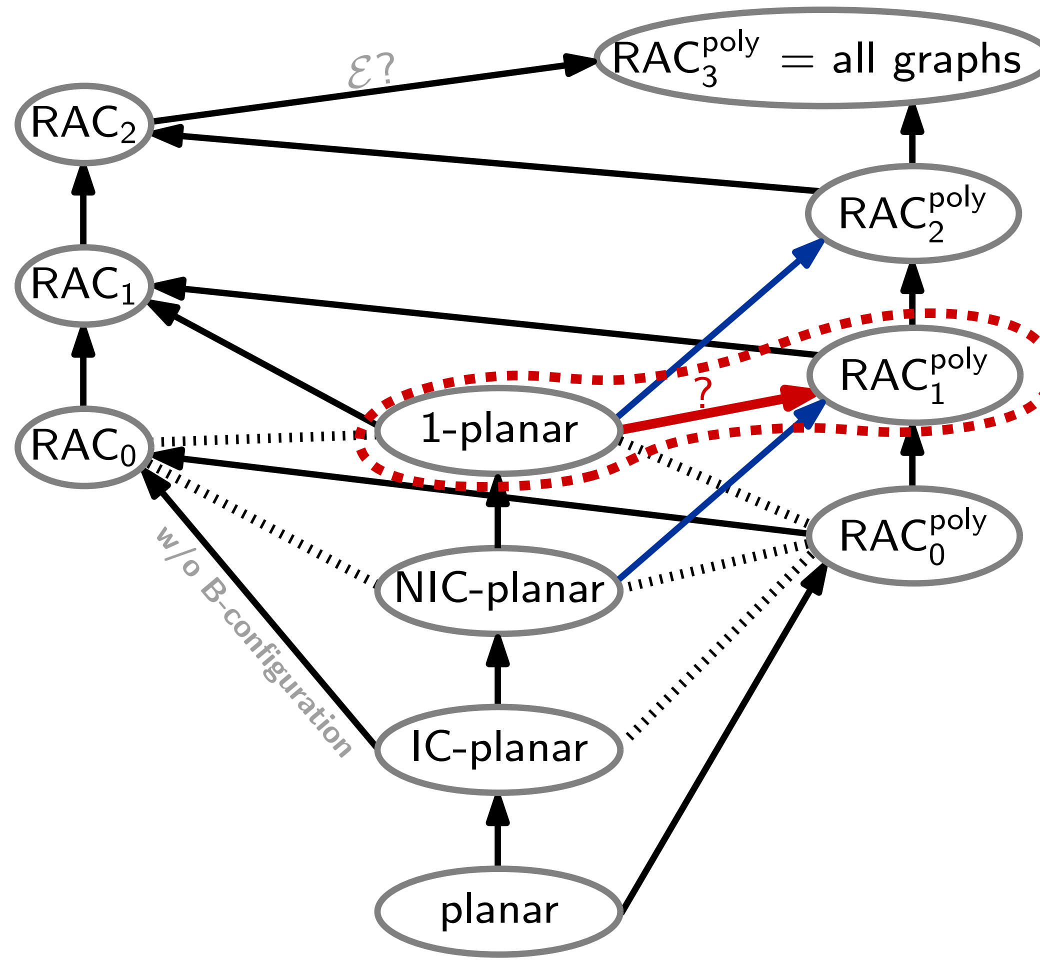

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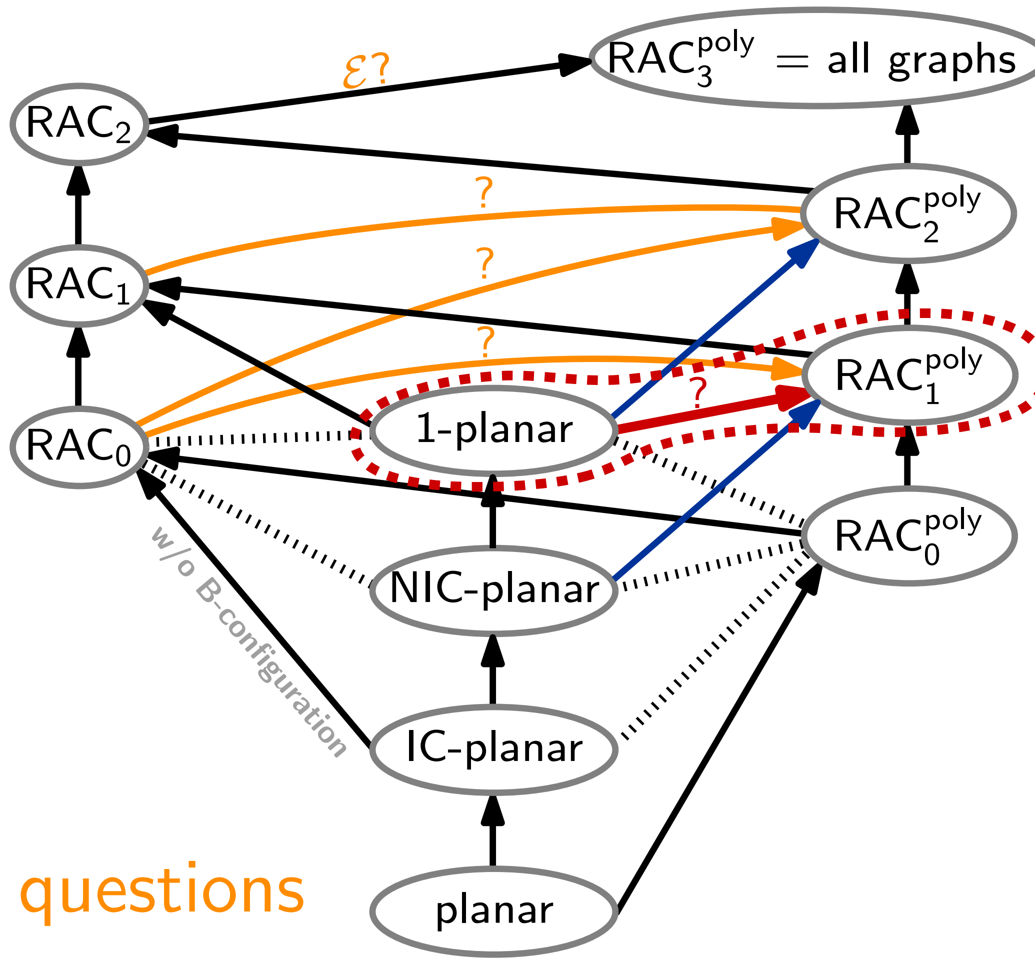

More open questions