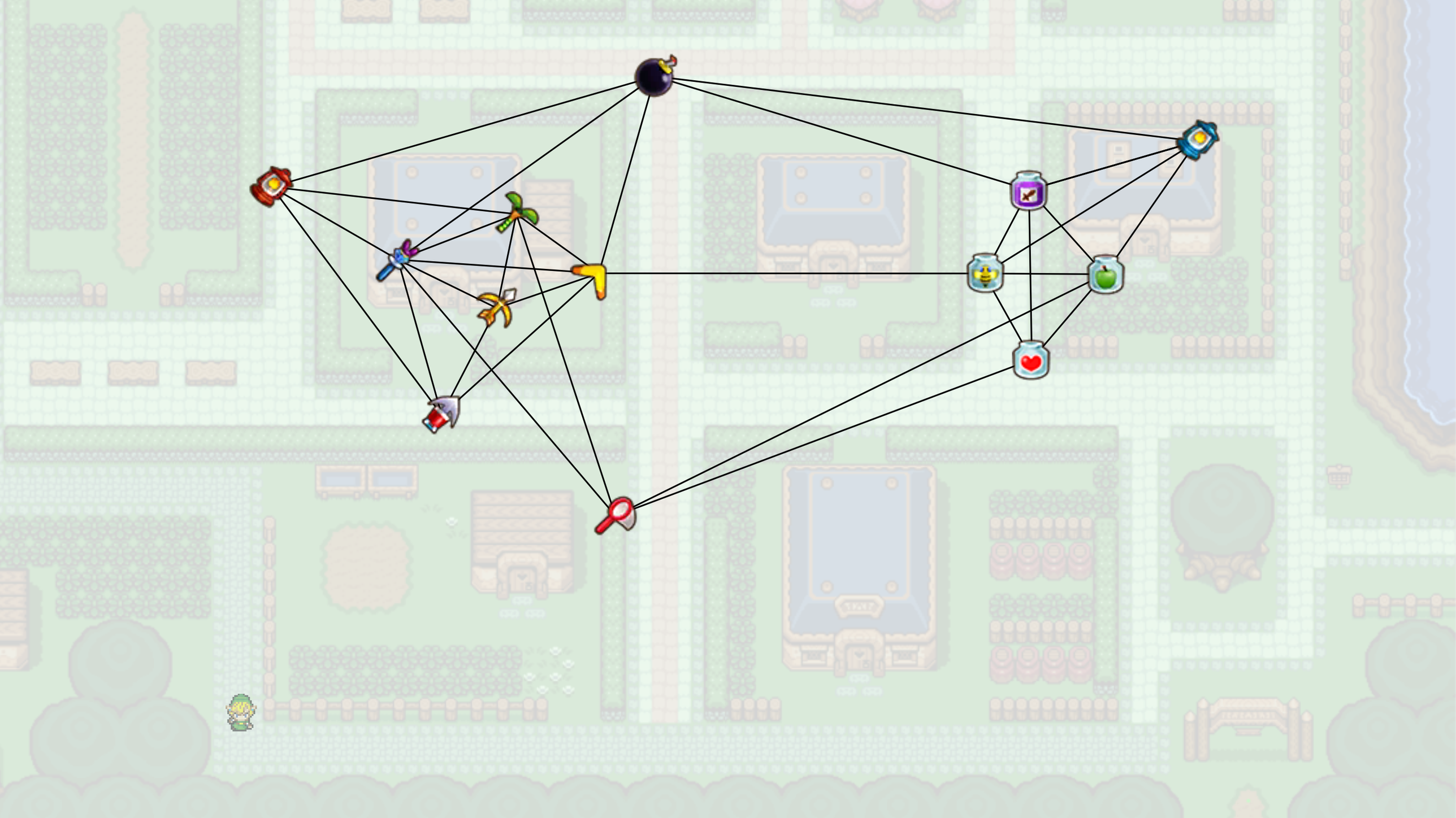
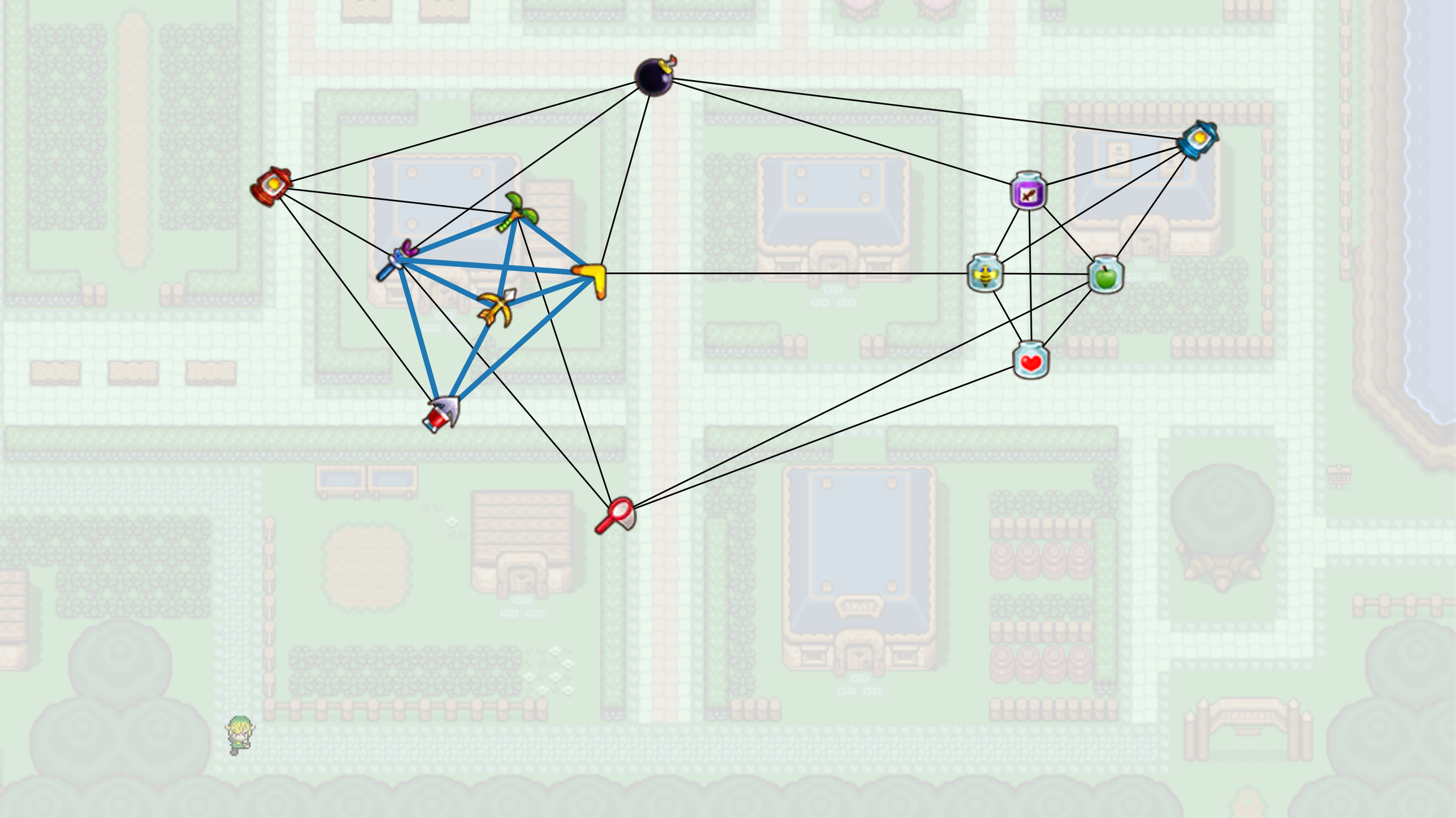
UNIVERSITÄT TRIER

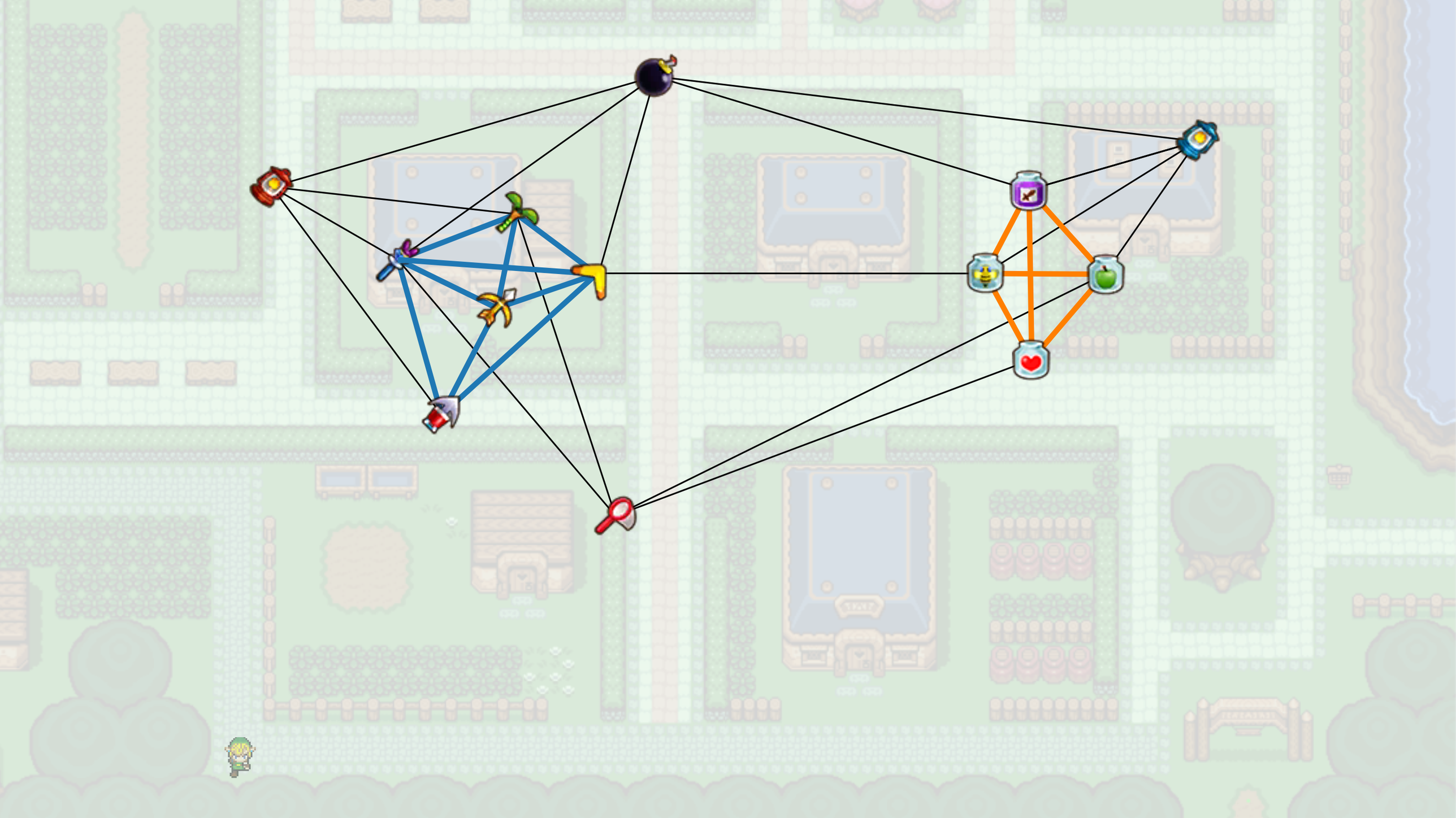
Julius-Maximilians-UNIVERSITÄT WÜRZBURG

The Computational Complexity of the ChordLink Model

Philipp Kindermann
Jan Sauer Alexander Wolff

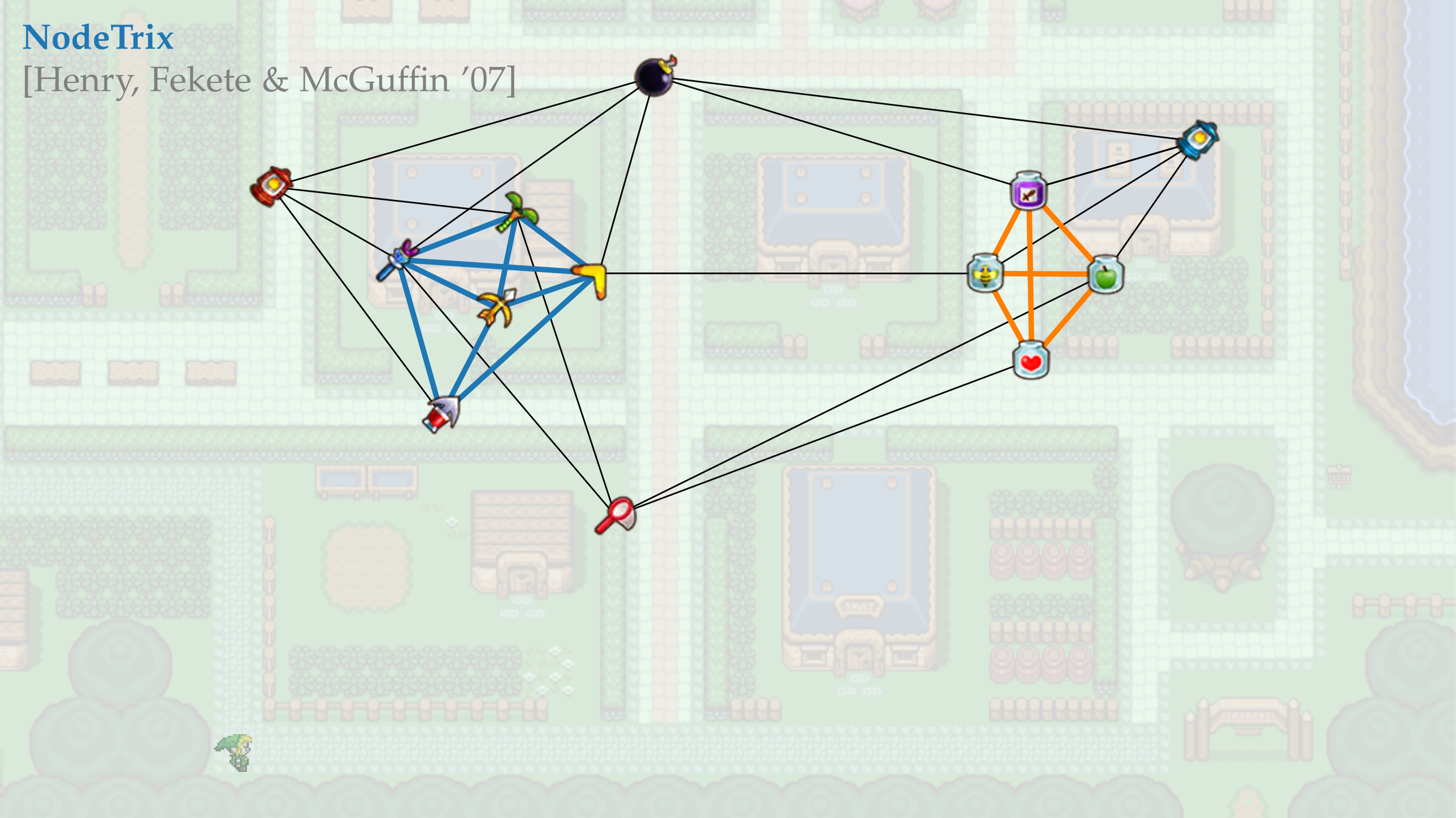






NodeTrix

[Henry, Fekete & McGuffin '07]



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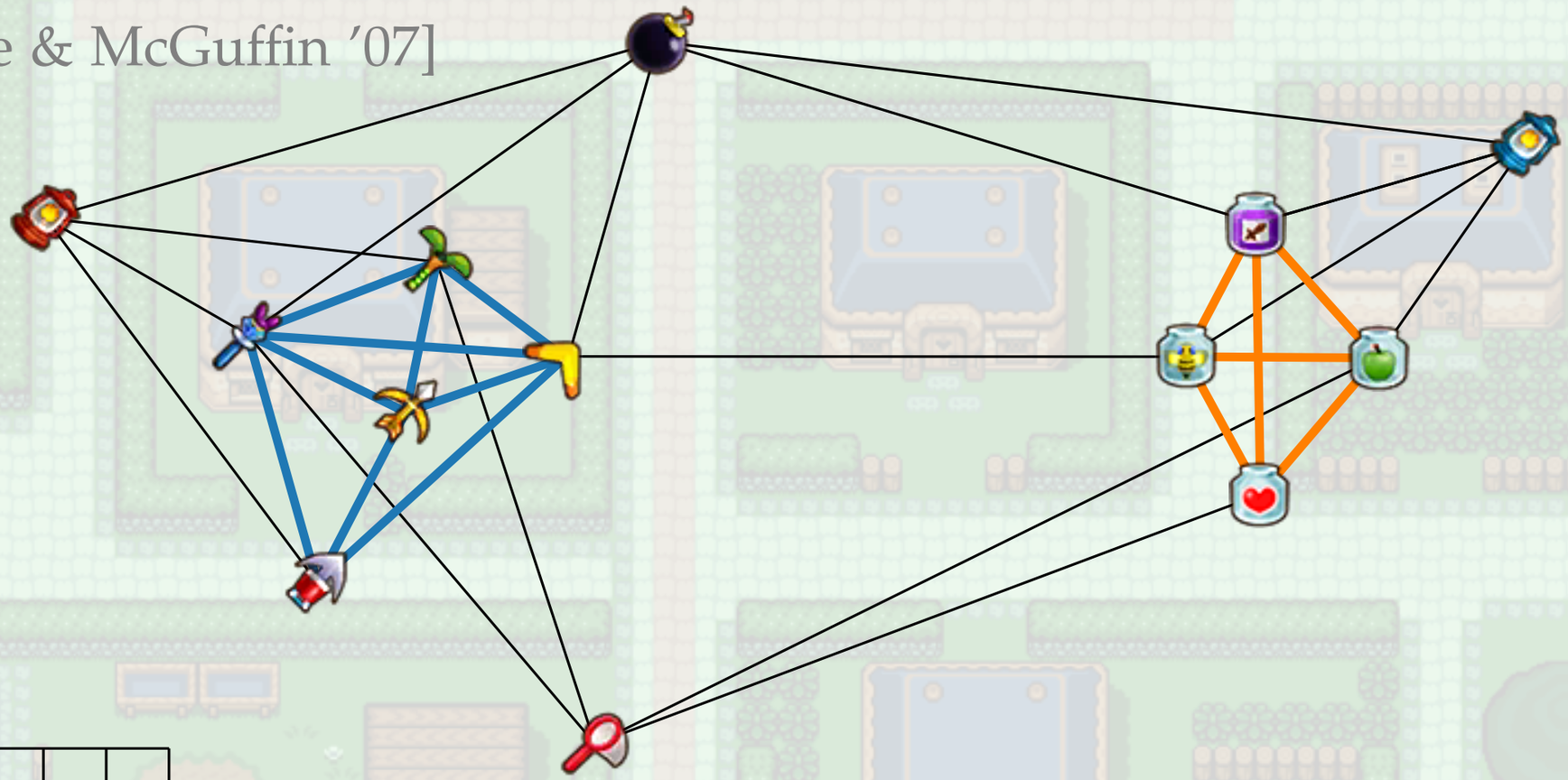

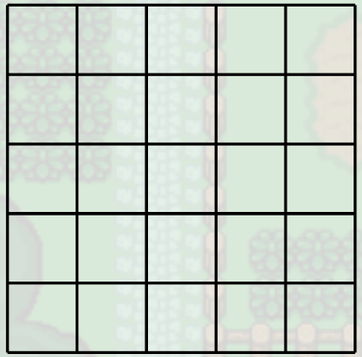

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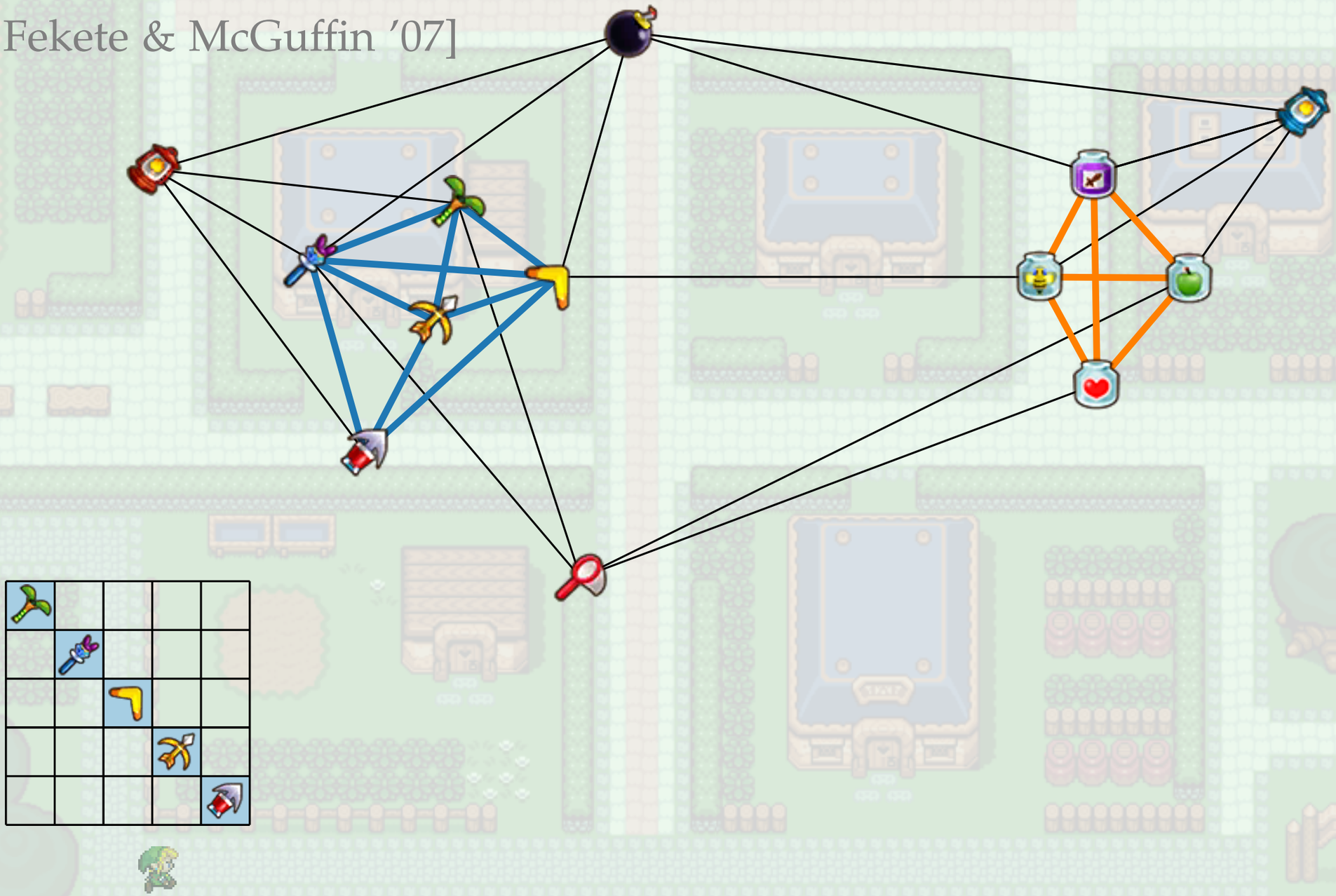

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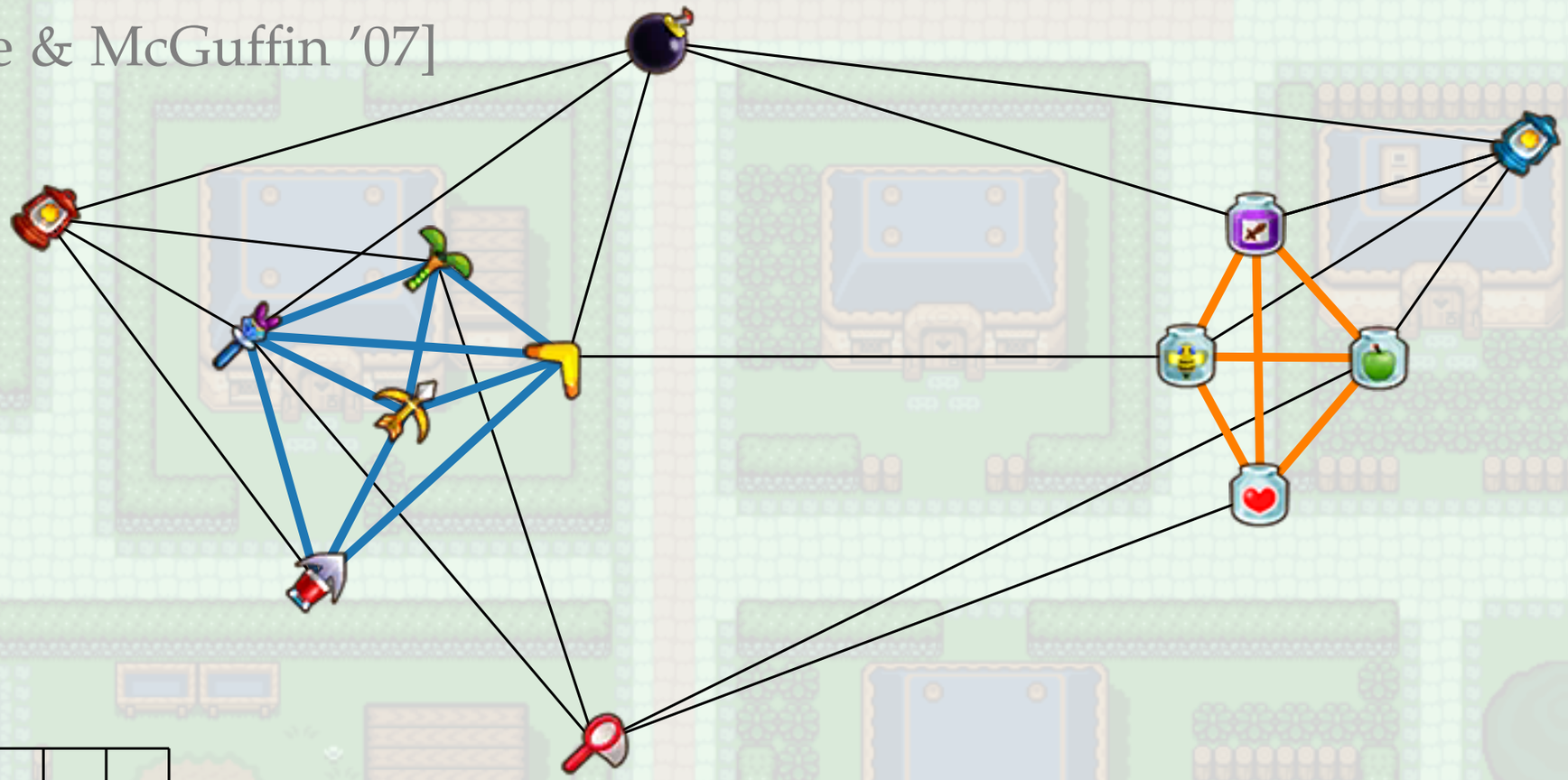

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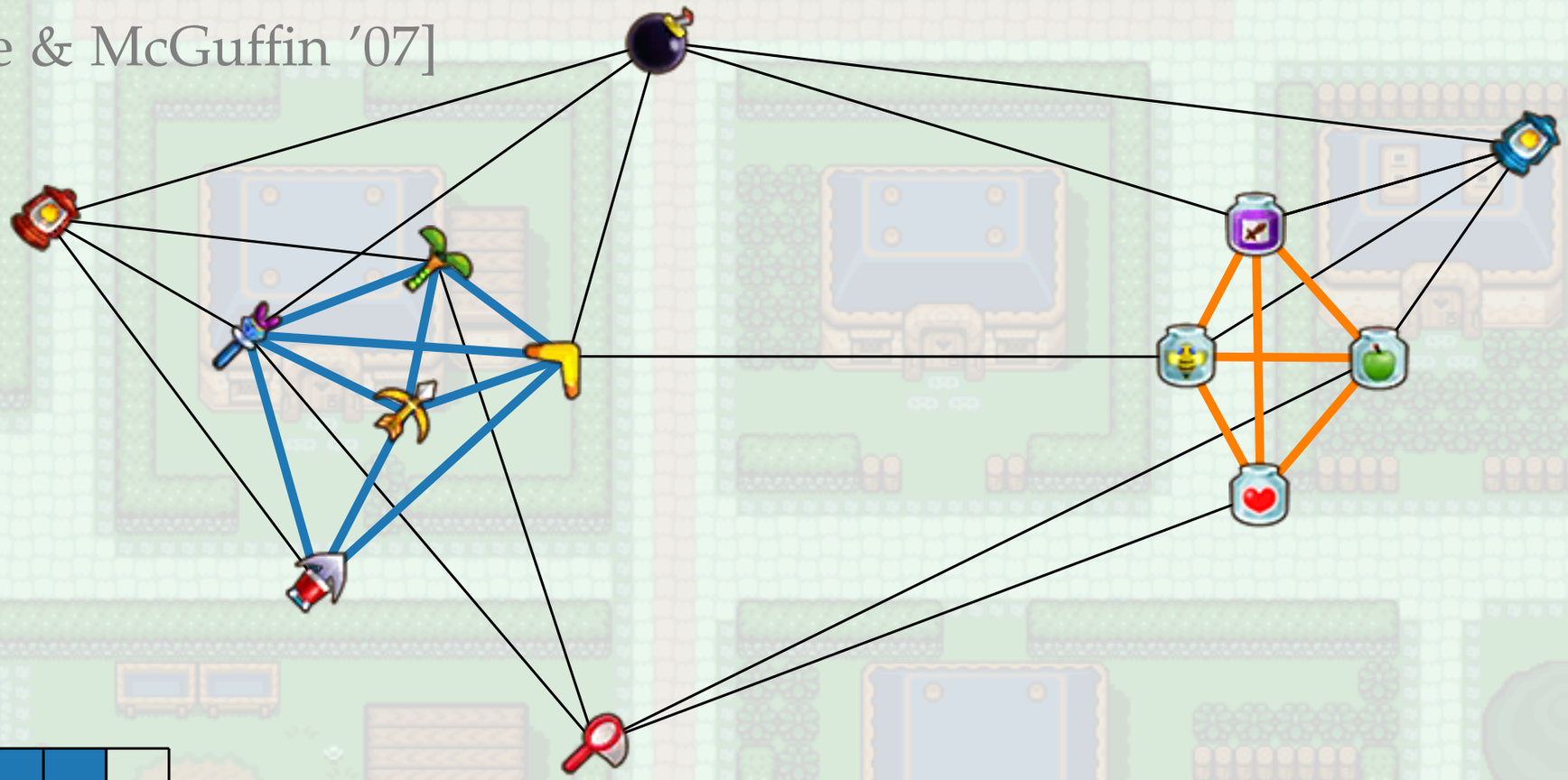

NodeTrix

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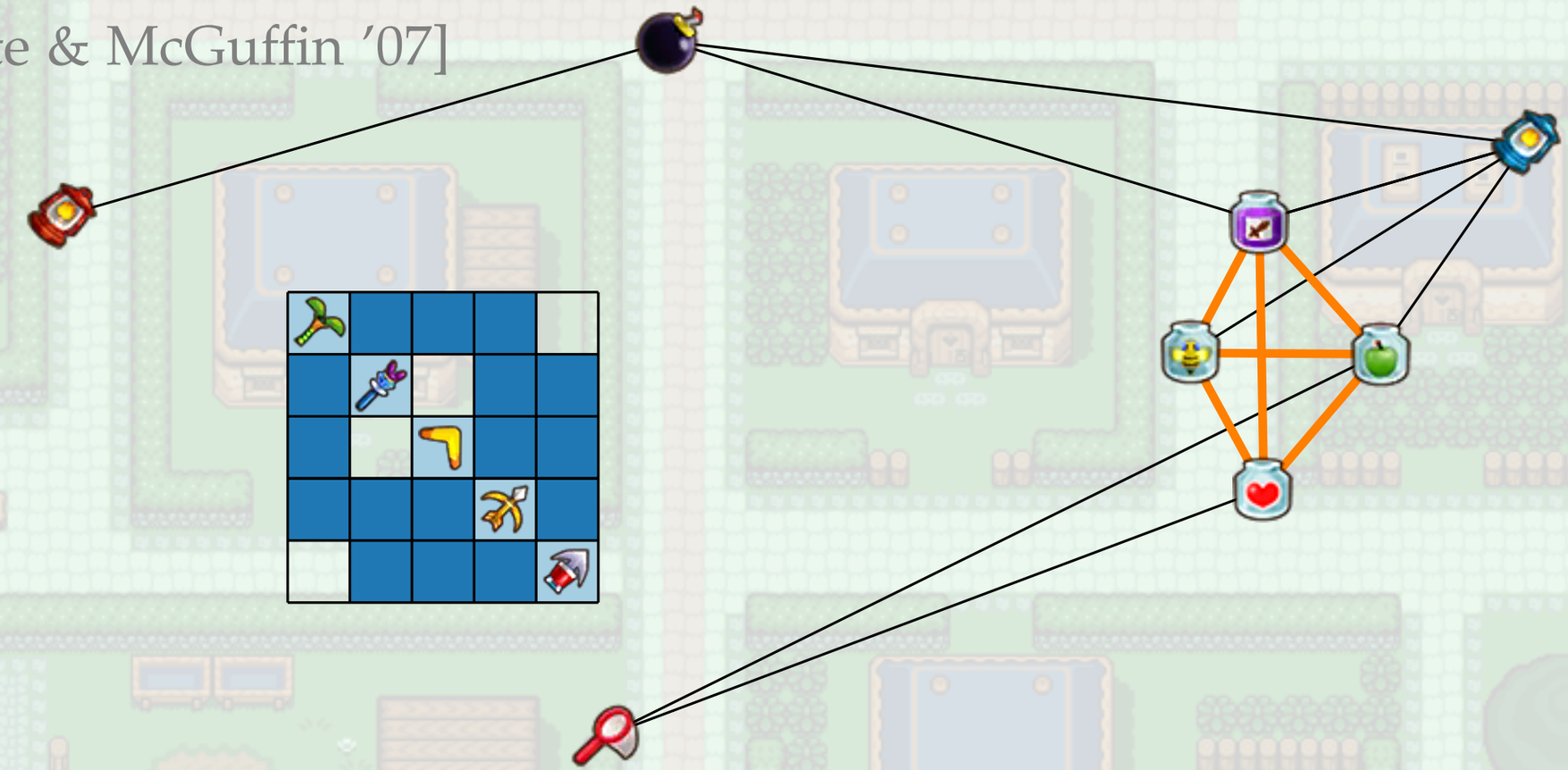

NodeTrix

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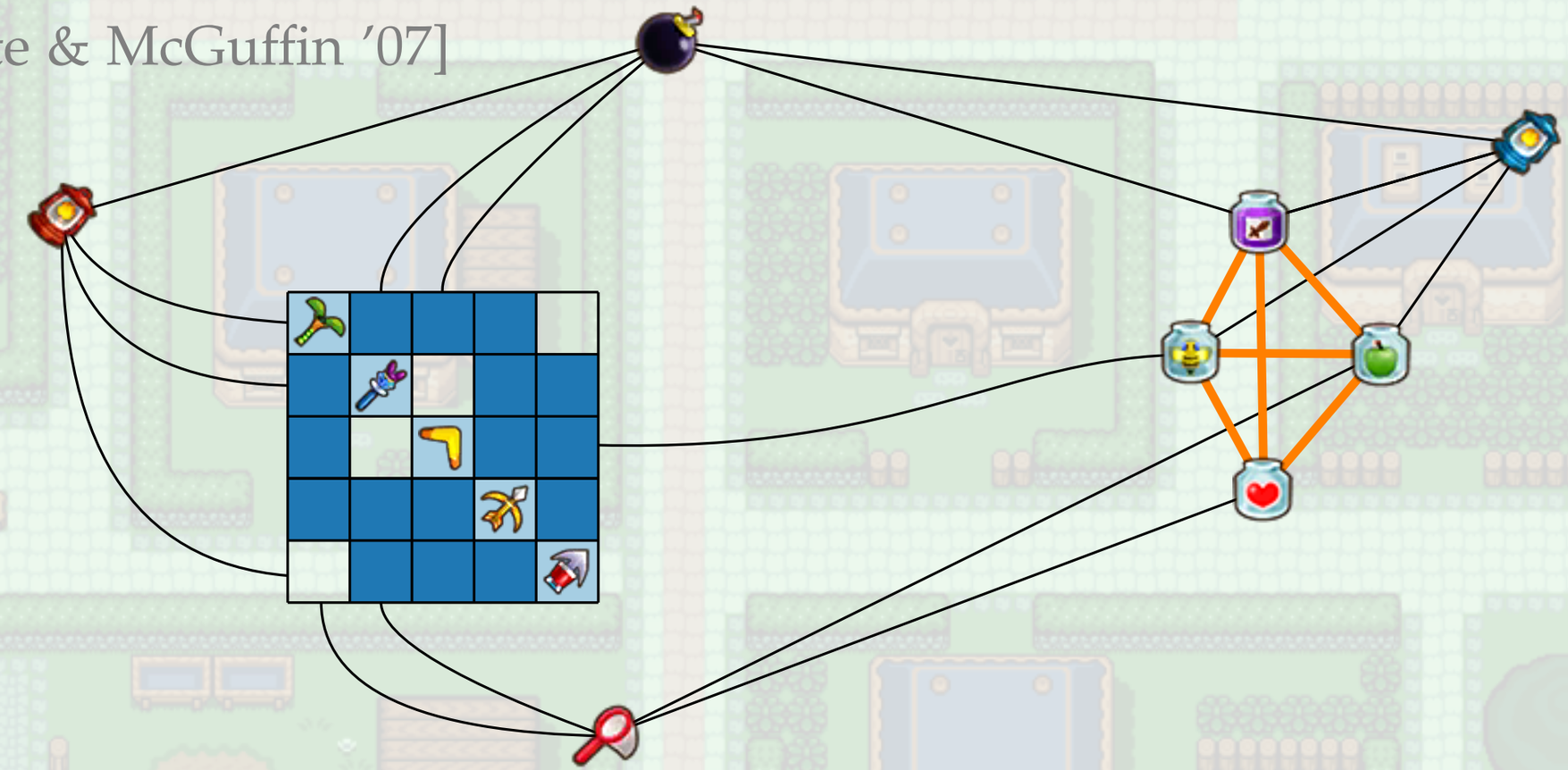

NodeTrix

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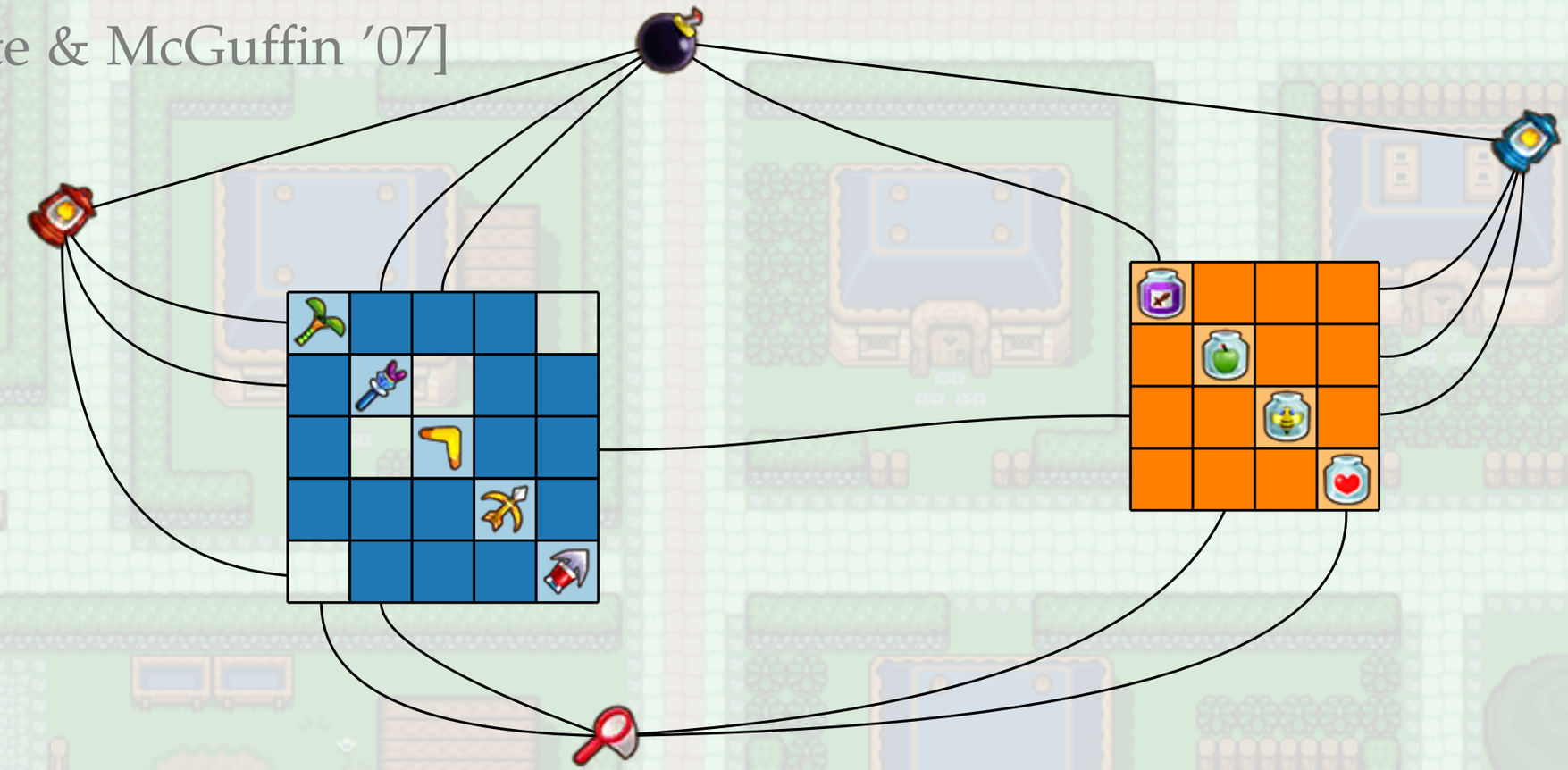
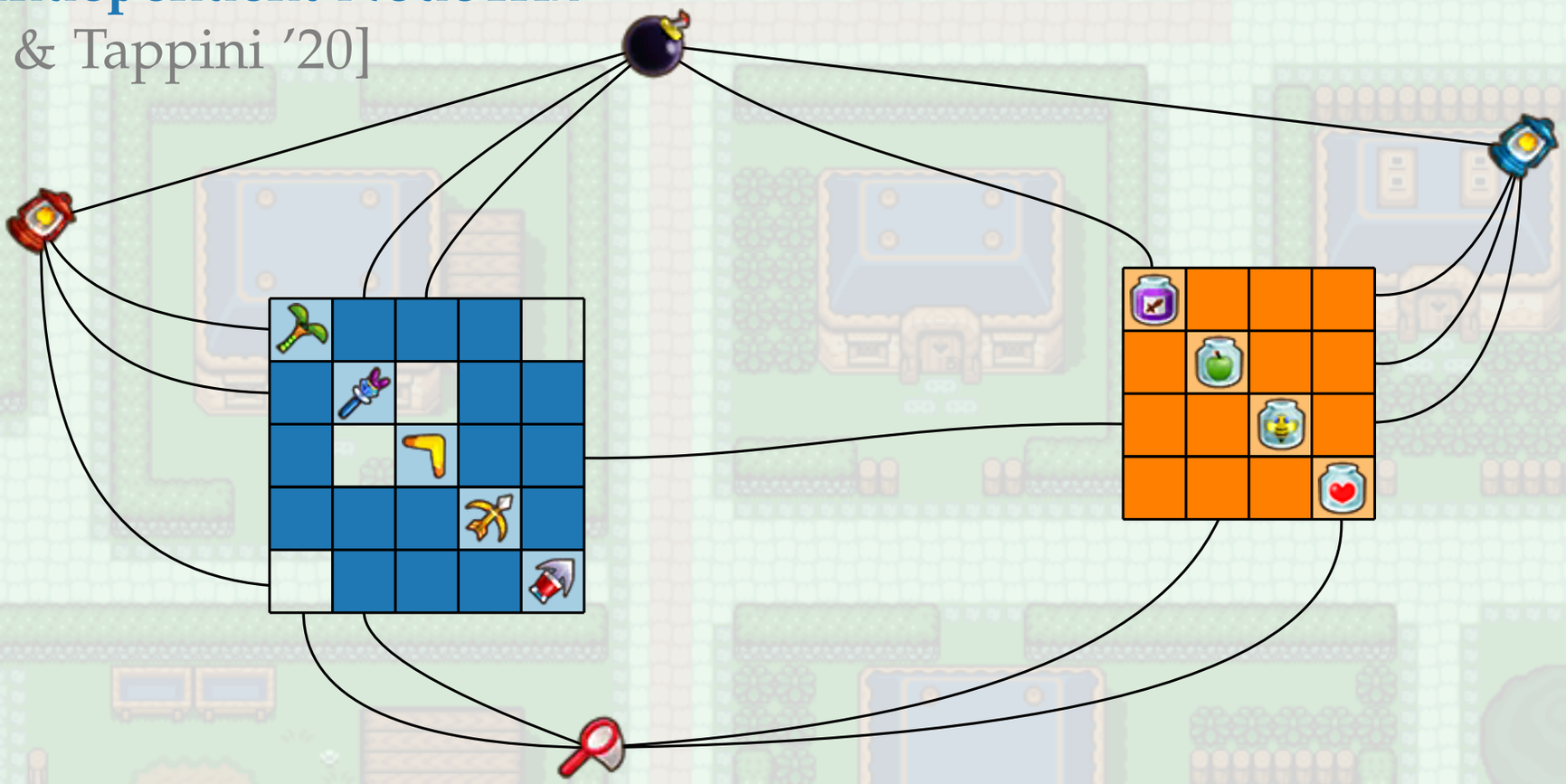


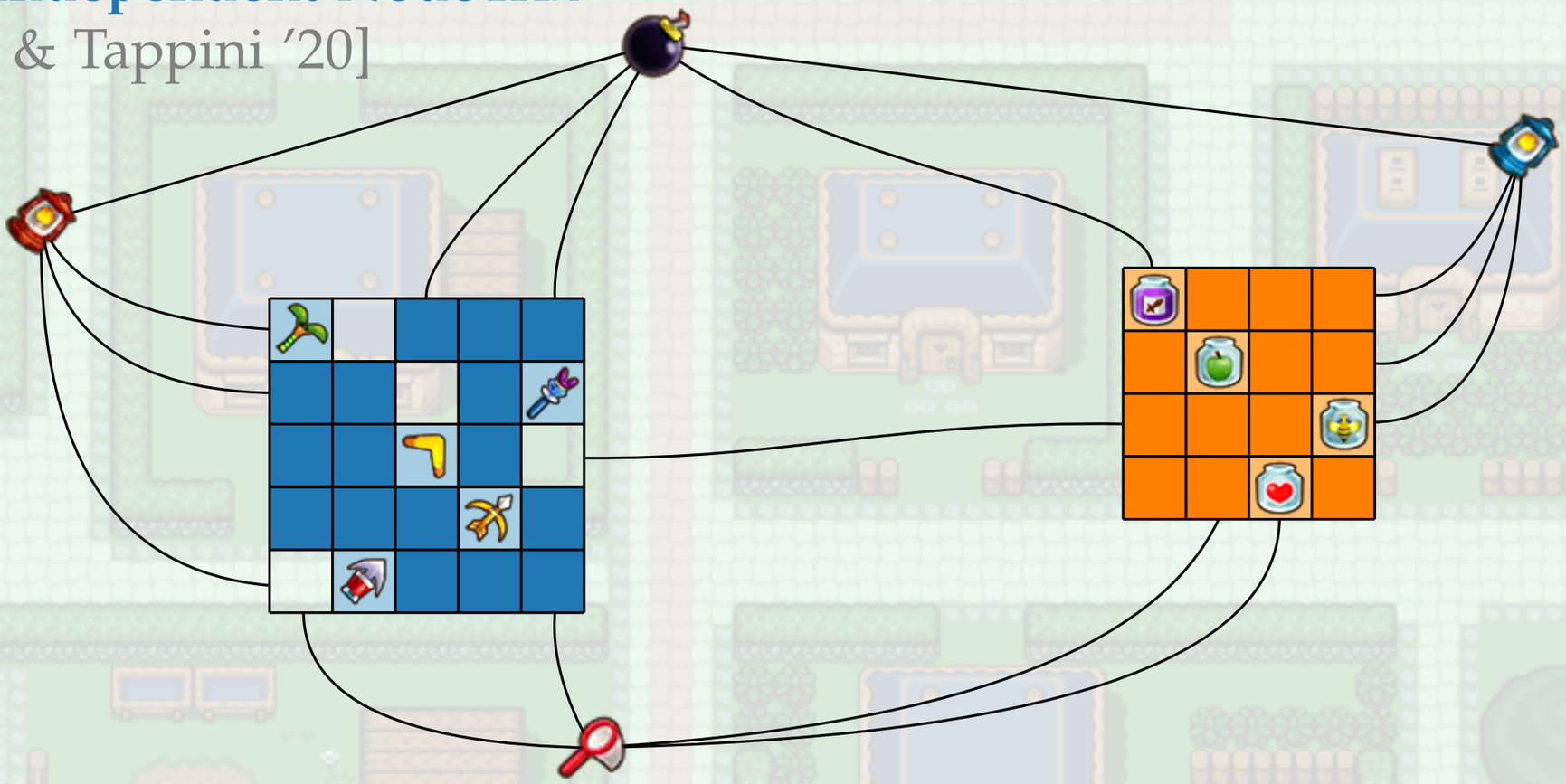
RowColumn Independent NodeTrix

[Liotta, Rutter & Tappini '20]



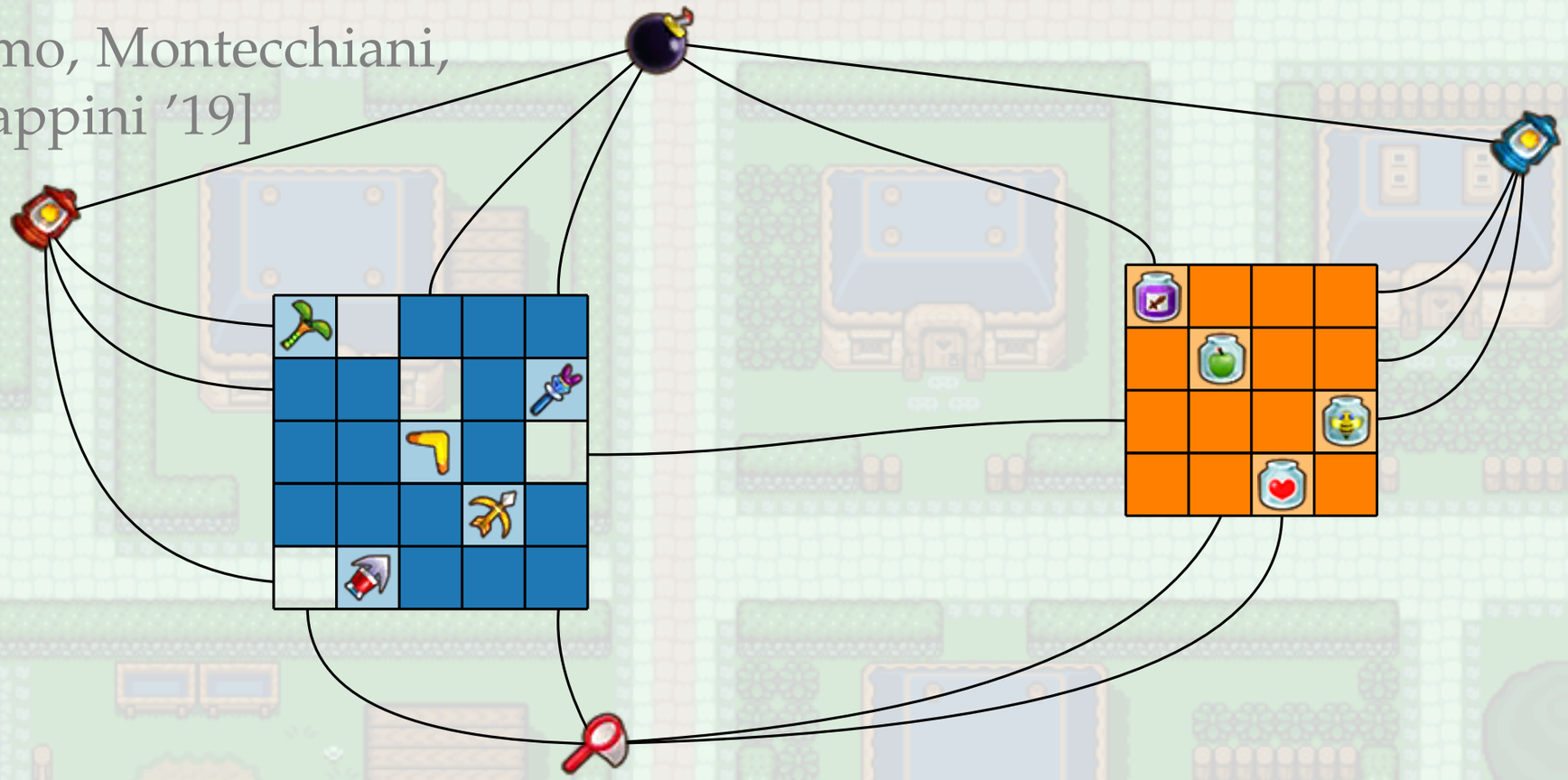
RowColumn Independent NodeTrix

[Liotta, Rutter & Tappini '20]



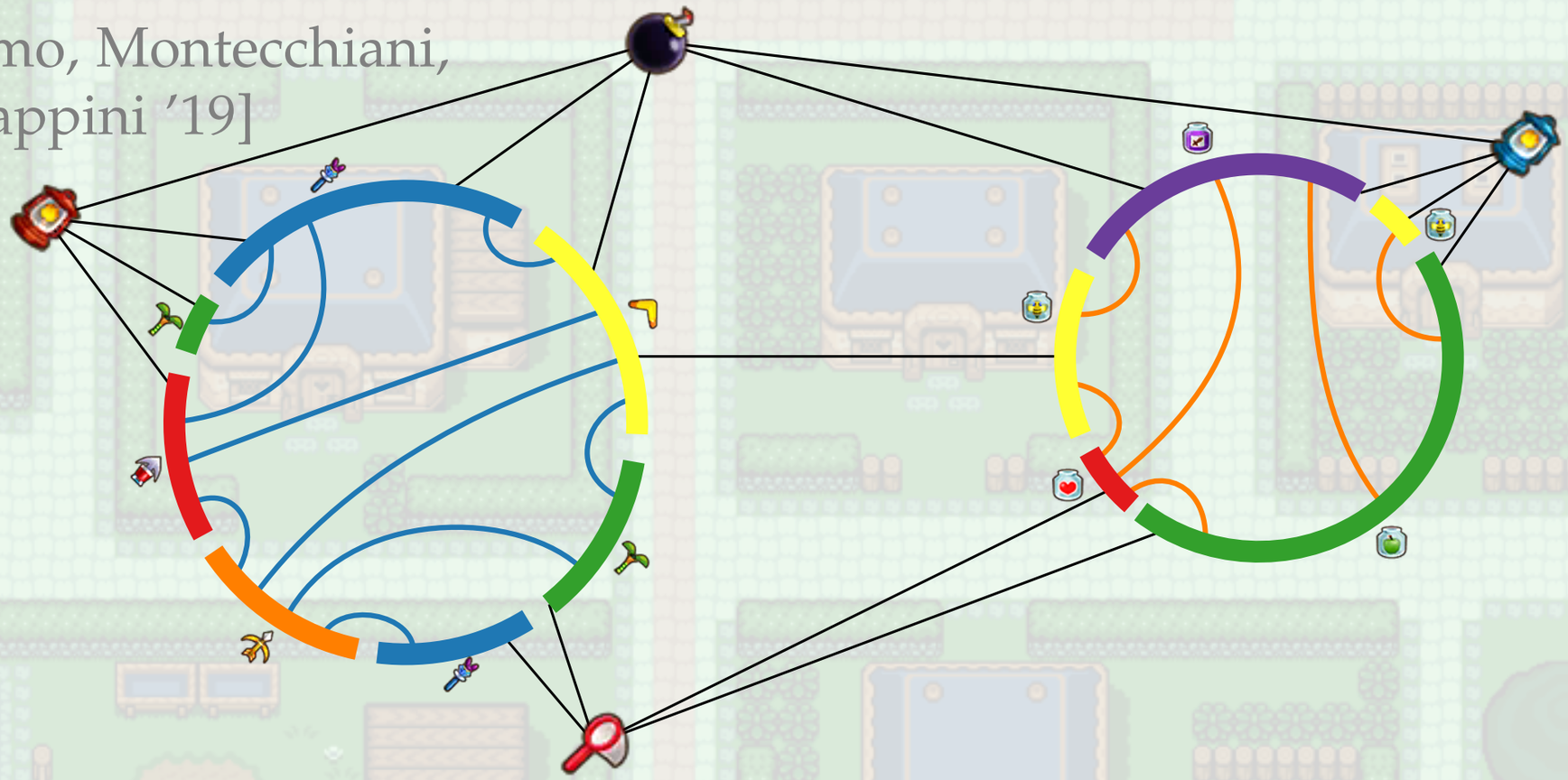
ChordLink

[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



ChordLink

[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



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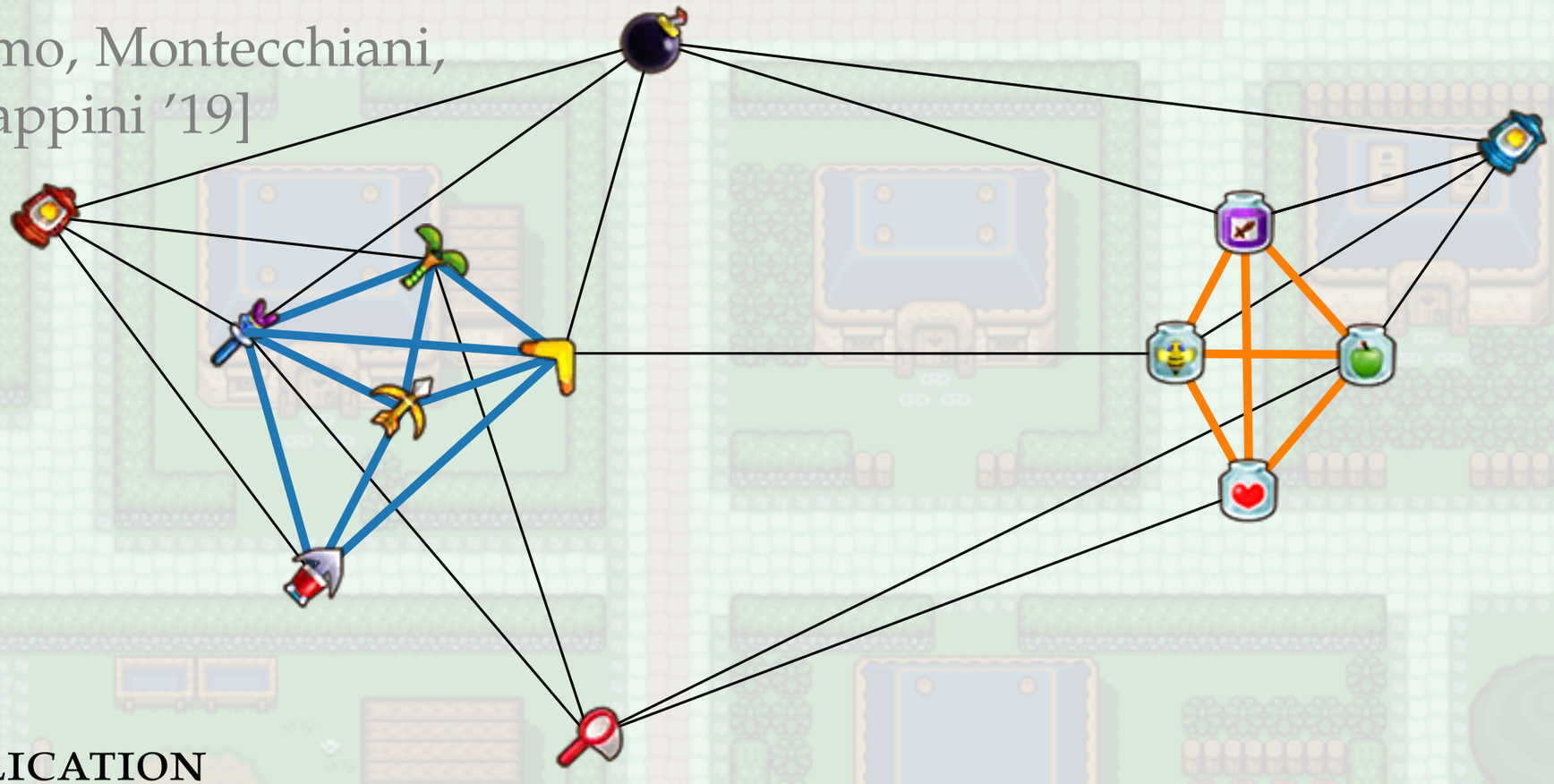
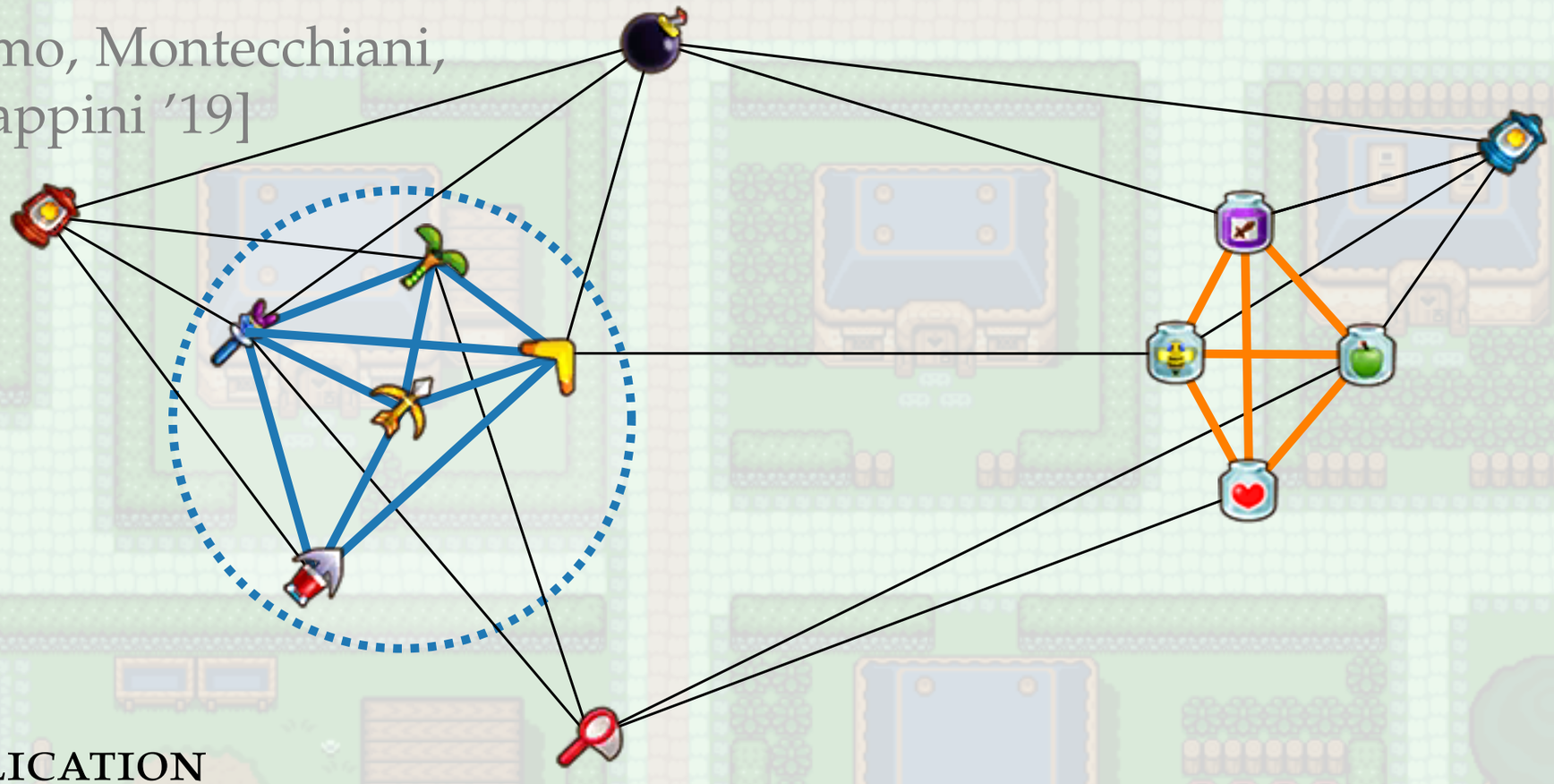
[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



1. NodeReplication

ChordLink

[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



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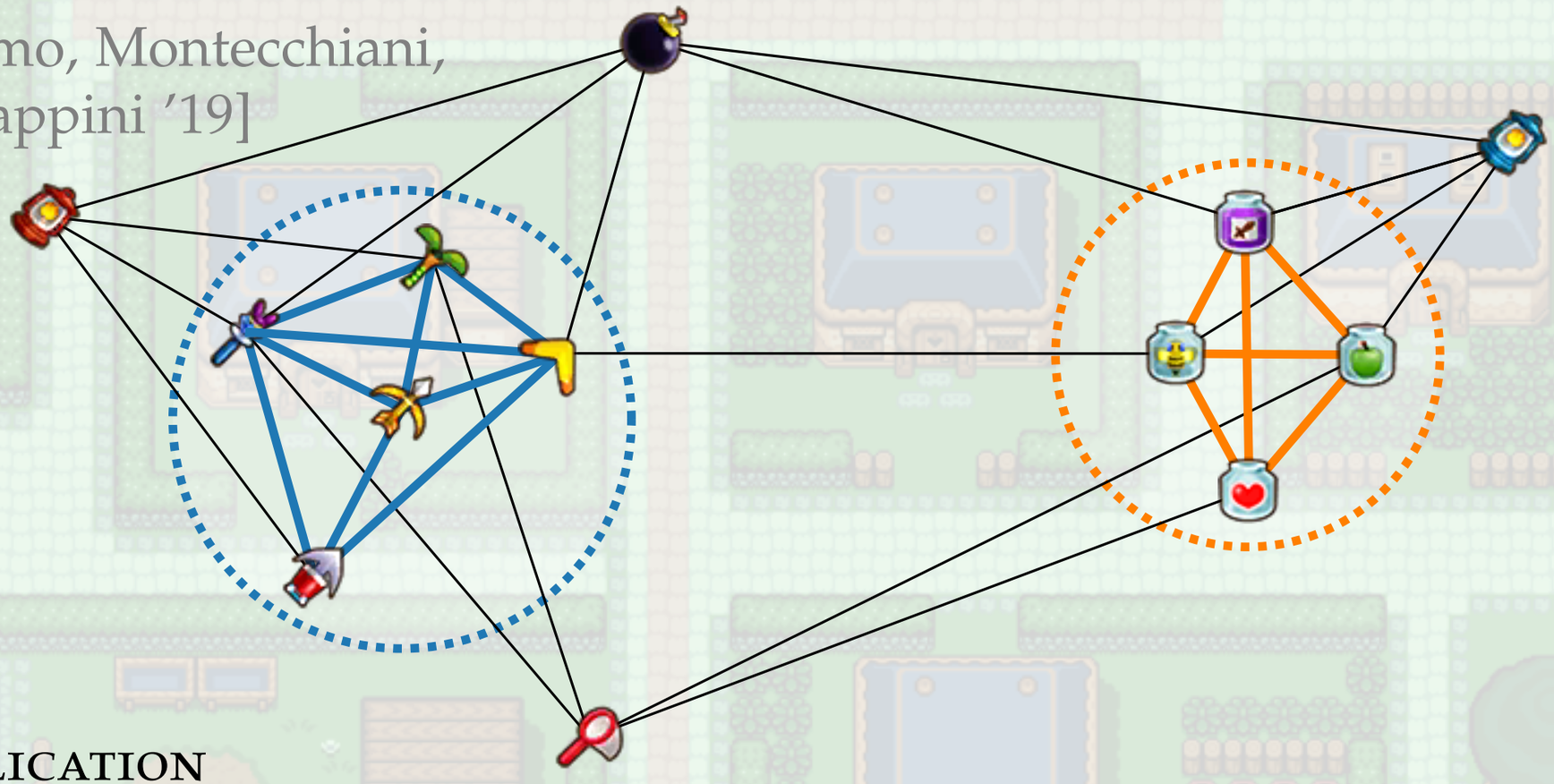

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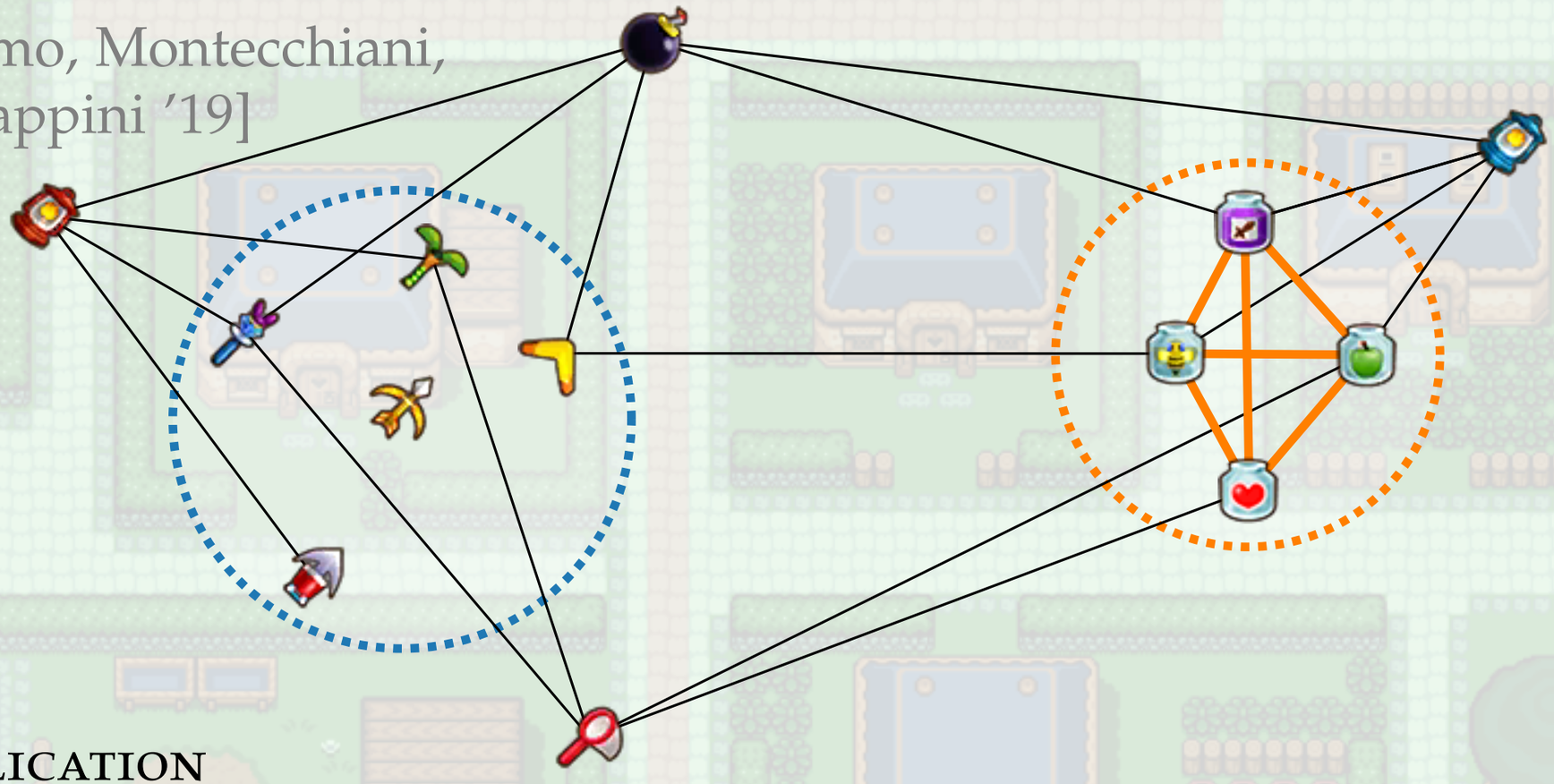

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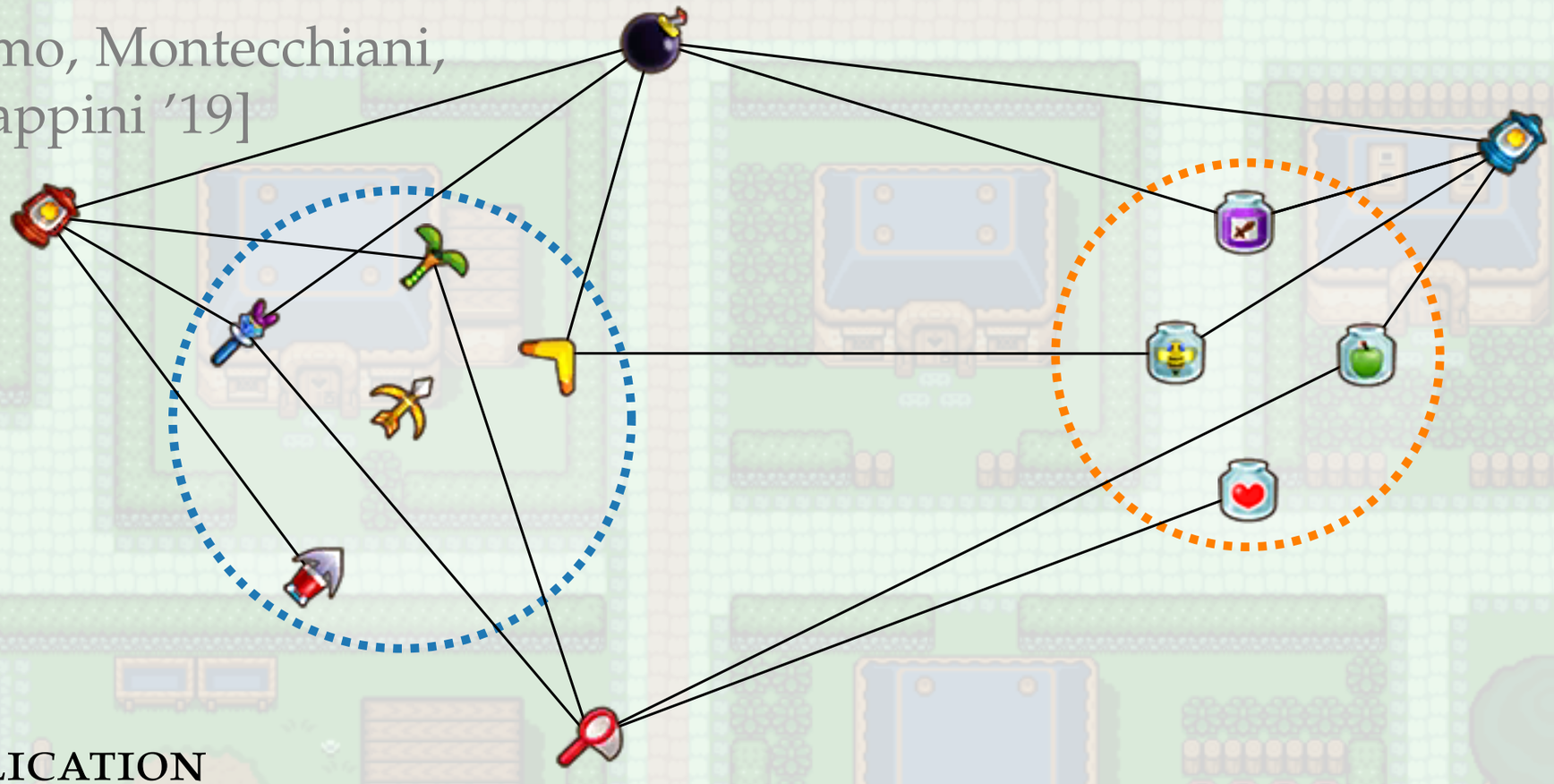

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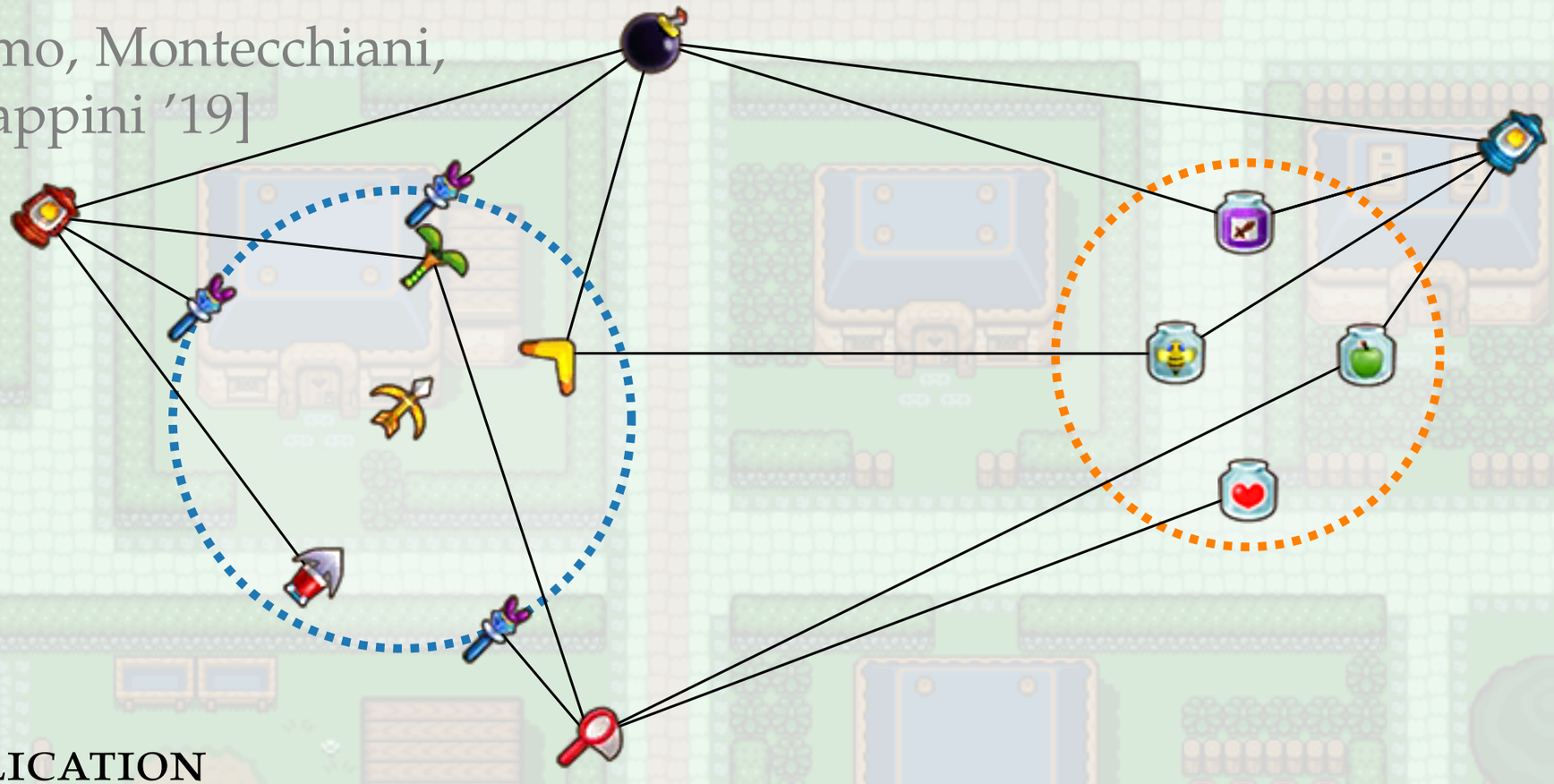

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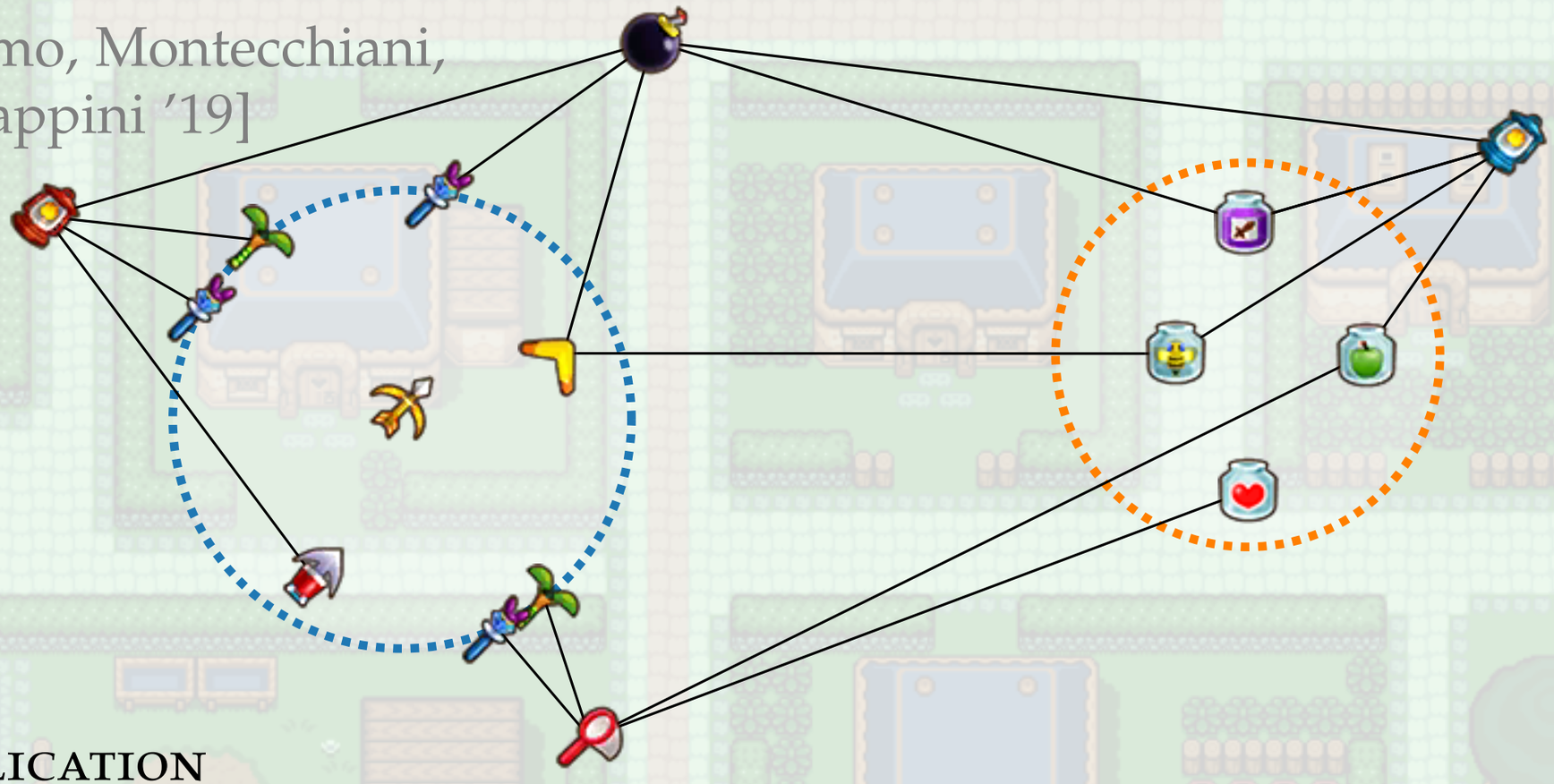

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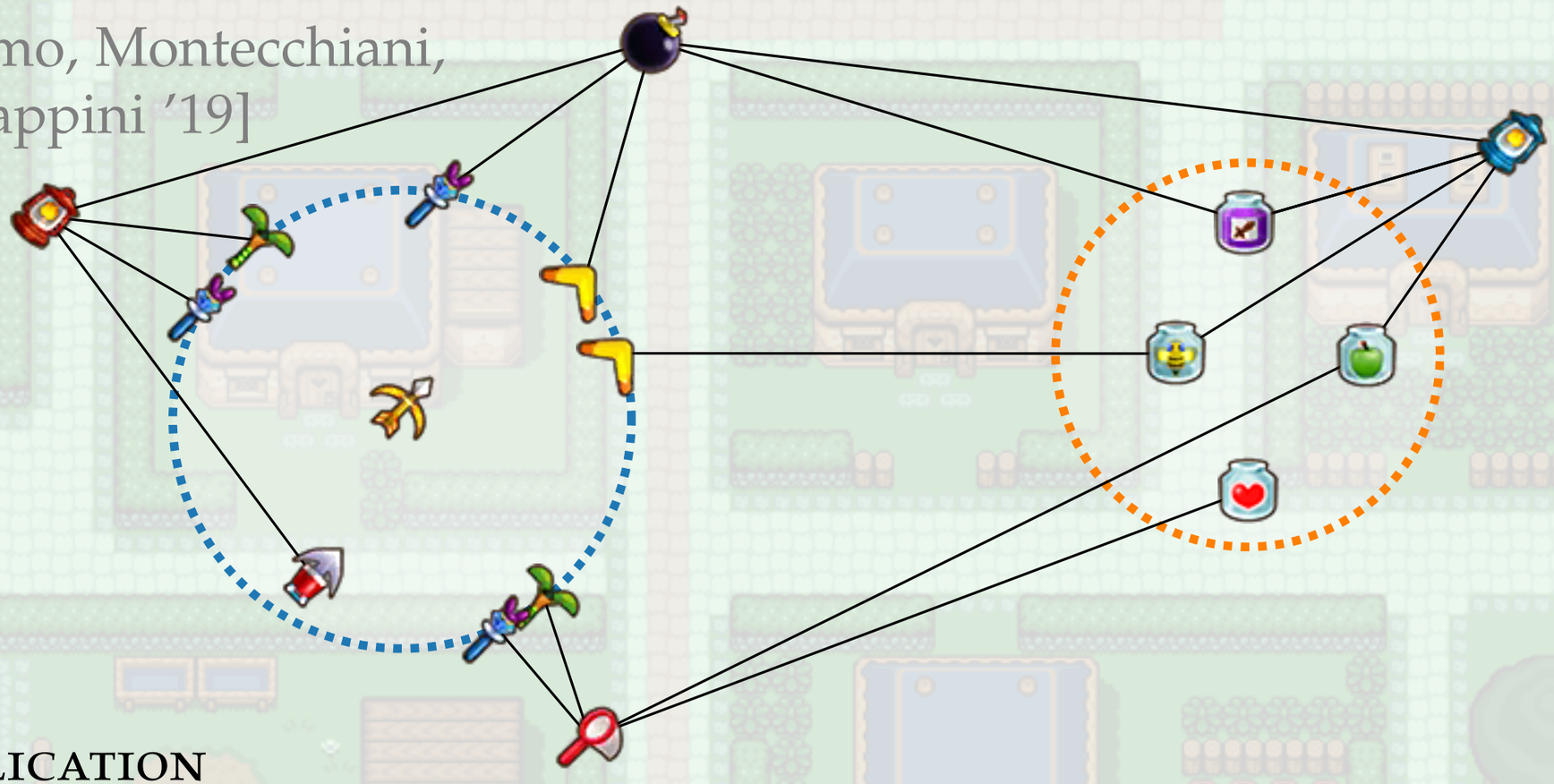

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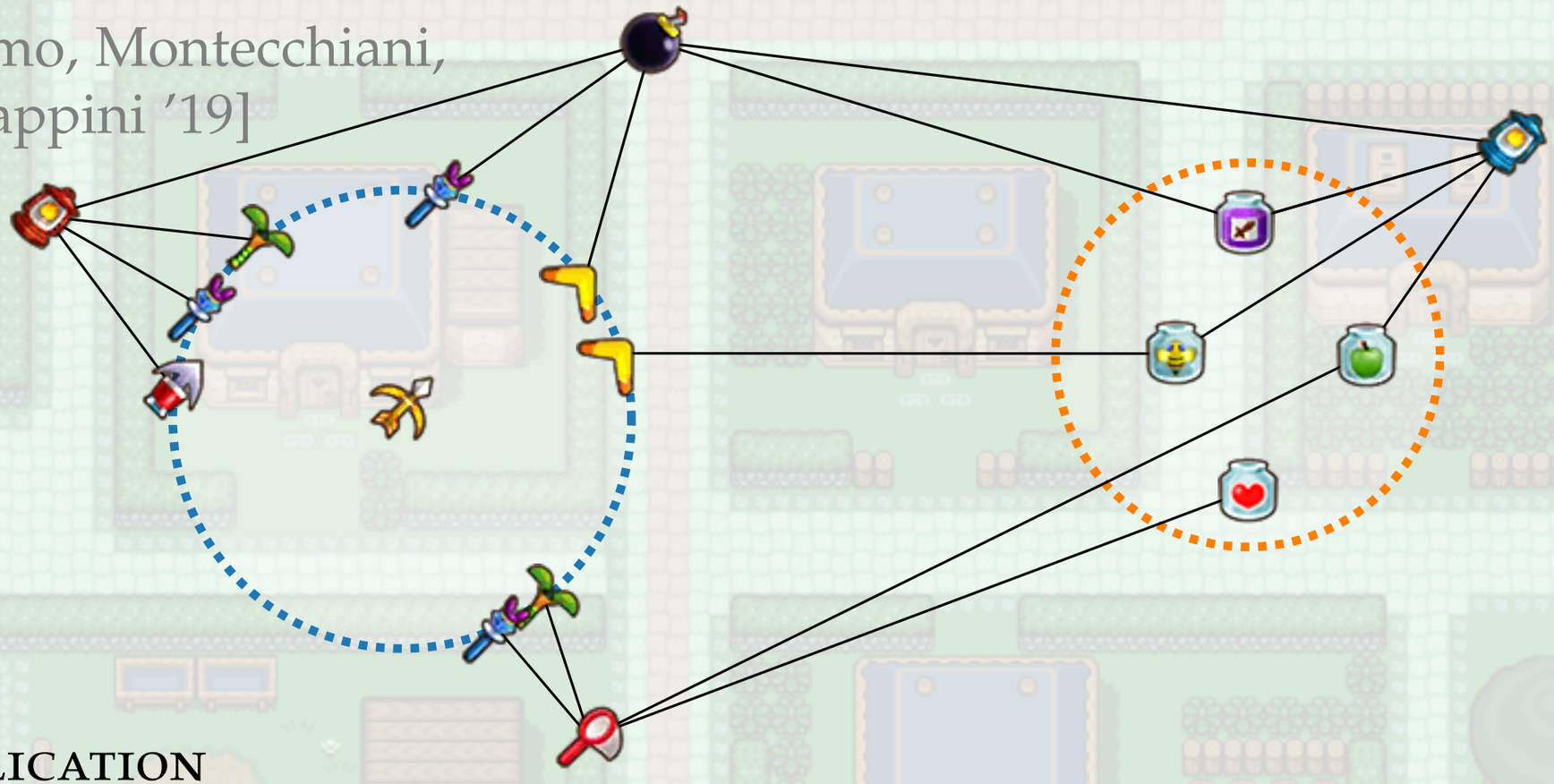

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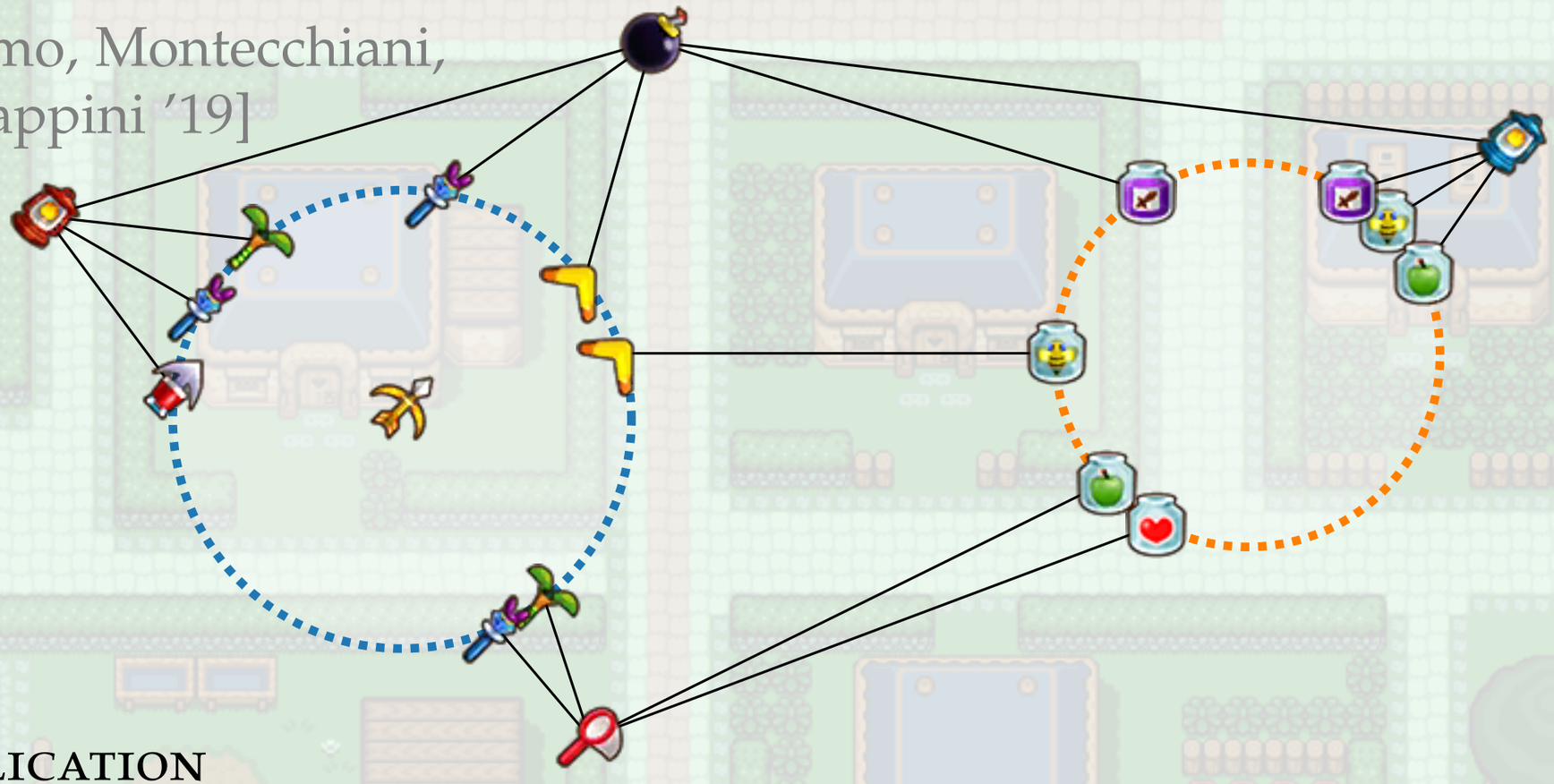

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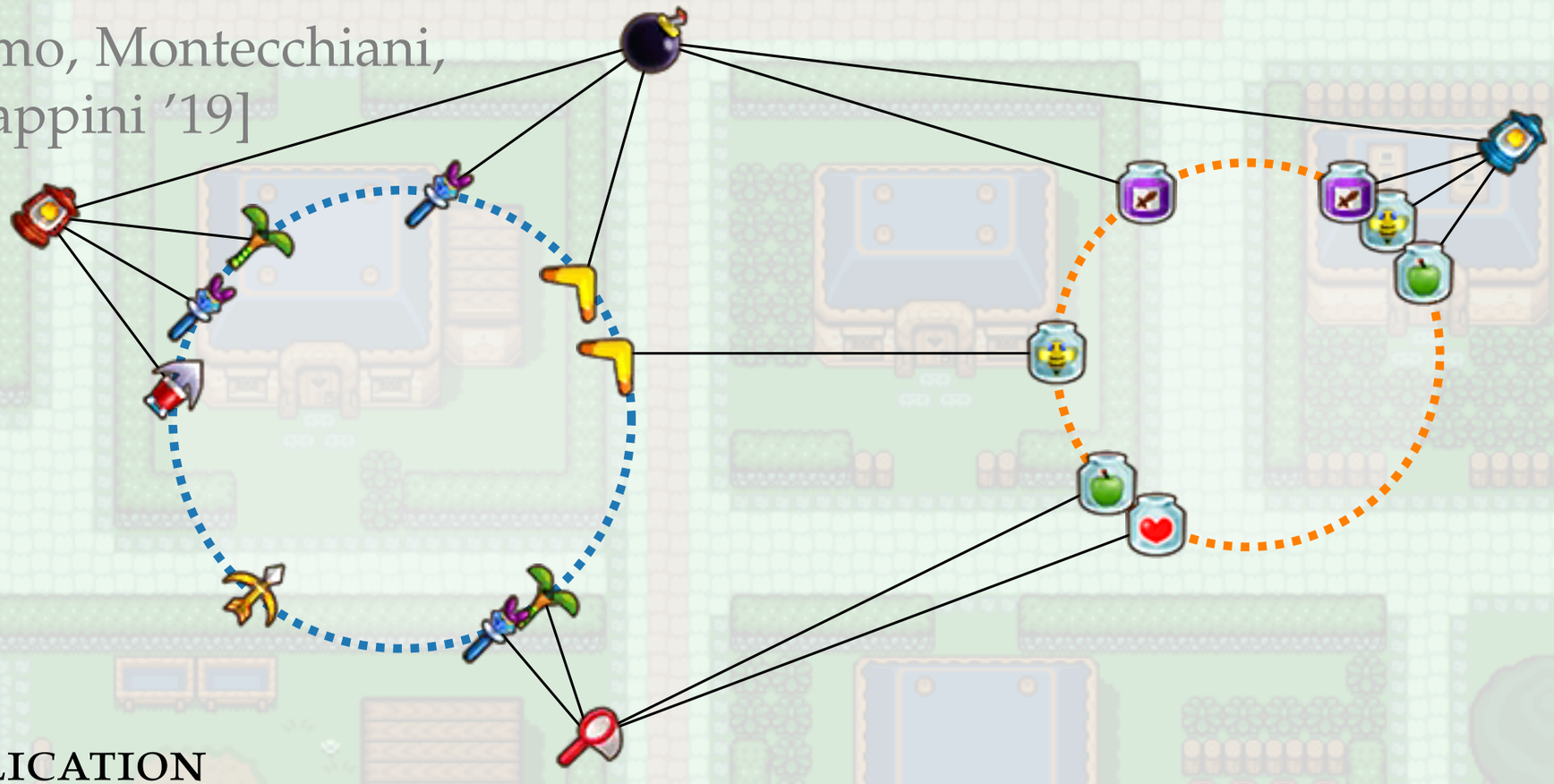

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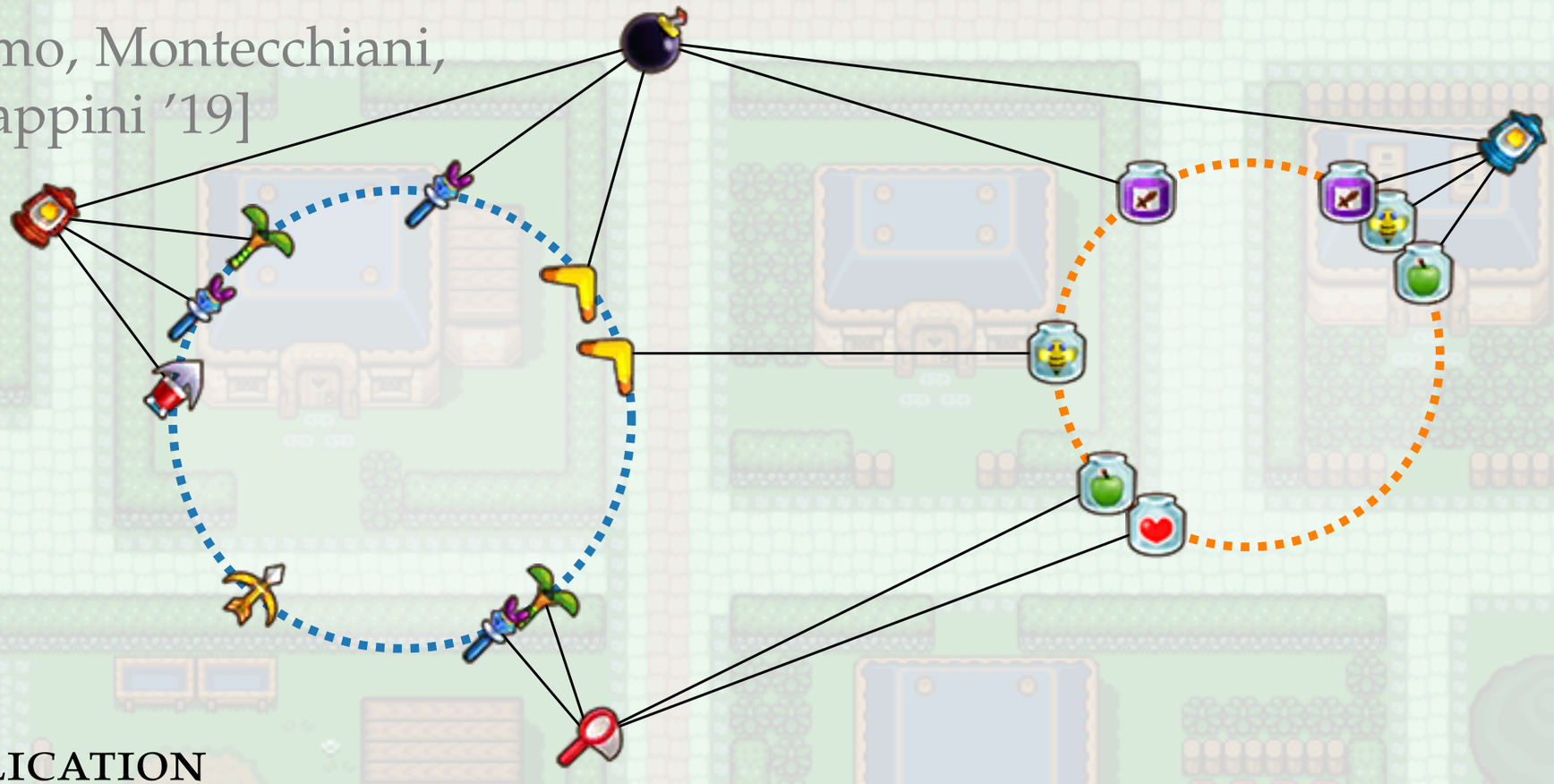
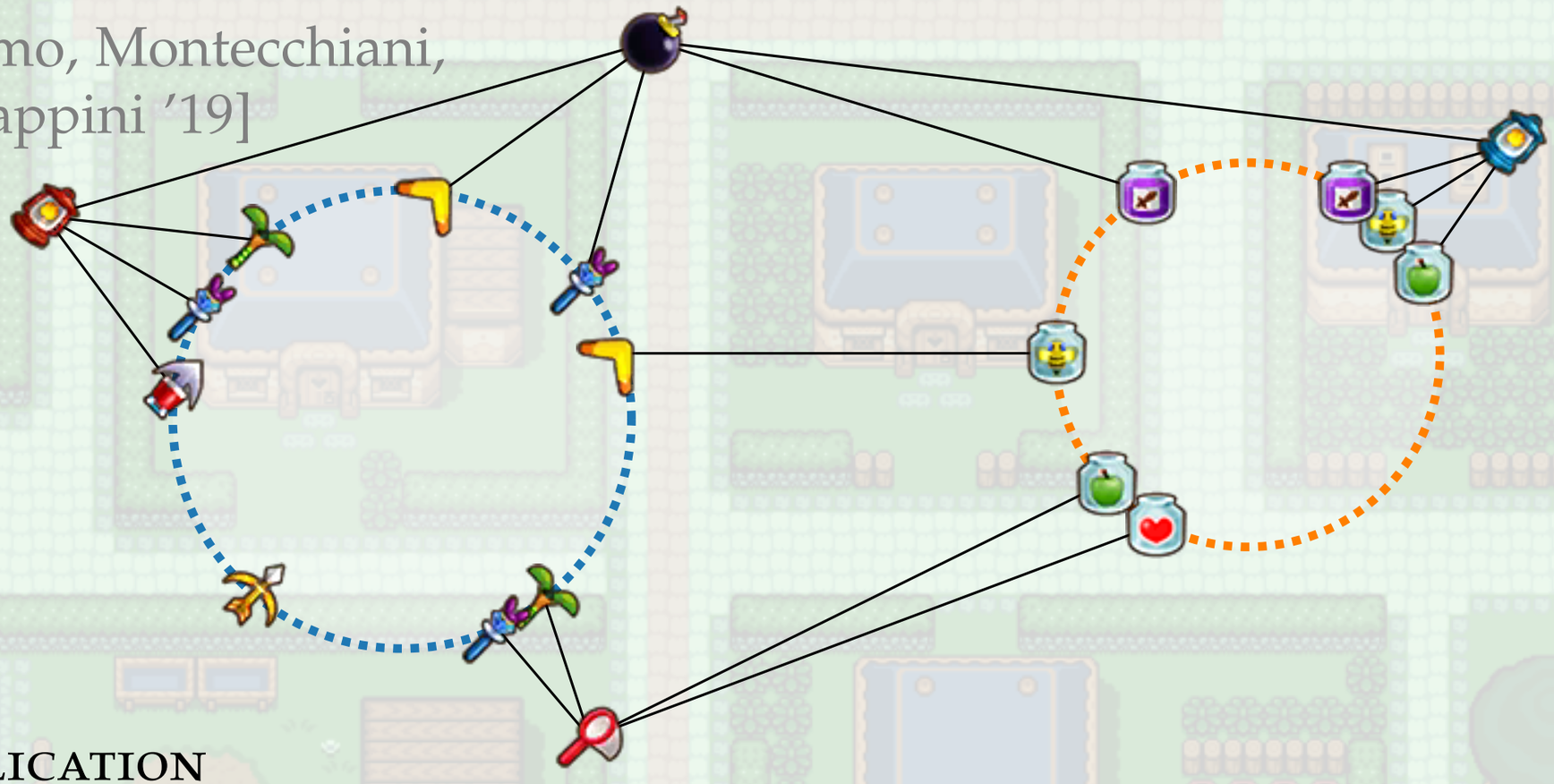


1. NodeReplication
2. NodePermutation

ChordLink

[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



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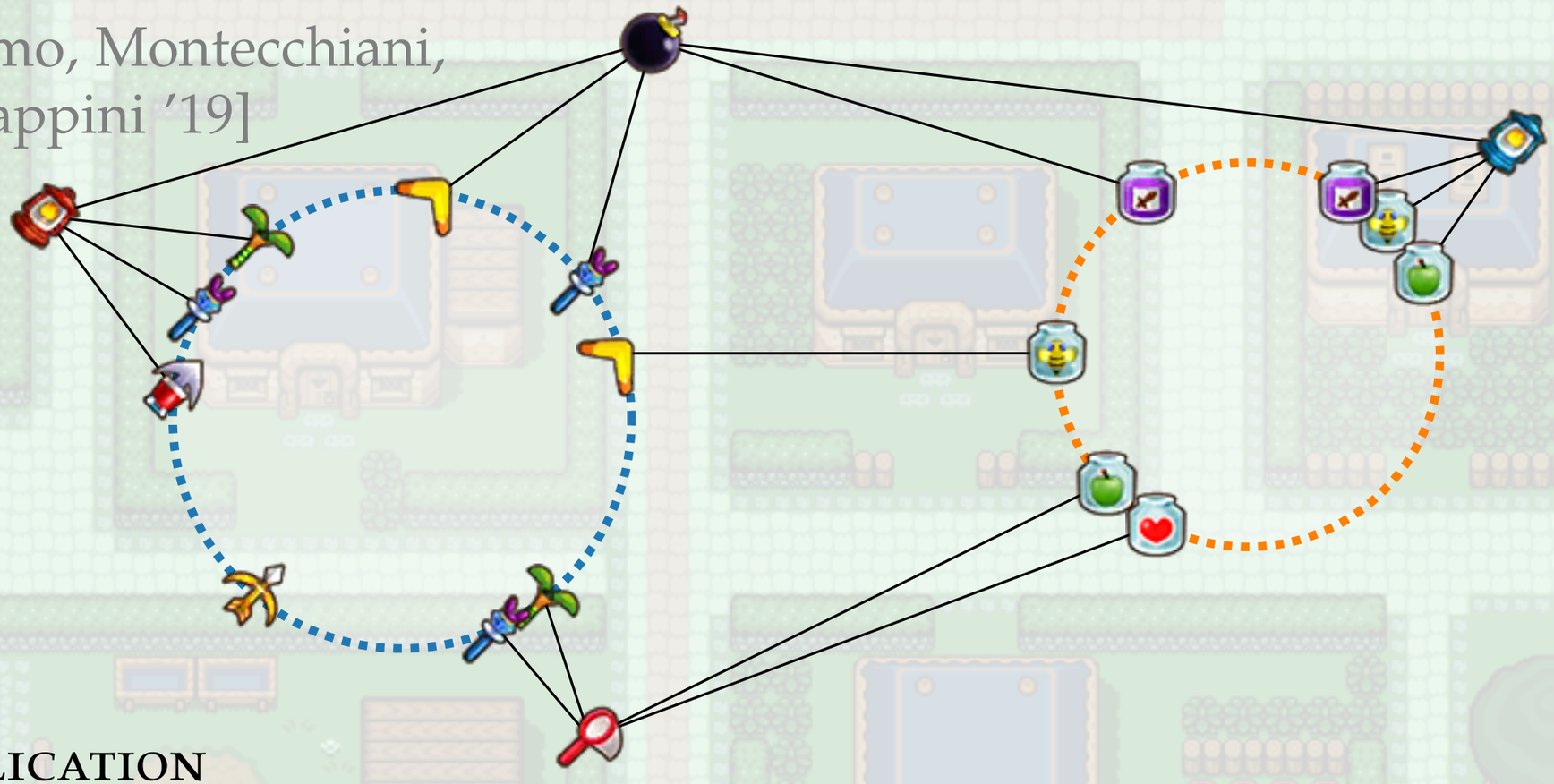
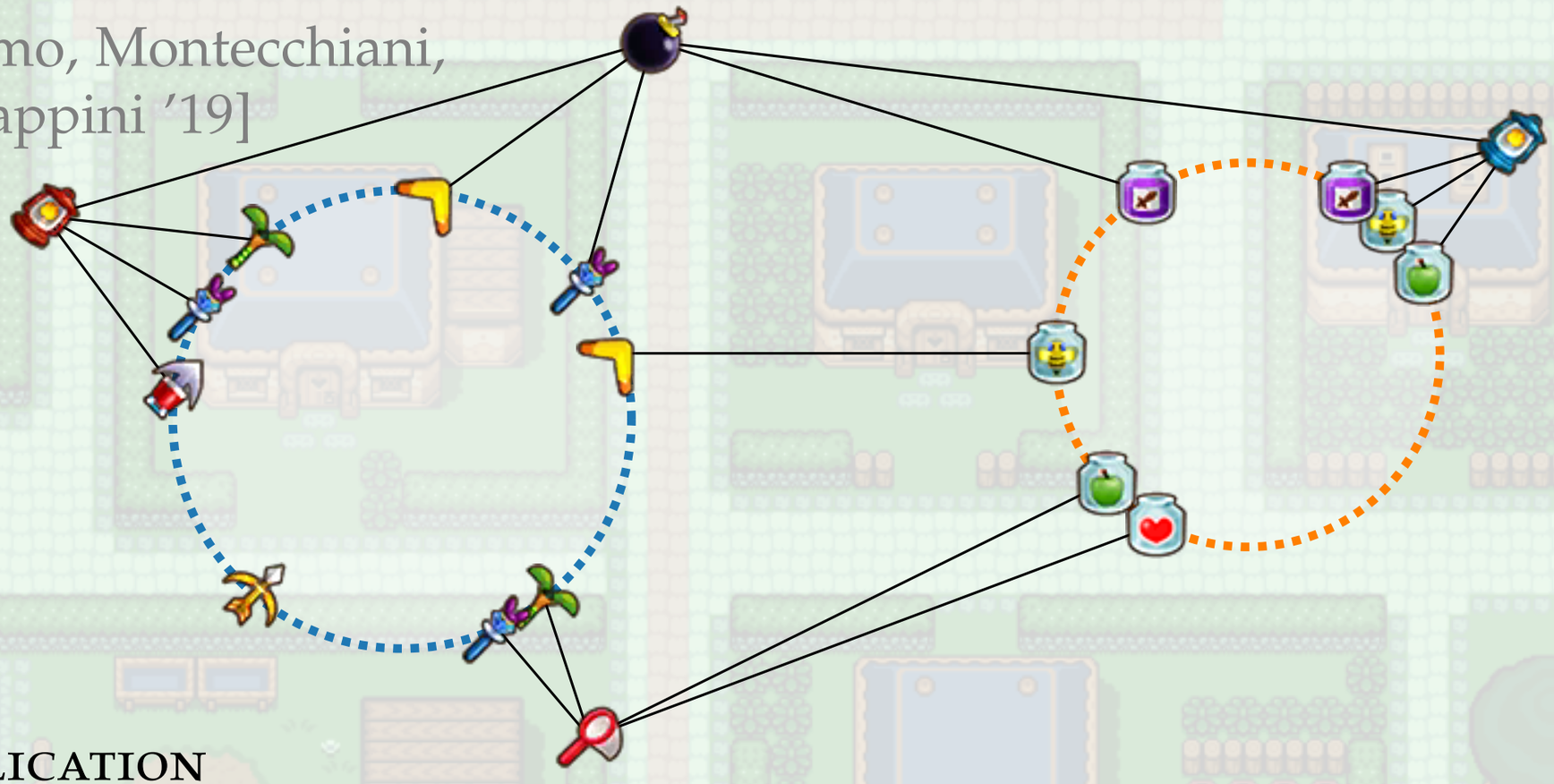


1. NodeReplication
2. NodePermutation Can flip nodes with common neighbor

ChordLink

[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



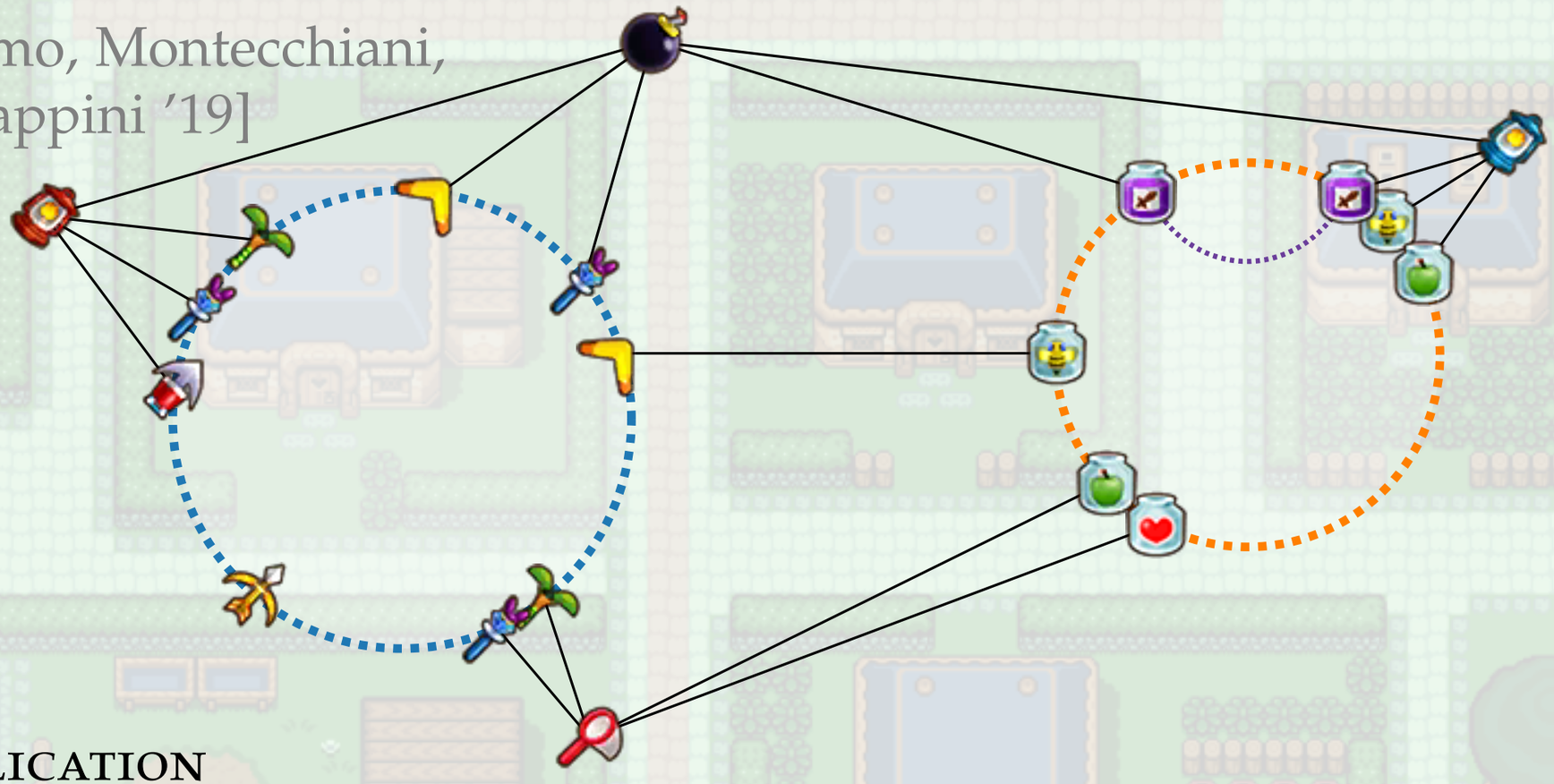
1. NodeReplication
2. NodePermutation

Can flip nodes with common neighbor

Goal: max. #successive pairs with same icon

ChordLink

[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



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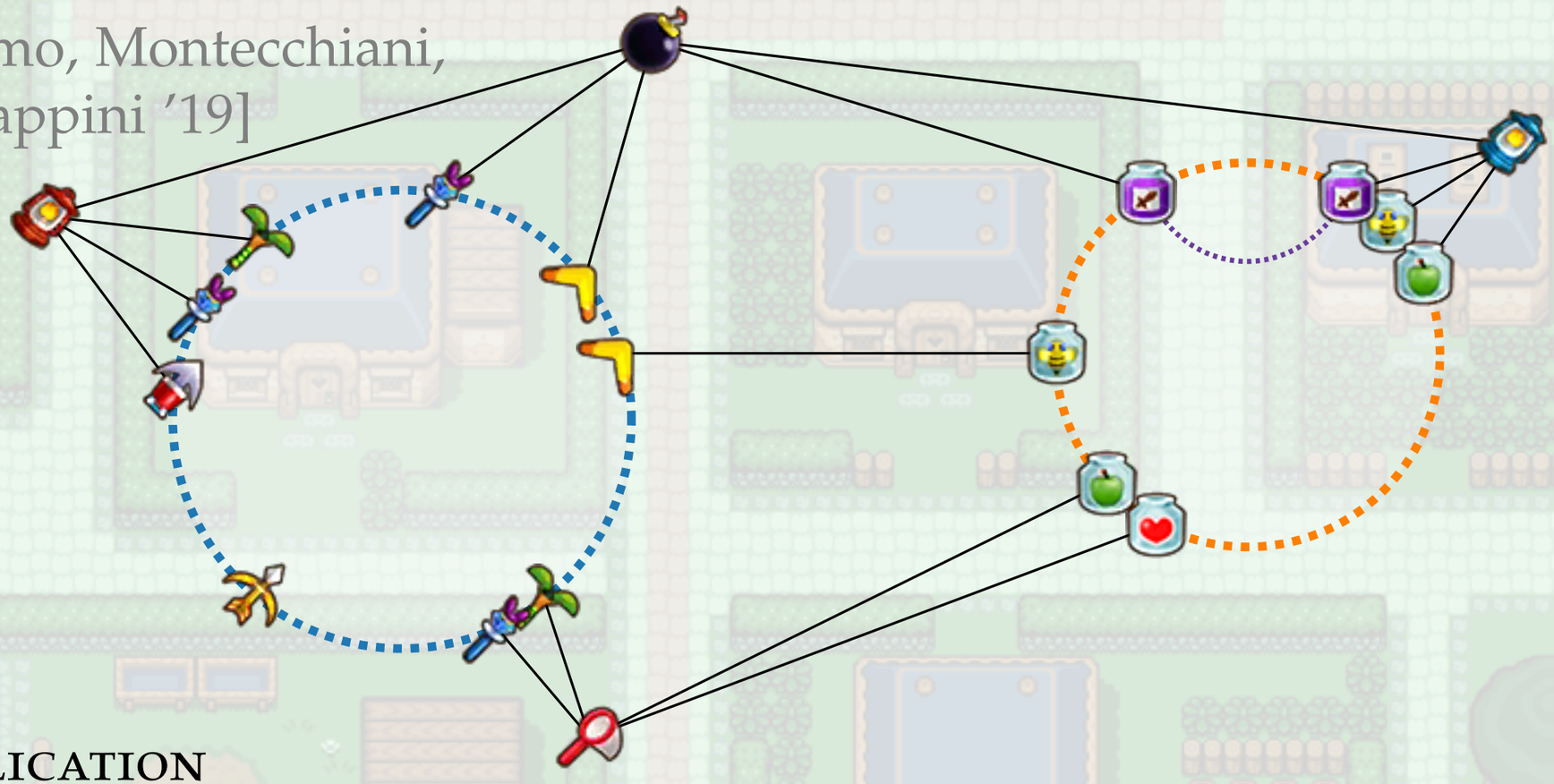

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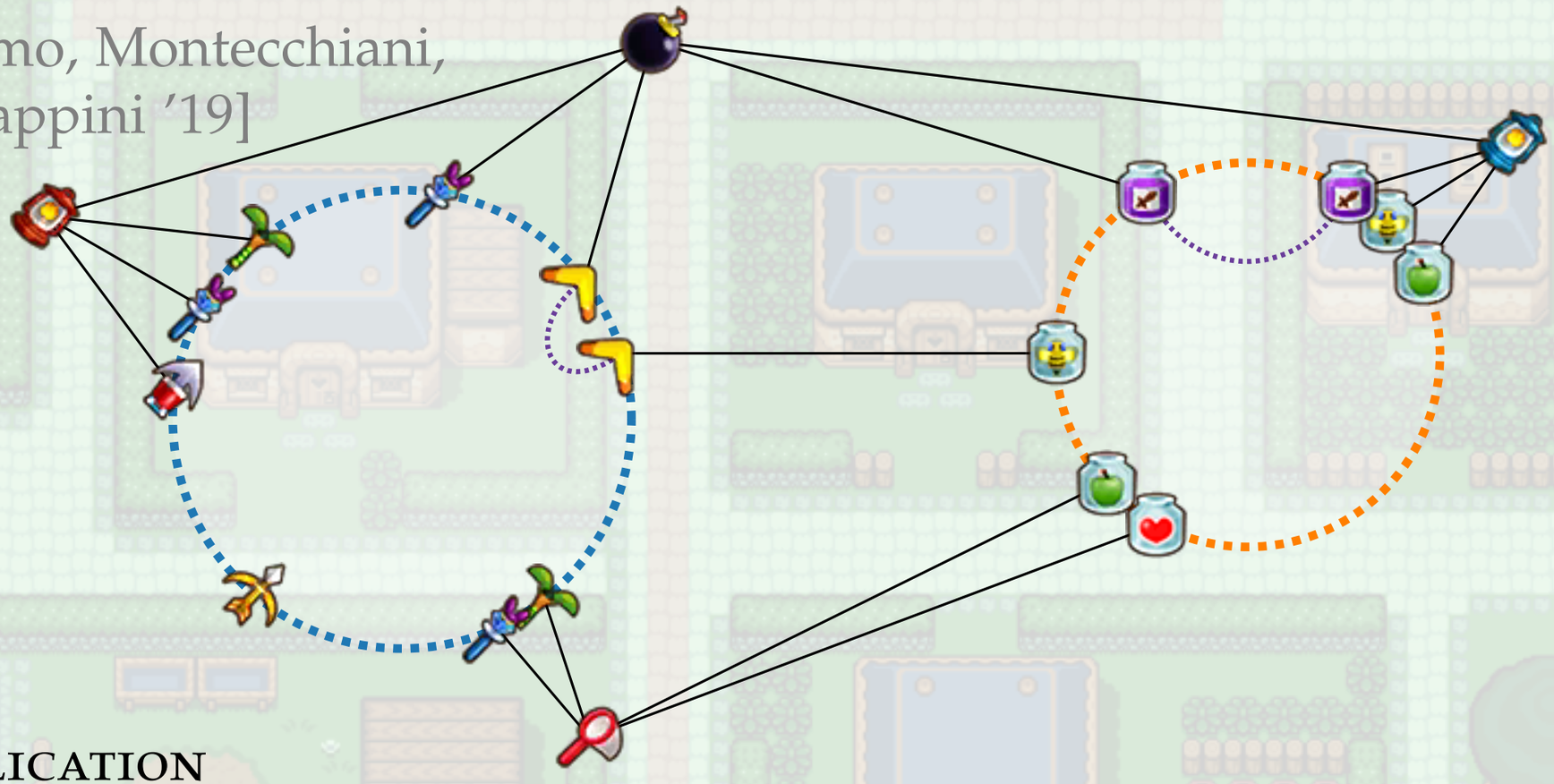

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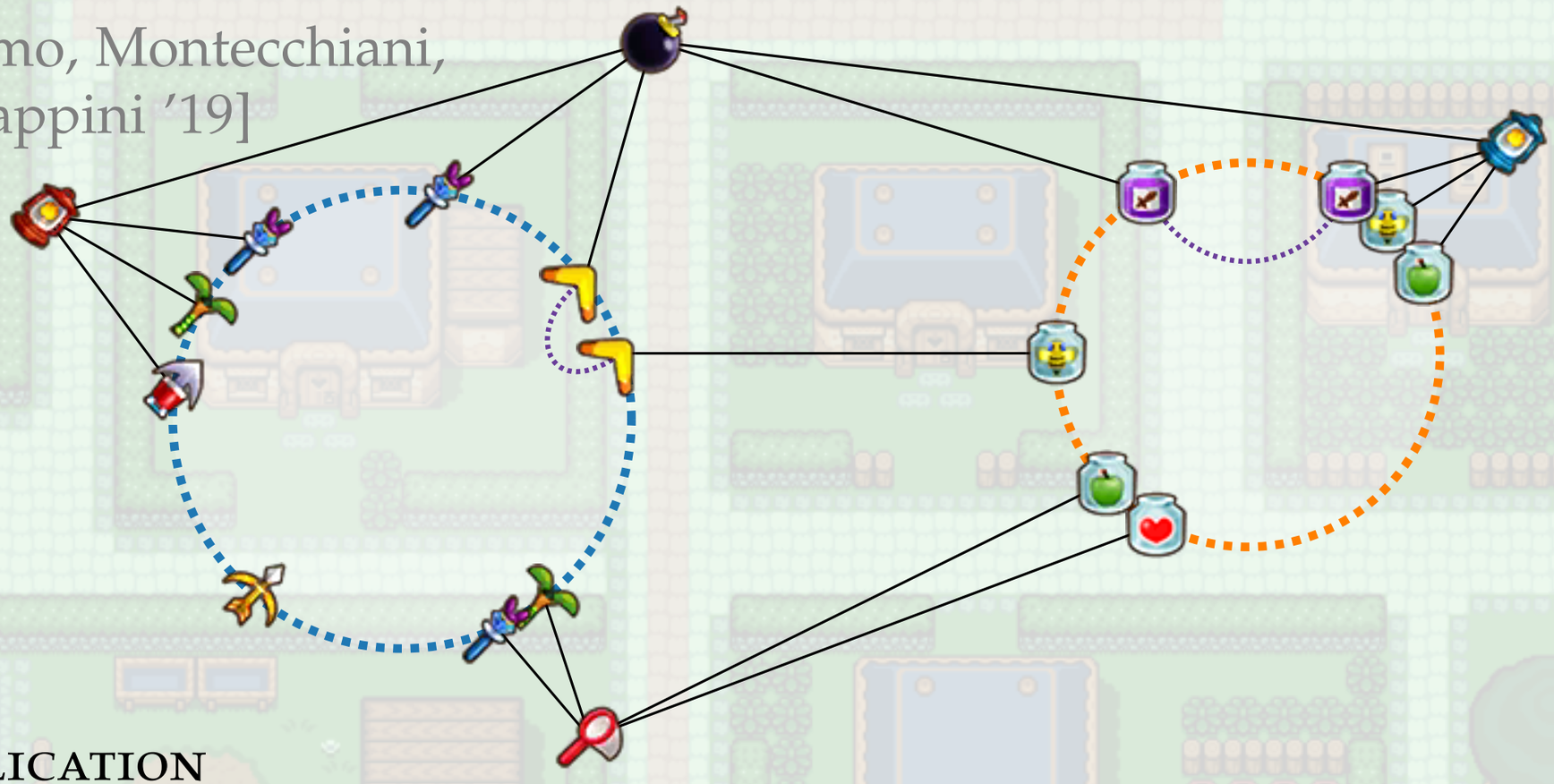

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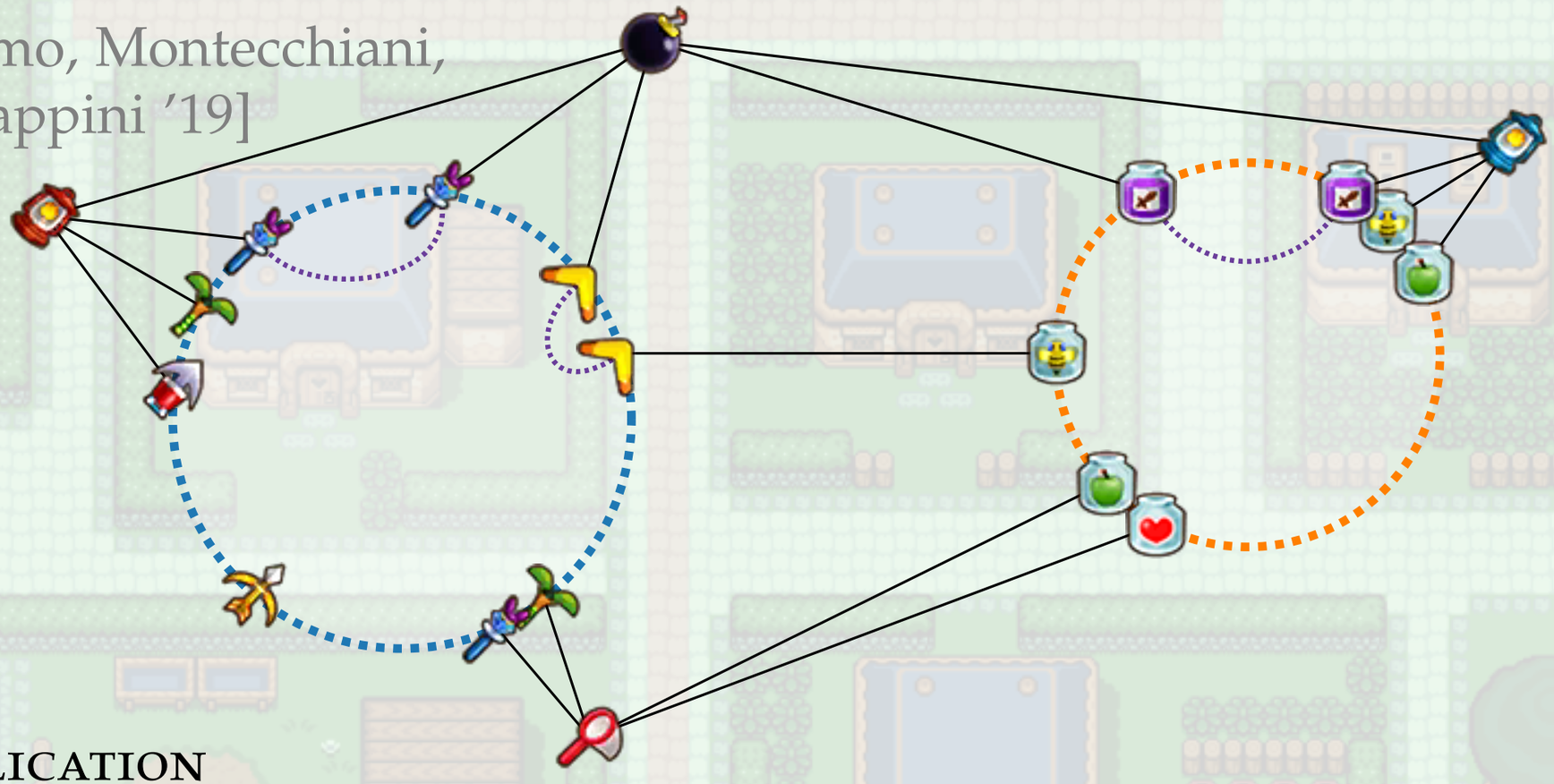

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Can flip nodes with common neighbor

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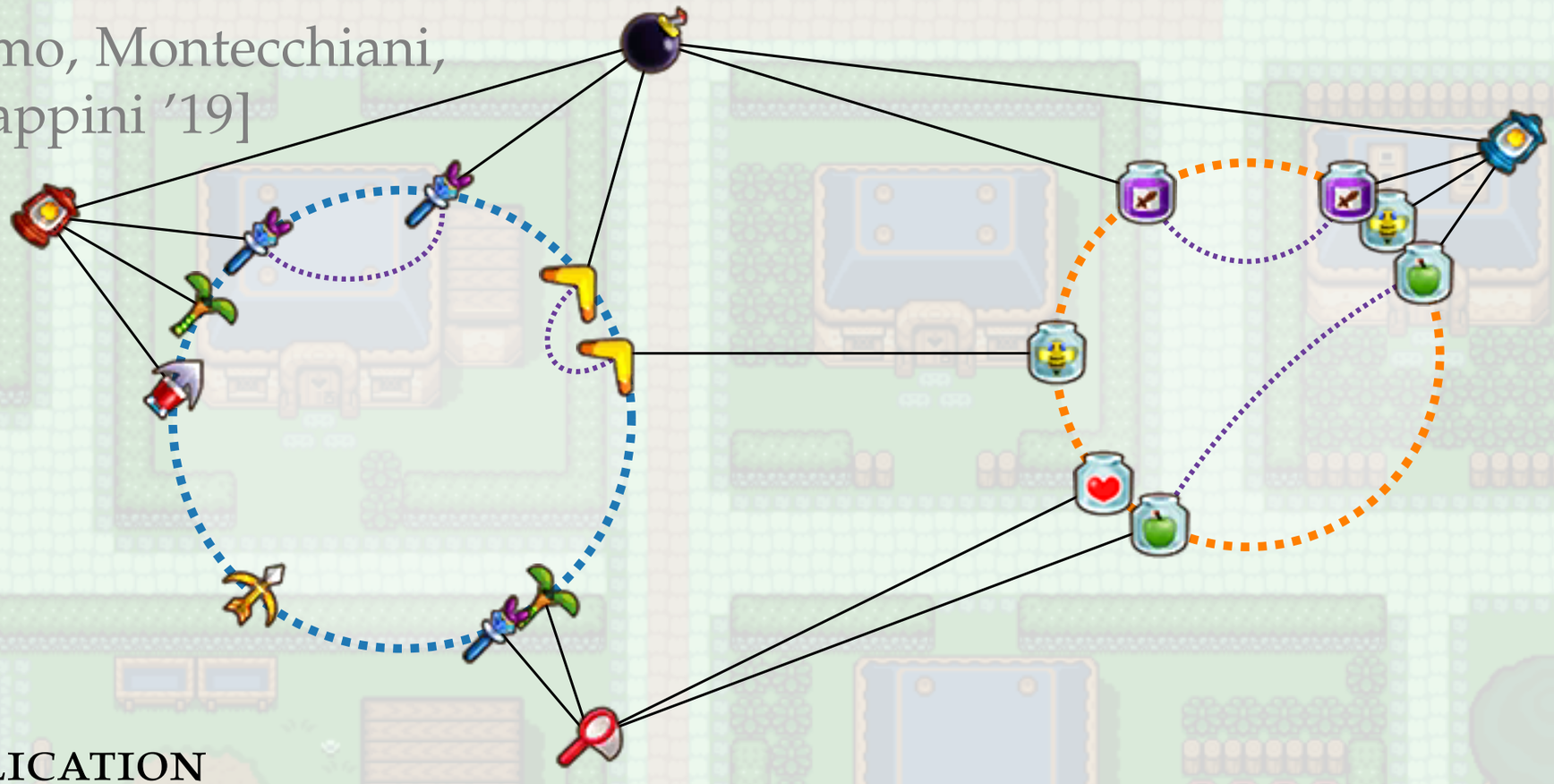

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Can flip nodes with common neighbor

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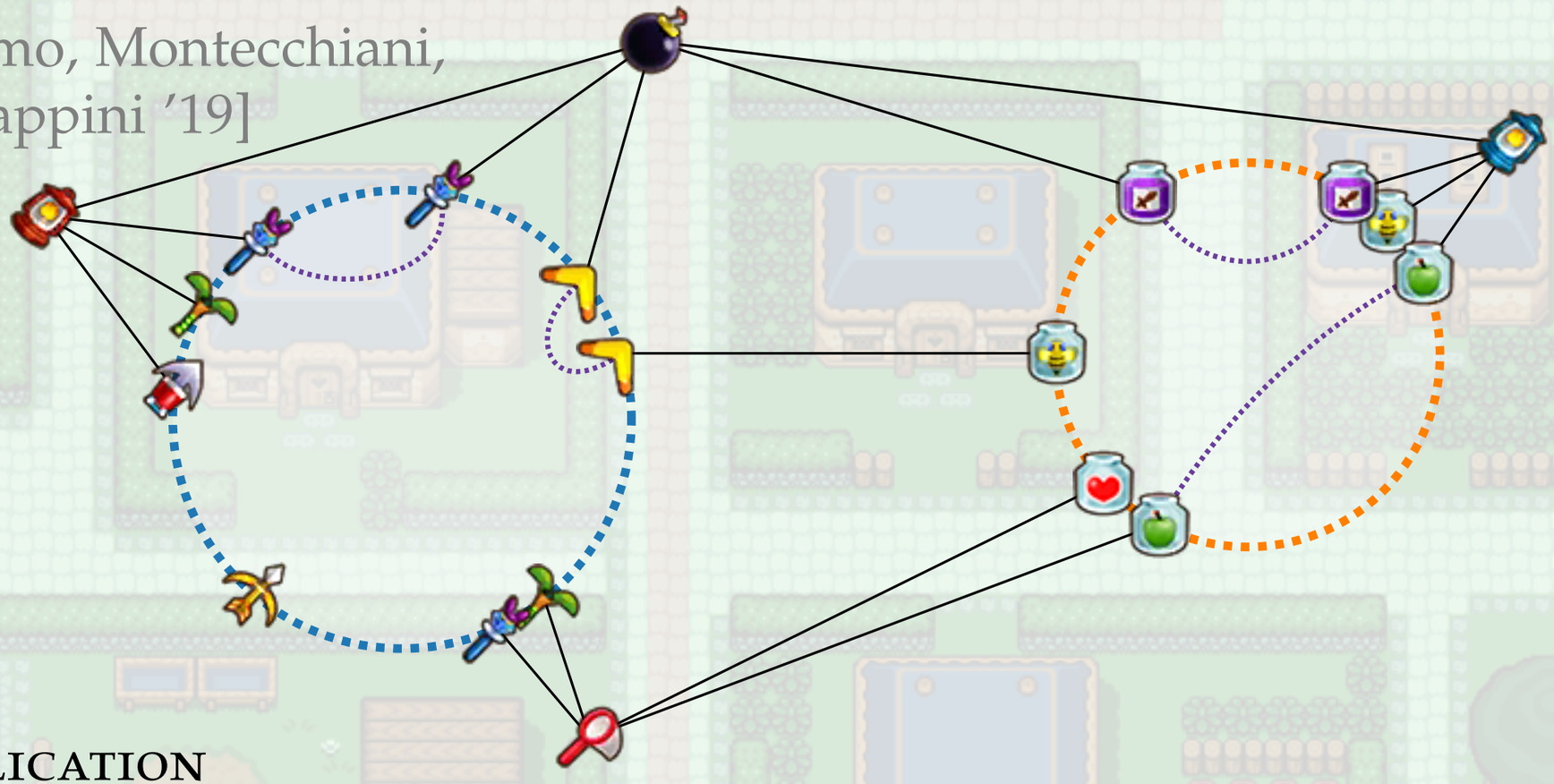
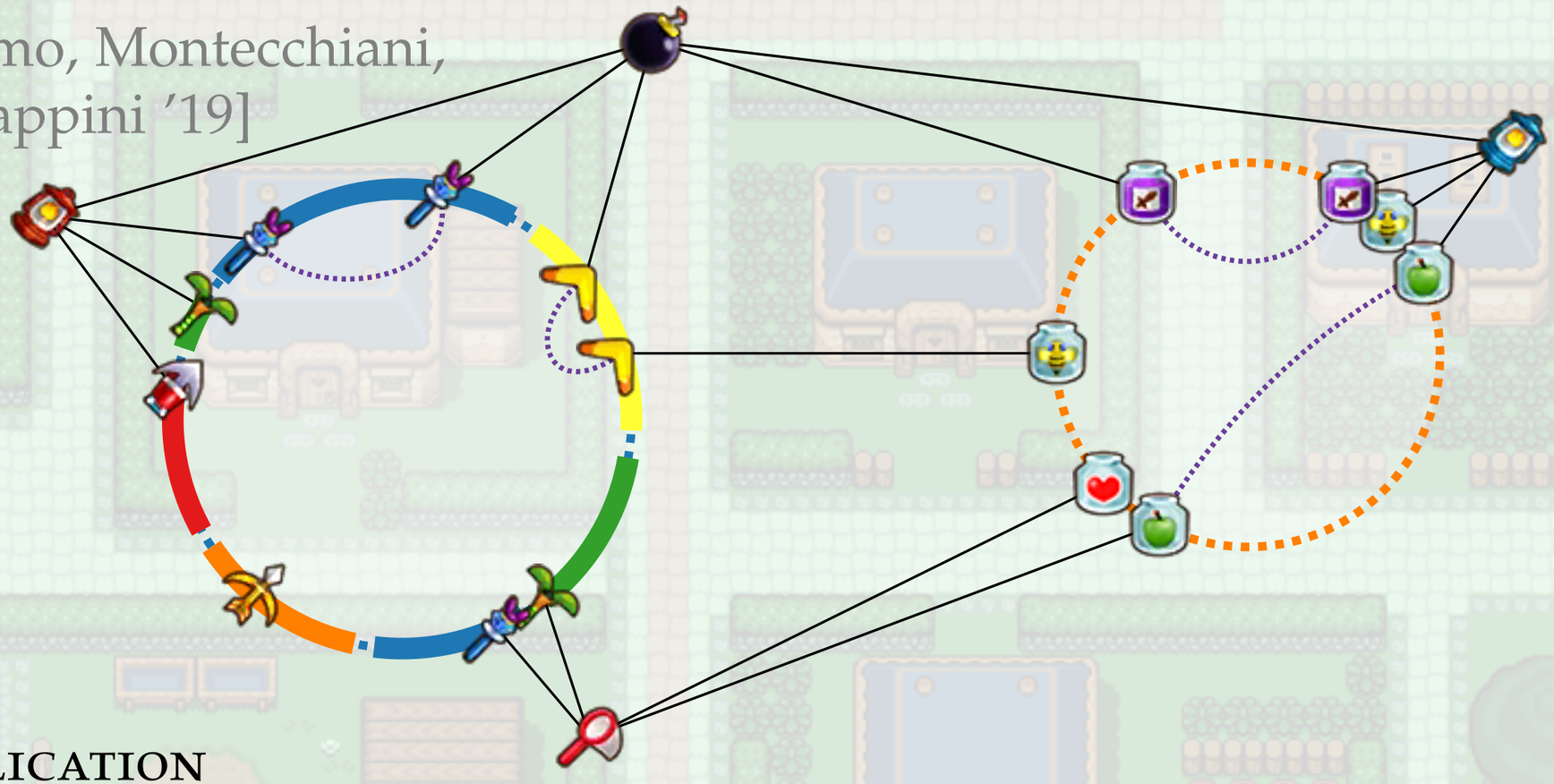


1. NodeReplication
2. NodePermutation — Can flip nodes with common neighbor
   Goal: max. #successive pairs with same icon
3. NodeMerging

ChordLink

[Angori, Didimo, Montecchiani, Pagliuca & Tappini '19]



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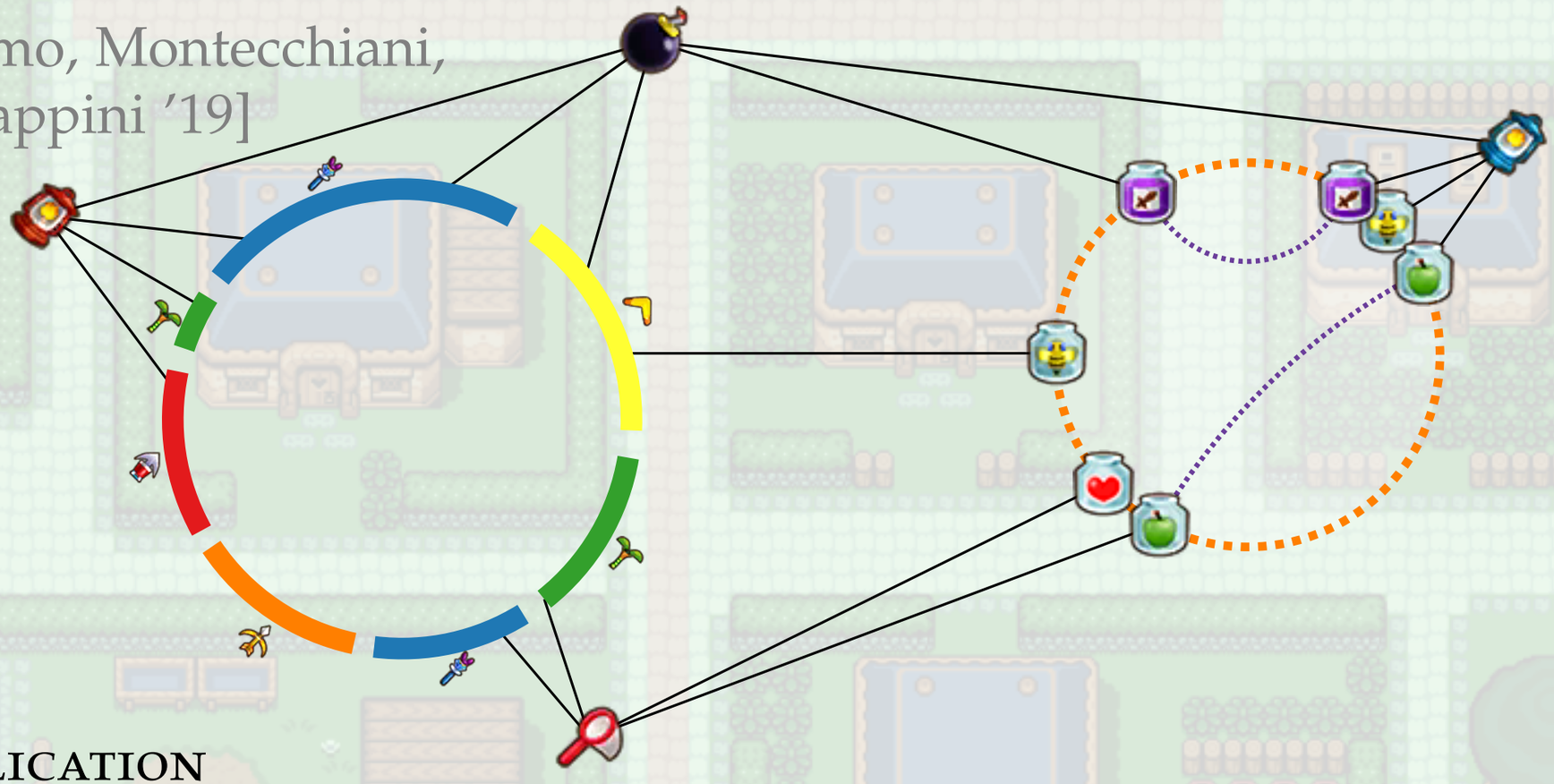

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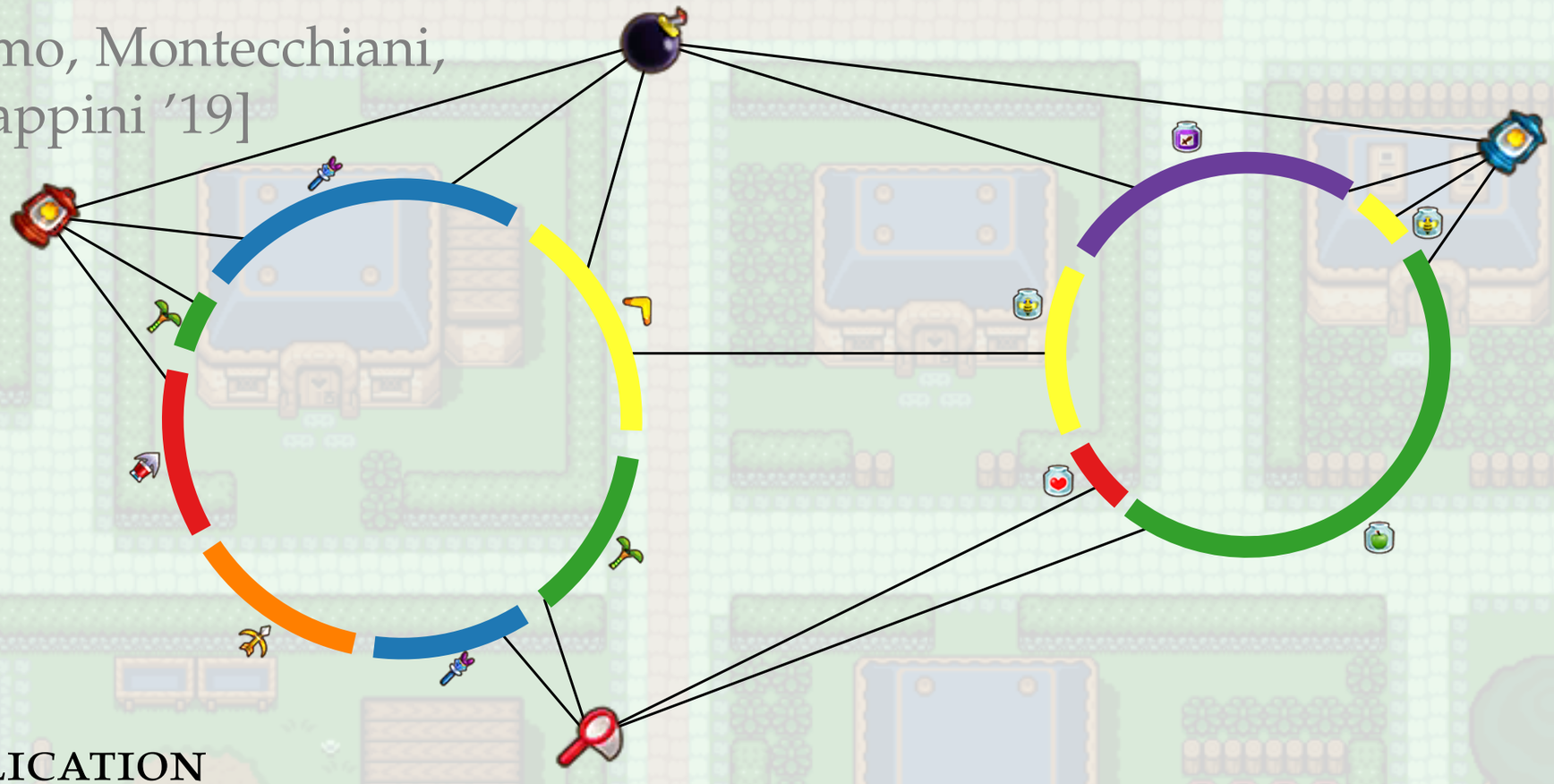

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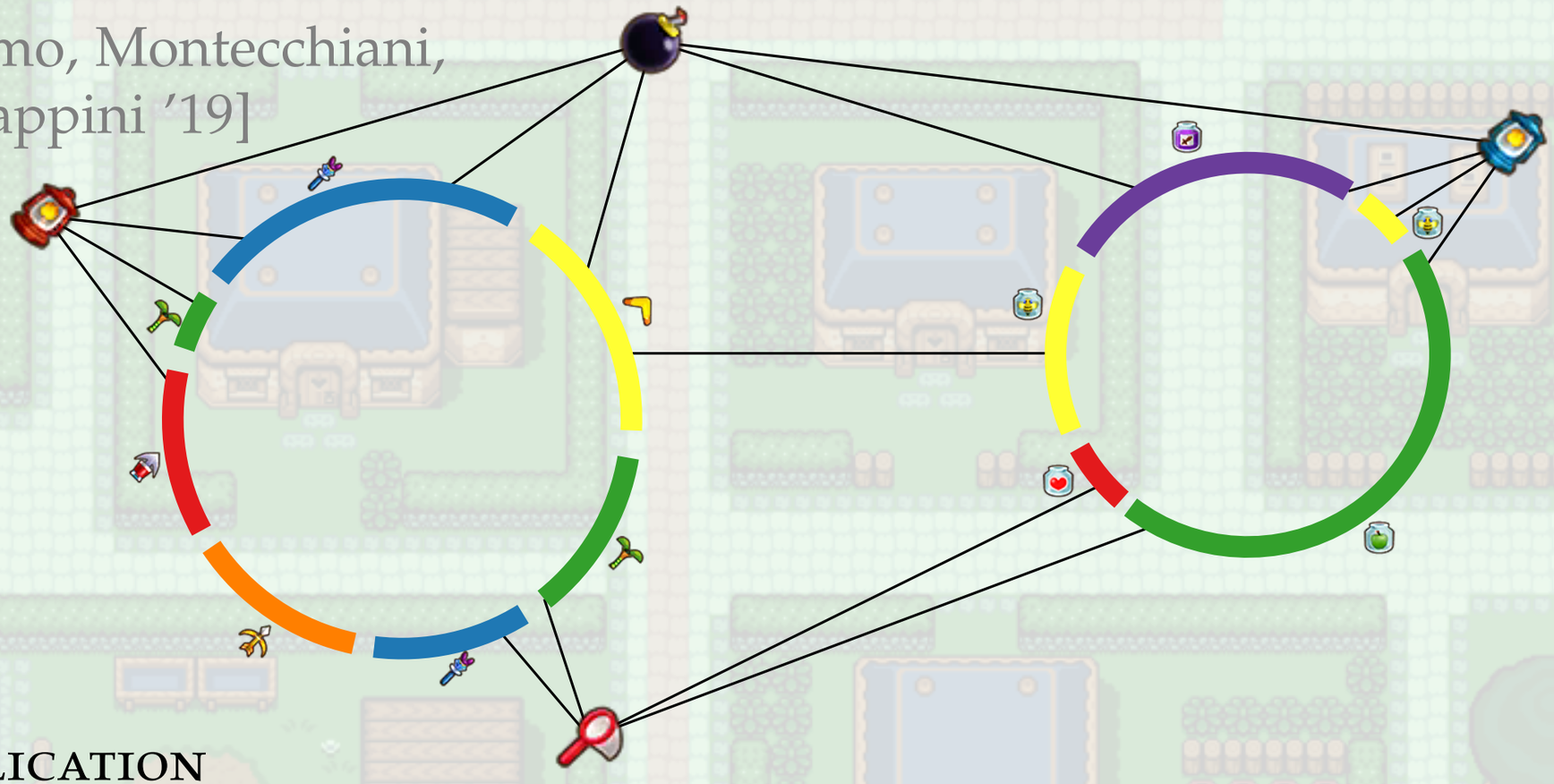

1. NodeReplication
2. NodePermutation — Can flip nodes with common neighbor
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3. NodeMerging
4. ChordInsertion

ChordLink

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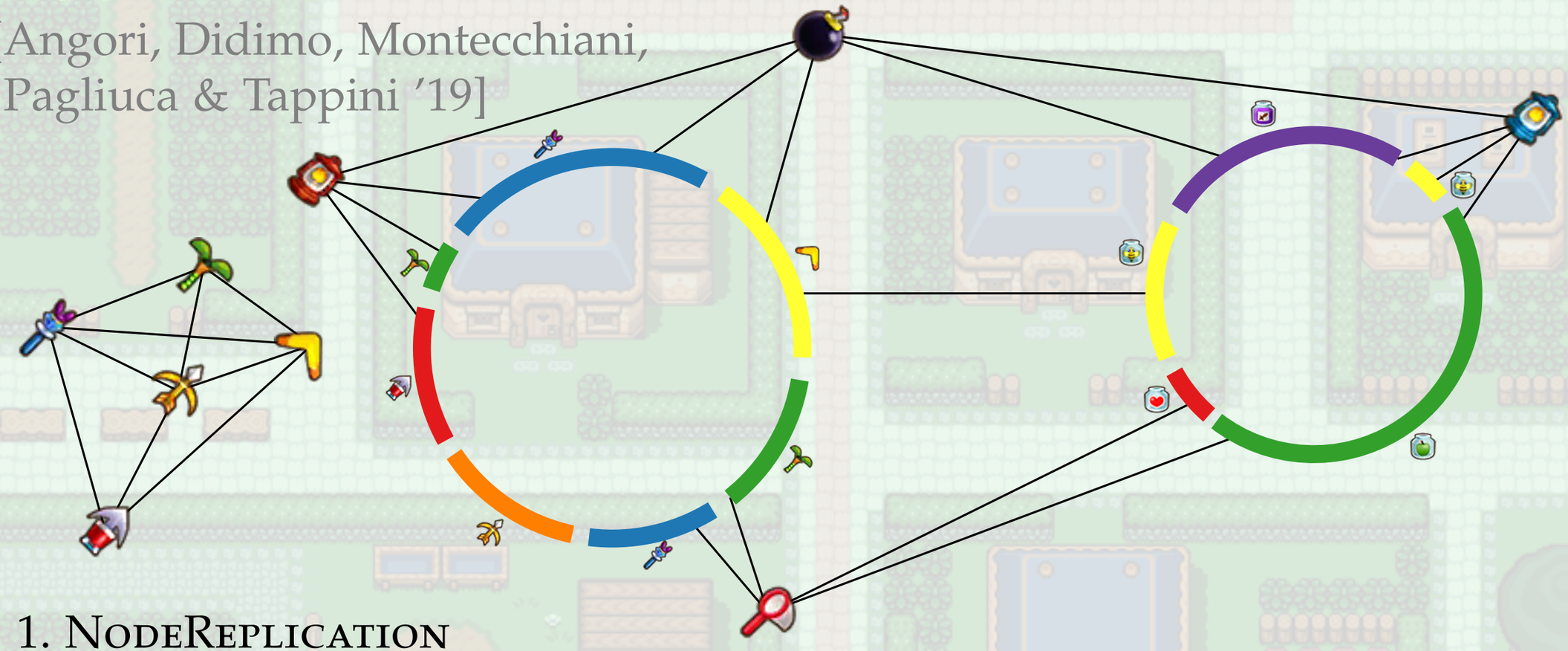

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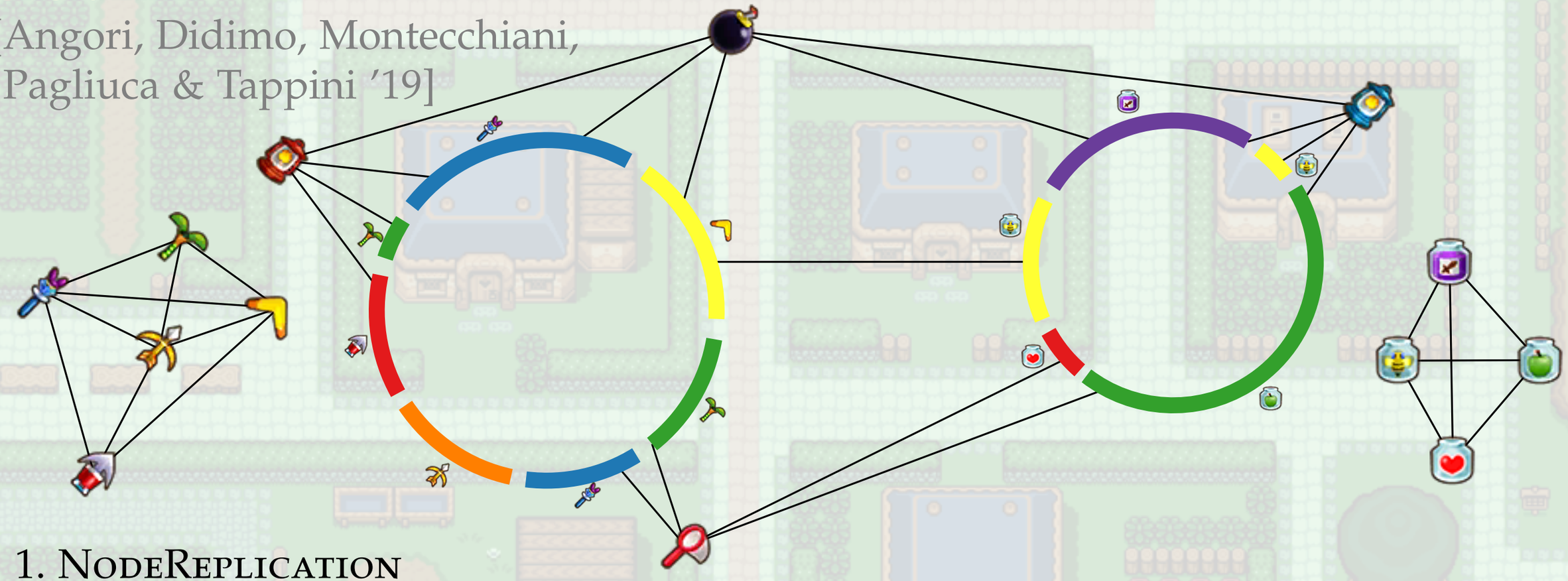

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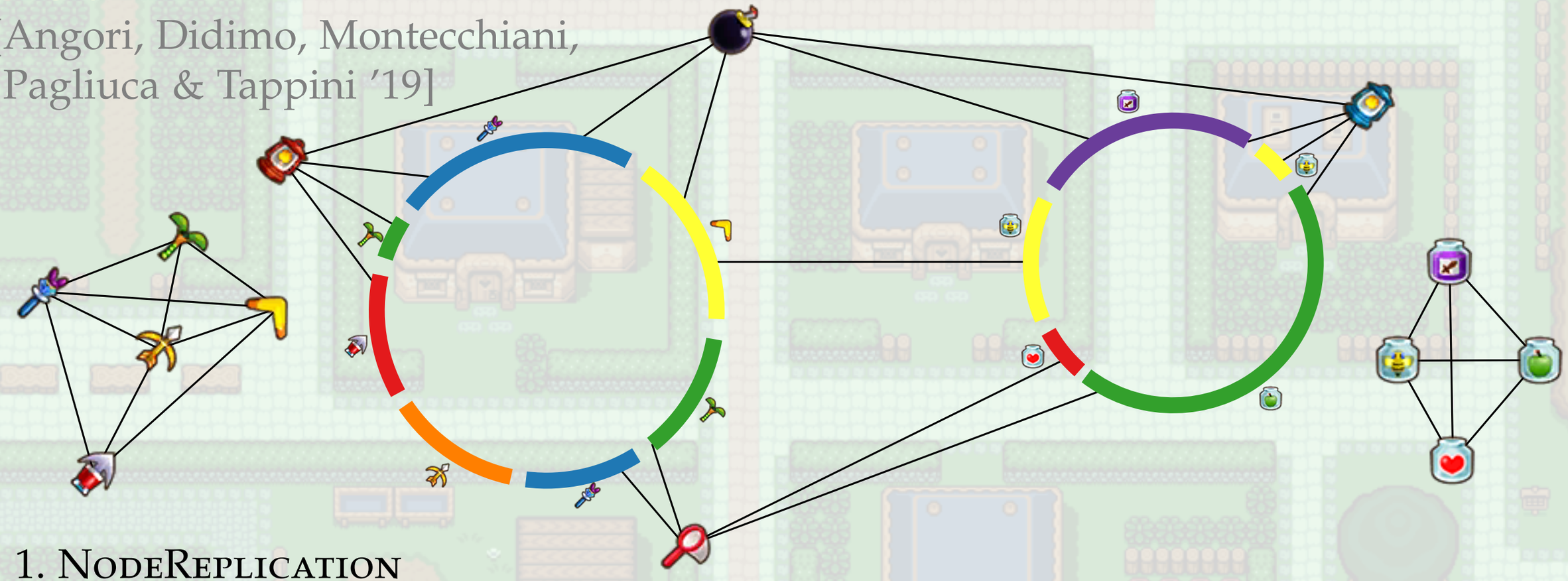

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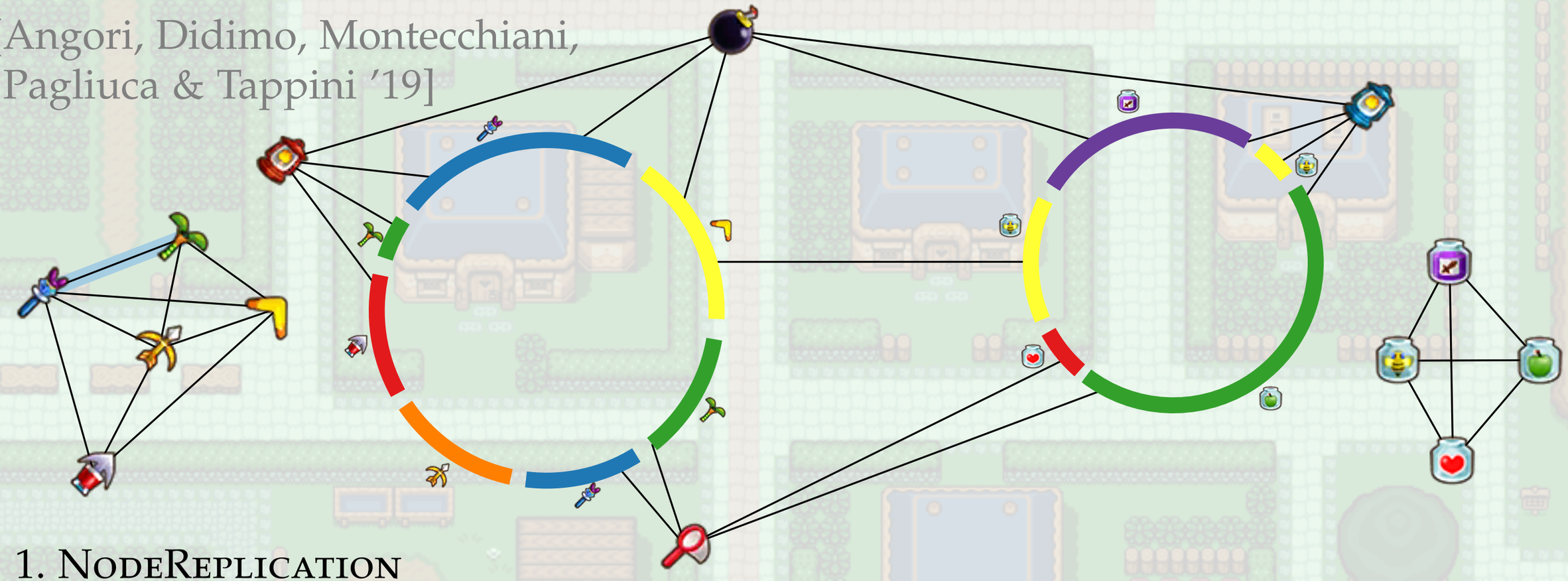

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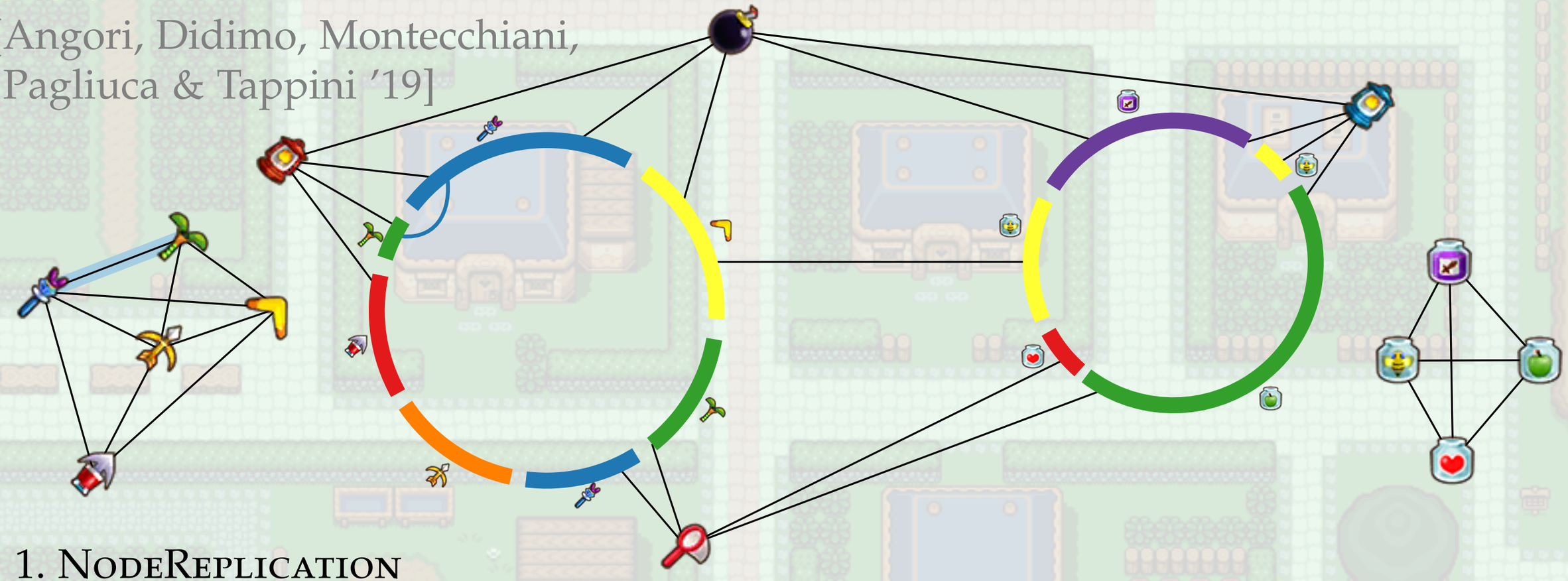

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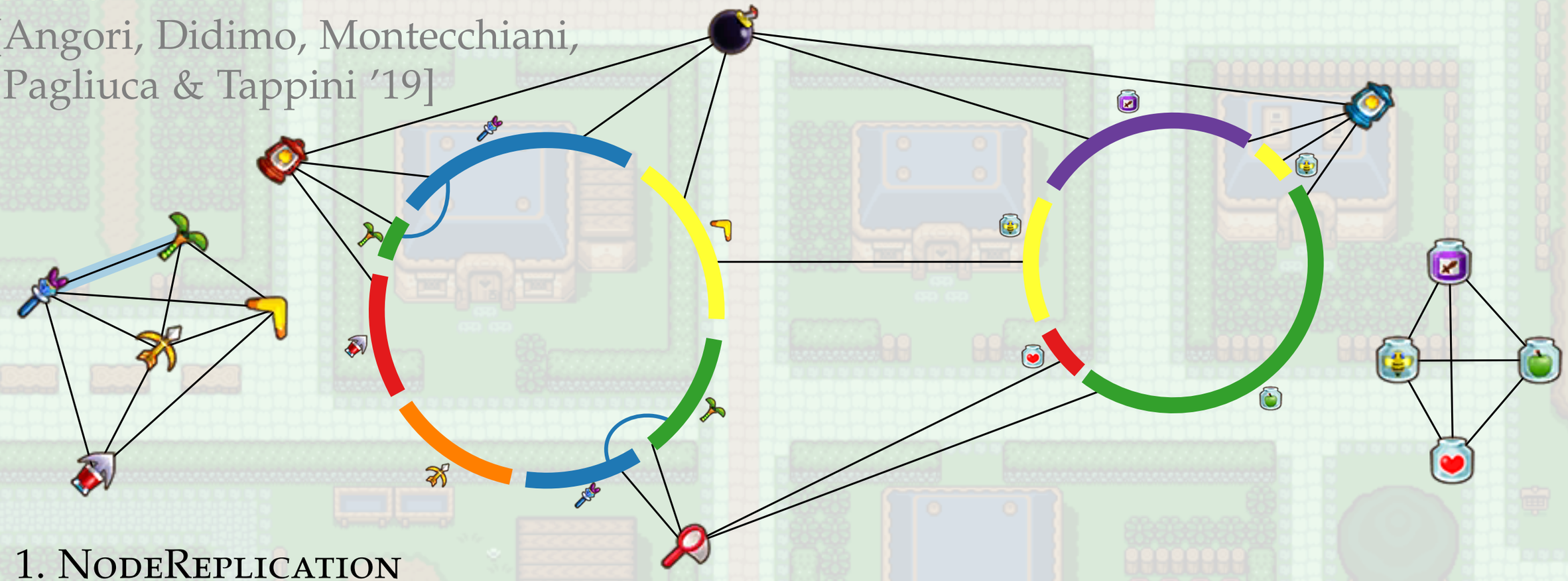

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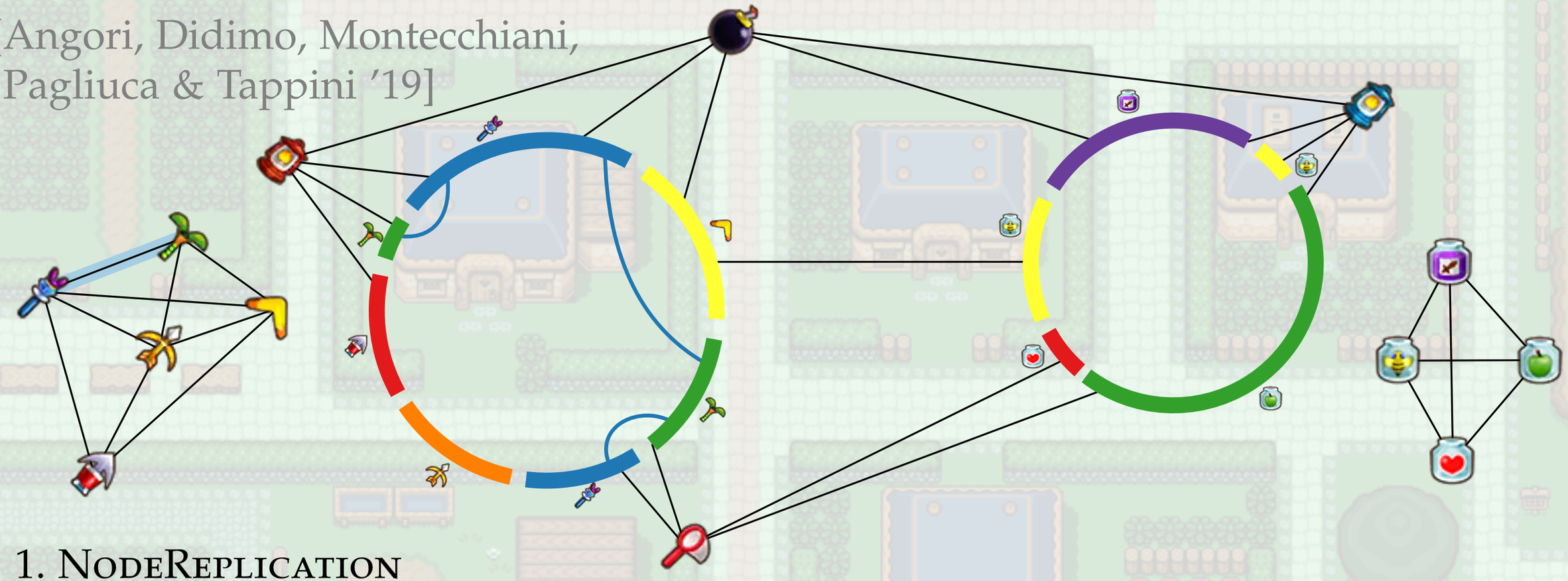

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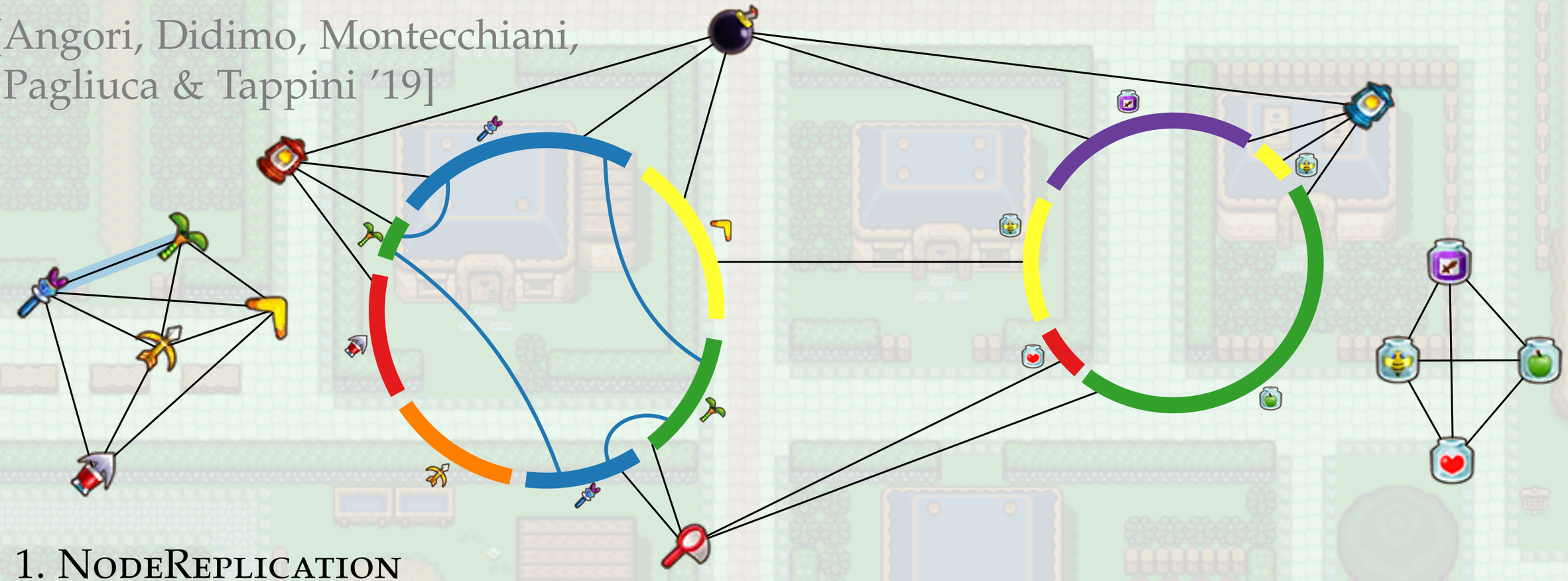

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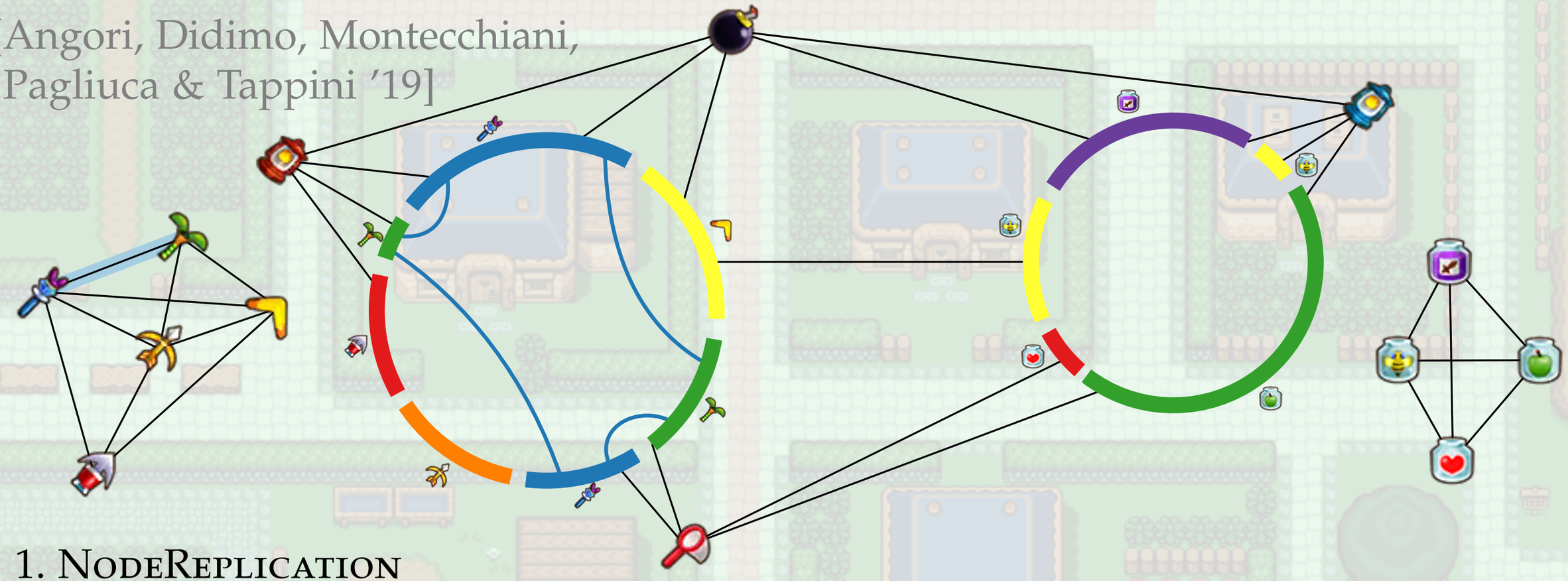

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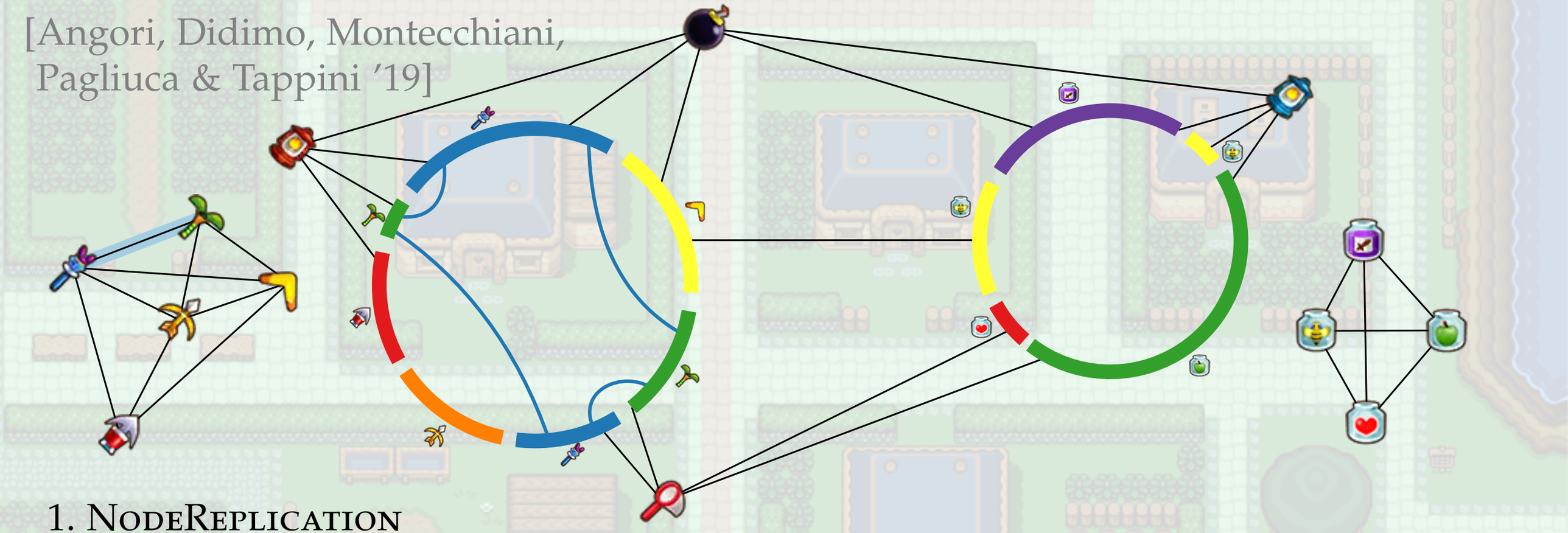

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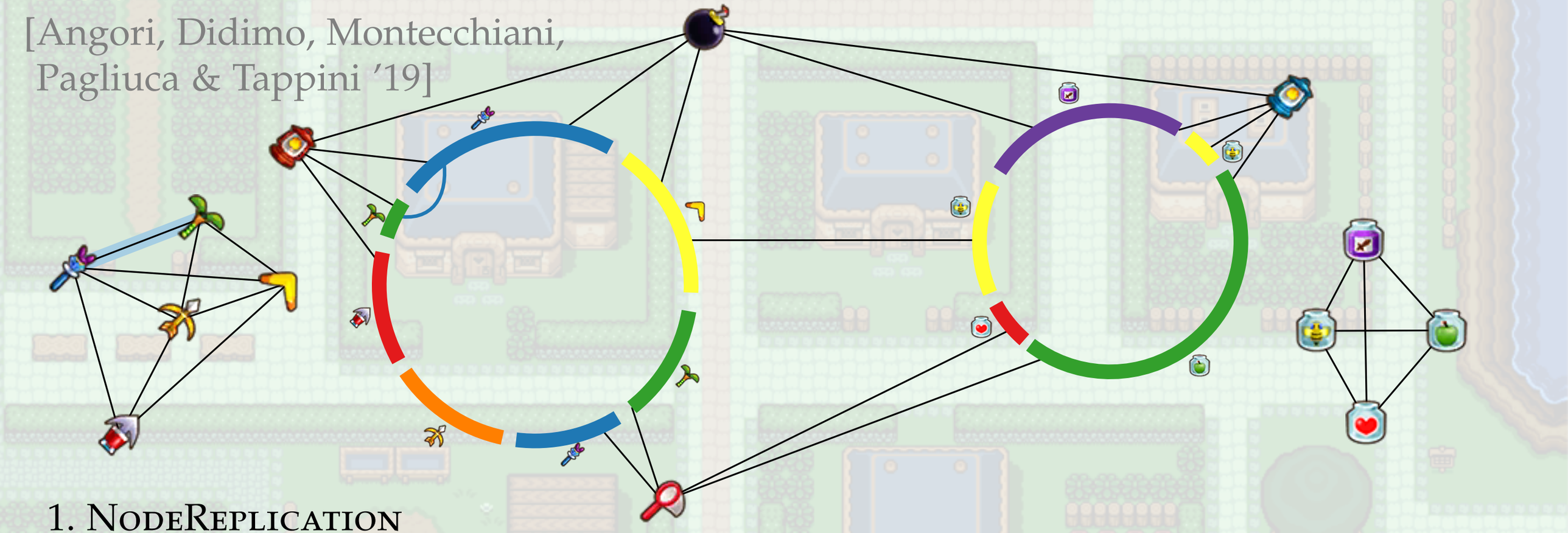

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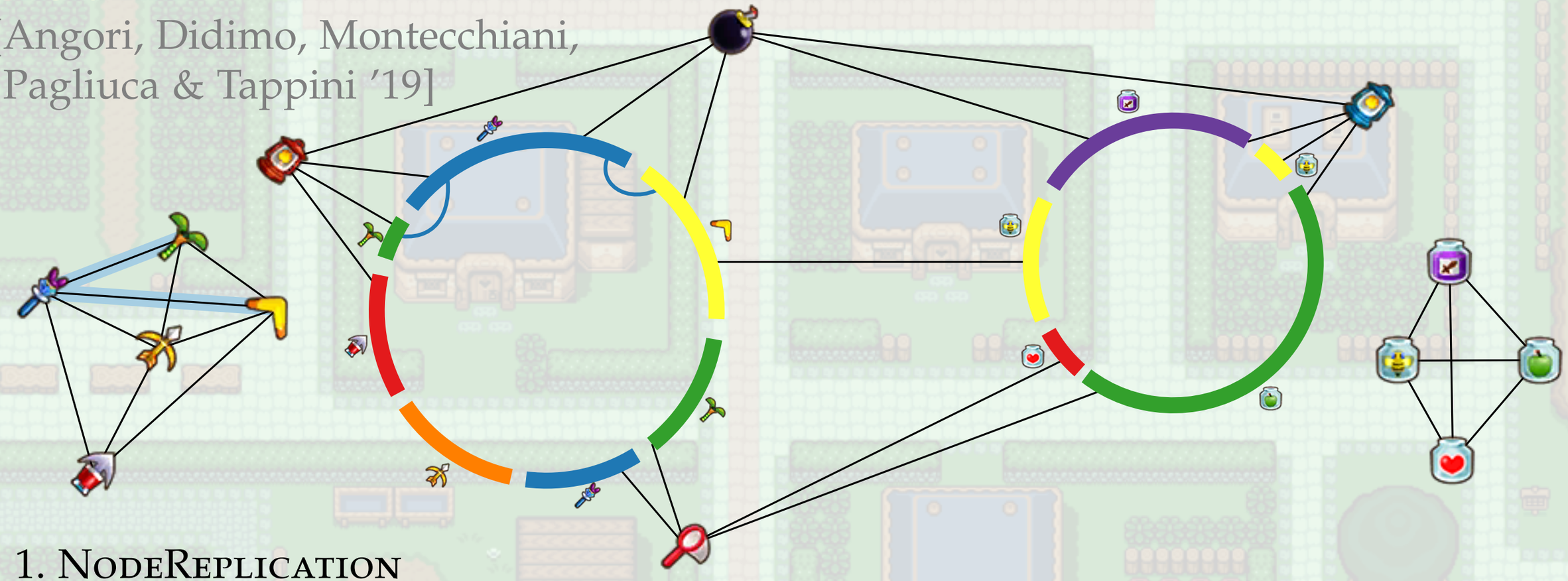

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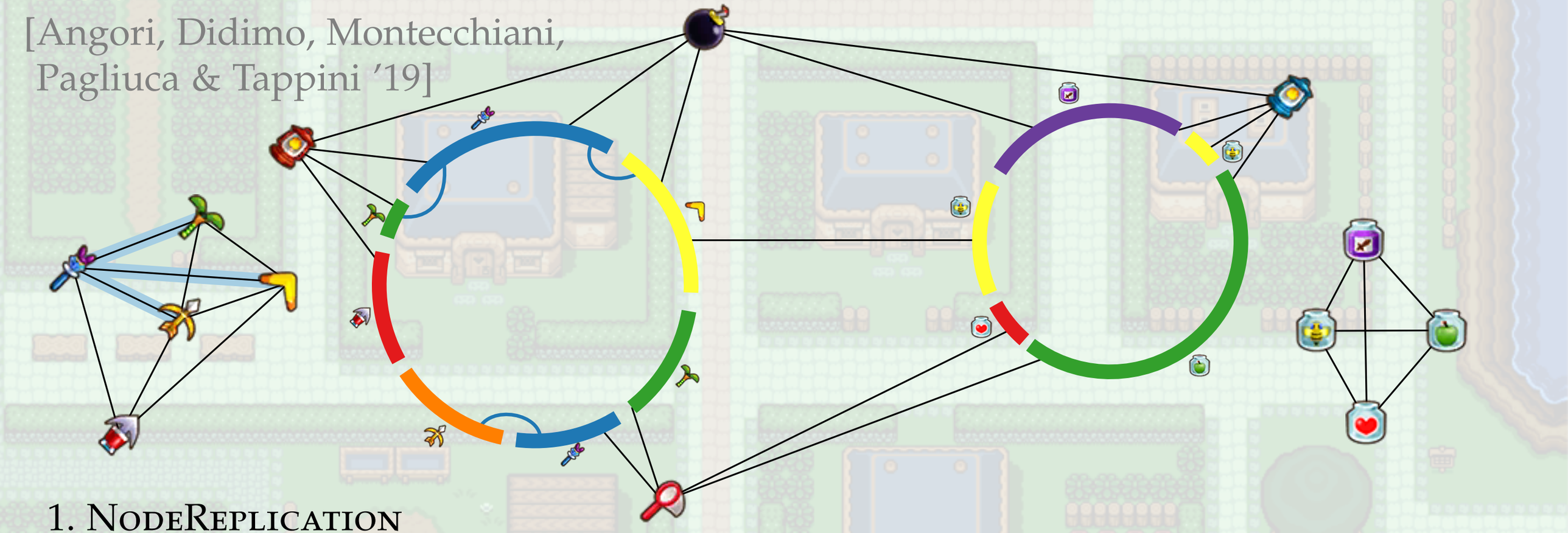

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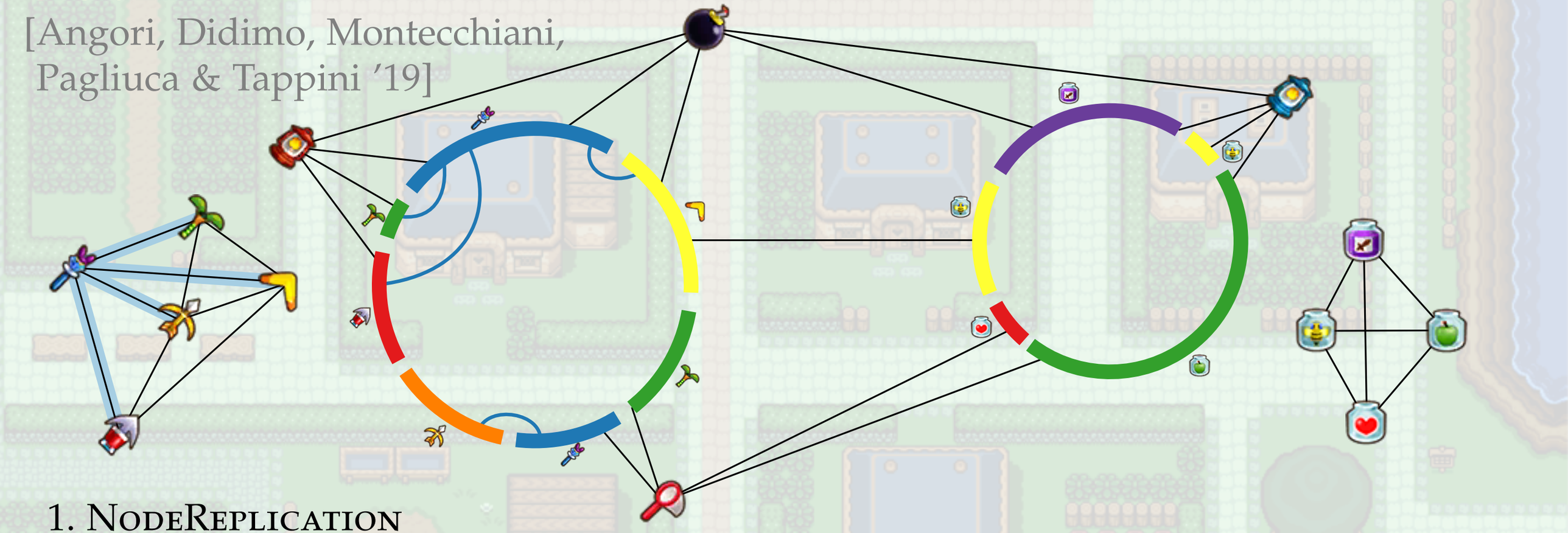

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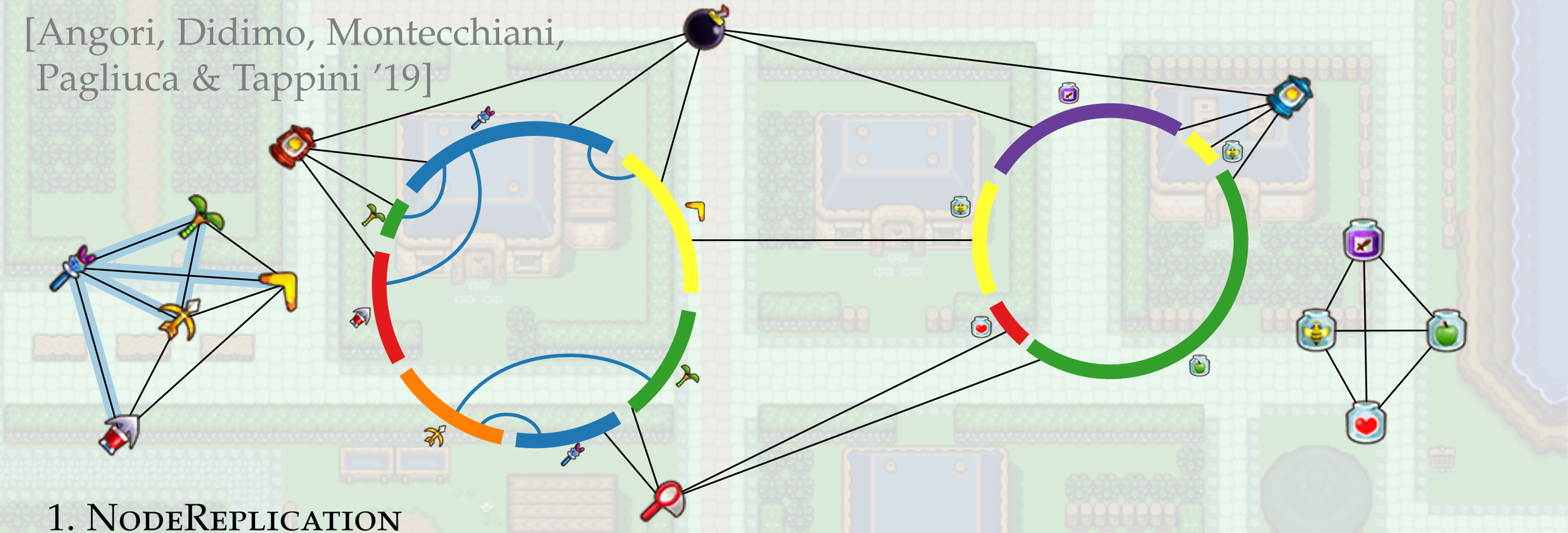

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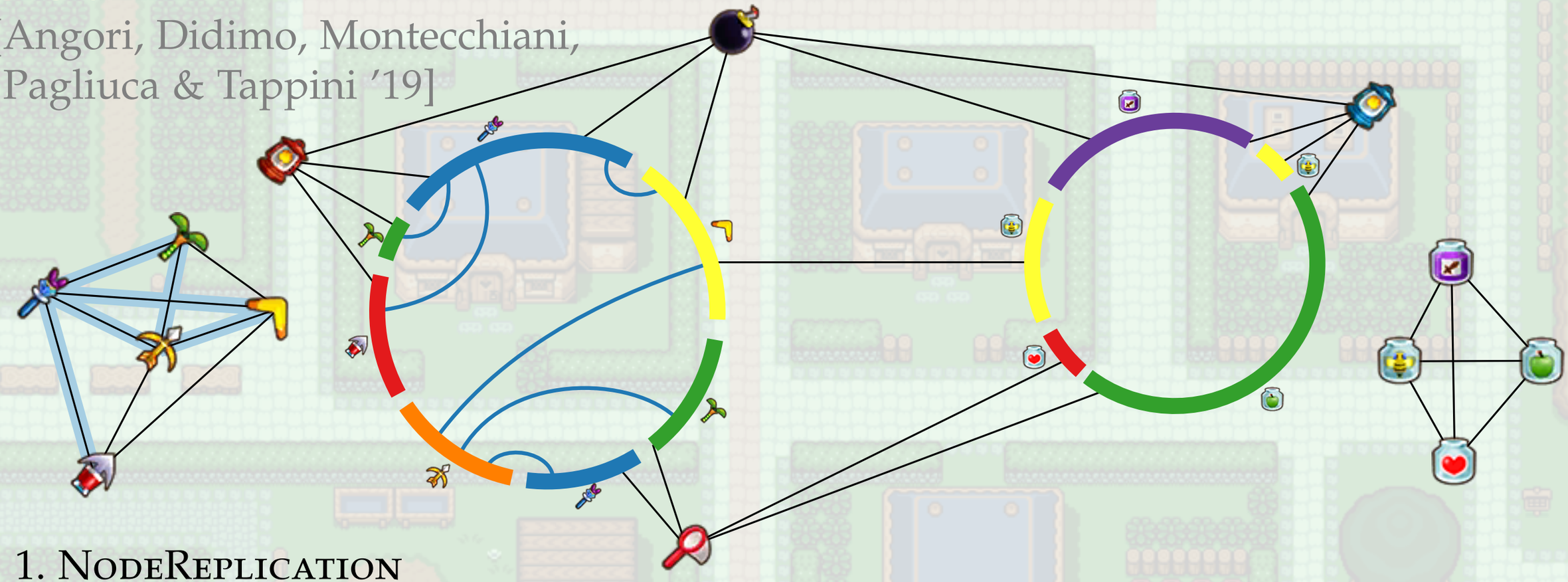

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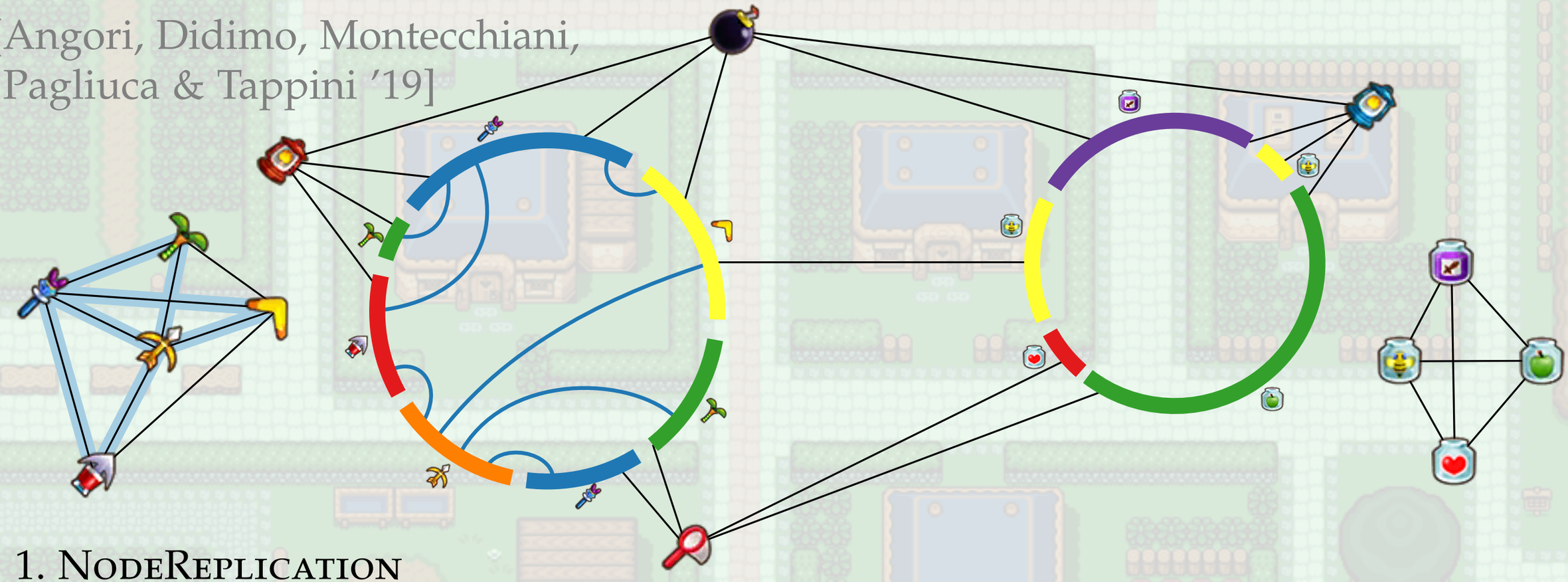

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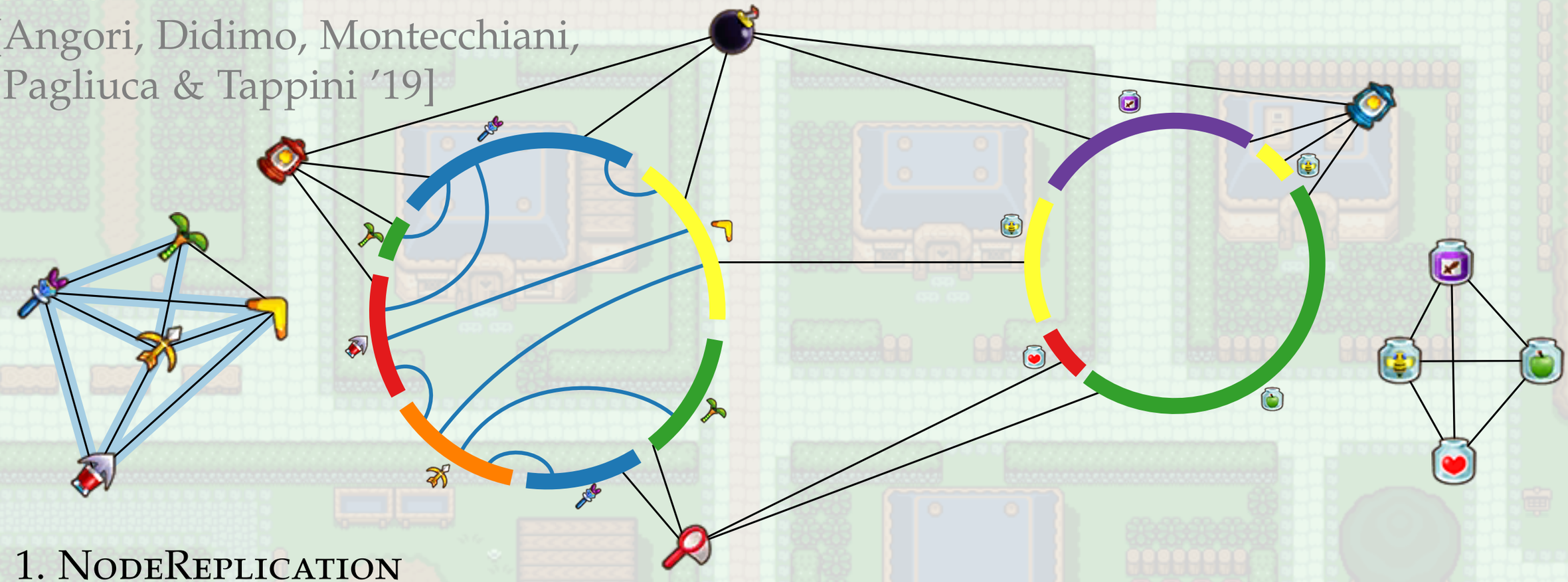

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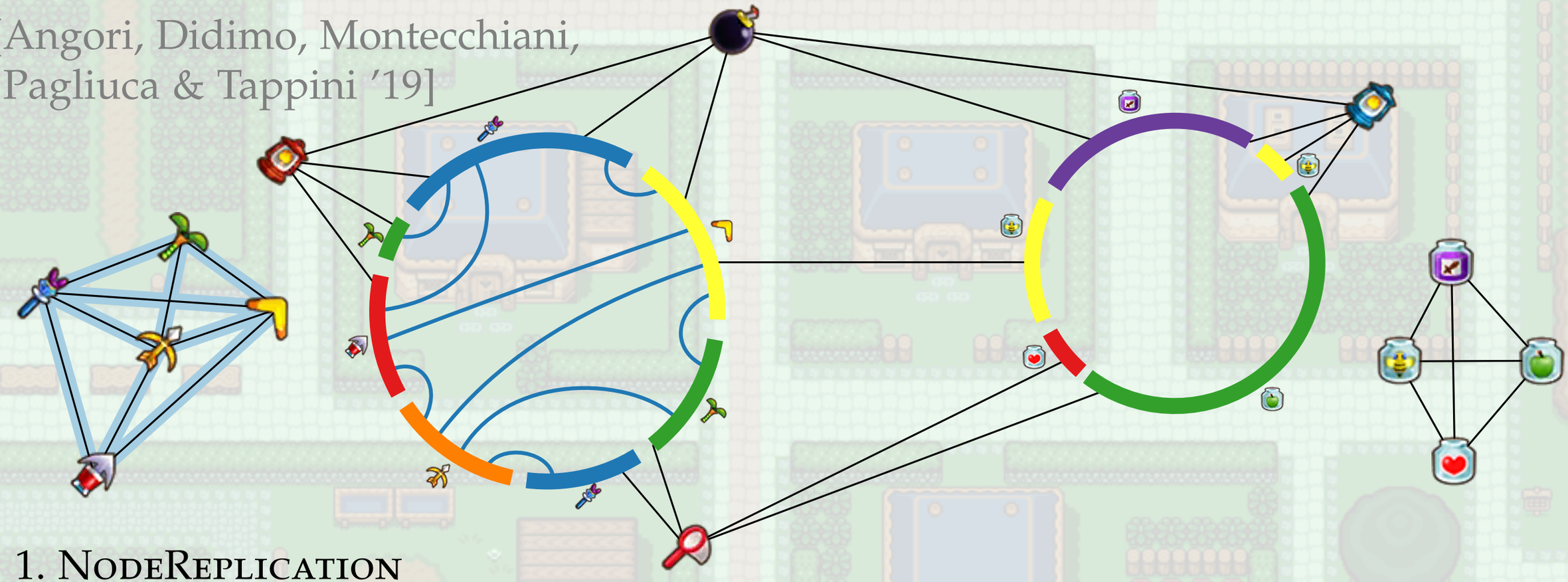

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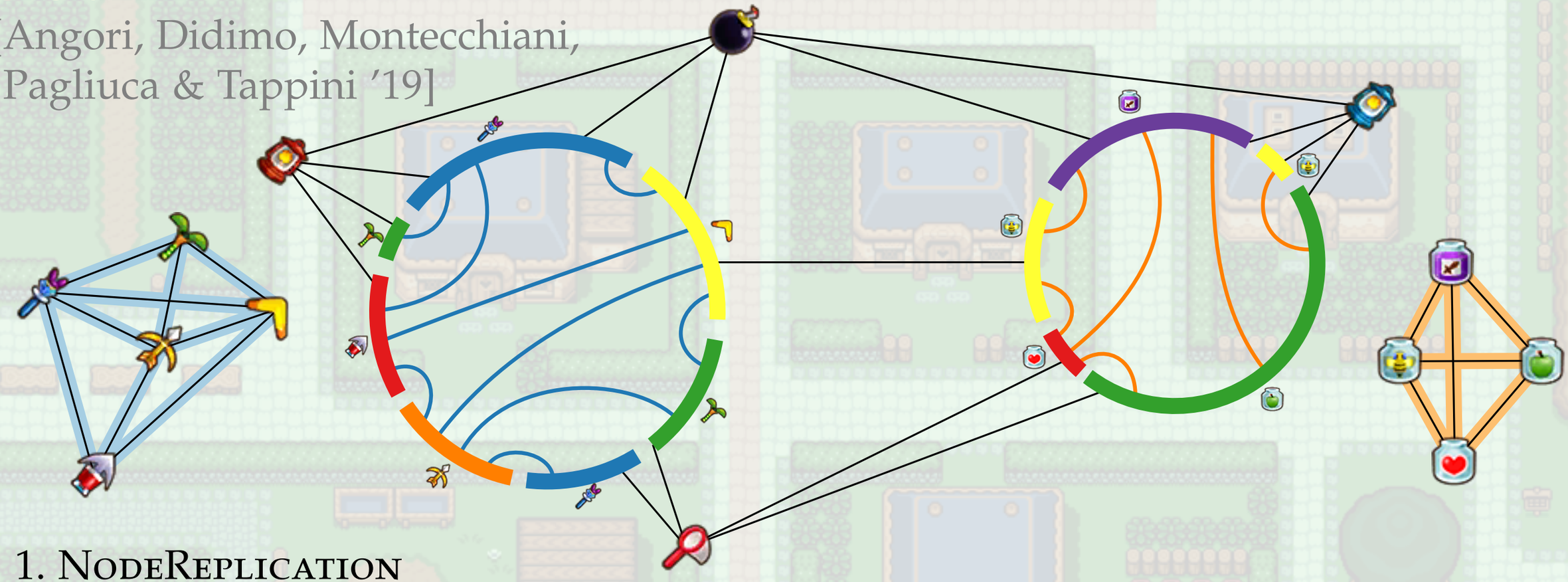

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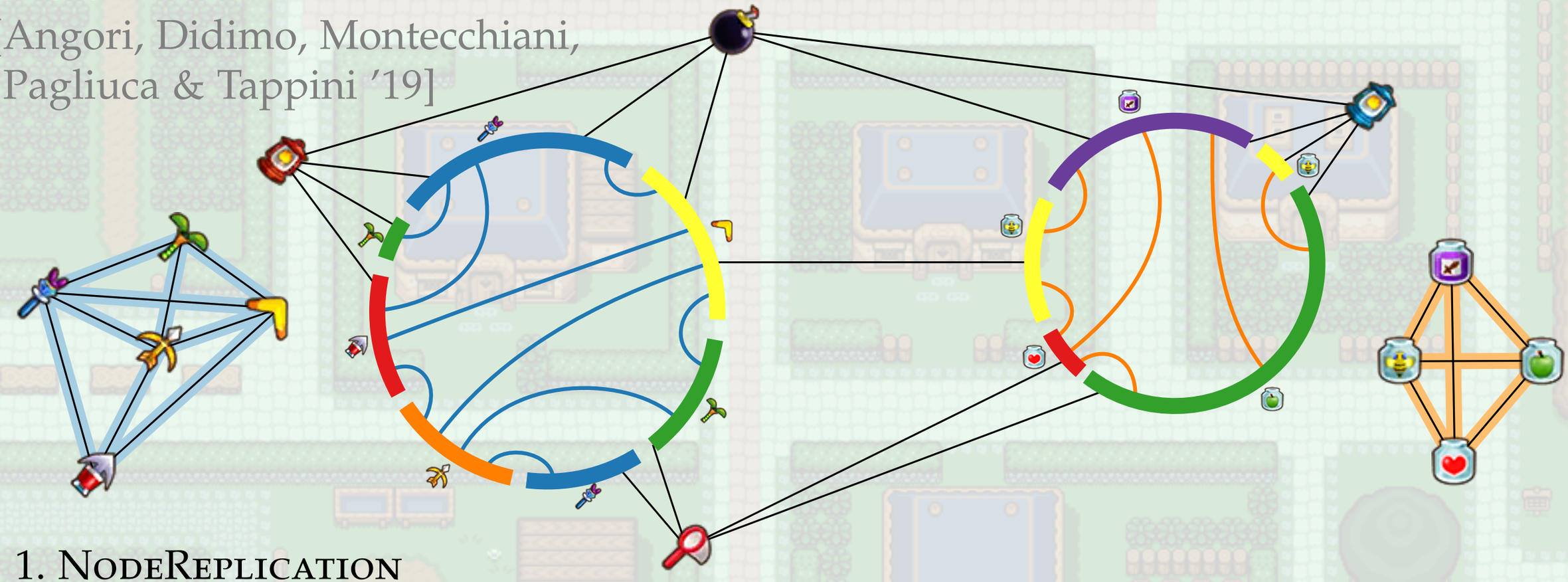

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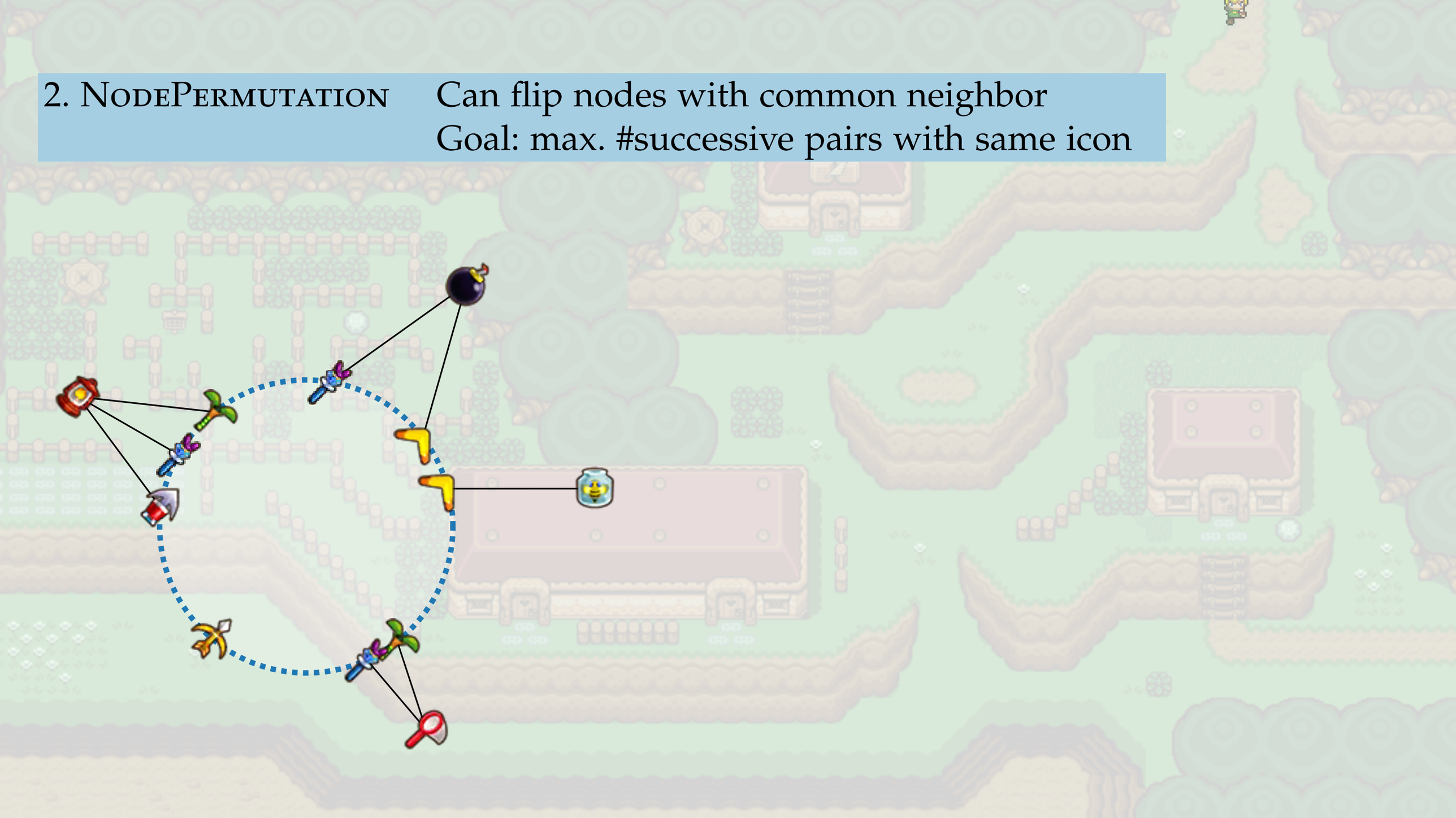

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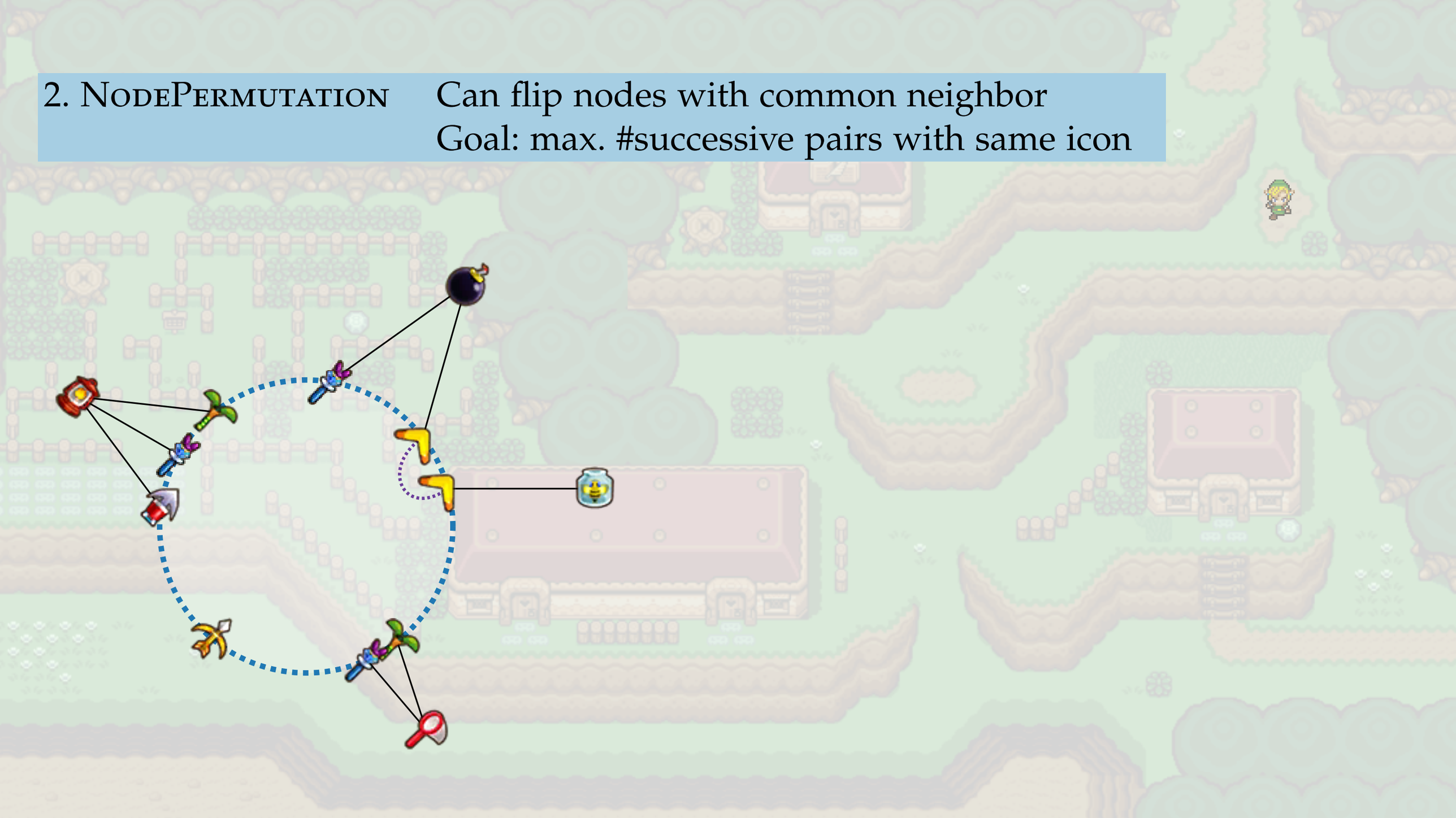

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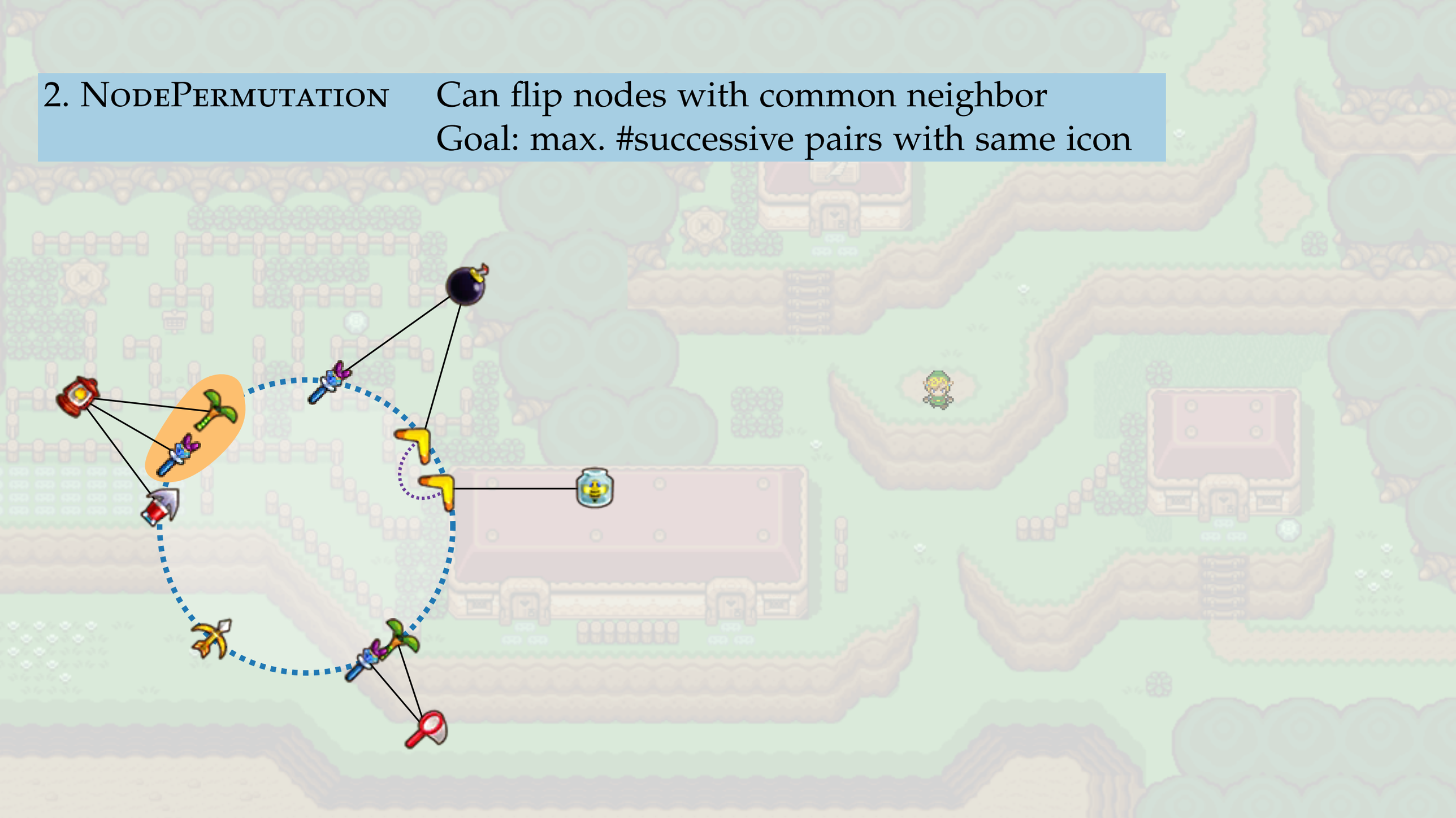

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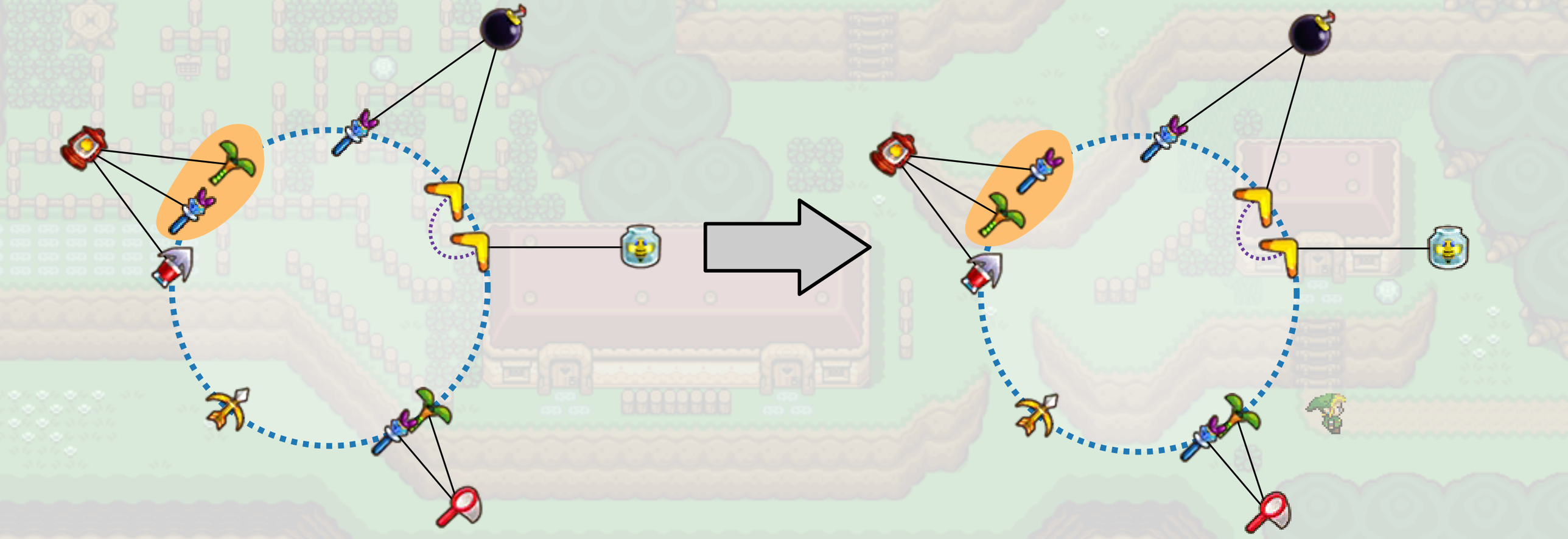

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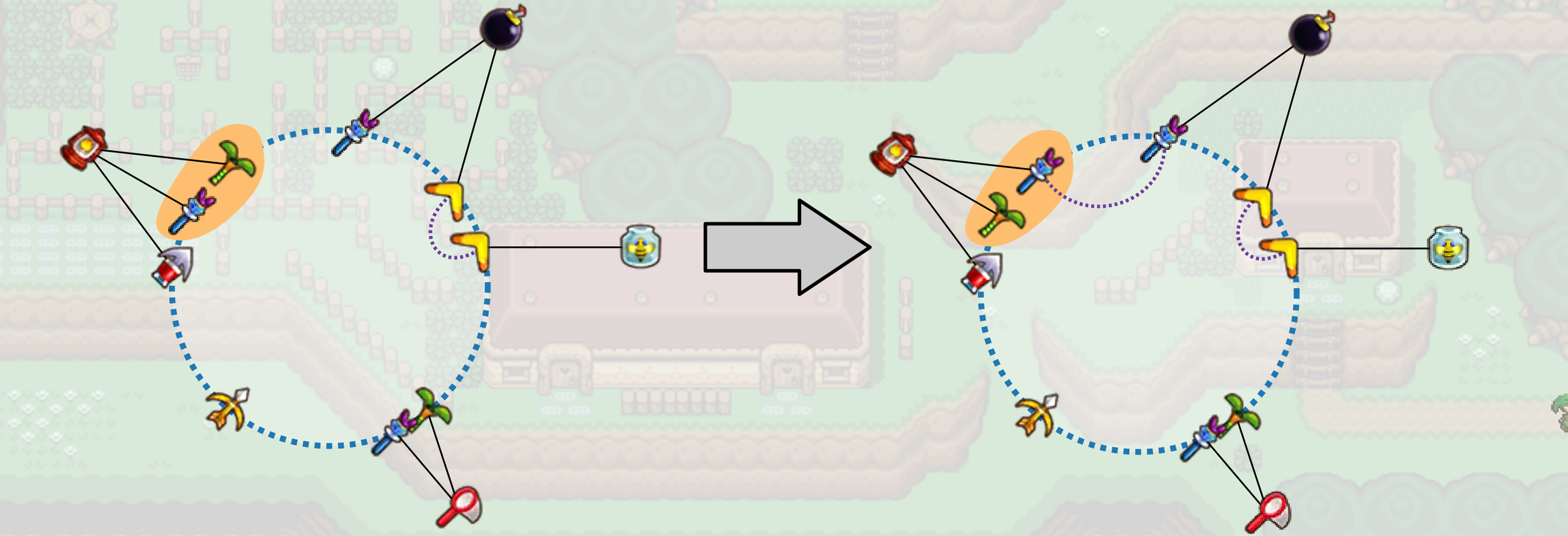
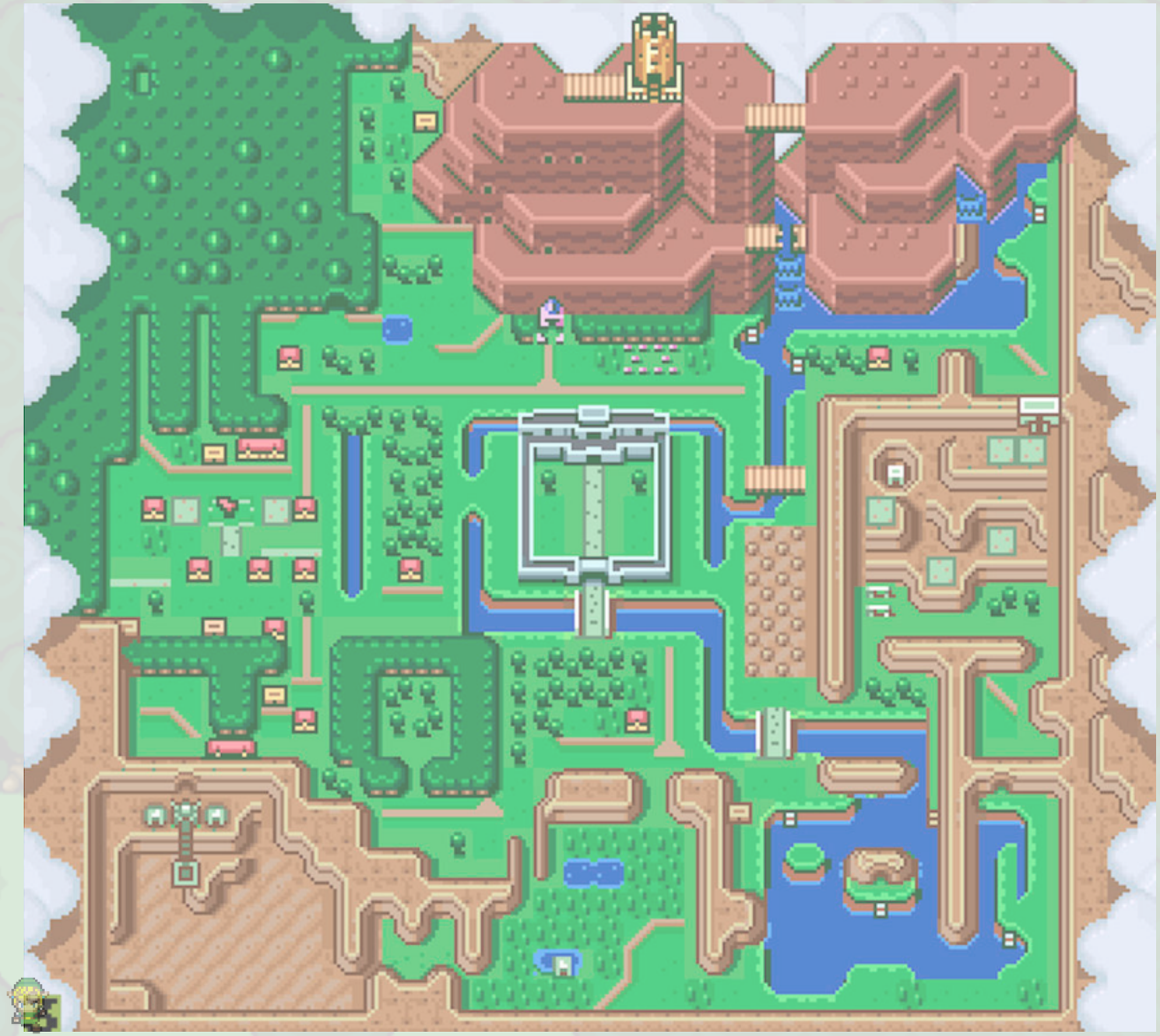
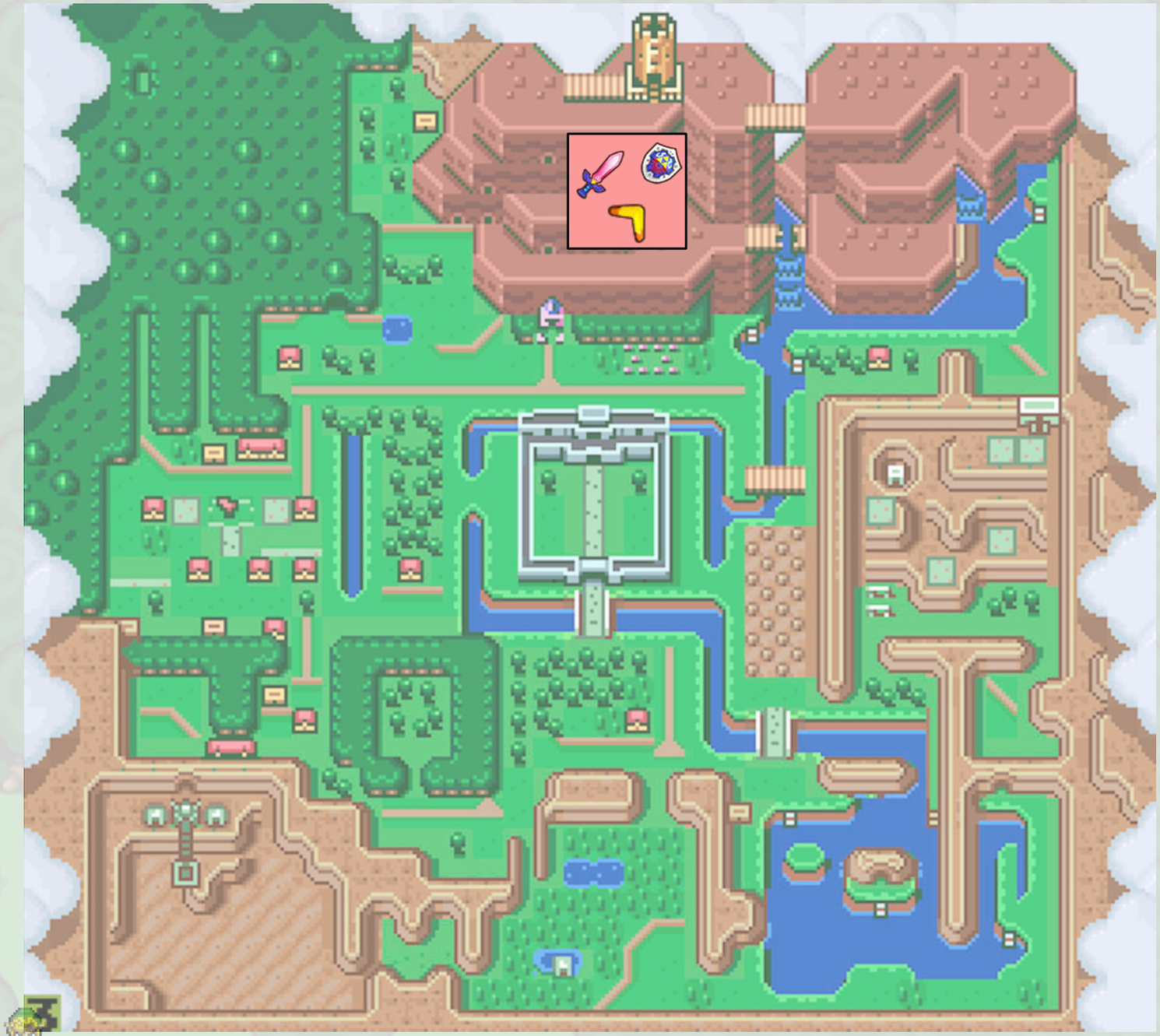
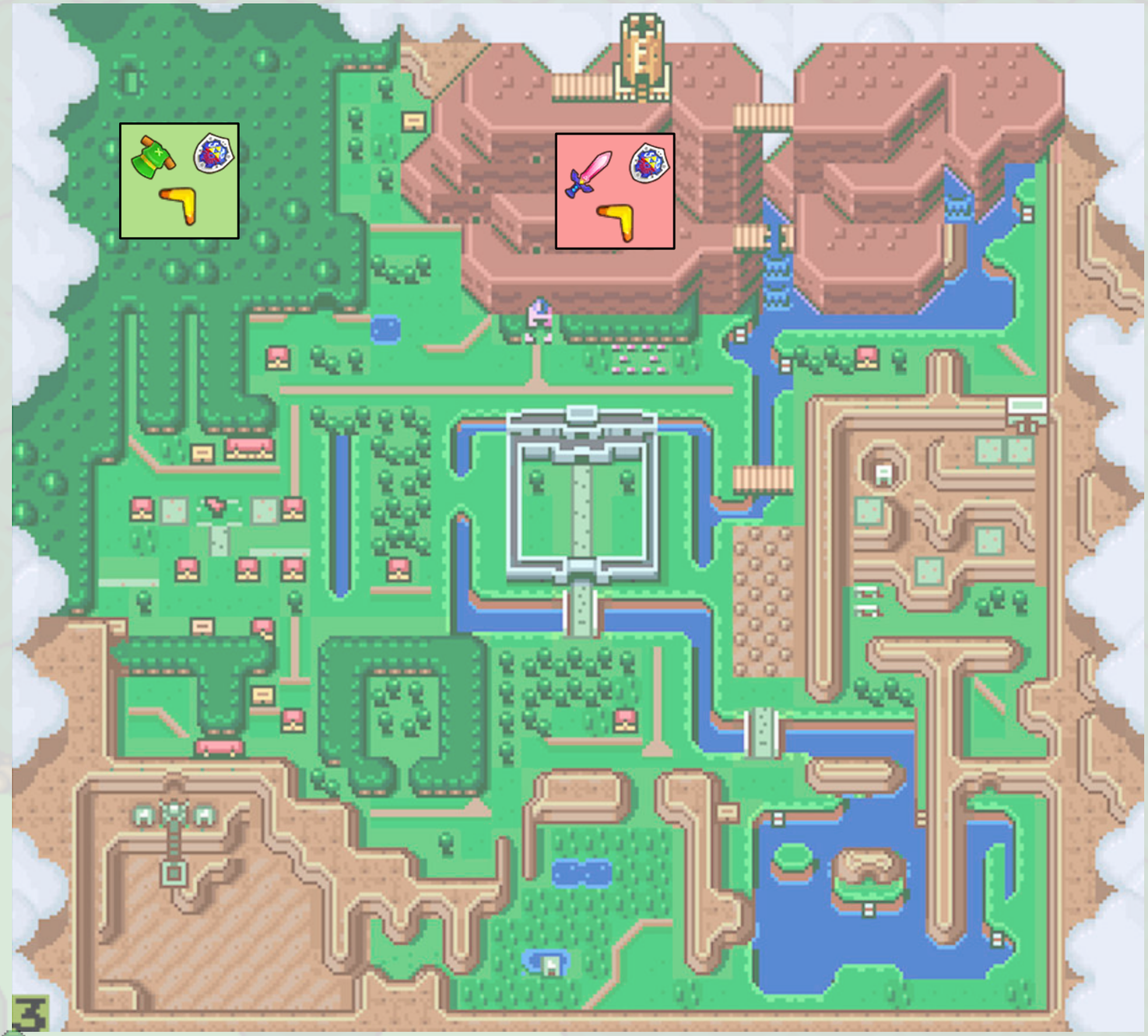
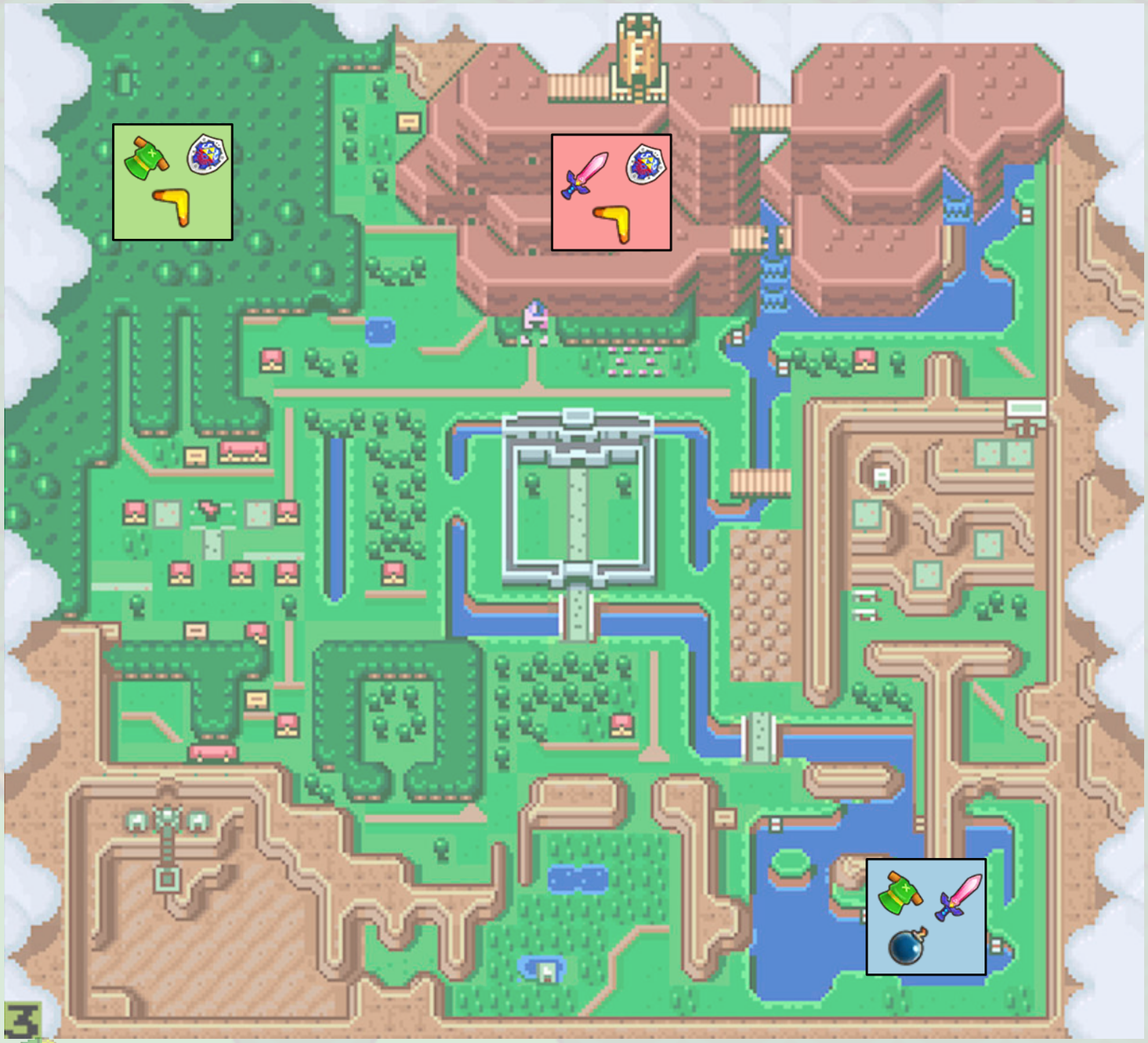
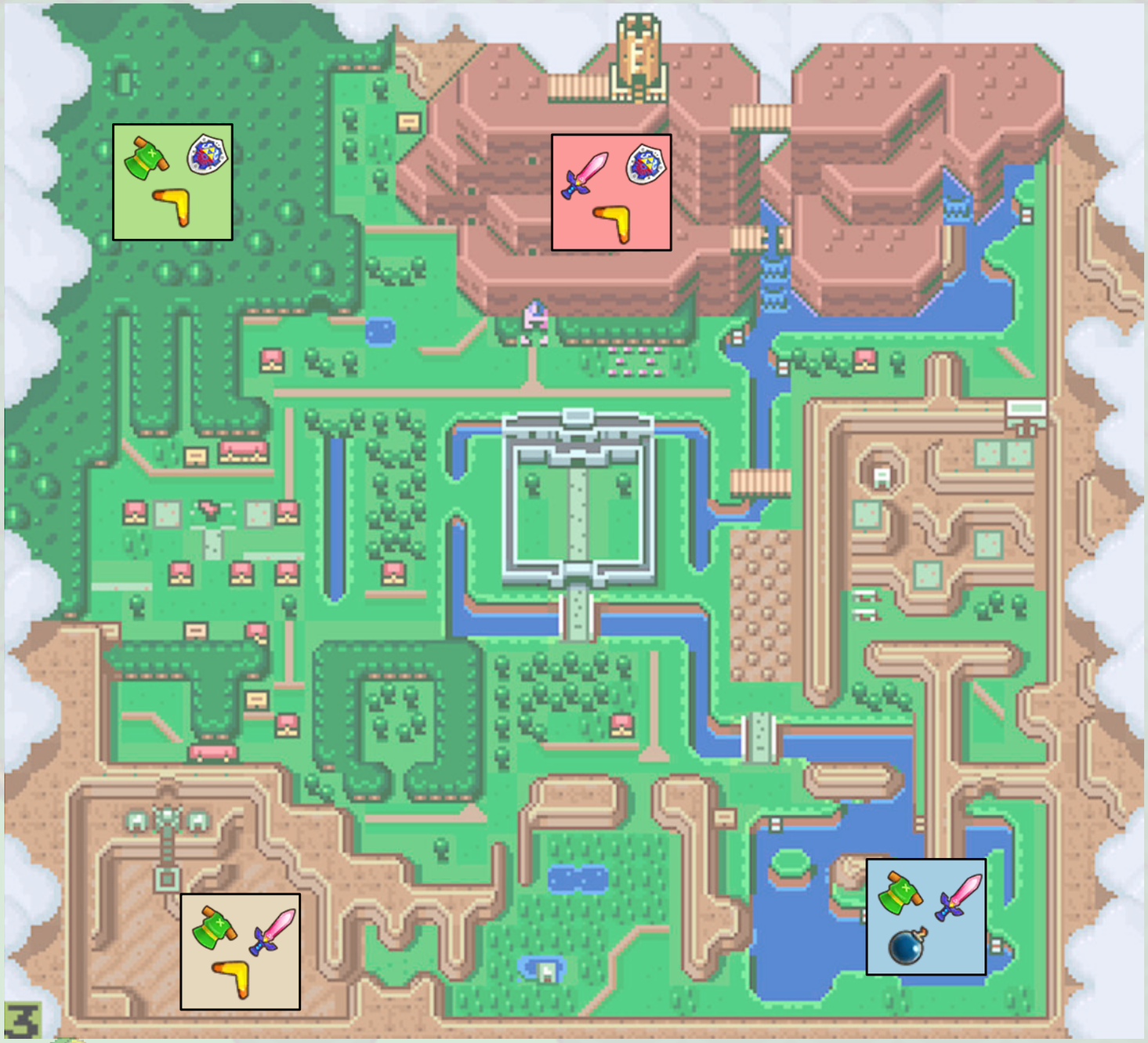


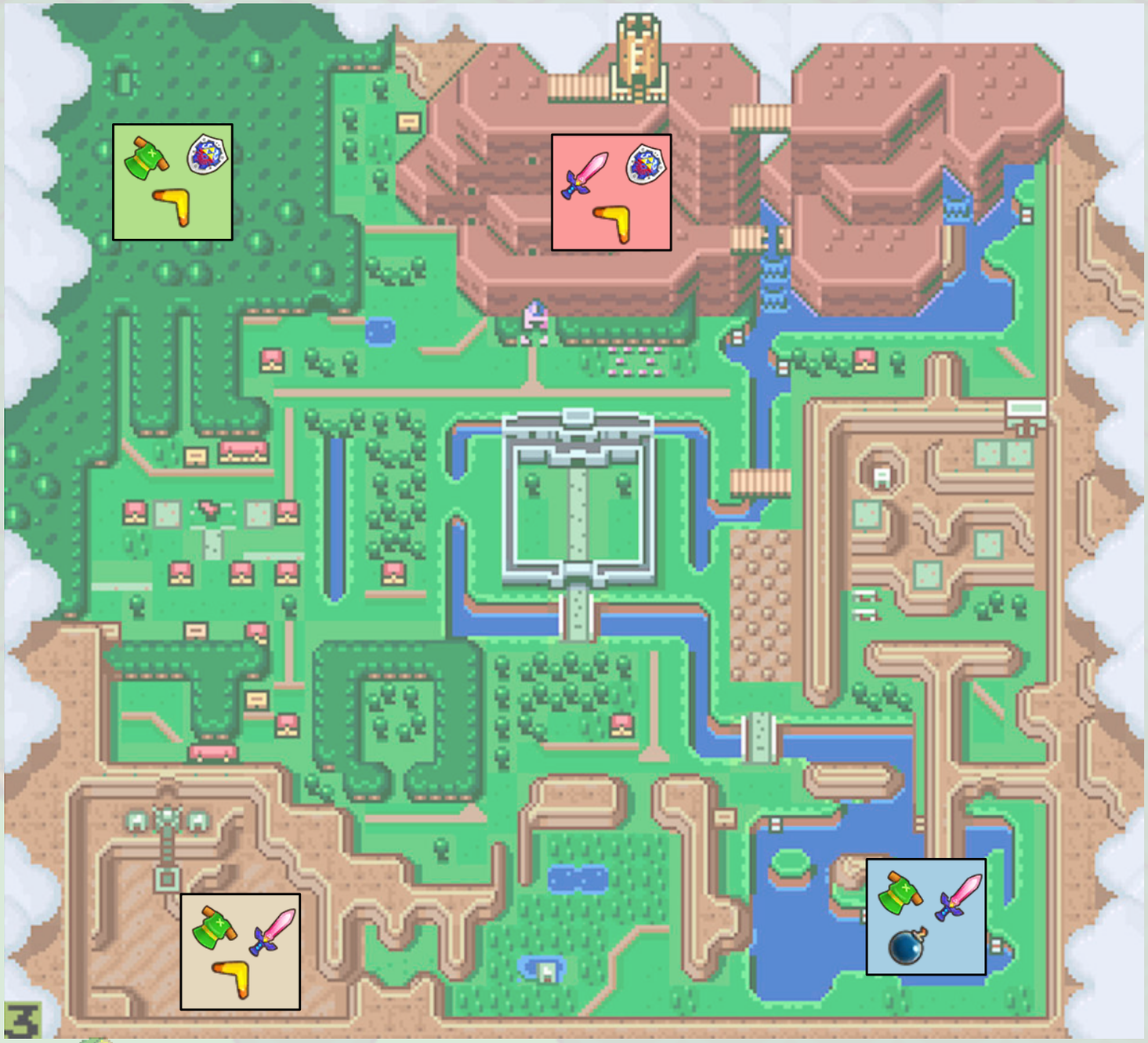




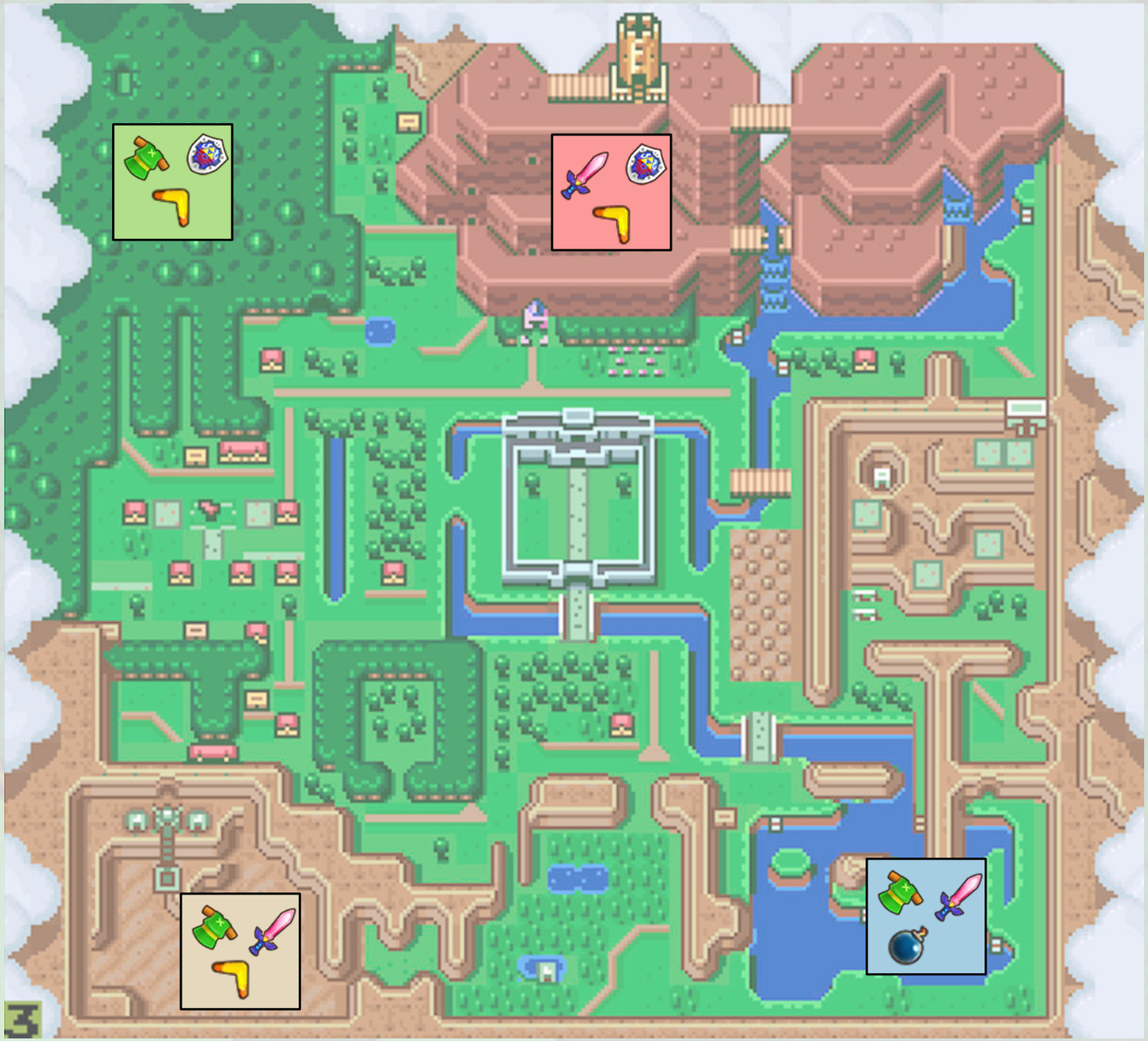








Where should I go to get all items with minimum effort?



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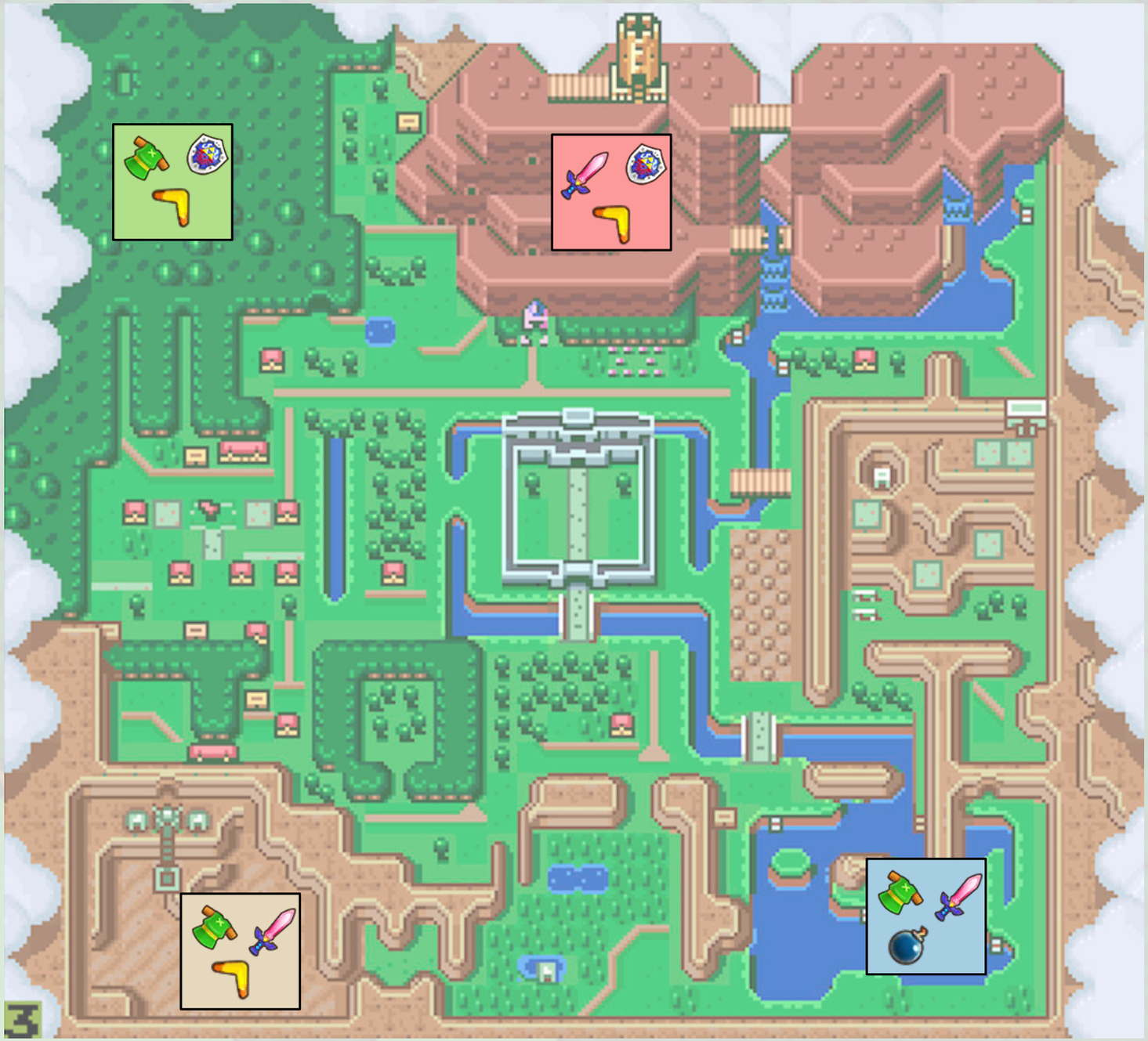

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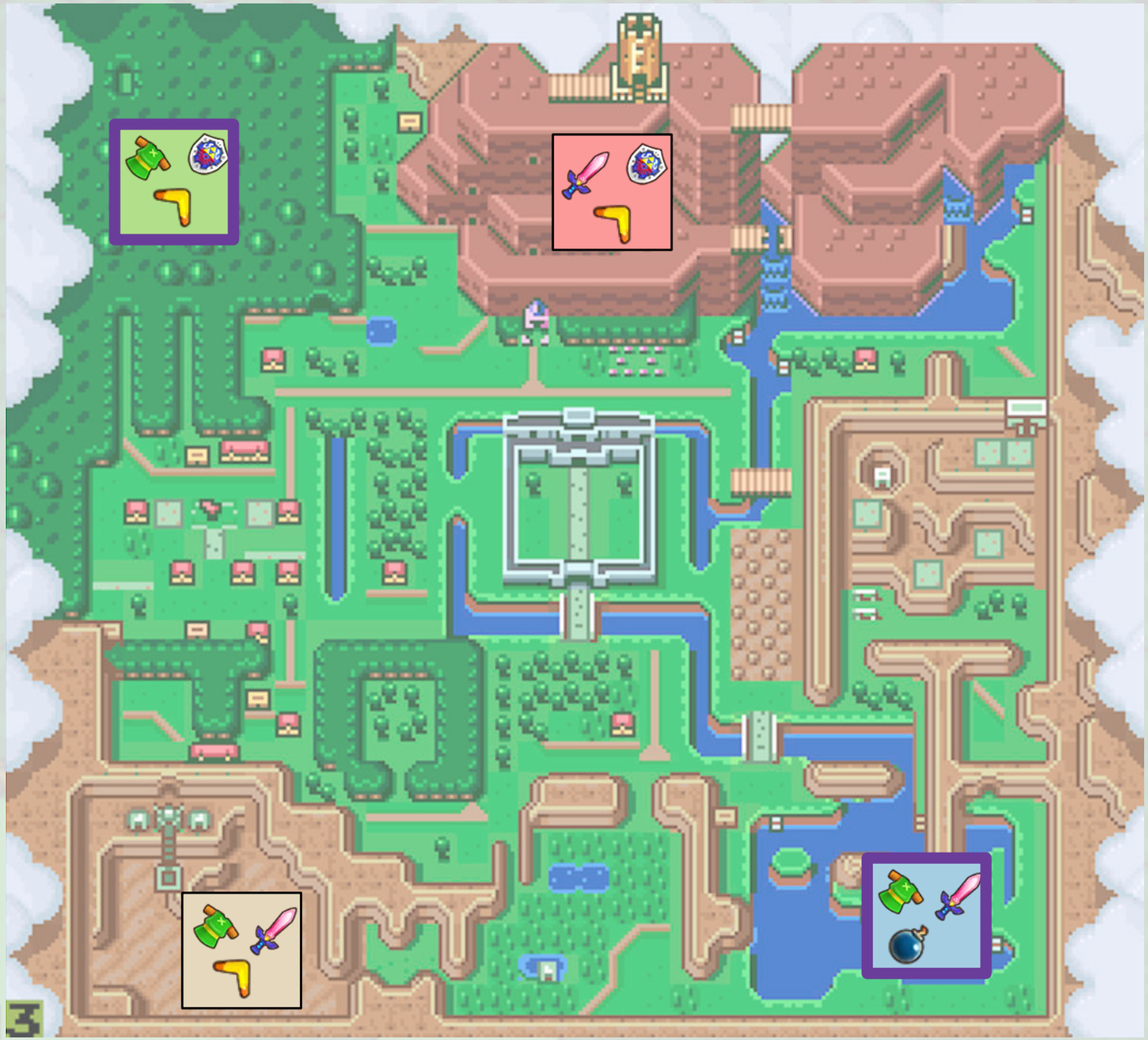
Is there a size-k SetCover?



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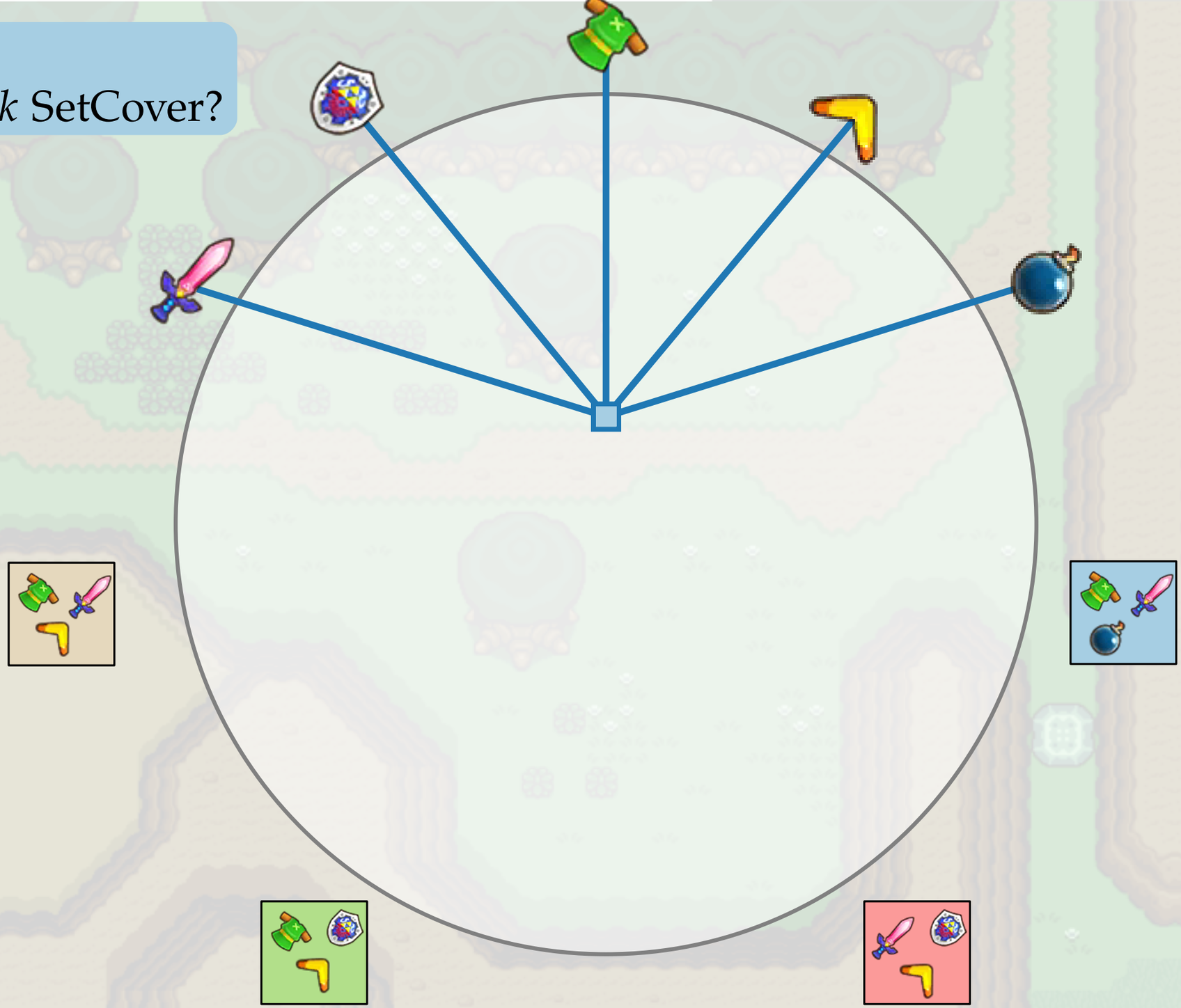

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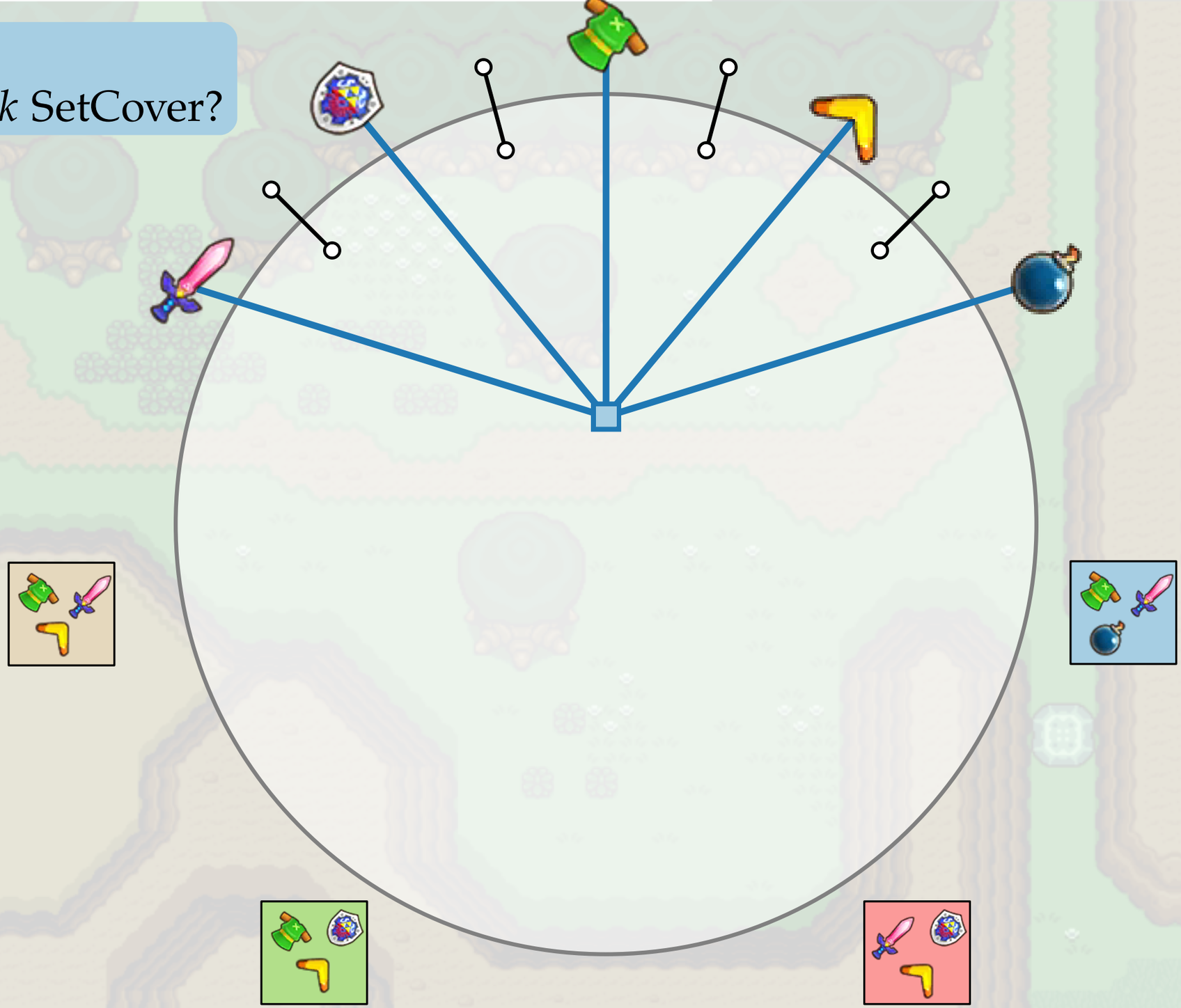

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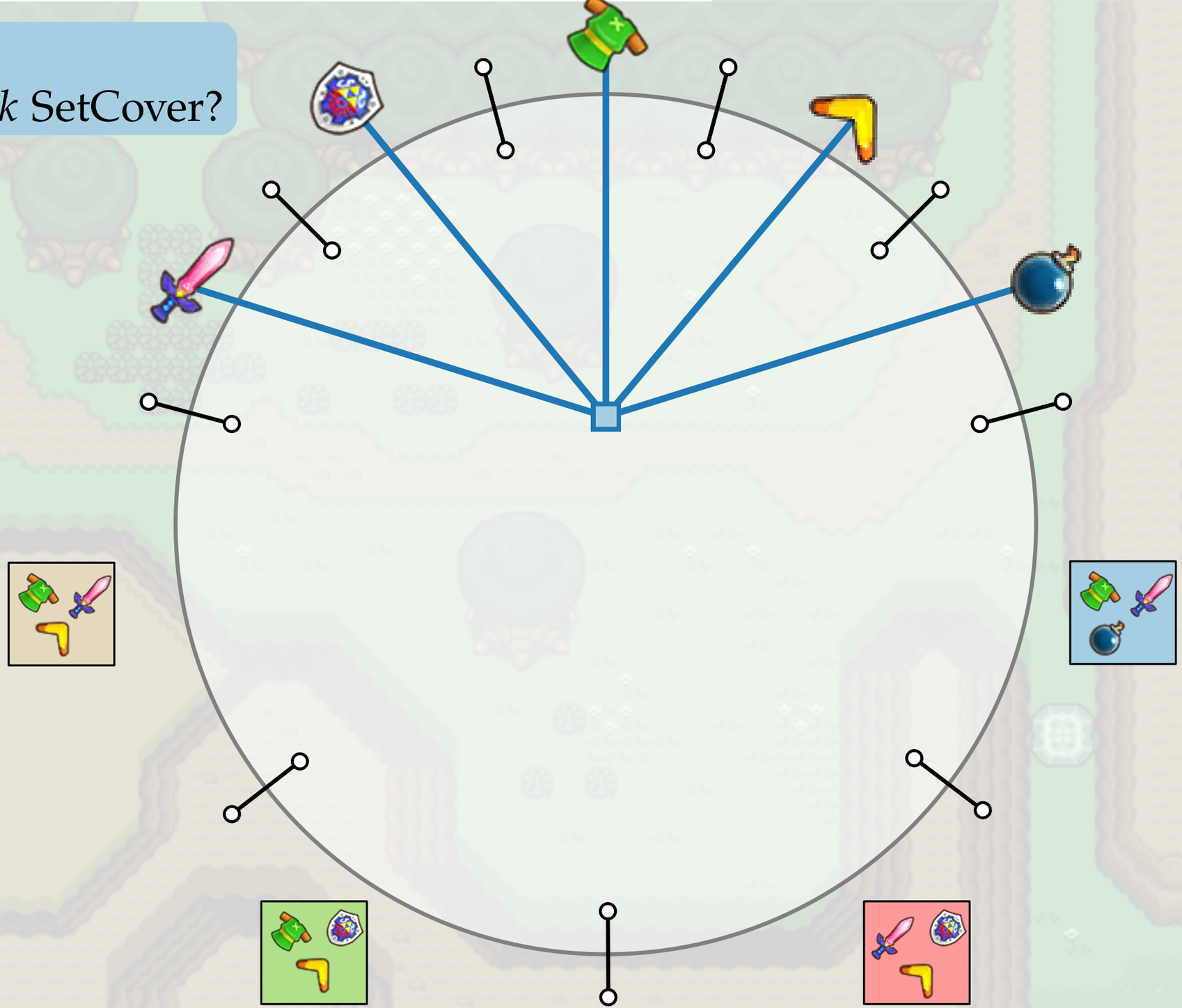

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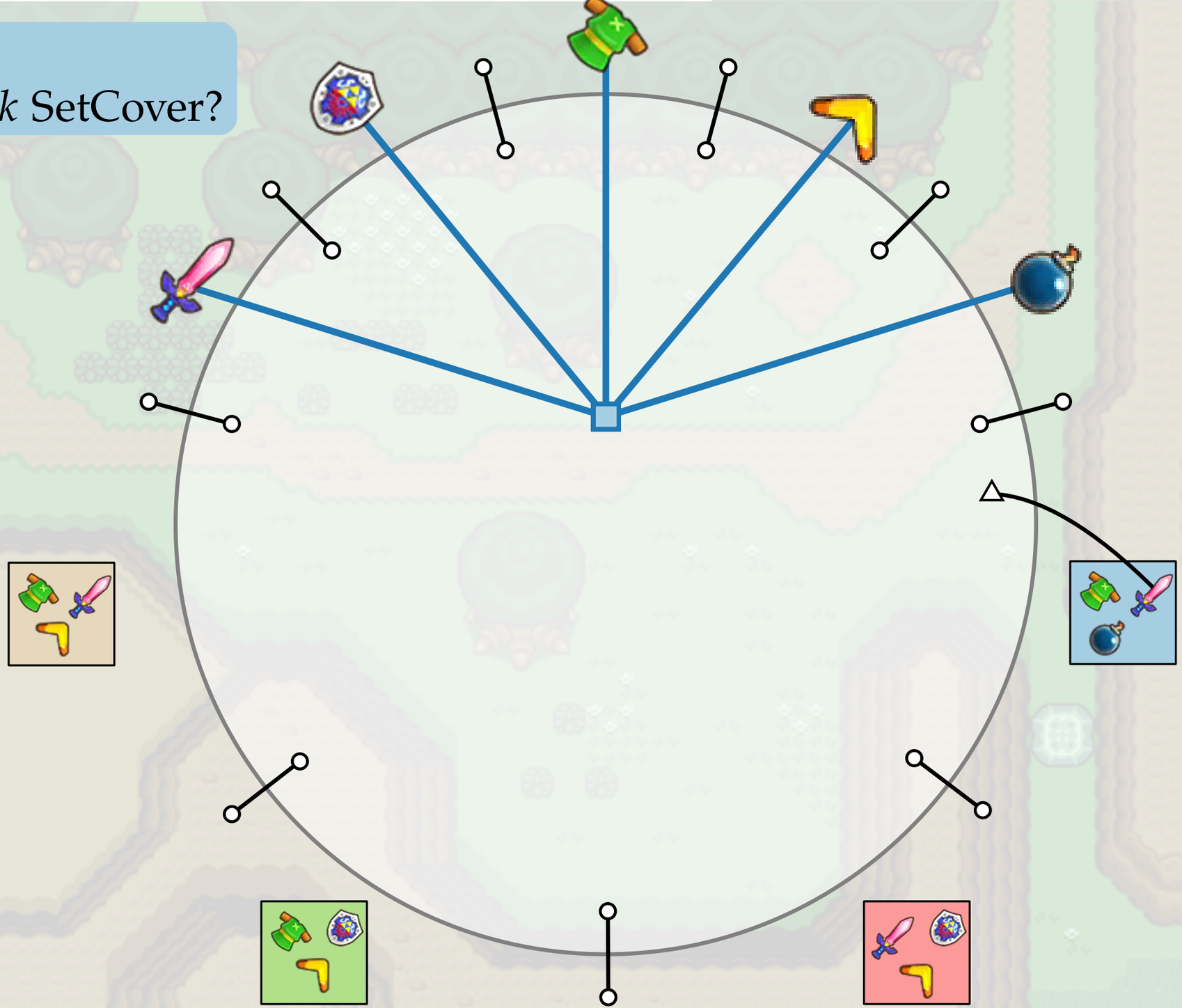

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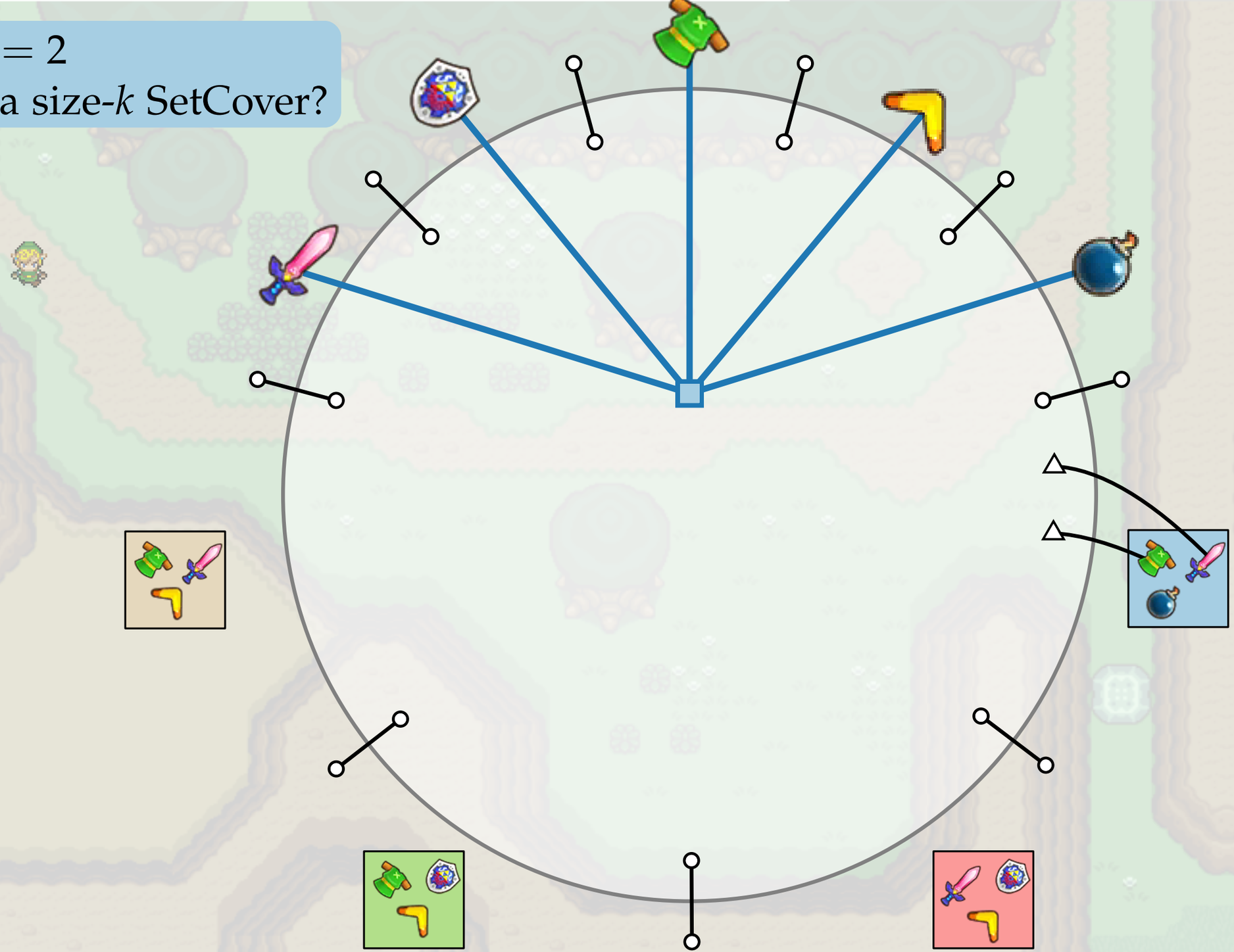

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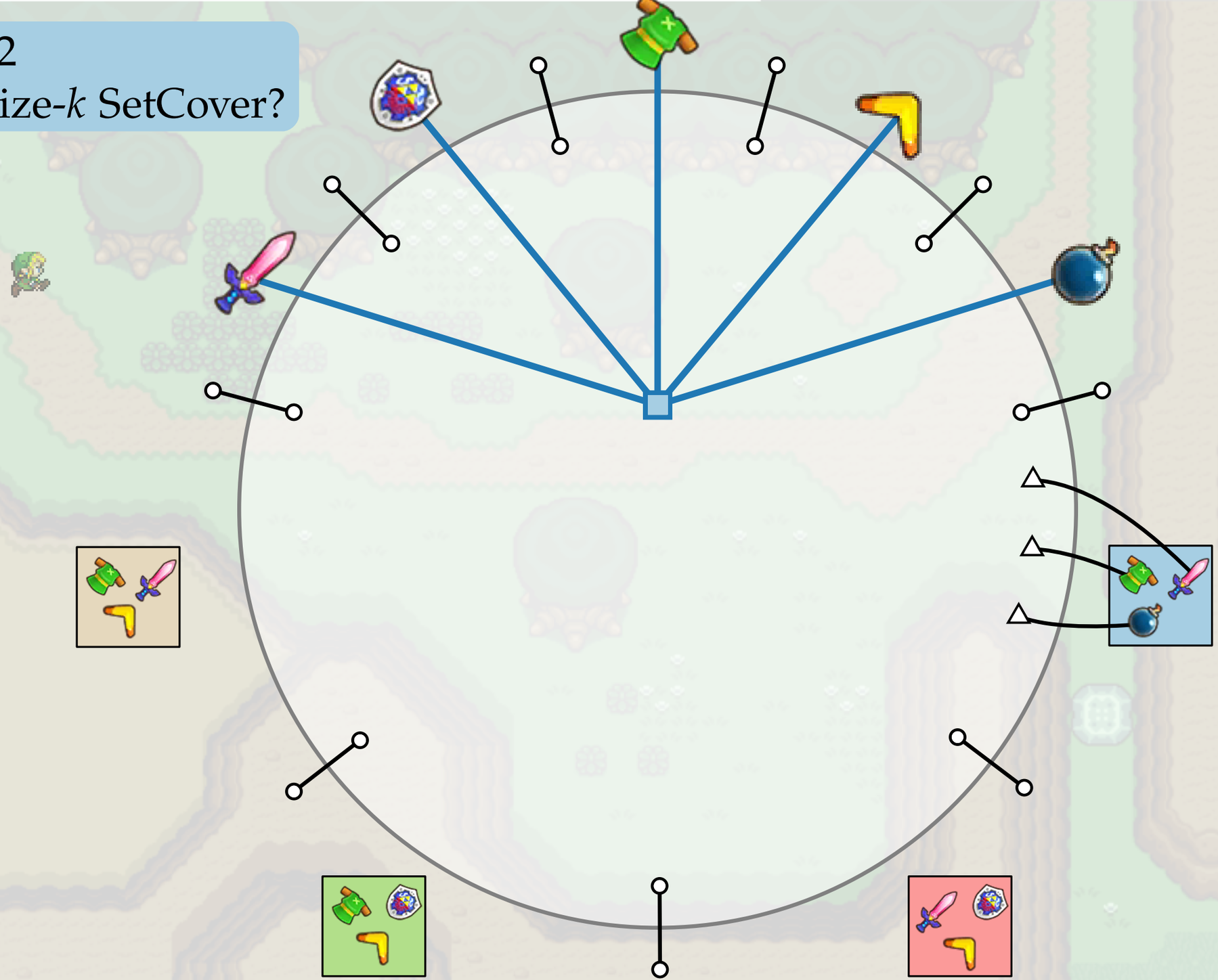

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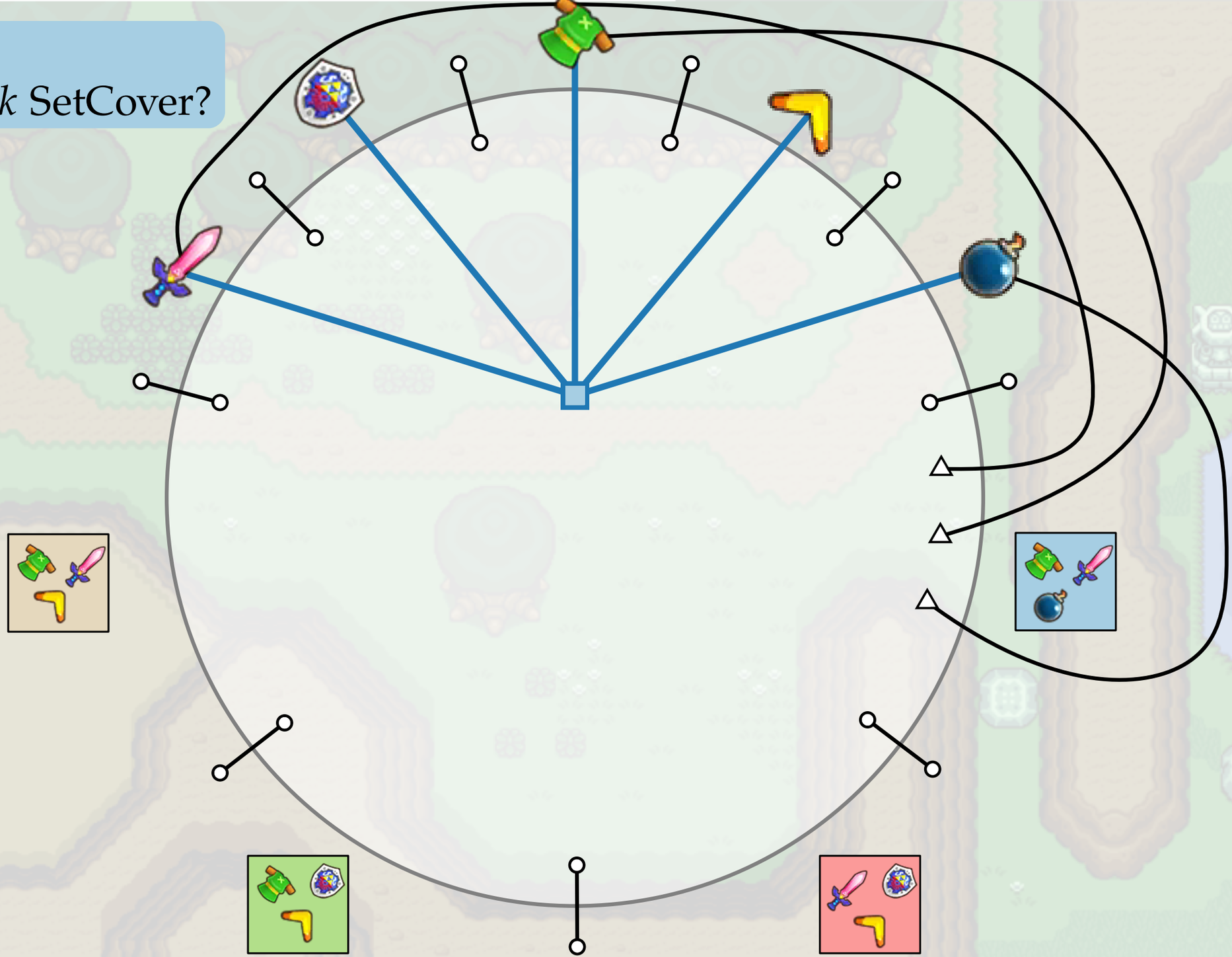

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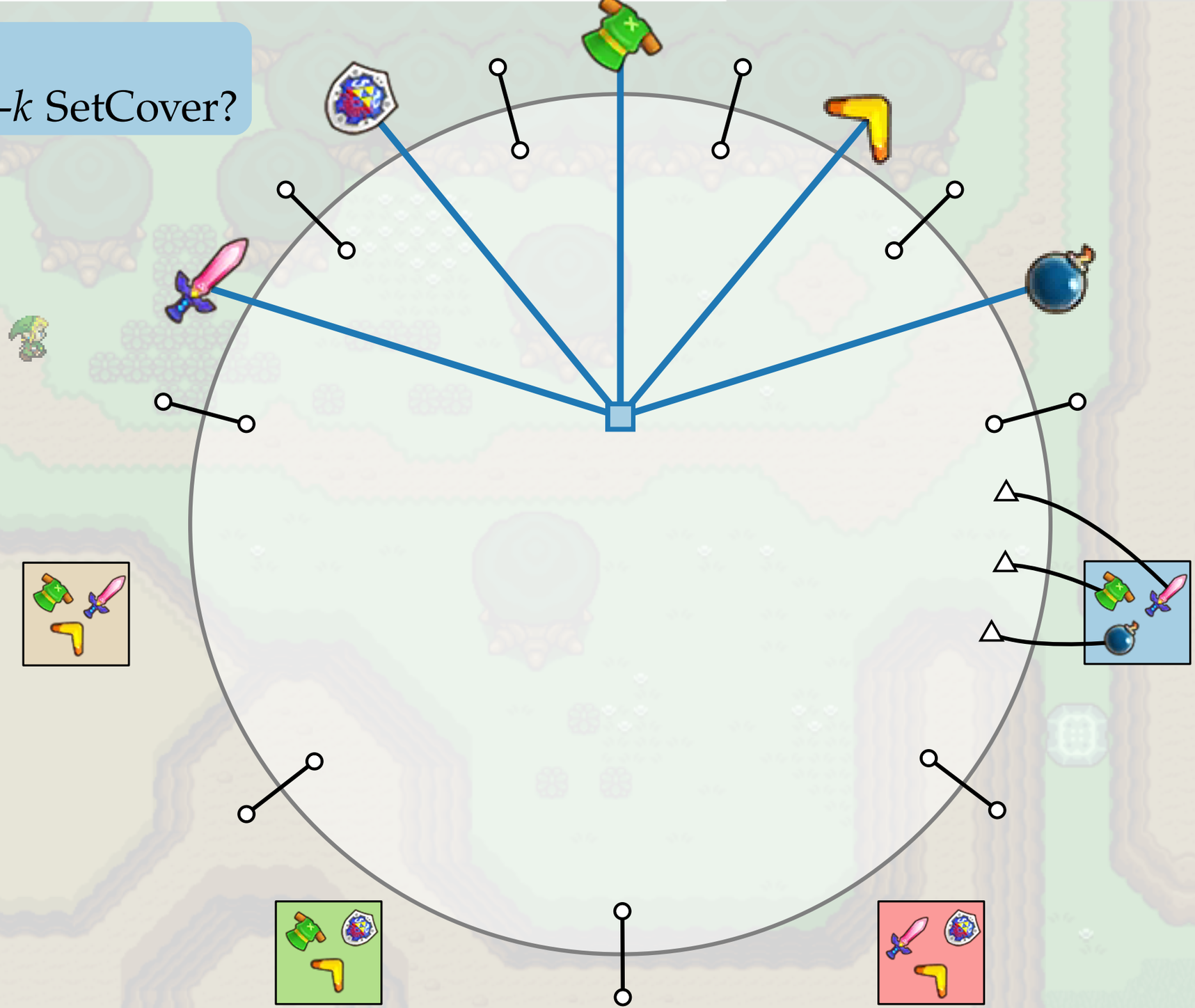

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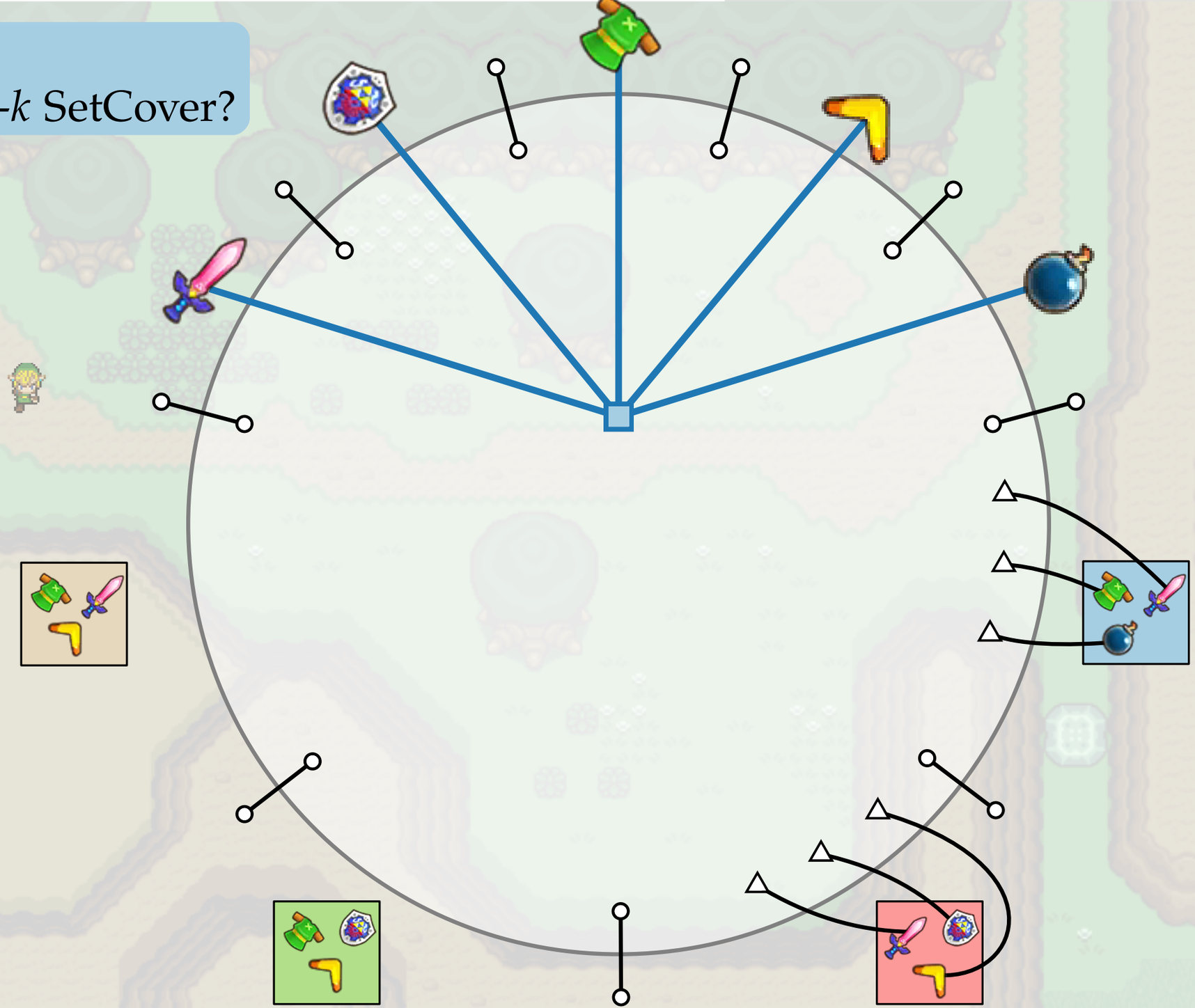
Is there a size-k SetCover?



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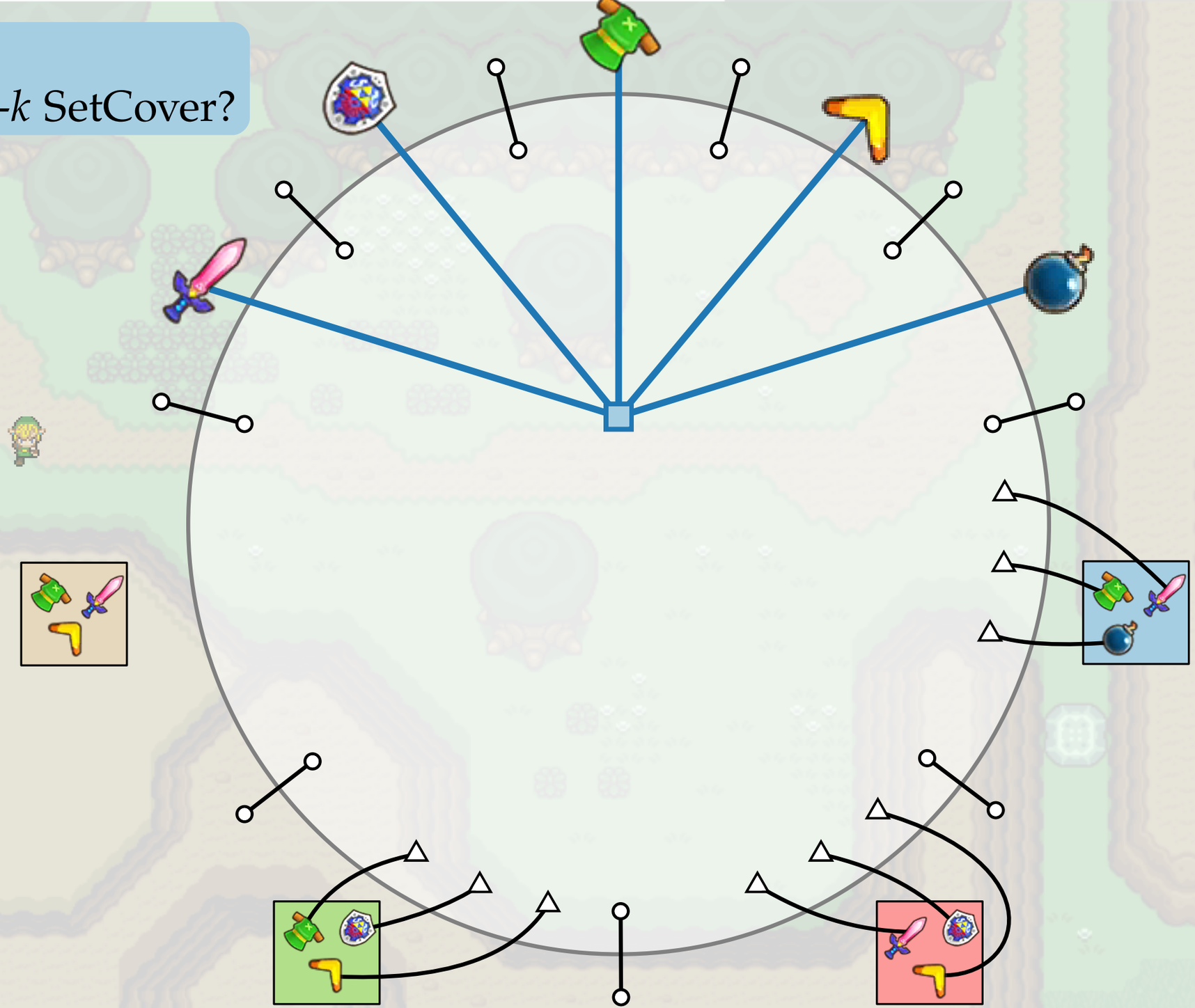
Is there a size-k SetCover?



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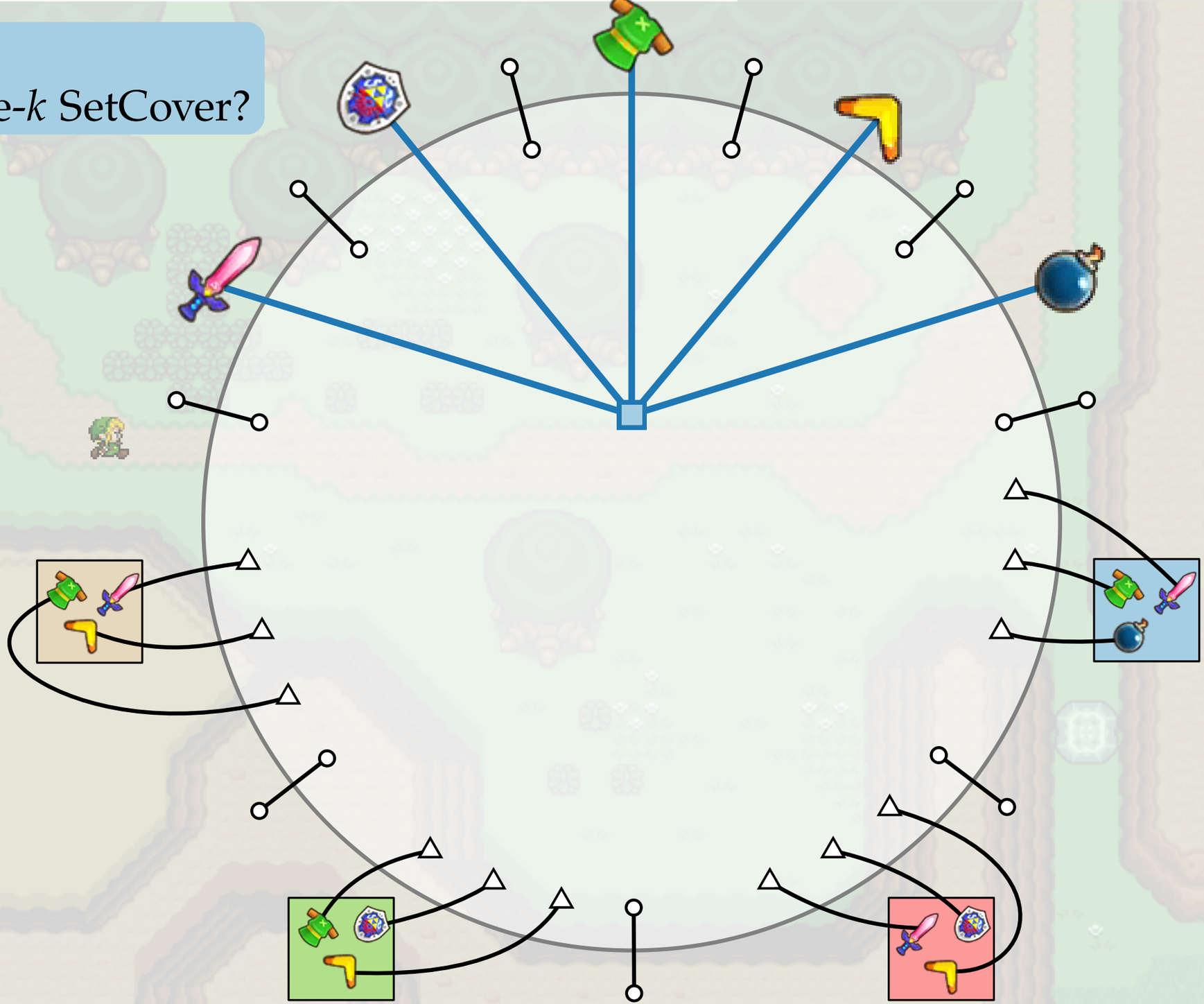
Is there a size-k SetCover?



$n = 5, k = 2$

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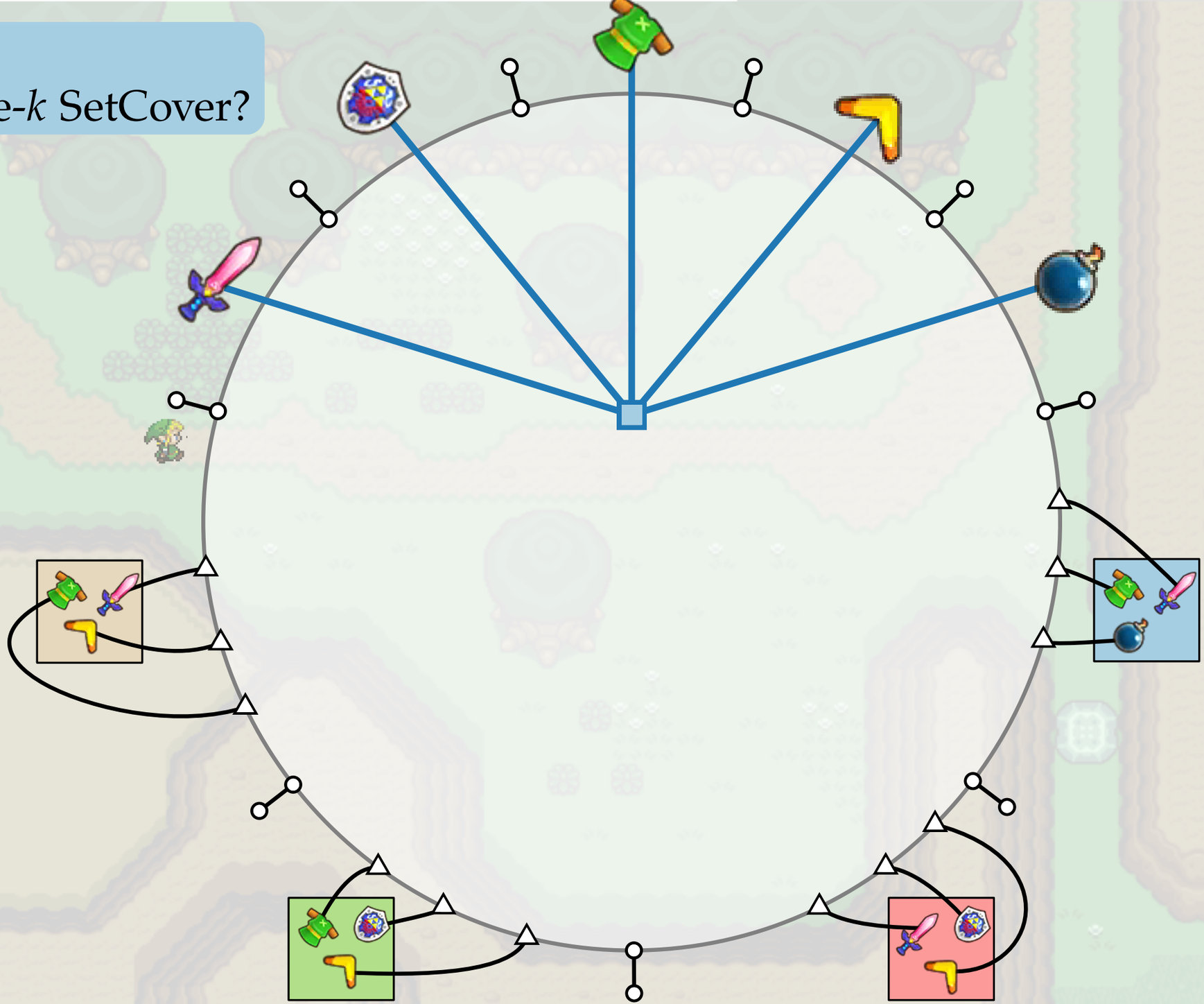
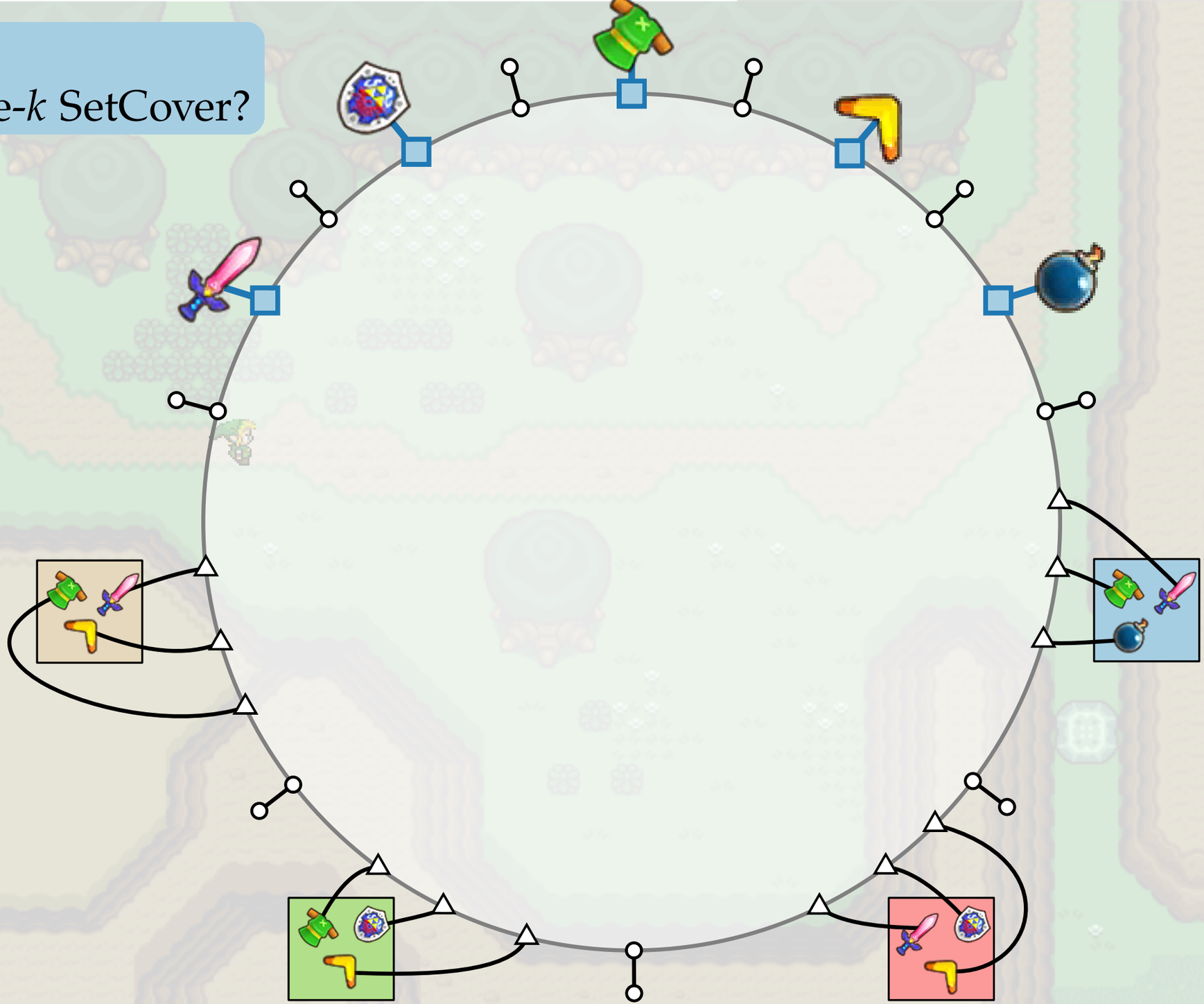
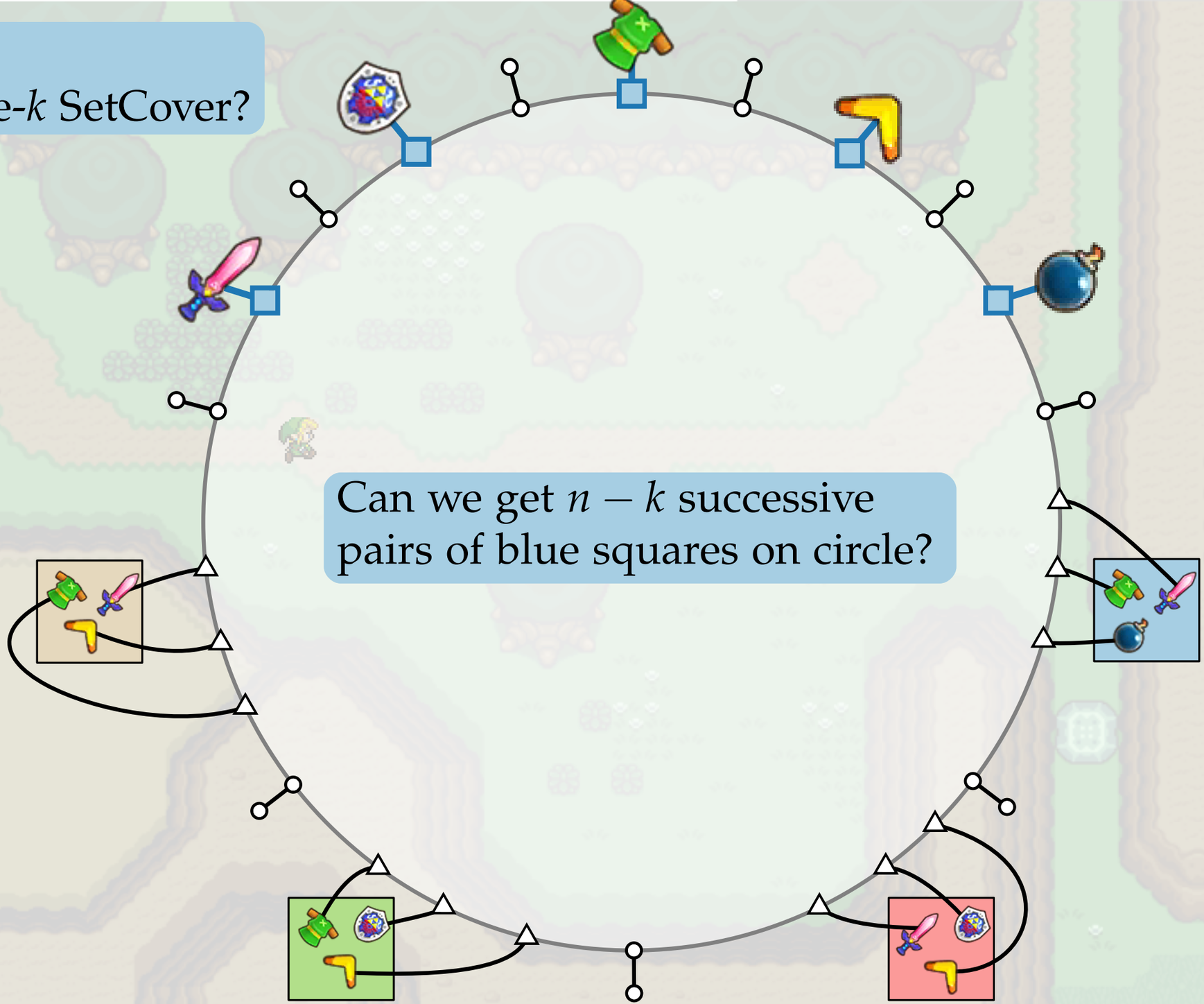


$n = 5, k = 2$

Is there a size-k SetCover?

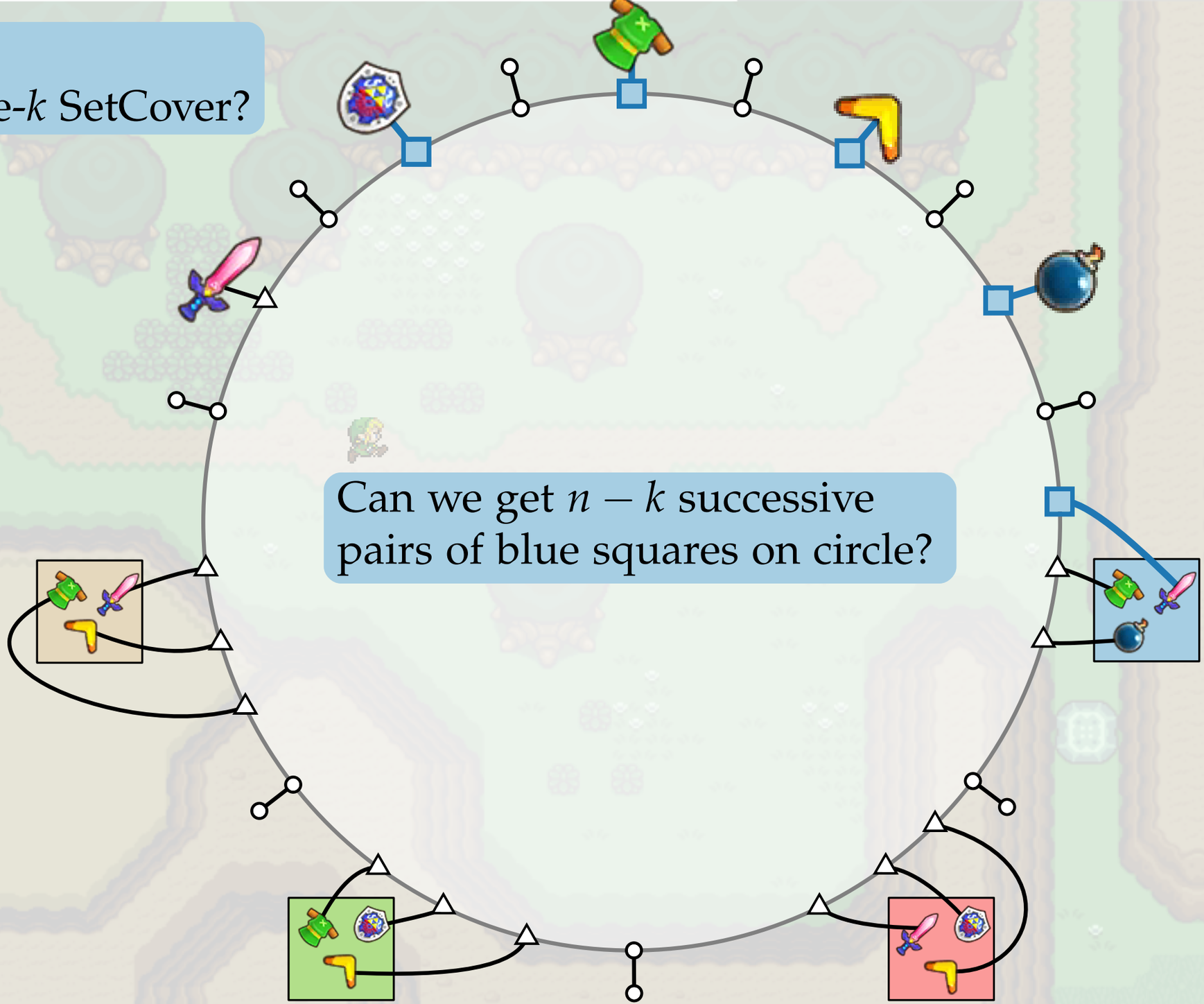


$n = 5, k = 2$
Is there a size-k SetCover?



Can we get $n - k$ successive pairs of blue squares on circle?

$n = 5, k = 2$
Is there a size-k SetCover?



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$n = 5, k = 2$
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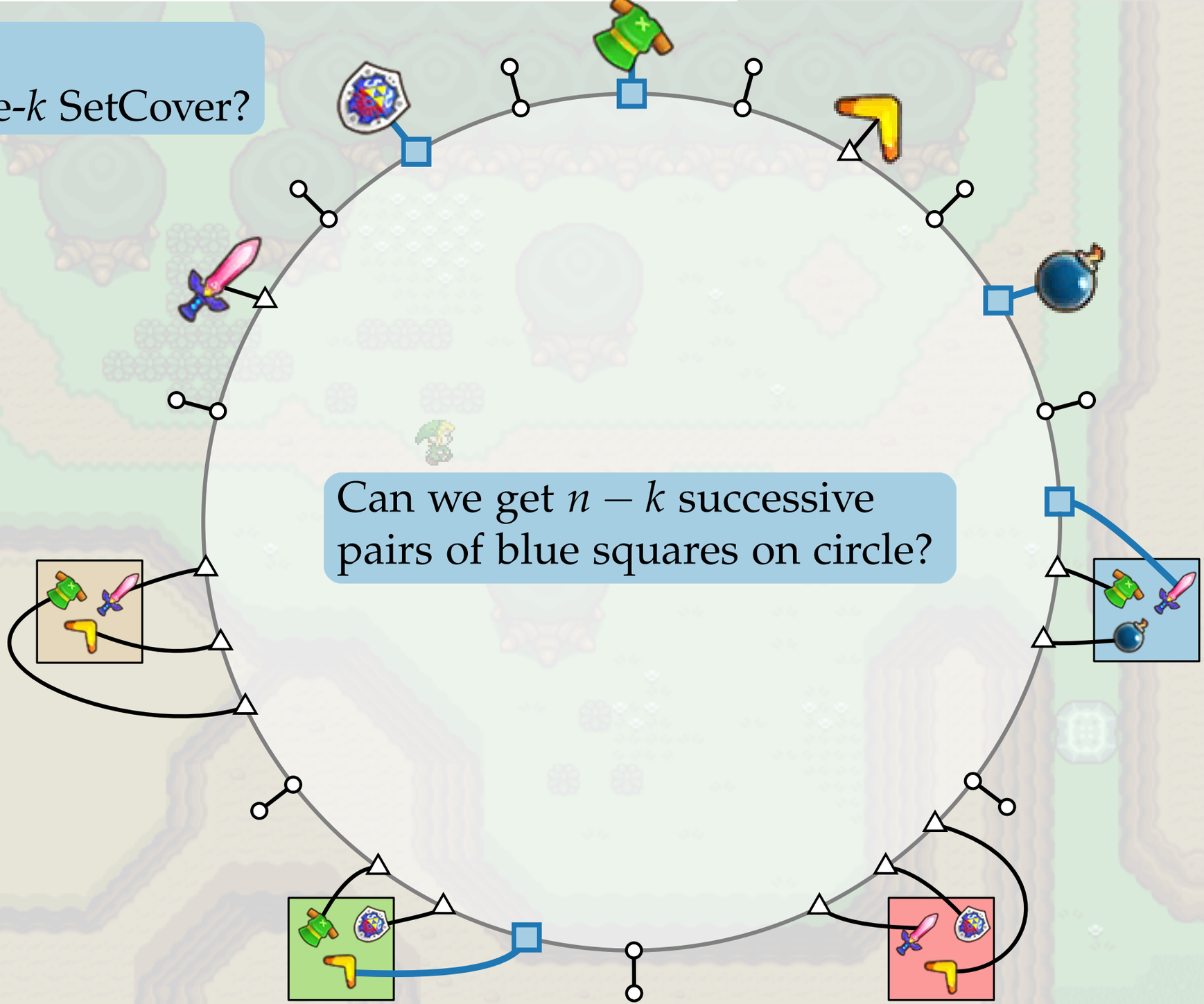

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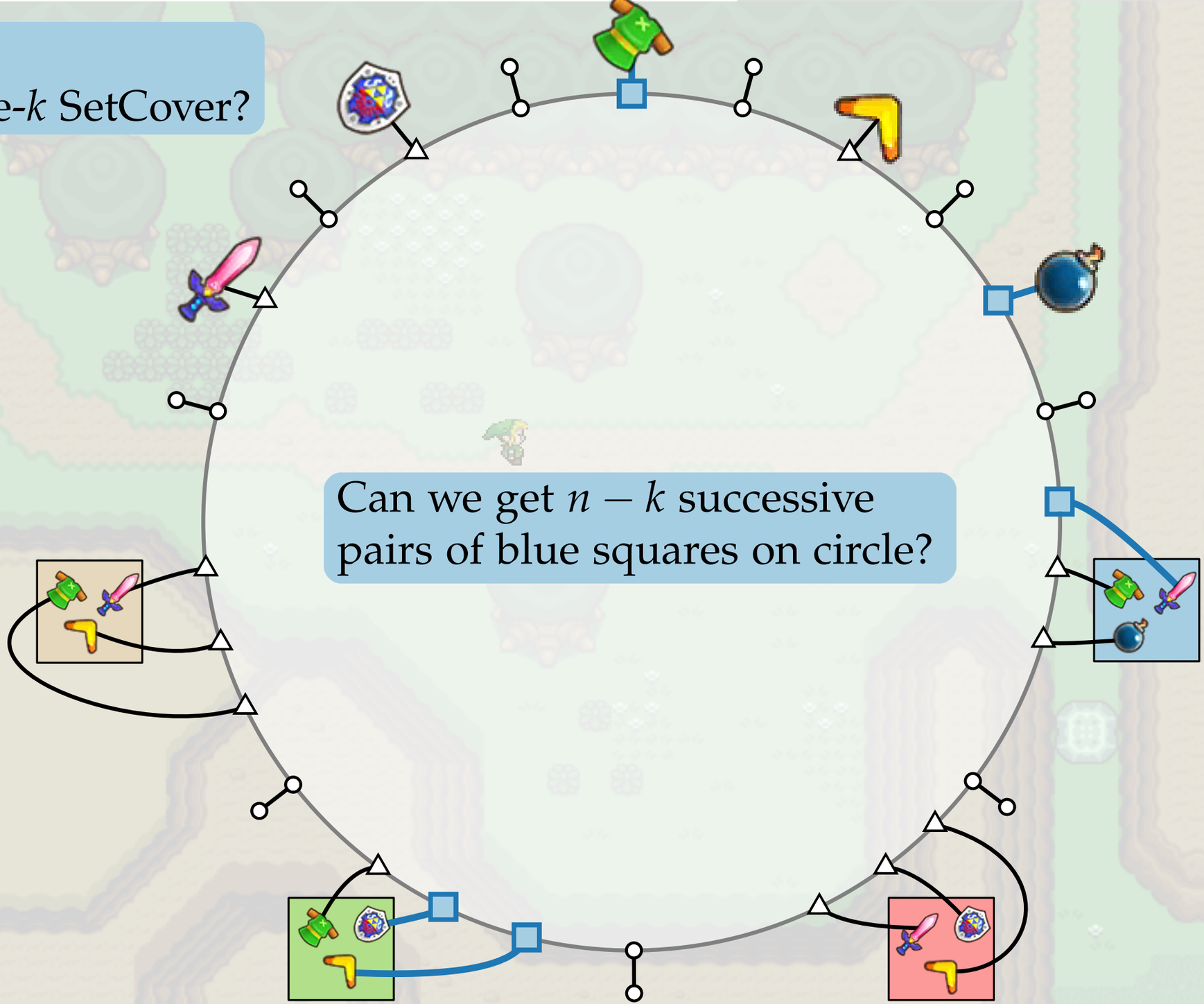
$n = 5, k = 2$
Is there a size-k SetCover?



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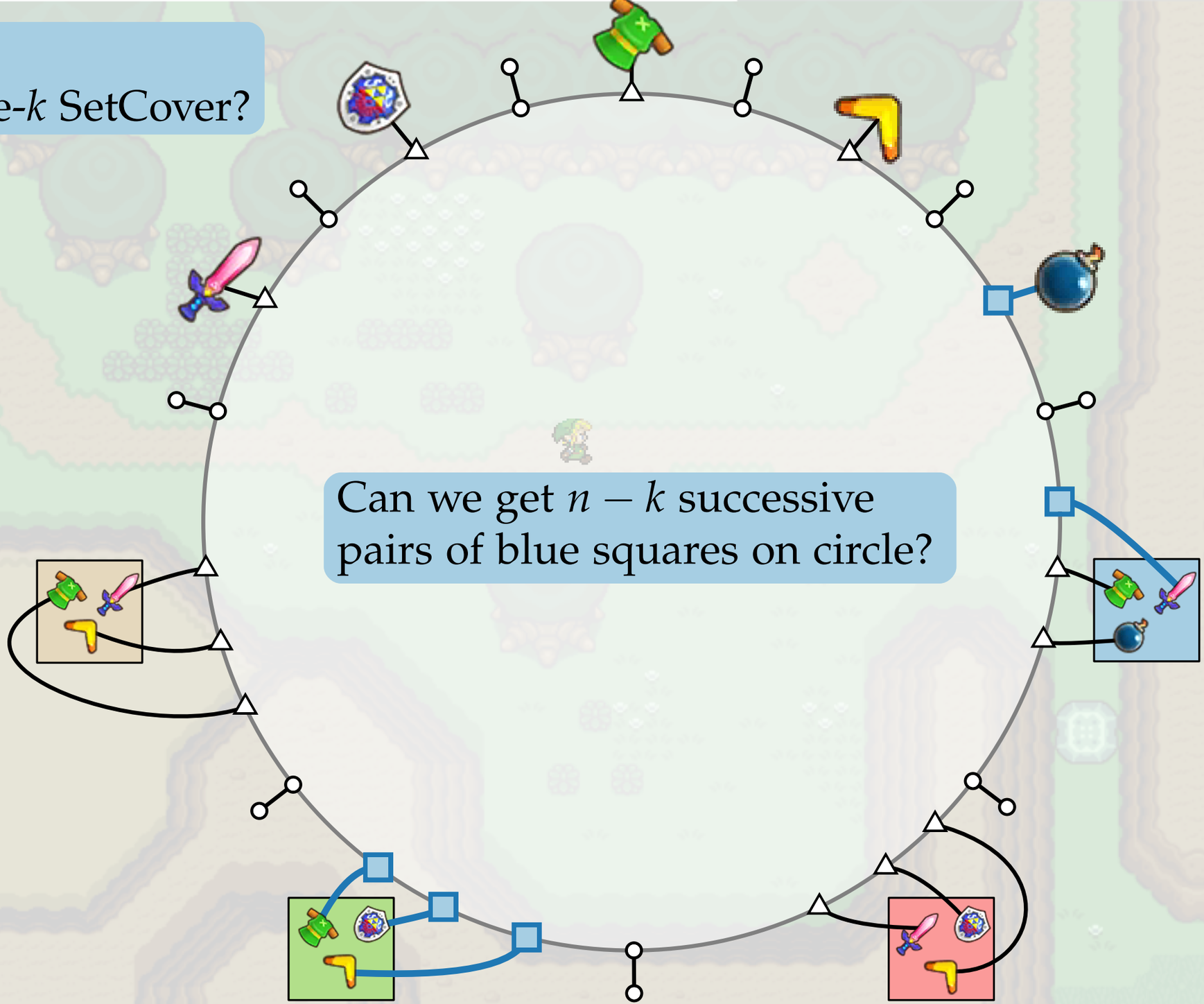
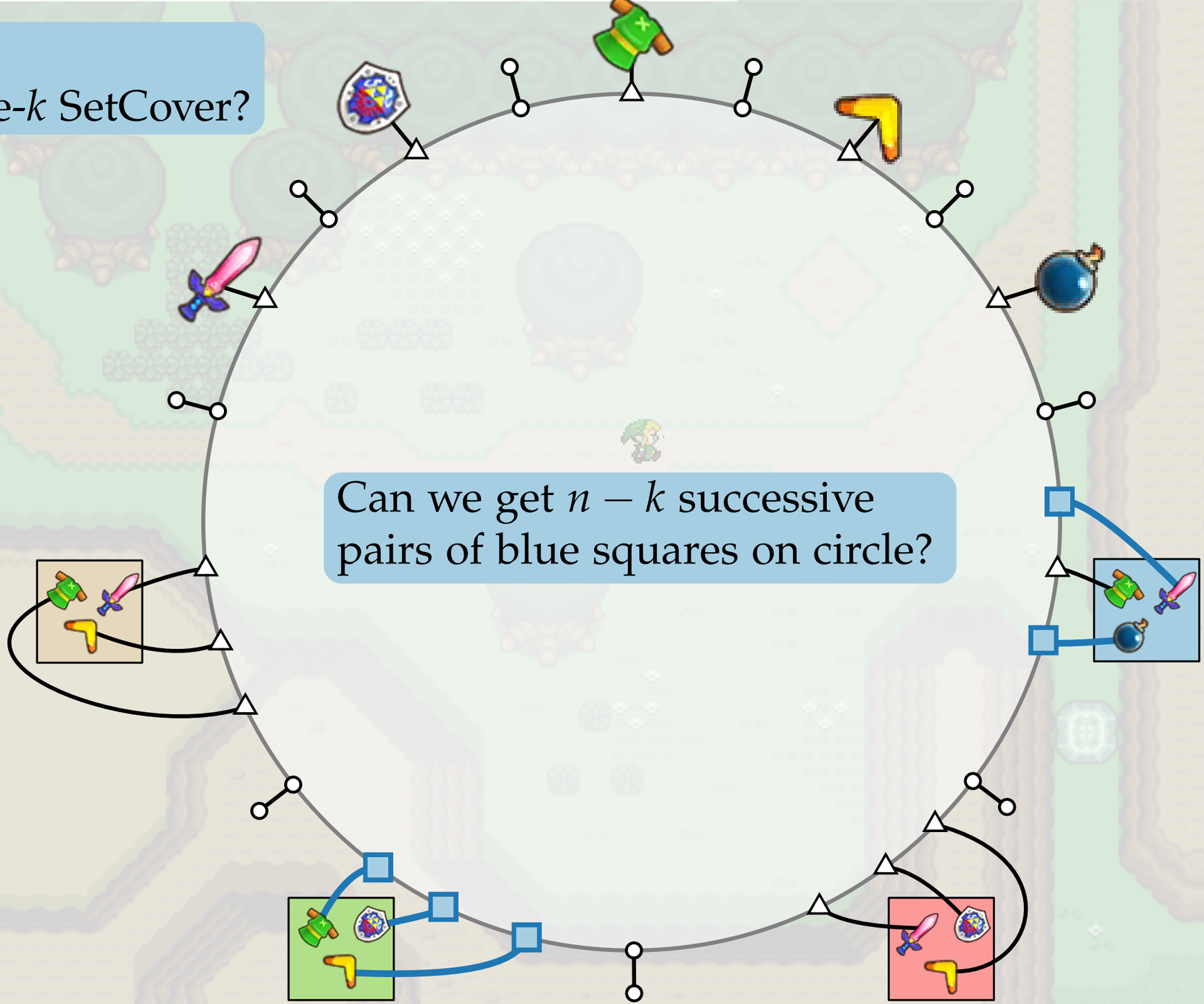
$n = 5, k = 2$
Is there a size-k SetCover?



Can we get $n - k$ successive pairs of blue squares on circle?

$n = 5, k = 2$
Is there a size-k SetCover?



Can we get $n - k$ successive pairs of blue squares on circle?

$n = 5, k = 2$
Is there a size-k SetCover?

Can we get $n - k$ successive pairs of blue squares on circle?

size-2 SetCover

$n = 5, k = 2$
Is there a size-k SetCover?

Can we get $n - k$ successive pairs of blue squares on circle?

size-2 SetCover

$n = 5, k = 2$
Is there a size-k SetCover?

$n = 5, k = 2$
Is there a size-k SetCover?

$n = 5, k = 2$
Is there a size-k SetCover?

$n = 5, k = 2$
Is there a size-k SetCover?

$n = 5, k = 2$
Is there a size-k SetCover?

Can we get $n - k$ successive pairs of blue squares on circle?

2 successive blue pairs

size-2 SetCover

$n = 5, k = 2$
Is there a size-k SetCover?

Can we get $n - k$ successive pairs of blue squares on circle?

2 successive blue pairs

size-2 SetCover

$n = 5, k = 2$
Is there a size-k SetCover?

$n = 5, k = 2$
Is there a size-k SetCover?

Can we get $n - k$ successive pairs of blue squares on circle?

2 successive blue pairs

size-2 SetCover

$n = 5, k = 2$
Is there a size-k SetCover?

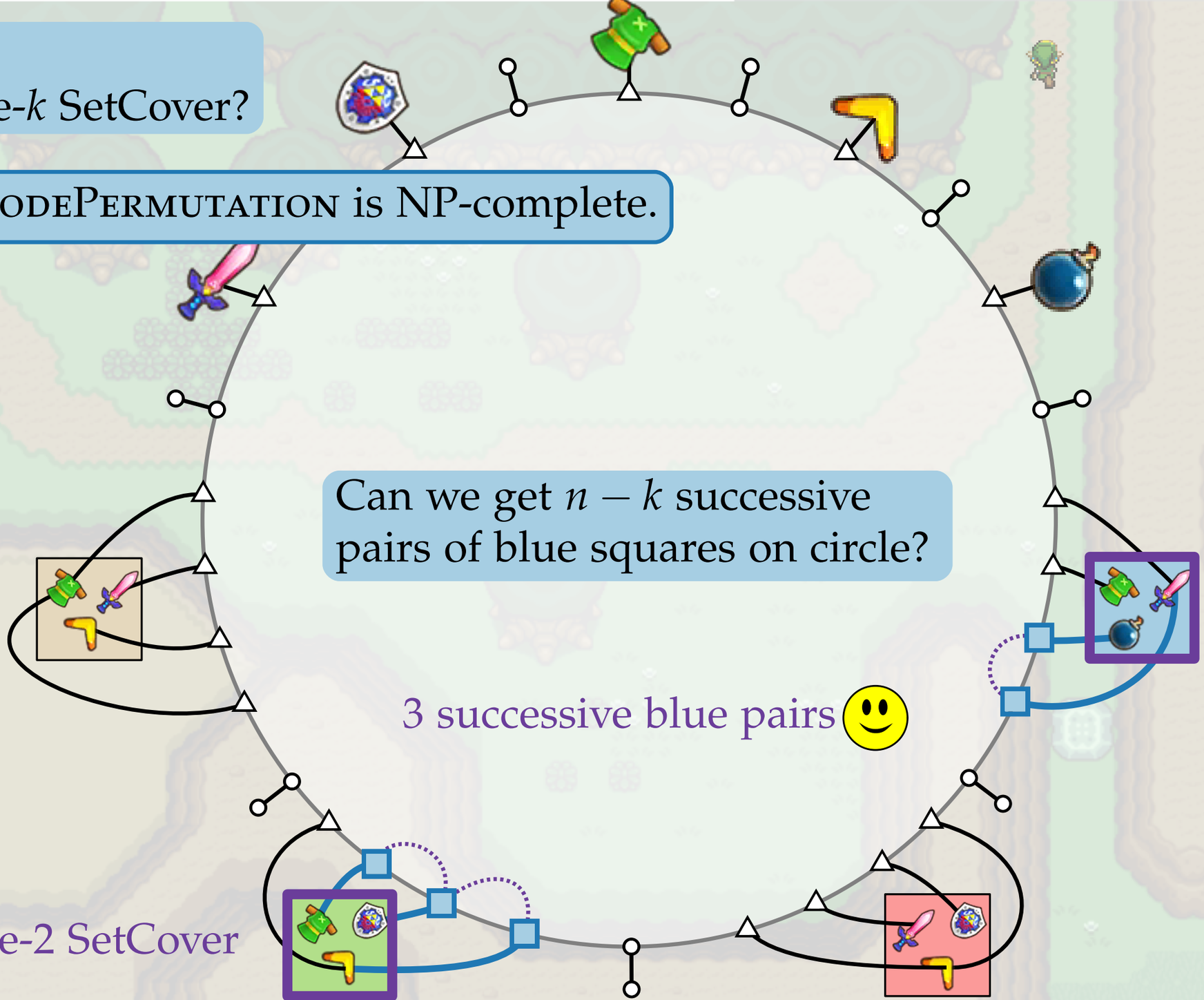
Can we get $n - k$ successive pairs of blue squares on circle?

3 successive blue pairs

size-2 SetCover

$n = 5, k = 2$
Is there a size-k SetCover?

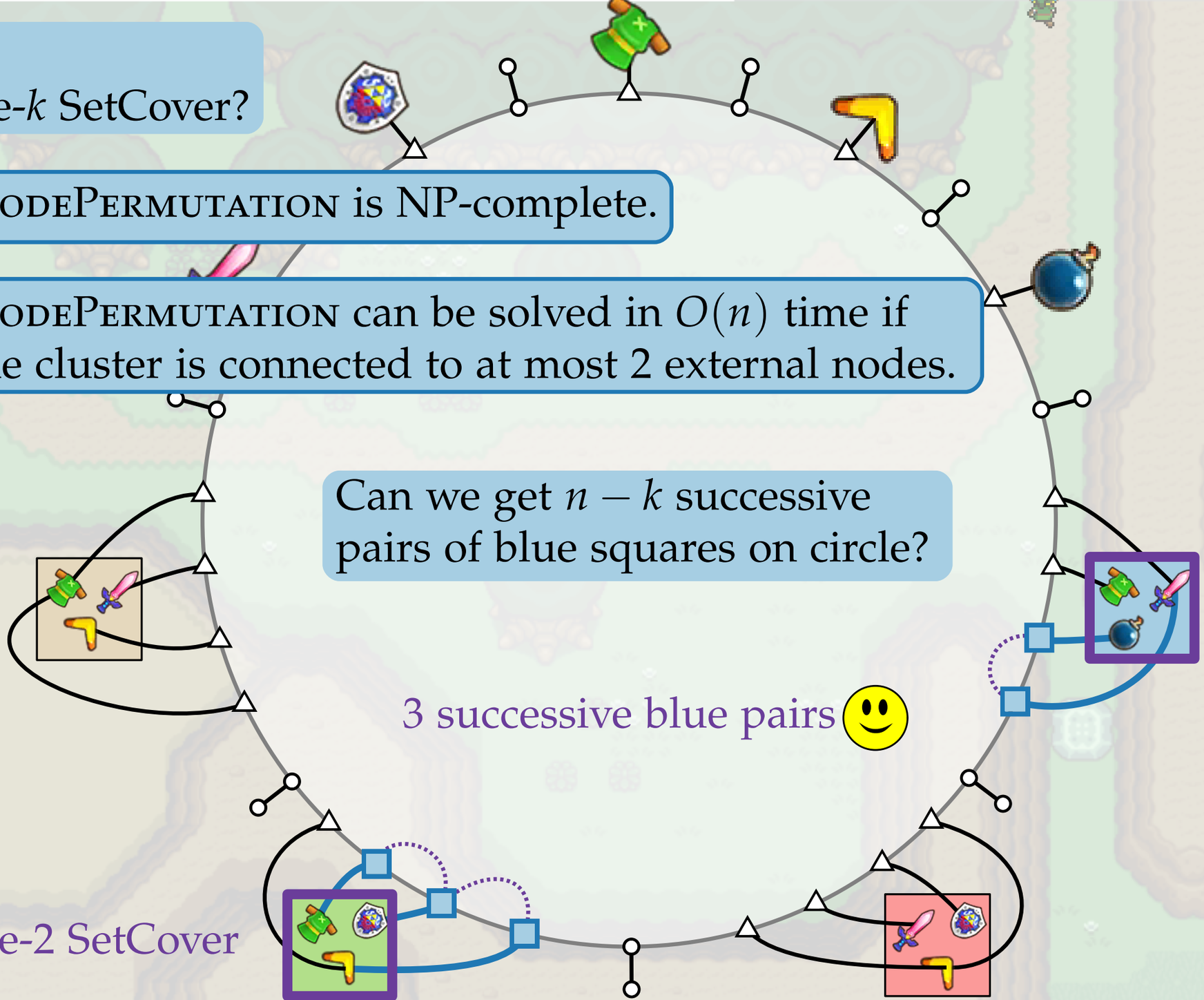
Theorem. NodePermutation is NP-complete.

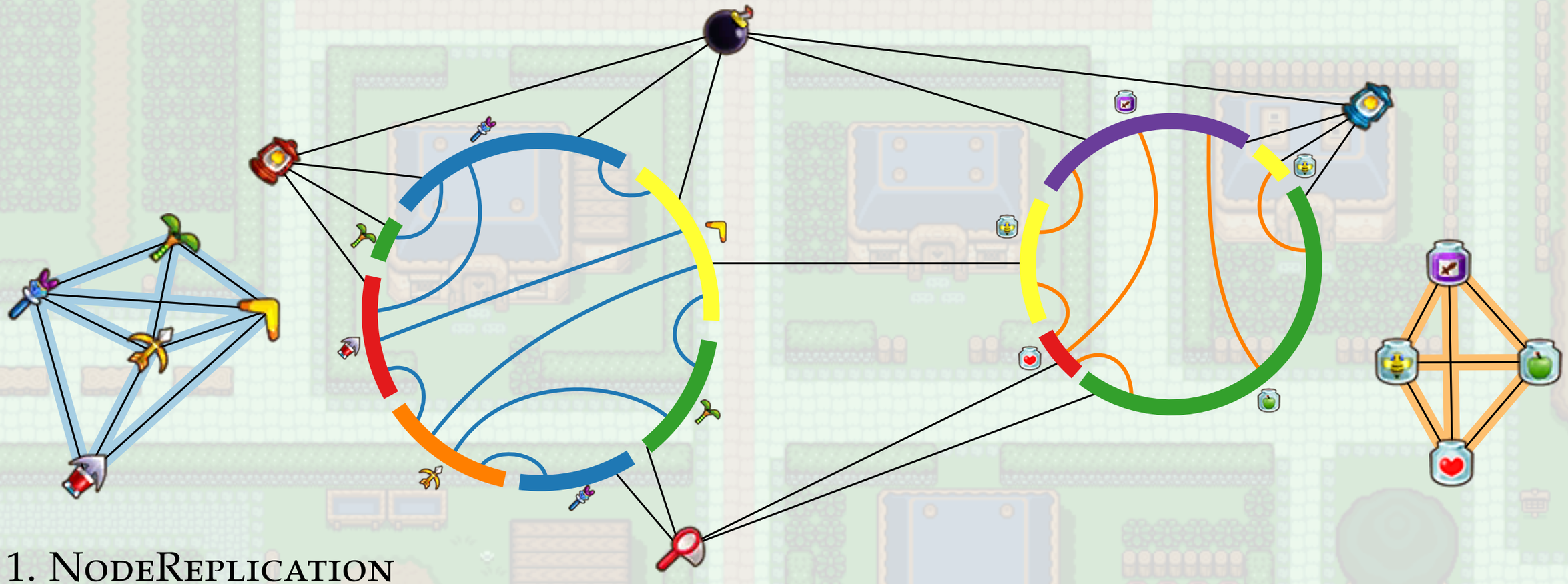


$n = 5, k = 2$
Is there a size-k SetCover?

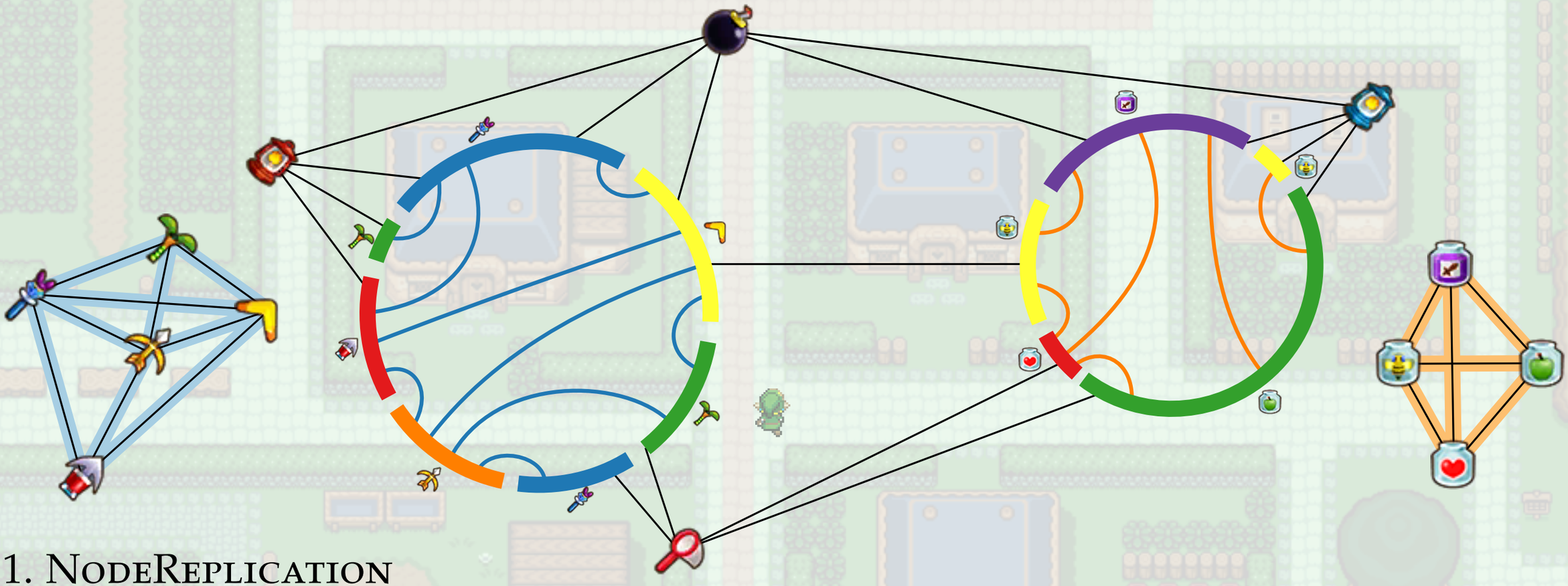
Theorem. NodePermutation is NP-complete.

Theorem. NodePermutation can be solved in $O(n)$ time if the cluster is connected to at most 2 external nodes.

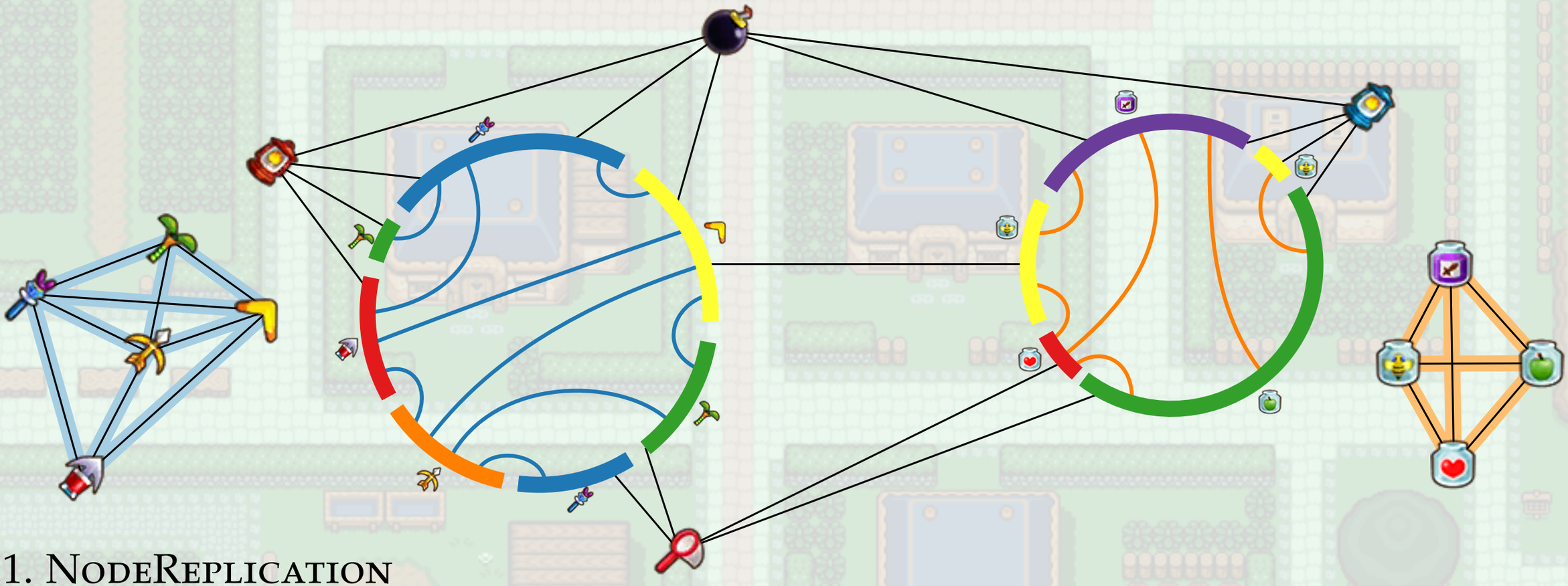




1. NodeReplication
2. NodePermutation — Can flip nodes with common neighbor
   Goal: max. #successive pairs with same icon
3. NodeMerging
4. ChordInsertion — Reintroduce chords in cluster
   Choose any copies of the adjacent vertices
   Goal: min. #crossings



1. NodeReplication
2. NodePermutation — Can flip nodes with common neighbor; Goal: max. #successive pairs with same icon — NP-complete
3. NodeMerging
4. ChordInsertion — Reintroduce chords in cluster; Choose any copies of the adjacent vertices; Goal: min. #crossings



1. NodeReplication		
2. NodePermutation	Can flip nodes with common neighbor Goal: max. #successive pairs with same icon	NP-complete
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4. ChordInsertion	Reintroduce chords in cluster Choose any copies of the adjacent vertices Goal: min. #crossings	

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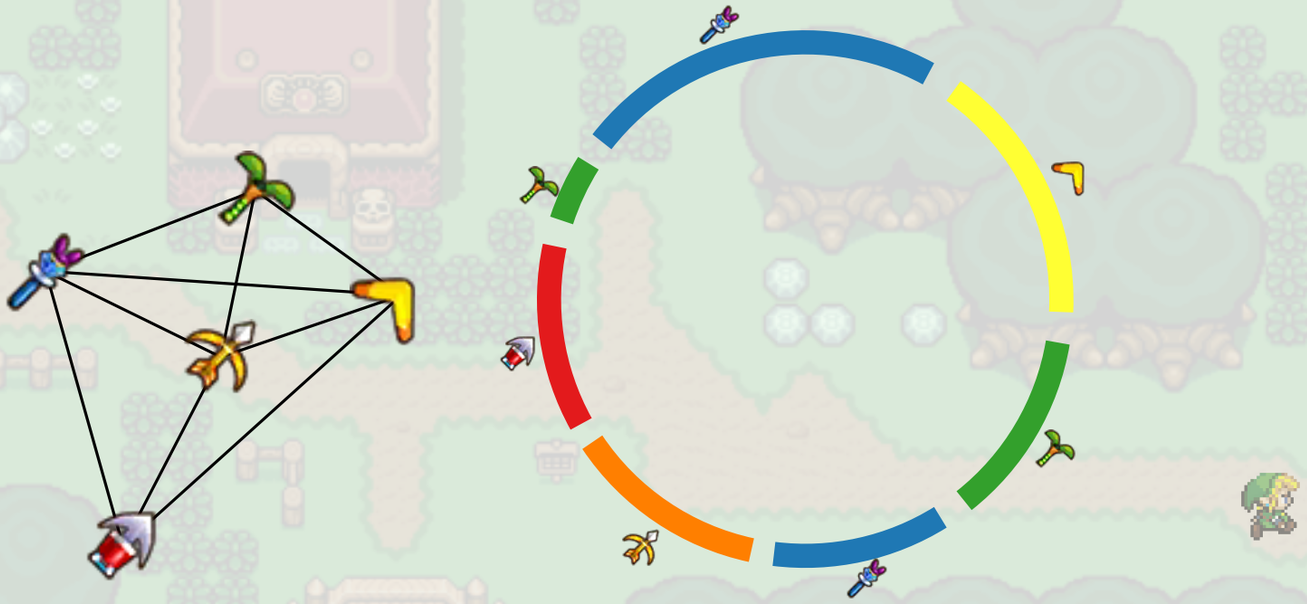
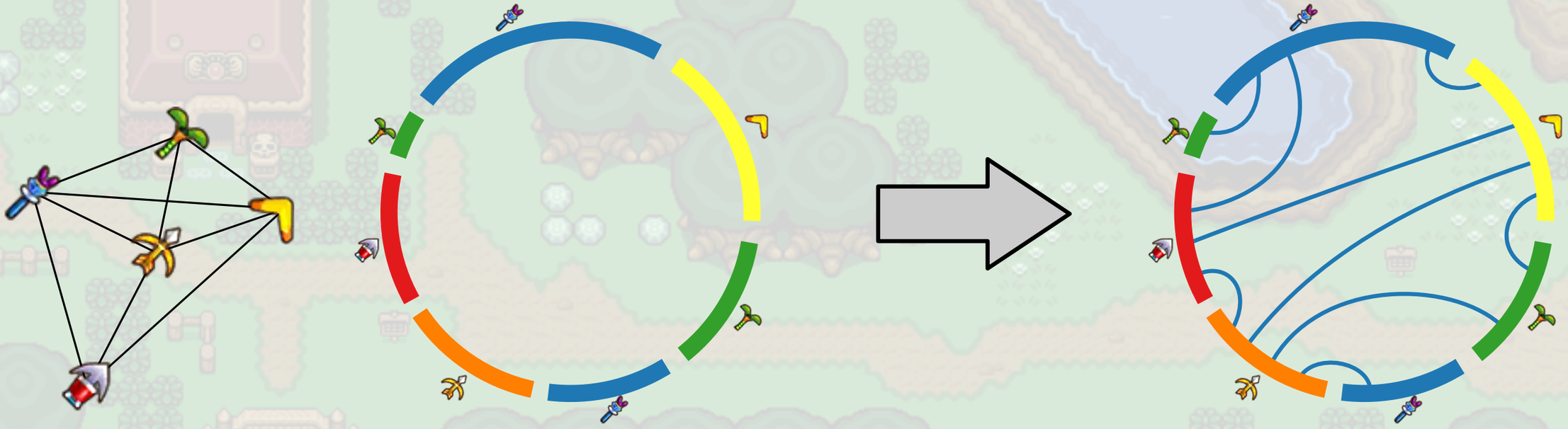
4. ChordInsertion

Reintroduce chords in cluster
Choose any copies of the adjacent vertices
Goal: min. #crossings



4. ChordInsertion

Reintroduce chords in cluster
Choose any copies of the adjacent vertices
Goal: min. #crossings



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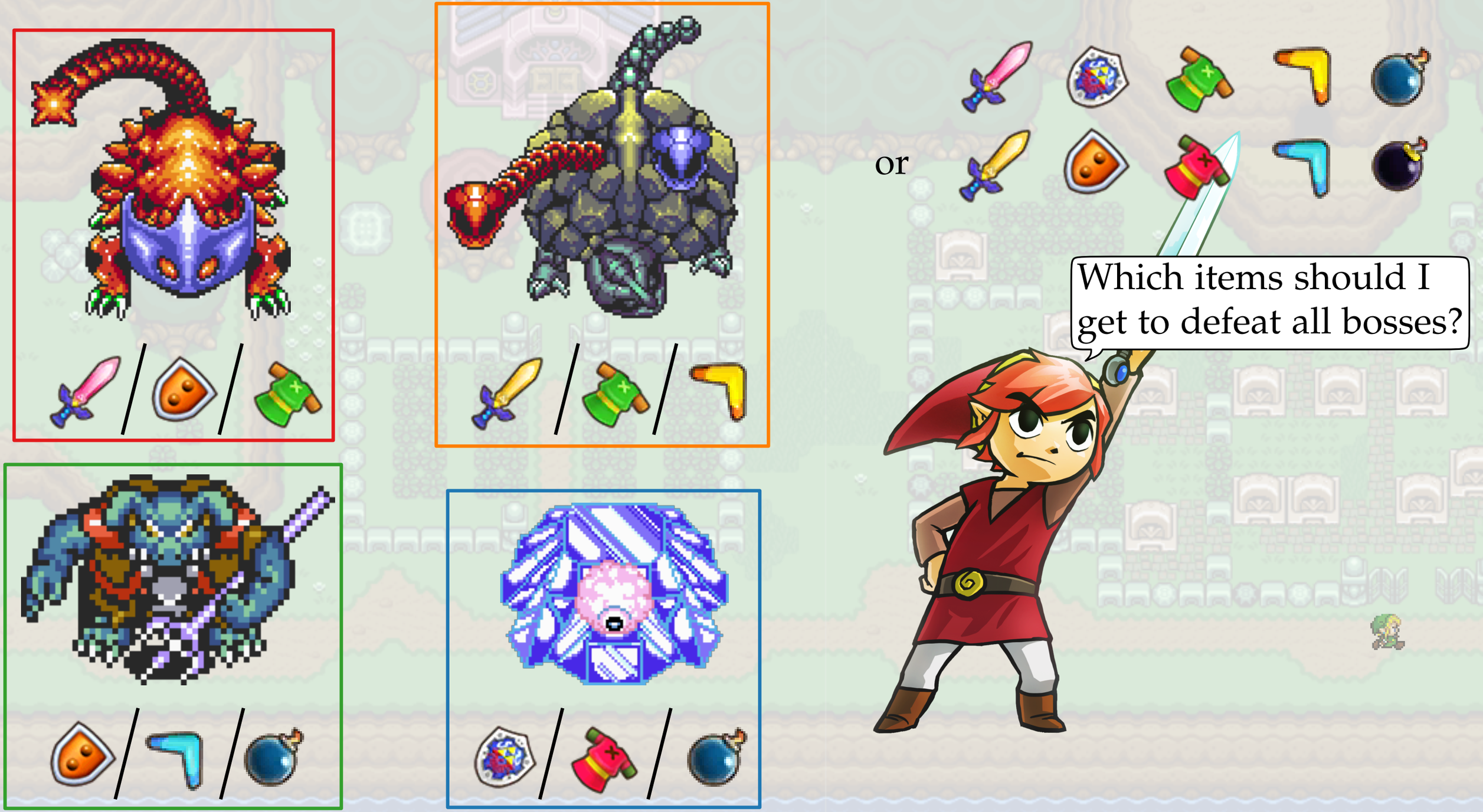
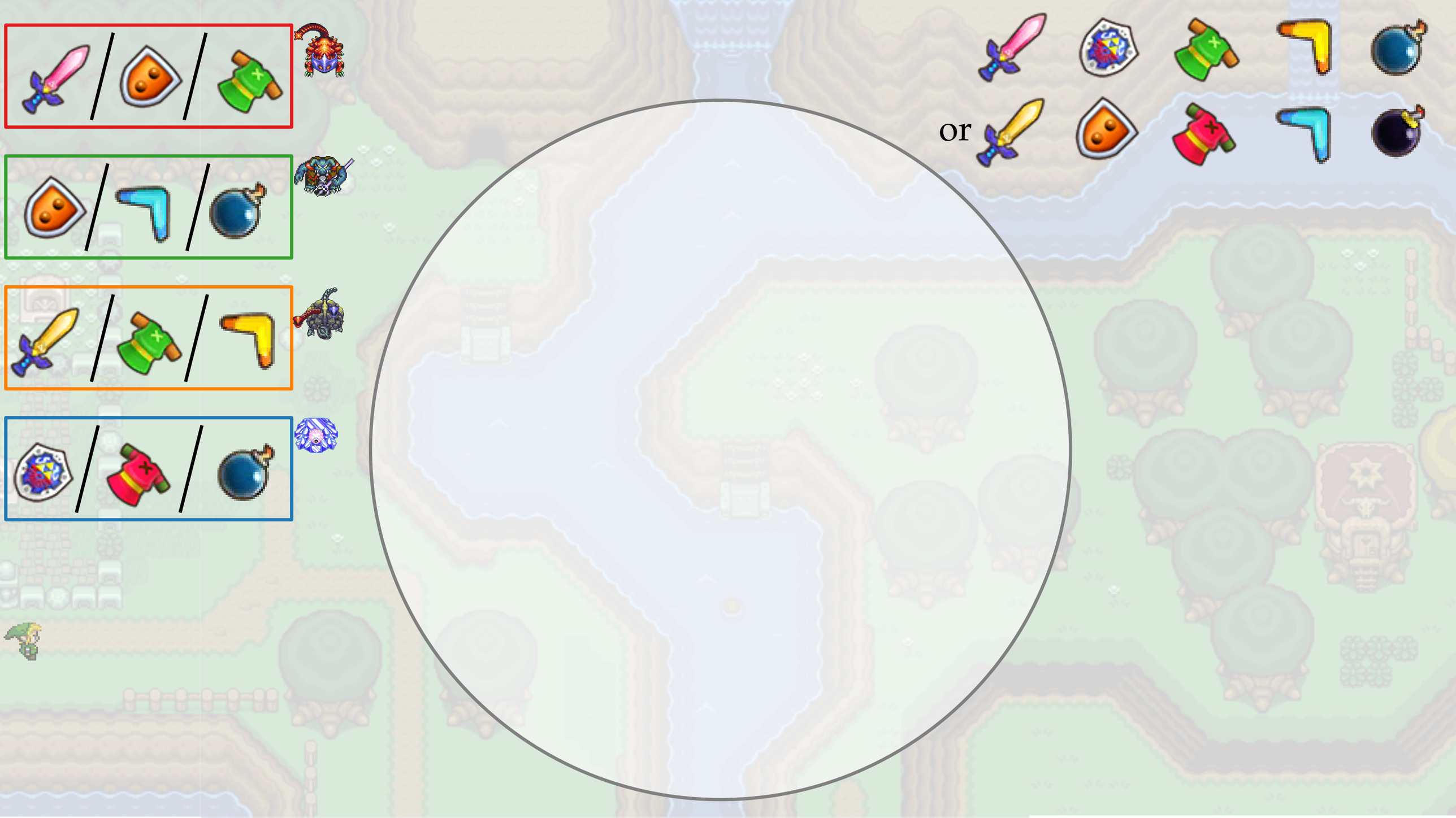

or
Which items should I get to defeat all bosses?

or
Which items should I get to defeat all bosses?
Is there a solution for the 3-SAT instance?

or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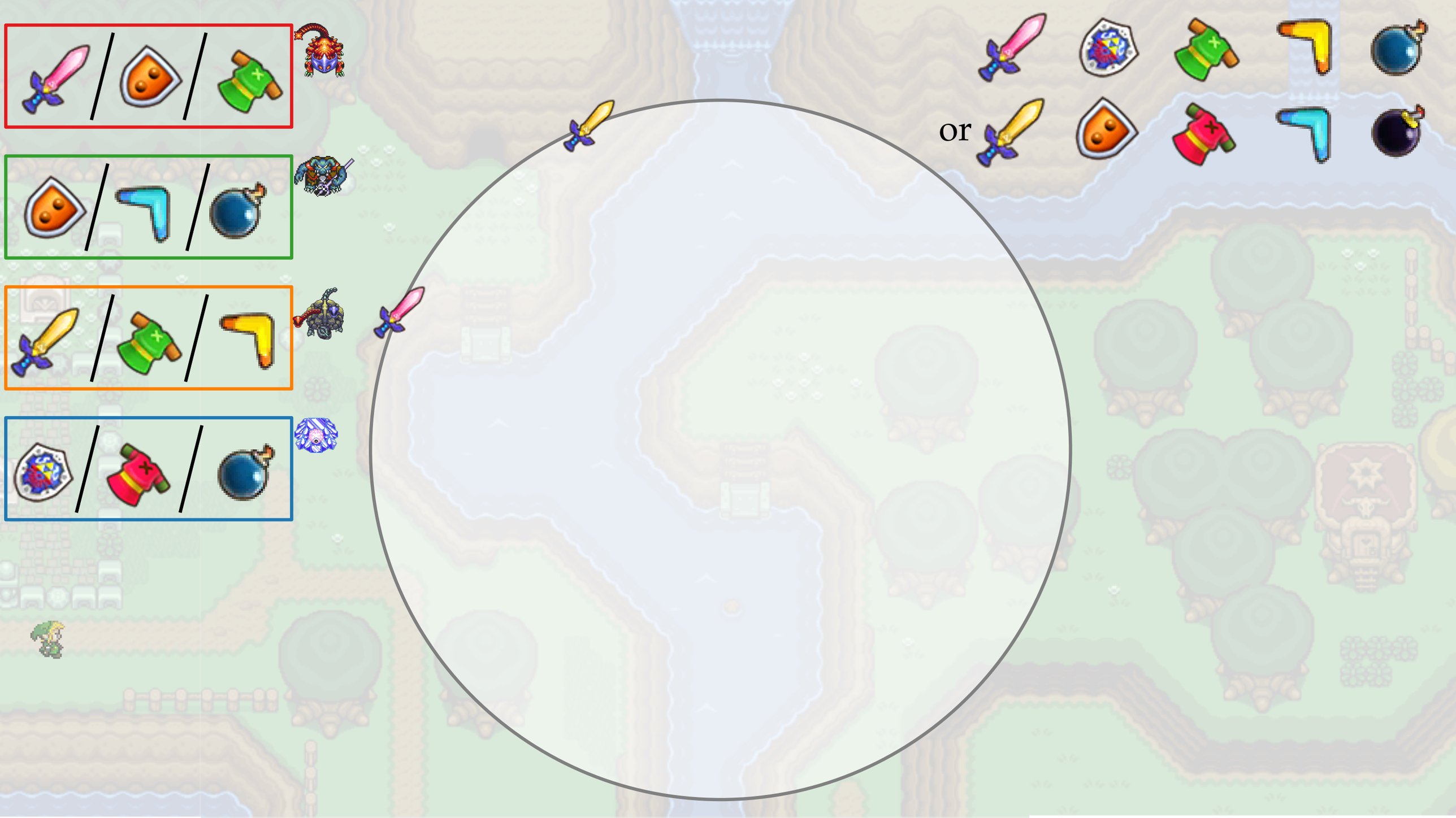
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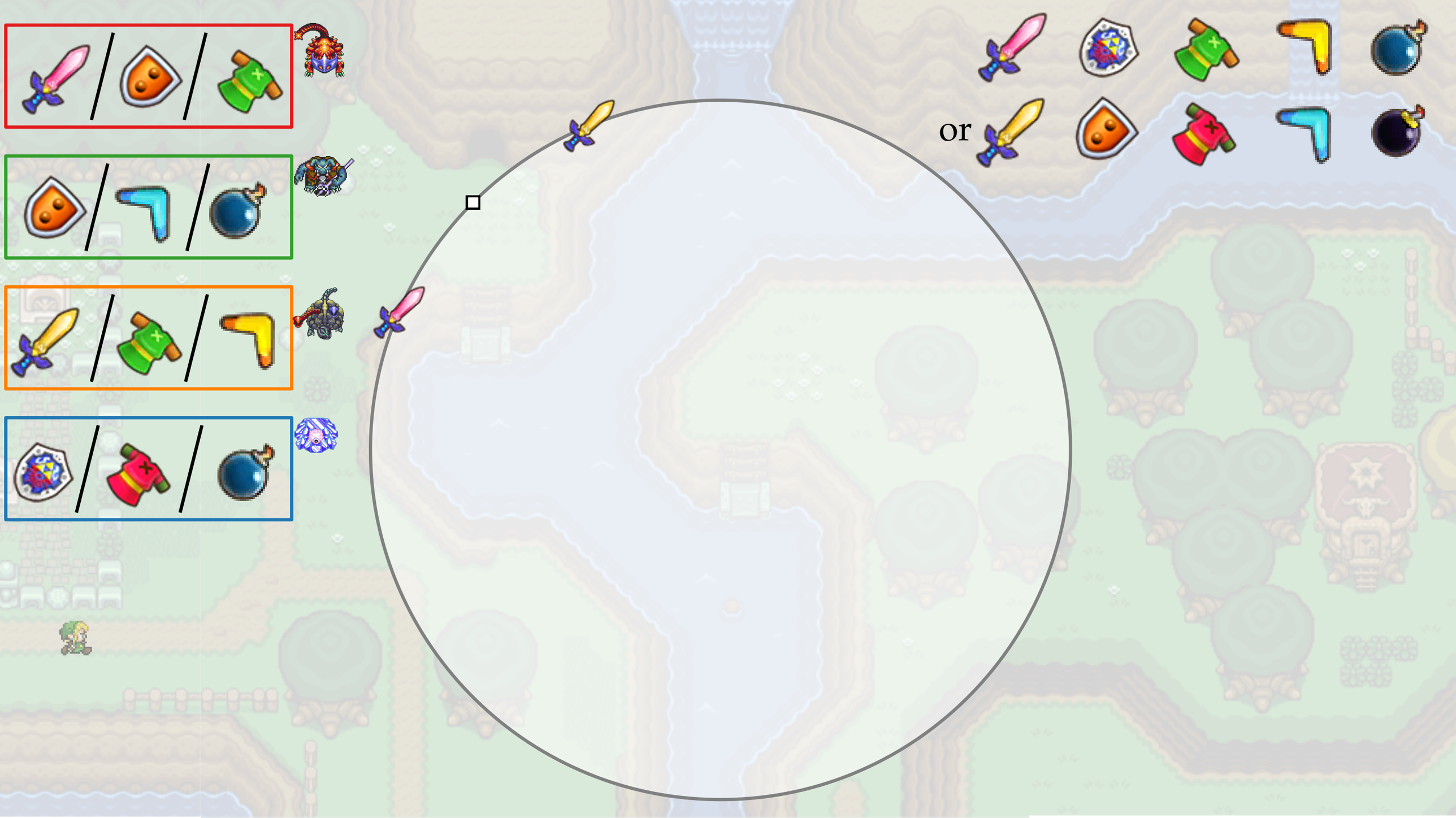
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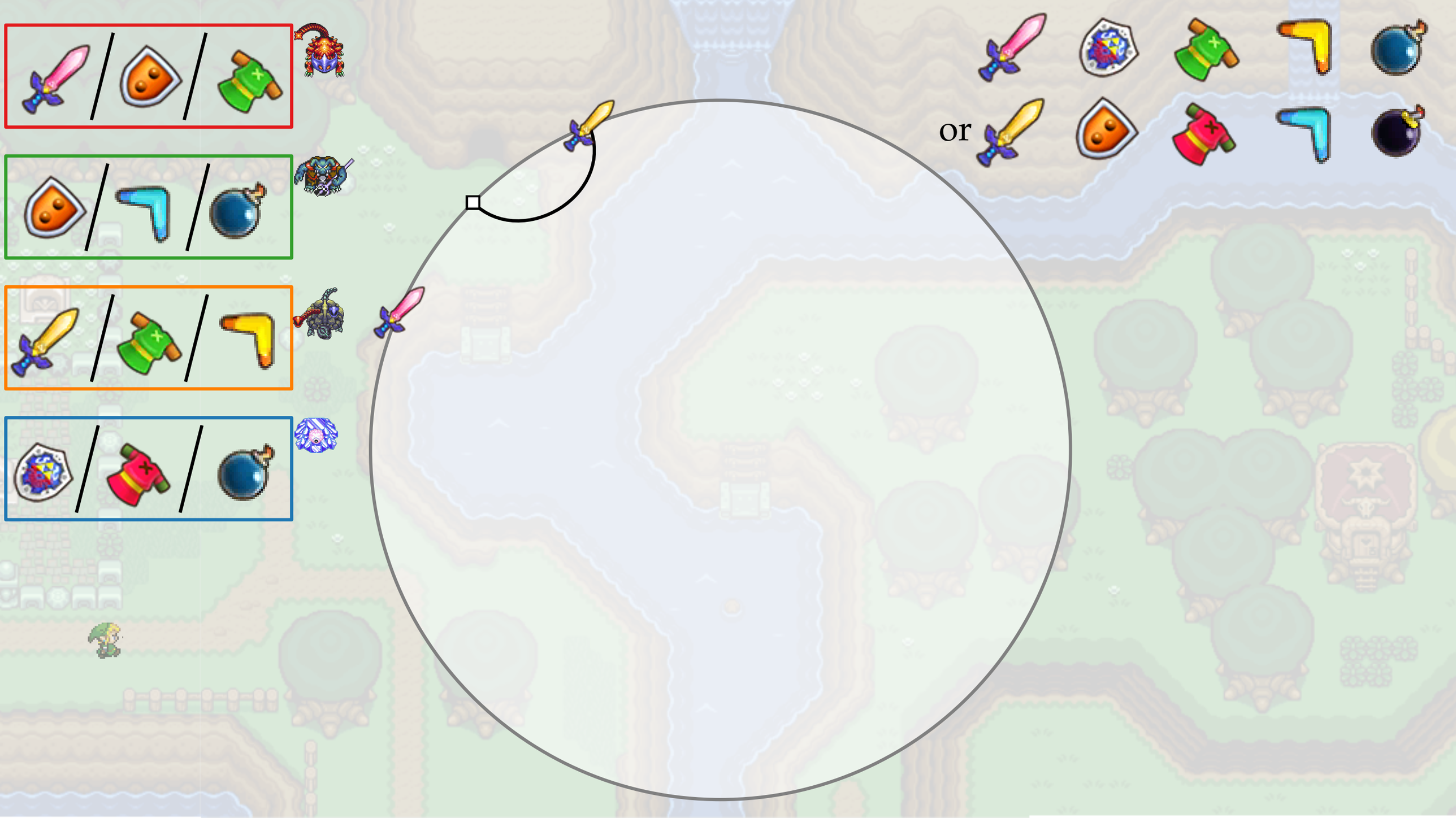
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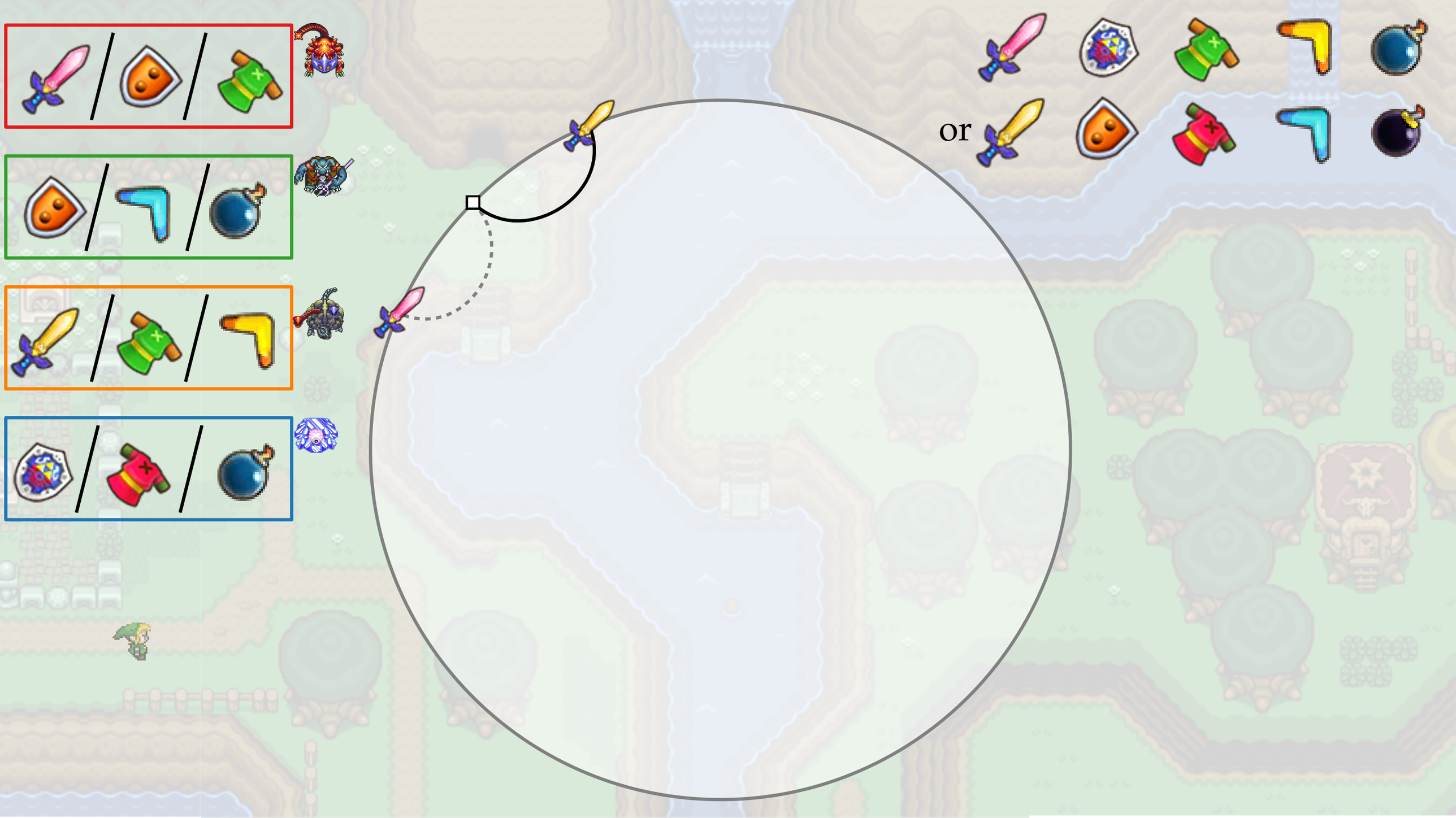
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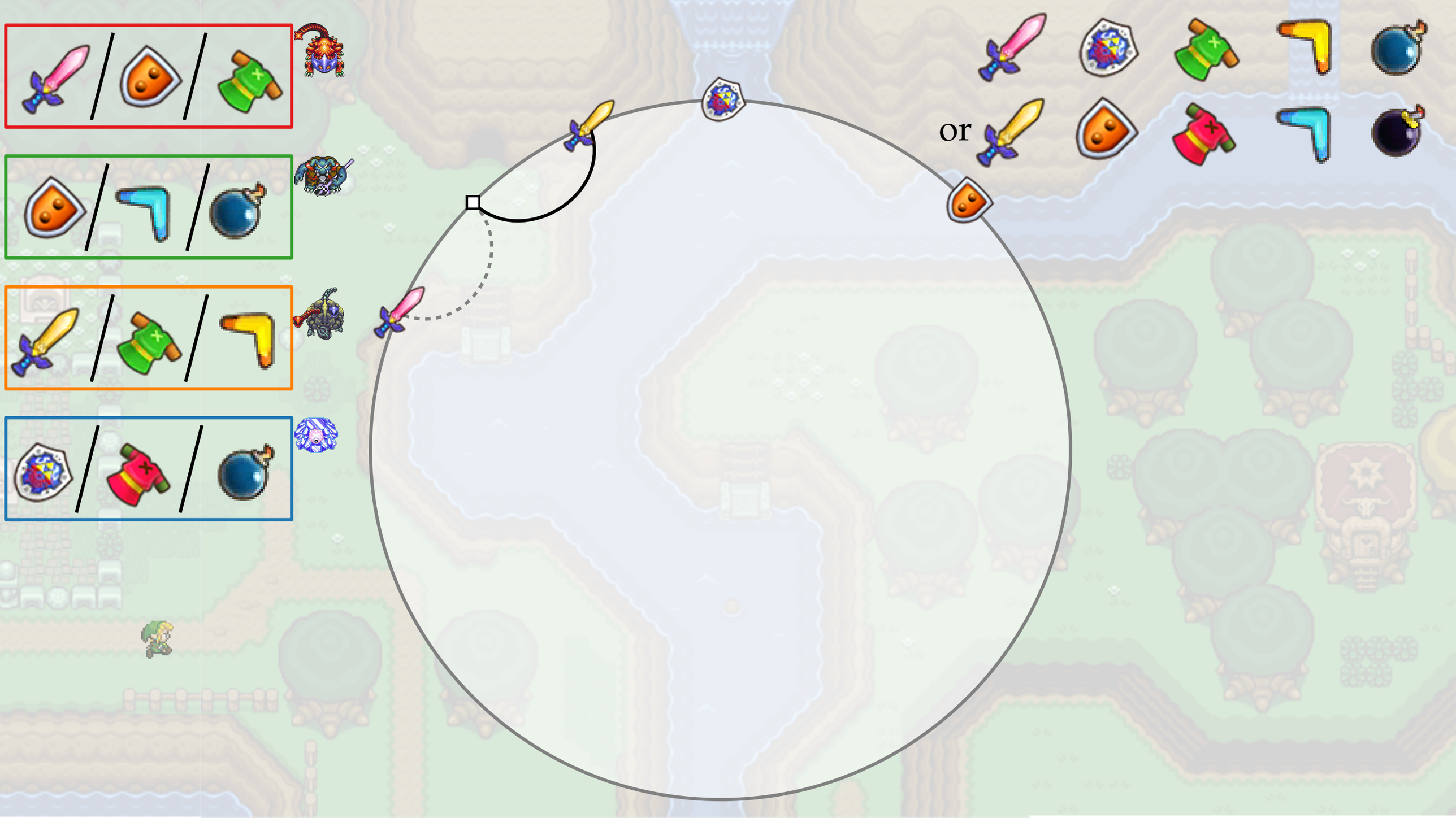
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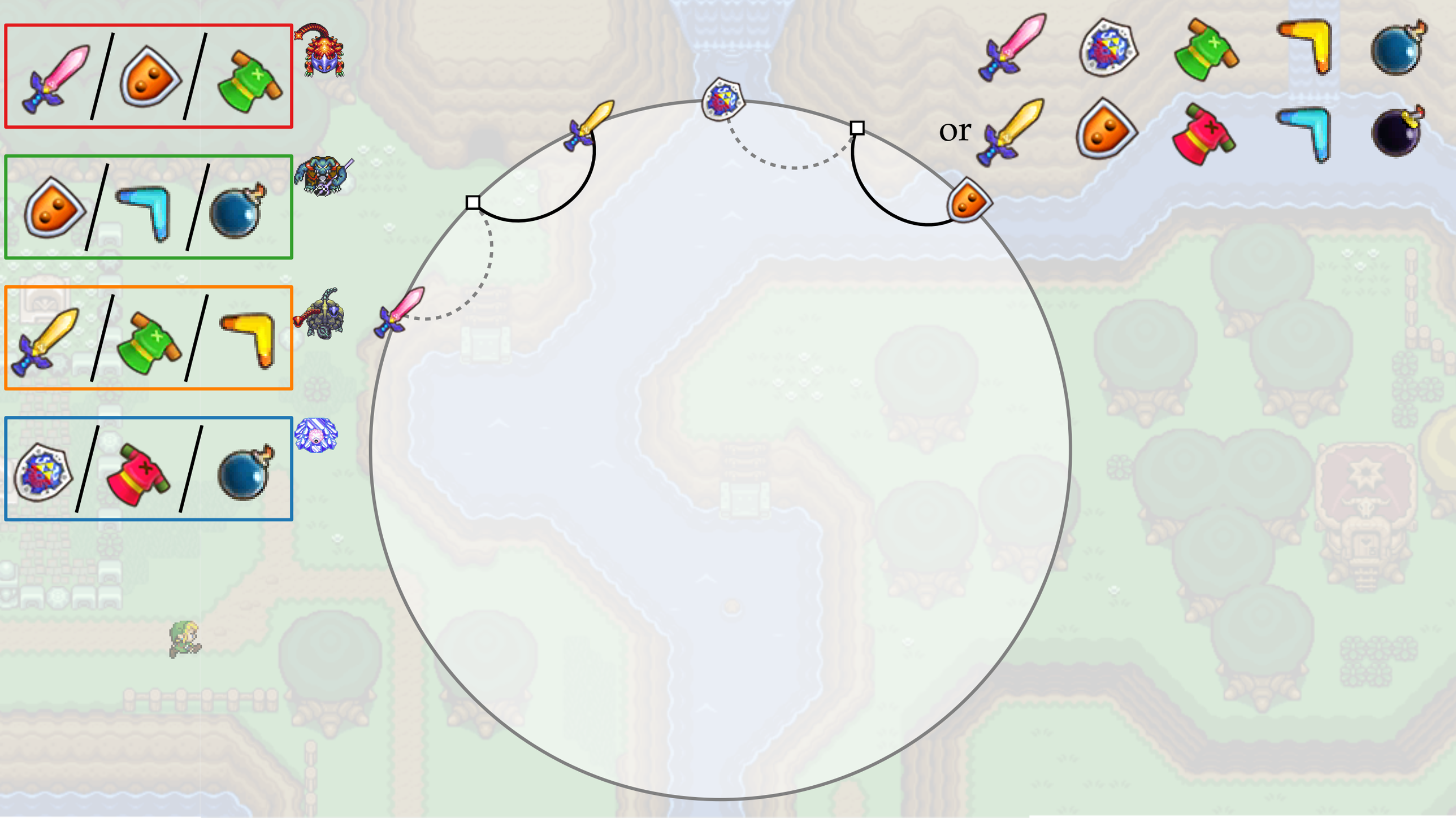
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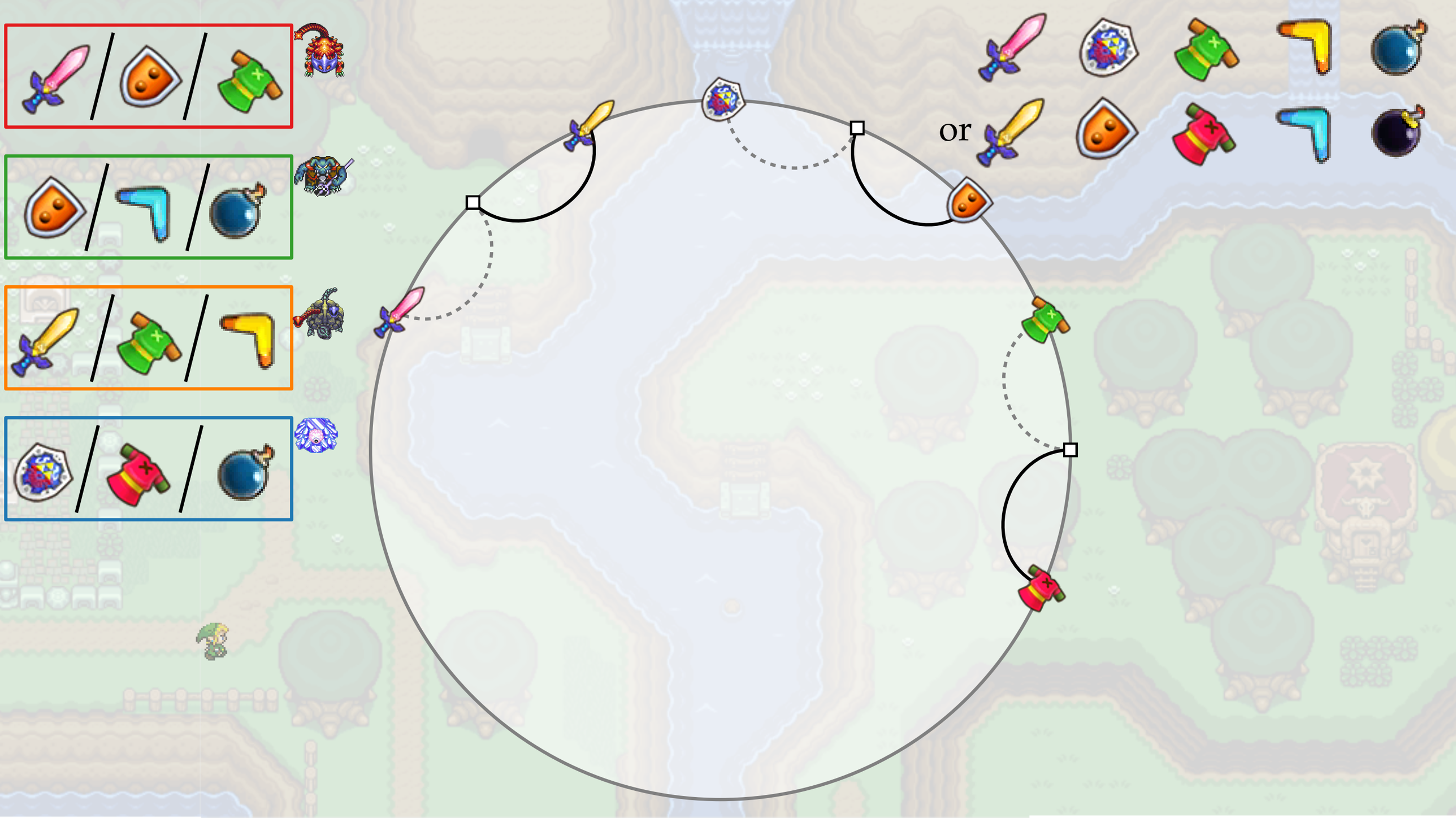
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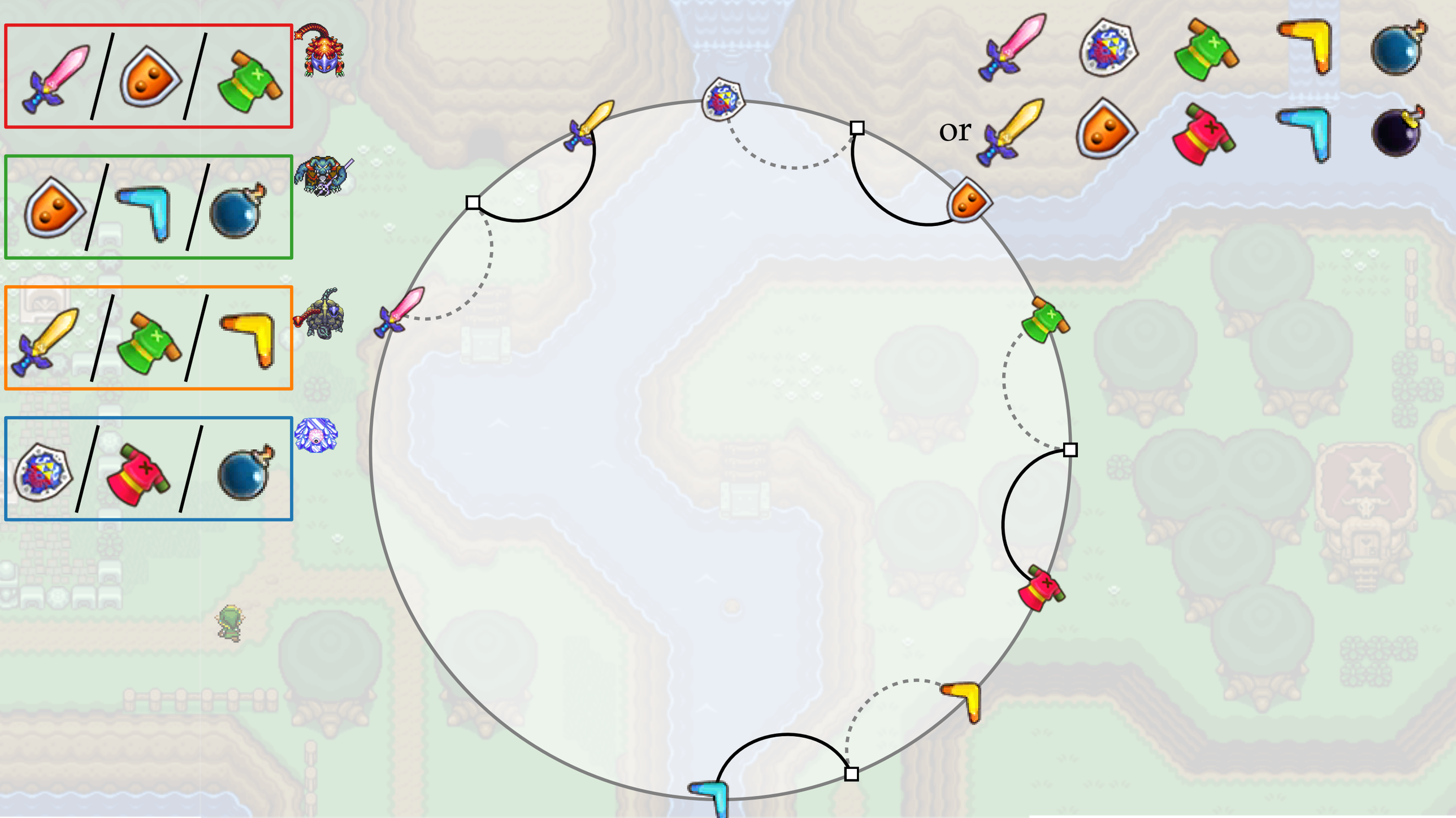
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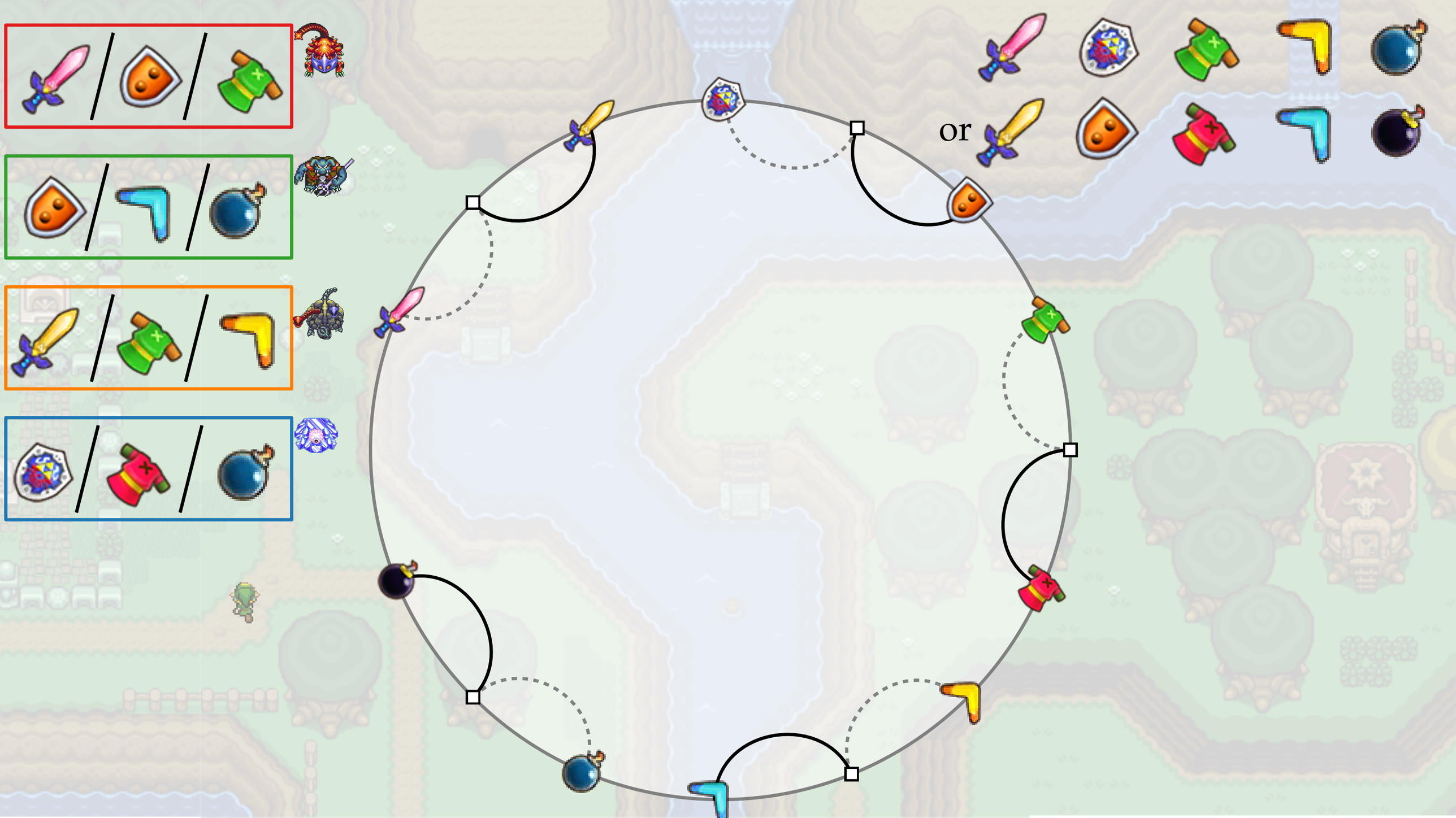
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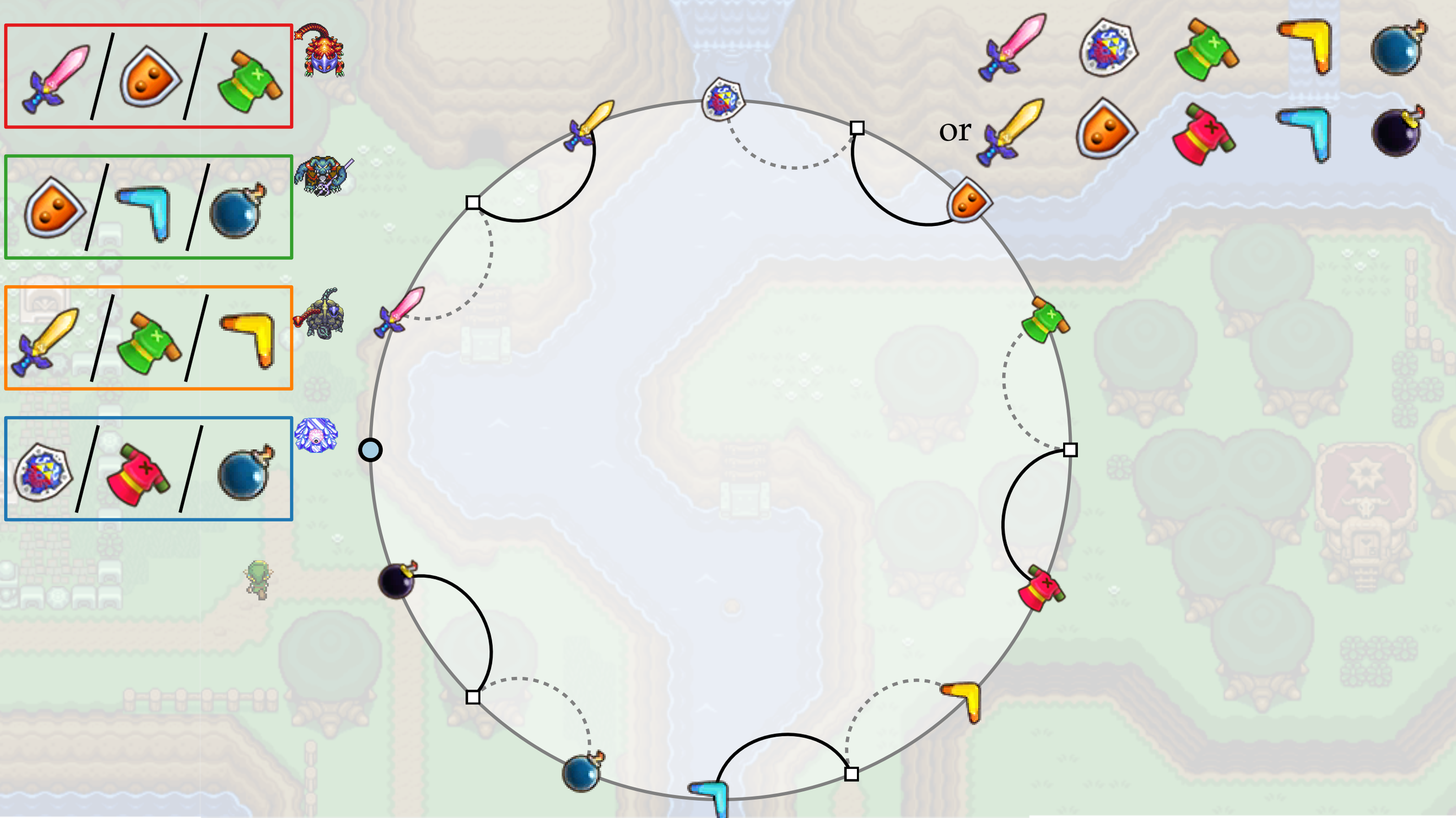
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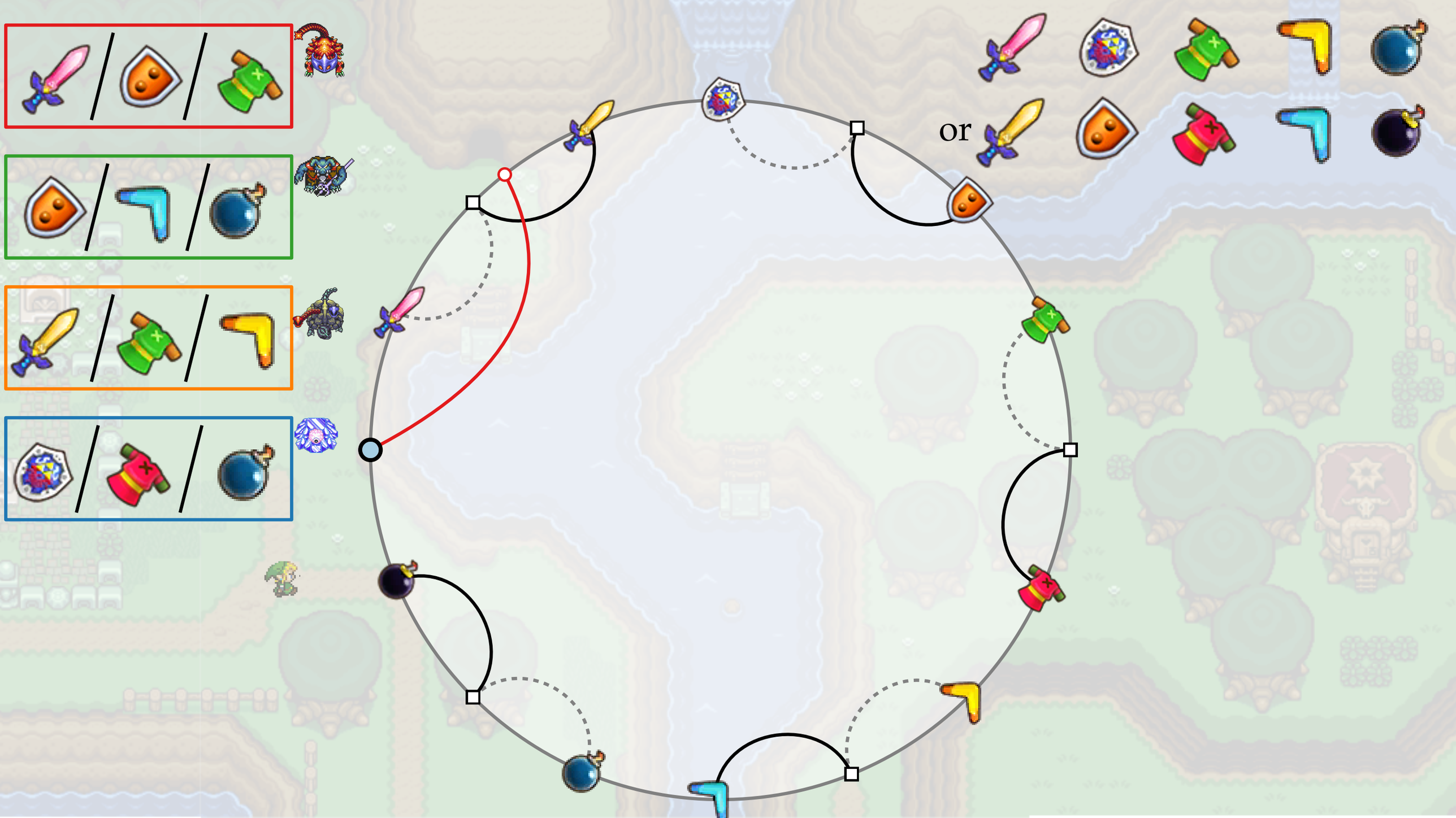
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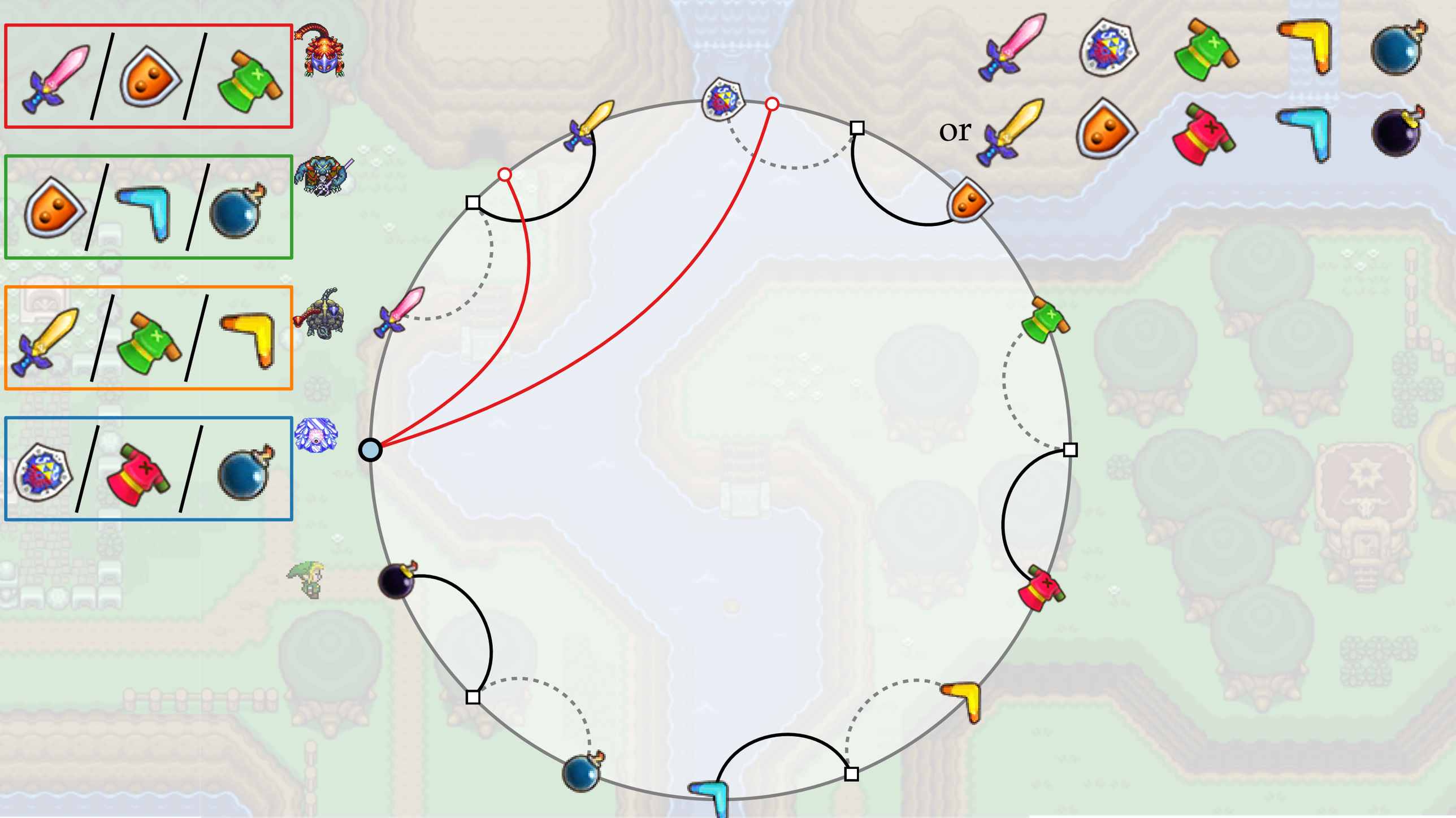
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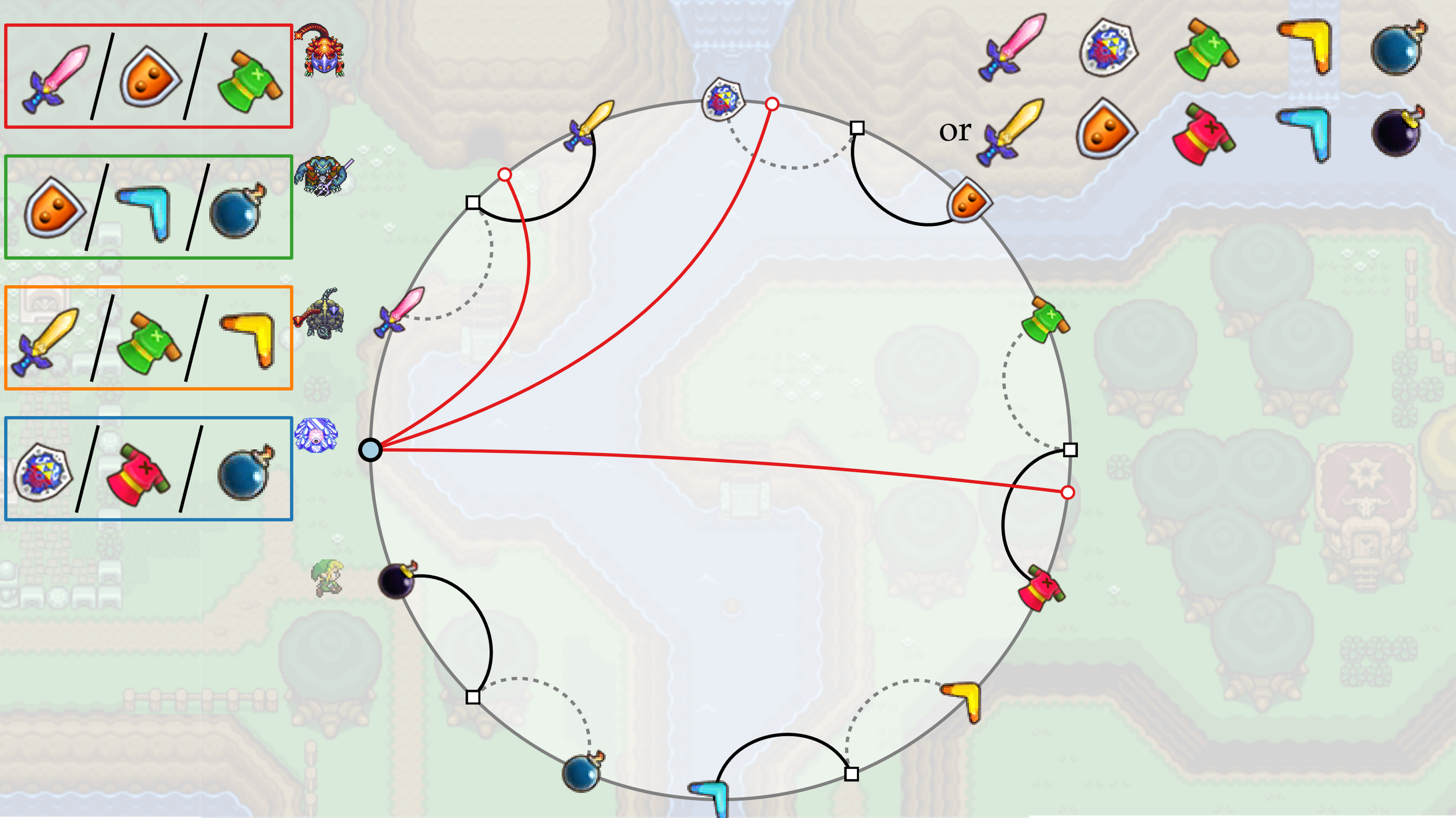
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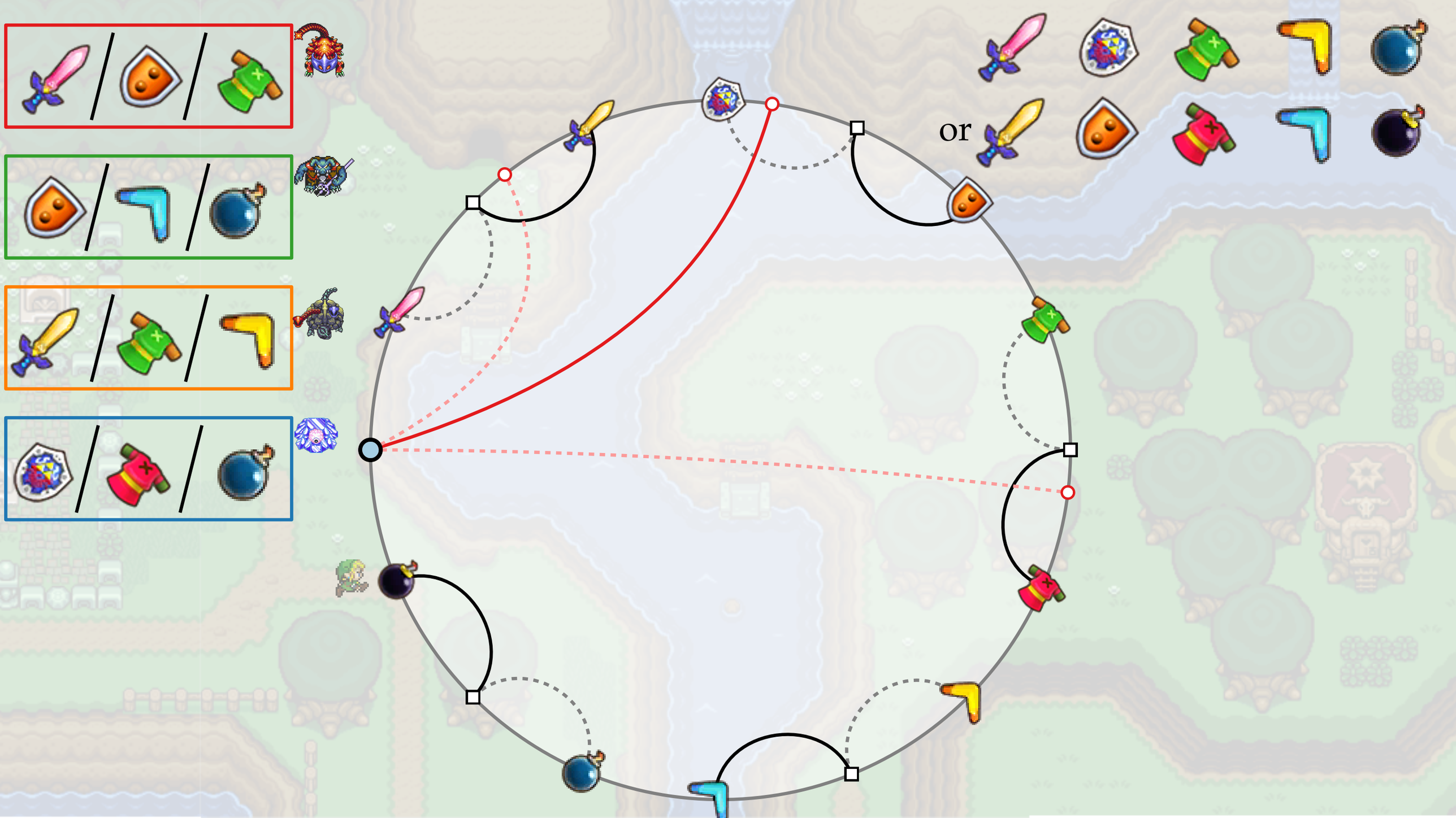
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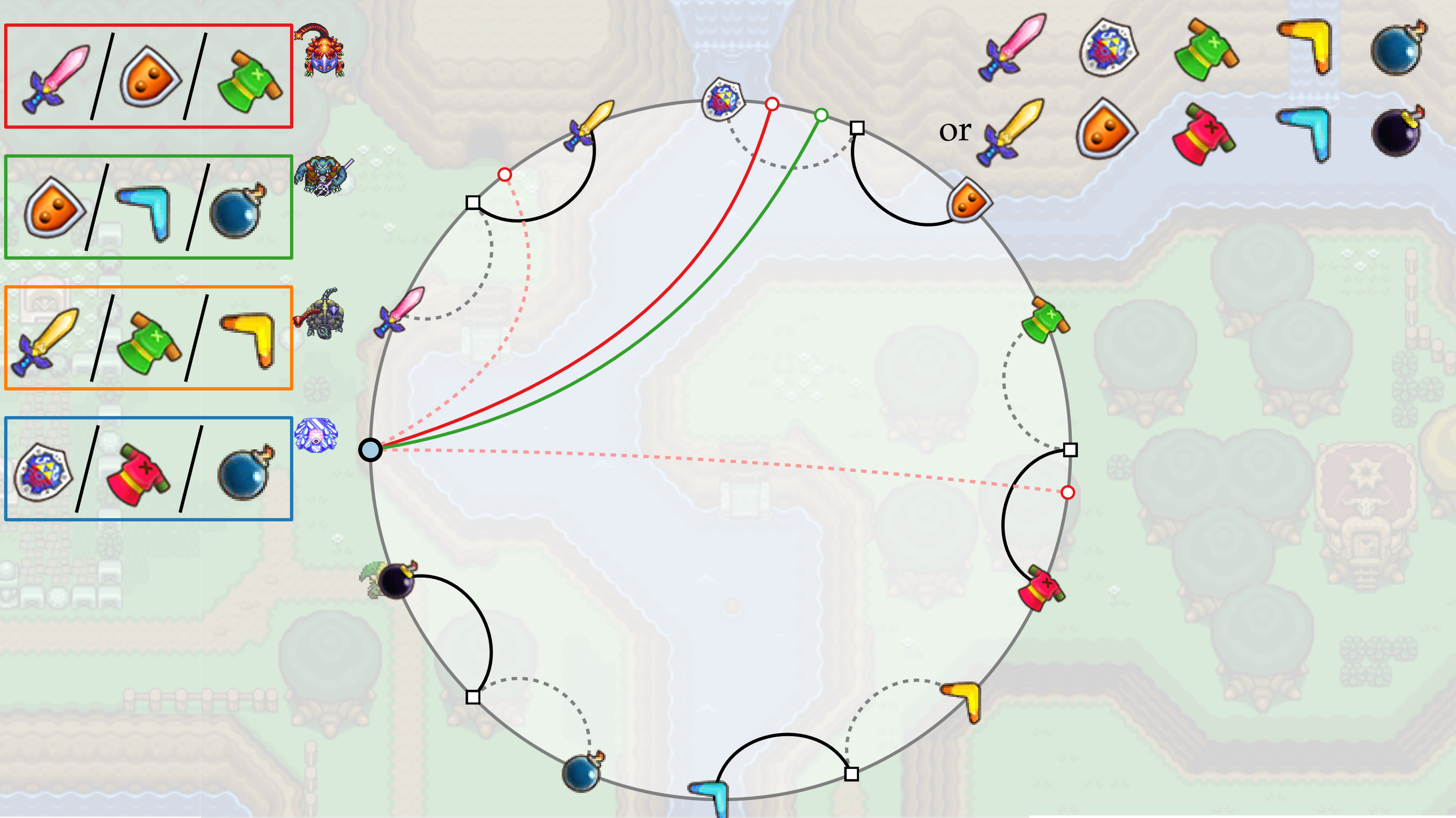
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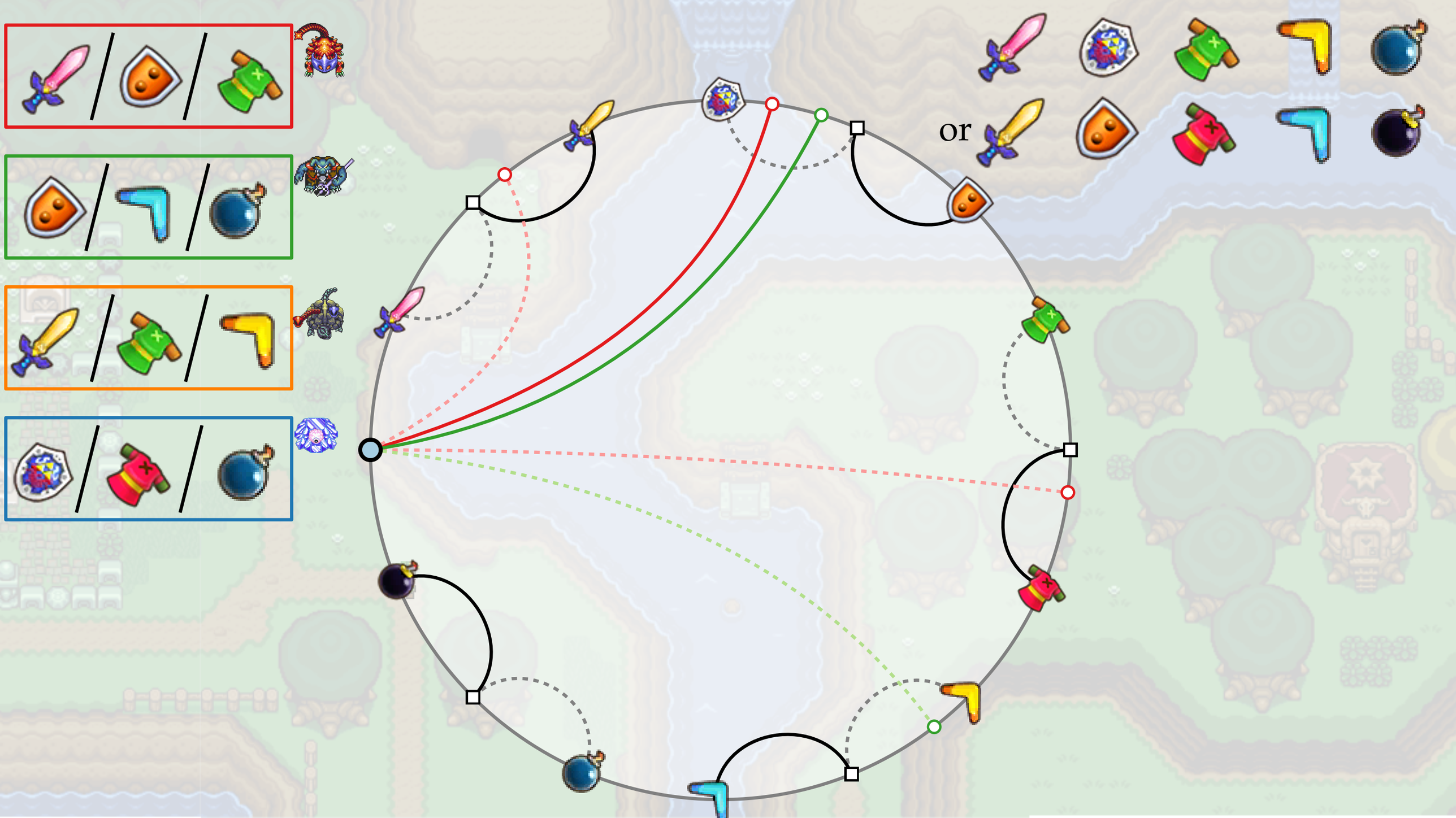
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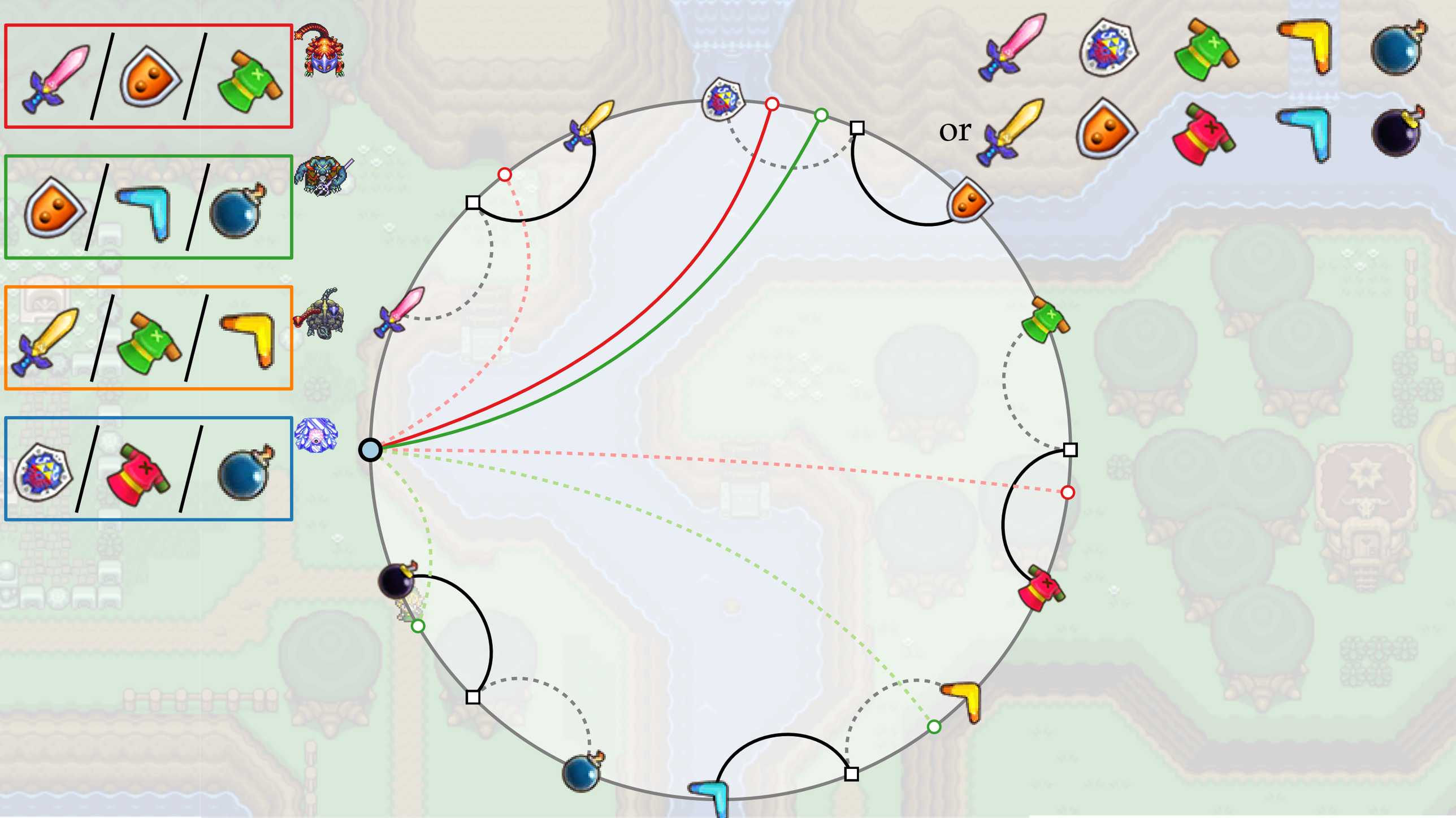
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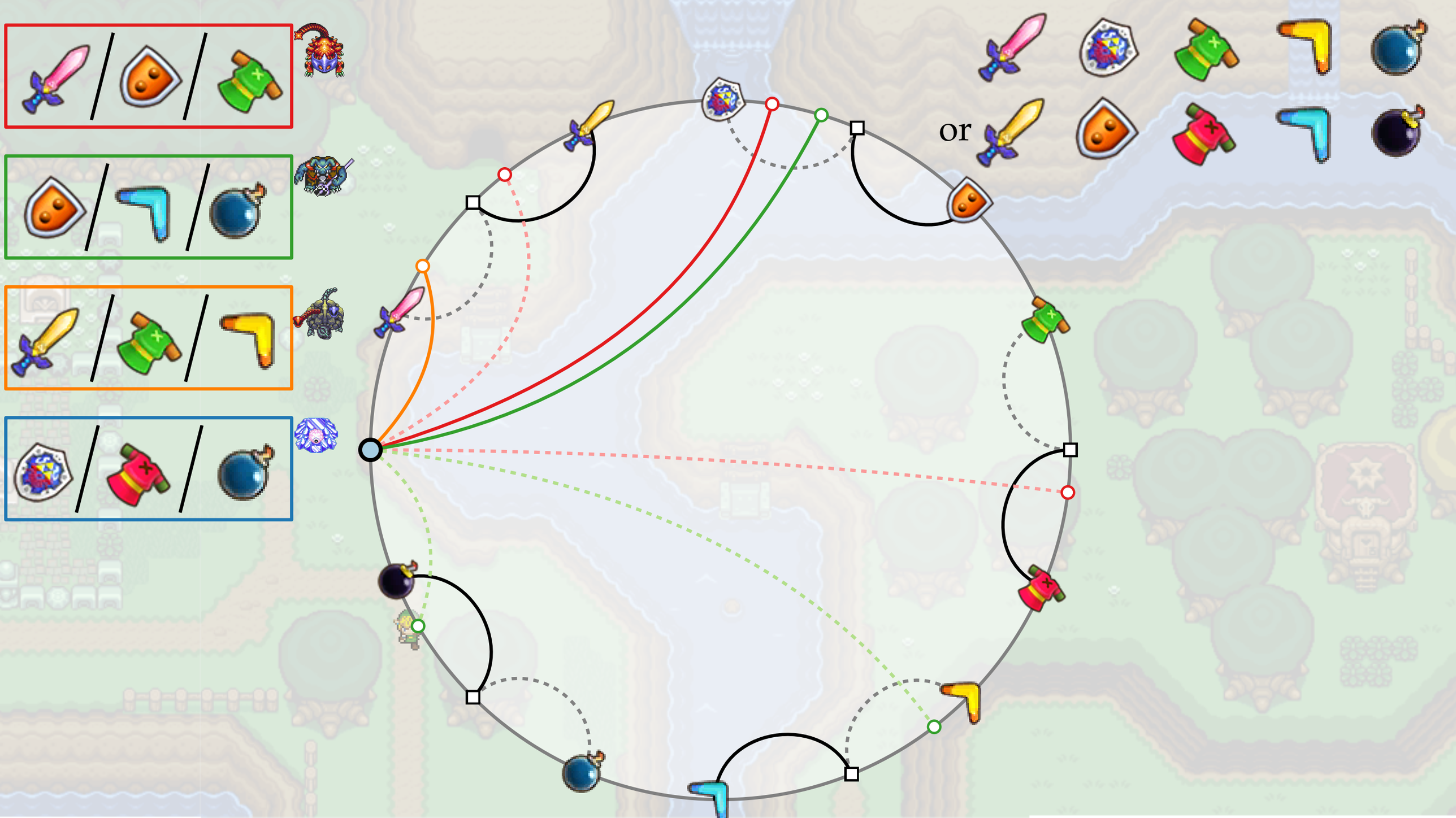
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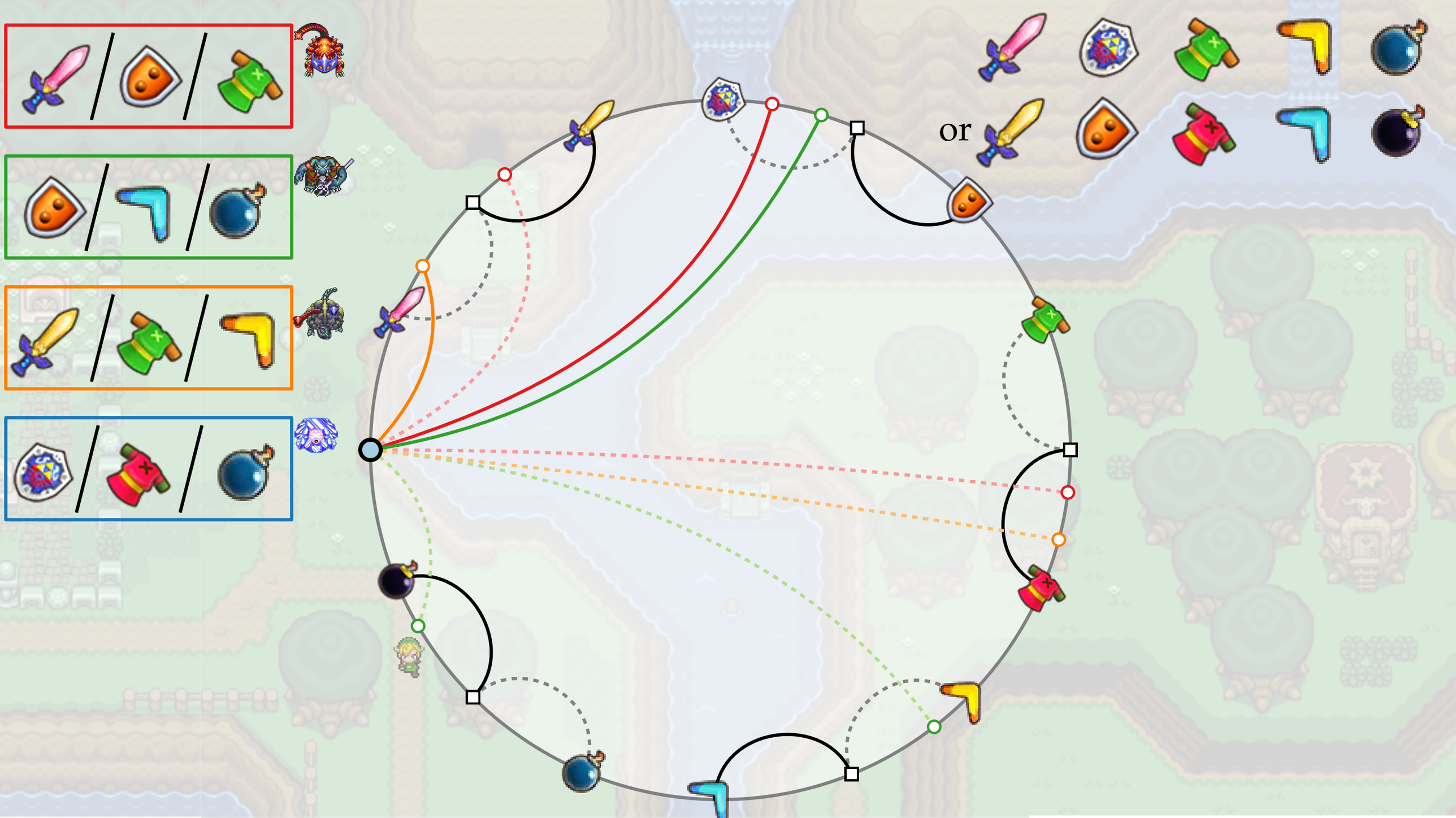
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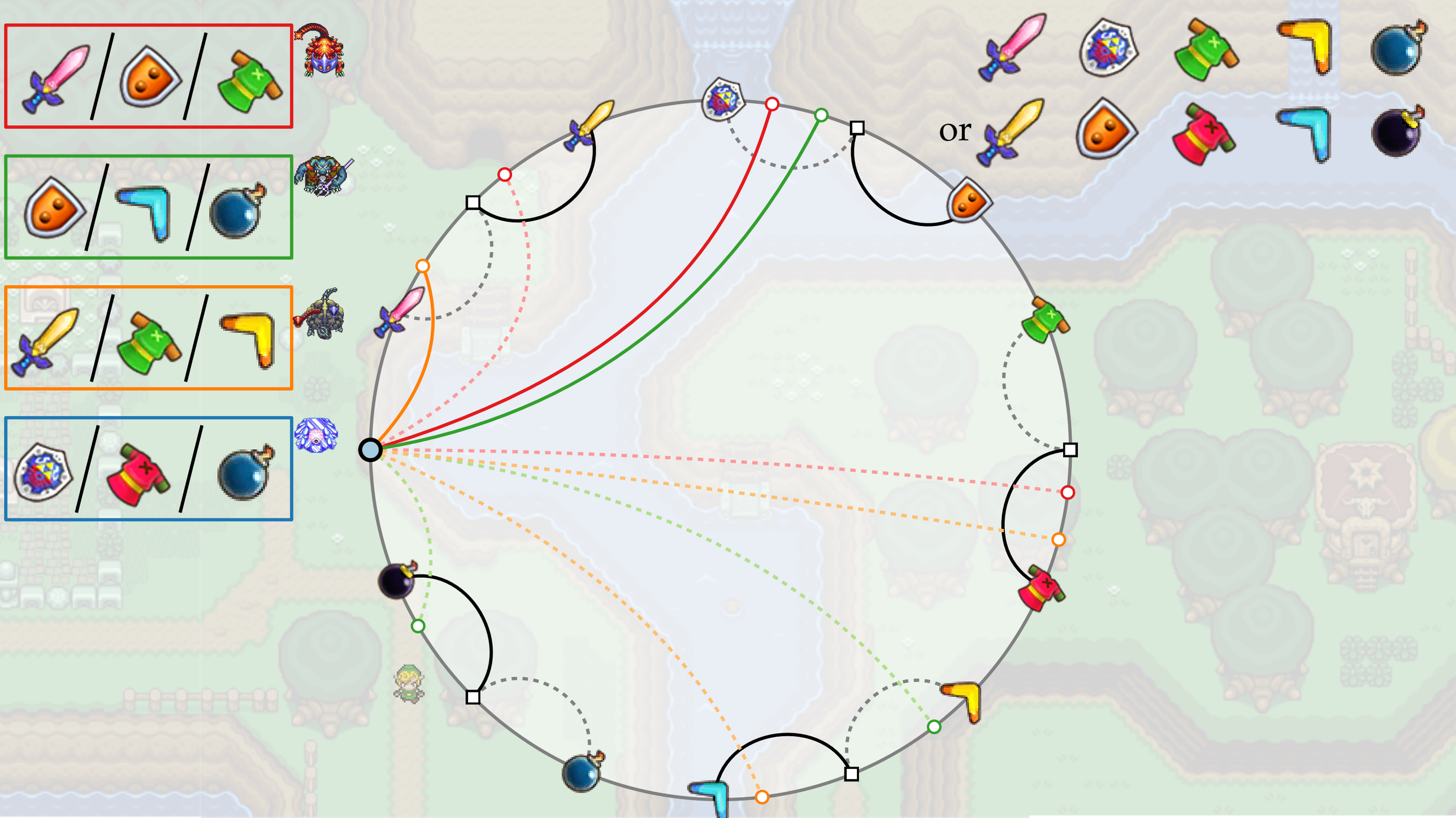
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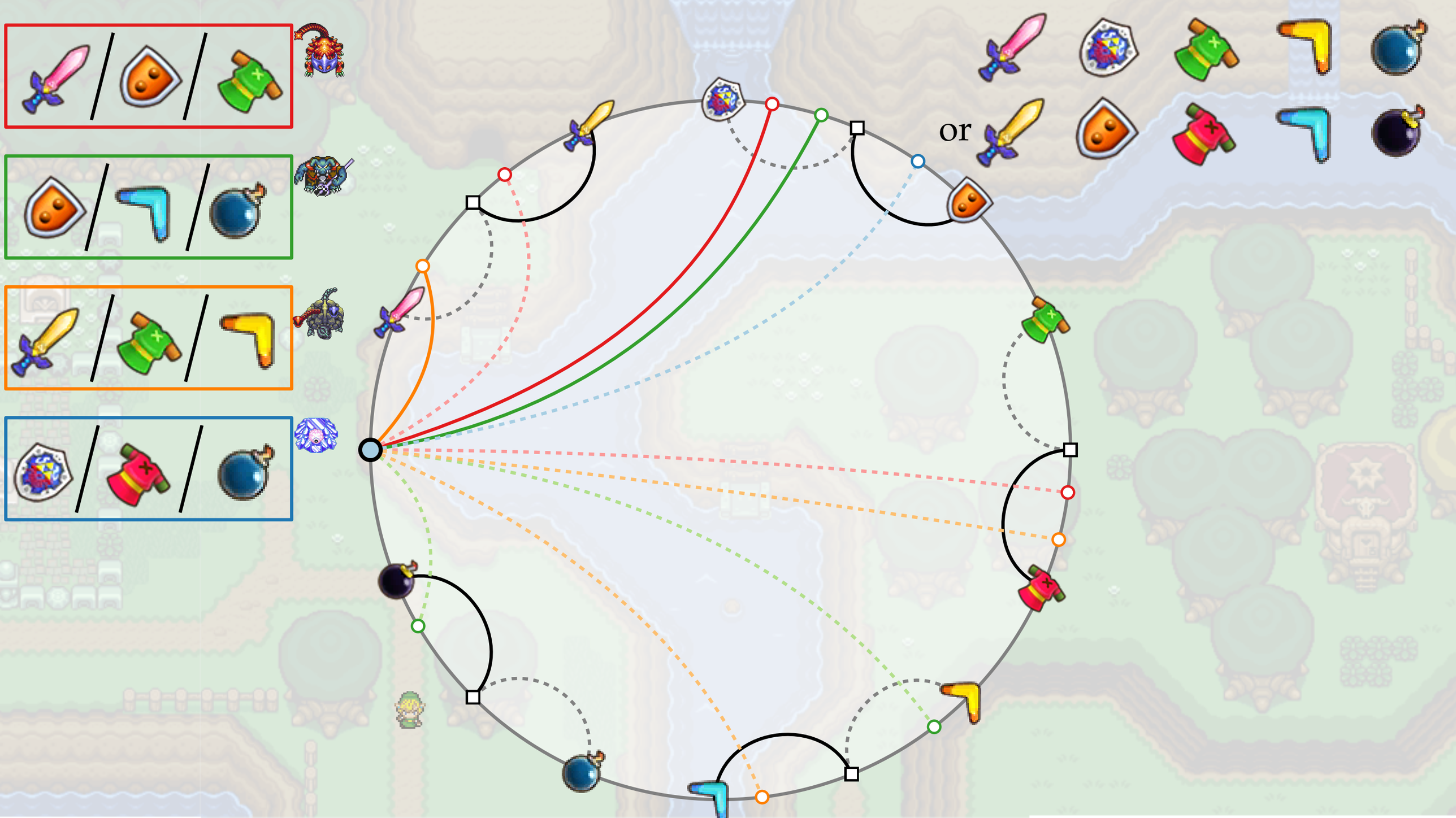
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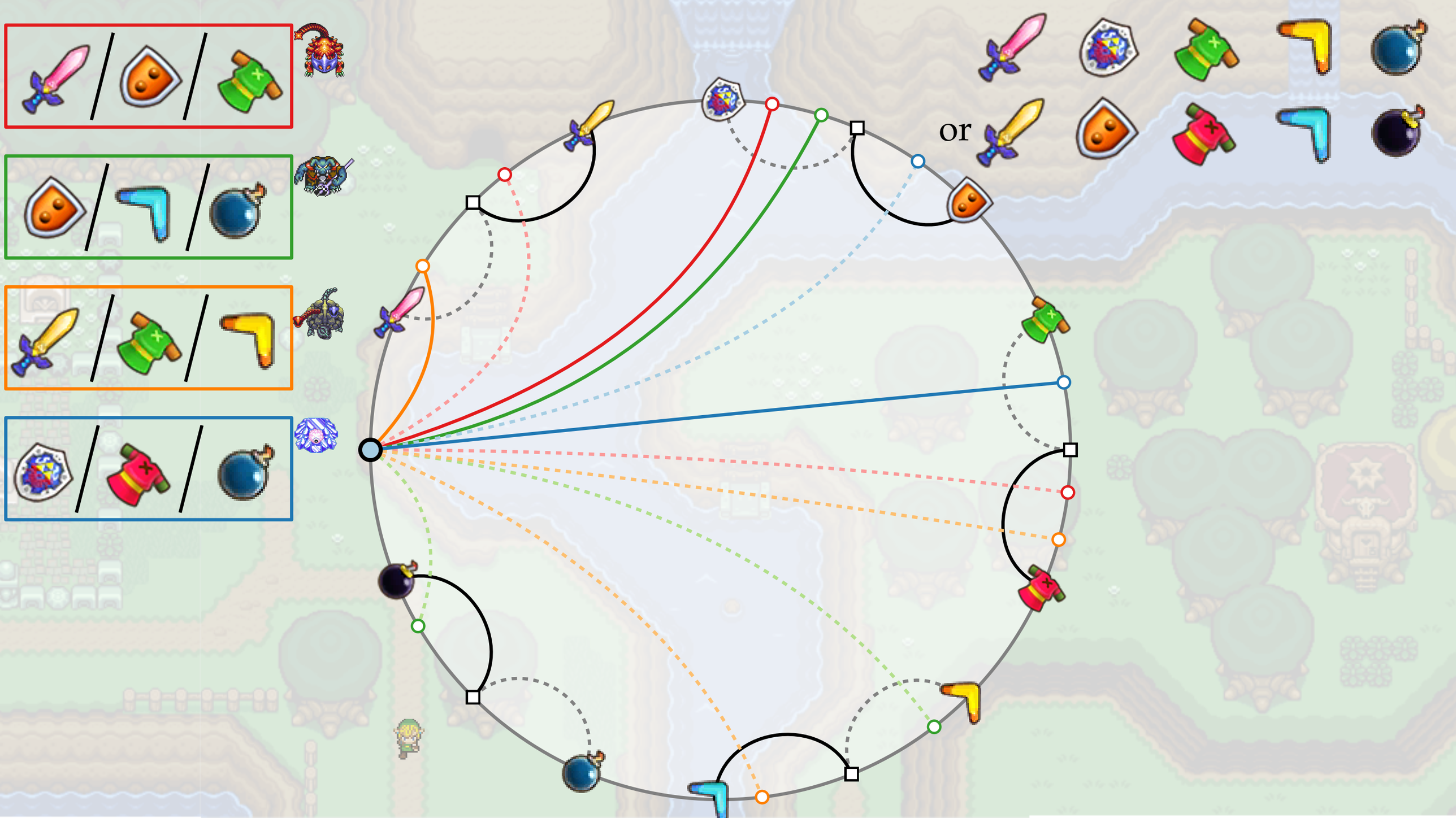
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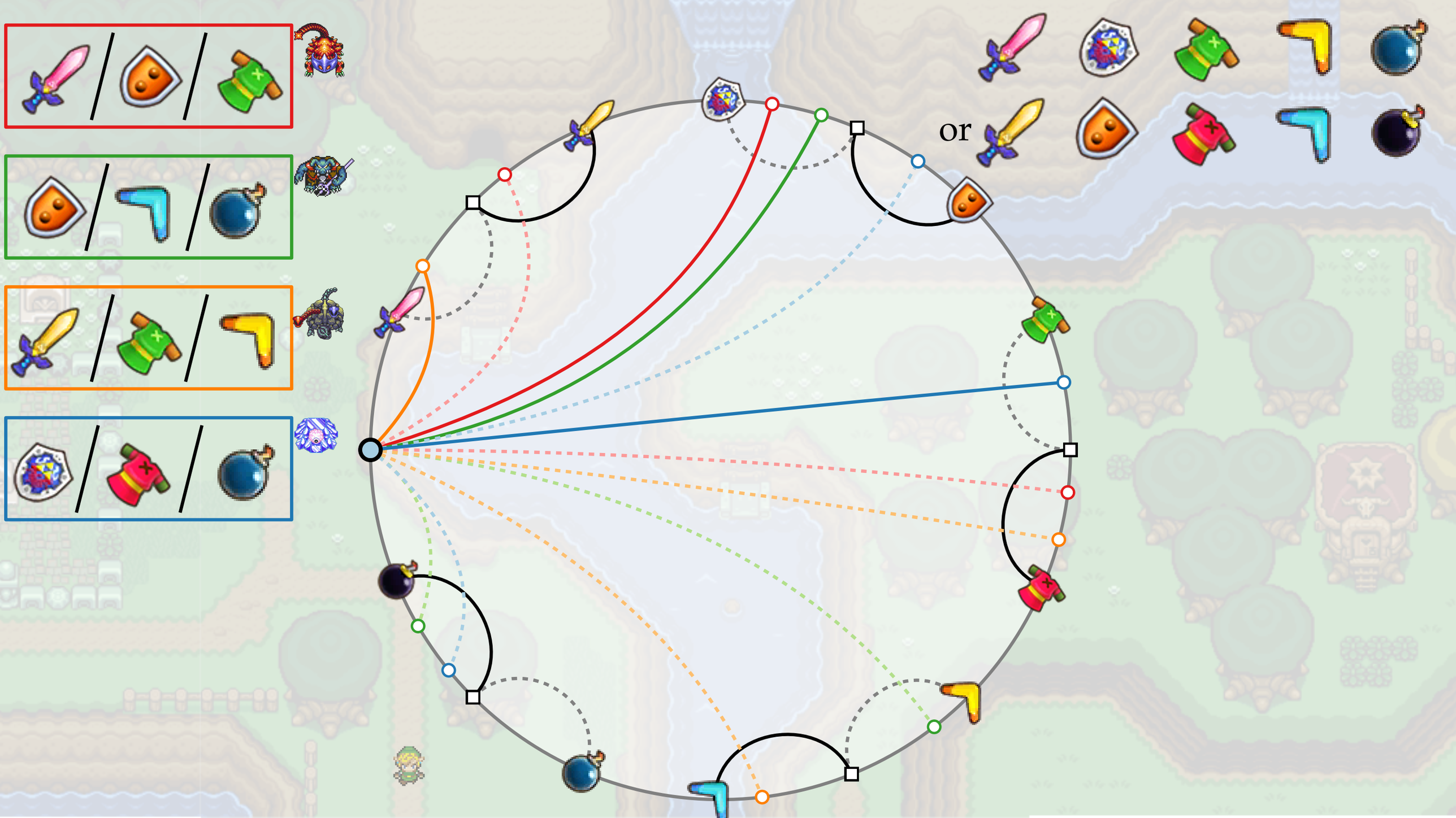
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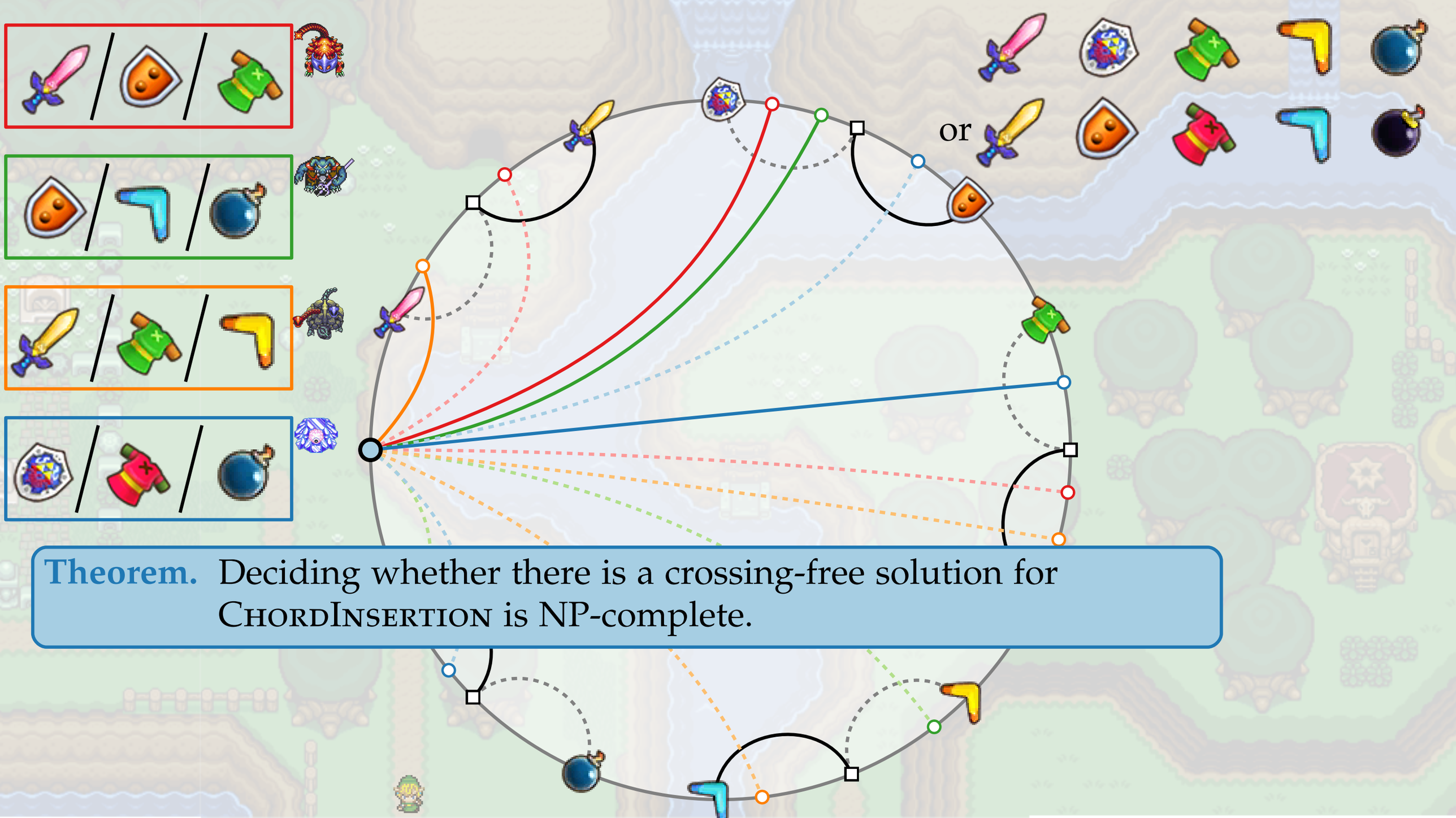
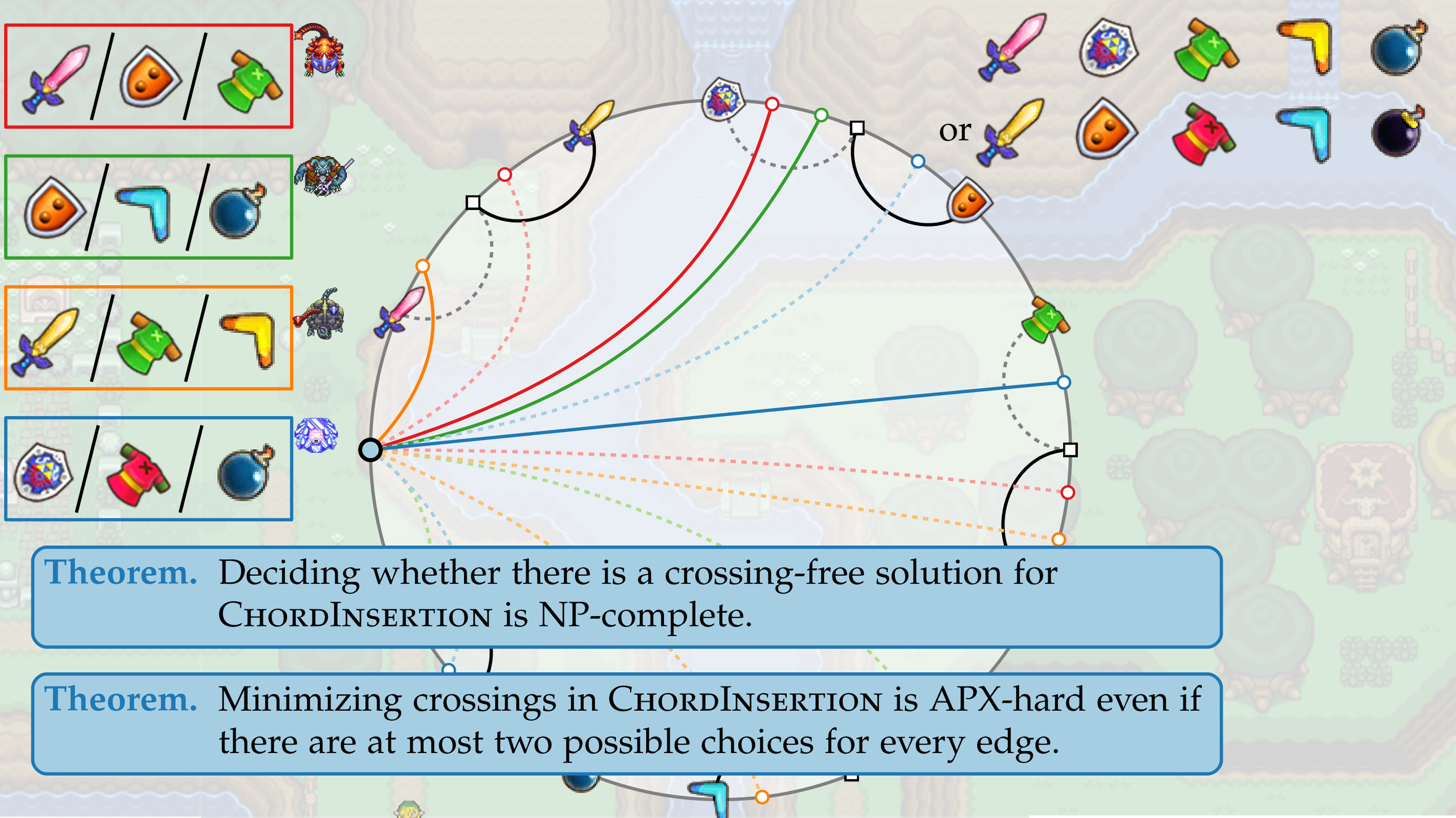
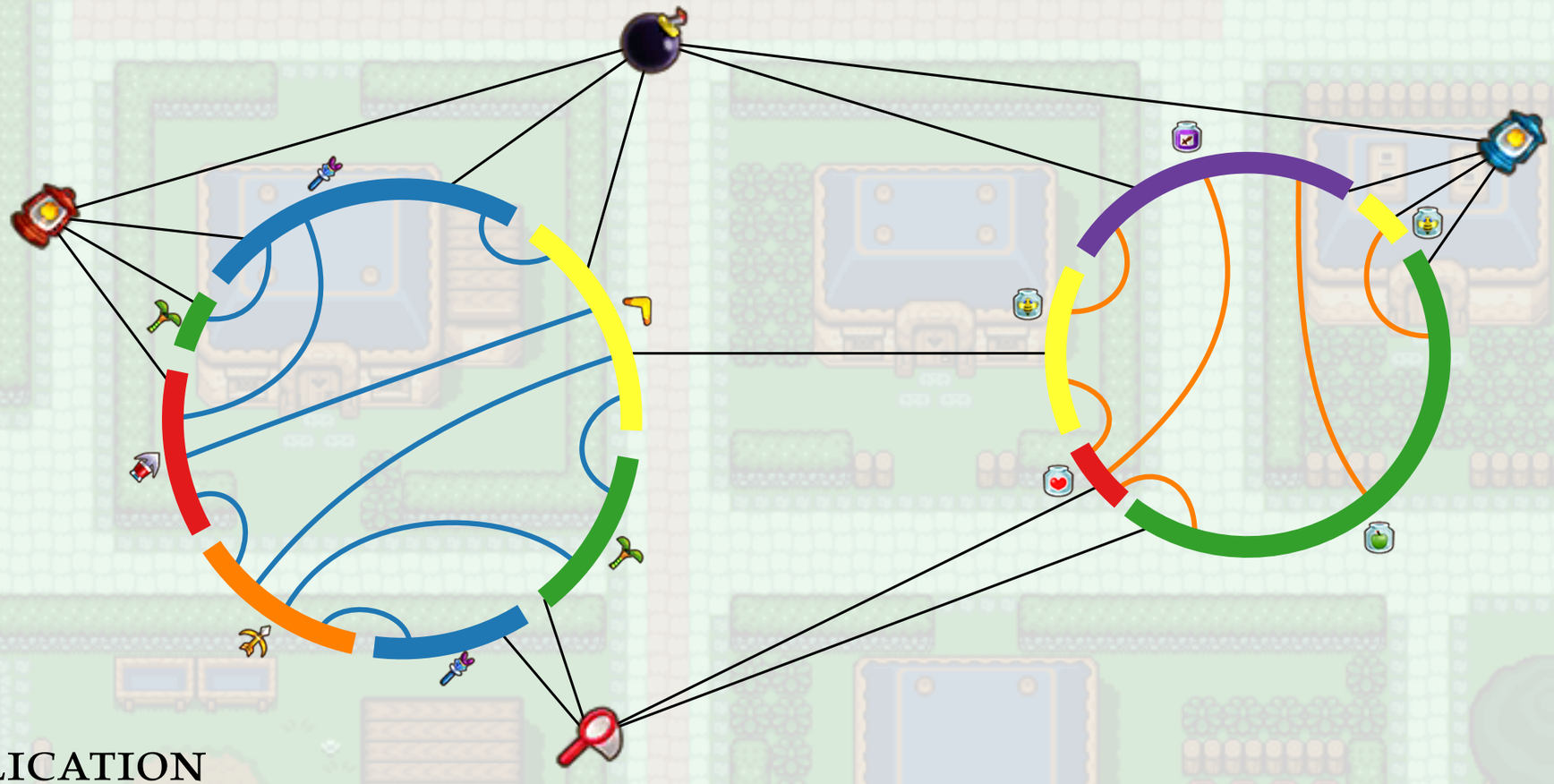


Theorem. Deciding whether there is a crossing-free solution for ChordInsertion is NP-complete.

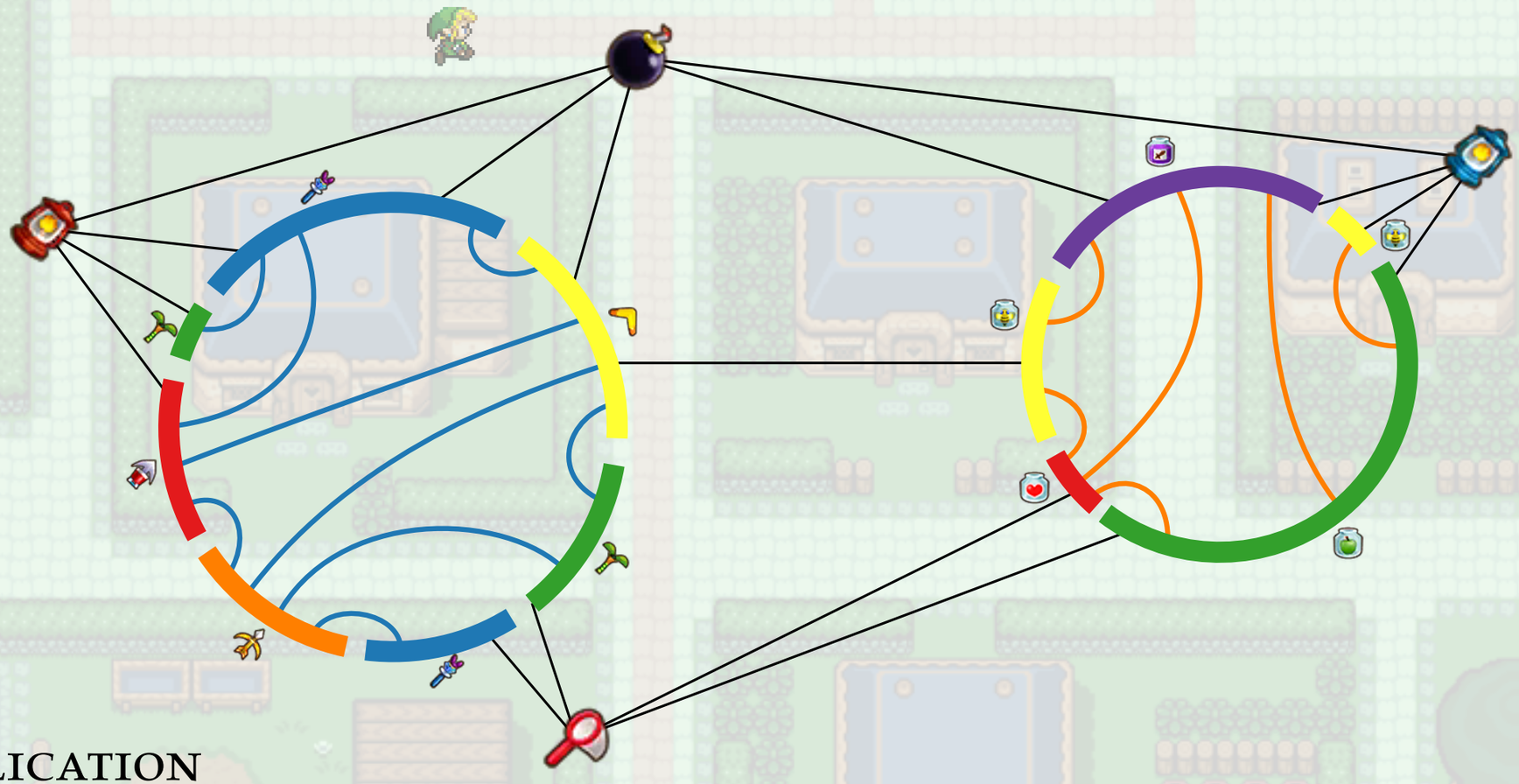


Theorem. Deciding whether there is a crossing-free solution for ChordInsertion is NP-complete.

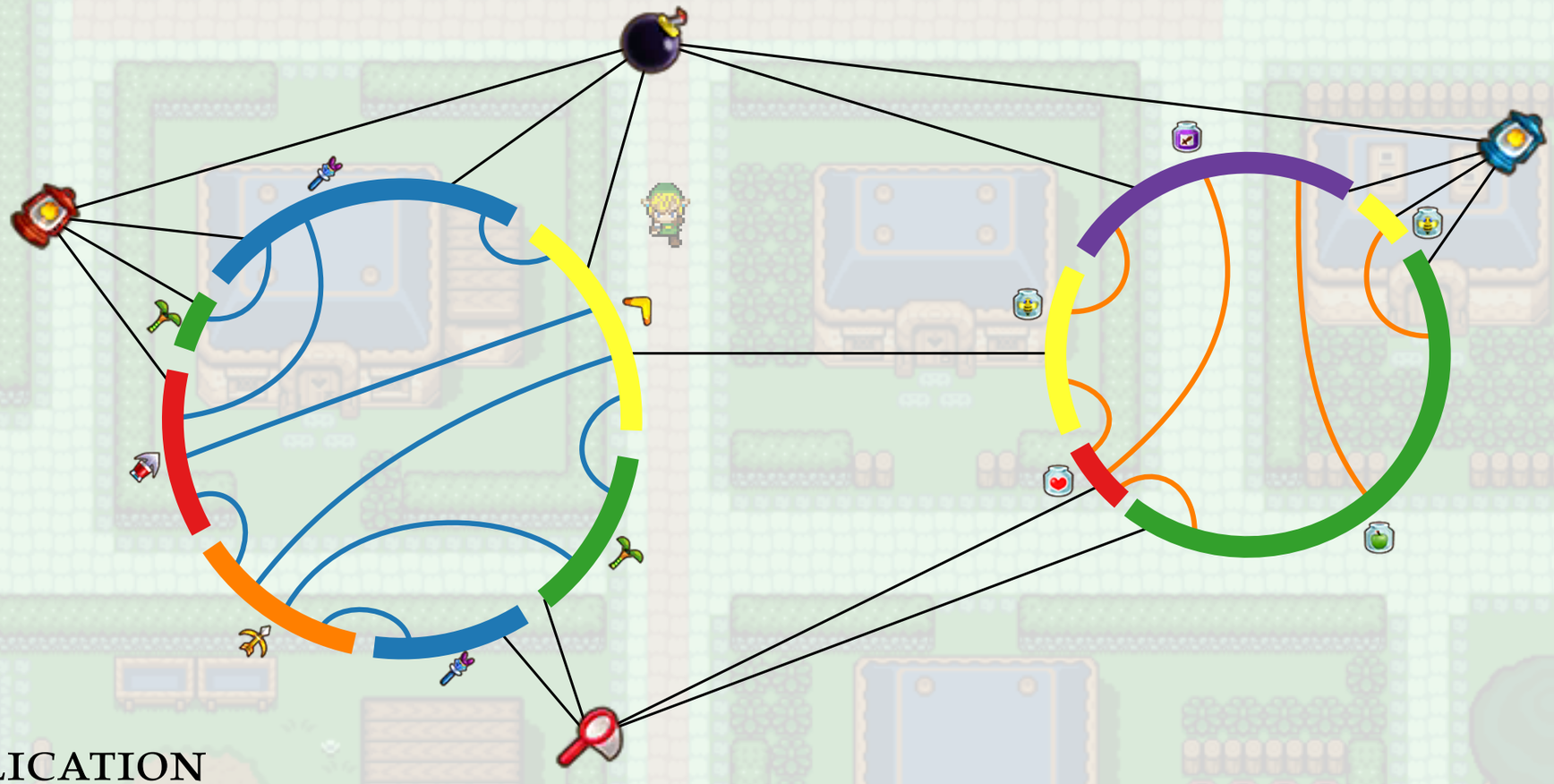
Theorem. Minimizing crossings in ChordInsertion is APX-hard even if there are at most two possible choices for every edge.



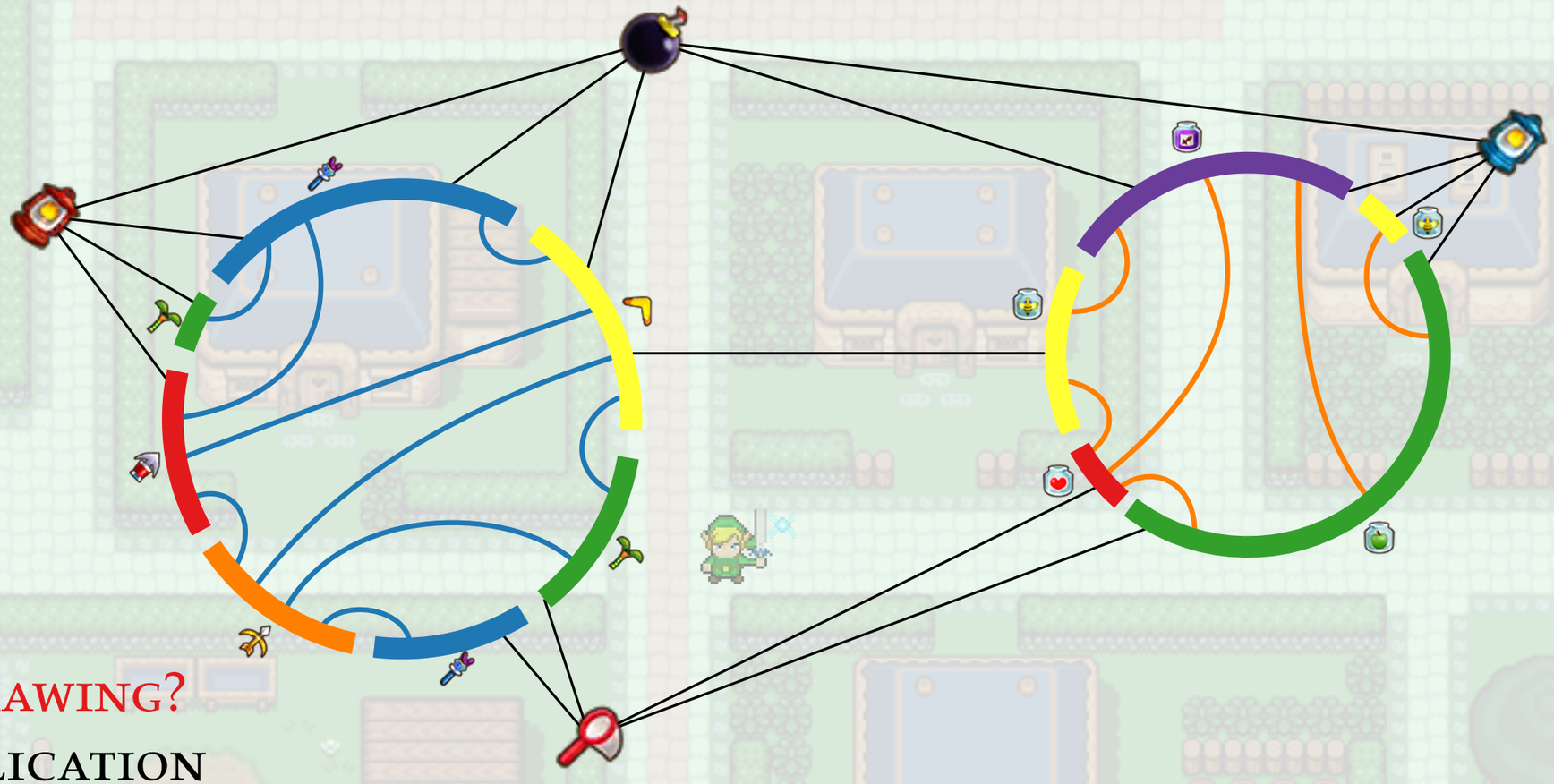
1. NodeReplication		
2. NodePermutation	Can flip nodes with common neighbor Goal: max. #successive pairs with same icon	NP-complete
3. NodeMerging		
4. ChordInsertion	Reintroduce chords in cluster Choose any copies of the adjacent vertices Goal: min. #crossings	APX-hard



1. NodeReplication		
2. NodePermutation	Can flip nodes with common neighbor Goal: max. #successive pairs with same icon	NP-complete
3. NodeMerging		
4. ChordInsertion	Reintroduce chords in cluster Choose any copies of the adjacent vertices Goal: min. #crossings / angular resolution	APX-hard



1. NodeReplication		
2. NodePermutation	Can flip nodes with common neighbor Goal: max. #successive pairs with same icon	NP-complete
3. NodeMerging		
4. ChordInsertion	Reintroduce chords in cluster Choose any copies of the adjacent vertices Goal: min. #crossings / angular resolution / ?	APX-hard



0. InitialDrawing?		
1. NodeReplication		
2. NodePermutation	Can flip nodes with common neighbor Goal: max. #successive pairs with same icon	NP-complete
3. NodeMerging		
4. ChordInsertion	Reintroduce chords in cluster Choose any copies of the adjacent vertices Goal: min. #crossings / angular resolution / ?	APX-hard