EUROVIS ZURICH 21

ClusterSets: Optimizing Planar Clusters in Categorical Point Data

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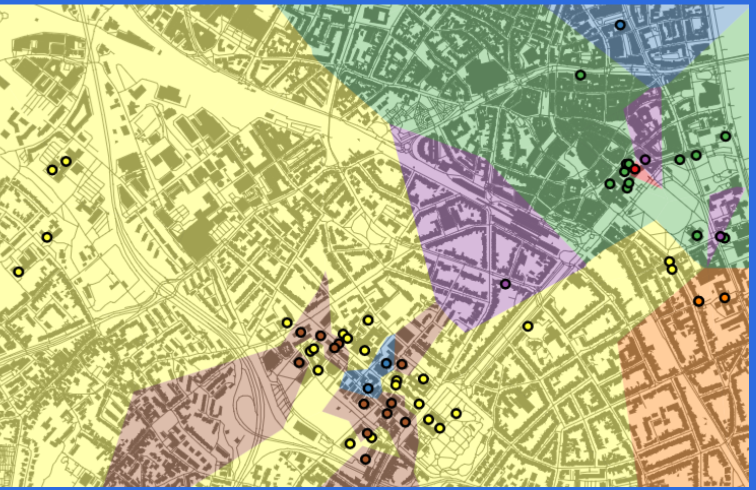

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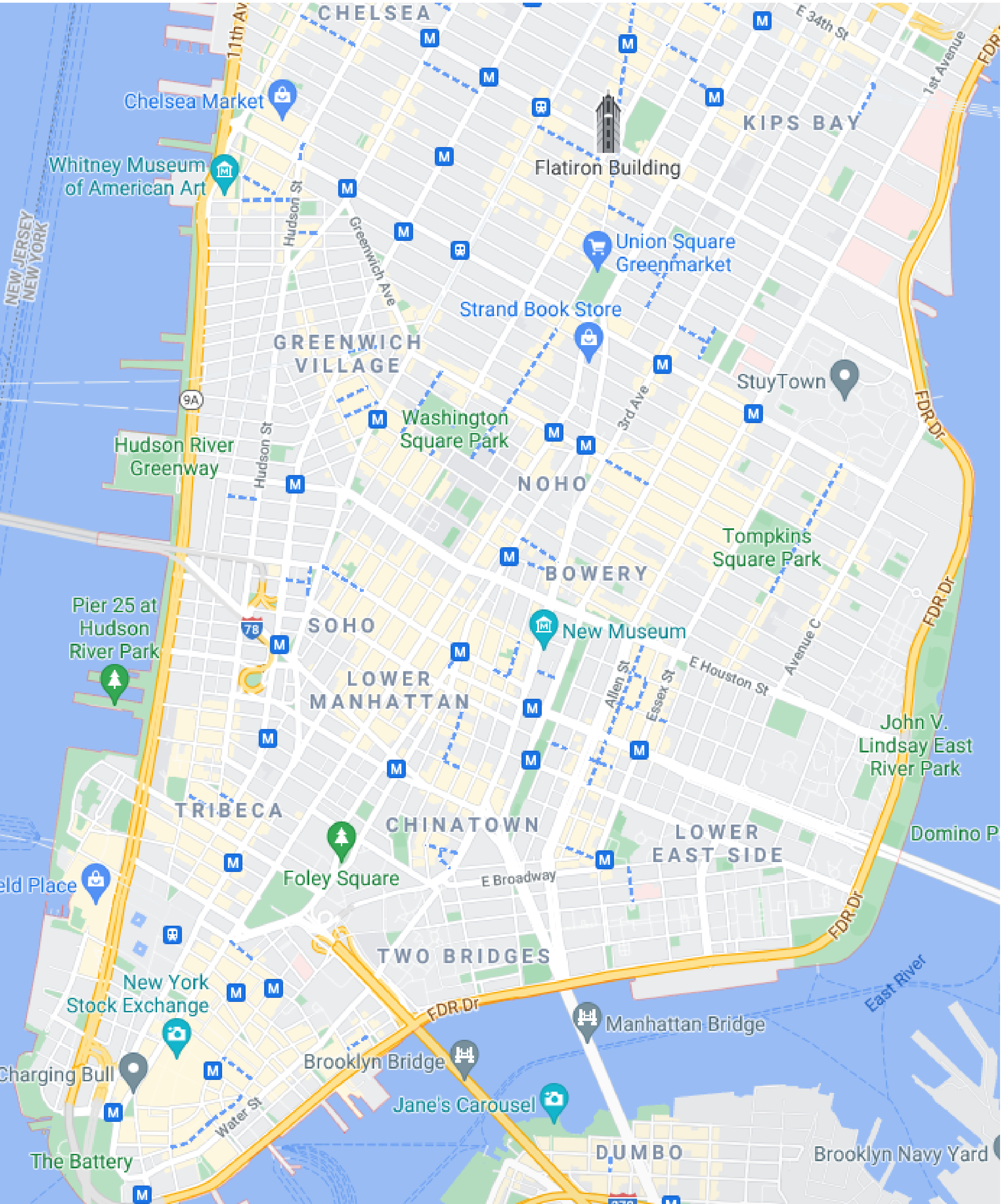
Sets of Points in Spatial Data



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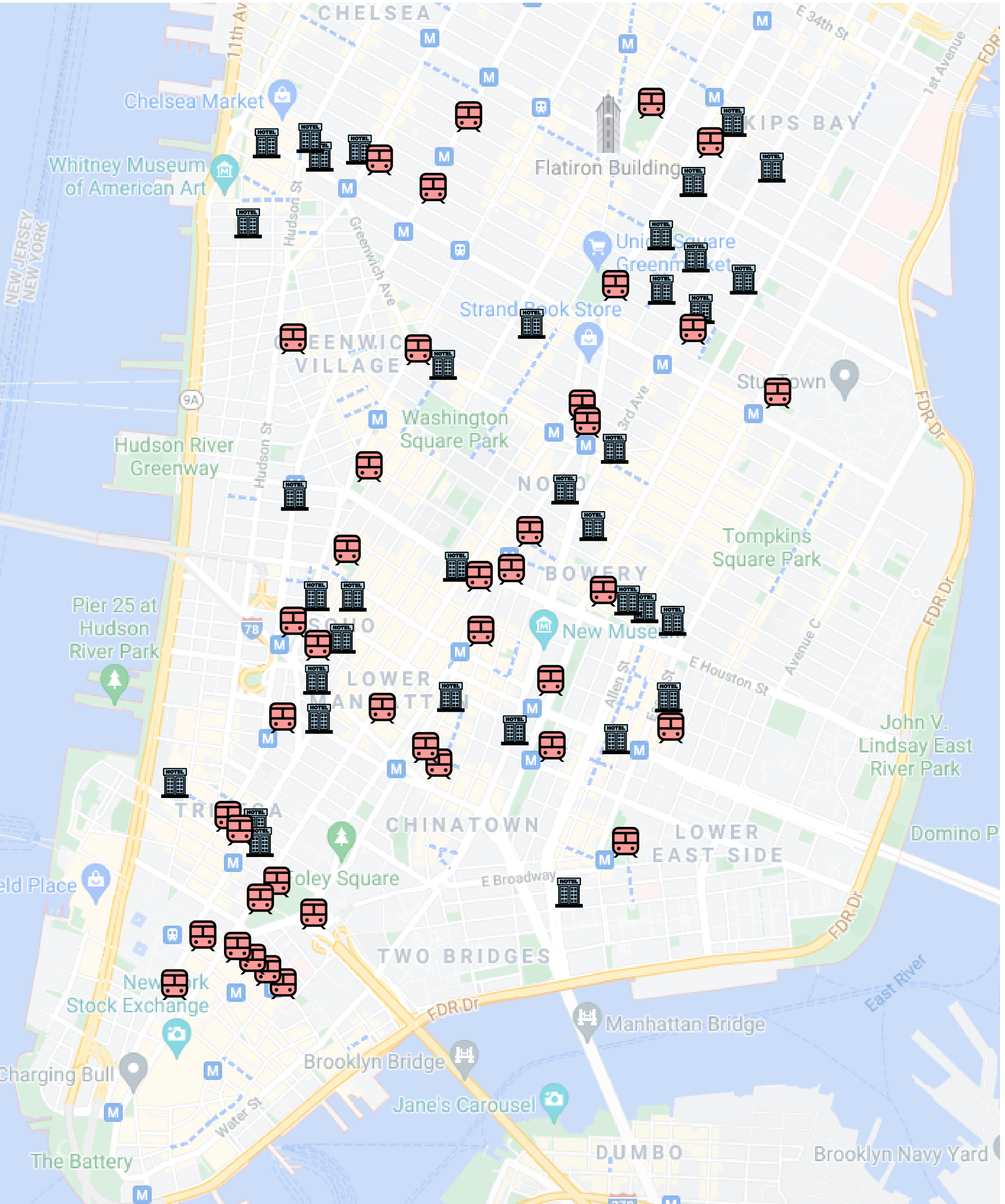

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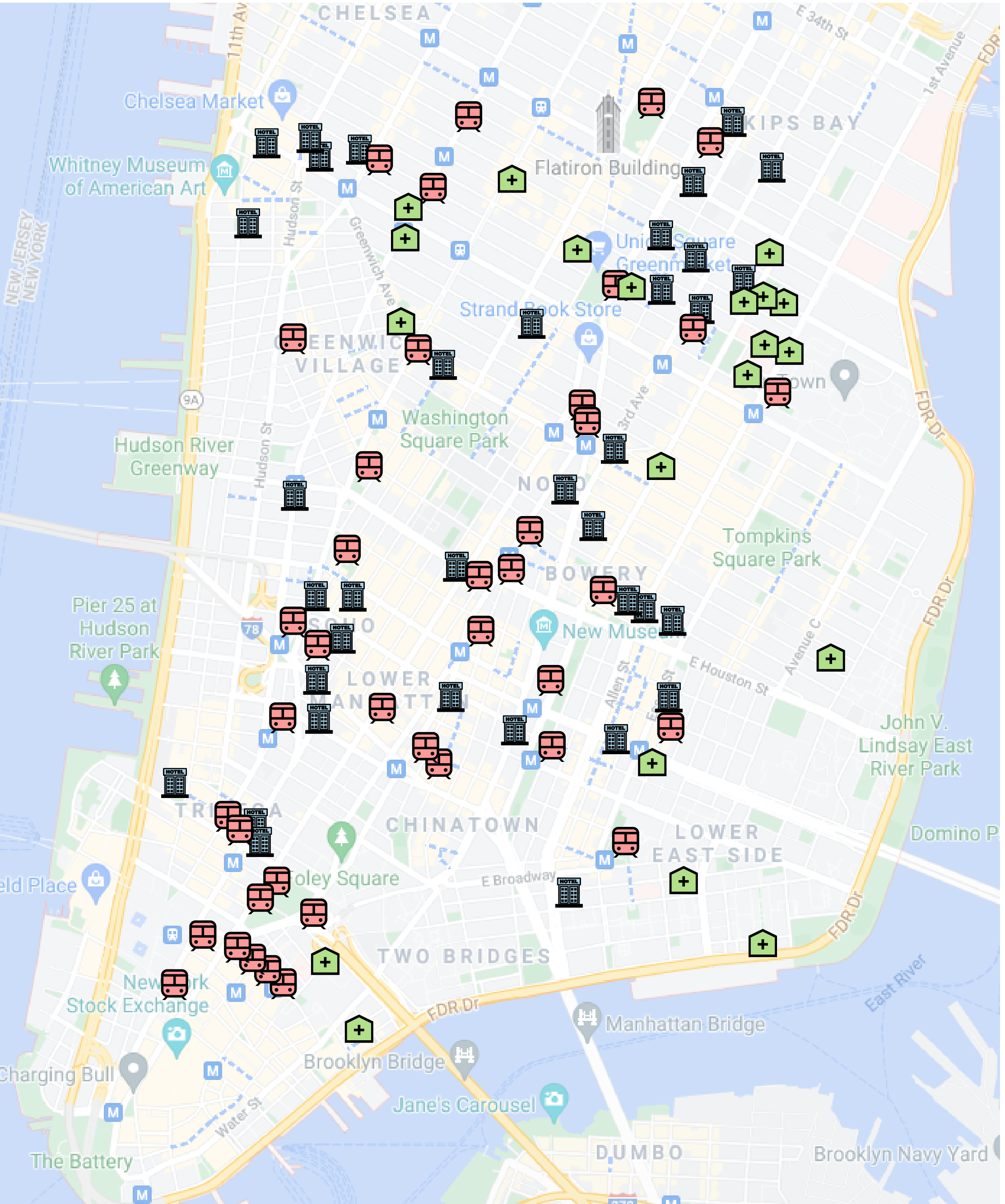

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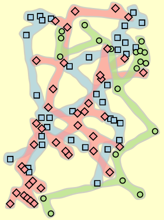
Sets of Points in Spatial Data

Bubble Sets

[Collins, Penn & Carpendal '09]



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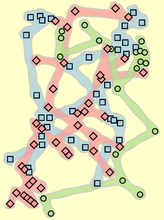
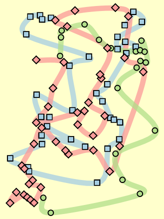


LineSets
[Alper, Henry Riche, Ramos & Czerwinski '11]



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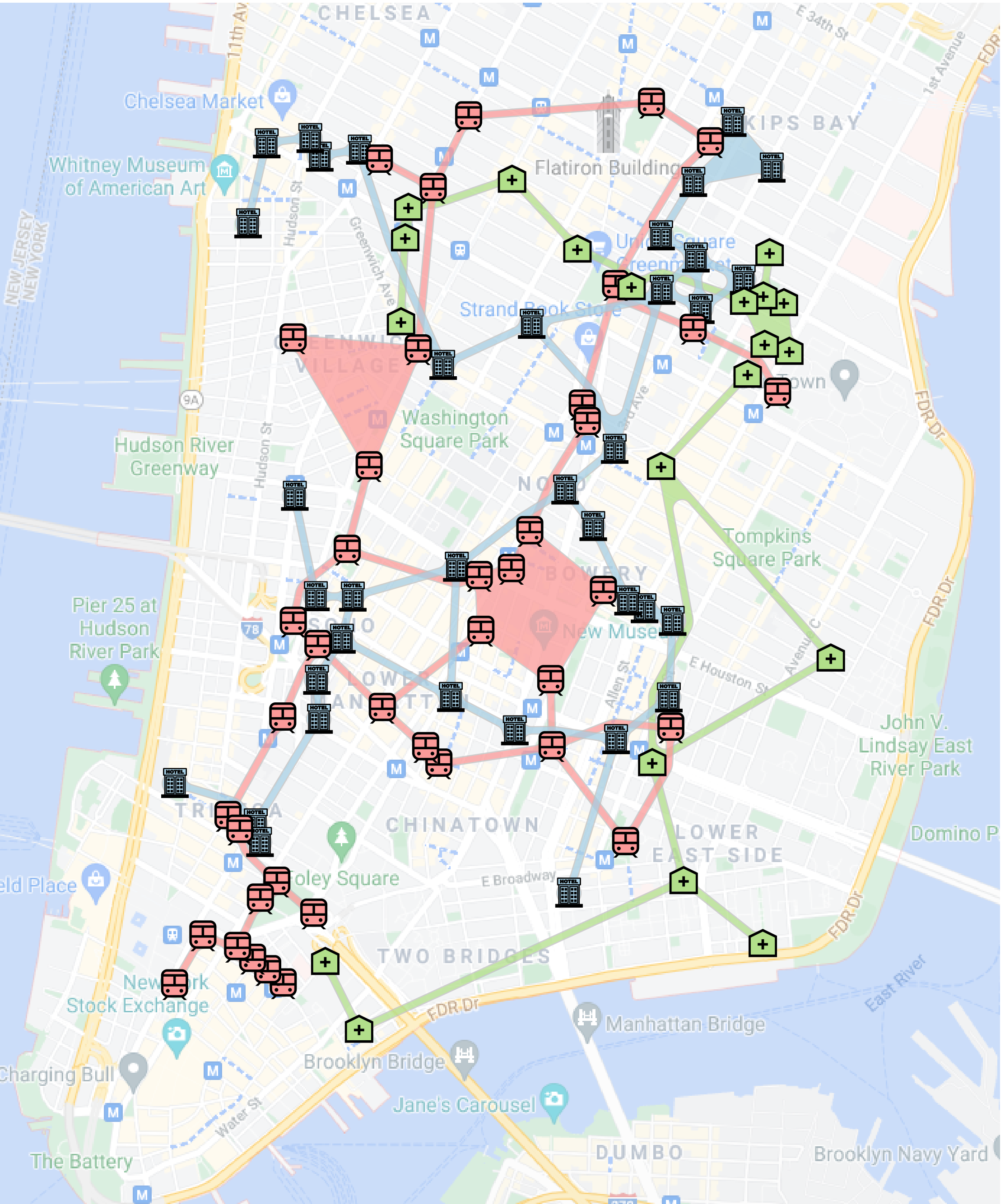

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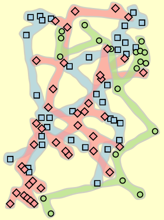

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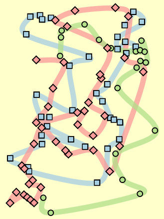
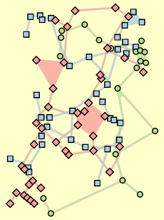


KelpFusion
[Meulemans, Henry Riche, Speckmann, Alper & Dwyer '13]



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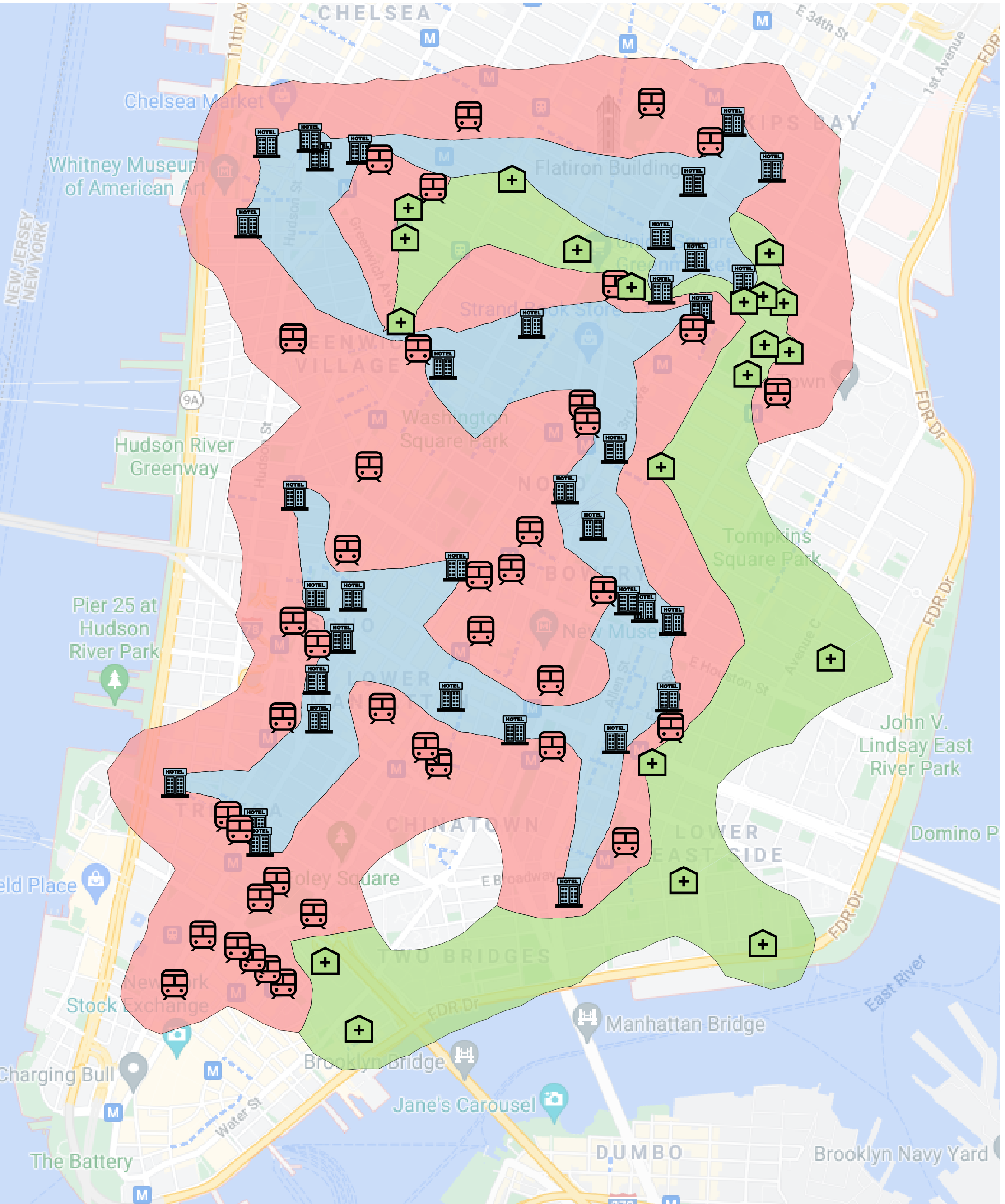

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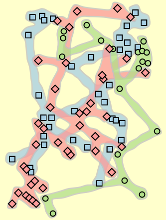

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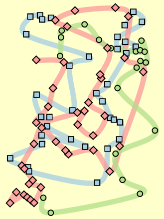

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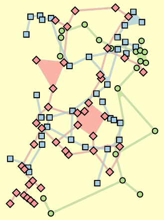
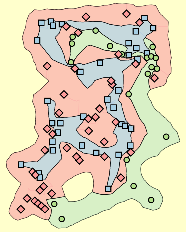


MapSets
[Efrat, Hu, Kobourov & Pupyrev '15]



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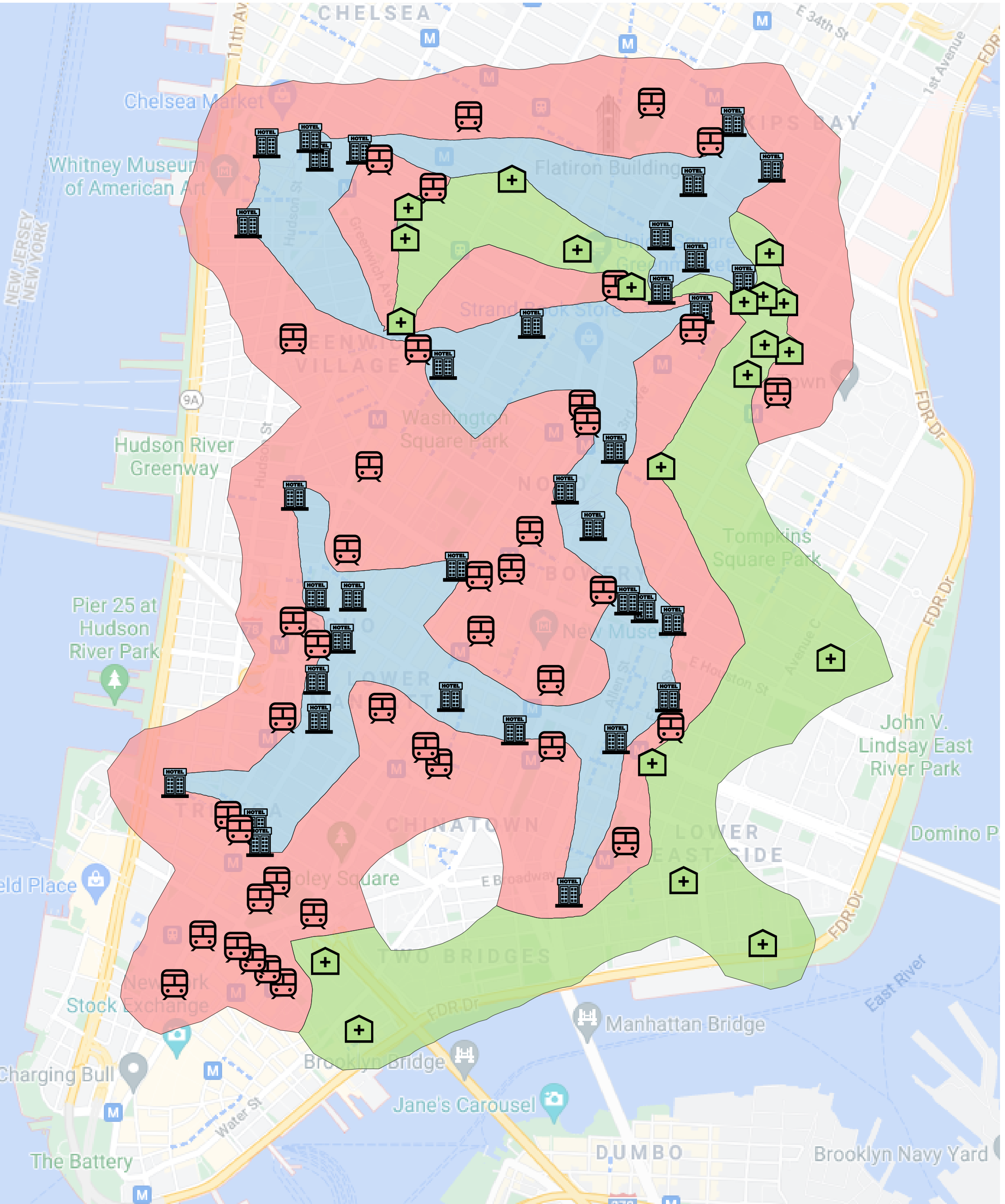

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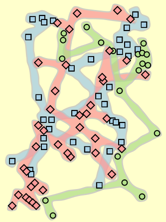

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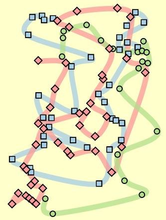

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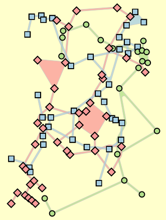

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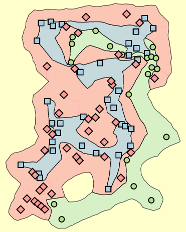
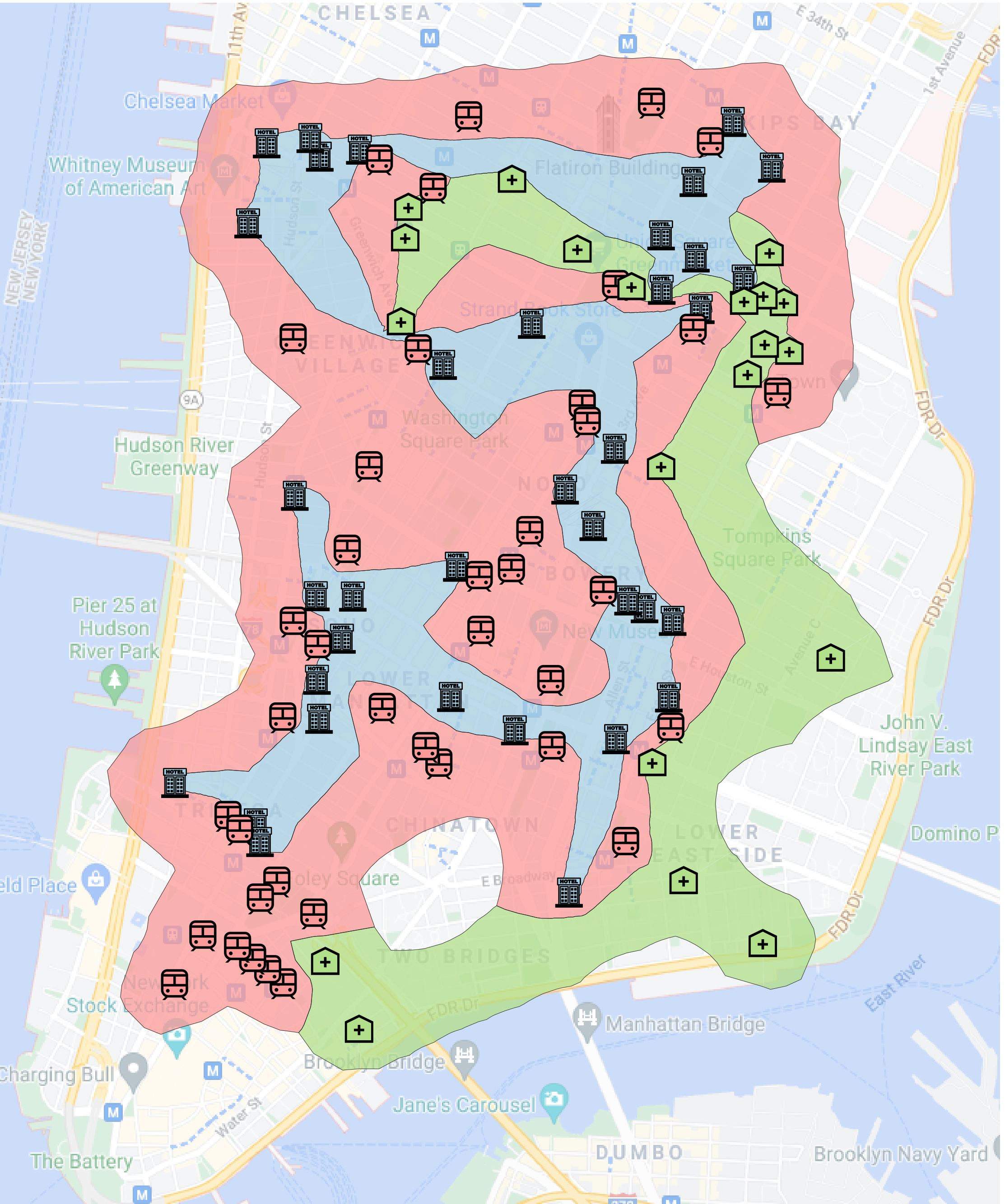


All points of the same category are connected.

Sets of Points in Spatial Data



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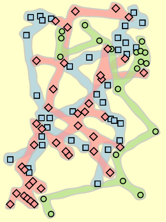

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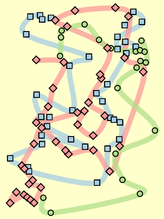

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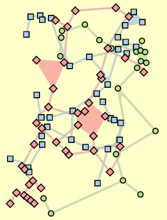

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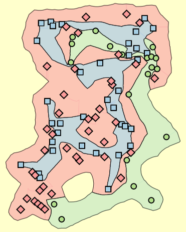

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Relax connectivity requirement
→ preservation of locality of clusters

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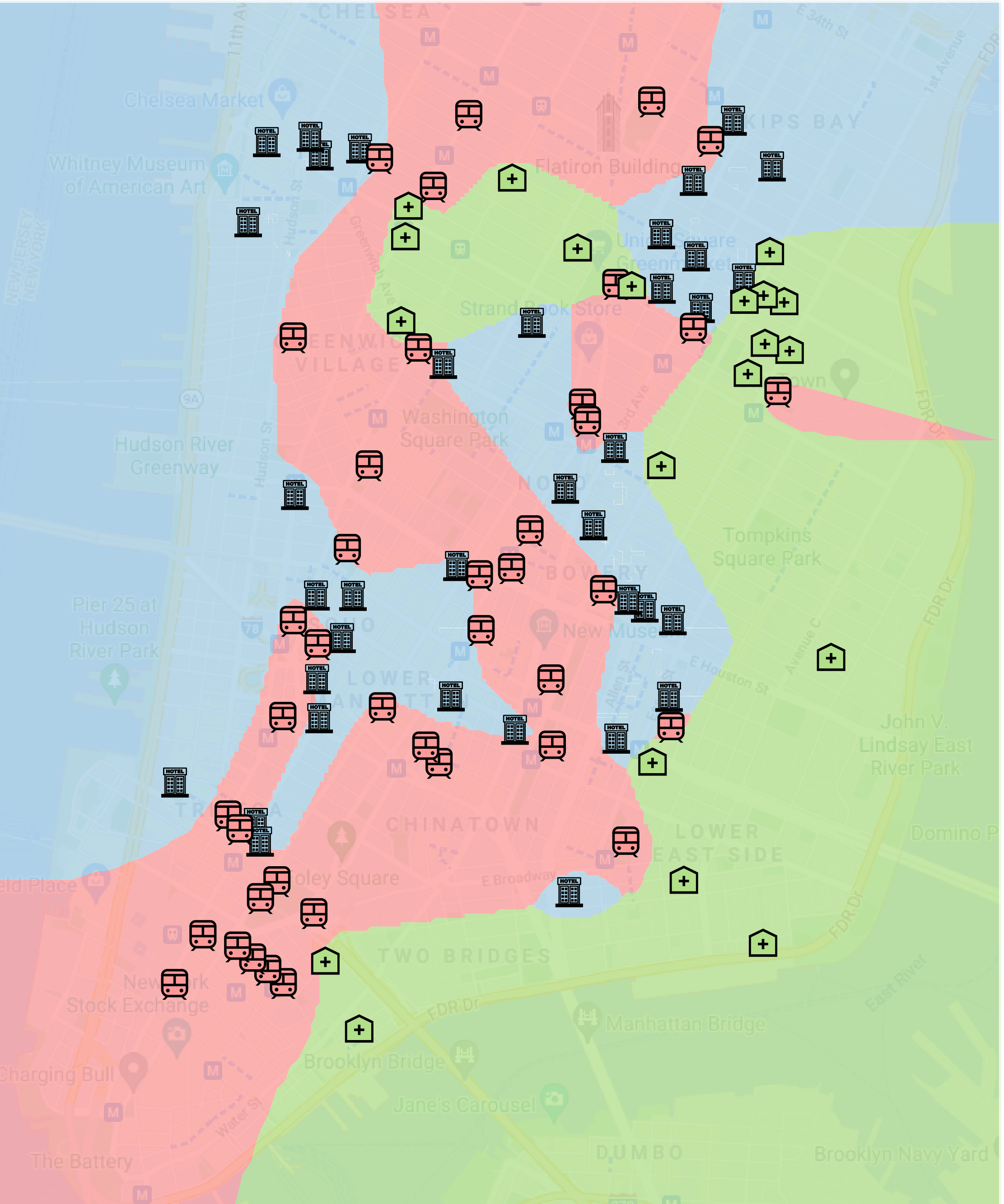

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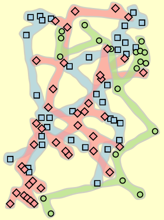

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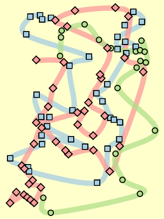

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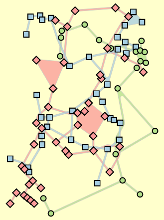

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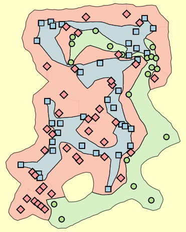

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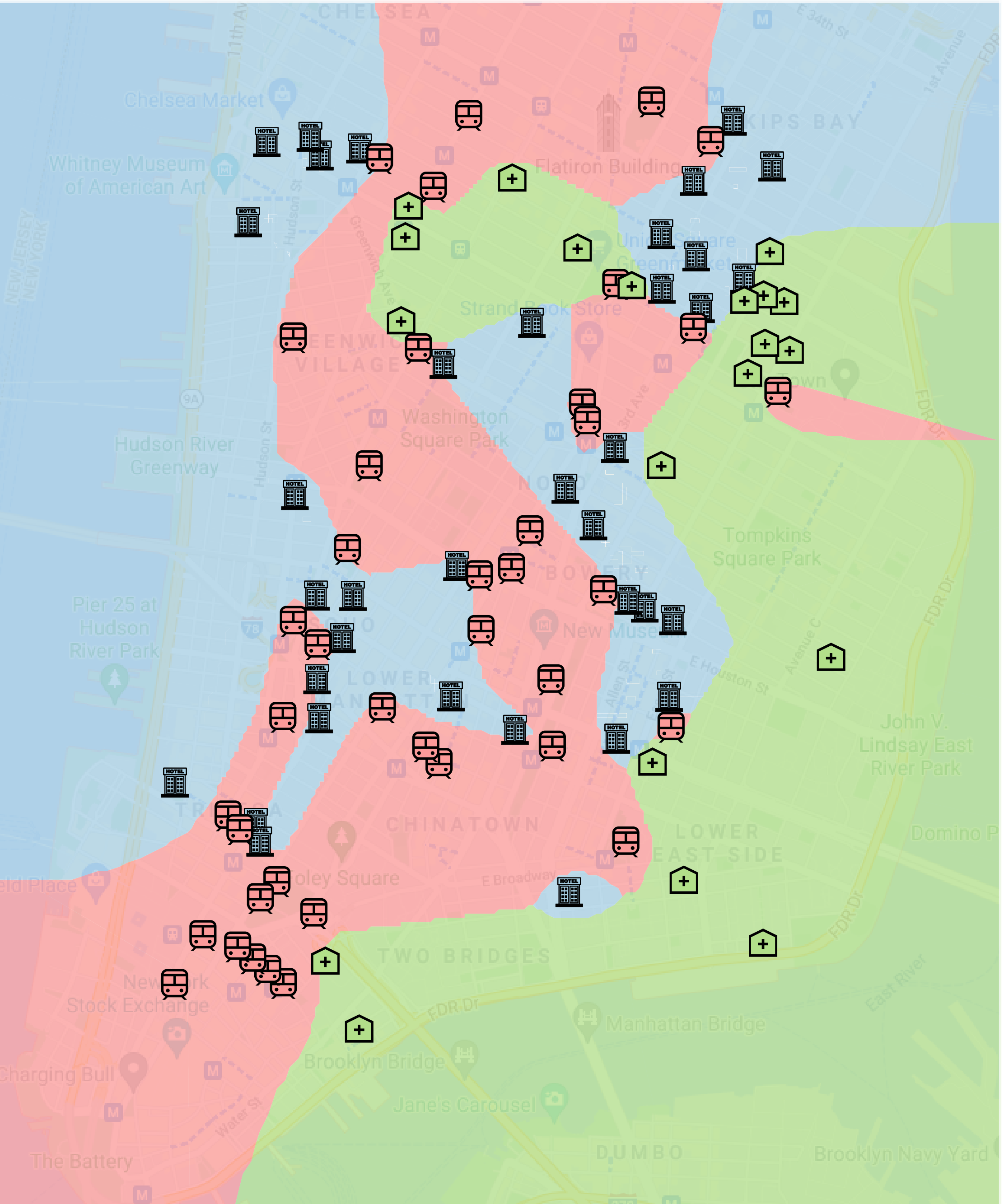

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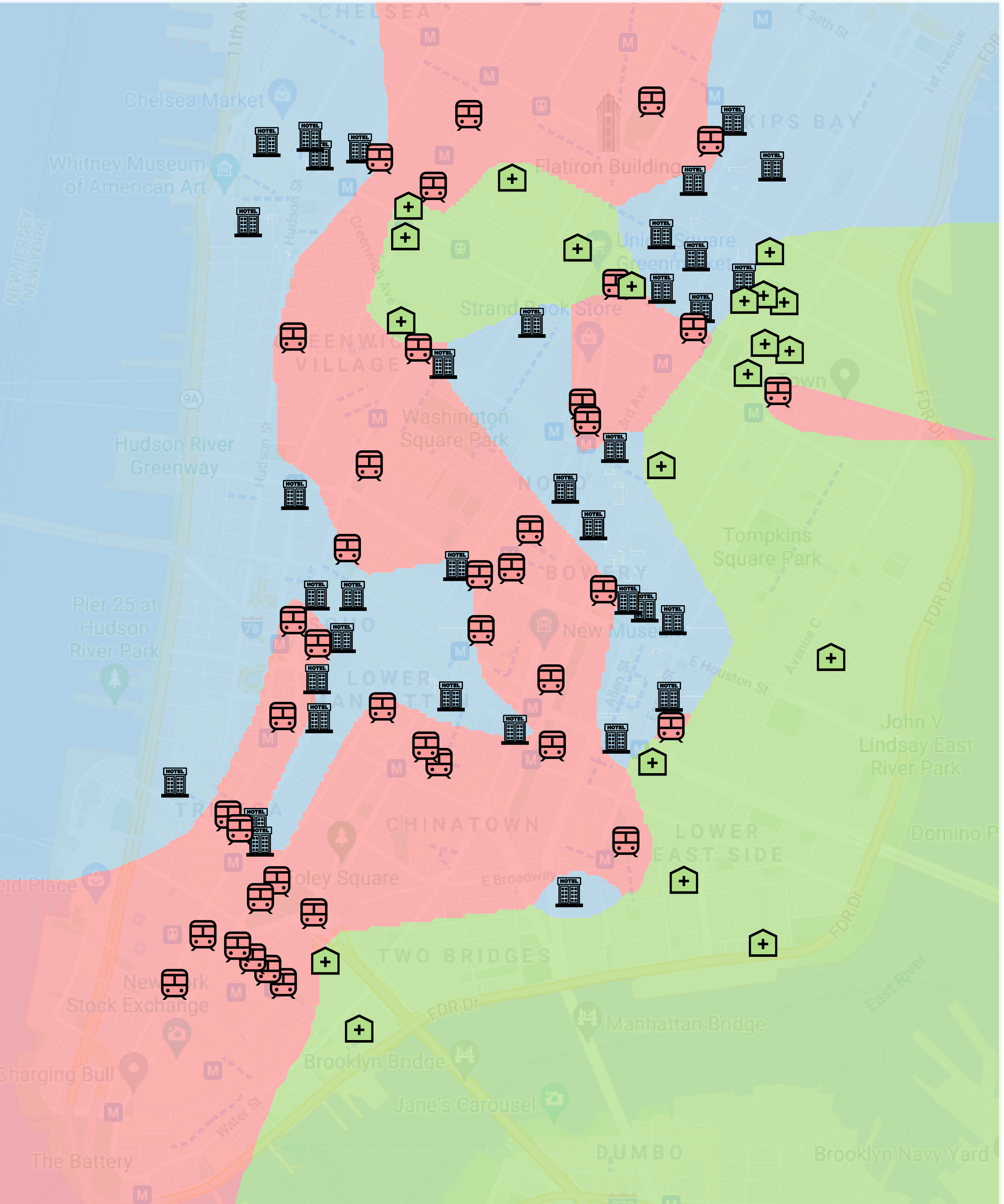

Relax connectivity requirement
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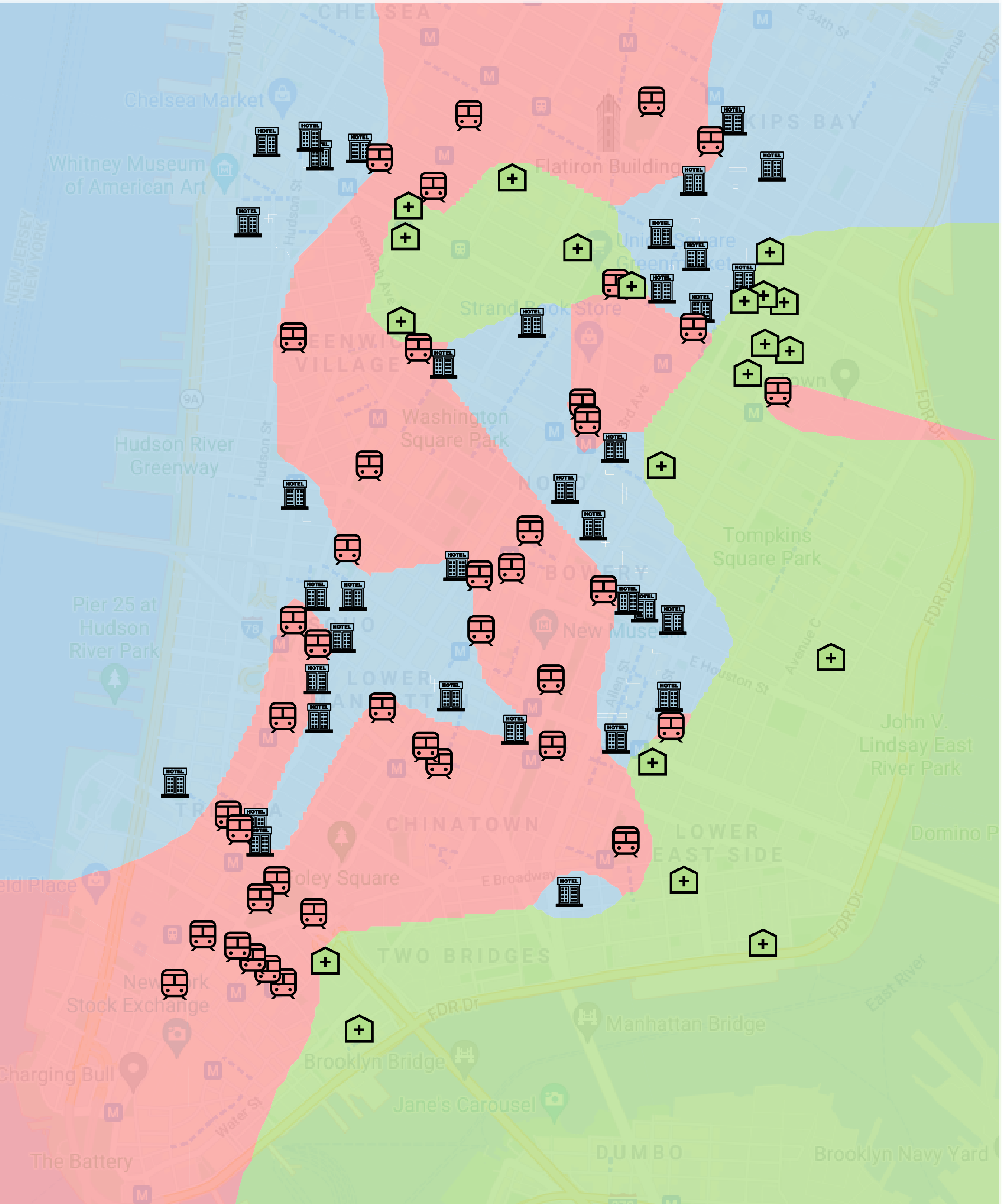
- Categories represented by distinct colors

Design Goals



Relax connectivity requirement
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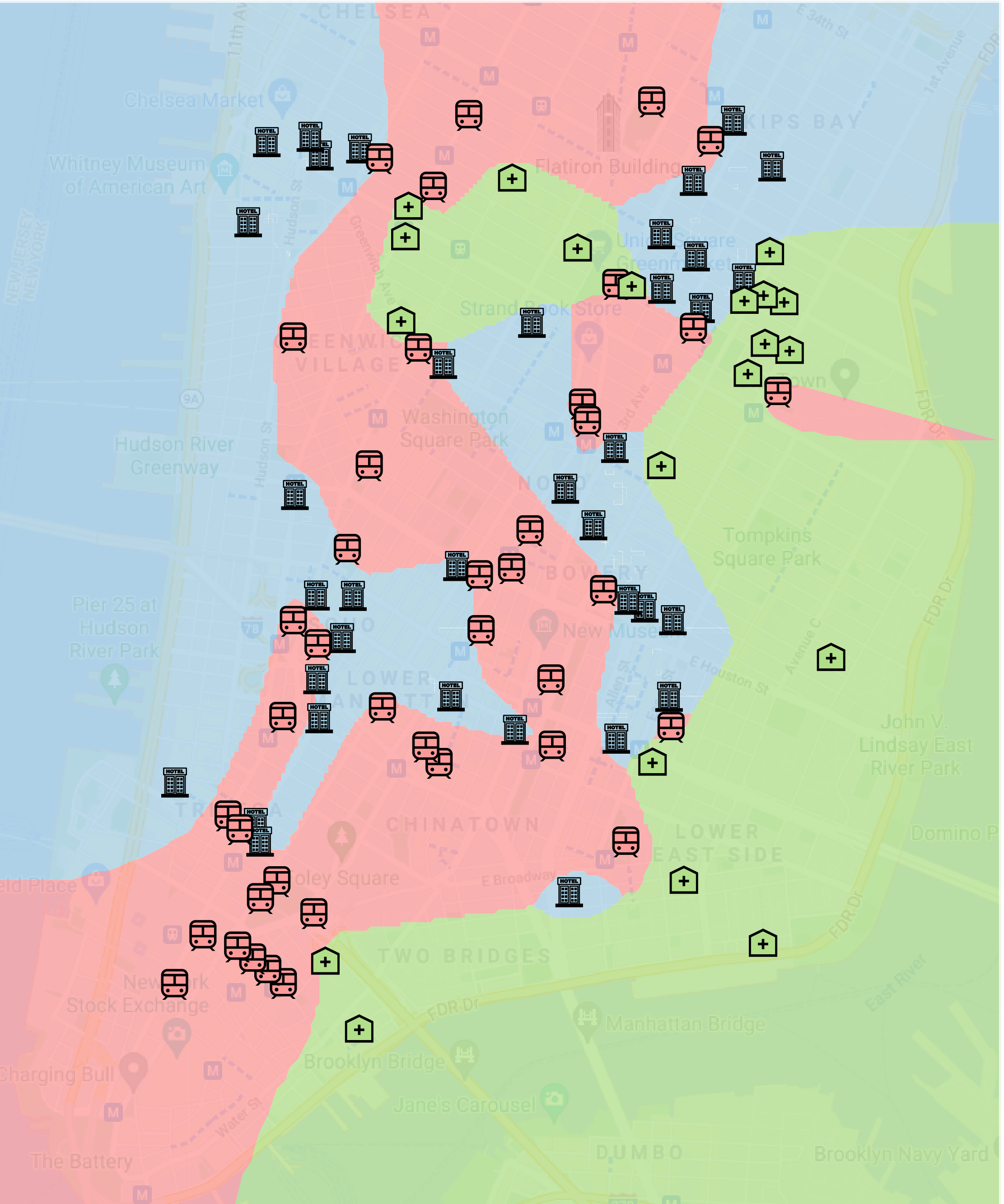

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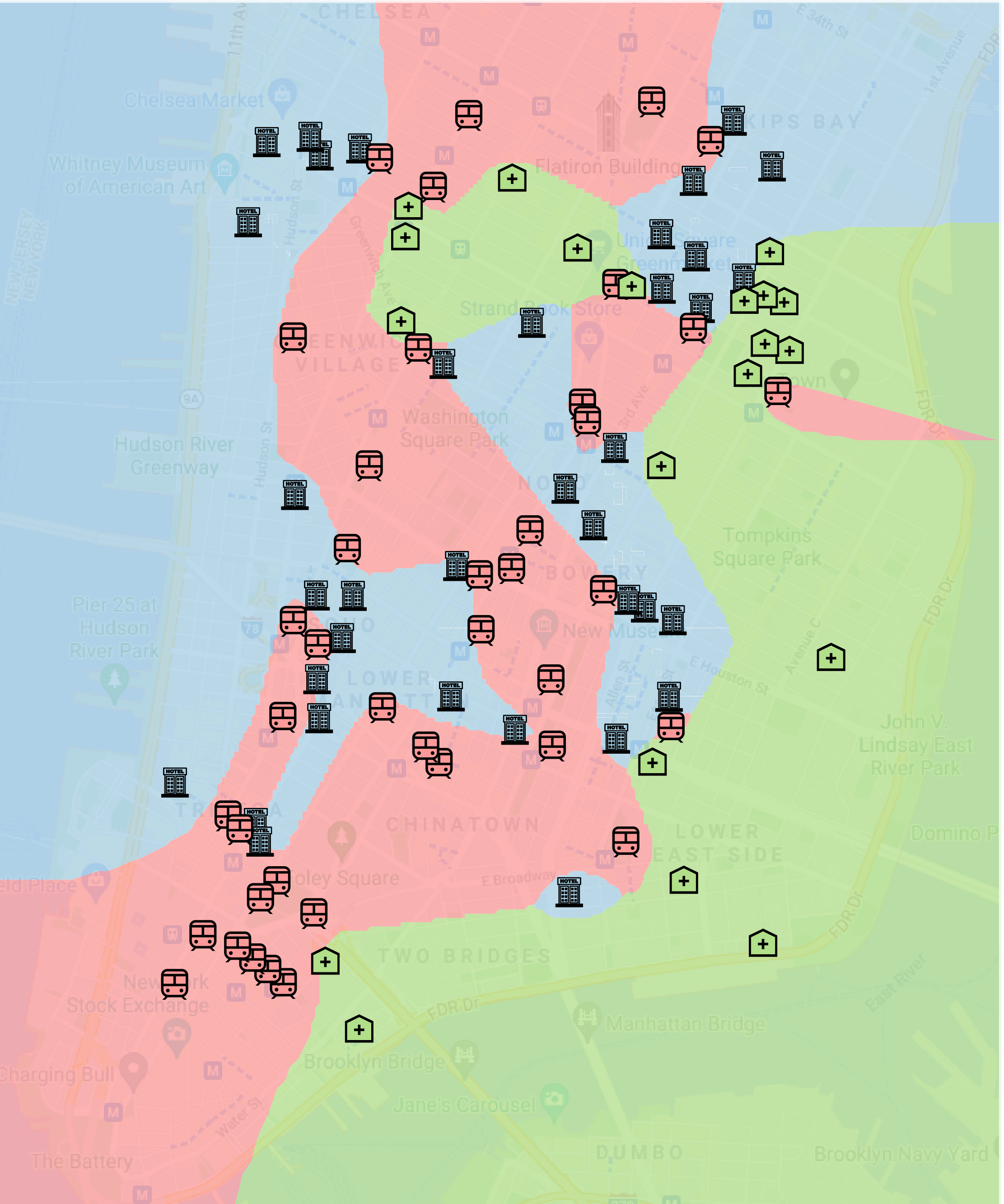

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- Points in cluster are sufficiently close

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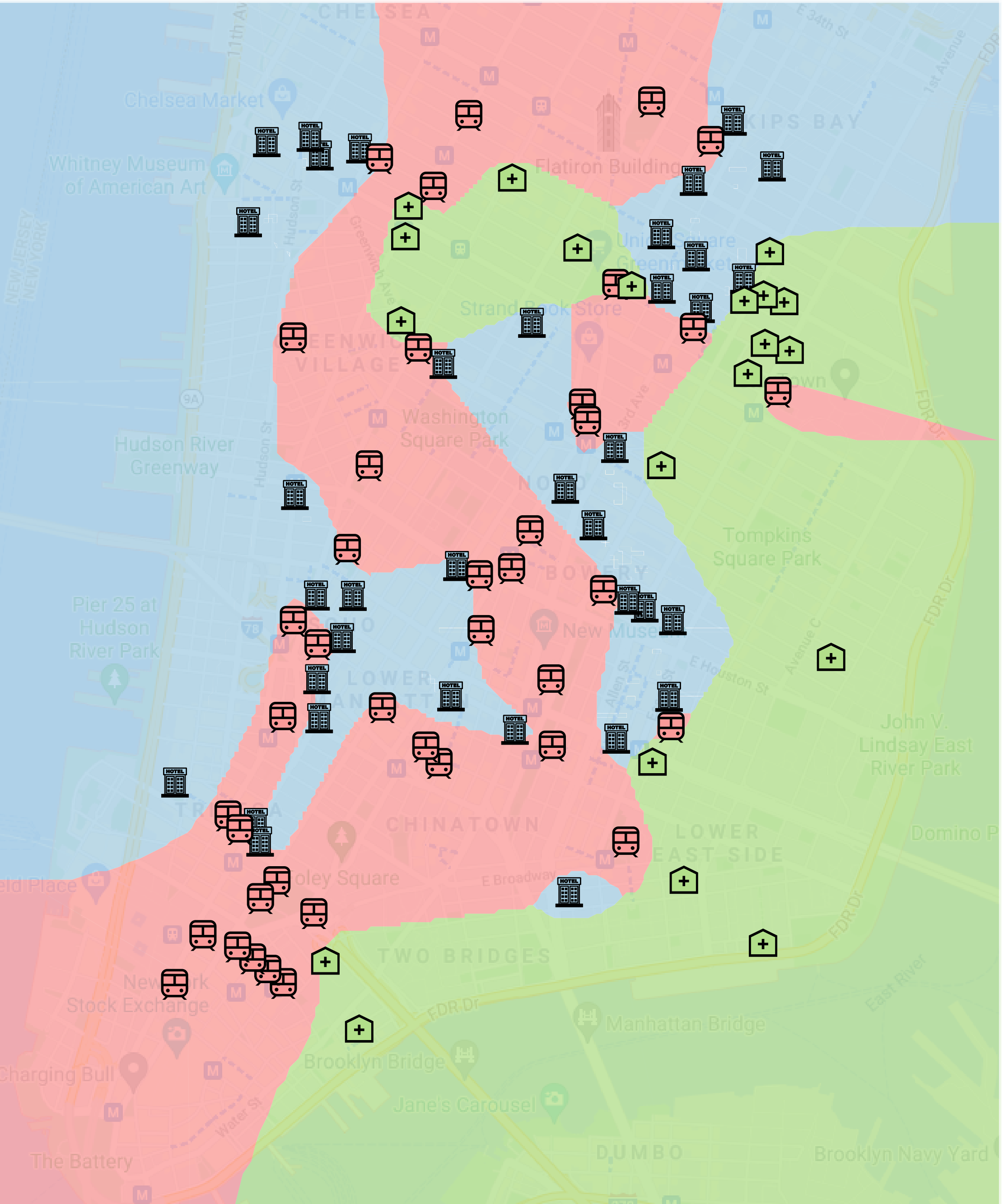

Relax connectivity requirement
$\rightarrow$ preservation of locality of clusters

- Categories represented by distinct colors
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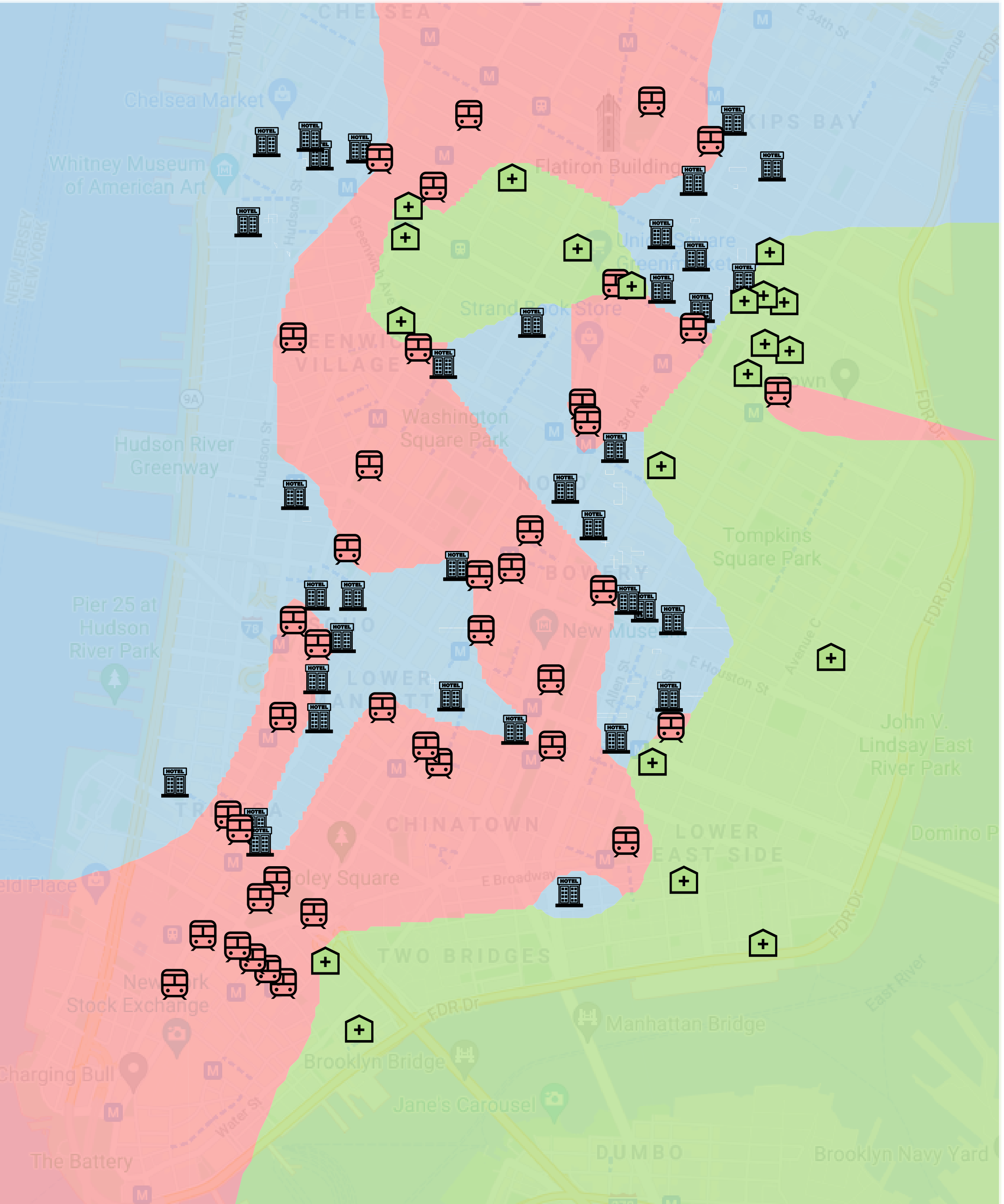
- Small number of clusters per category

Design Goals



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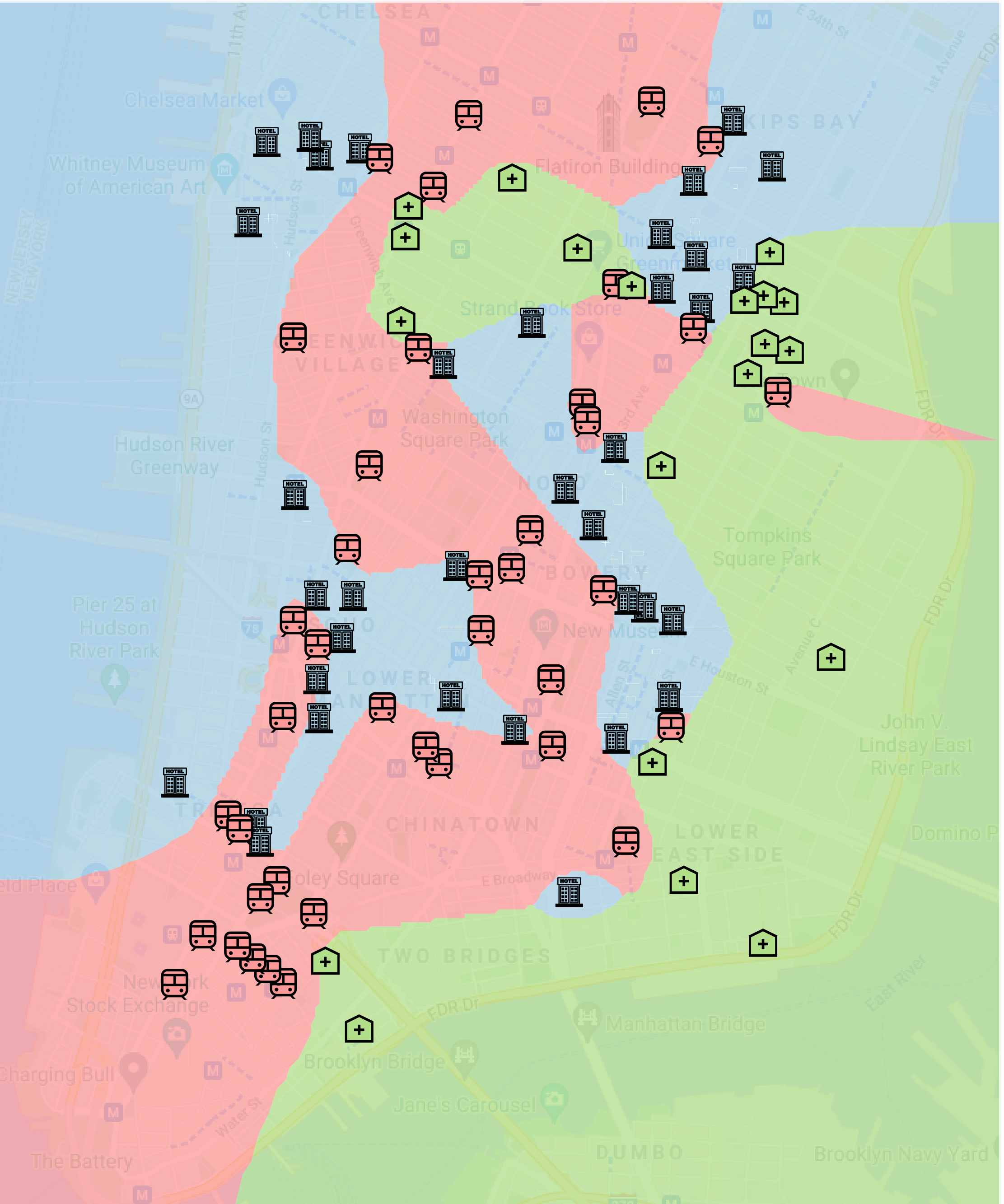

Relax connectivity requirement
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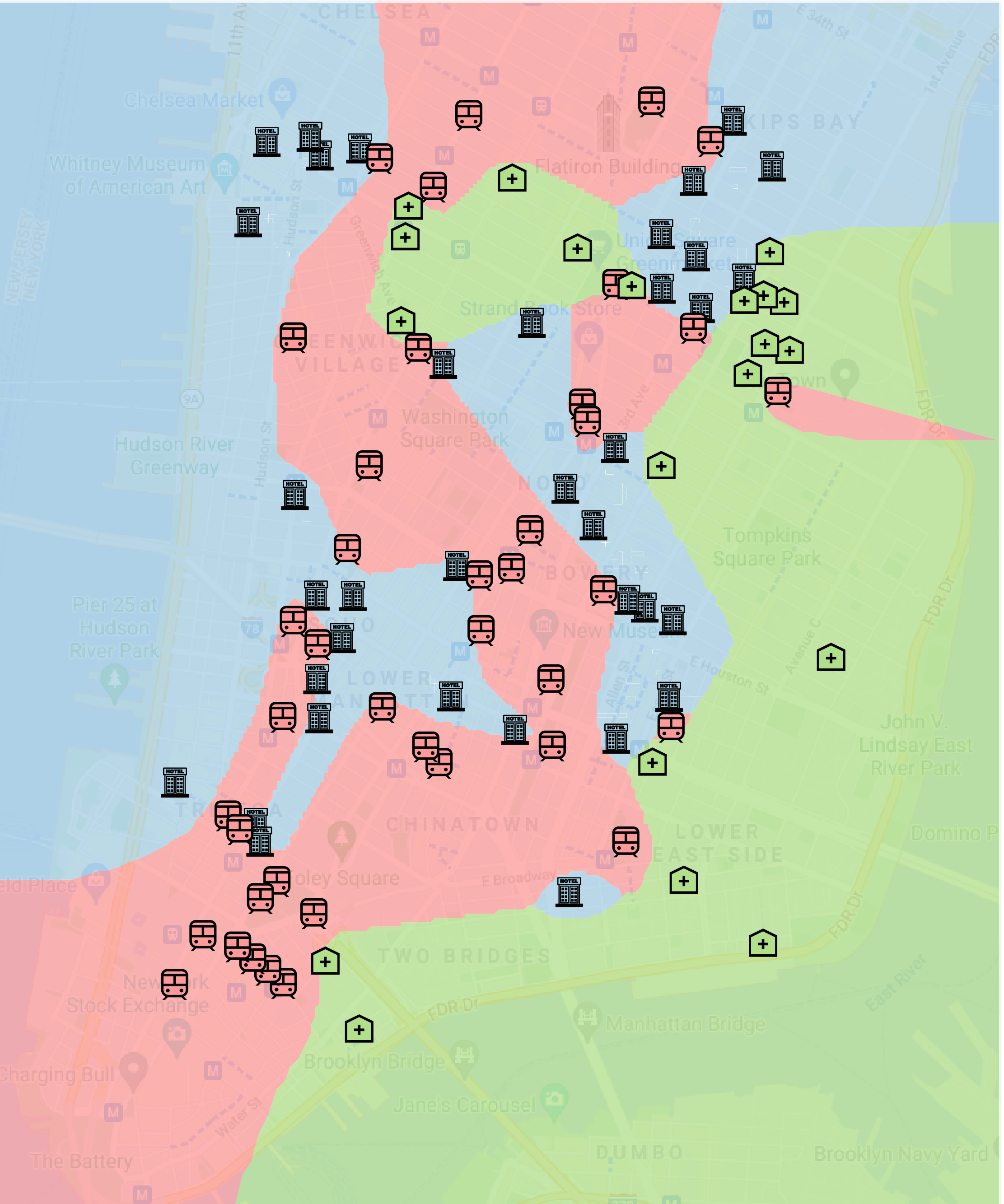
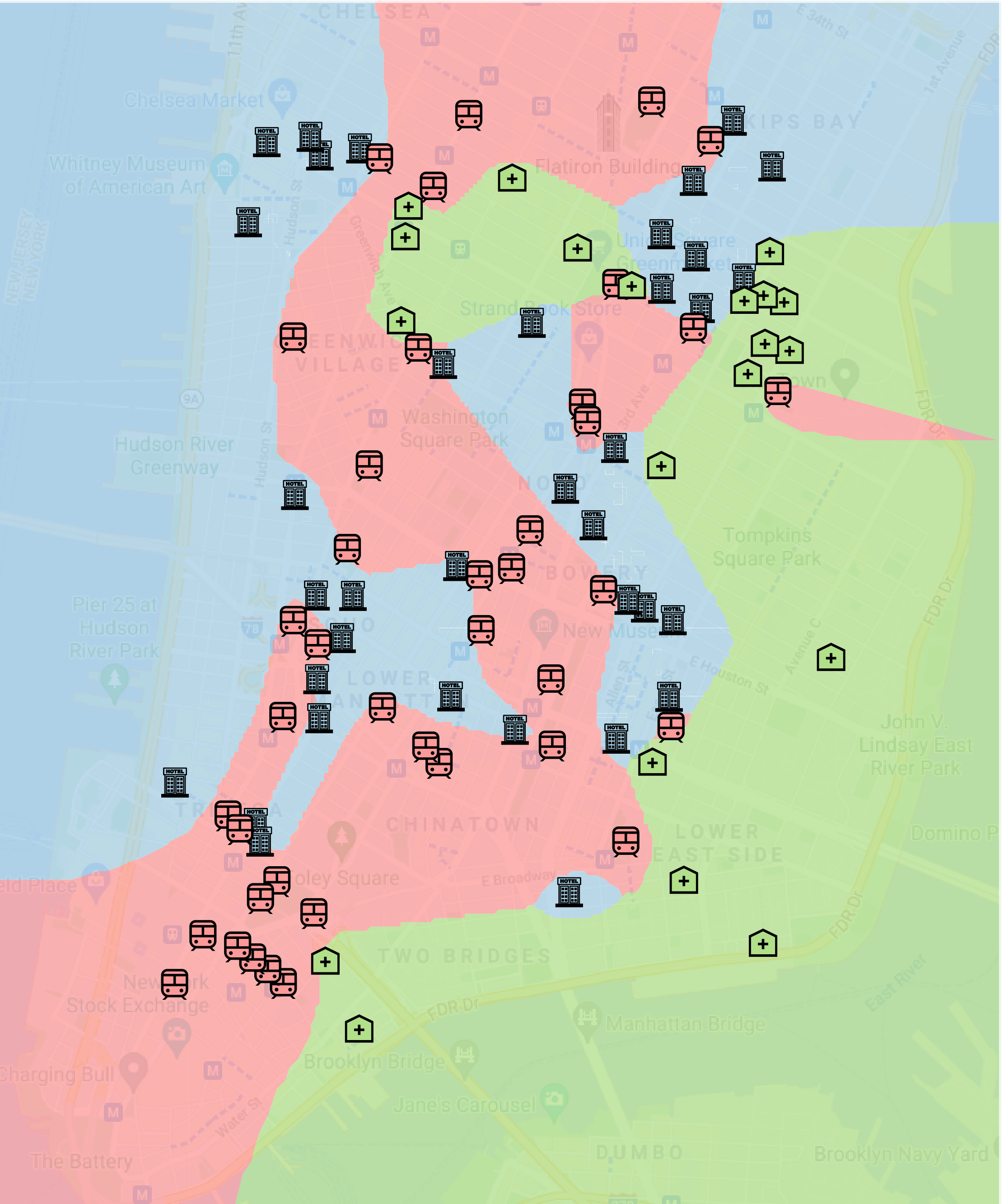
Points in same cluster connected
in a suitable proximity graph

Pipeline



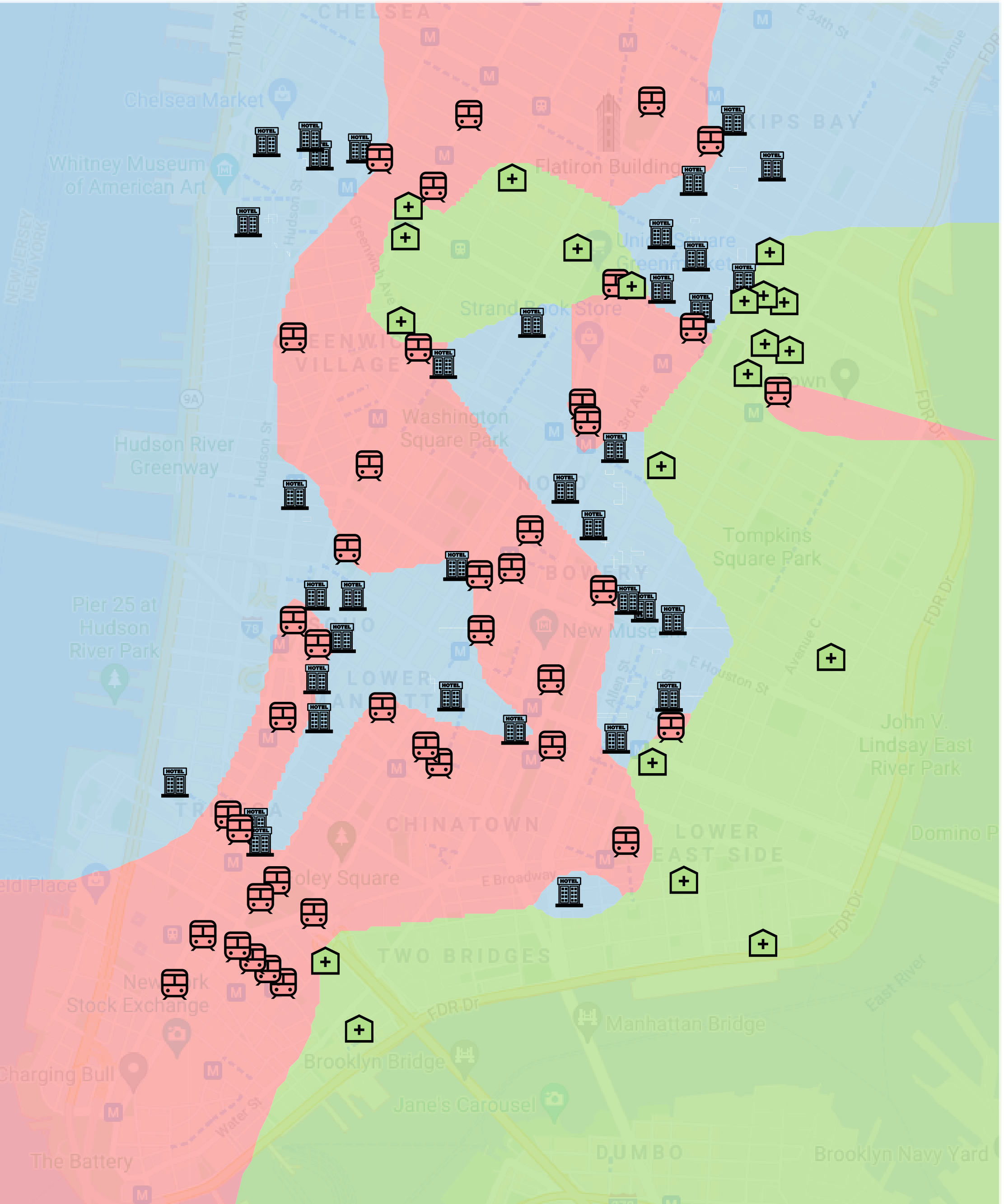
CHELSEA
Chelsea Market
Whitney Museum of American Art
Flatiron Building
KIPS BAY
Strand Book Store
GREENWICH VILLAGE
Washington Square Park
Hudson River Greenway
Tompkins Square Park
BOWERY
Pier 25 at Hudson River Park
New Museum
LOWER MANHATTAN
John V. Lindsay East River Park
CHINATOWN
LOWER EAST SIDE
Foley Square
TWO BRIDGES
New York Stock Exchange
Manhattan Bridge
Brooklyn Bridge
Charging Bull
Jane's Carousel
DUMBO
The Battery
Brooklyn Navy Yard

Pipeline



1. **Proximity Graph**

Pipeline



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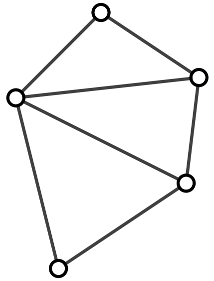
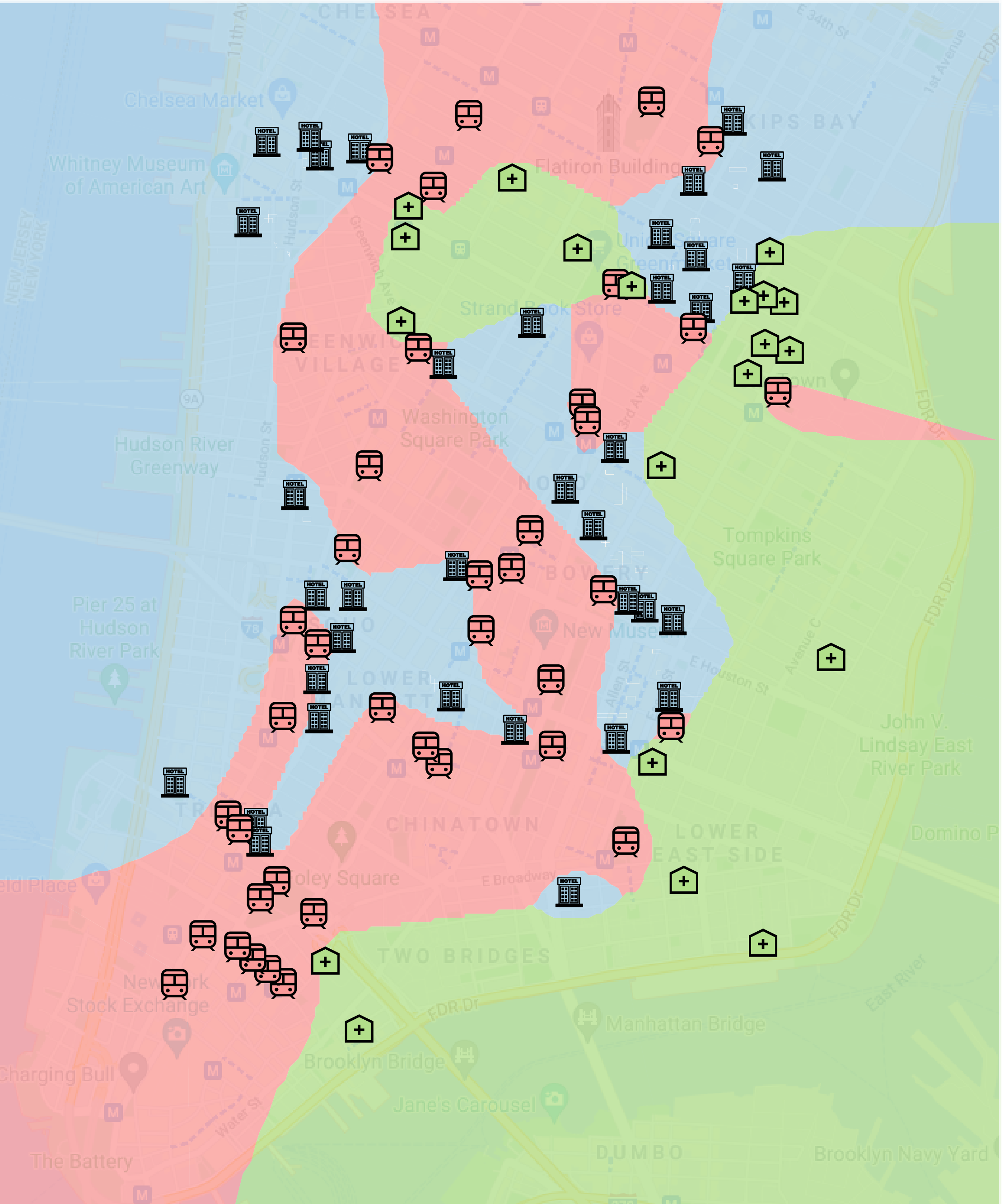


Delaunay Triangulation

Pipeline



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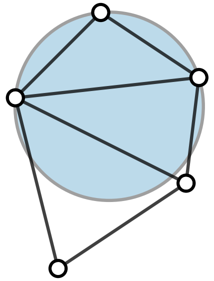

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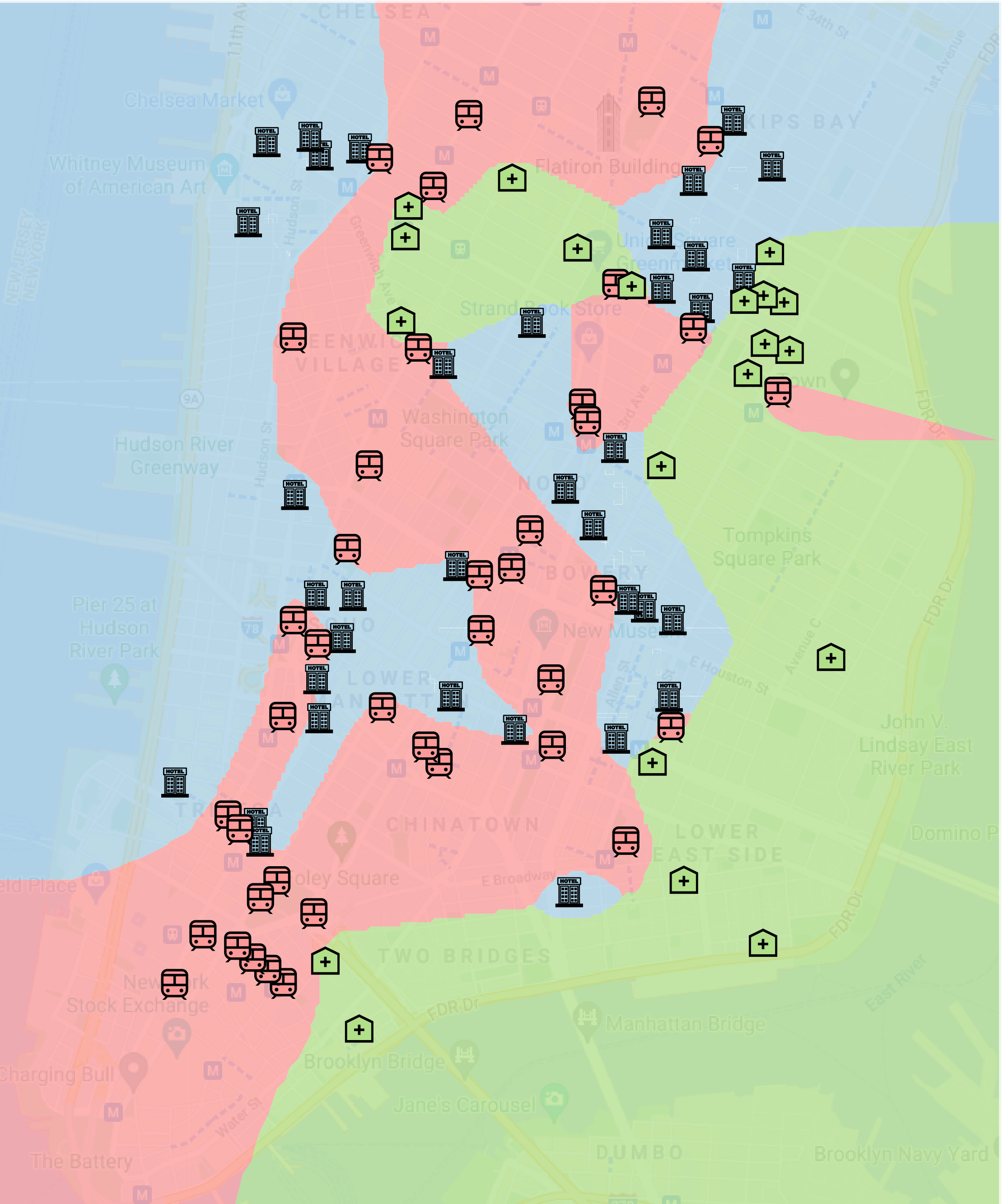

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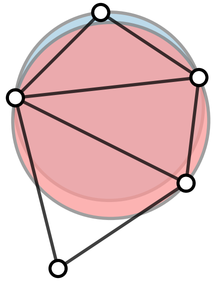

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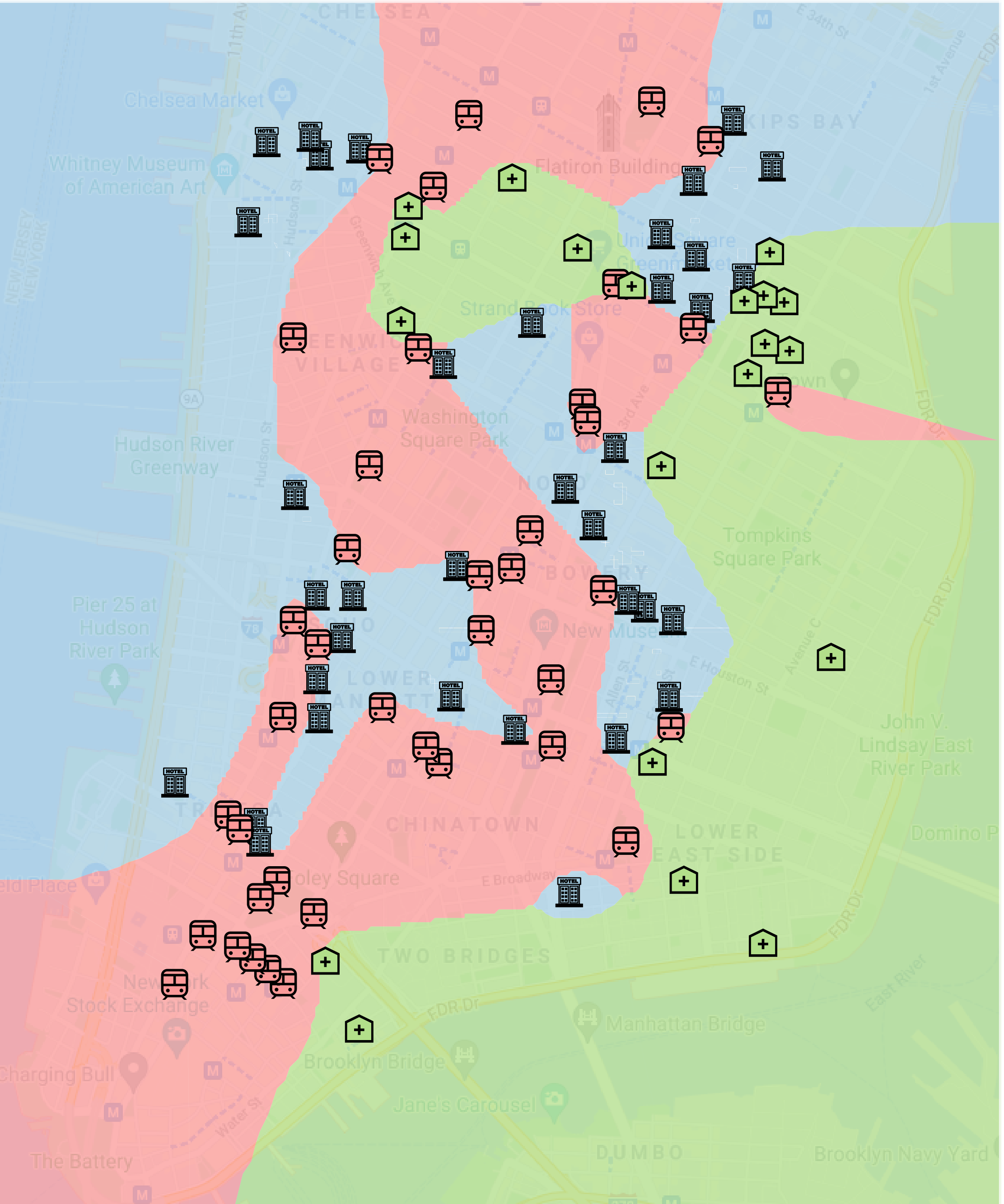

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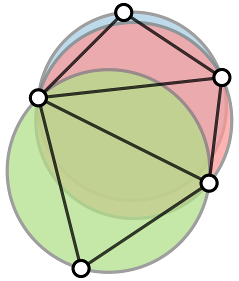

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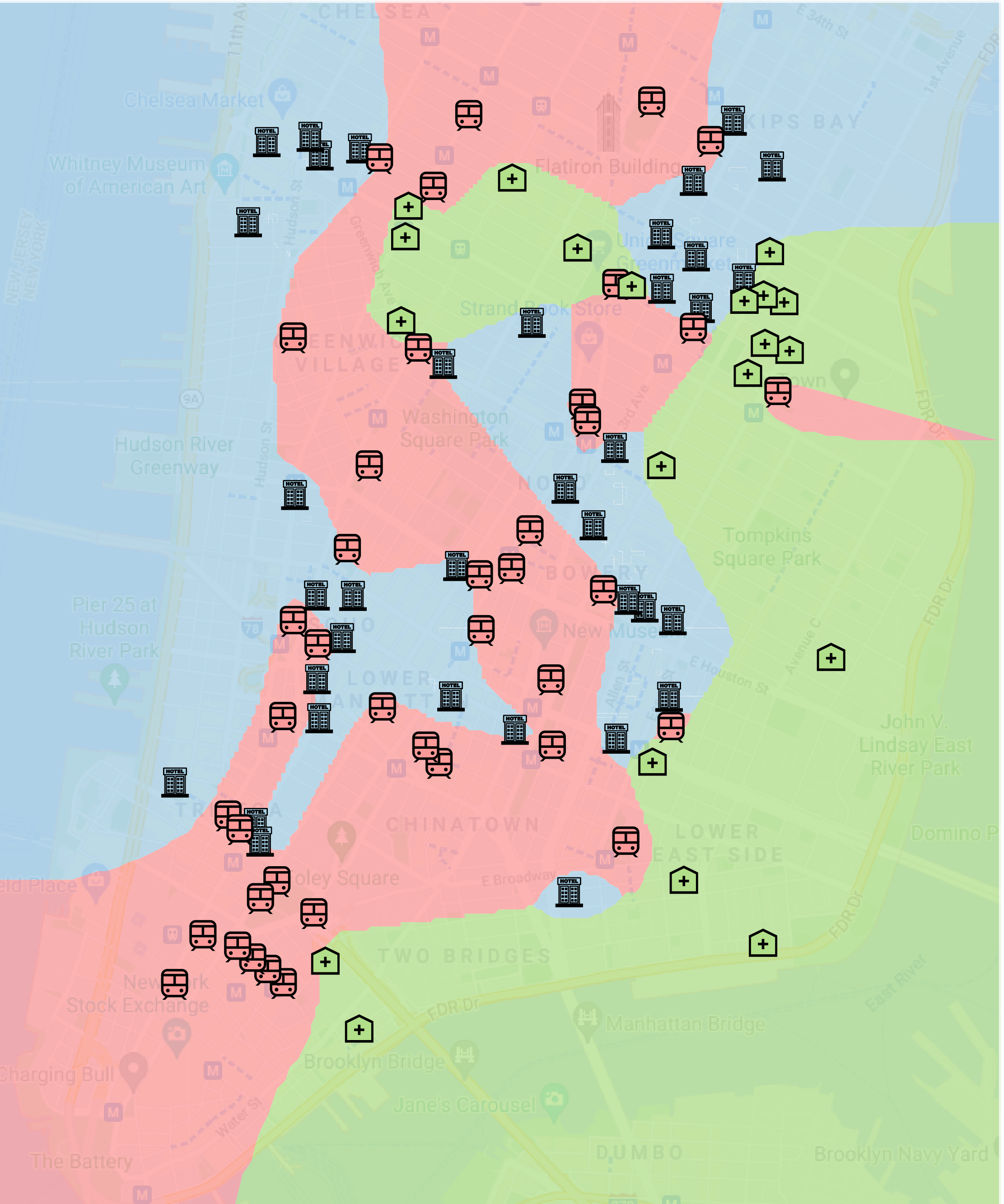

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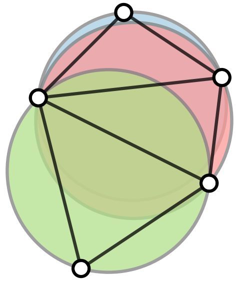

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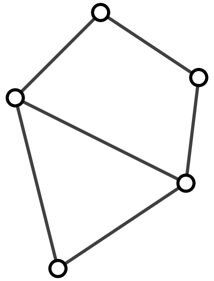
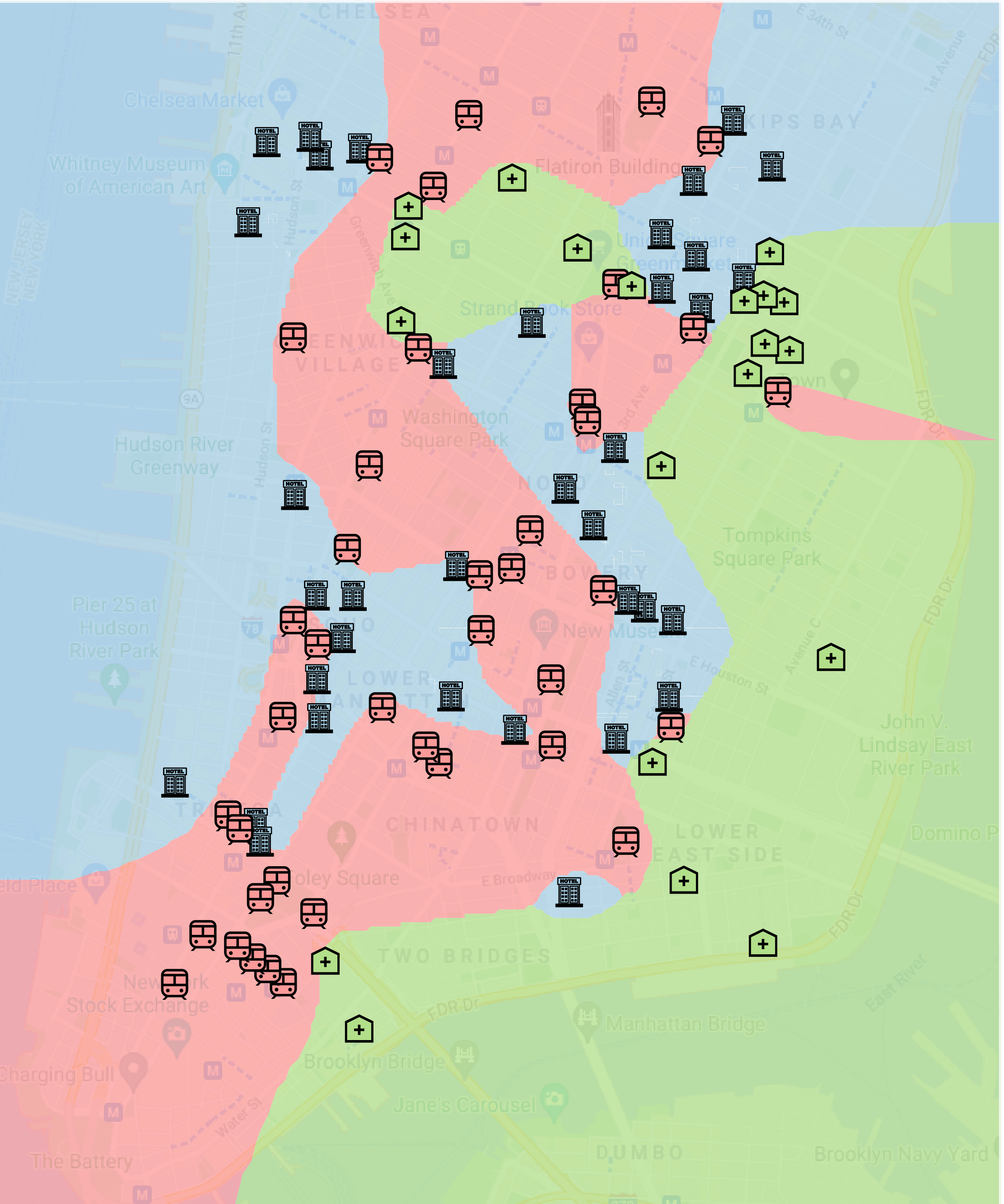


Gabriel Graph

Pipeline



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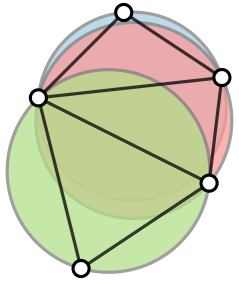

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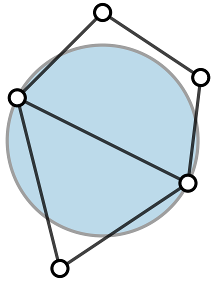

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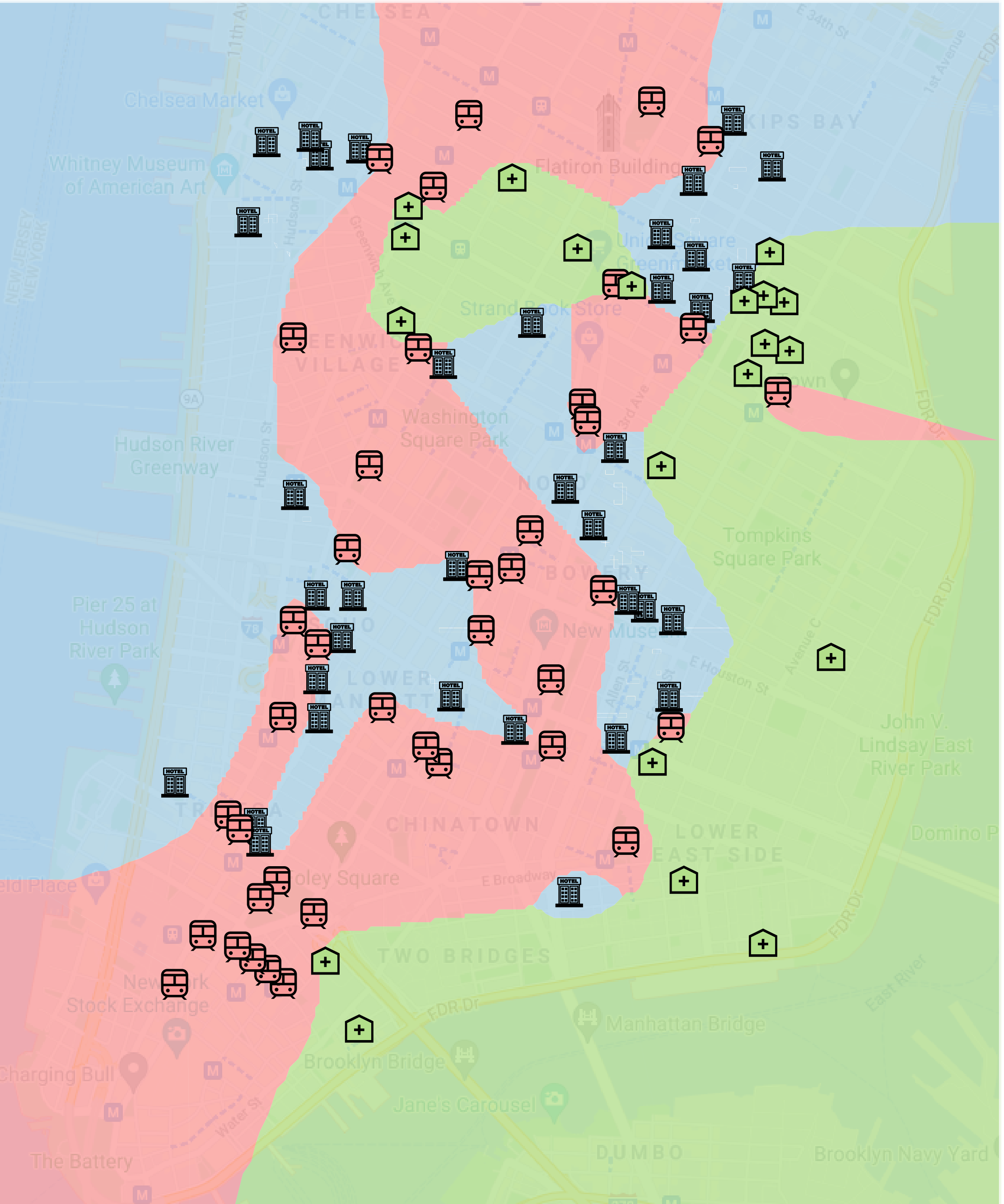

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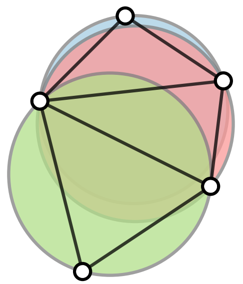

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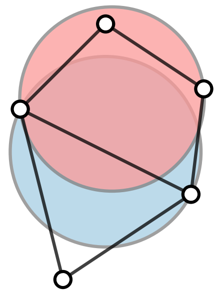

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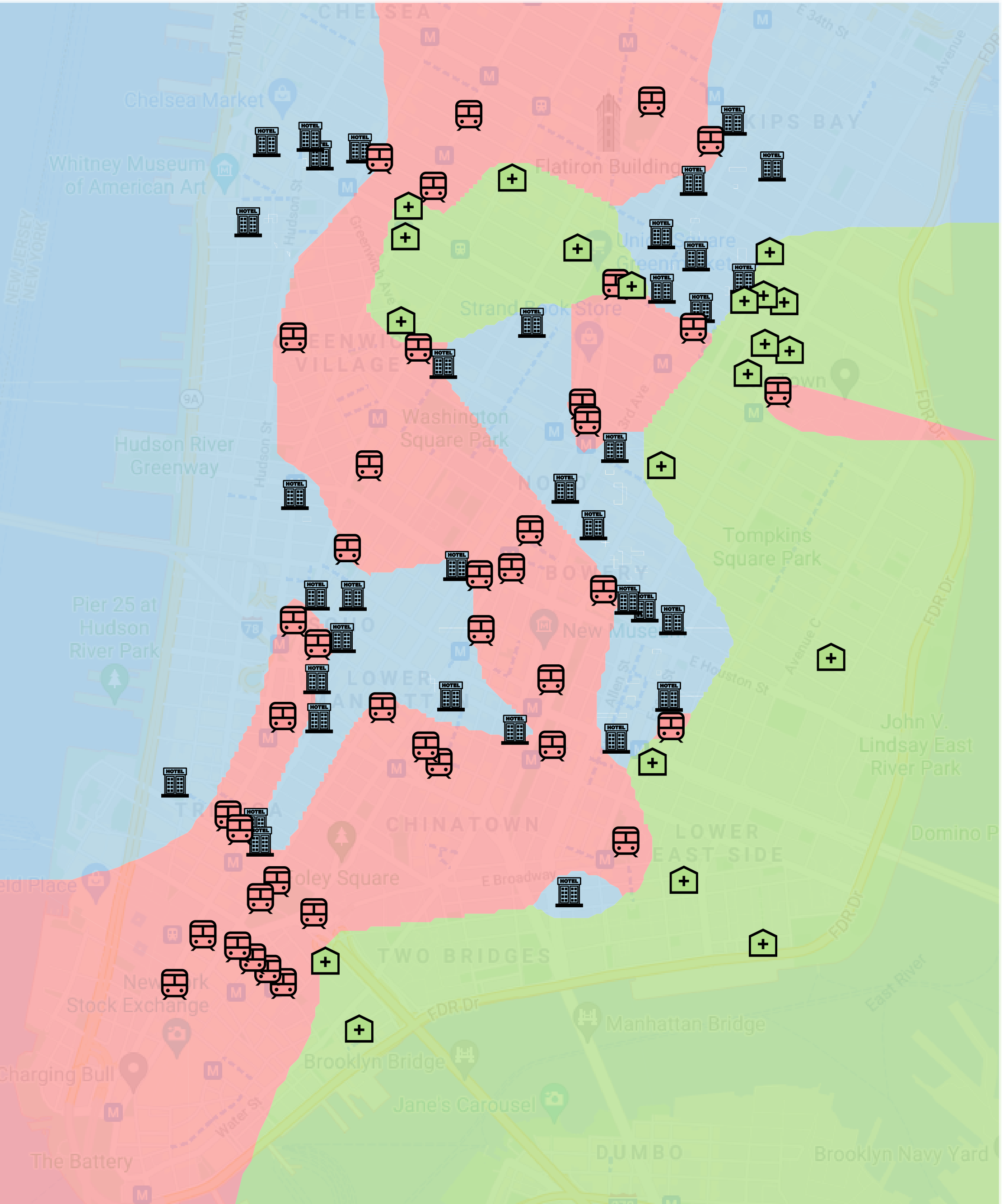

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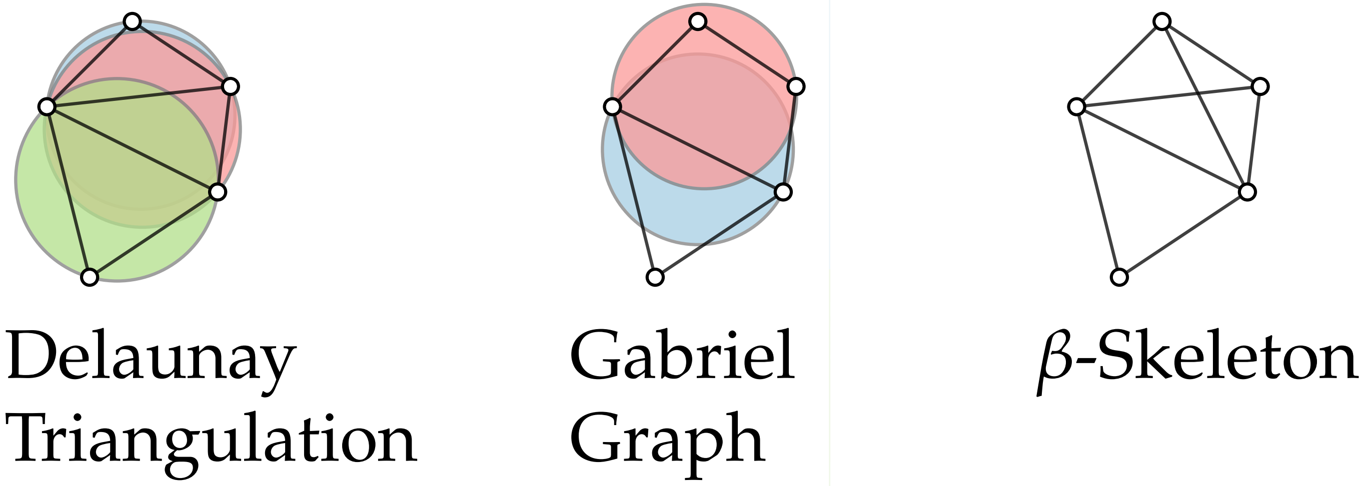

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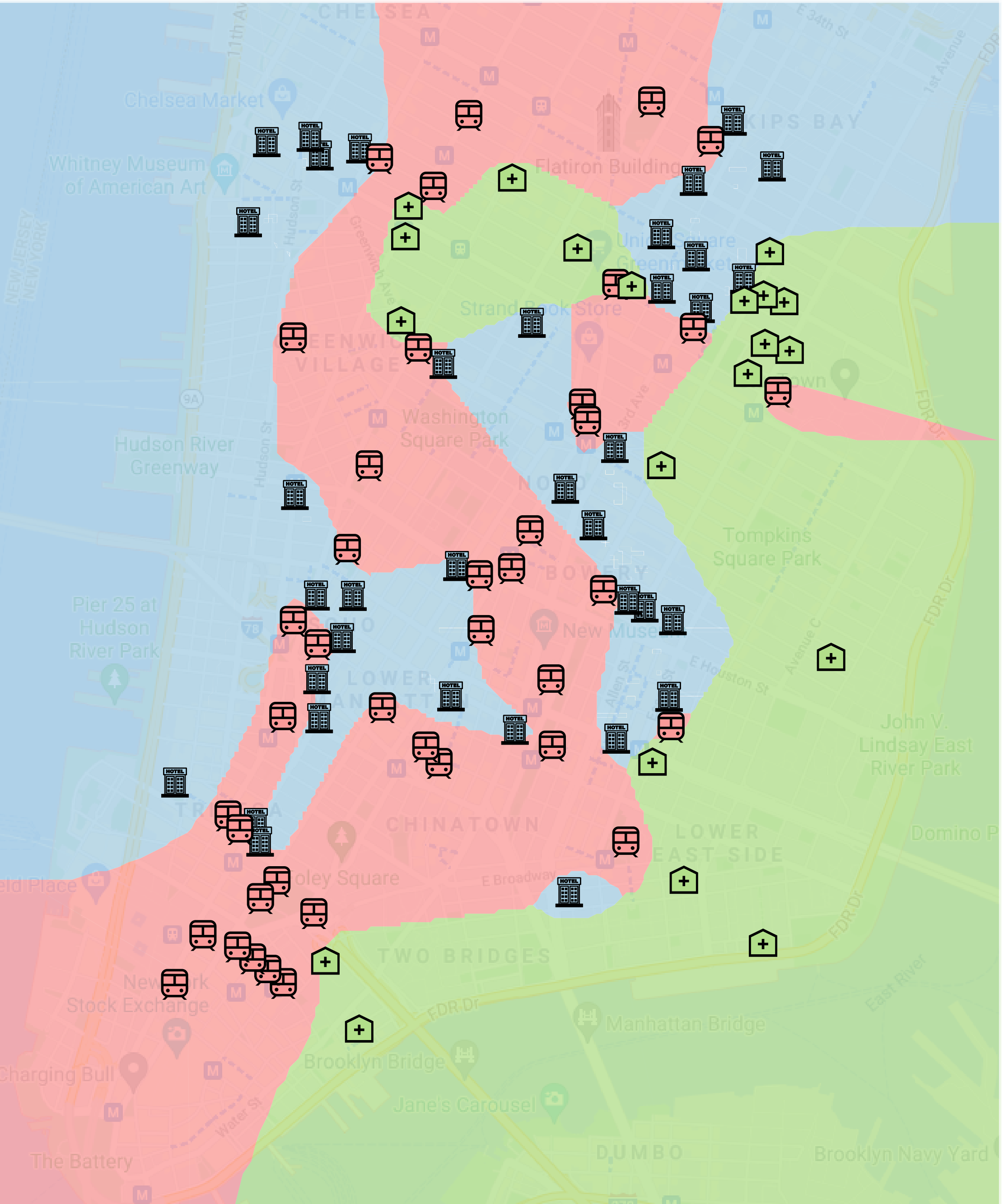

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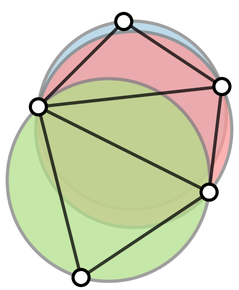
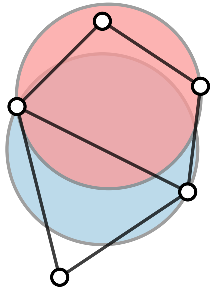
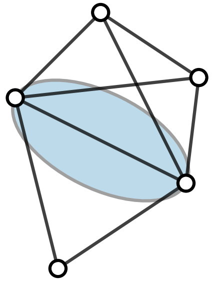


Delaunay Triangulation

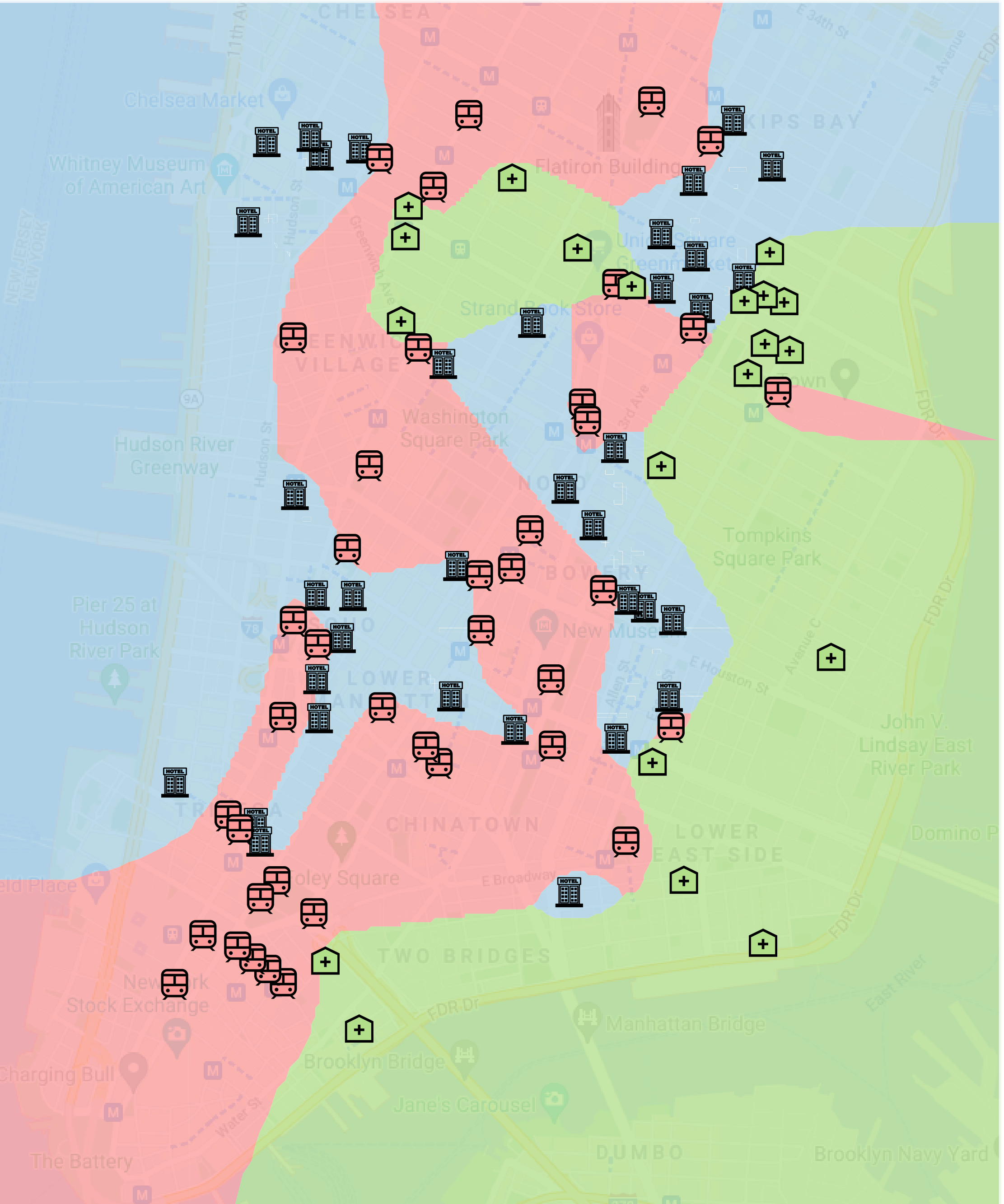


Gabriel Graph

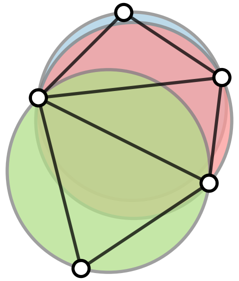


β-Skeleton

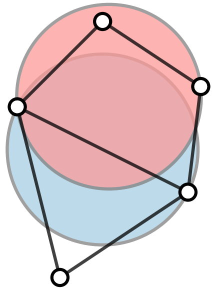
Pipeline



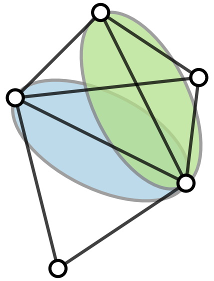
1. **Proximity Graph**



Delaunay Triangulation

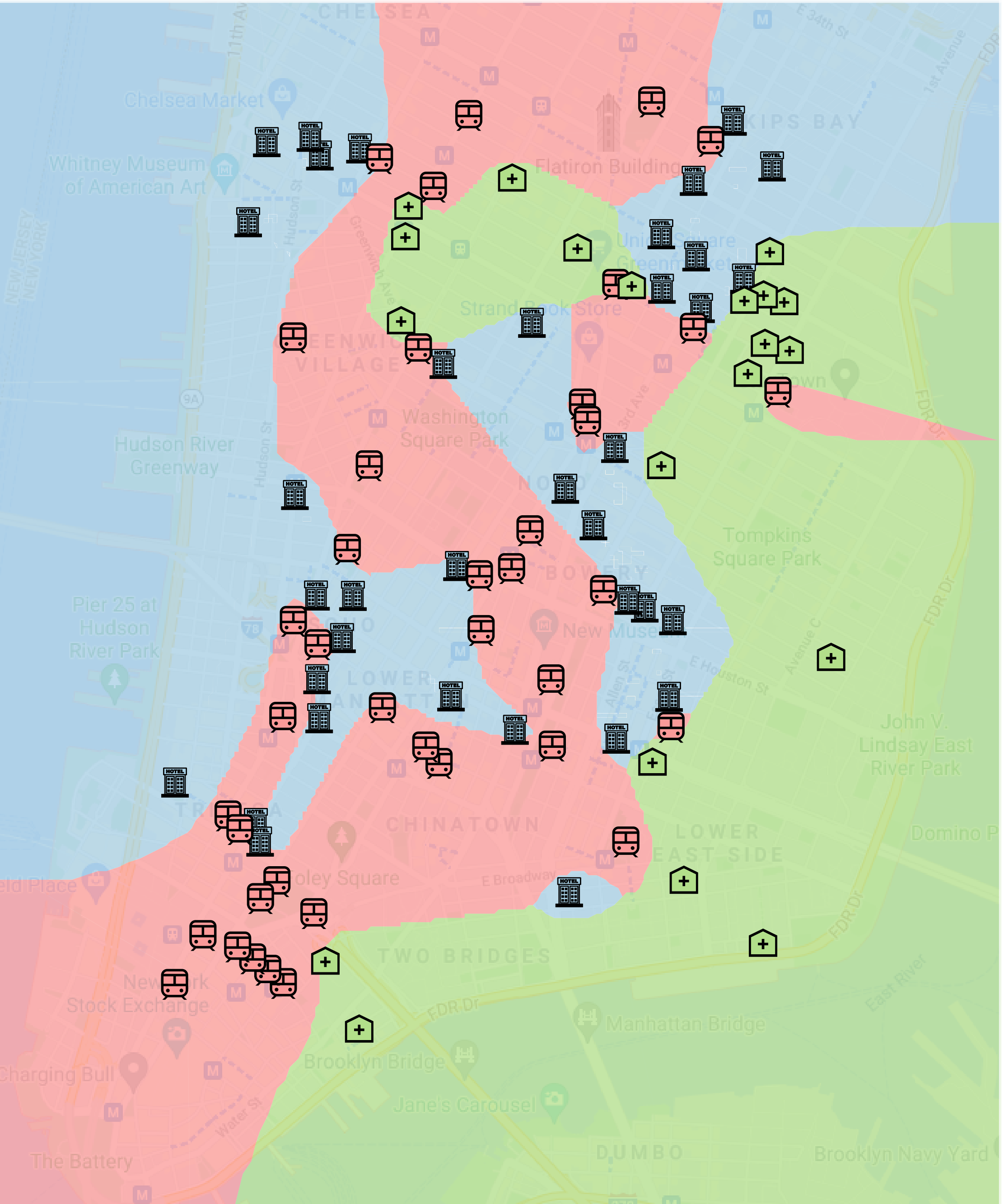


Gabriel Graph

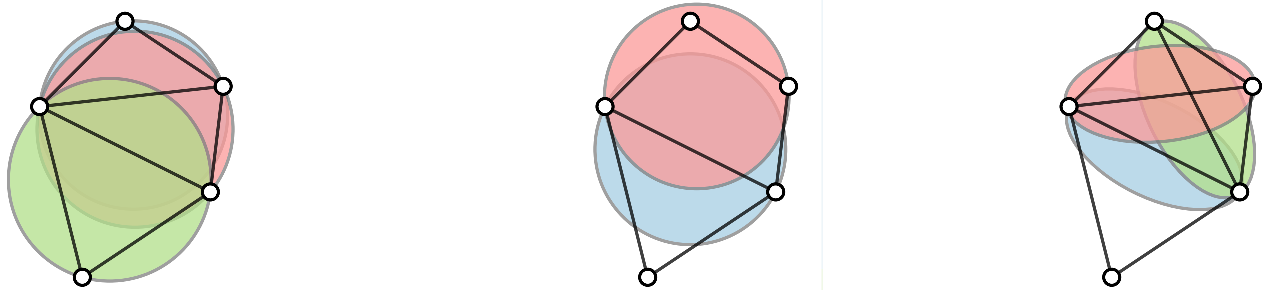


β-Skeleton

Pipeline



1. **Proximity Graph**

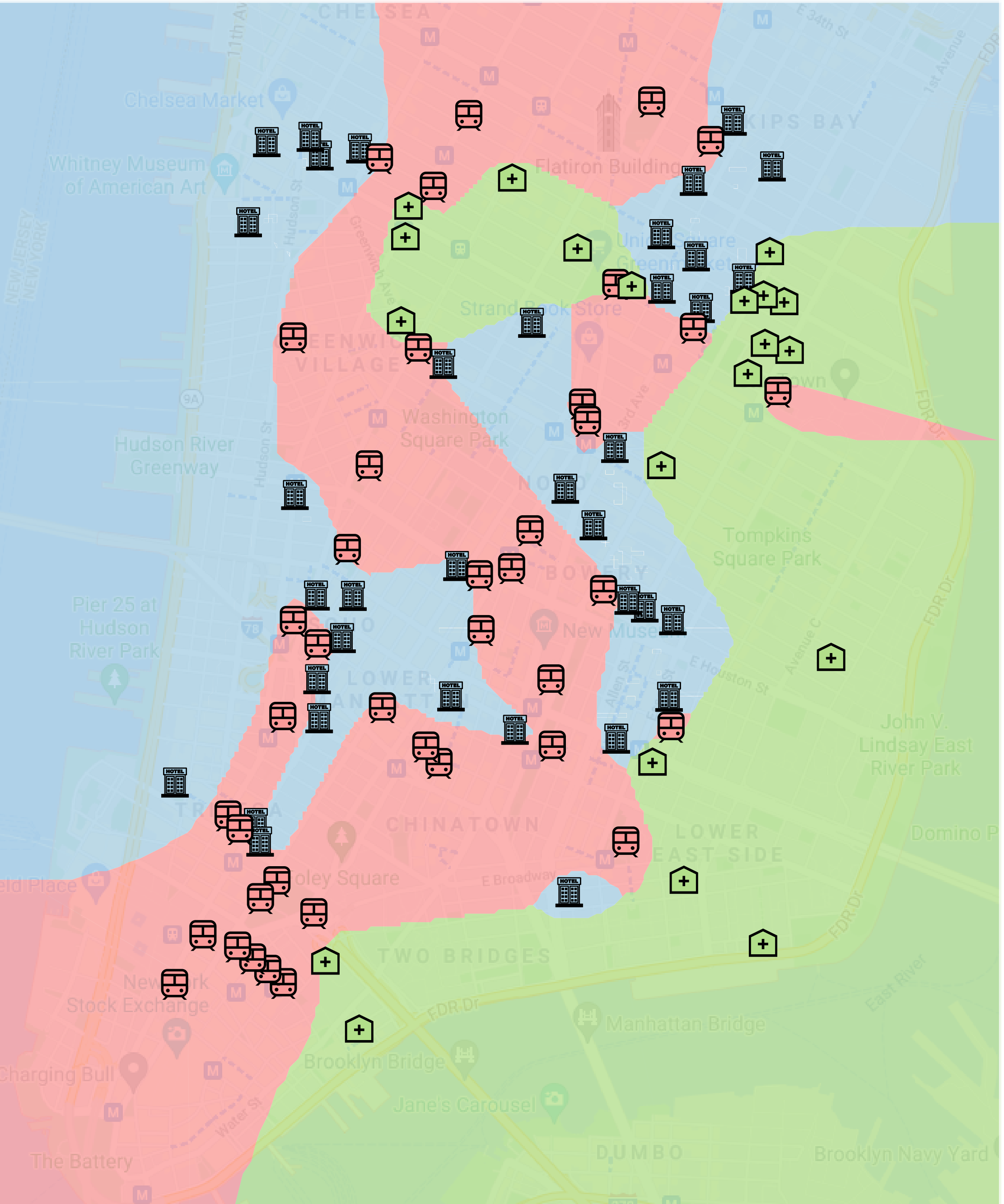


Delaunay Triangulation

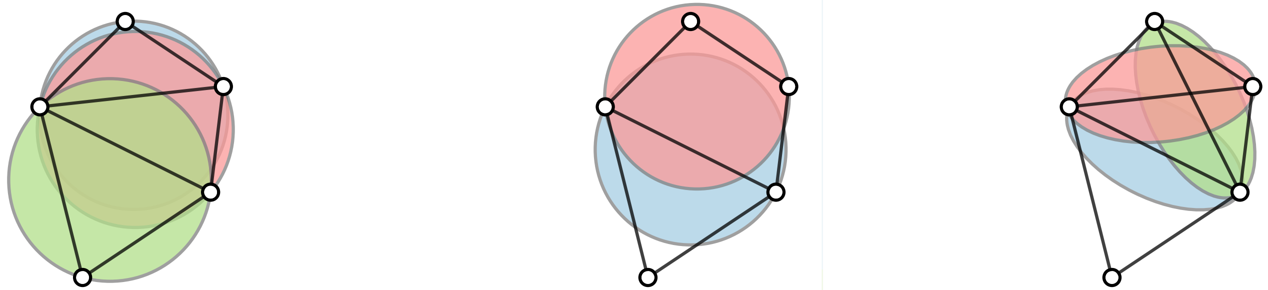
Gabriel Graph

β-Skeleton

Pipeline



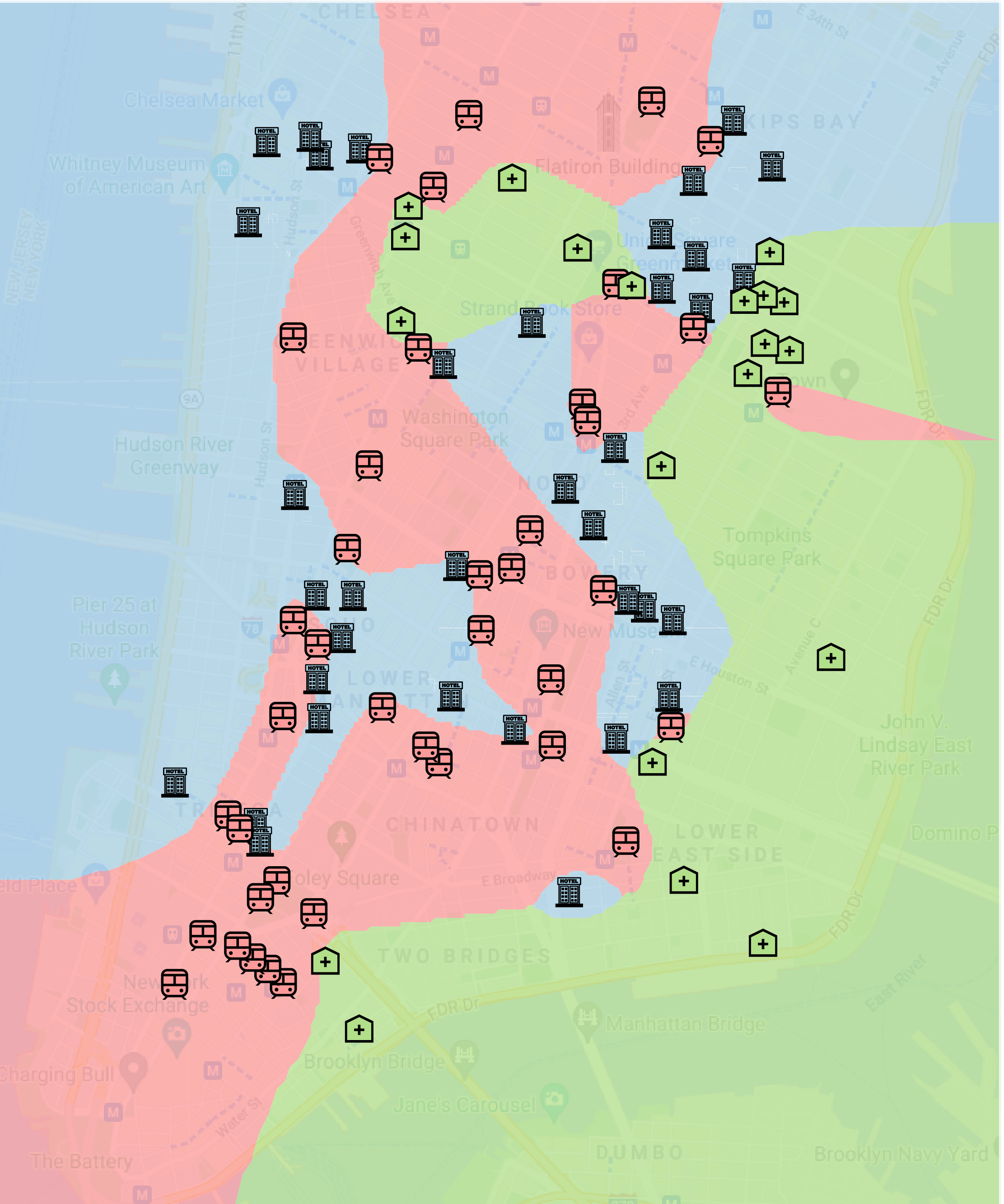
1. **Proximity Graph**



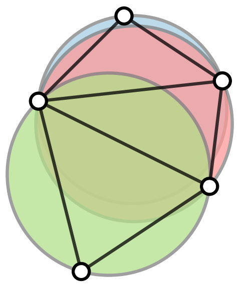
Delaunay Triangulation | Gabriel Graph | β-Skeleton

2. **Planar Spanning Forest**

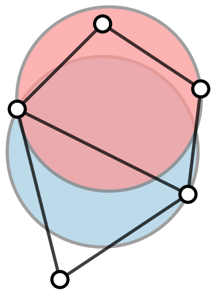
Pipeline



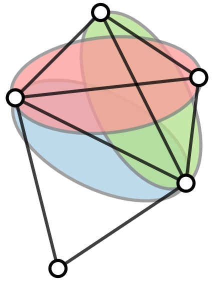
1. **Proximity Graph**



Delaunay Triangulation

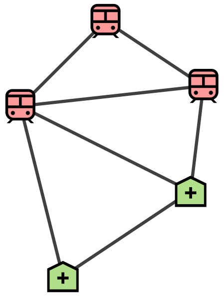


Gabriel Graph

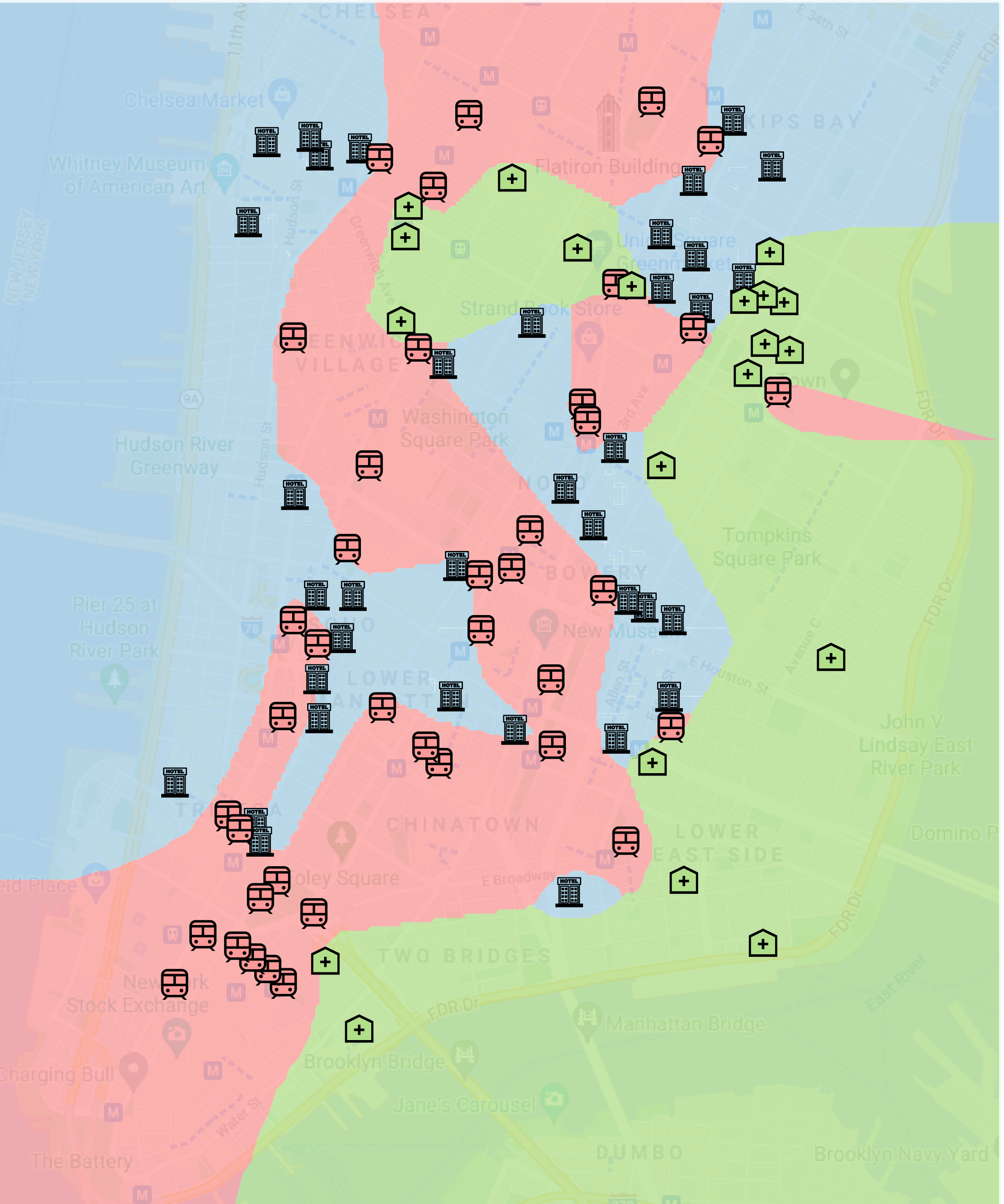


β-Skeleton

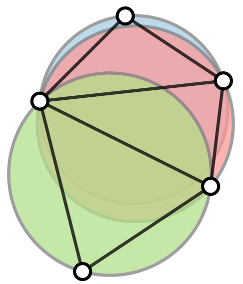
2. **Planar Spanning Forest**



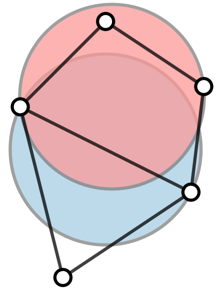
Pipeline



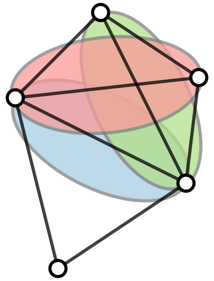
1. **Proximity Graph**



Delaunay Triangulation

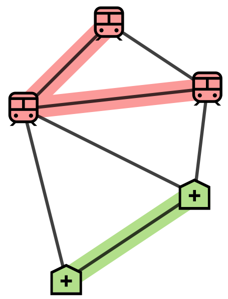


Gabriel Graph

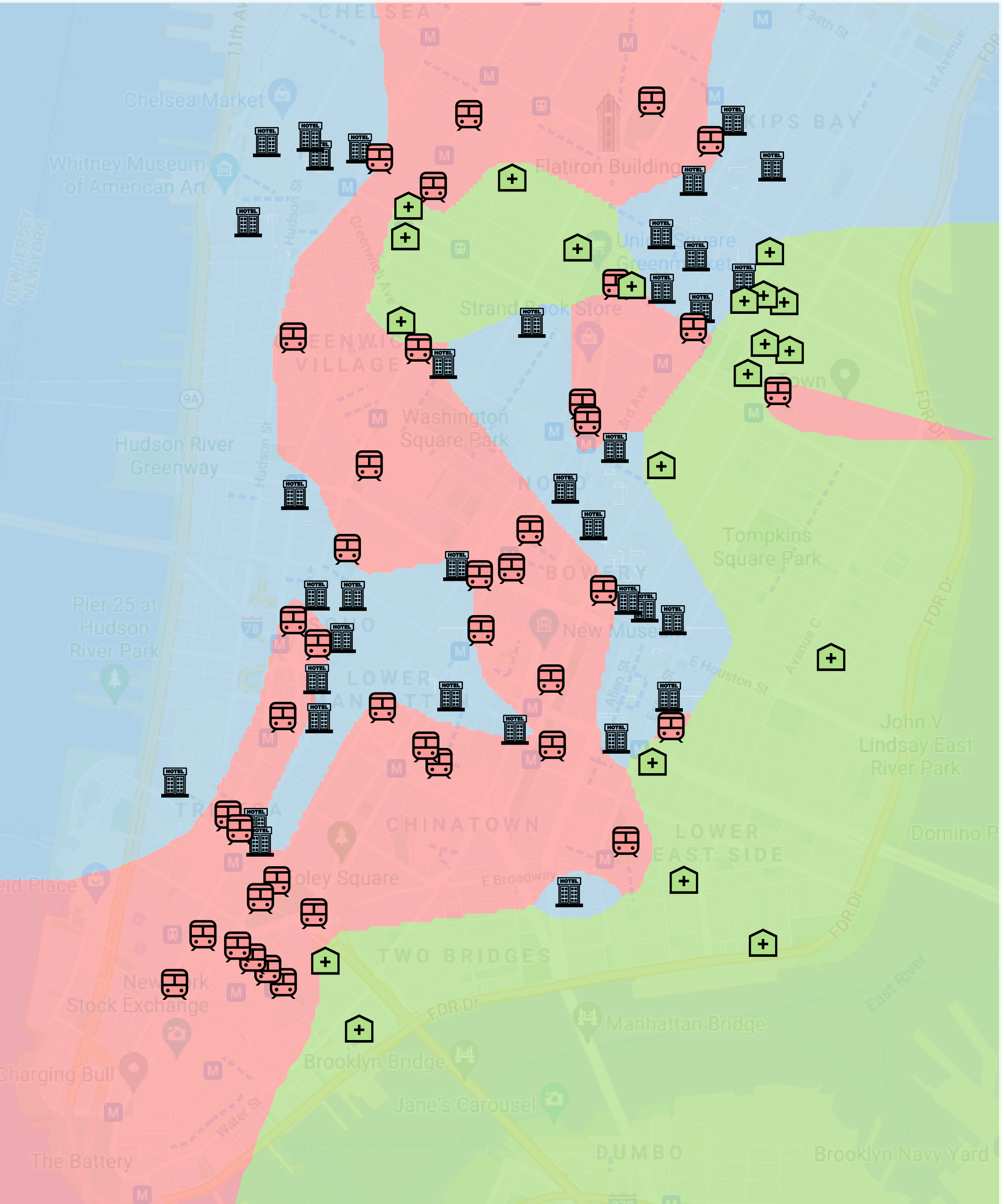


β-Skeleton

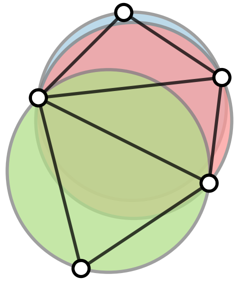
2. **Planar Spanning Forest**



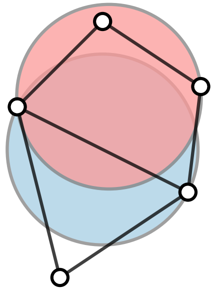
Pipeline



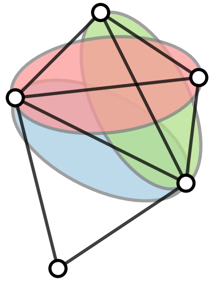
1. **Proximity Graph**



Delaunay Triangulation

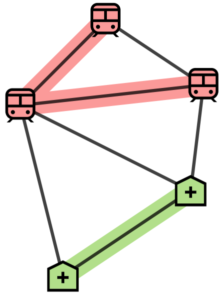


Gabriel Graph



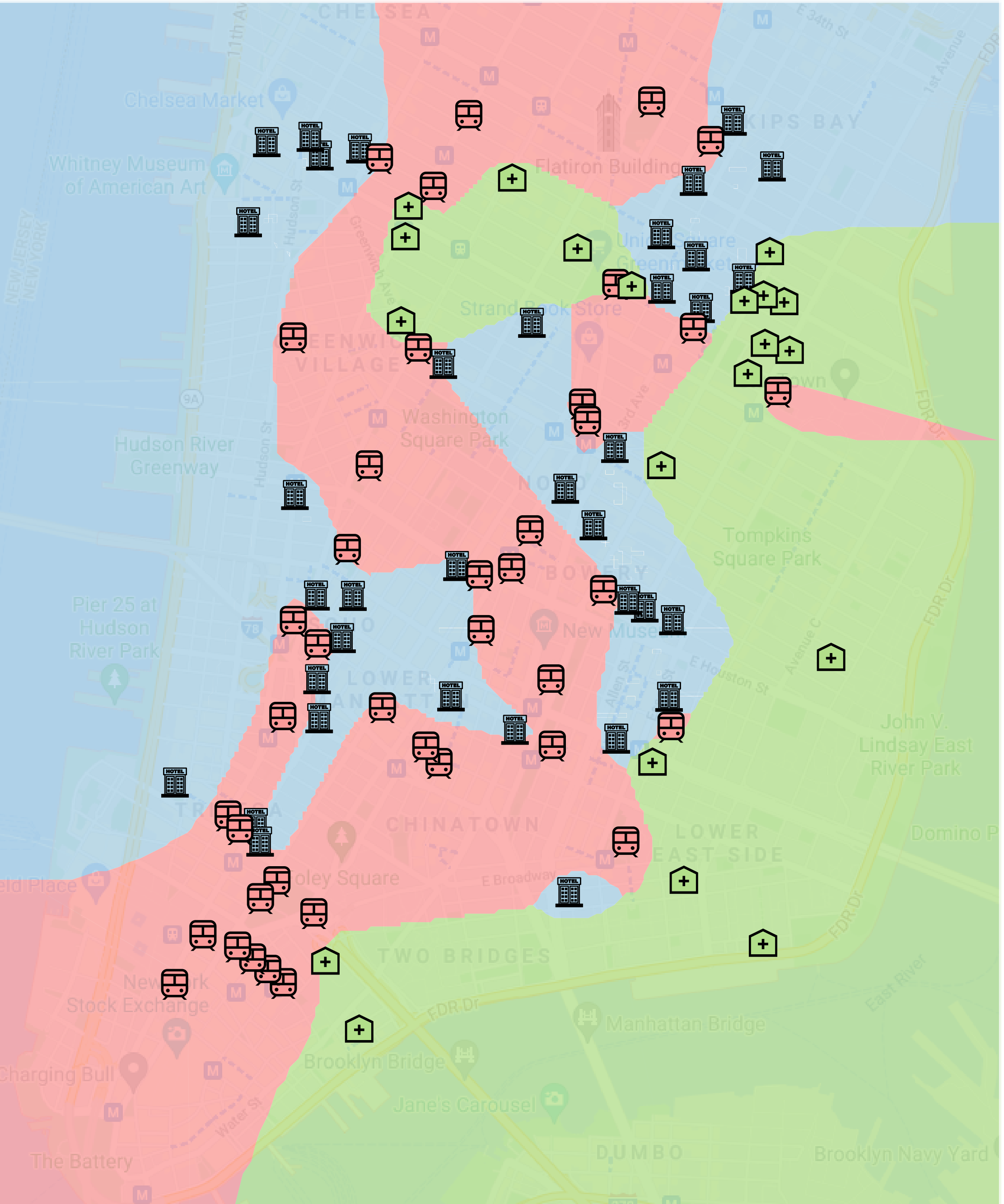
β-Skeleton

2. **Planar Spanning Forest**

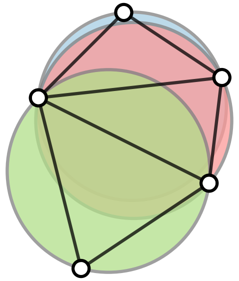


3. **Edge Augmentation**

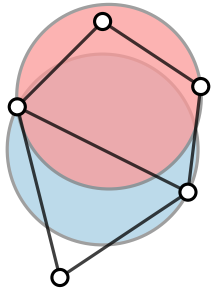
Pipeline



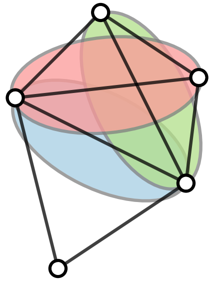
1. **Proximity Graph**



Delaunay Triangulation

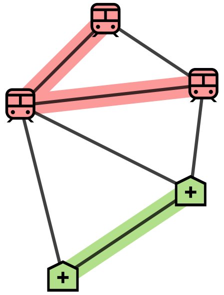


Gabriel Graph

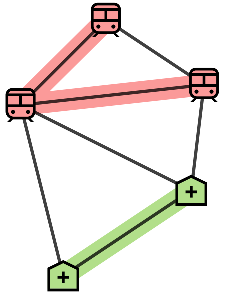


β-Skeleton

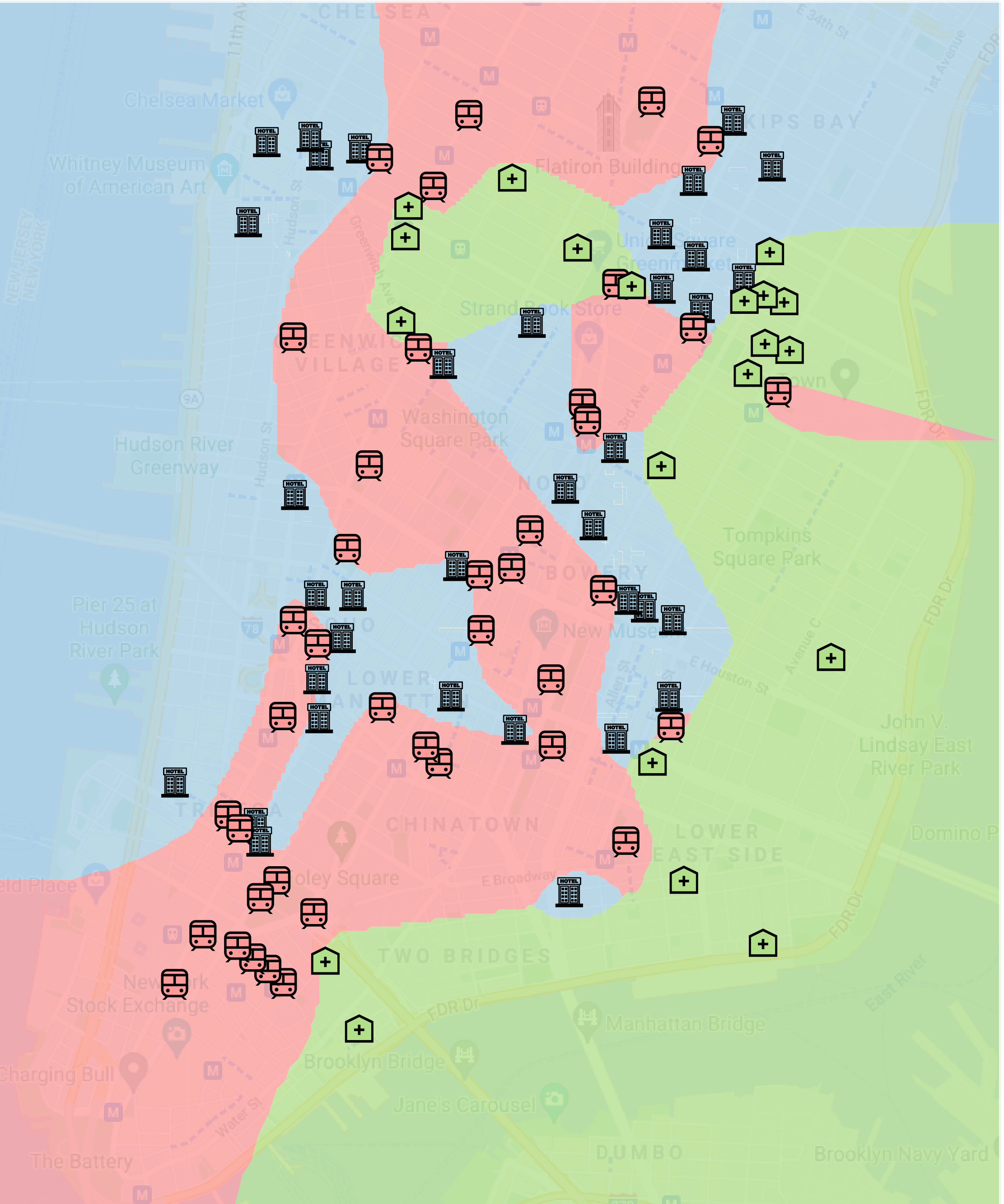
2. **Planar Spanning Forest**



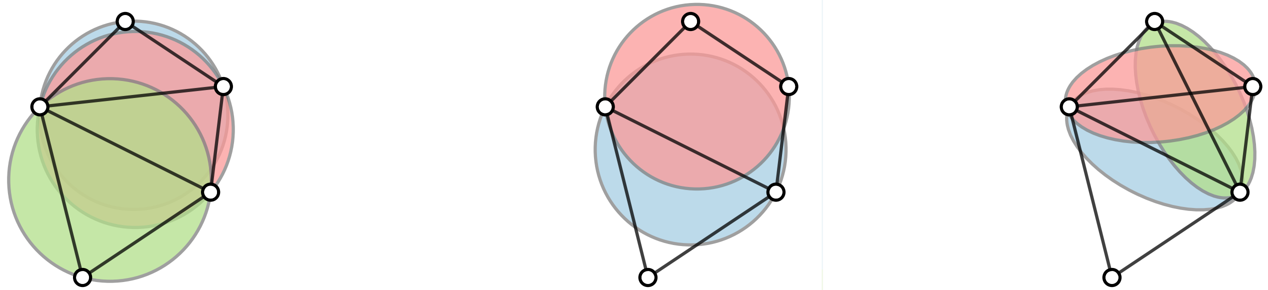
3. **Edge Augmentation**



Pipeline



1. **Proximity Graph**

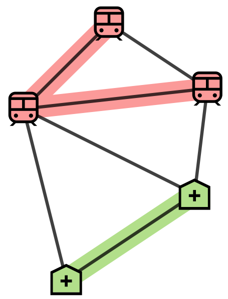


Delaunay Triangulation

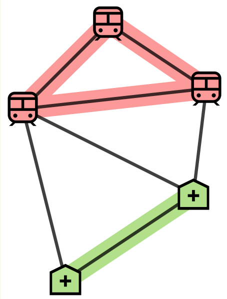
Gabriel Graph

β-Skeleton

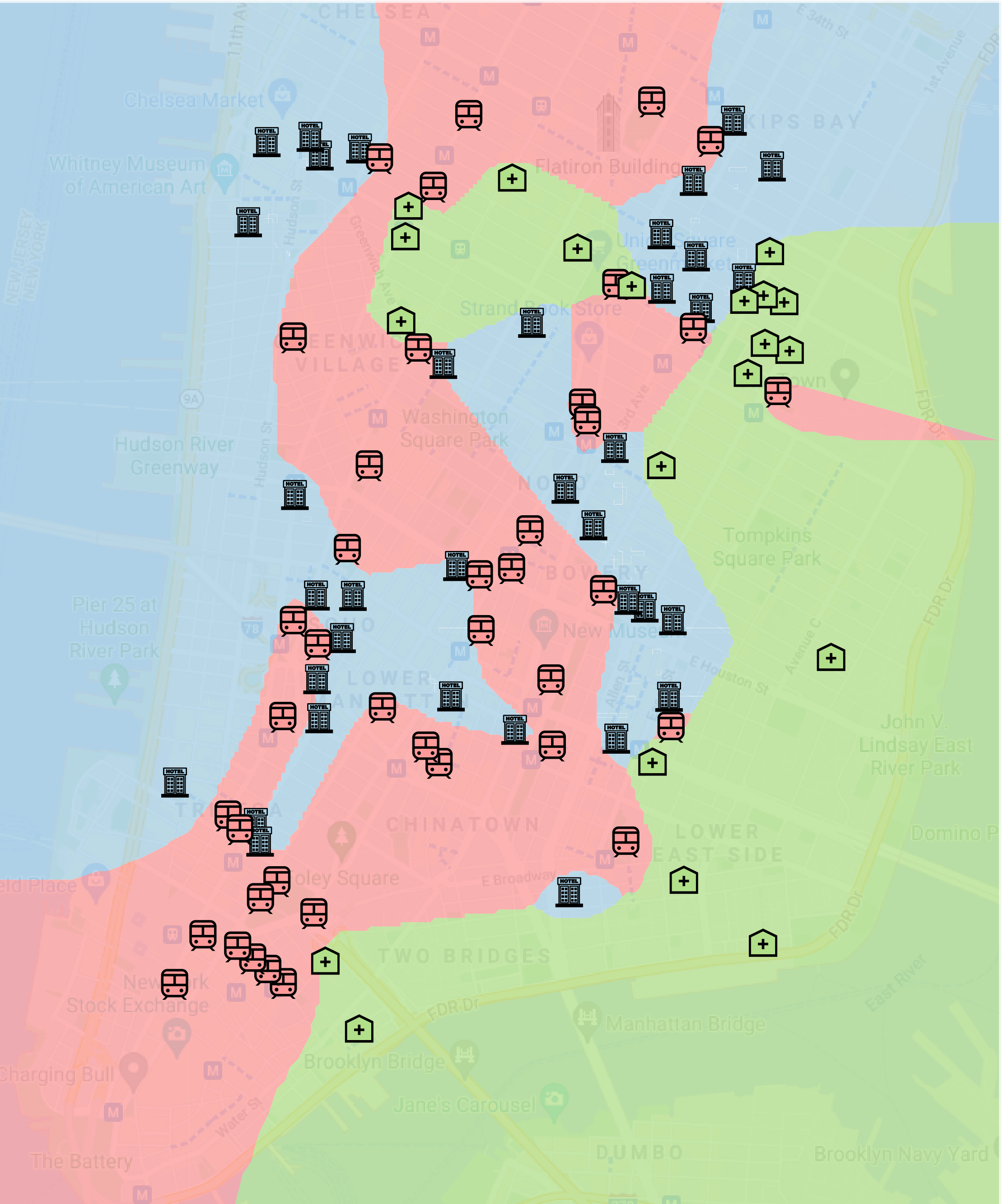
2. **Planar Spanning Forest**



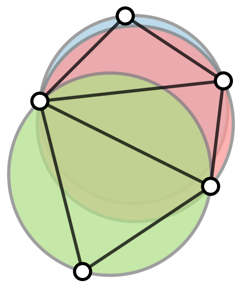
3. **Edge Augmentation**



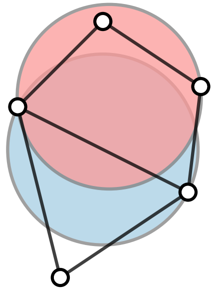
Pipeline



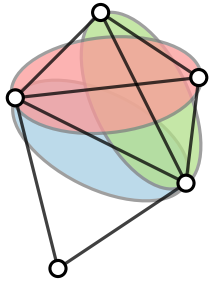
1. Proximity Graph



Delaunay Triangulation

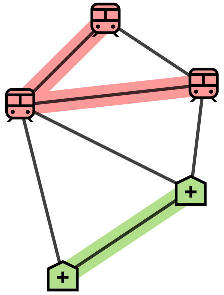


Gabriel Graph

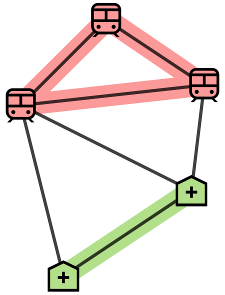


β-Skeleton

2. Planar Spanning Forest

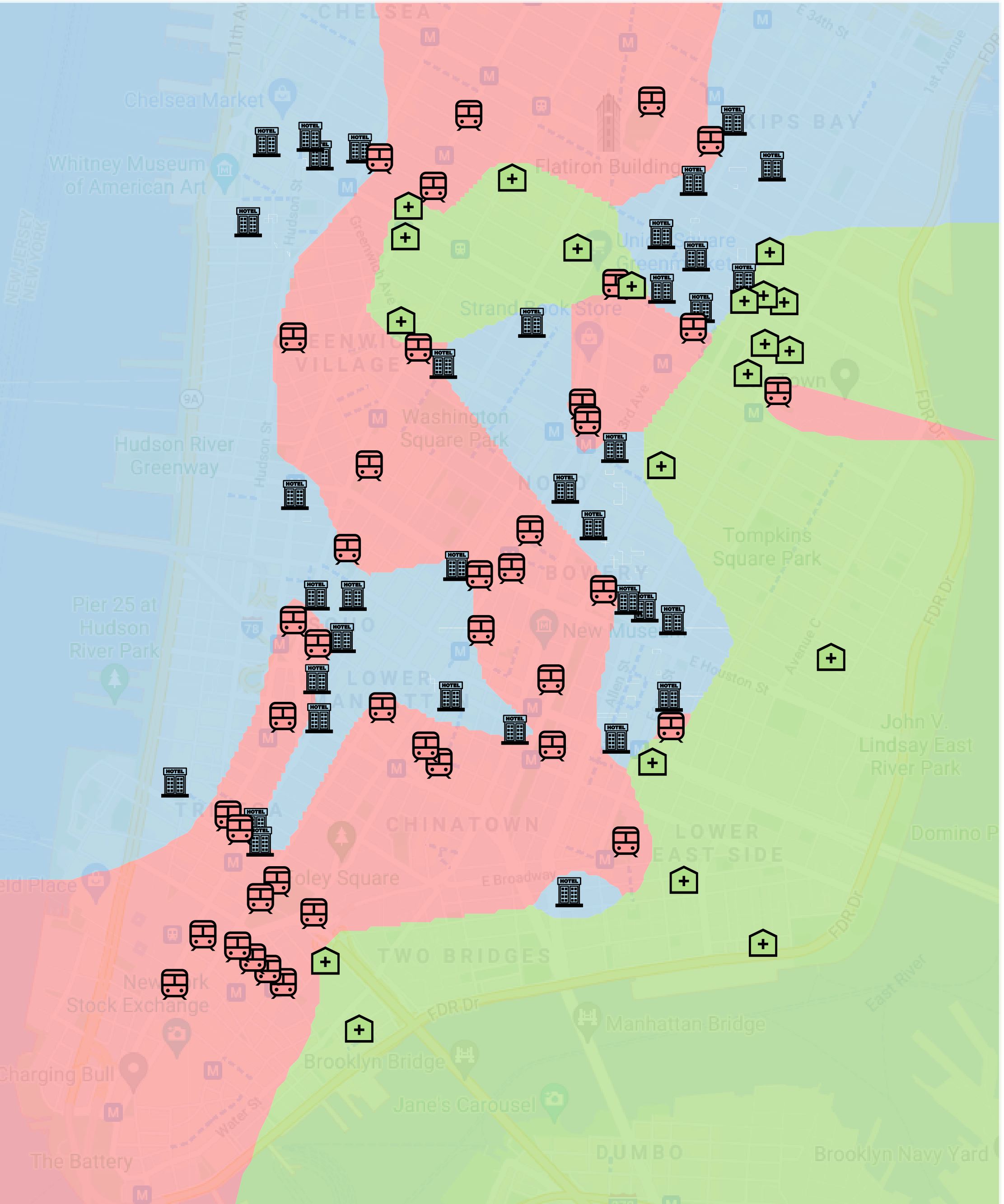


3. Edge Augmentation

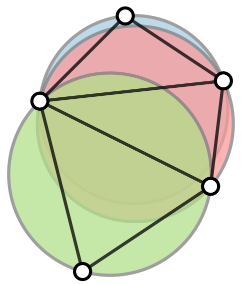


4. Rendering

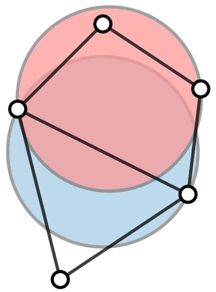
Pipeline



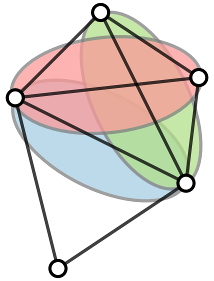
1. Proximity Graph



Delaunay Triangulation

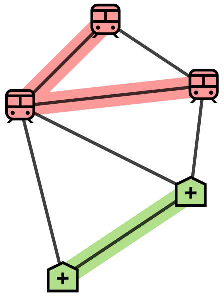


Gabriel Graph

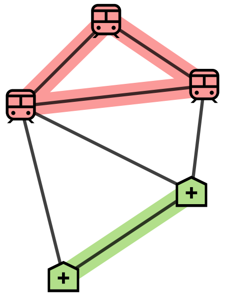


β-Skeleton

2. Planar Spanning Forest



3. Edge Augmentation

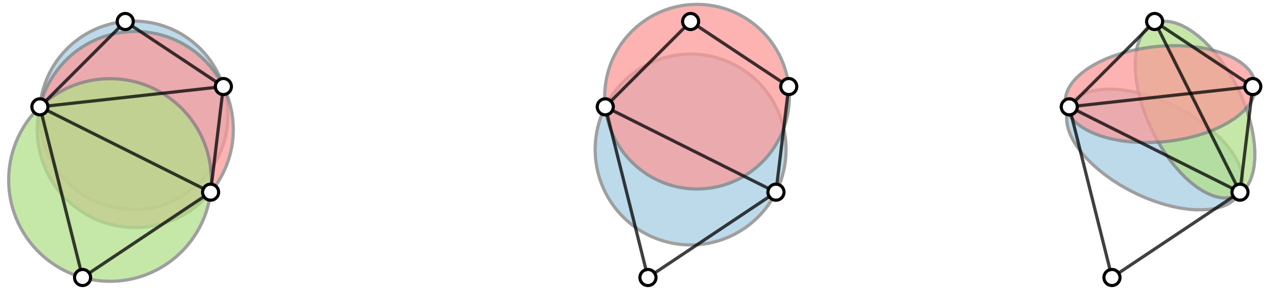


4. Rendering

- Line Voronoi Diagram

Pipeline

1. Proximity Graph

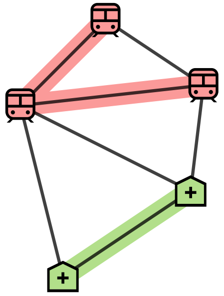


Delaunay Triangulation

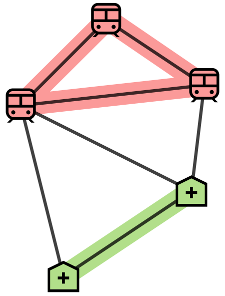
Gabriel Graph

β-Skeleton

2. Planar Spanning Forest



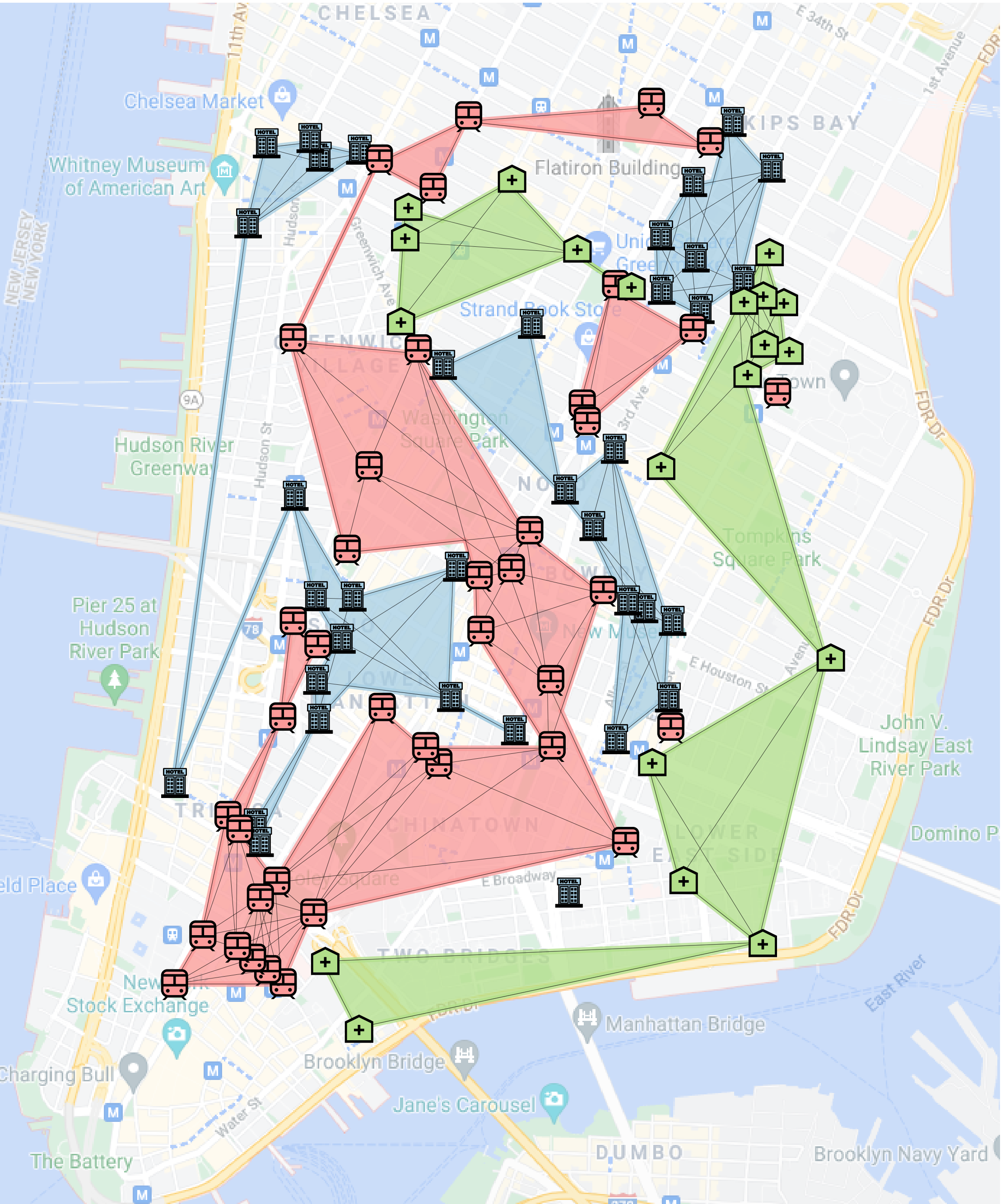
3. Edge Augmentation



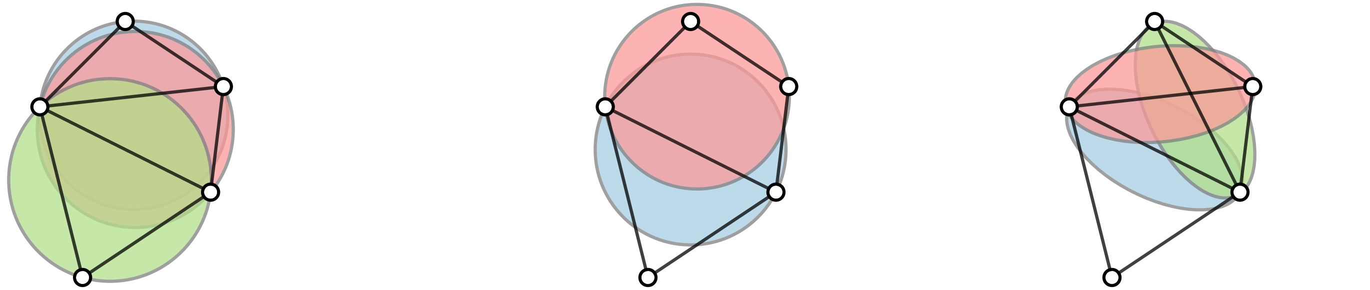
4. Rendering

- Line Voronoi Diagram
- Tree Representation

Pipeline



1. Proximity Graph

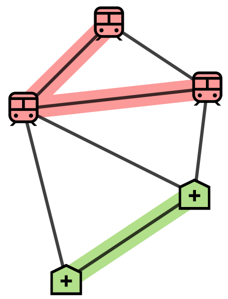


Delaunay Triangulation

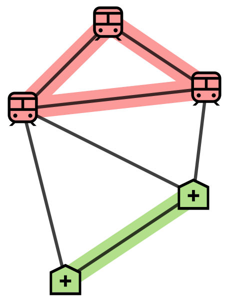
Gabriel Graph

β-Skeleton

2. Planar Spanning Forest



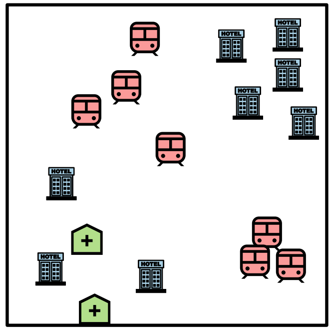
3. Edge Augmentation



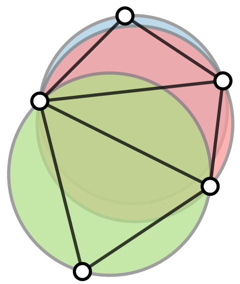
4. Rendering

- Line Voronoi Diagram
- Tree Representation
- Polygon Representation

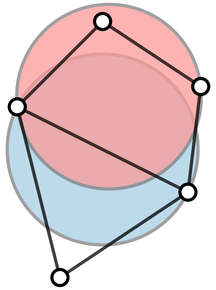
Pipeline



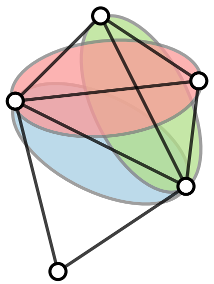
1. **Proximity Graph**



Delaunay Triangulation

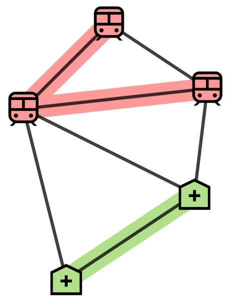


Gabriel Graph

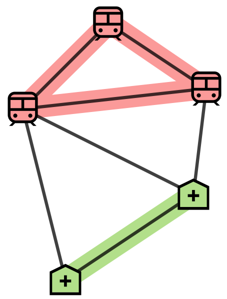


β-Skeleton

2. **Planar Spanning Forest**

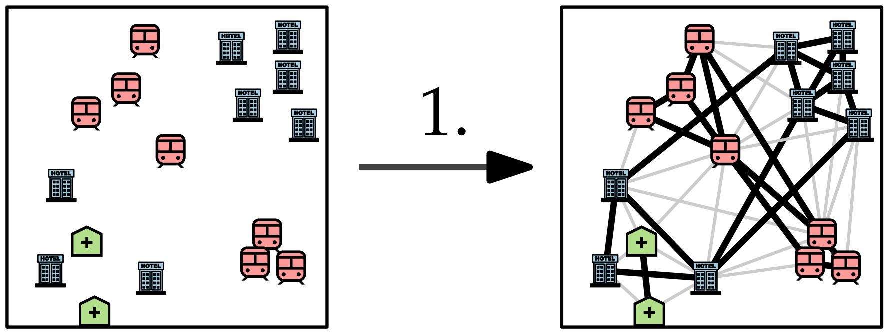


3. **Edge Augmentation**

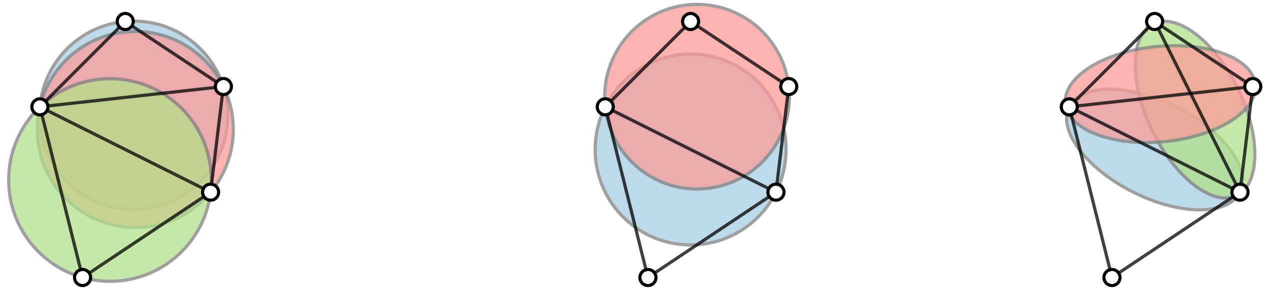


4. **Rendering**
   - Line Voronoi Diagram
   - Tree Representation
   - Polygon Representation

Pipeline

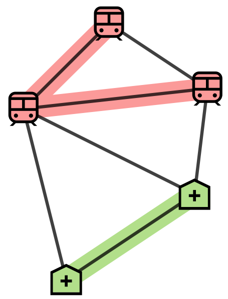


1. **Proximity Graph**

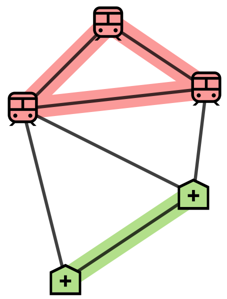


Delaunay Triangulation | Gabriel Graph | β-Skeleton

2. **Planar Spanning Forest**



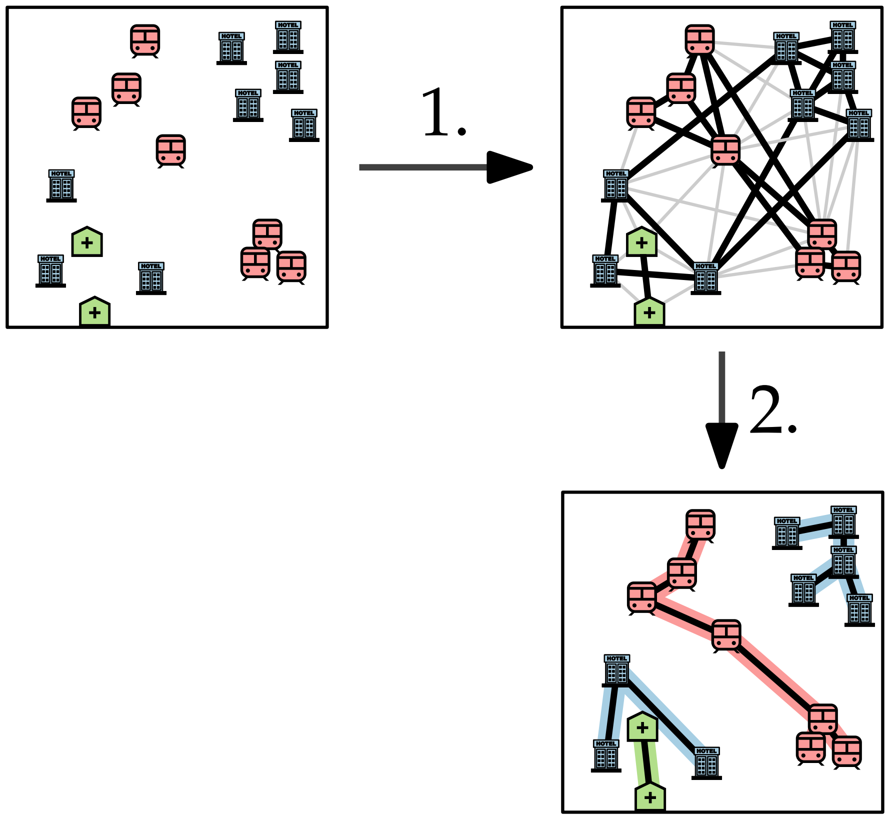
3. **Edge Augmentation**



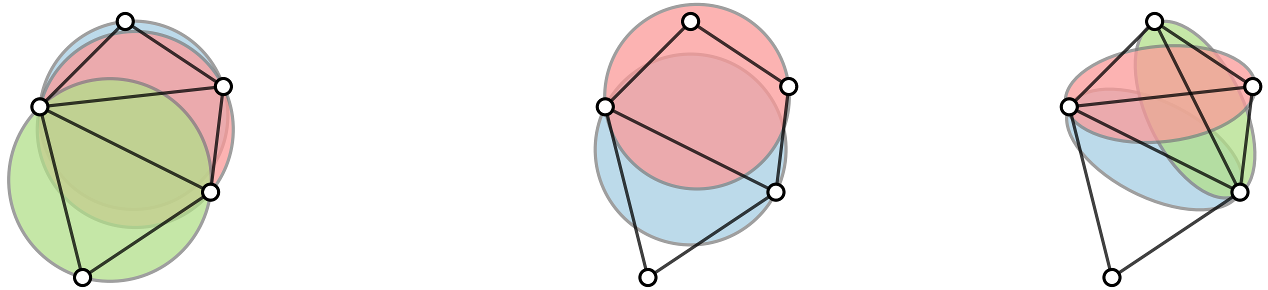
4. **Rendering**

- Line Voronoi Diagram
- Tree Representation
- Polygon Representation

Pipeline



1. **Proximity Graph**

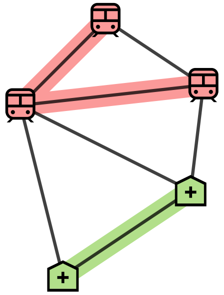


Delaunay Triangulation

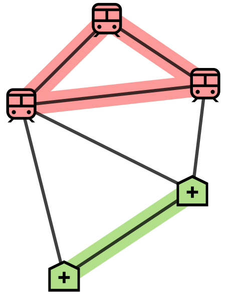
Gabriel Graph

β-Skeleton

2. **Planar Spanning Forest**



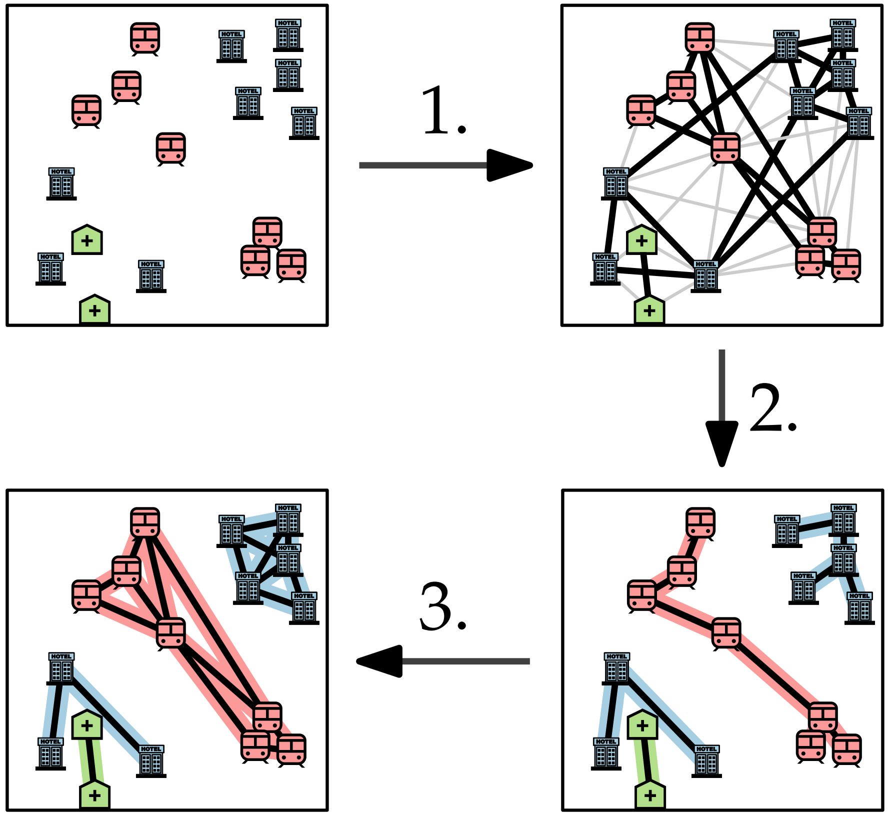
3. **Edge Augmentation**



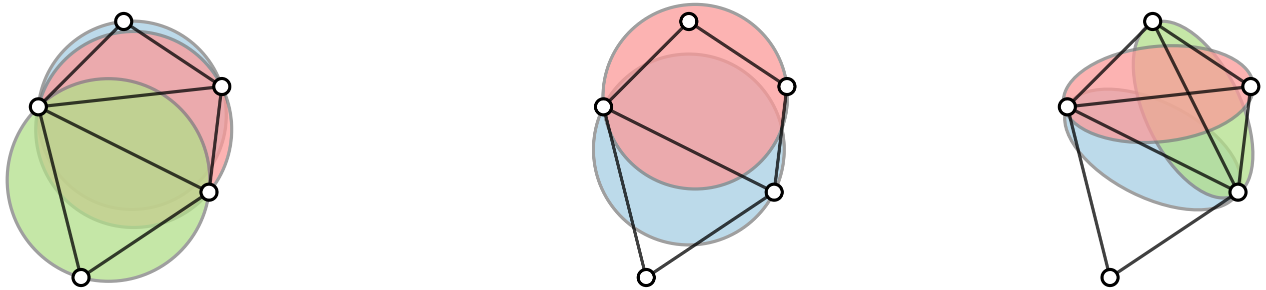
4. **Rendering**

- Line Voronoi Diagram
- Tree Representation
- Polygon Representation

Pipeline



1. Proximity Graph

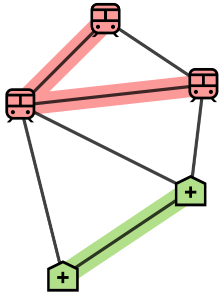


Delaunay Triangulation

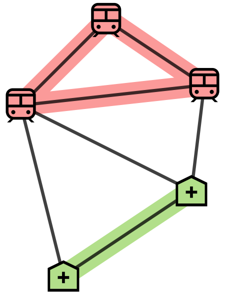
Gabriel Graph

β-Skeleton

2. Planar Spanning Forest

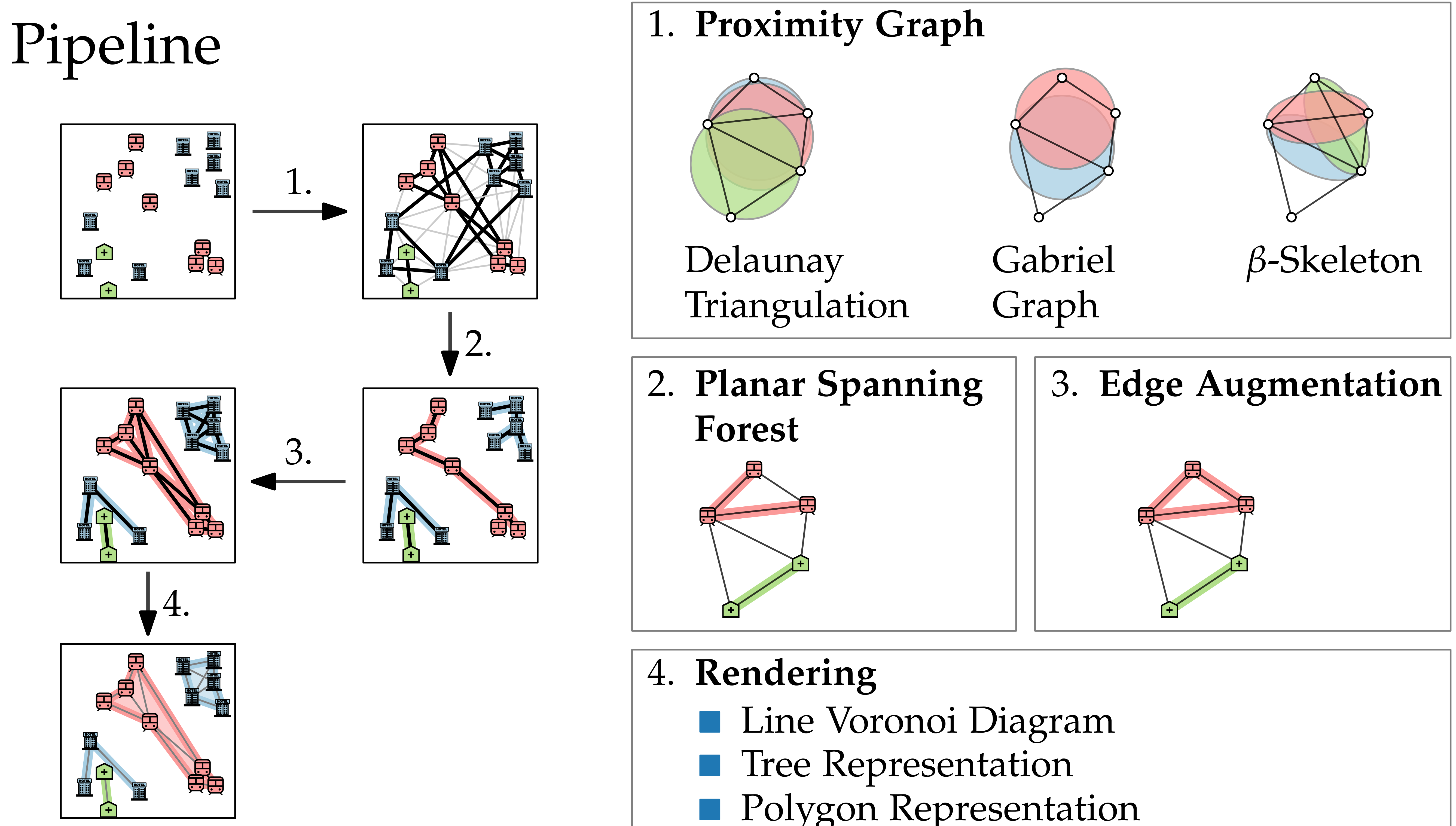


3. Edge Augmentation



4. Rendering

- Line Voronoi Diagram
- Tree Representation
- Polygon Representation


Pipeline
1.
2.
3.
4.
1. Proximity Graph
Delaunay Triangulation
Gabriel Graph
β-Skeleton
2. Planar Spanning Forest
3. Edge Augmentation
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Line Voronoi Diagram
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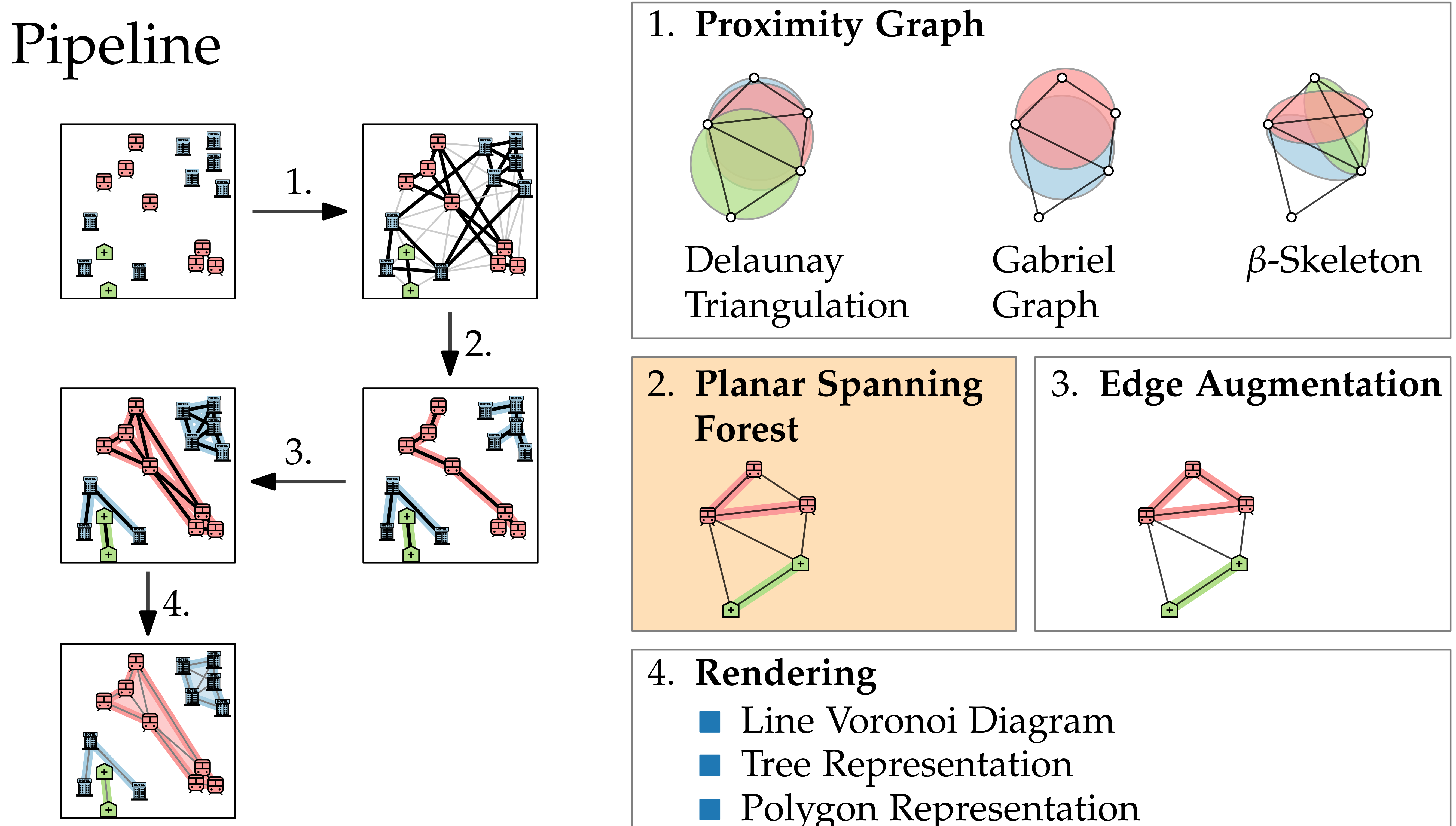
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Planar Spanning Forest: Heuristics

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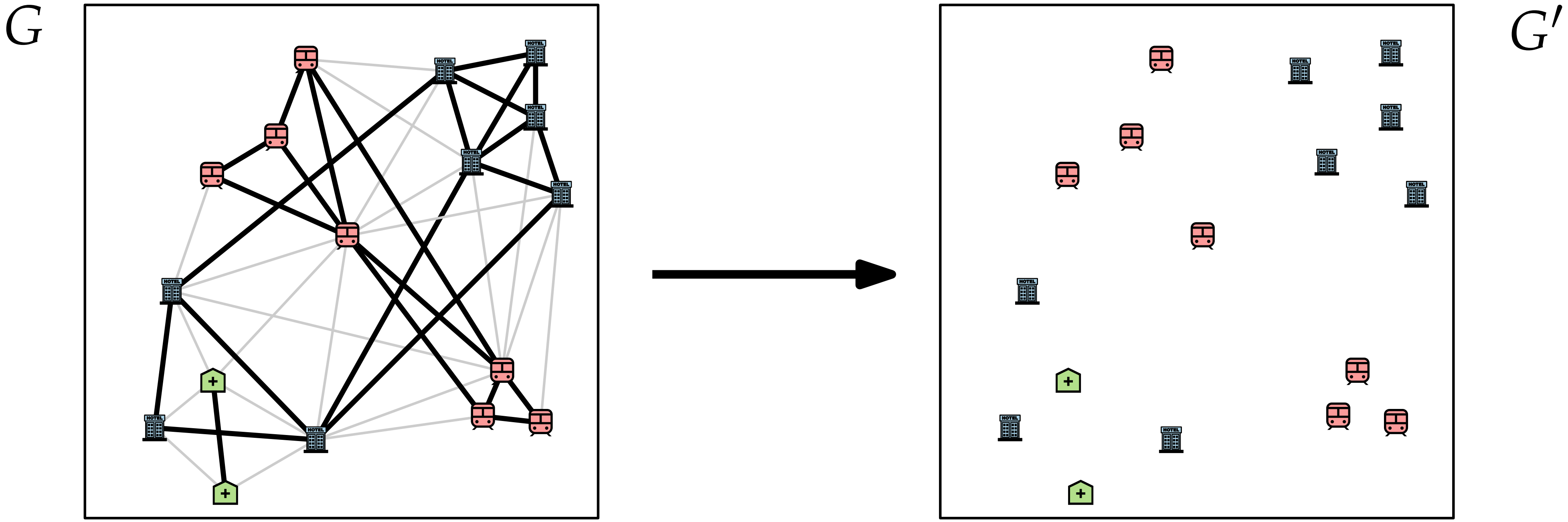

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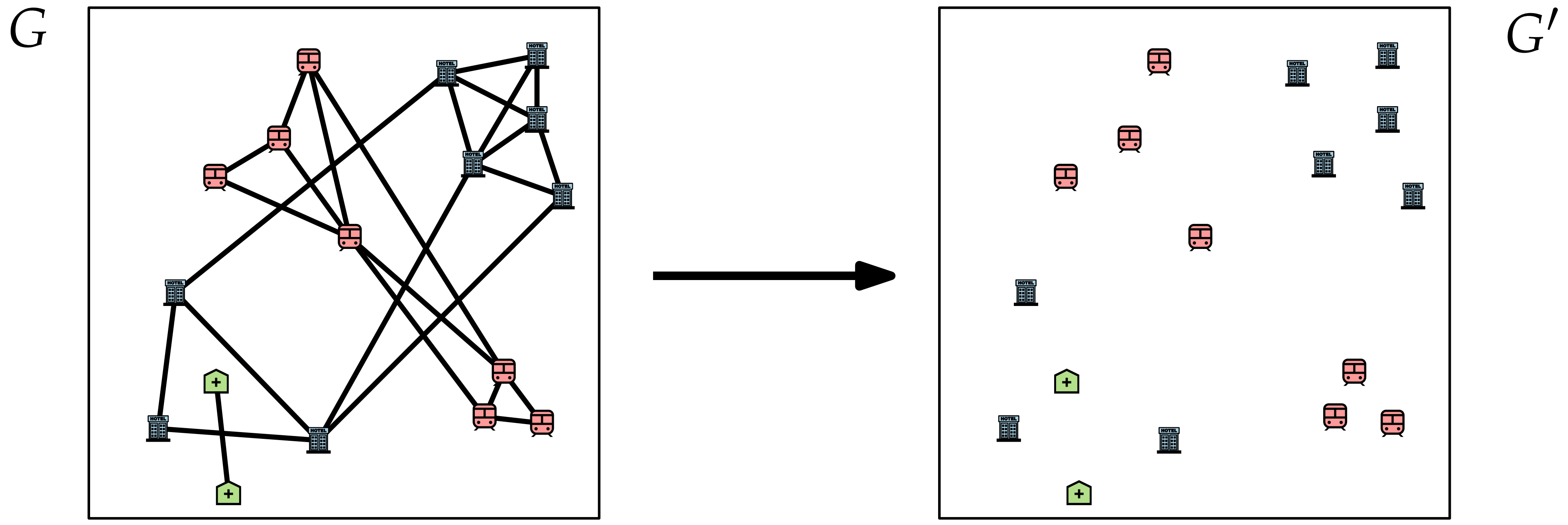
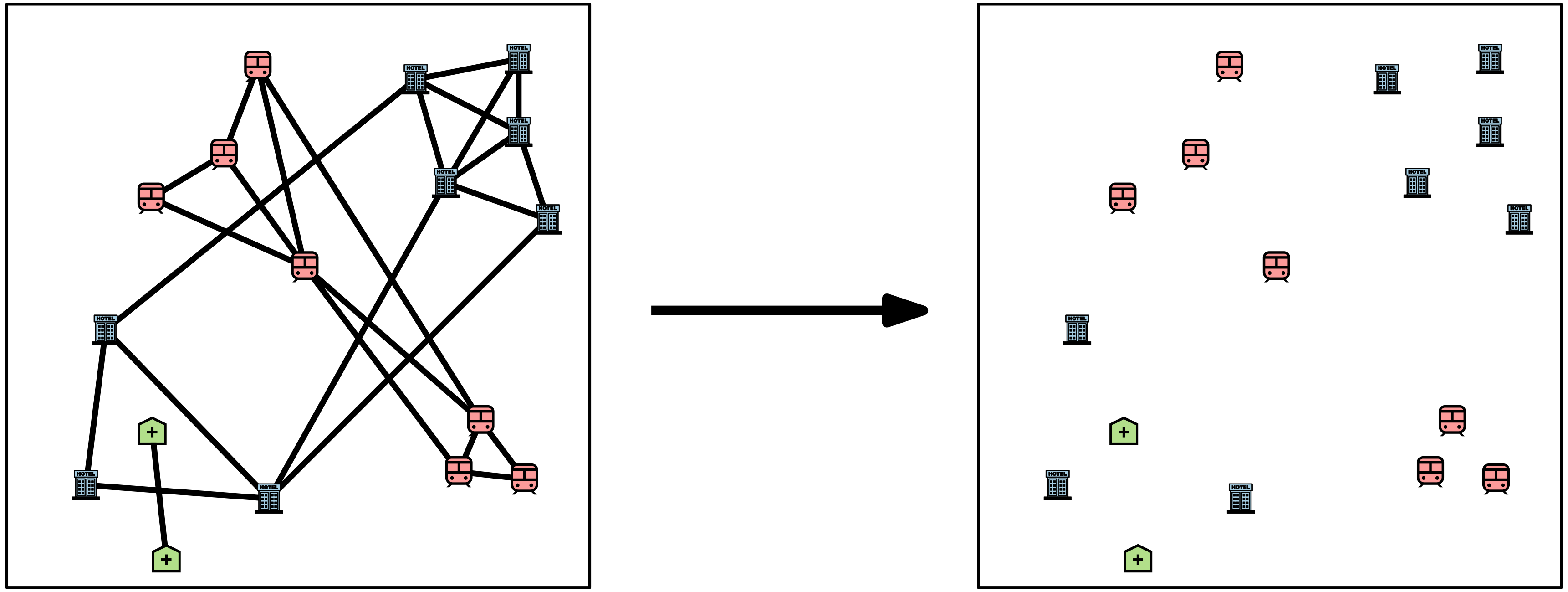


Planar Spanning Forest: Heuristics

Greedy:

G



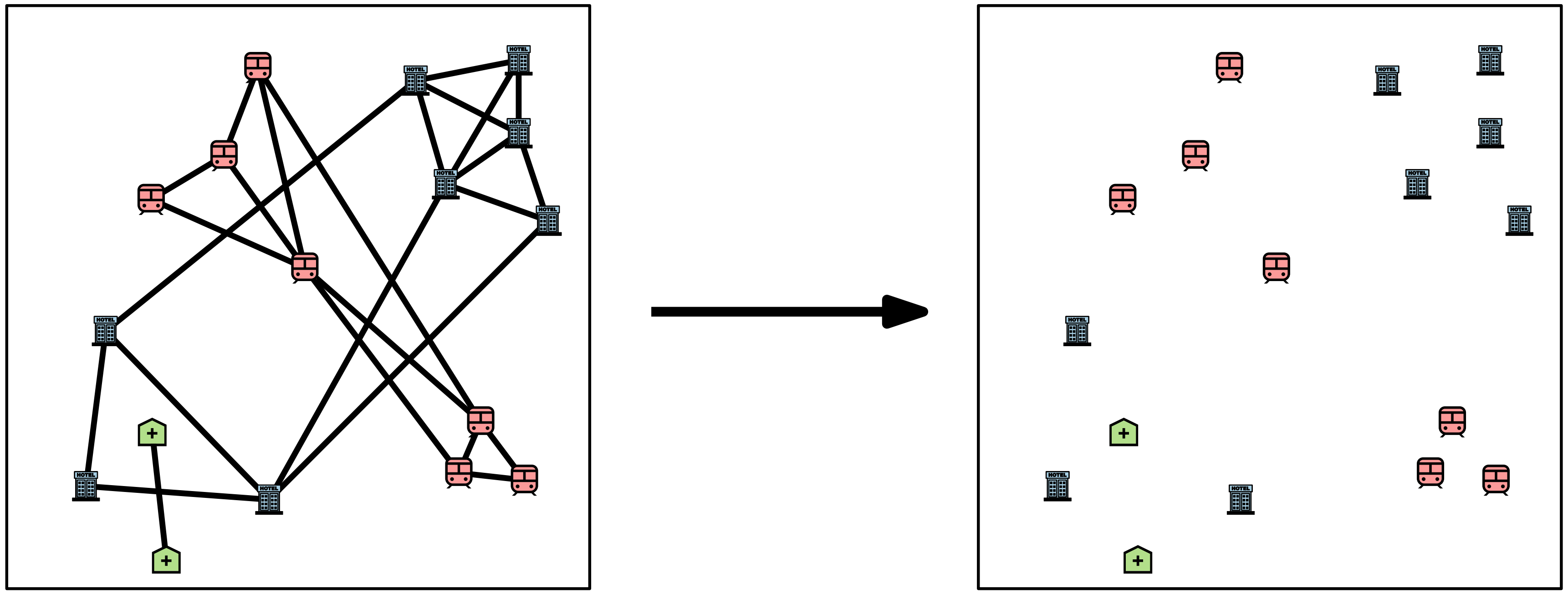
G'

Planar Spanning Forest: Heuristics

GREEDY:

1. Remove from G every edge that lies within a connected component of G'.

G

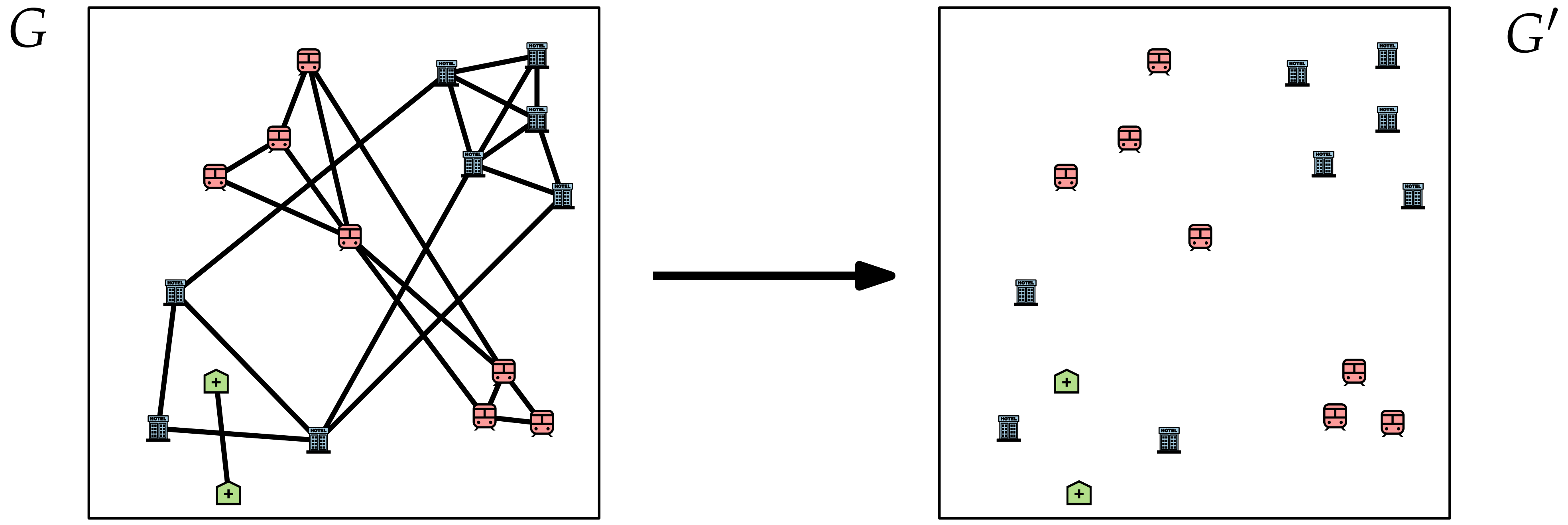


G'

Planar Spanning Forest: Heuristics

Greedy:

1. Remove from G every edge that lies within a connected component of G'.
2. Move edge e that is crossed by min. number of edges to G'.



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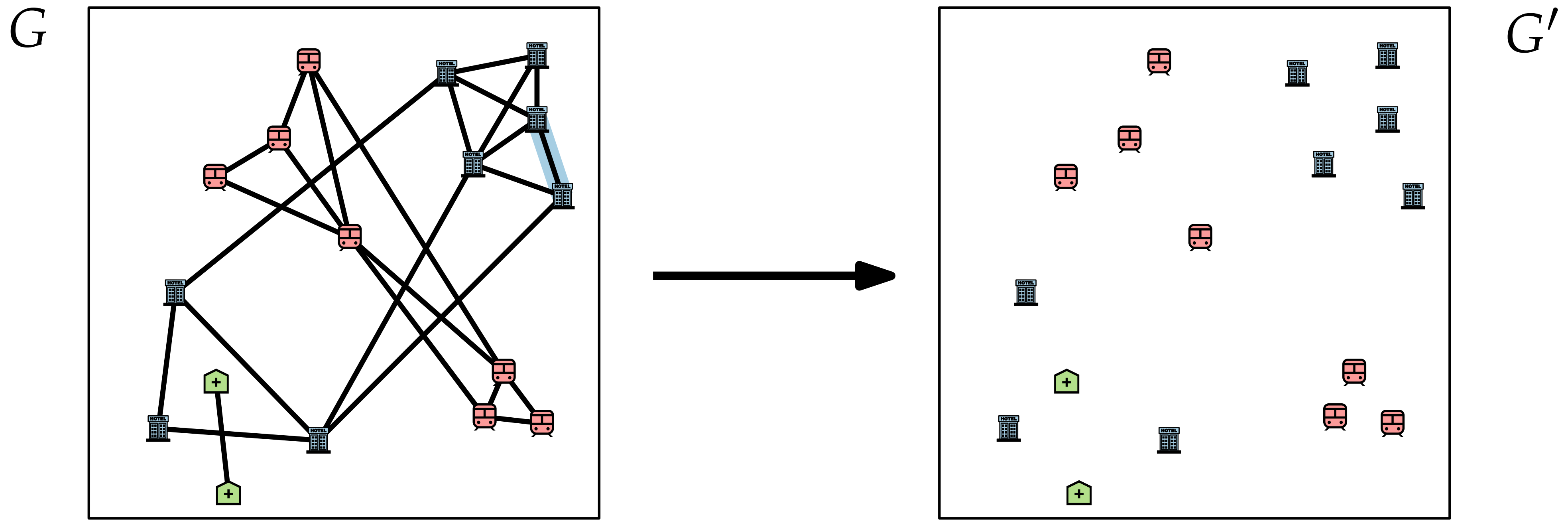

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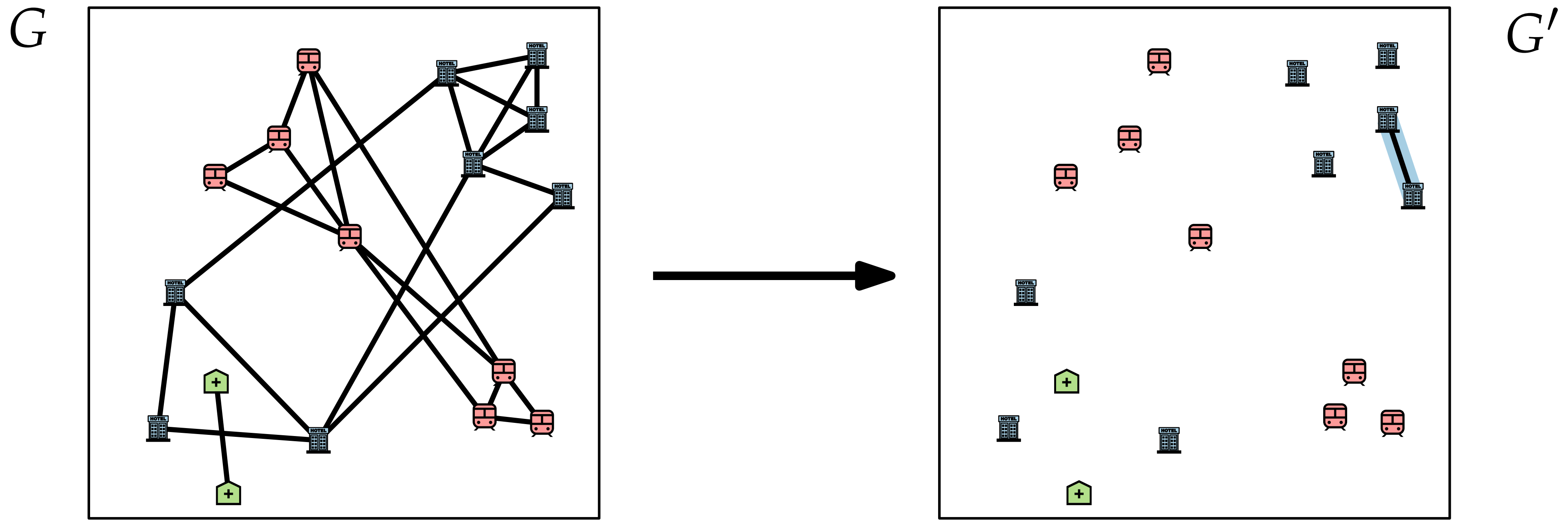

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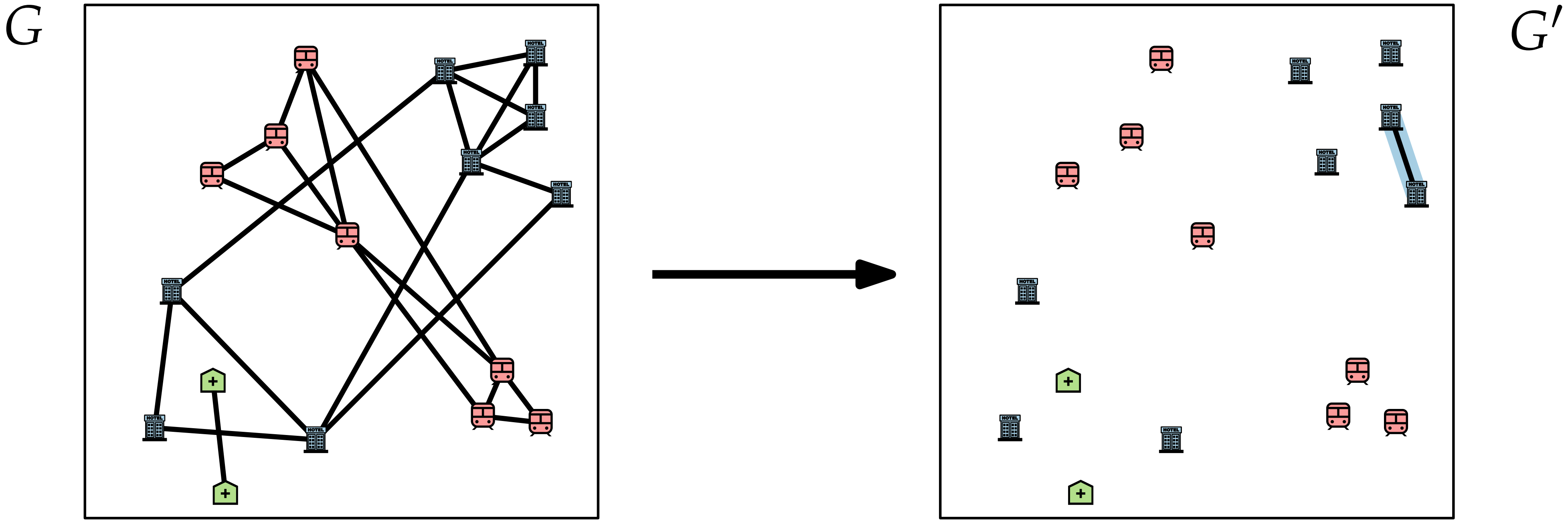

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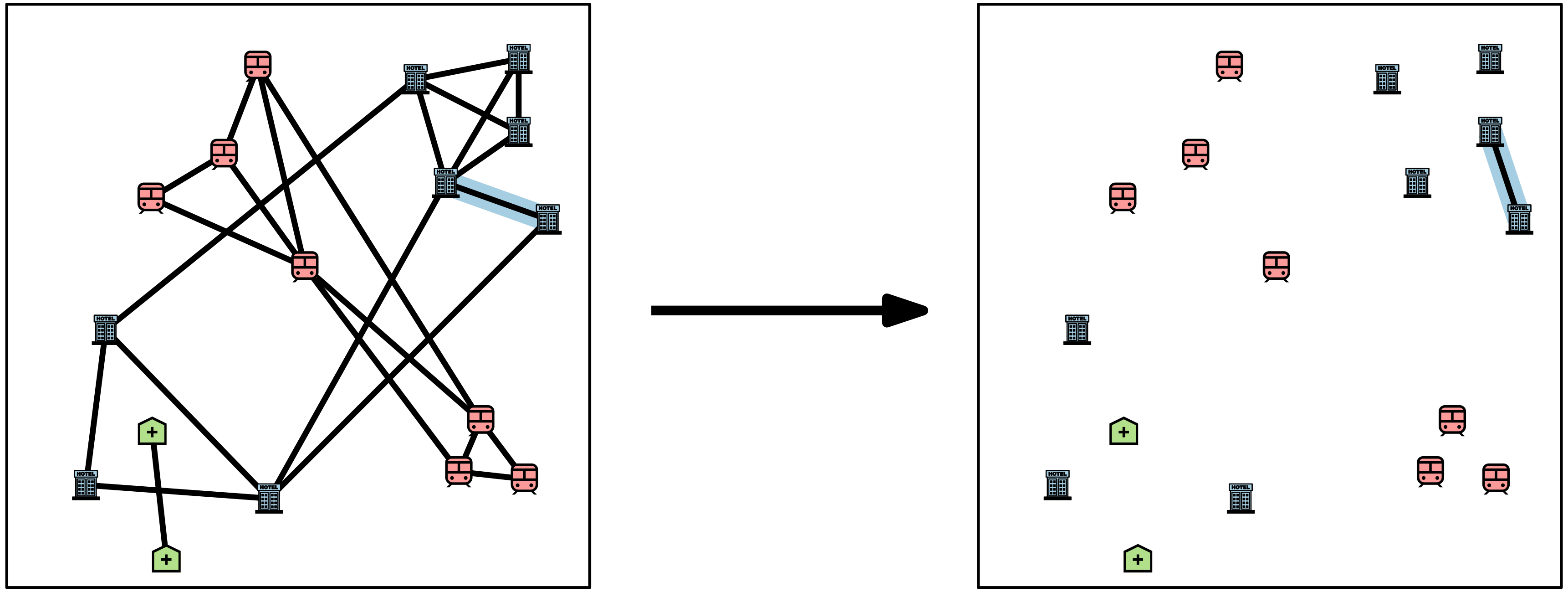

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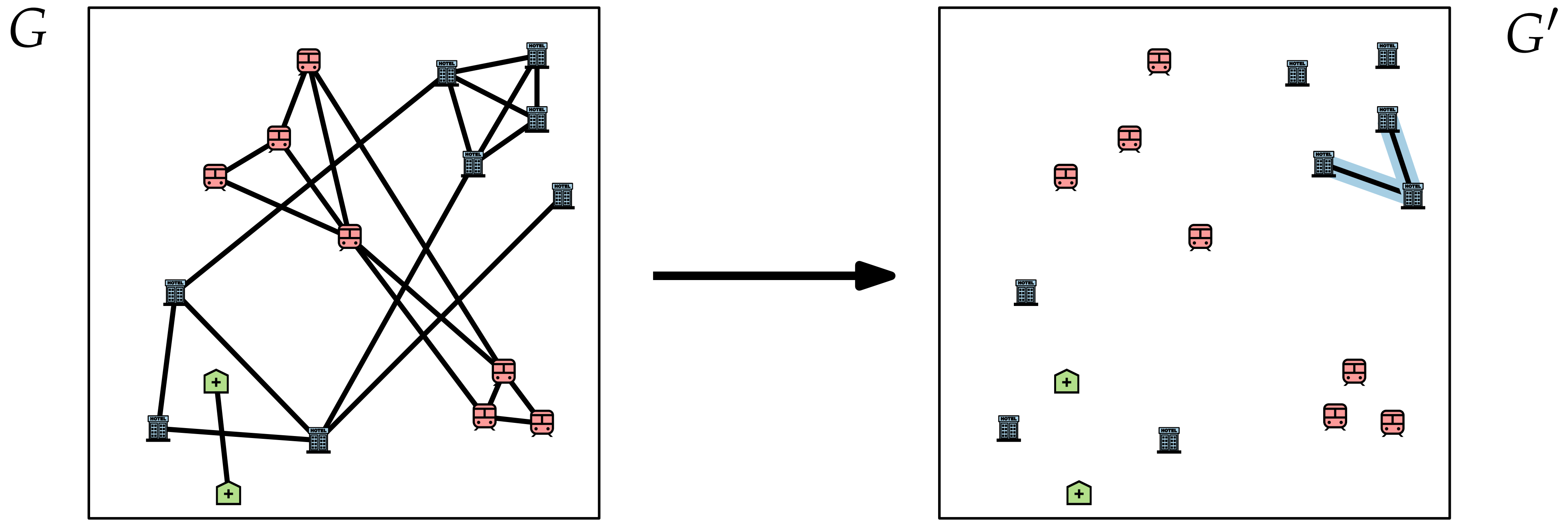

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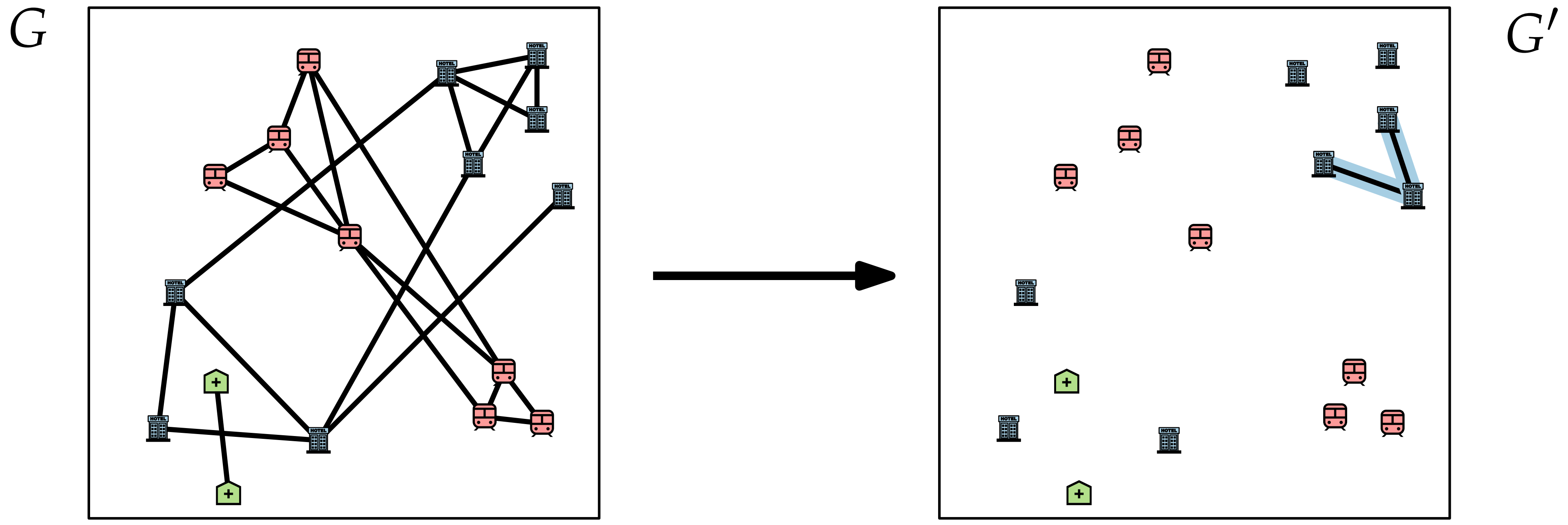

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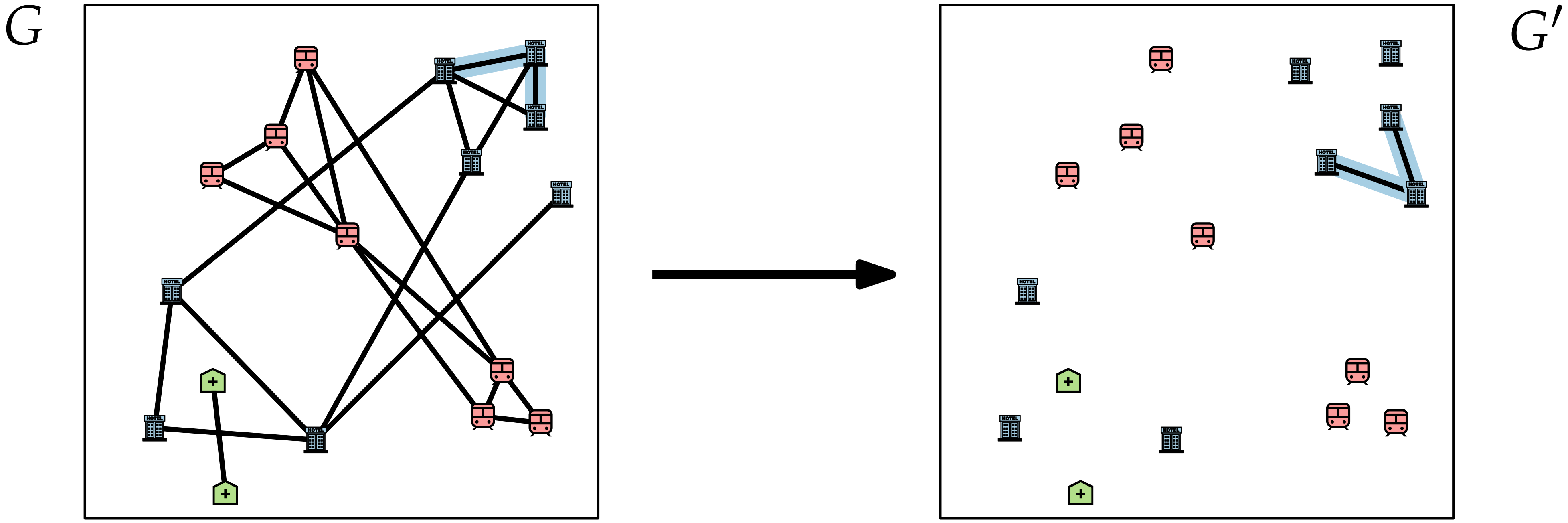

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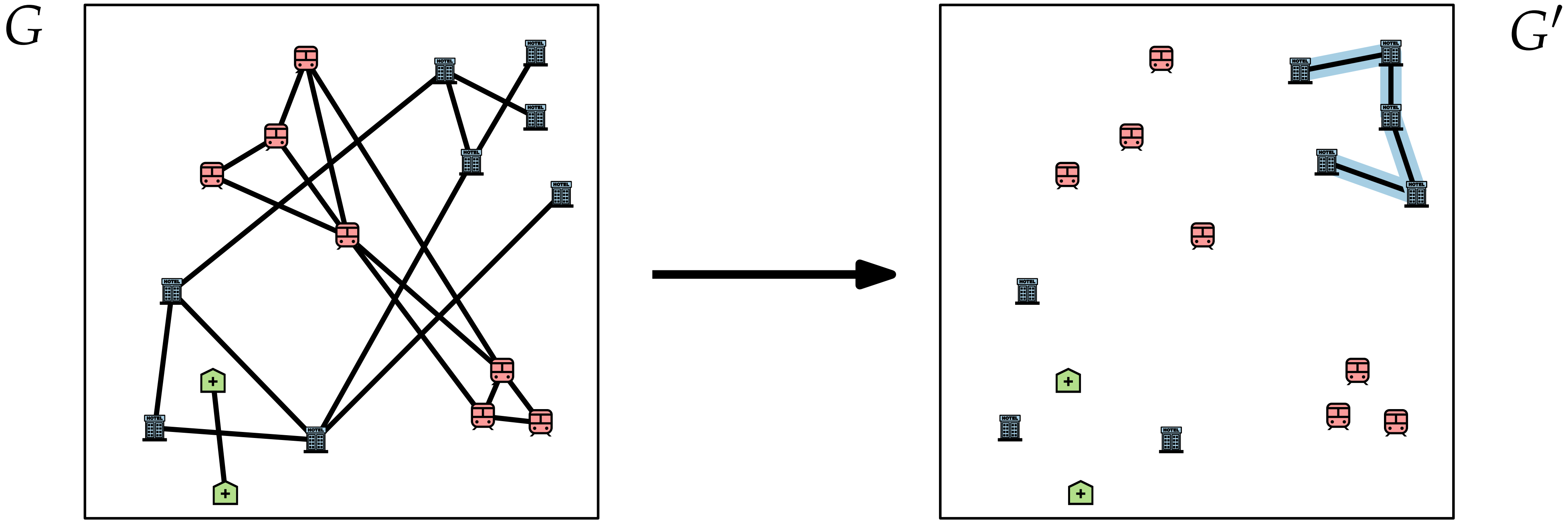

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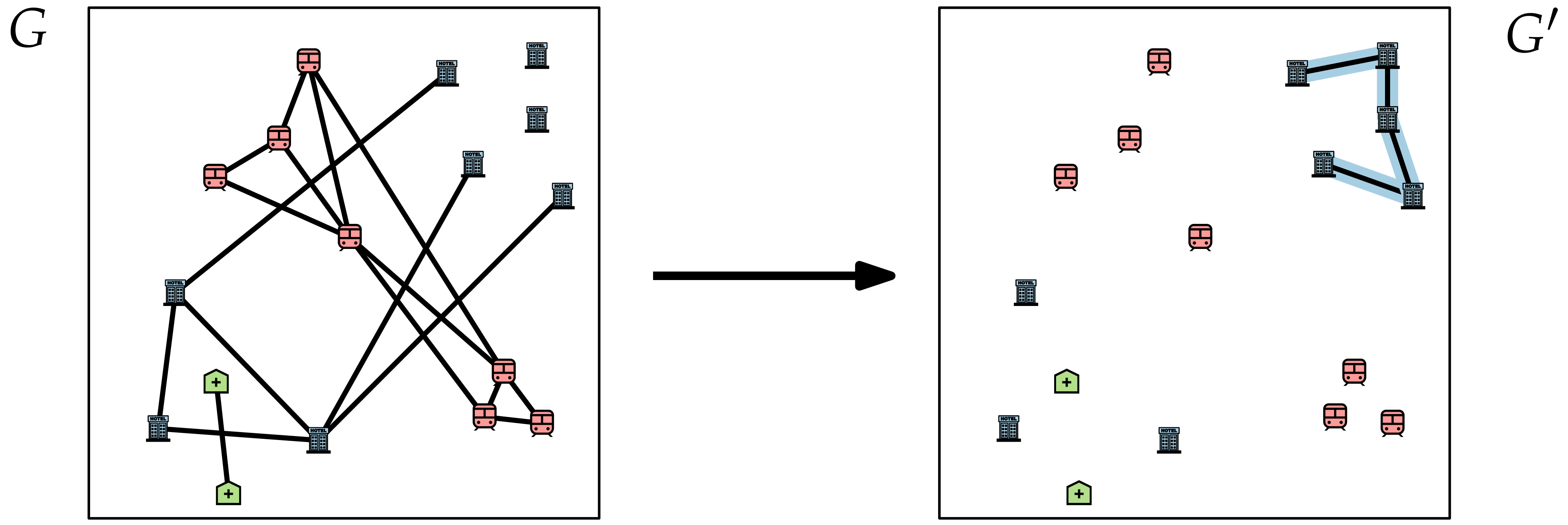

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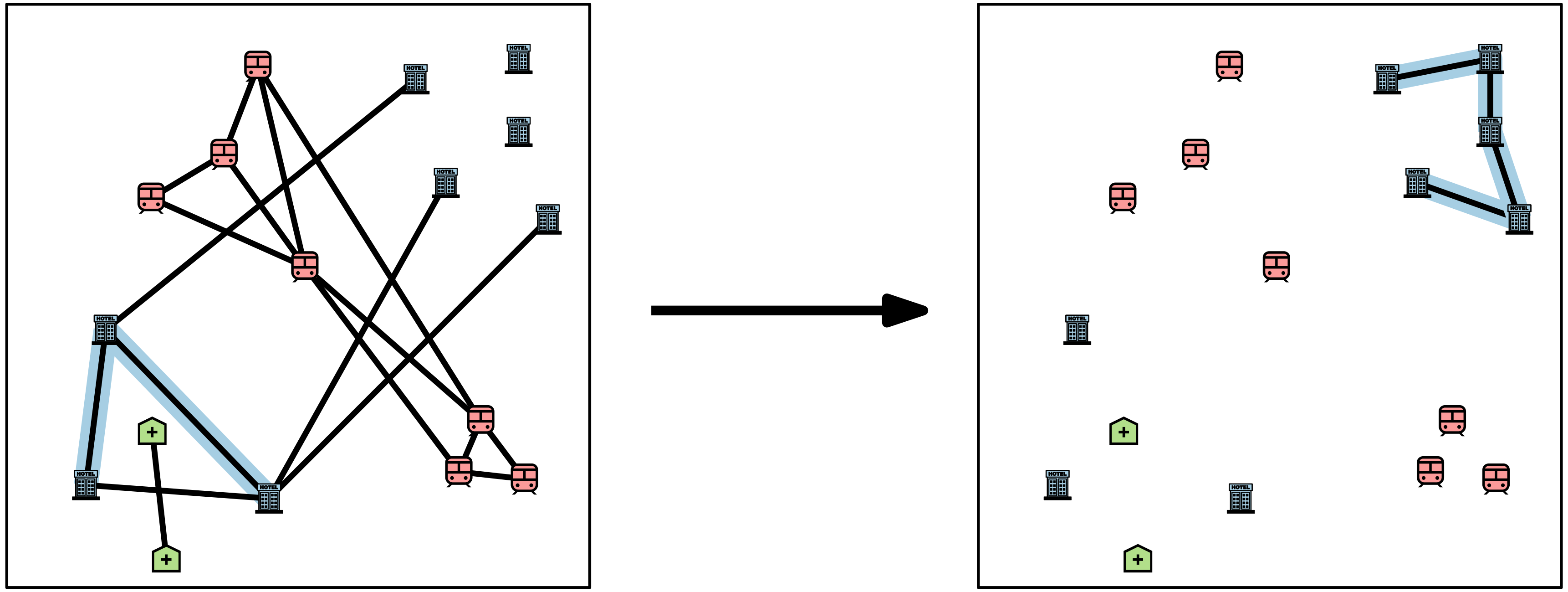

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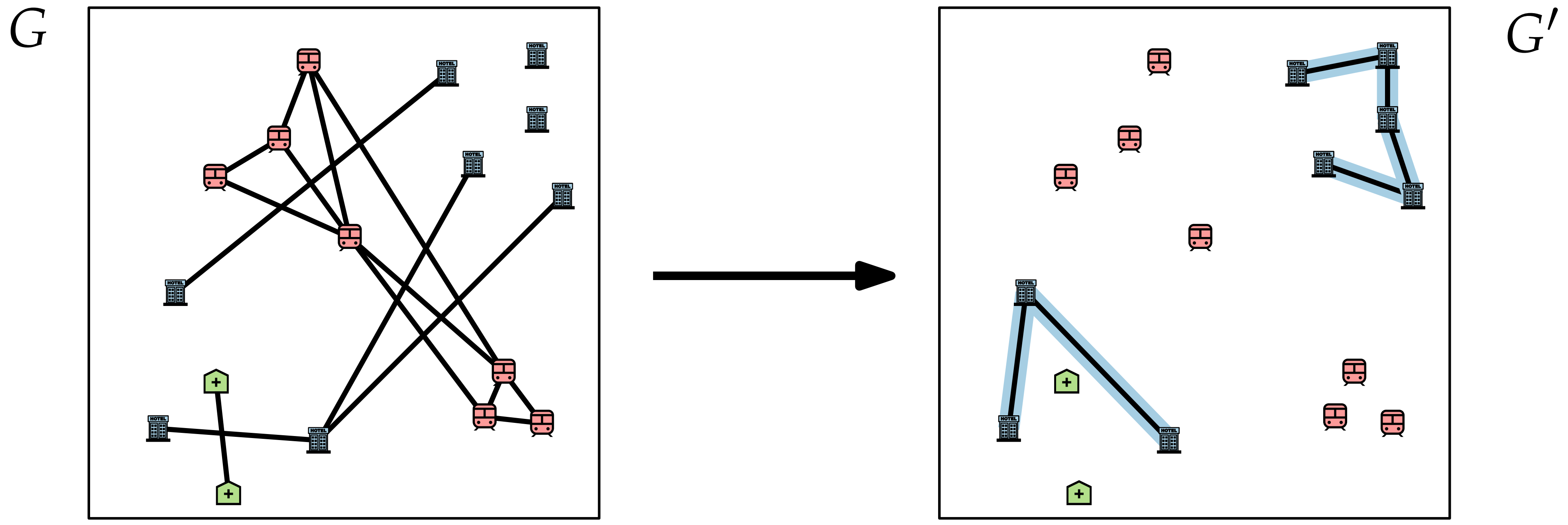

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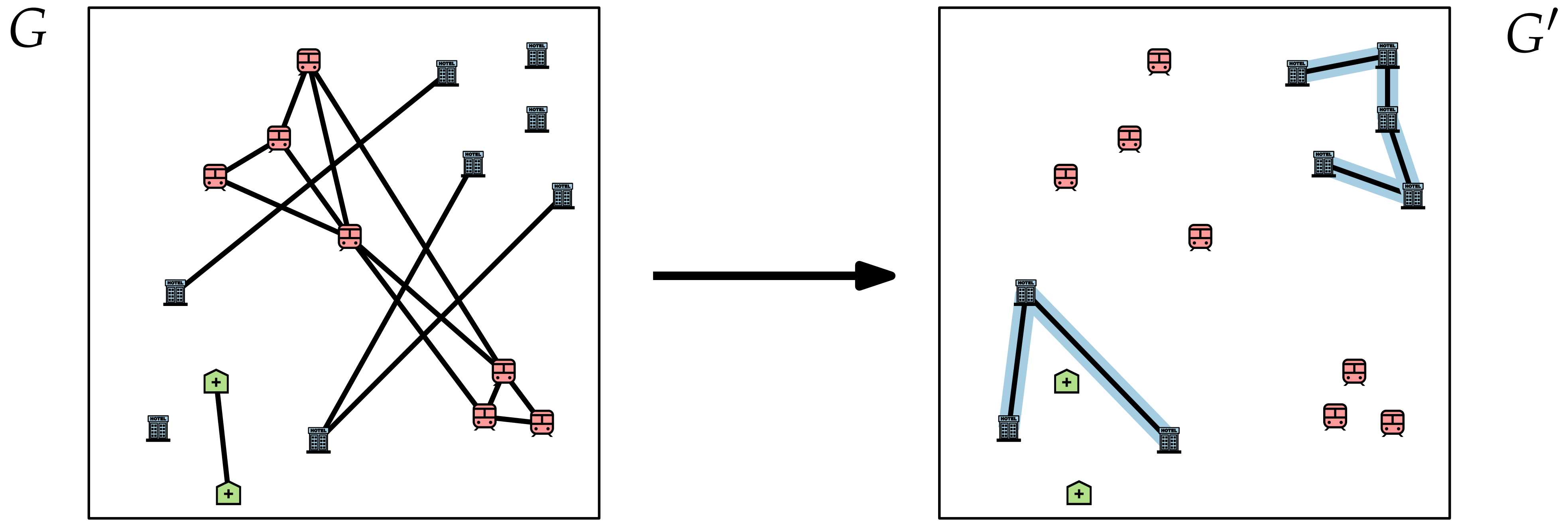

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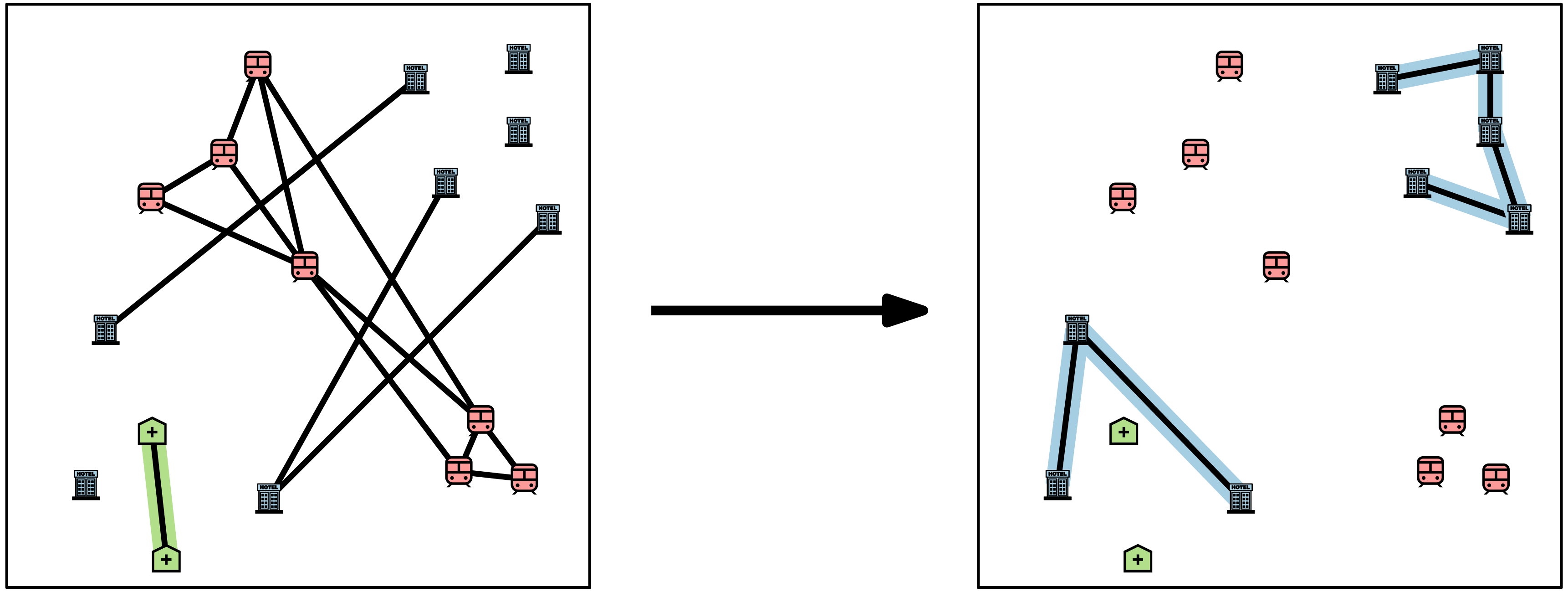

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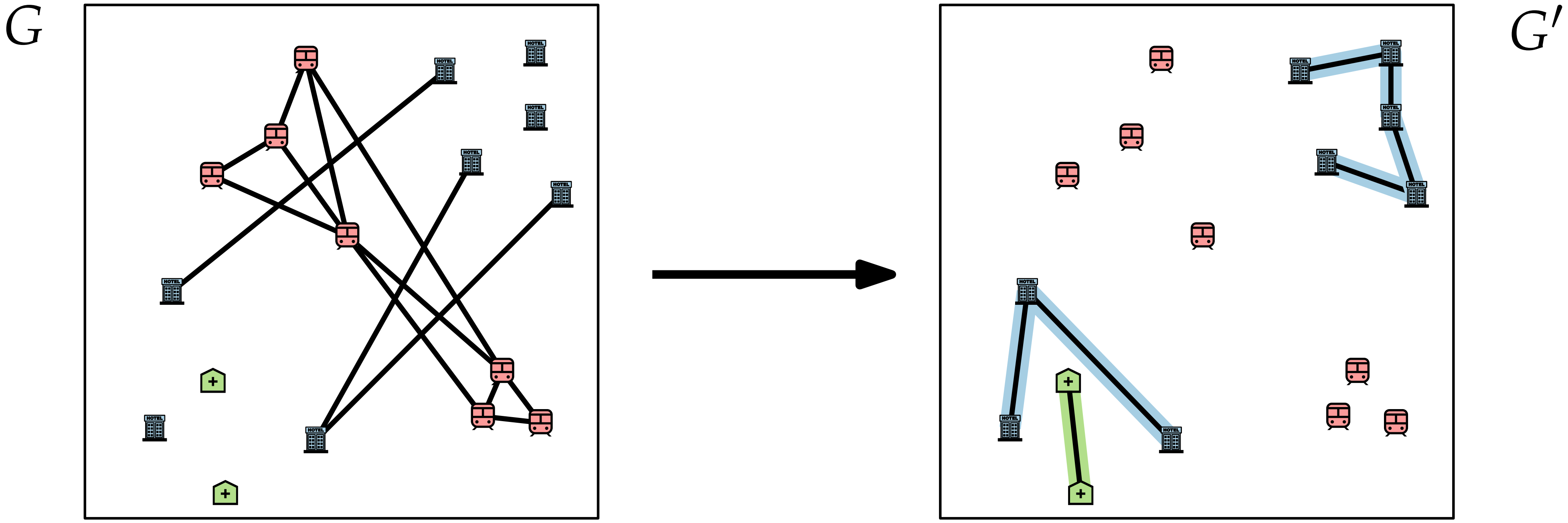

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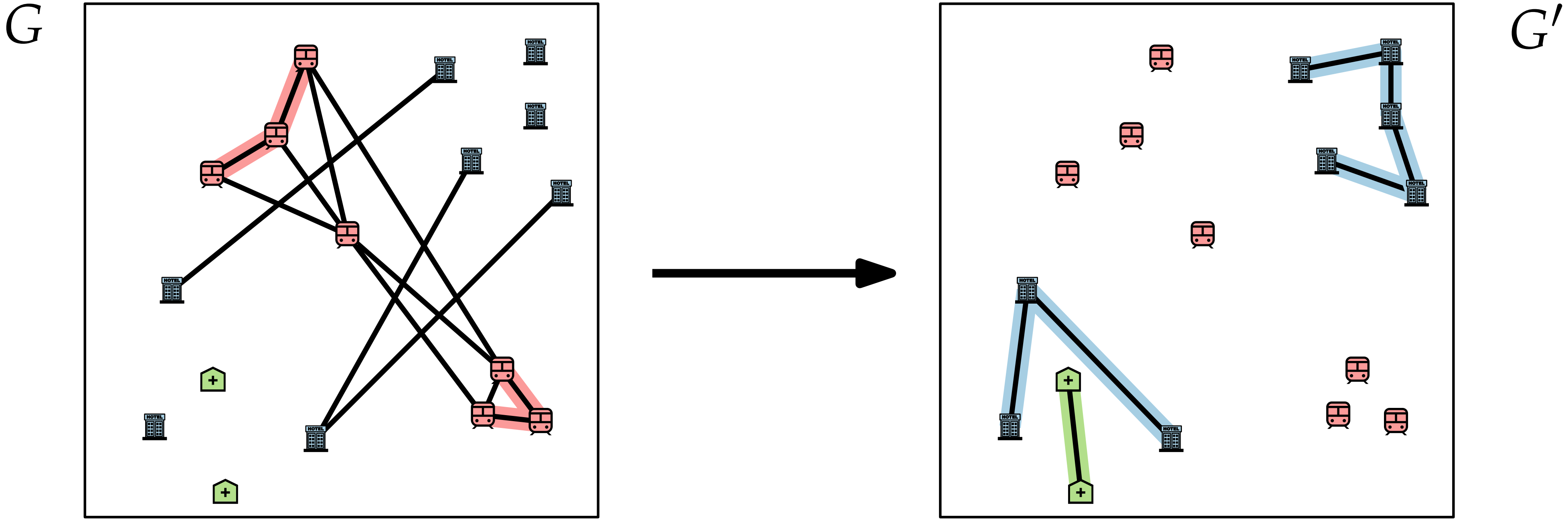

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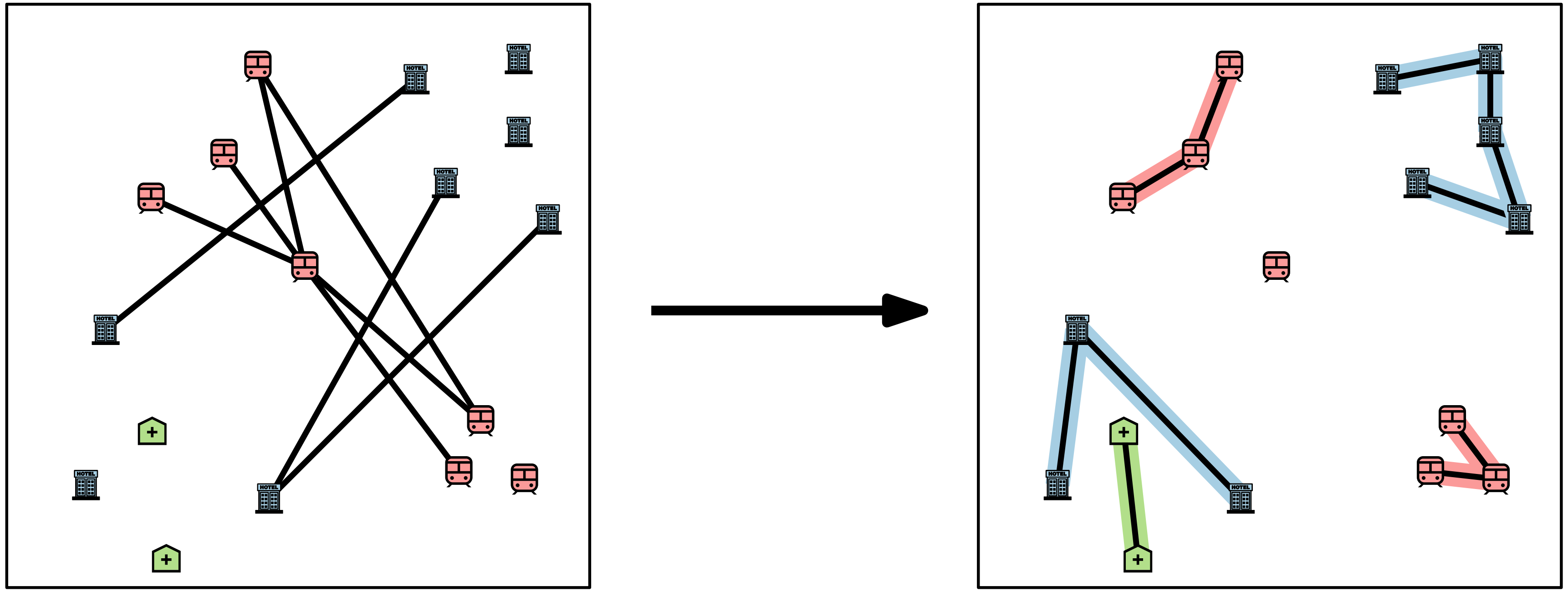

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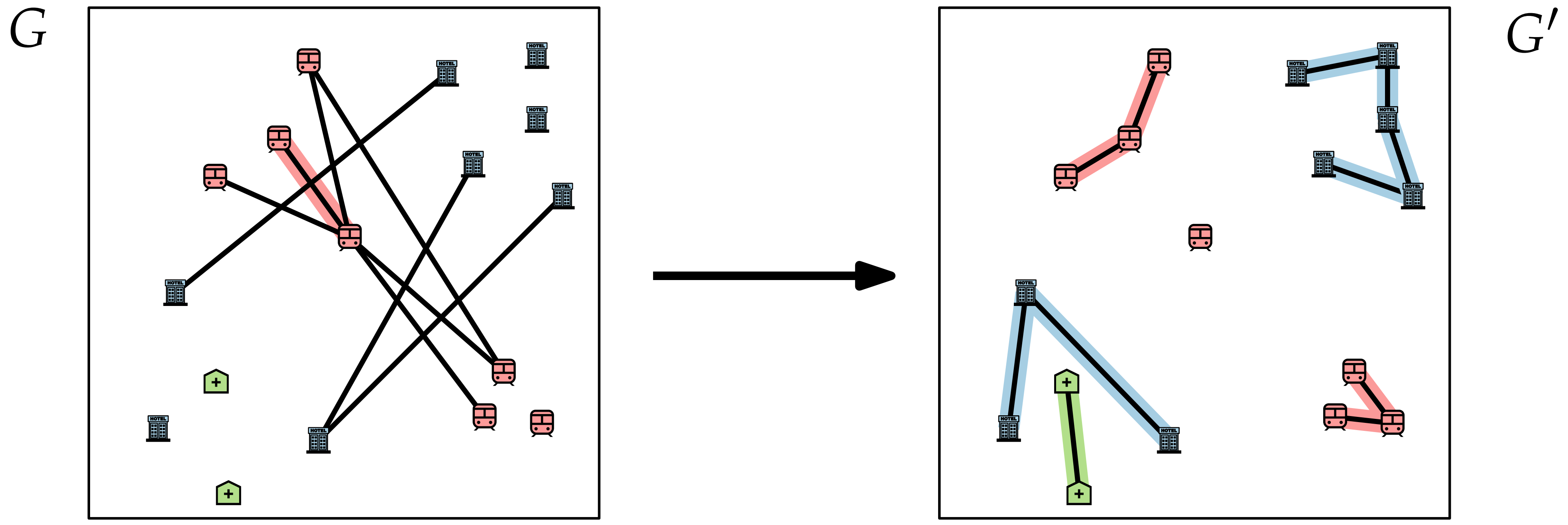

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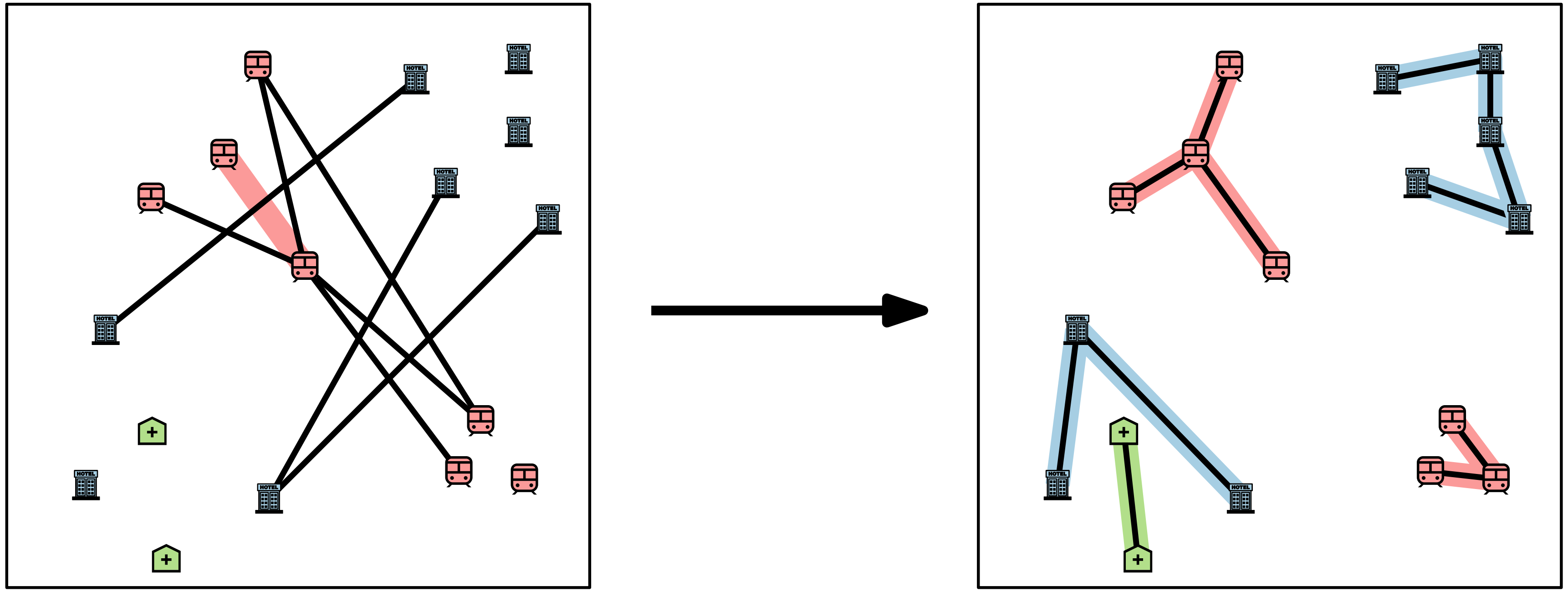

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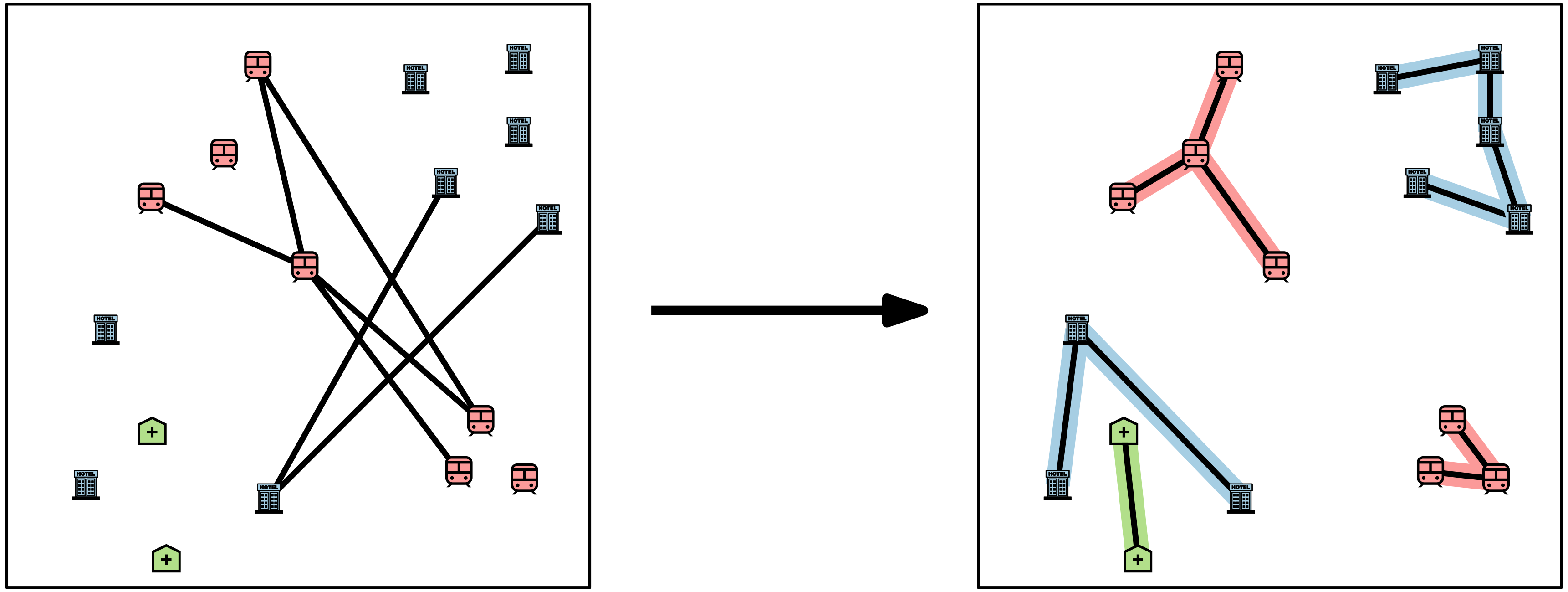

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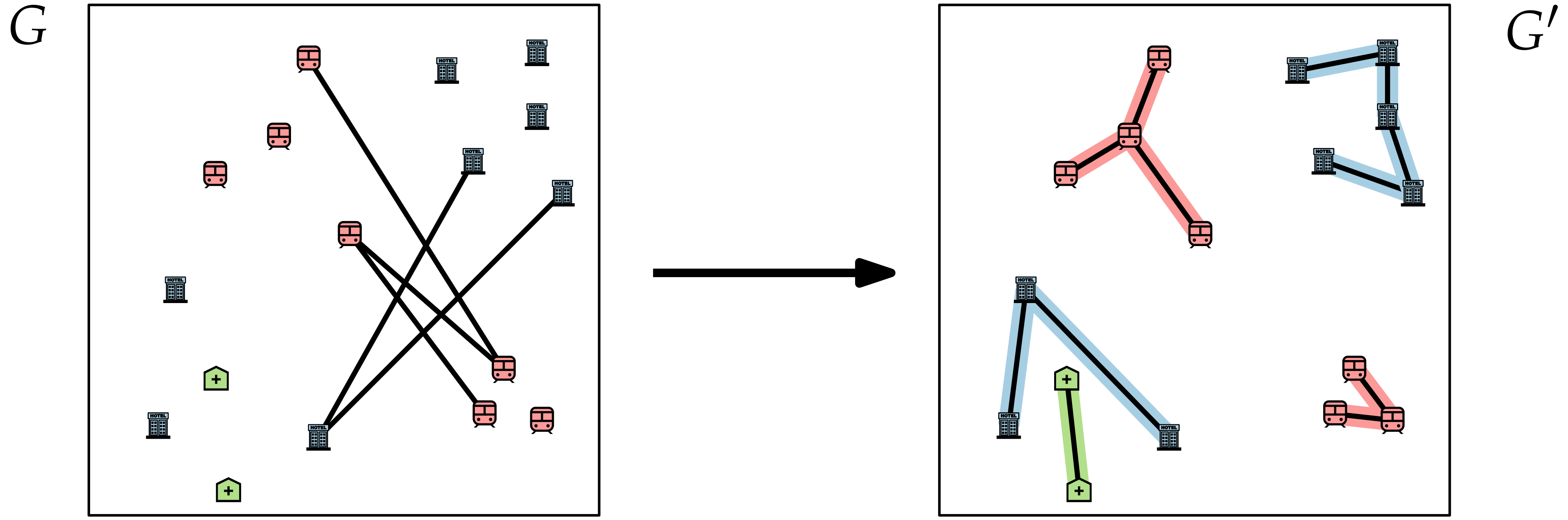

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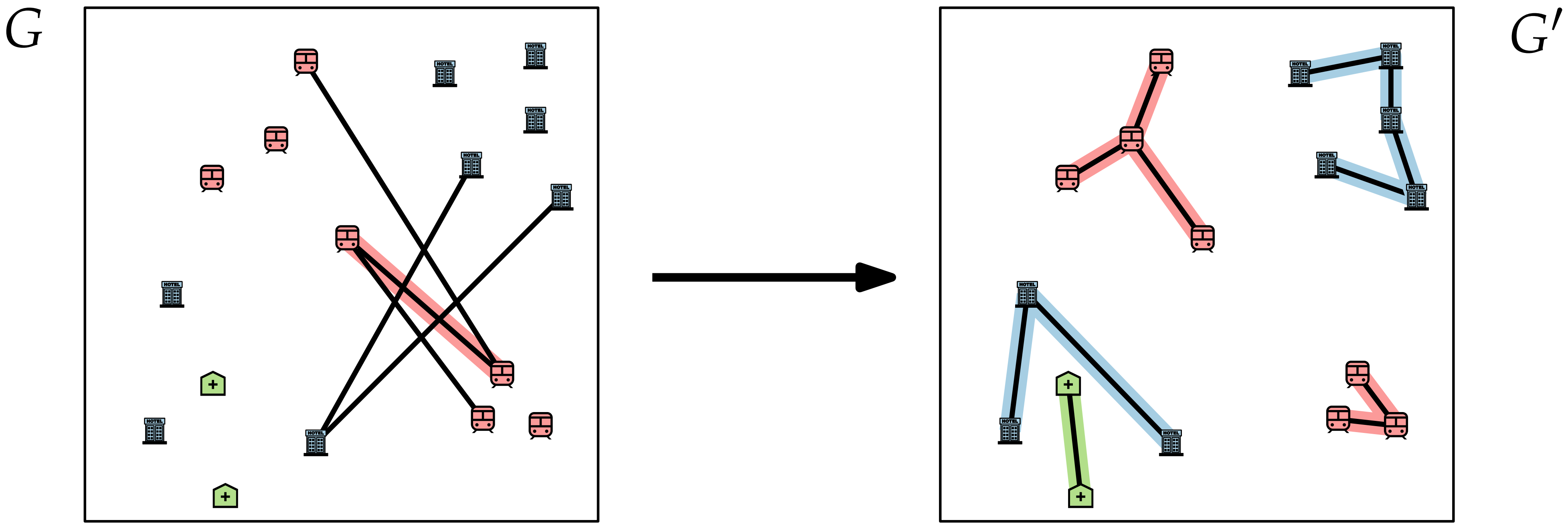

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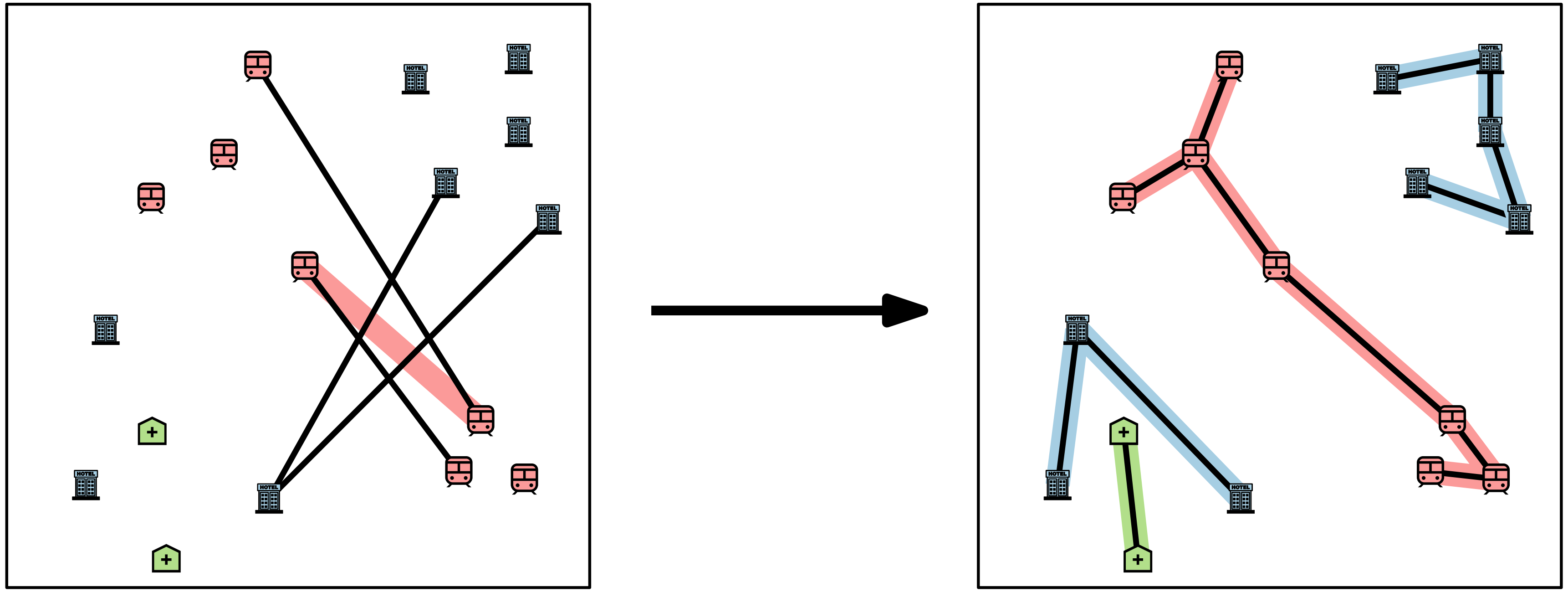

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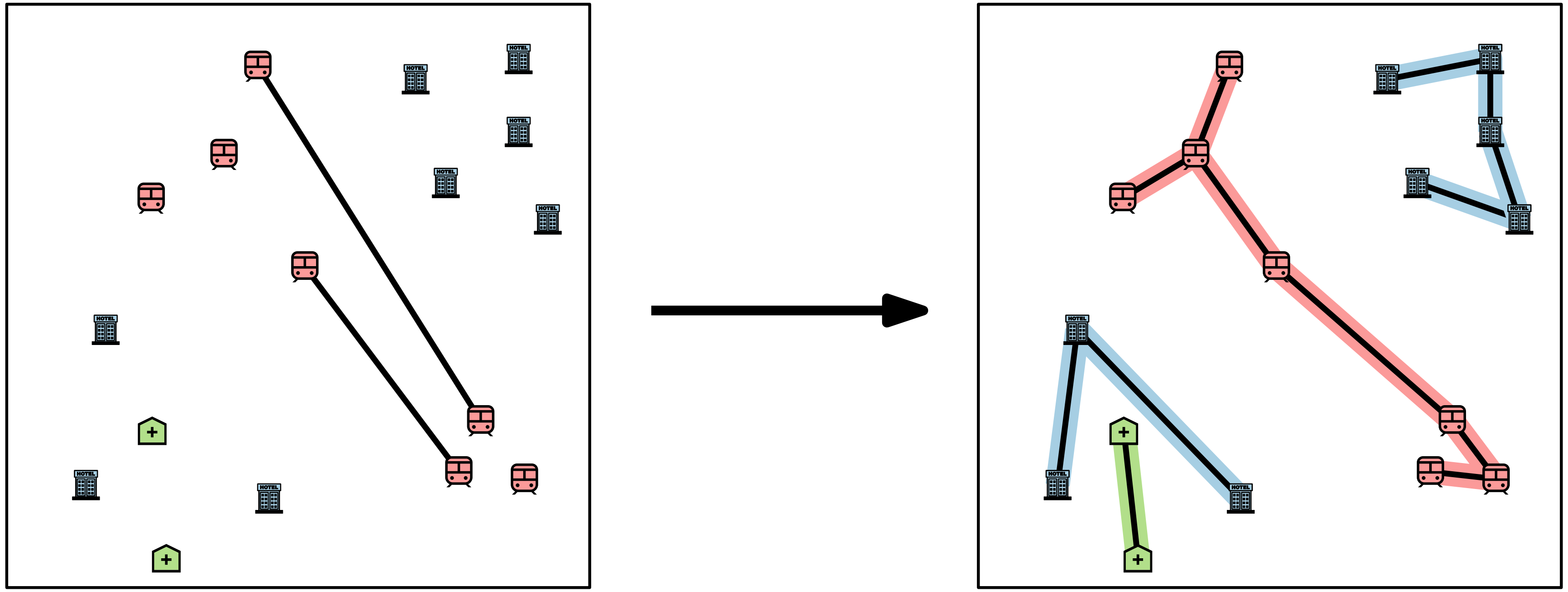

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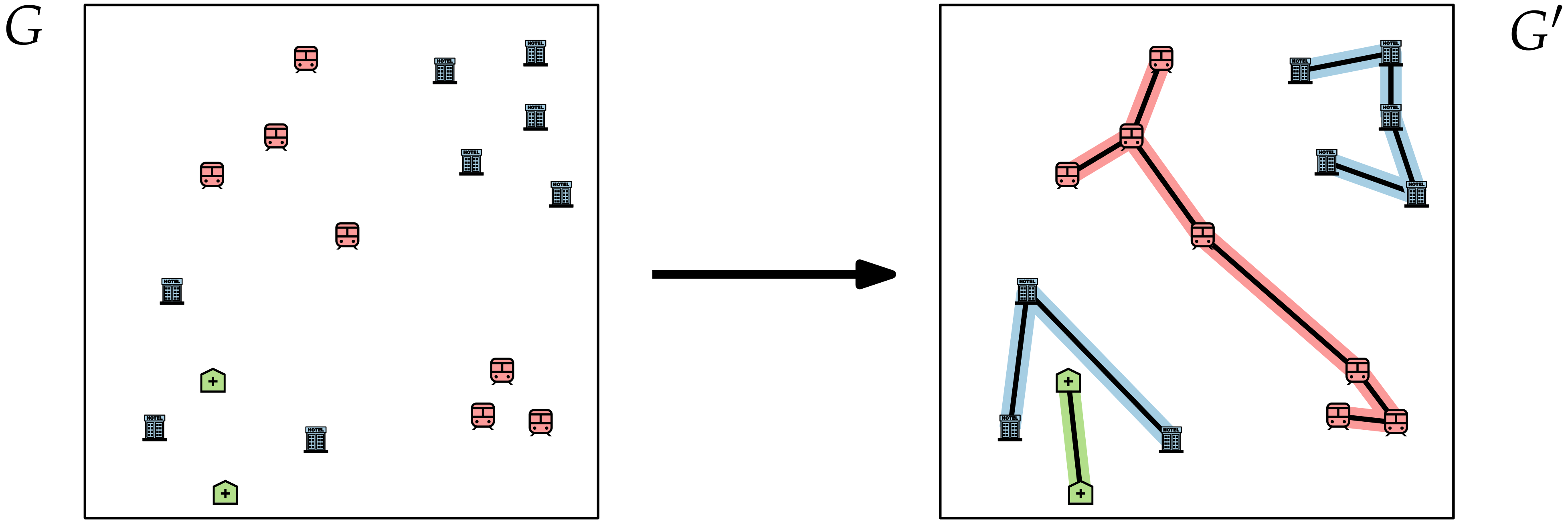
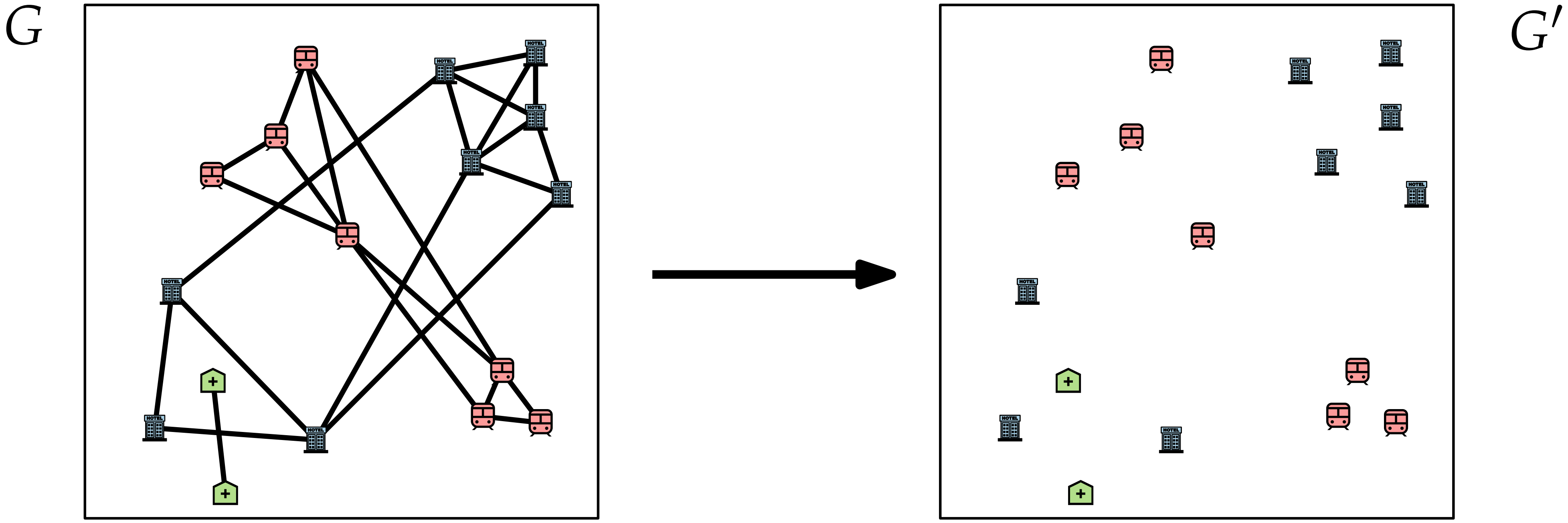
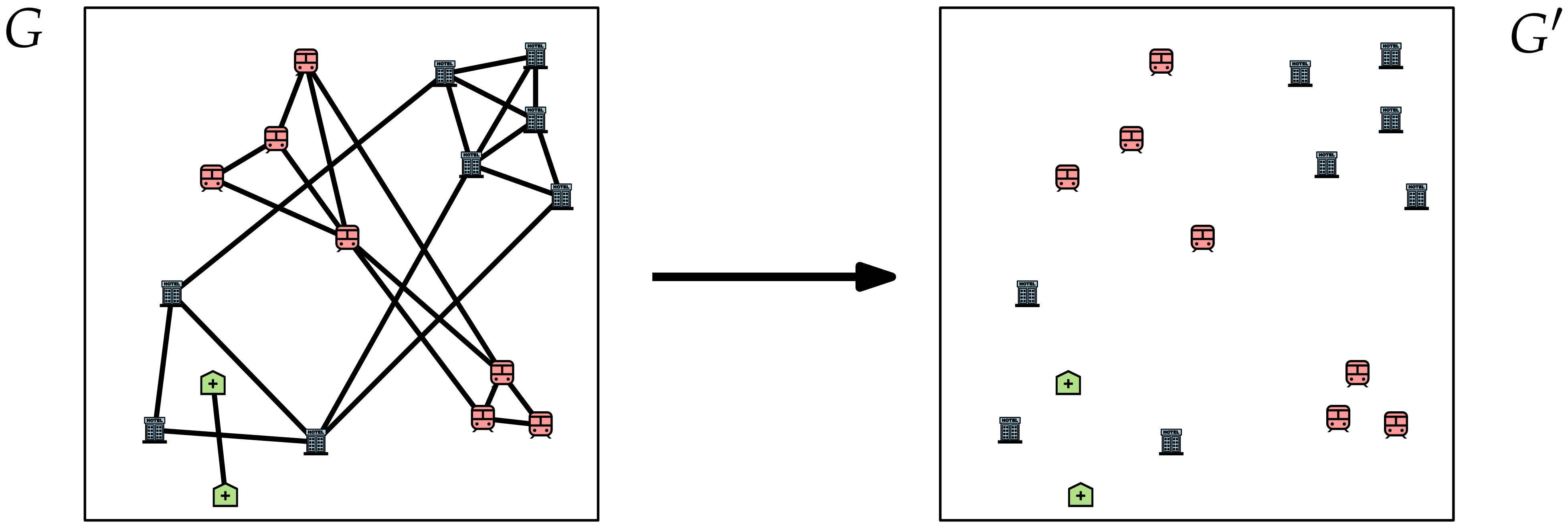


Planar Spanning Forest: Heuristics



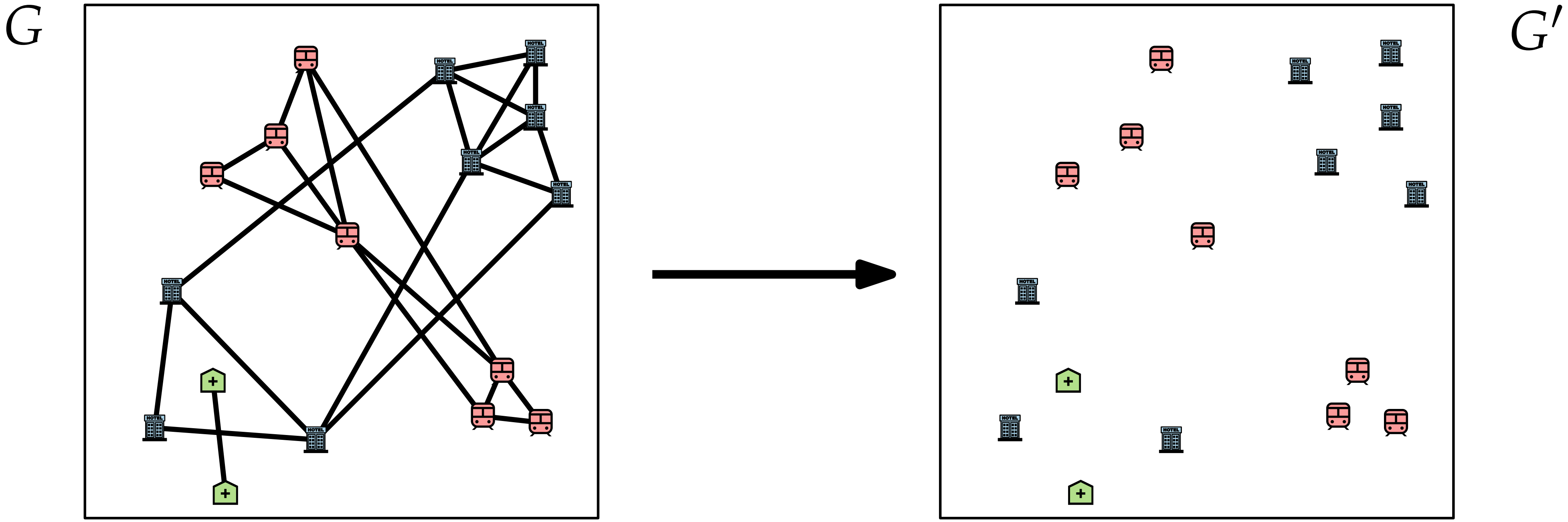
Planar Spanning Forest: Heuristics

REVERSEGREEDY:



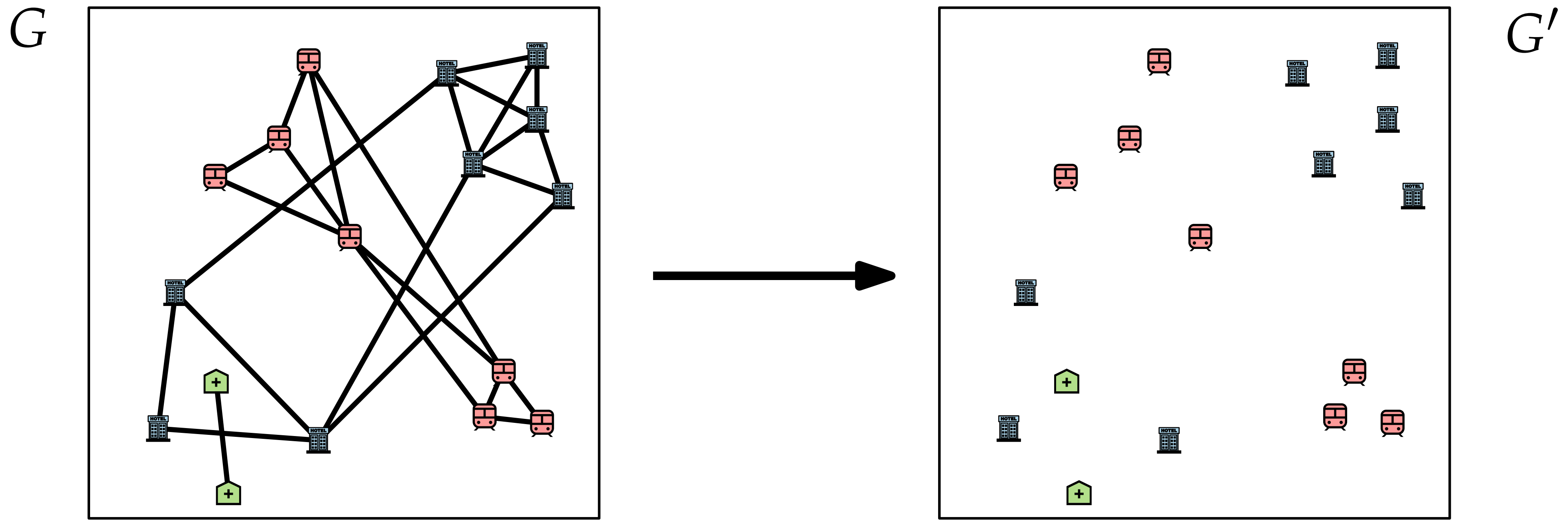
Planar Spanning Forest: Heuristics

ReverseGreedy: 1. Remove from G every edge that lies within a connected component of G'.



Planar Spanning Forest: Heuristics

ReverseGreedy:
1. Remove from G every edge that lies within a connected component of G'.
2. Move all non-crossed edges to G' (unless they close a cycle).



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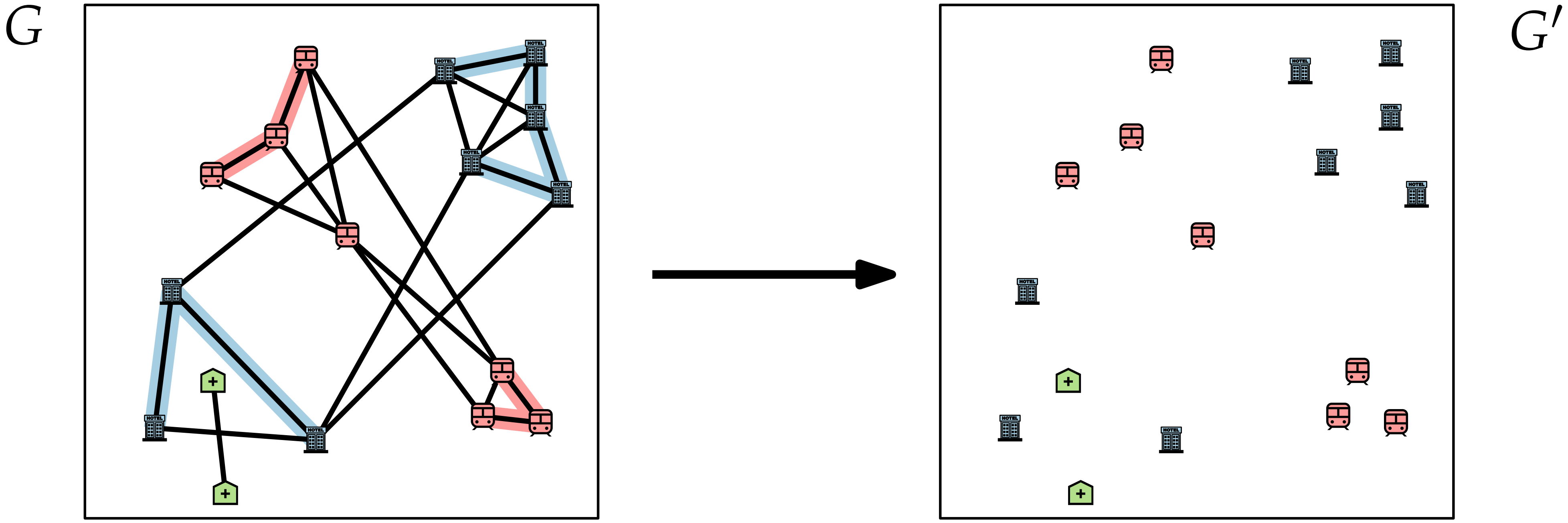

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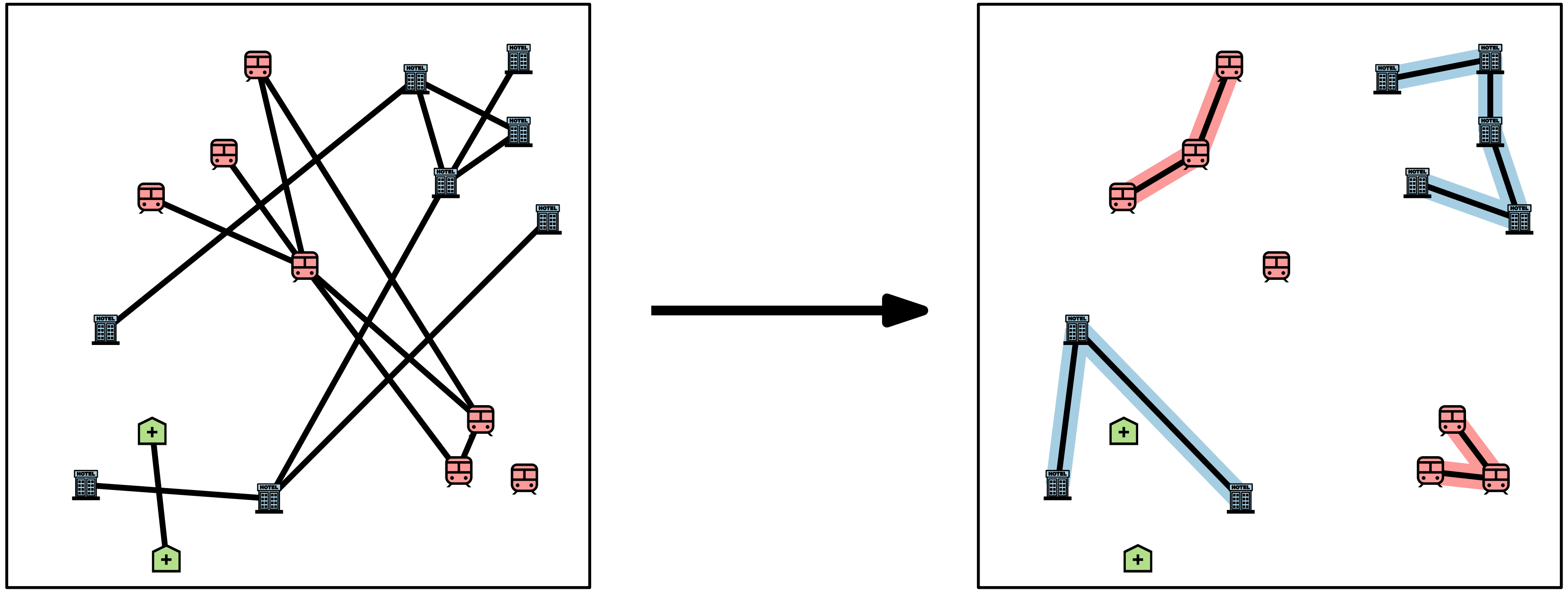

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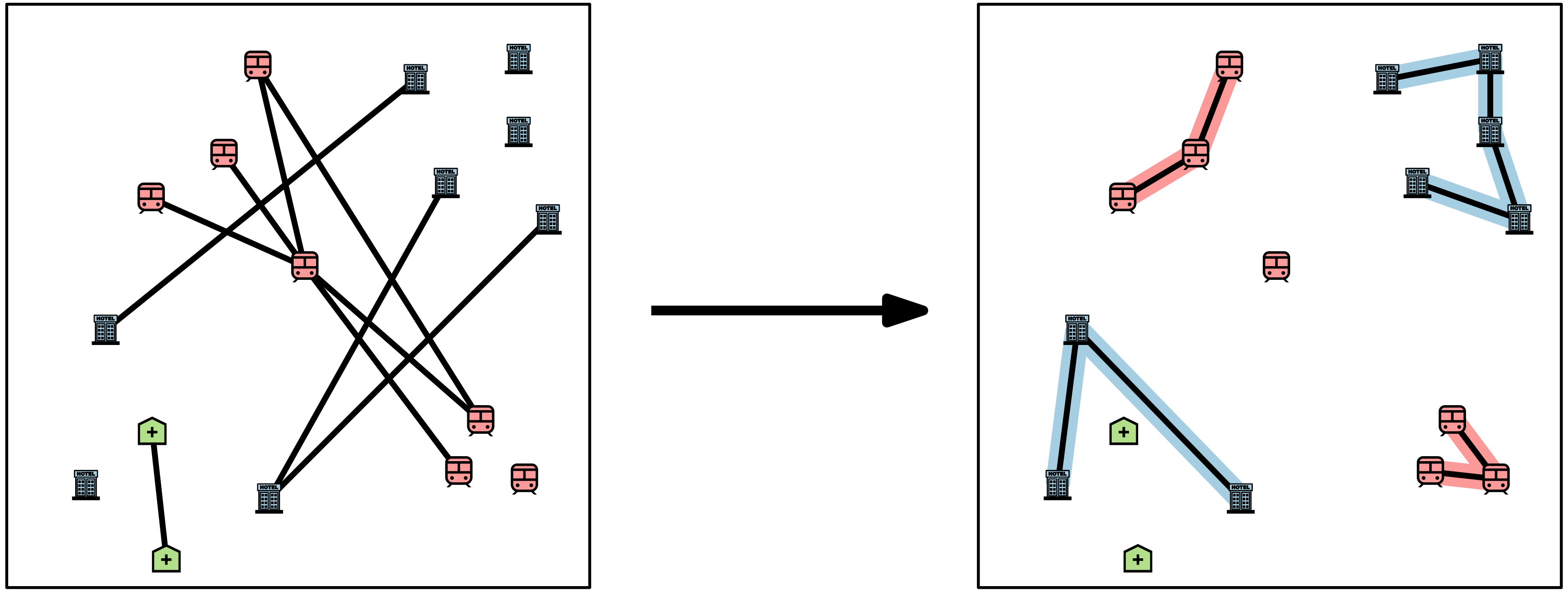

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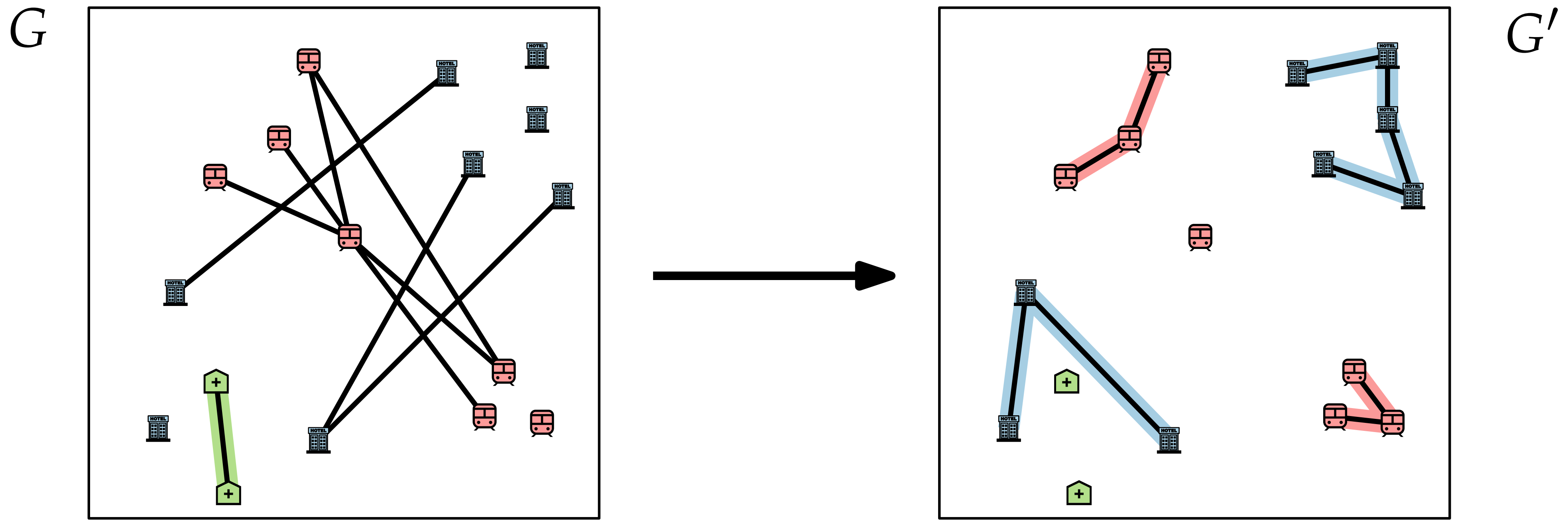

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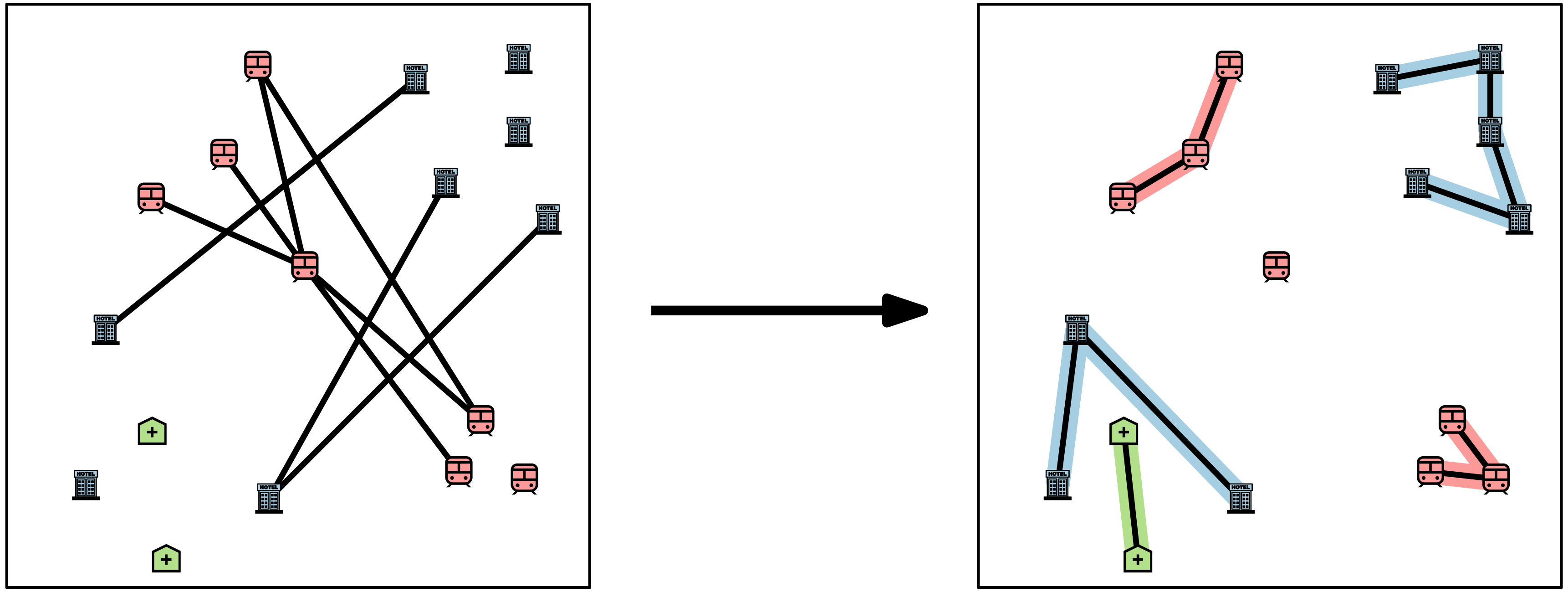

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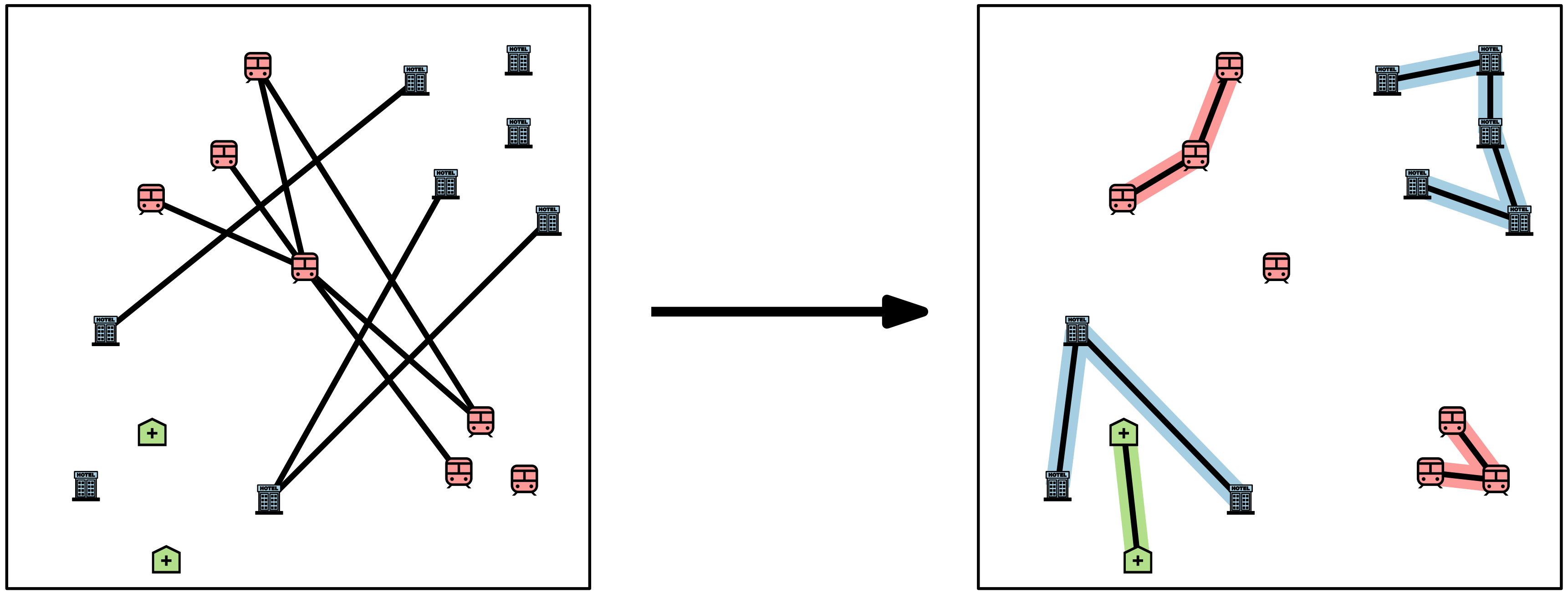

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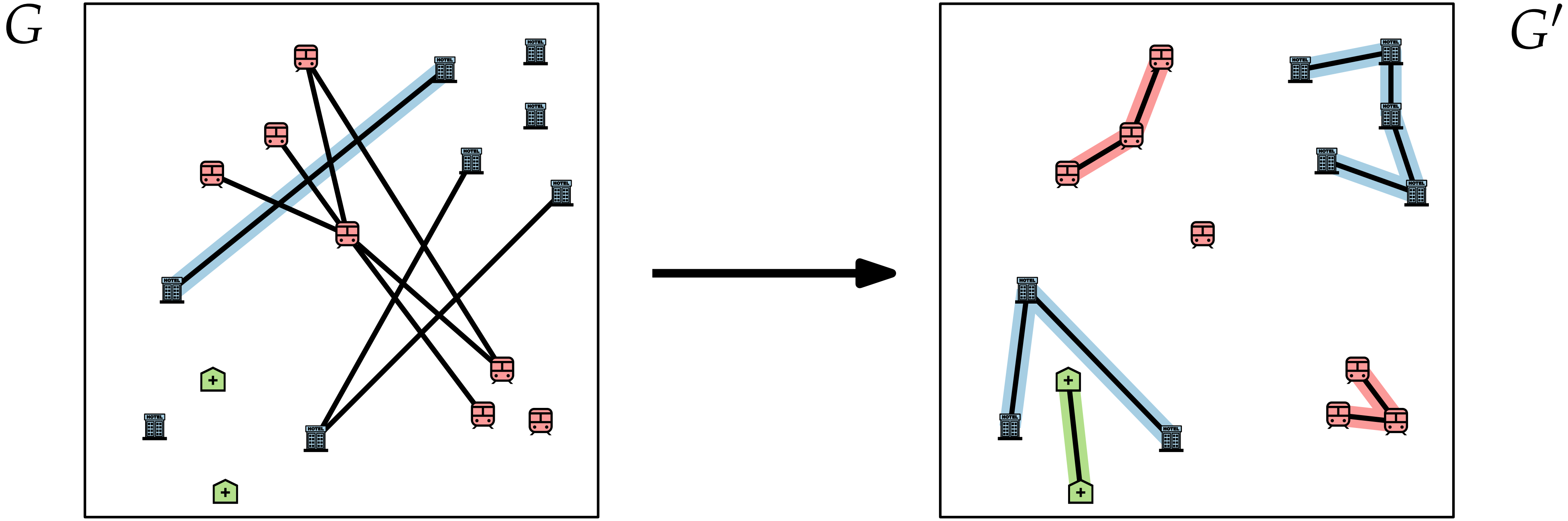

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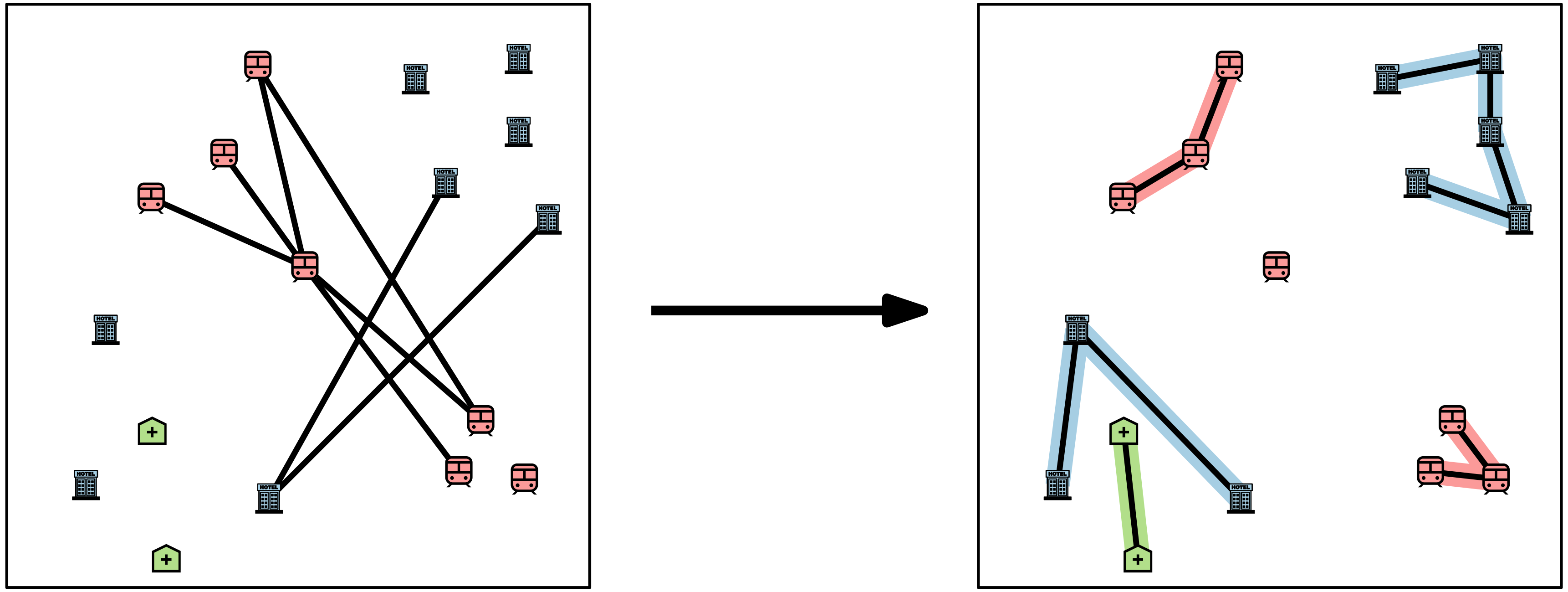

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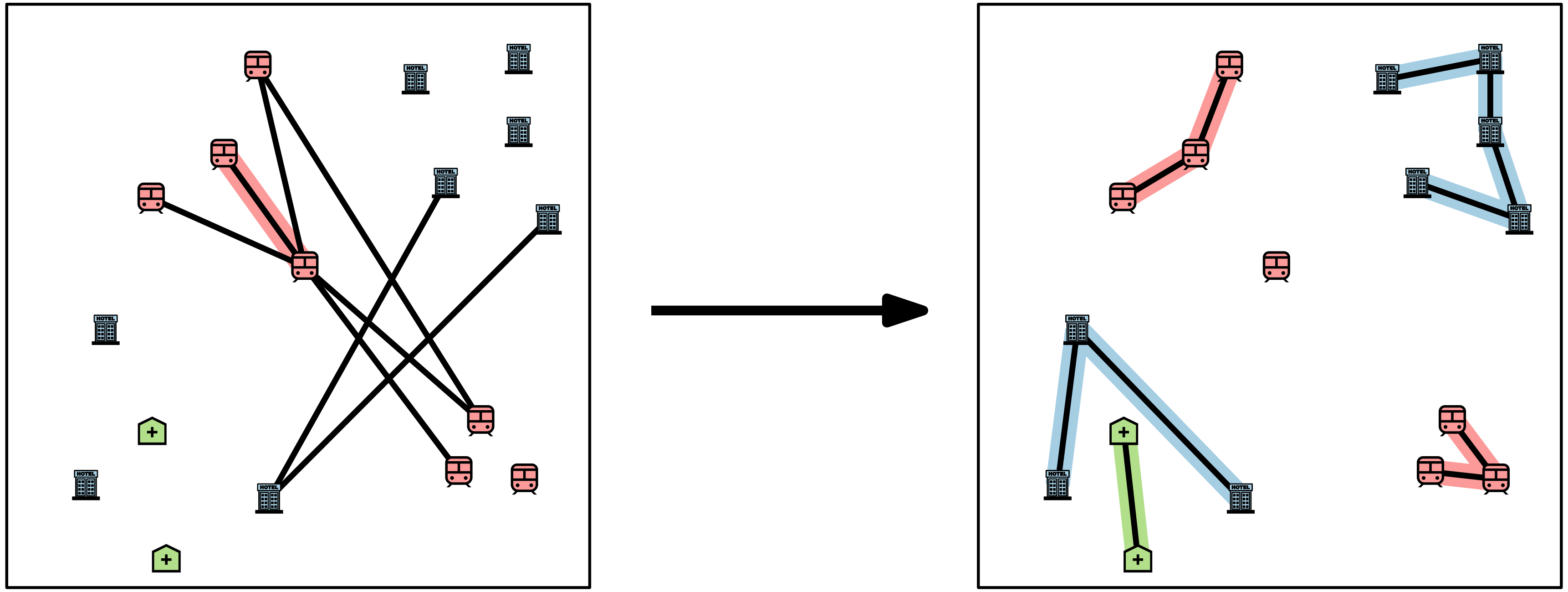

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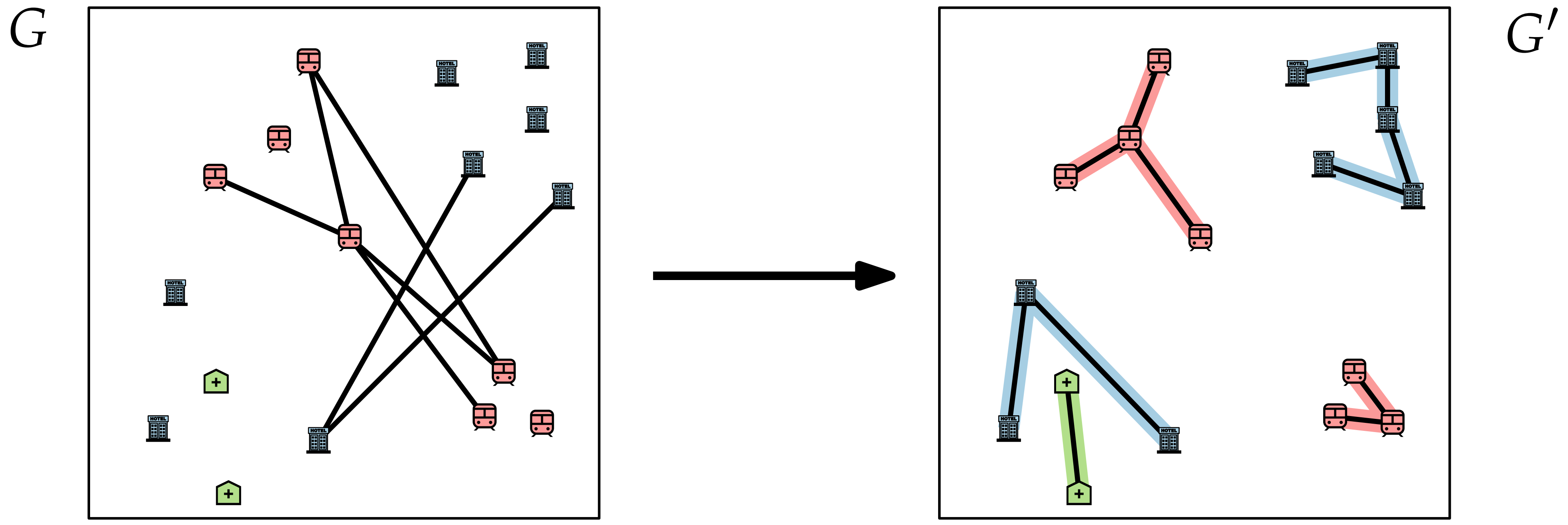

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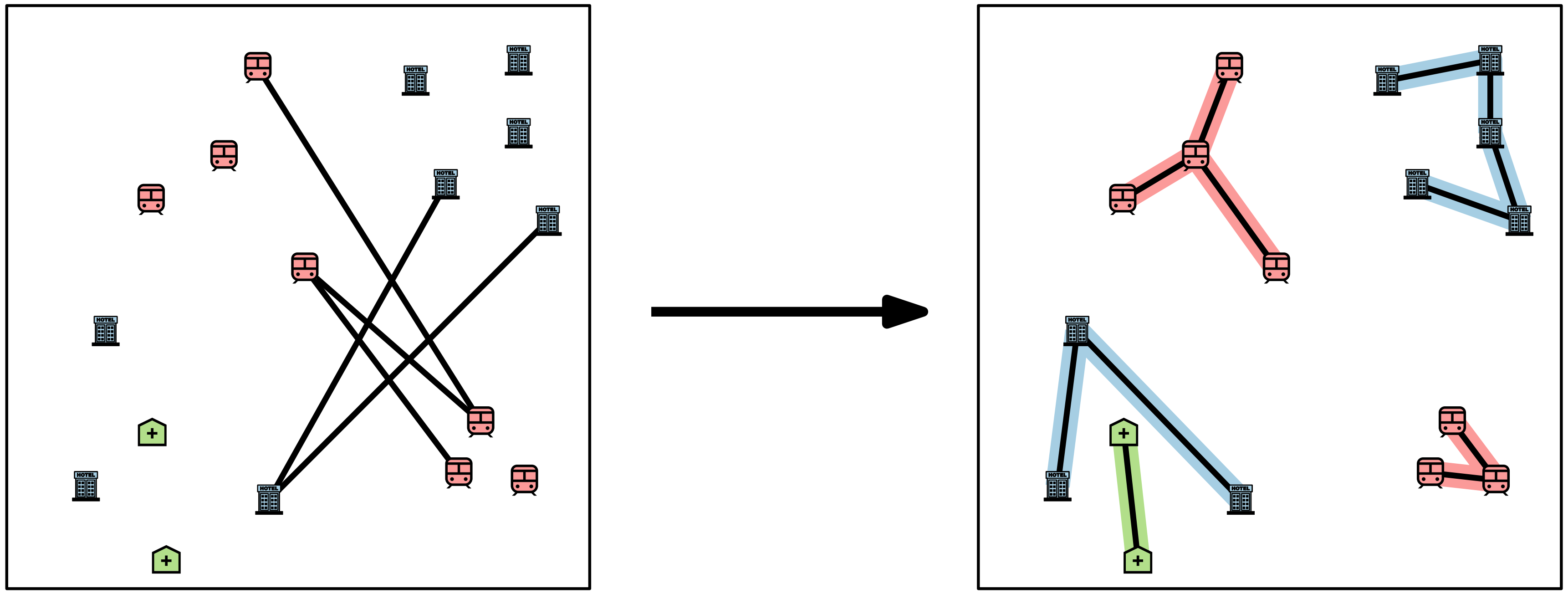

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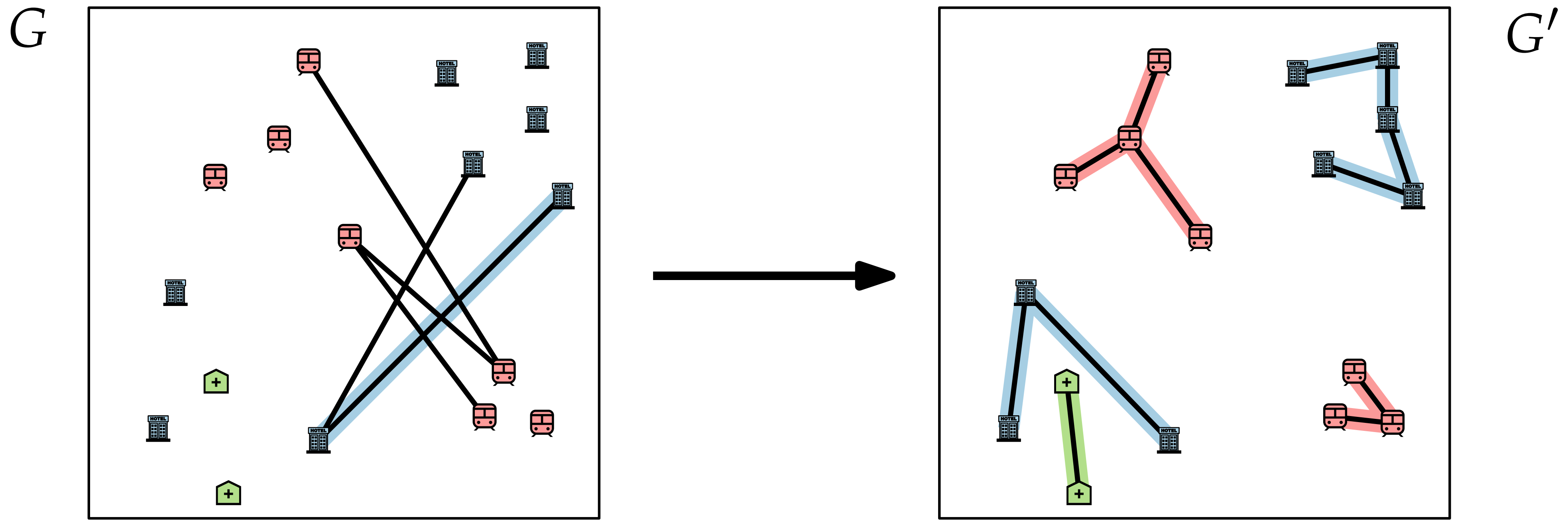
3. Remove an edge that is crossed by max. number of edges



Planar Spanning Forest: Heuristics

ReverseGreedy:

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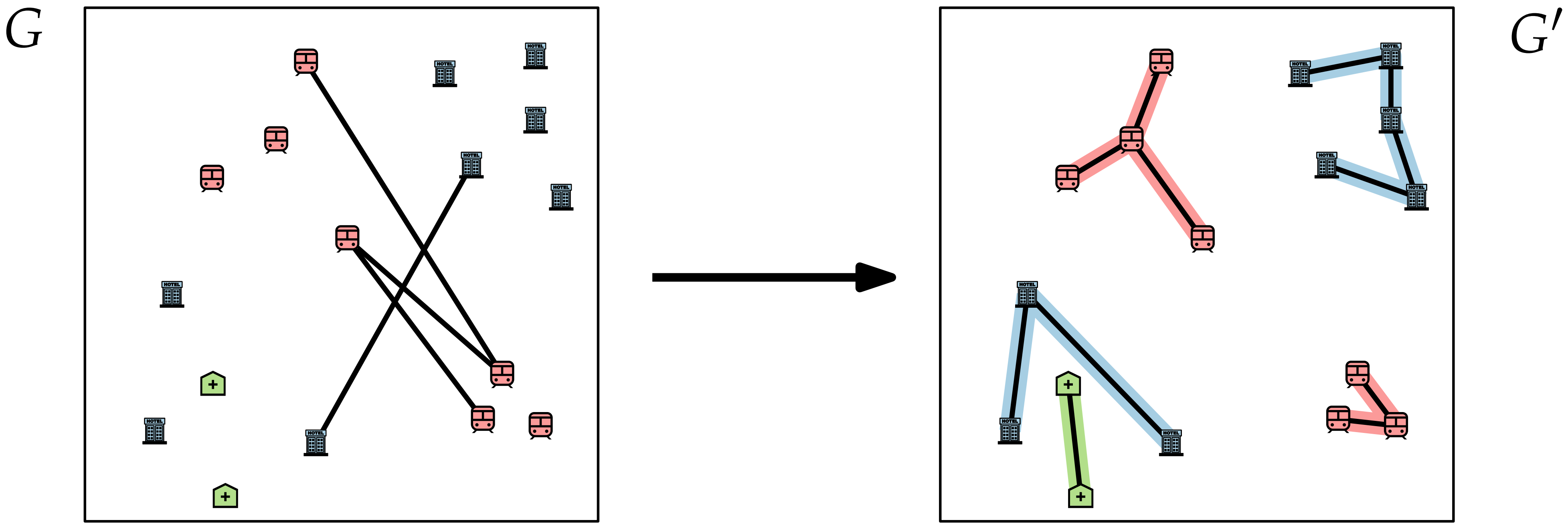

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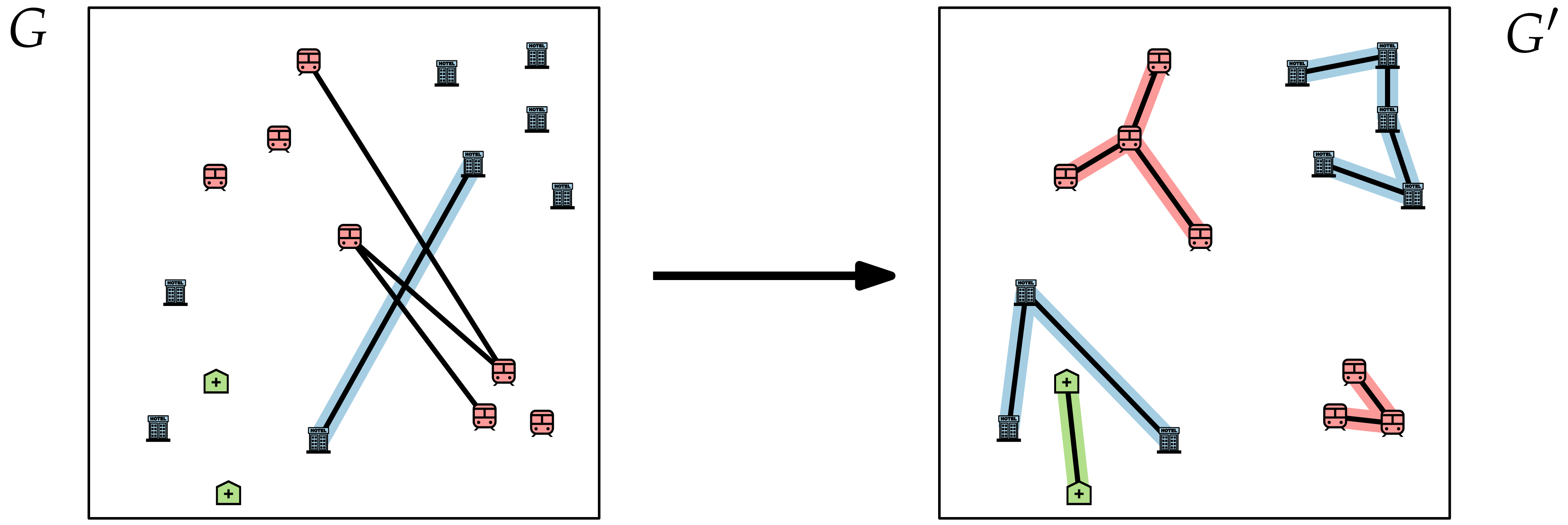

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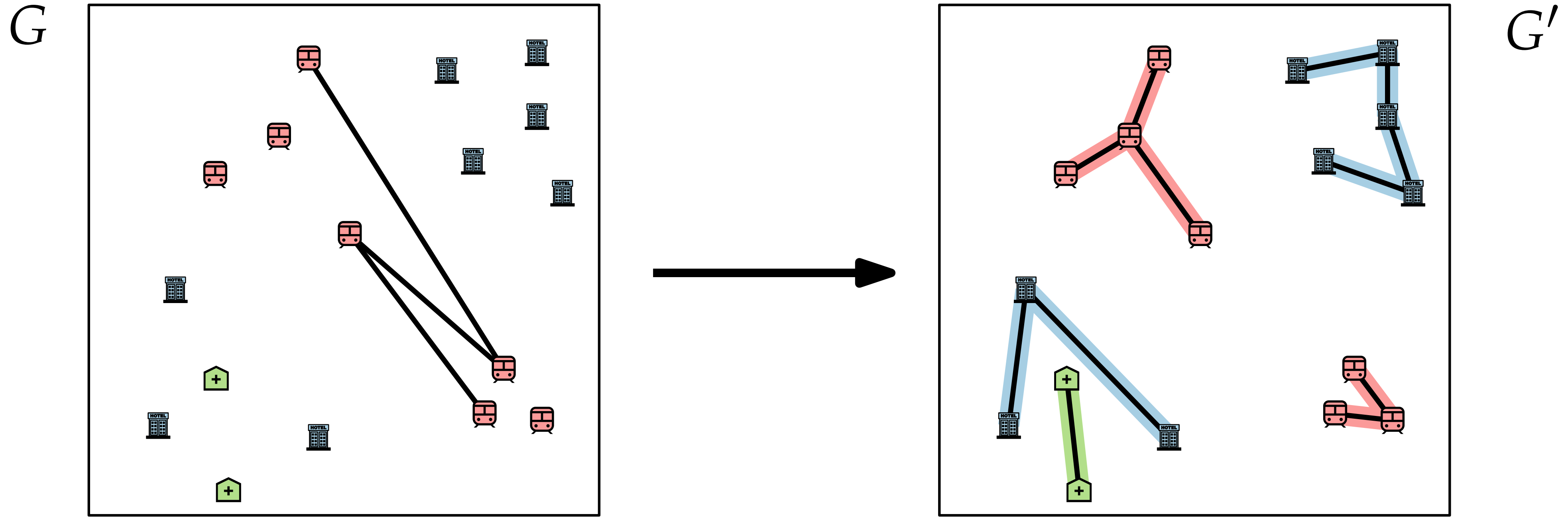

Planar Spanning Forest: Heuristics

REVERSEGREEDY:

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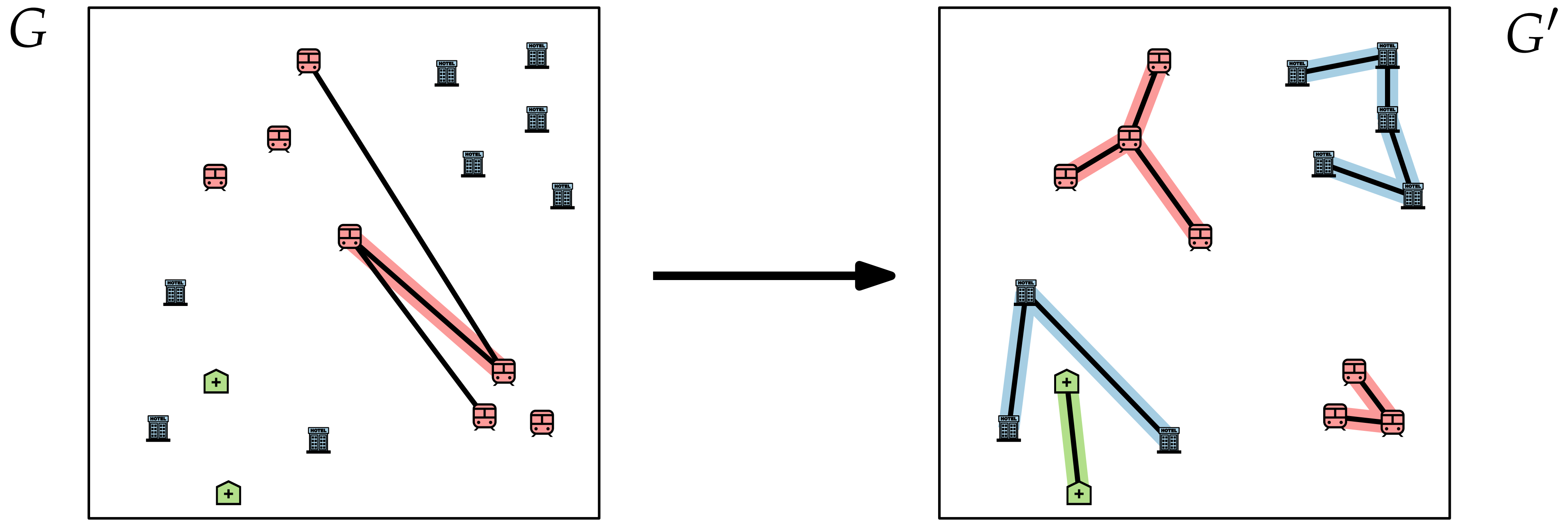

Planar Spanning Forest: Heuristics

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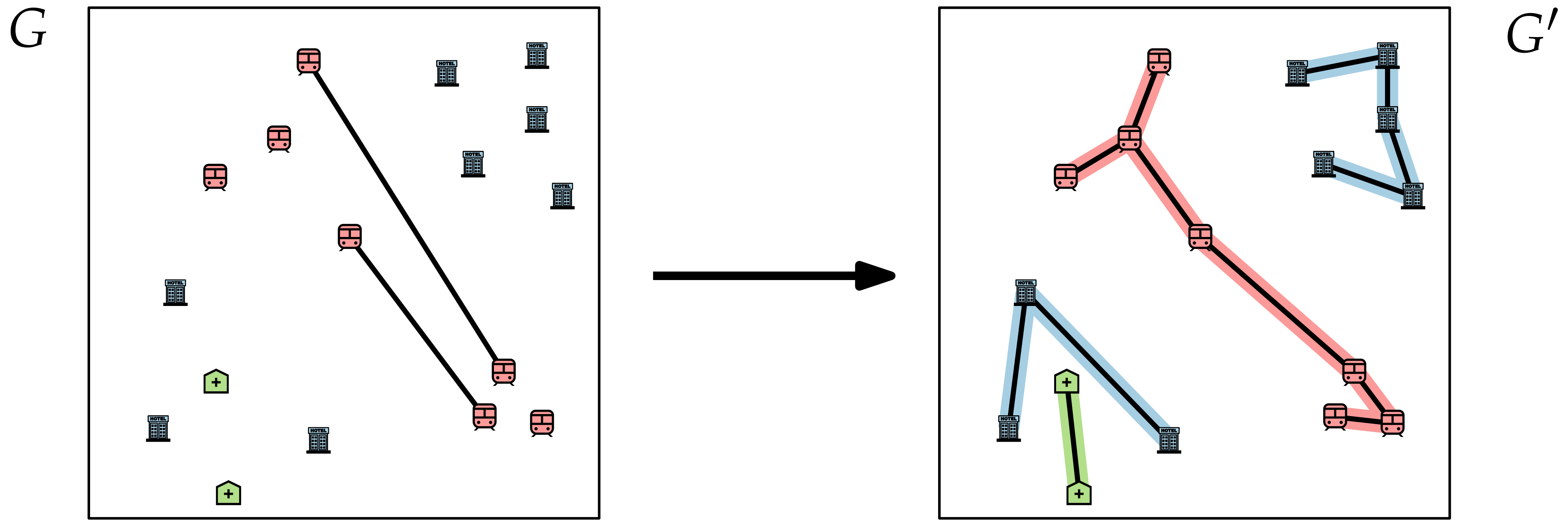

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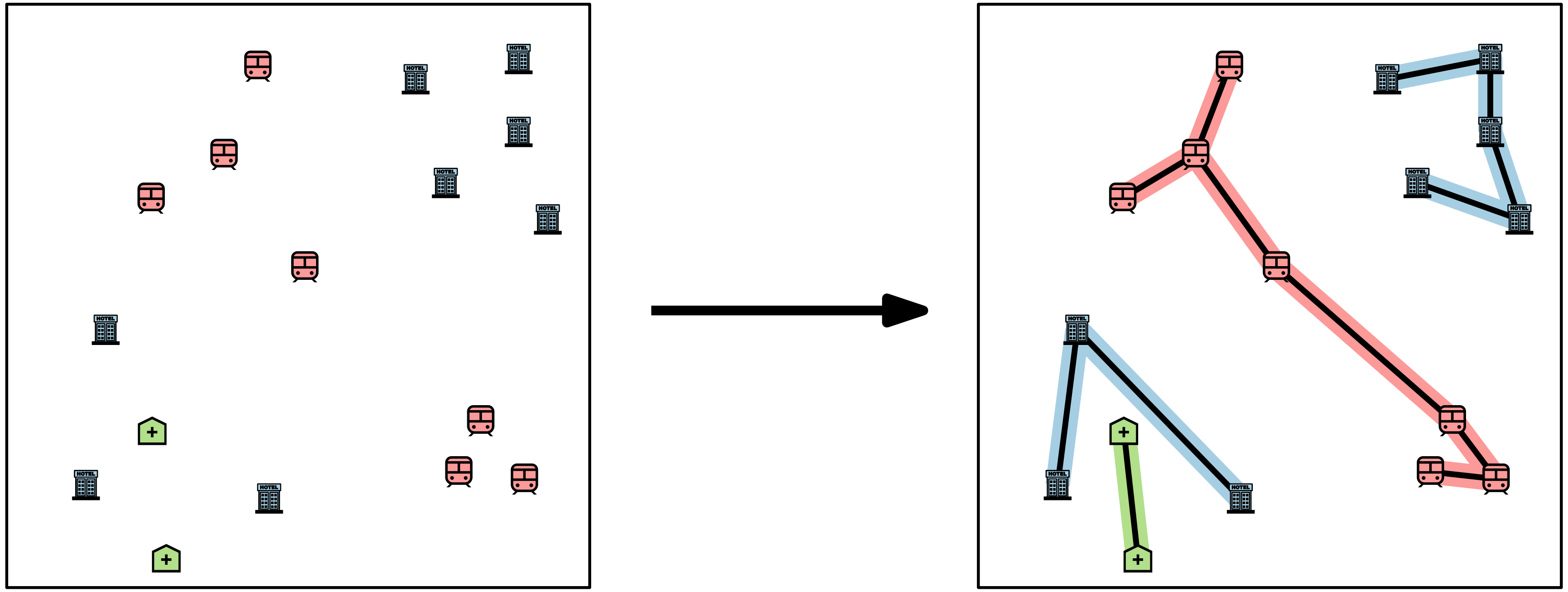
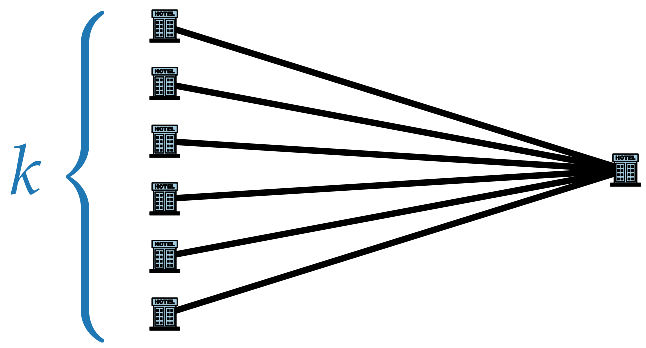
G



G'

Bad Examples



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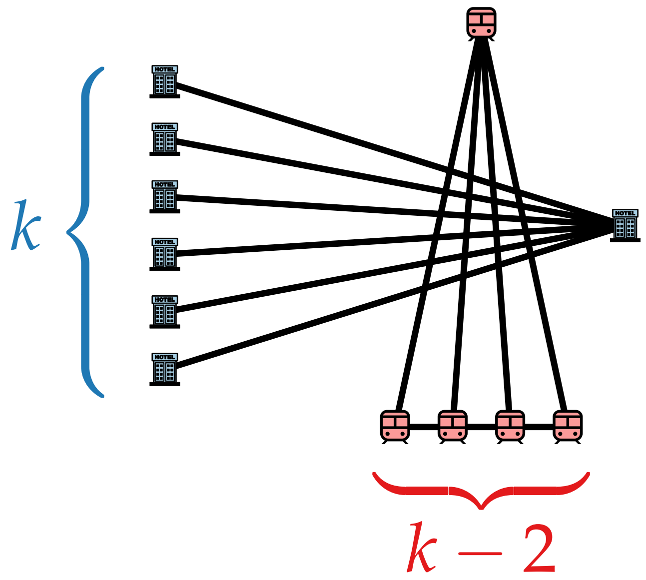

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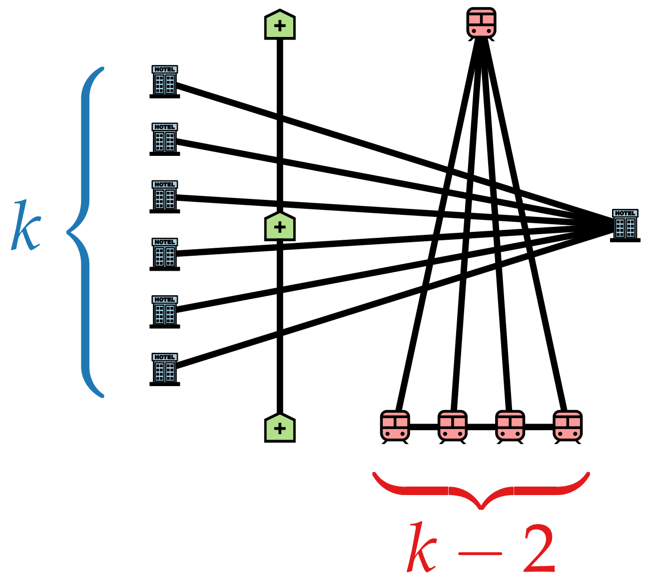

Bad Examples

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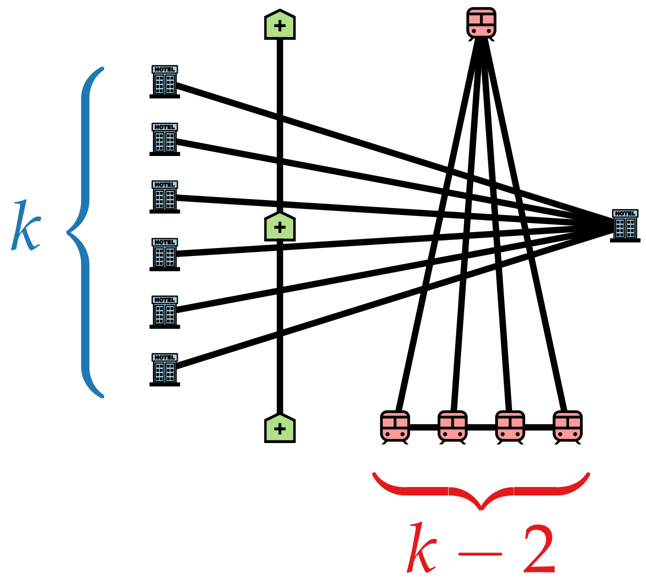

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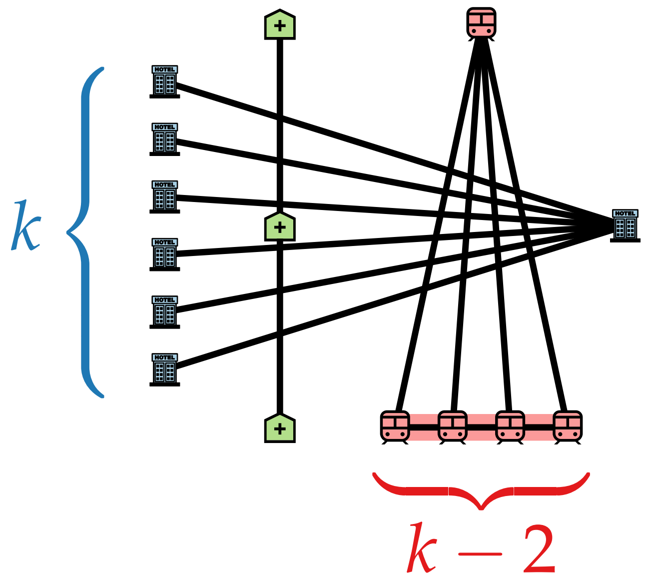

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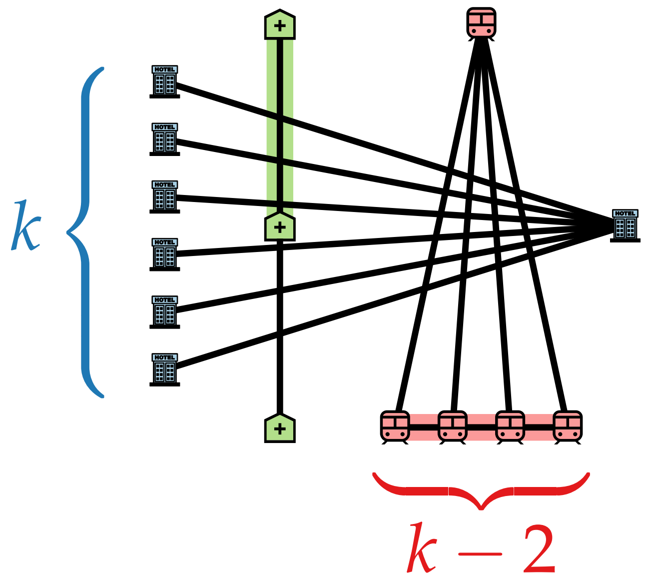

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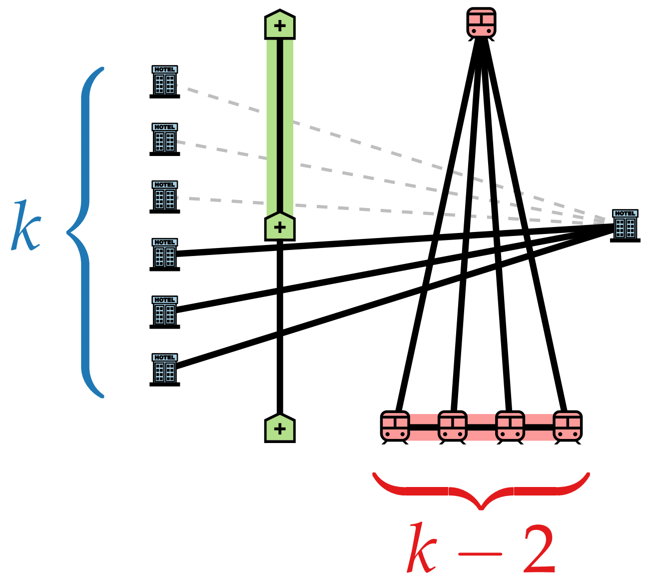

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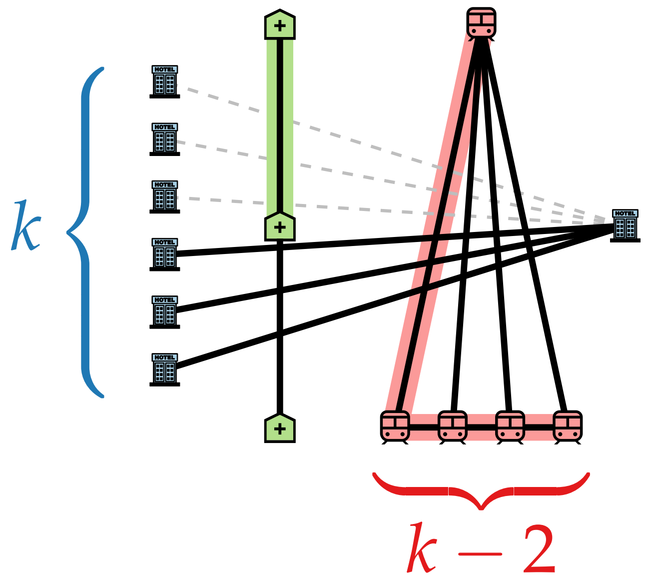

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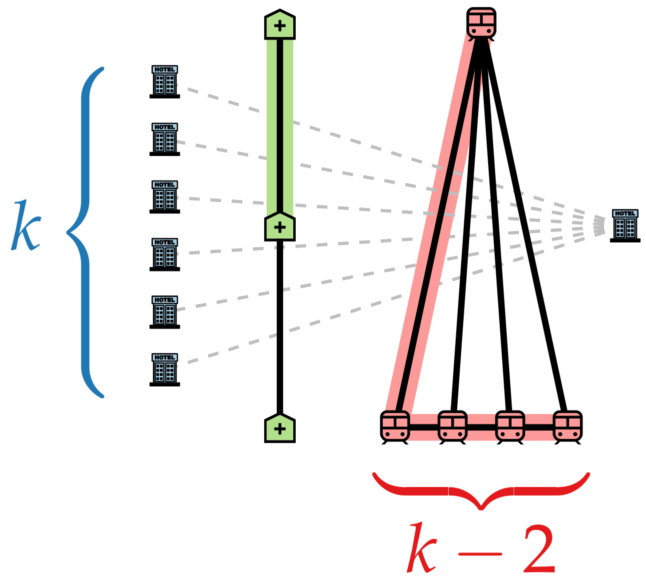

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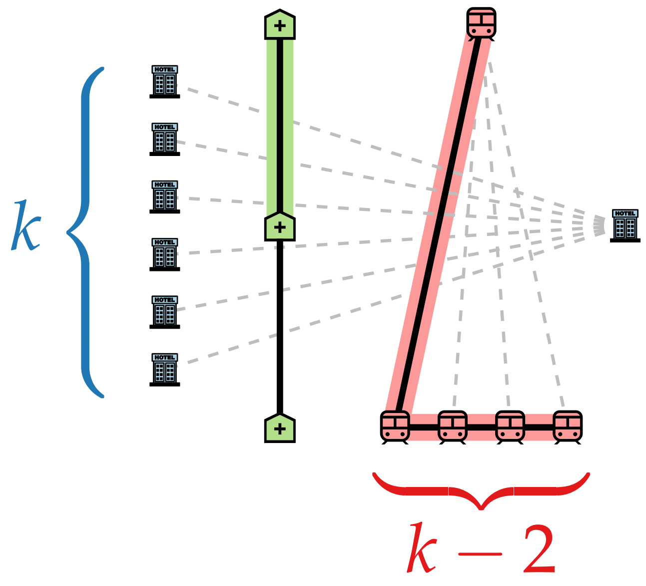

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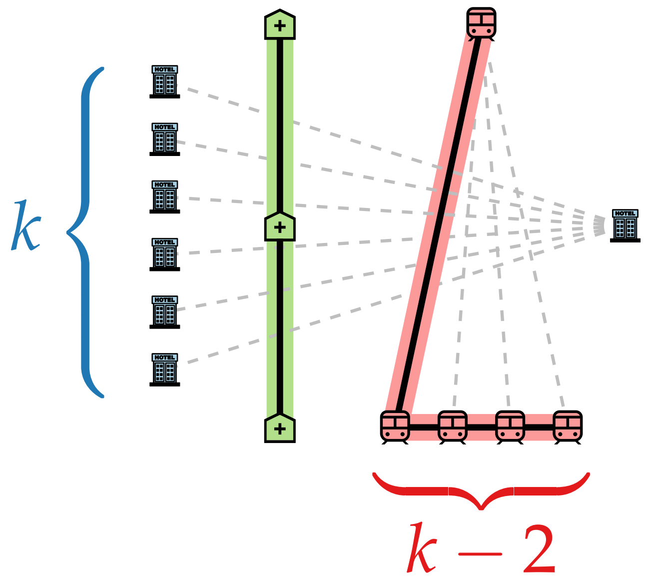

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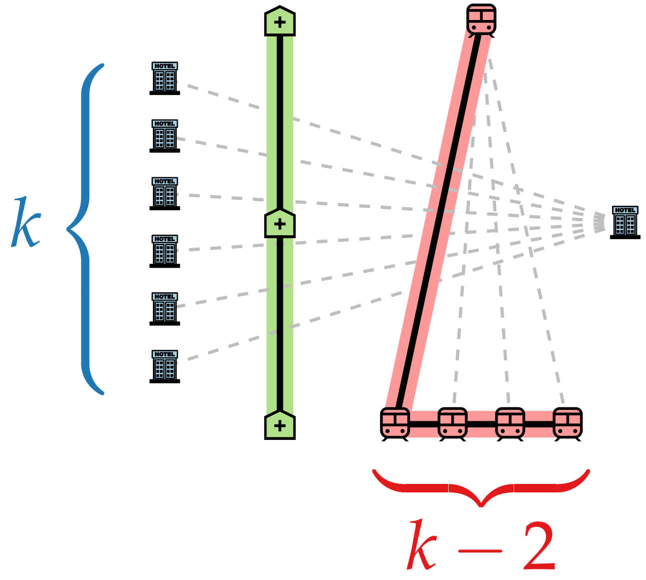

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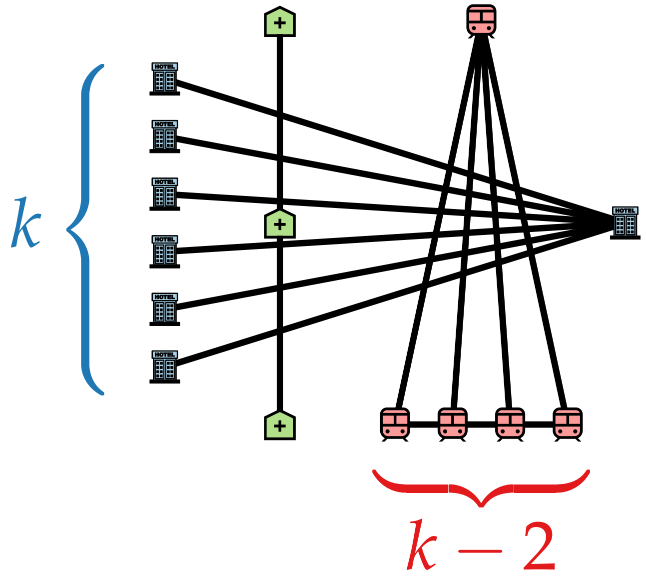

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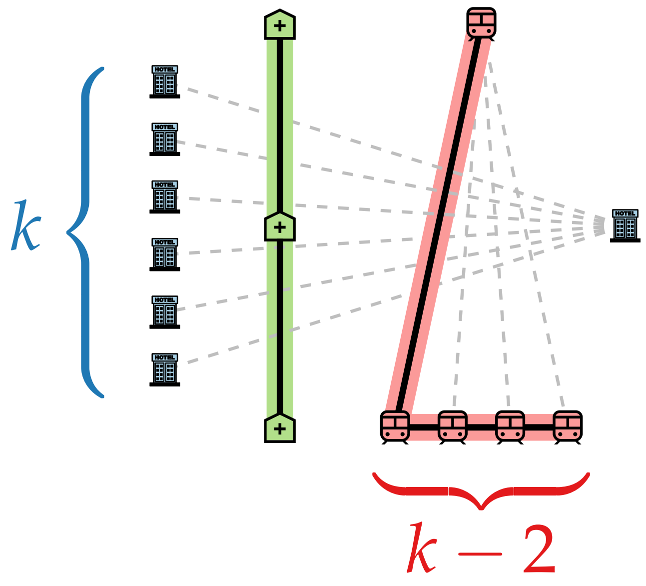

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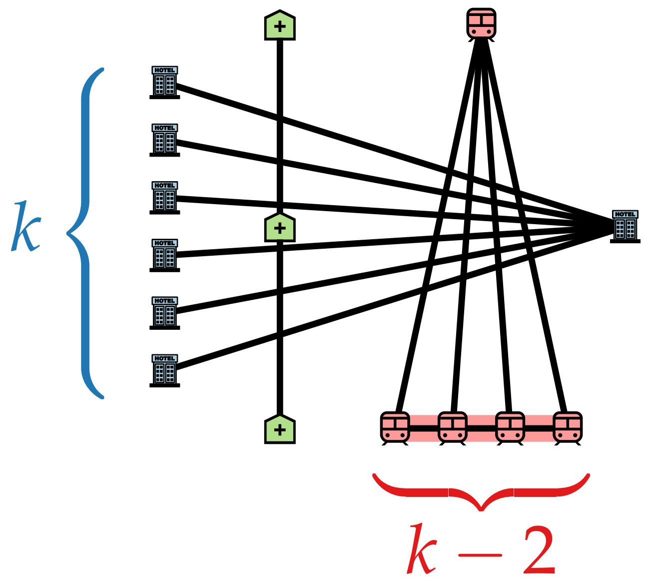

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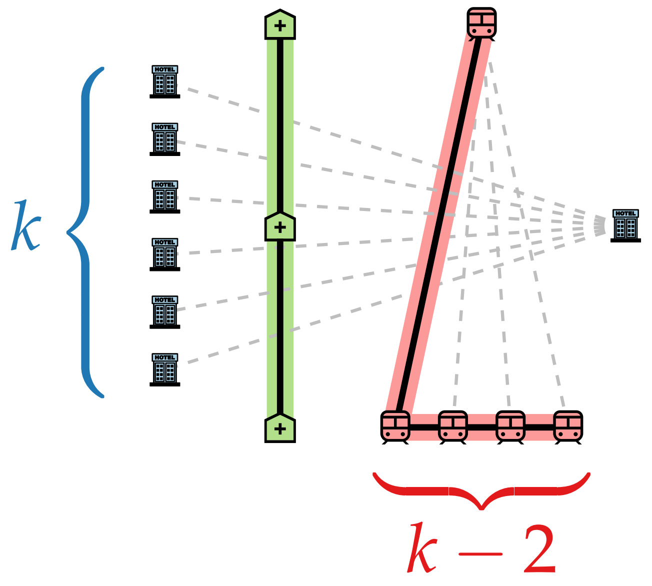

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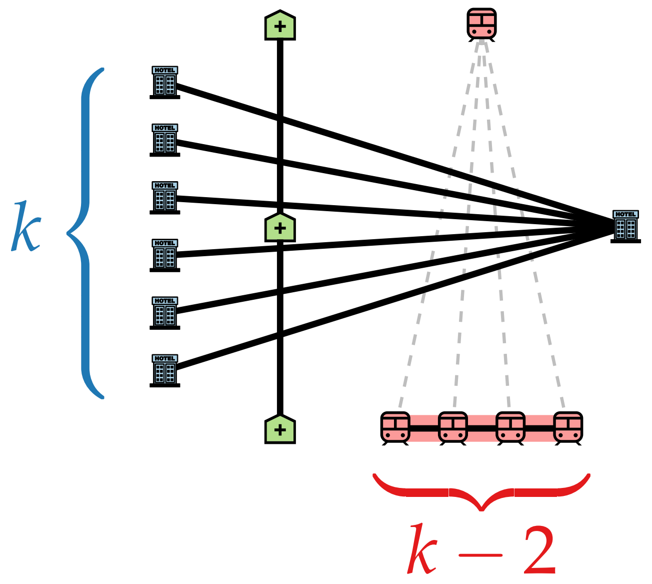

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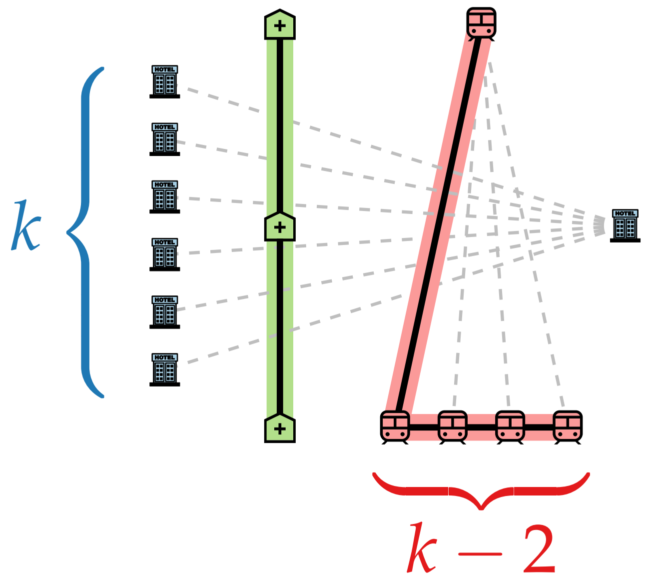
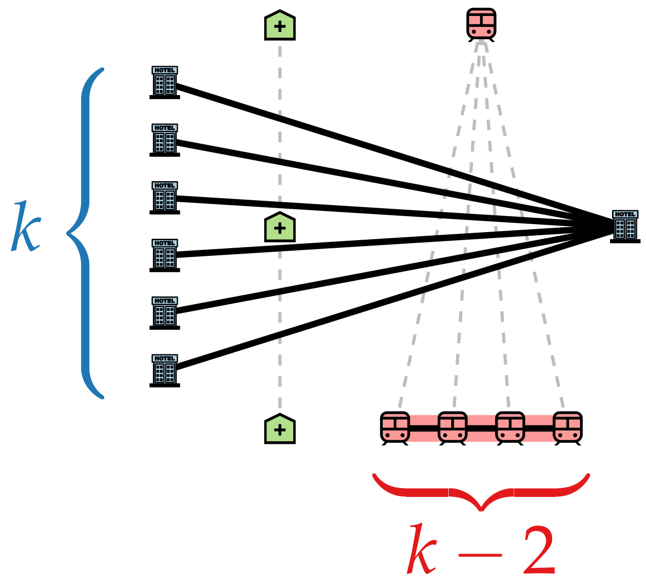


REVERSEGREEDY:



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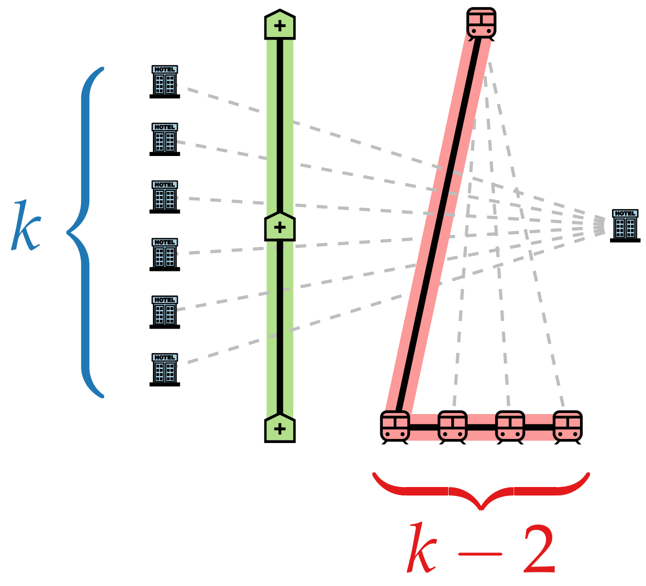

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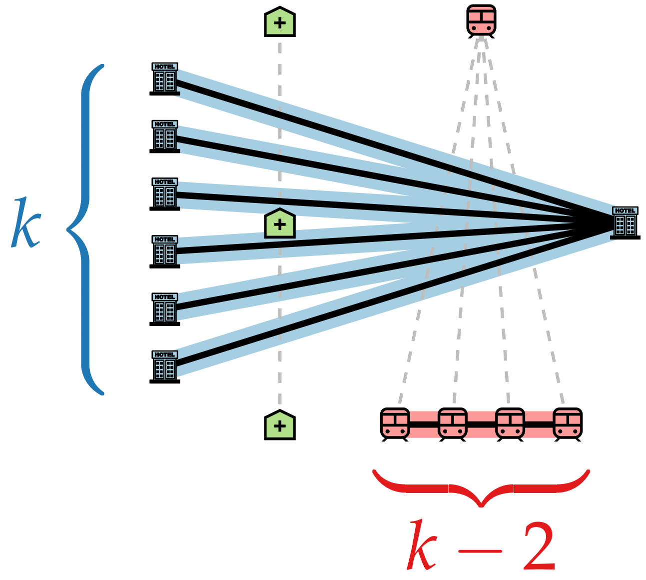

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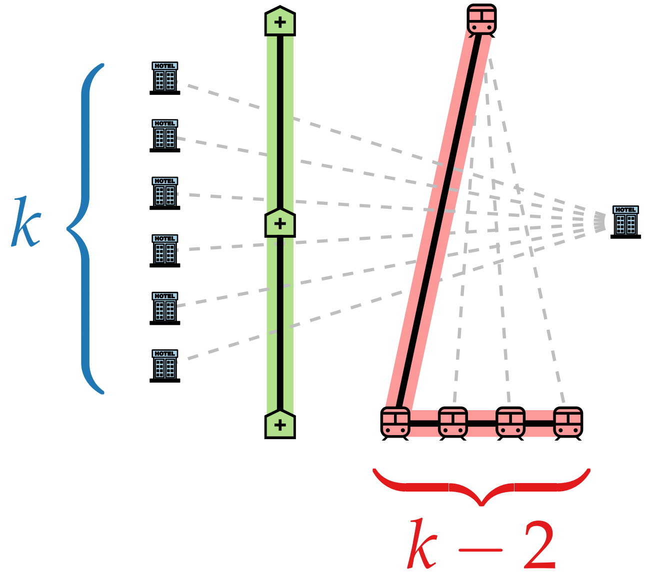

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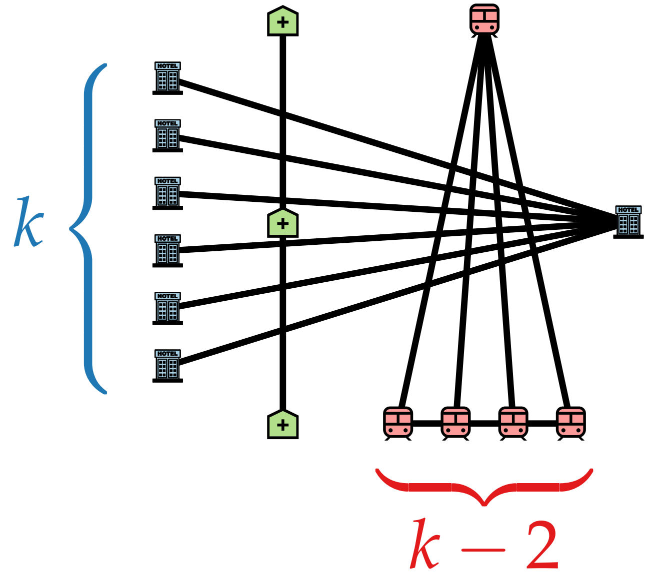

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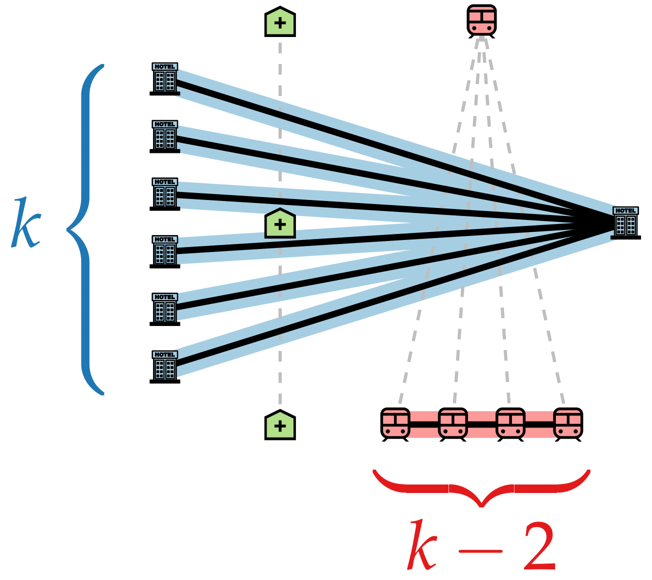

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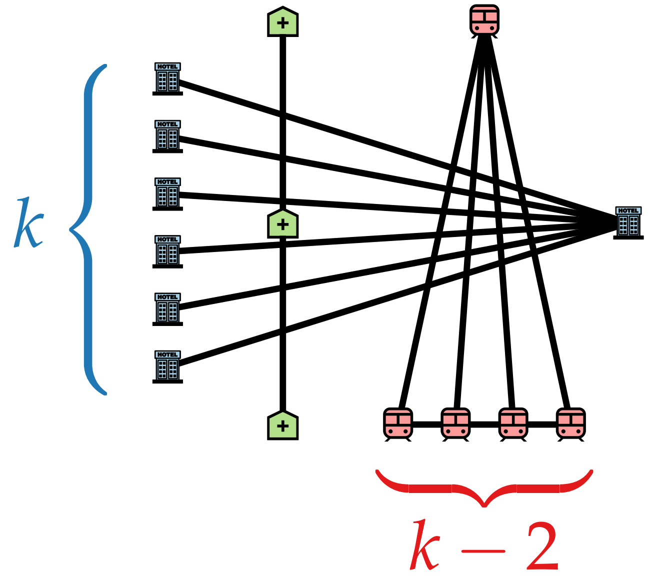

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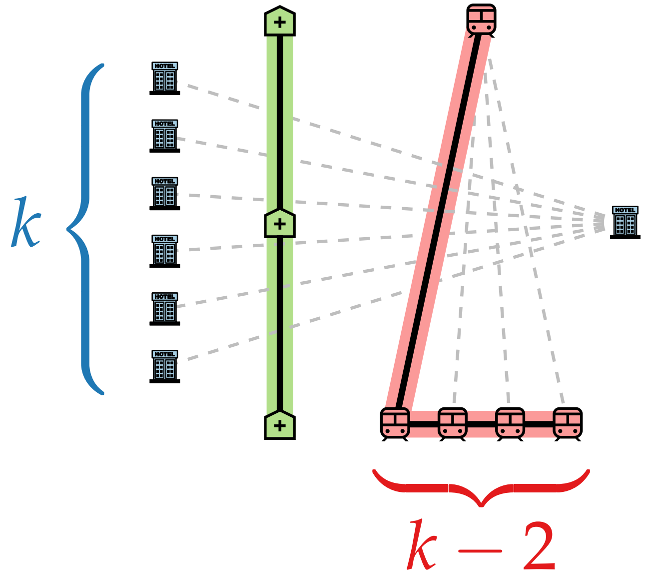

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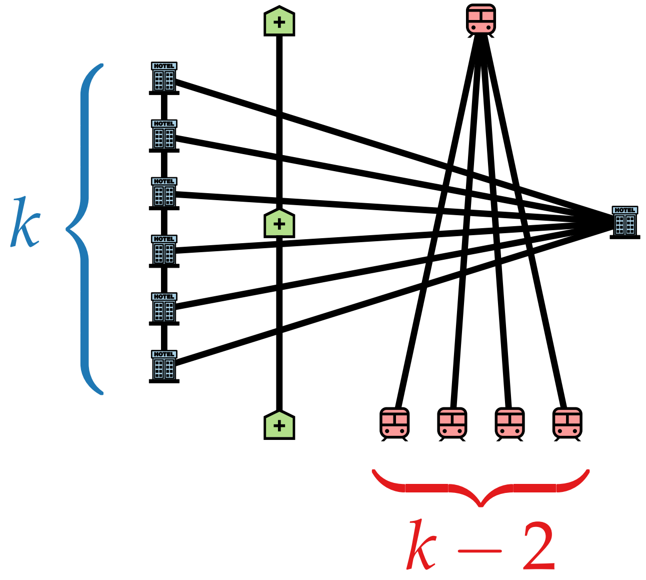

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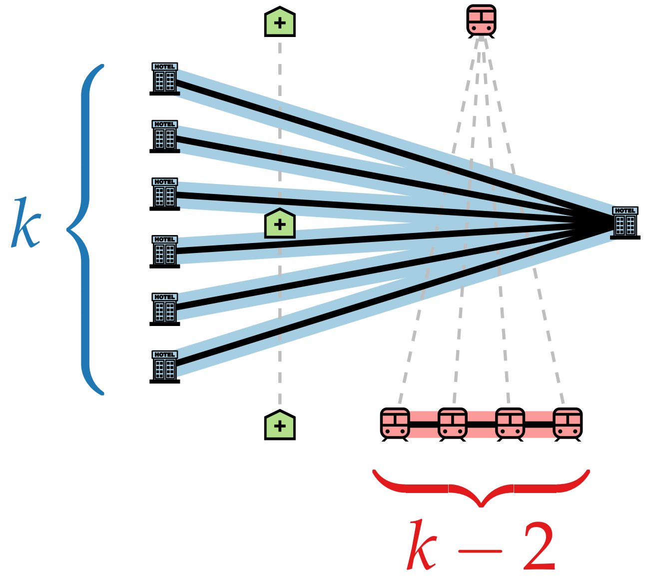

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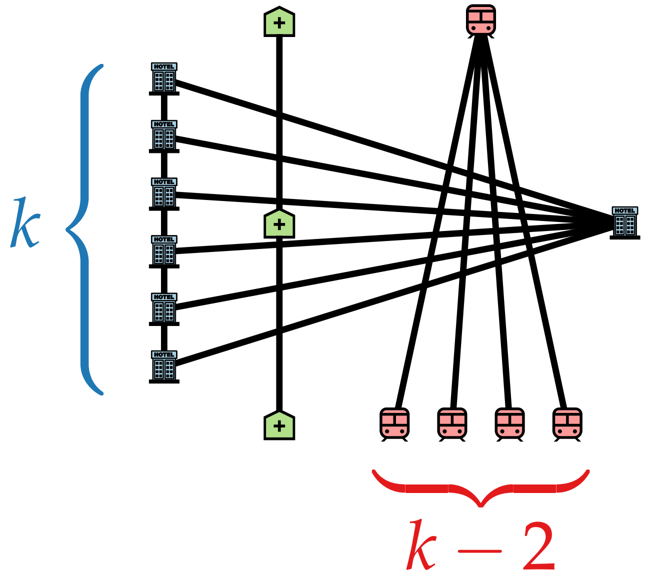

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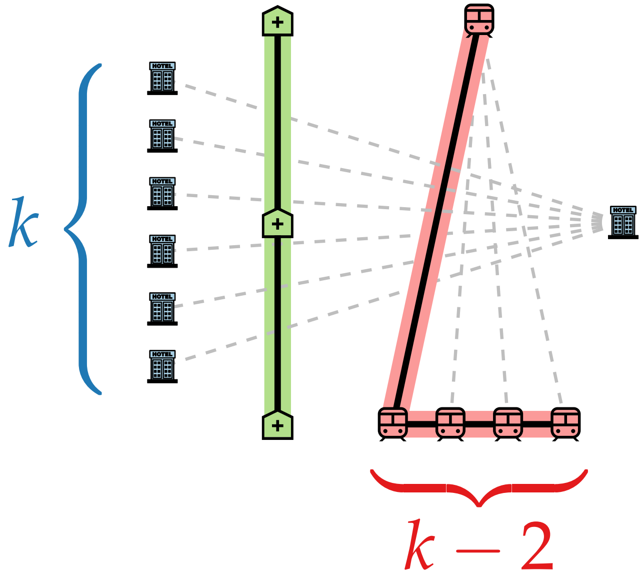

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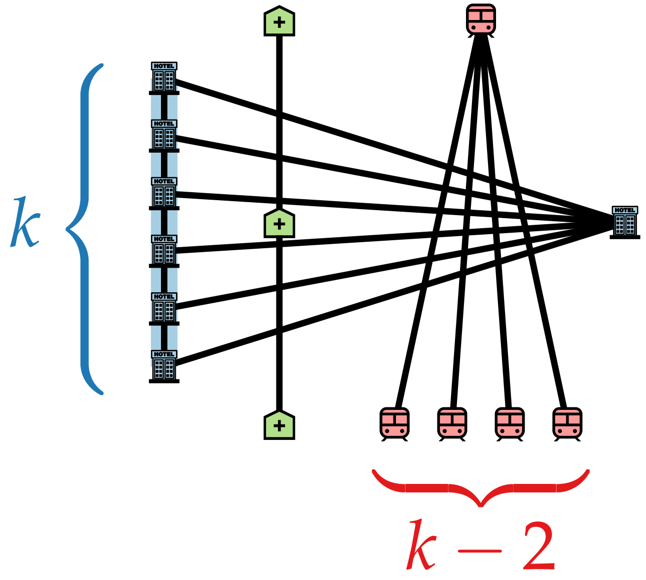

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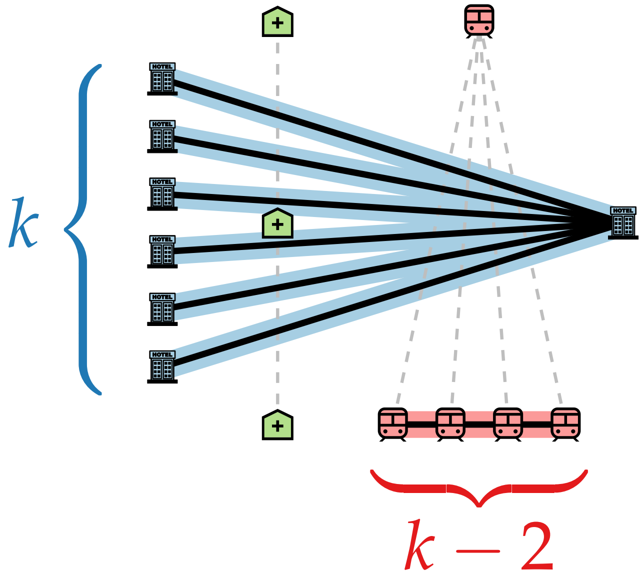

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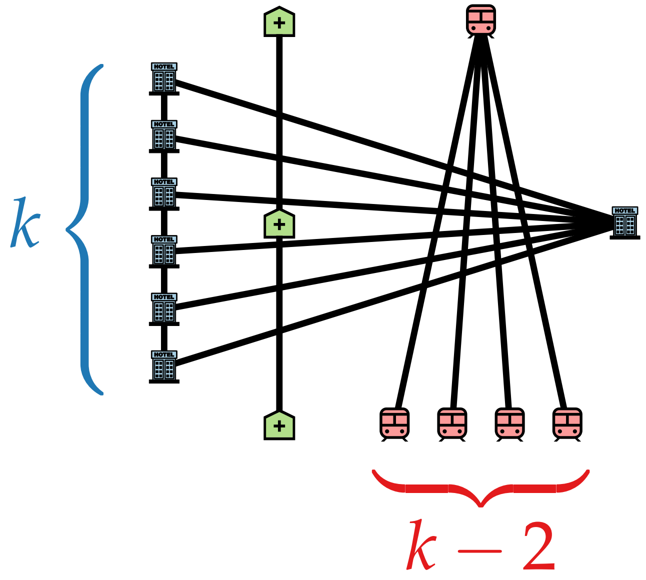

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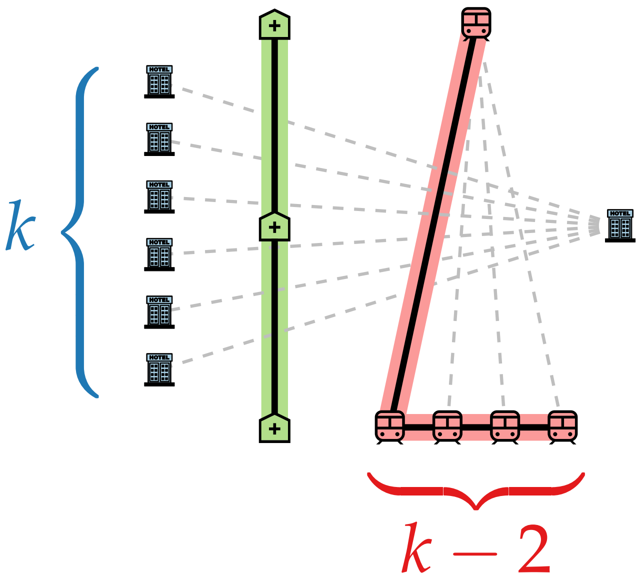

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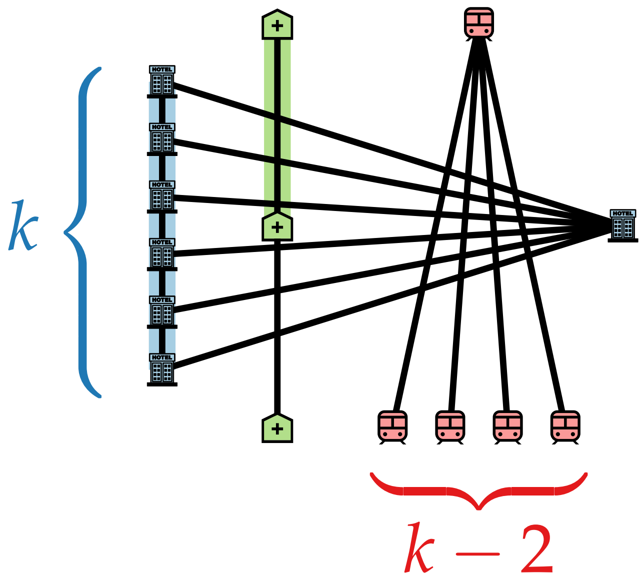

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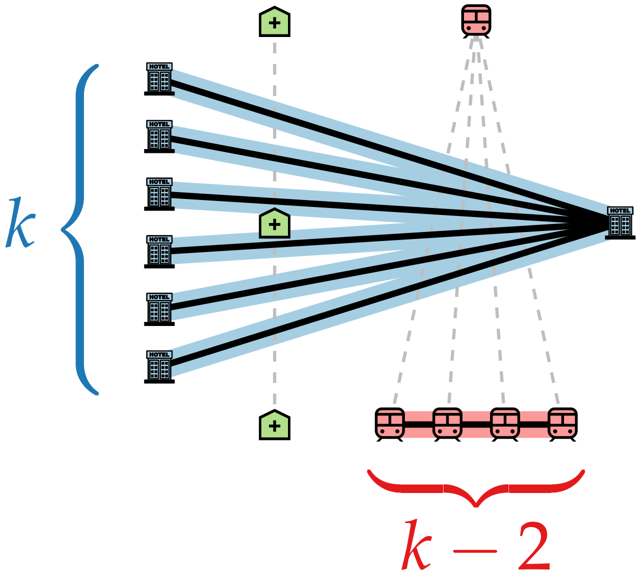

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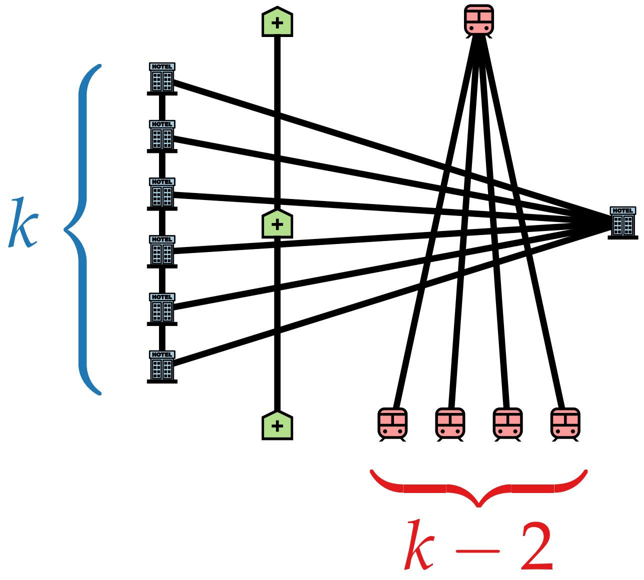

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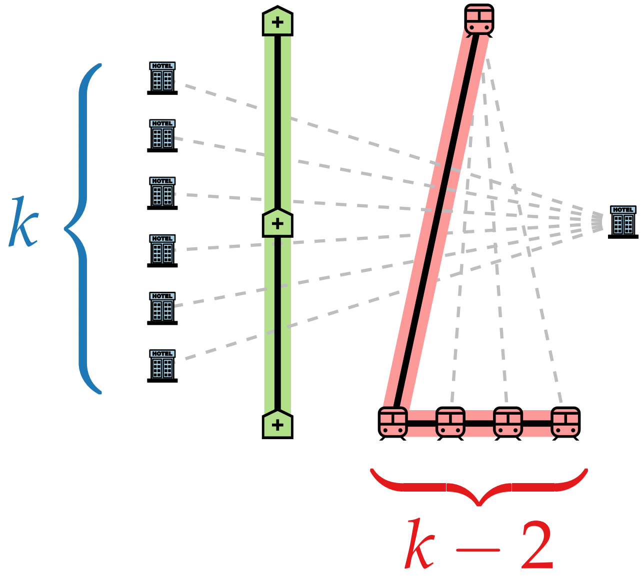

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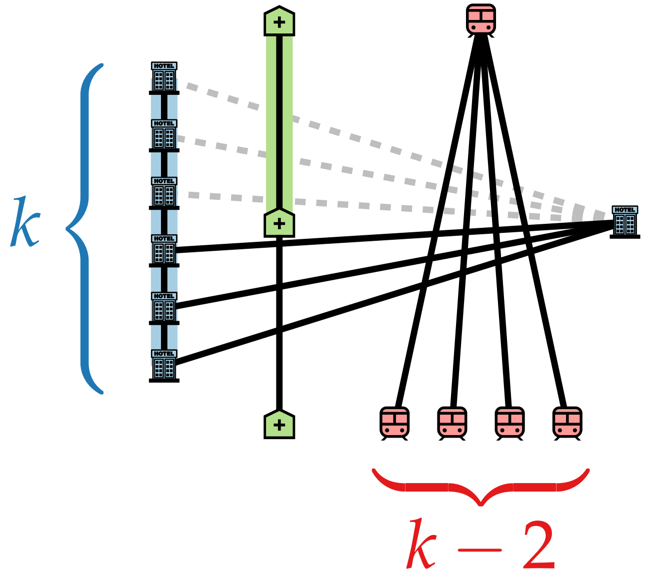

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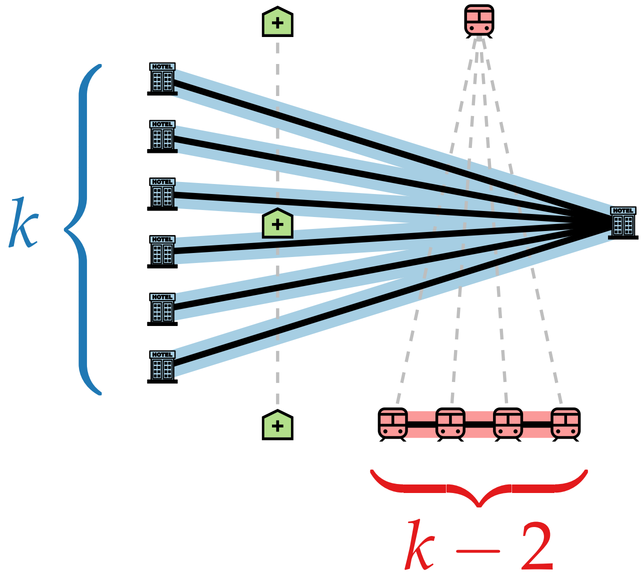

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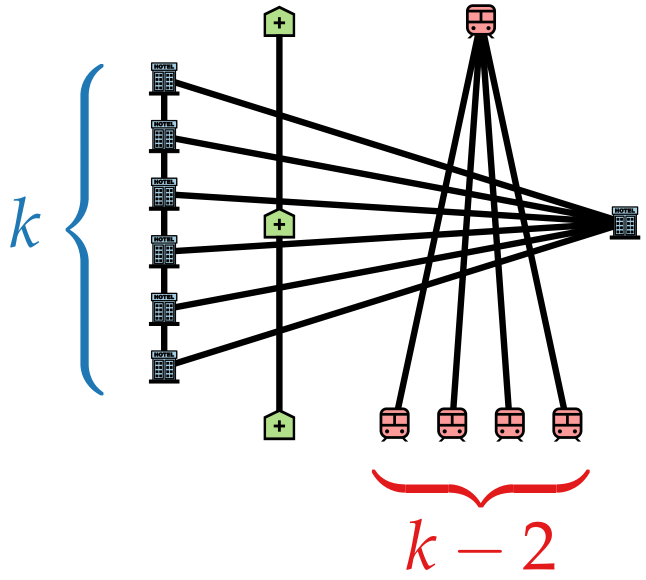

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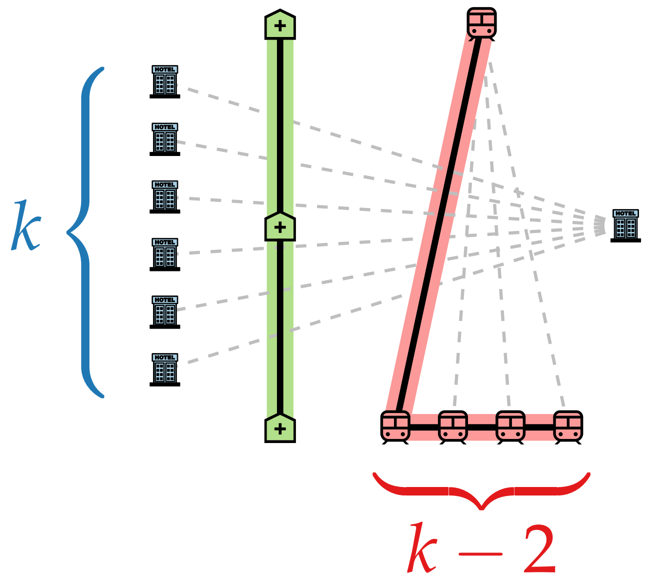

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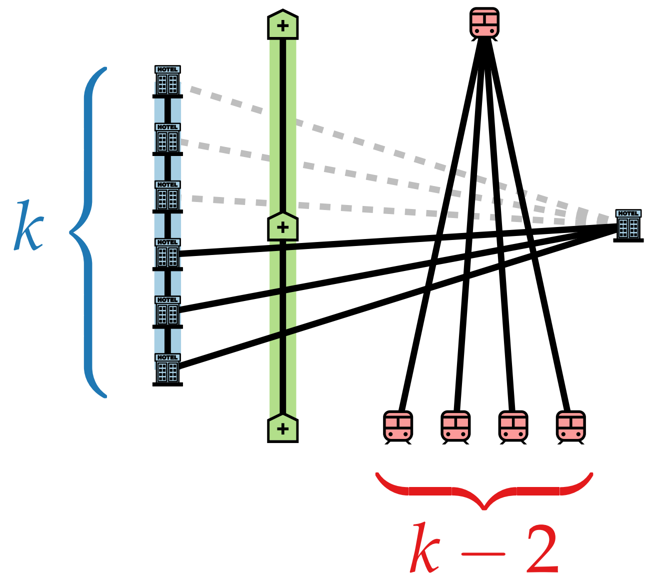

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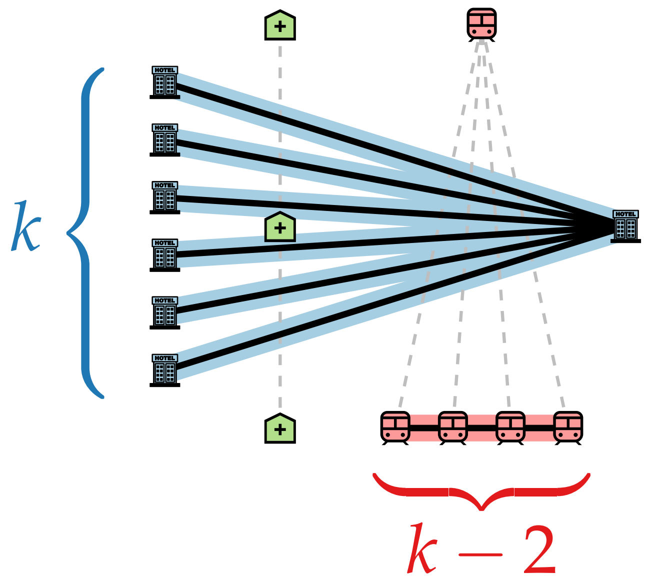

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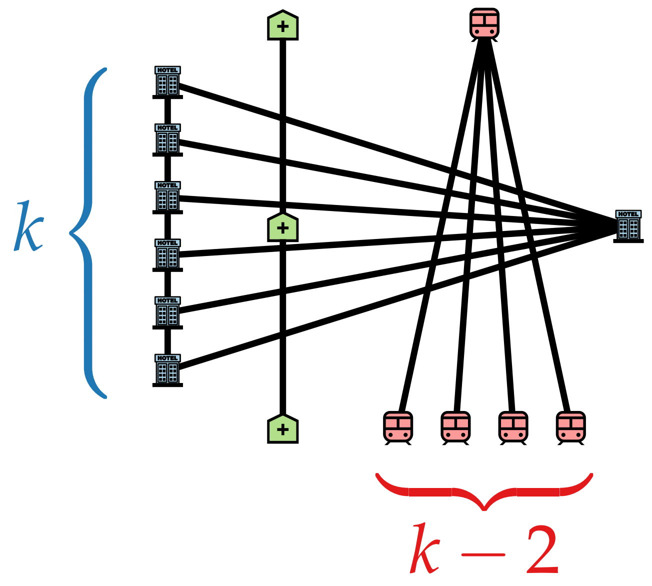

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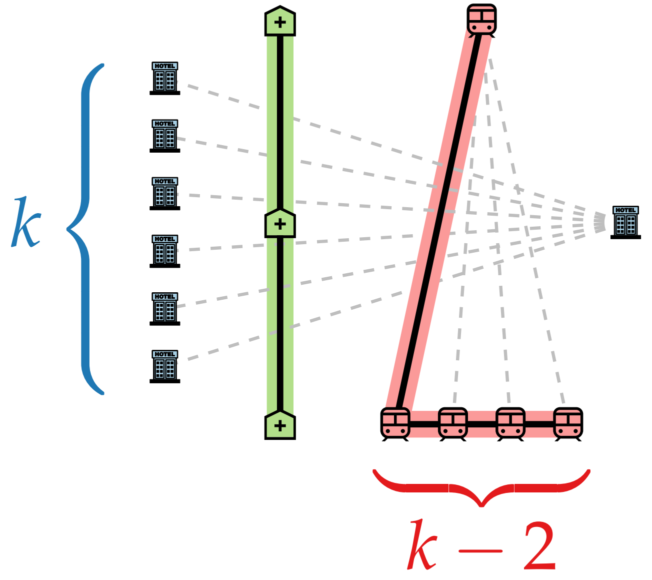

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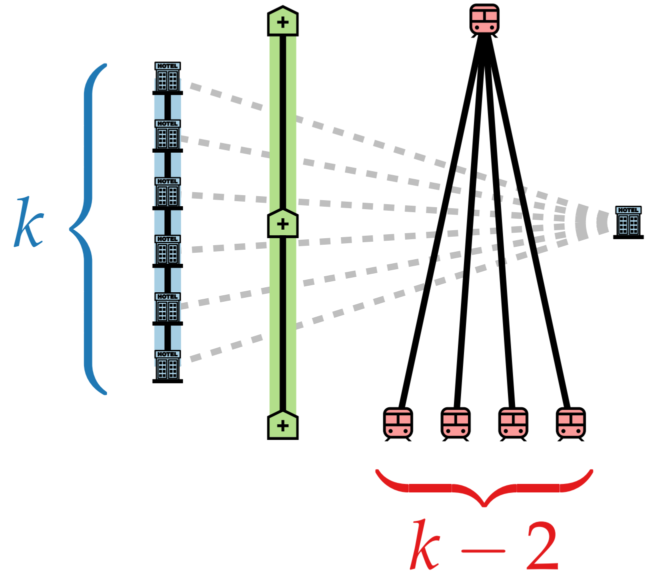

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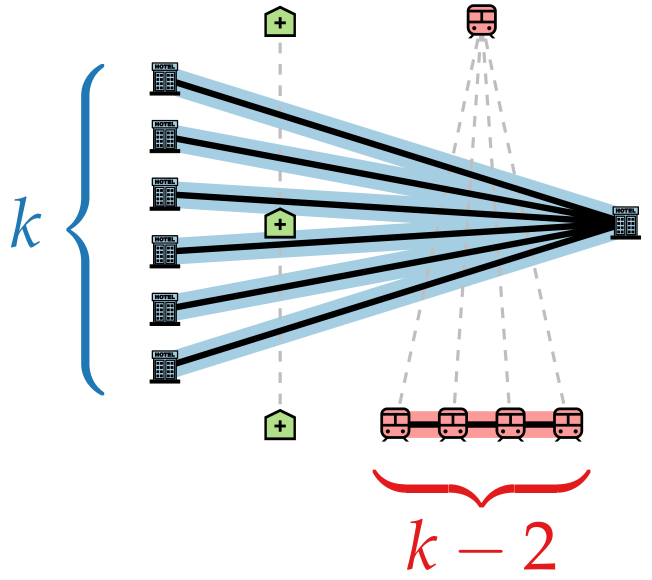

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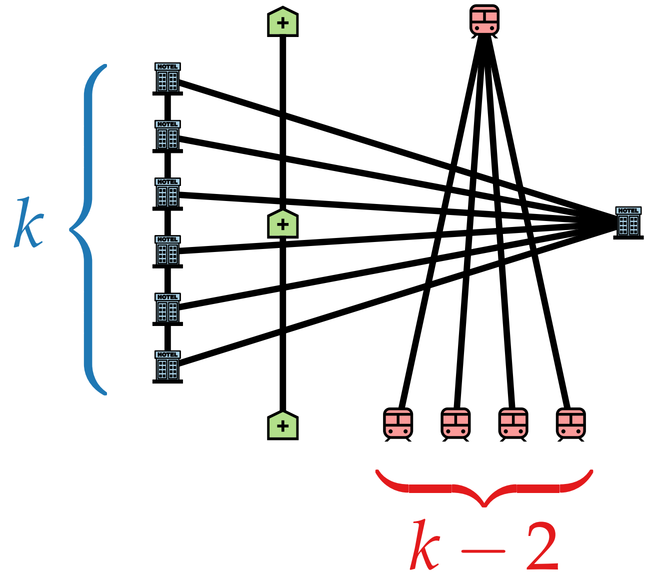

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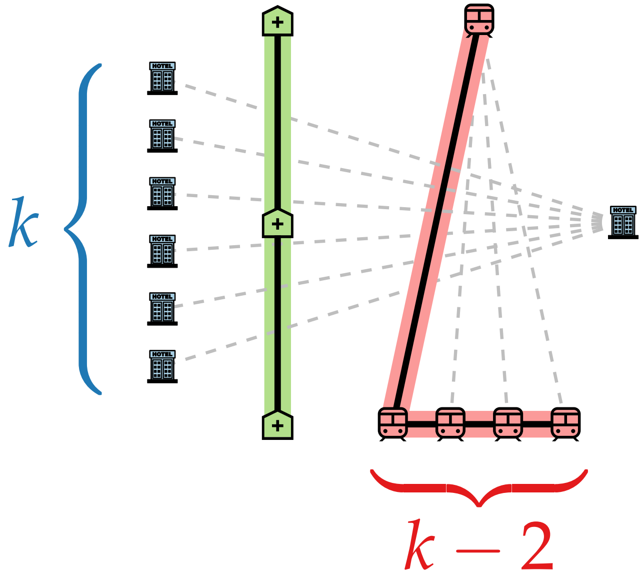

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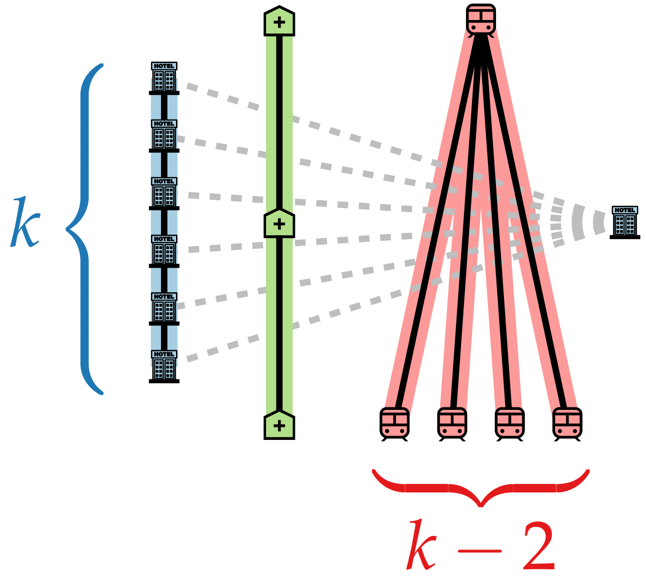

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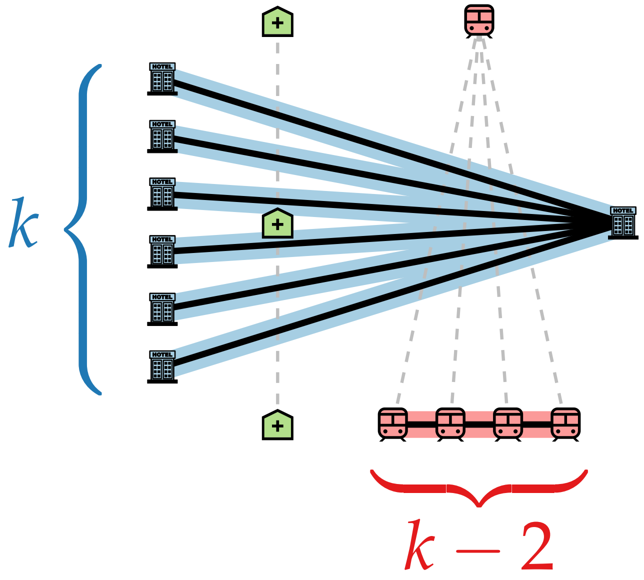

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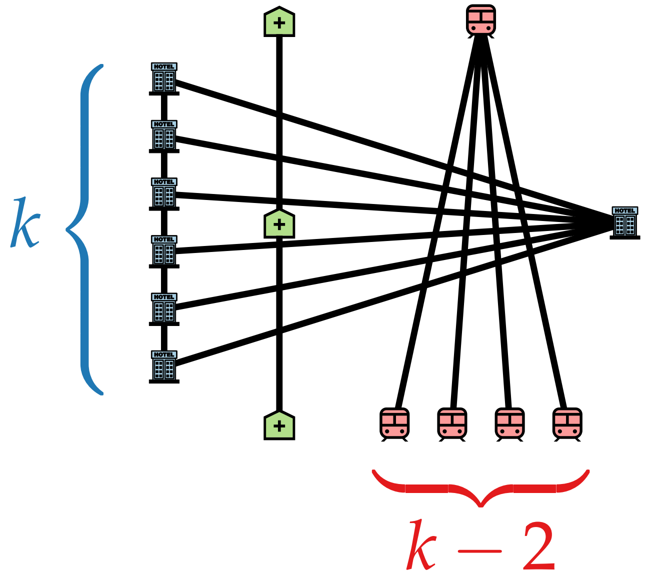

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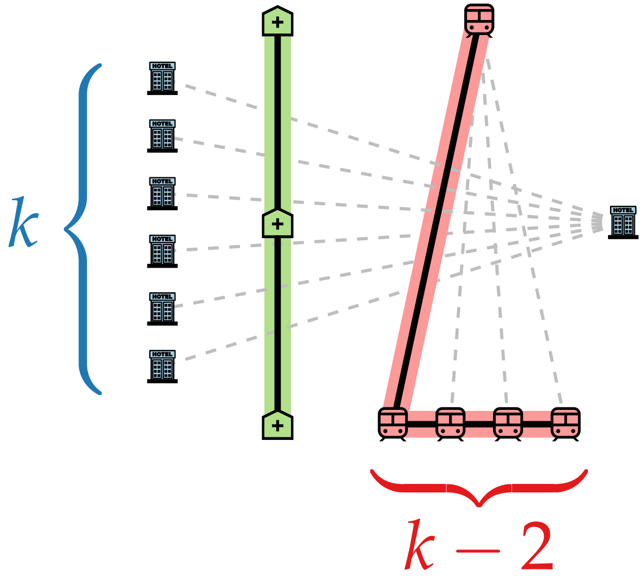

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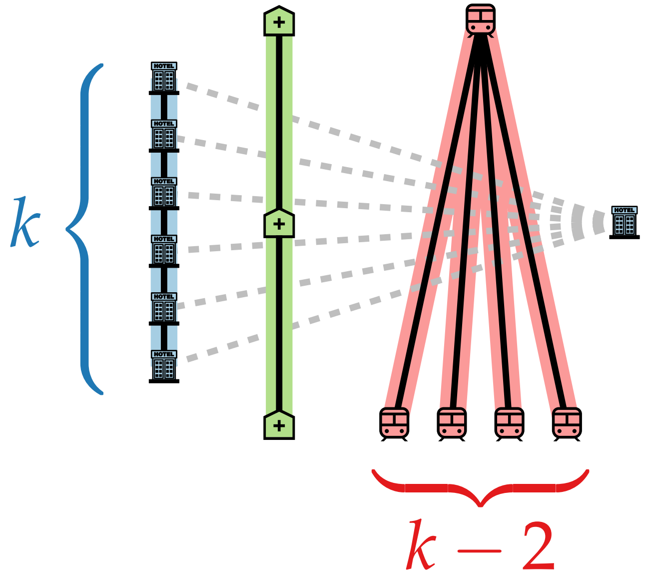

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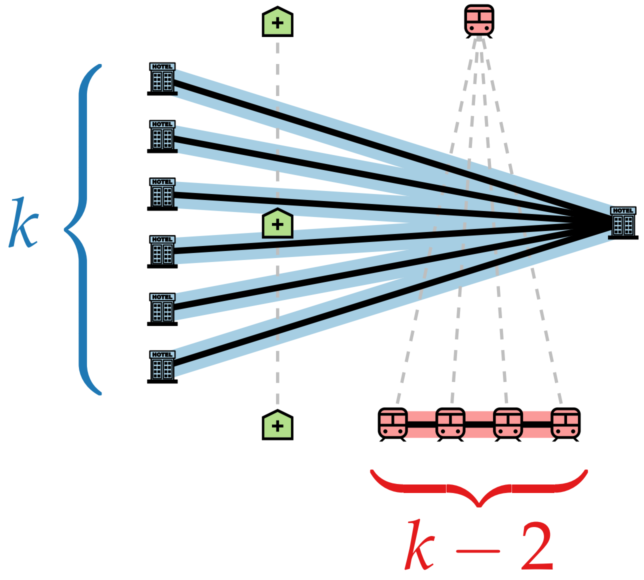

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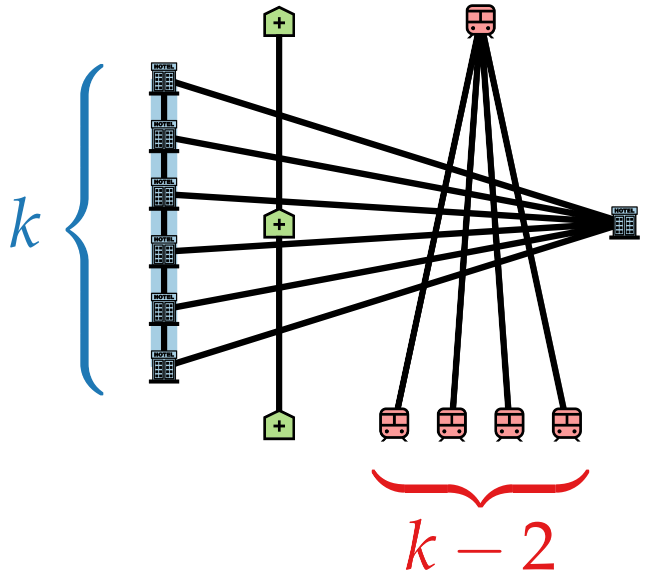

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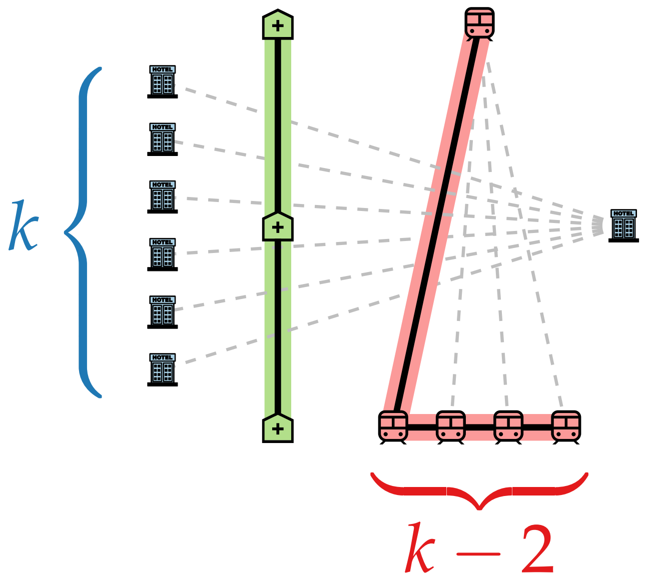

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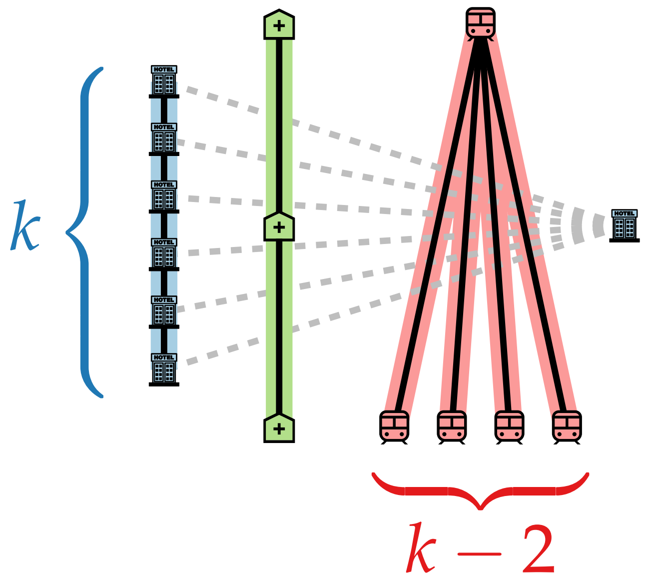

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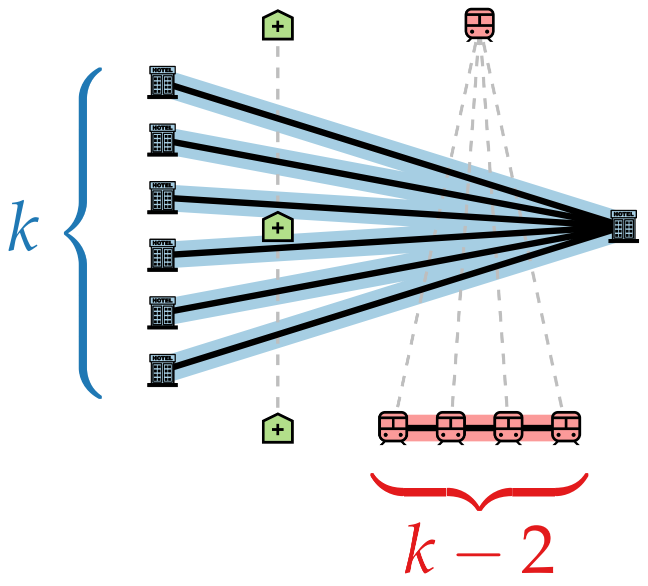

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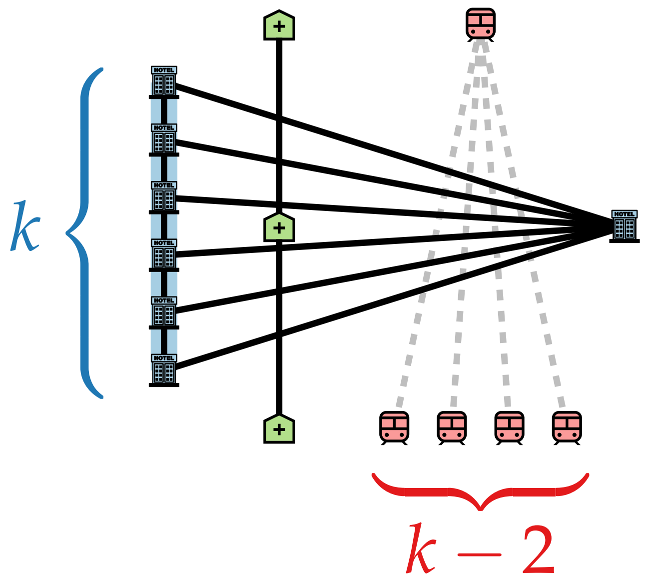

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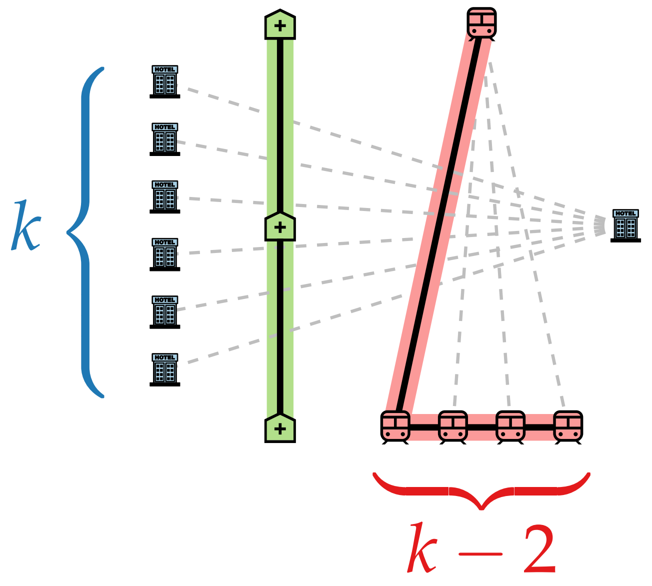
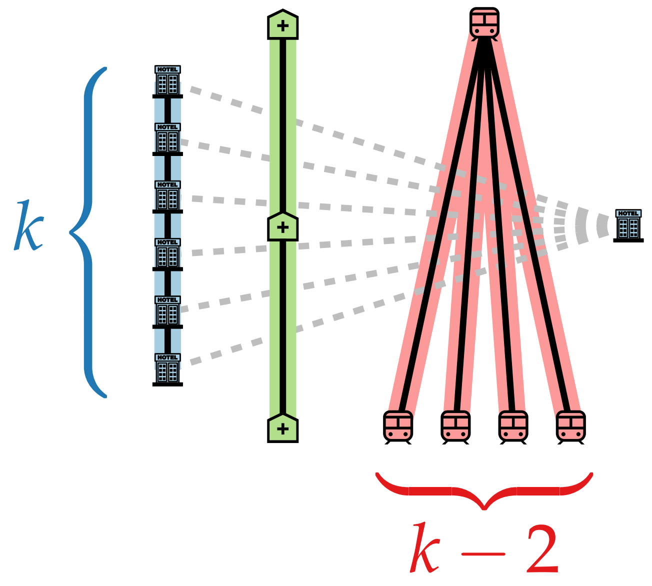
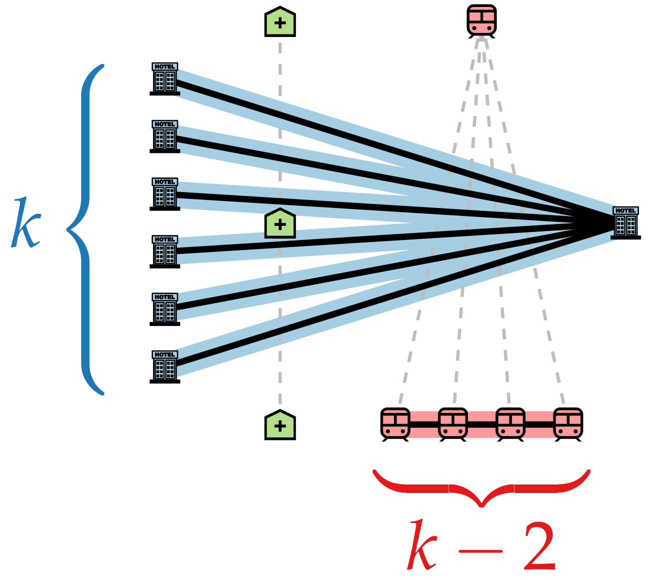


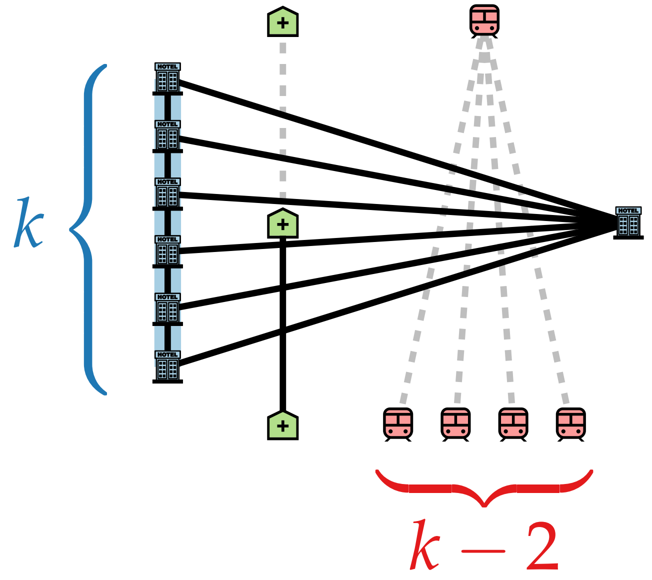
GREEDY:



REVERSEGREEDY:

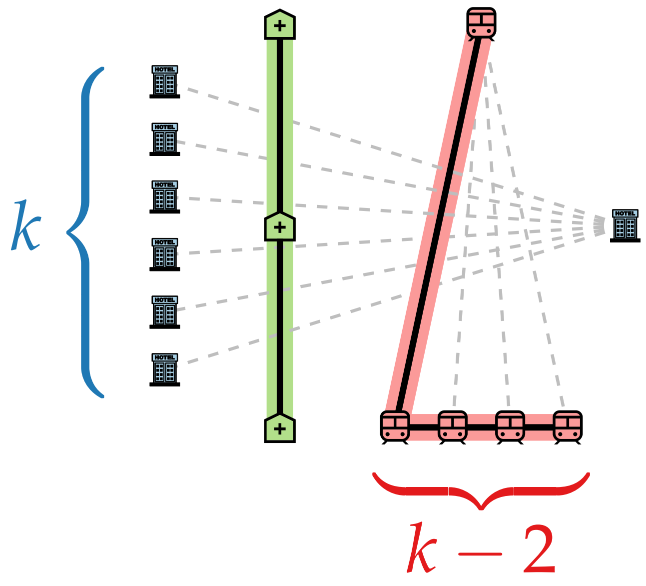


REVERSEGREEDY:

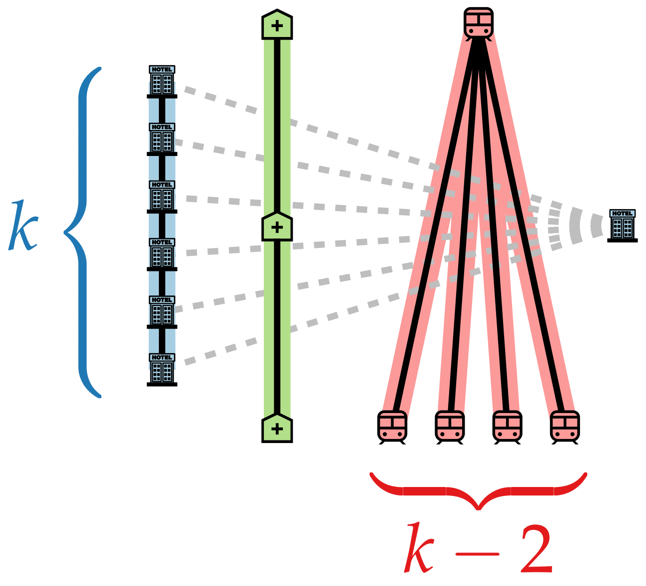


Bad Examples

Greedy:



Greedy:



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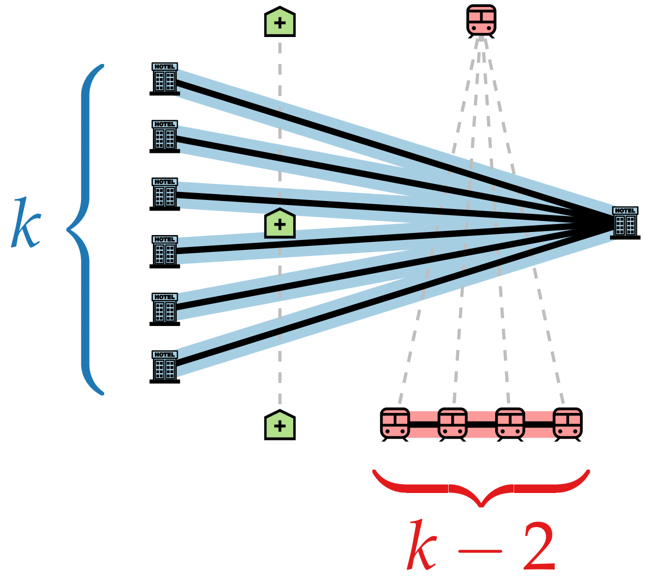

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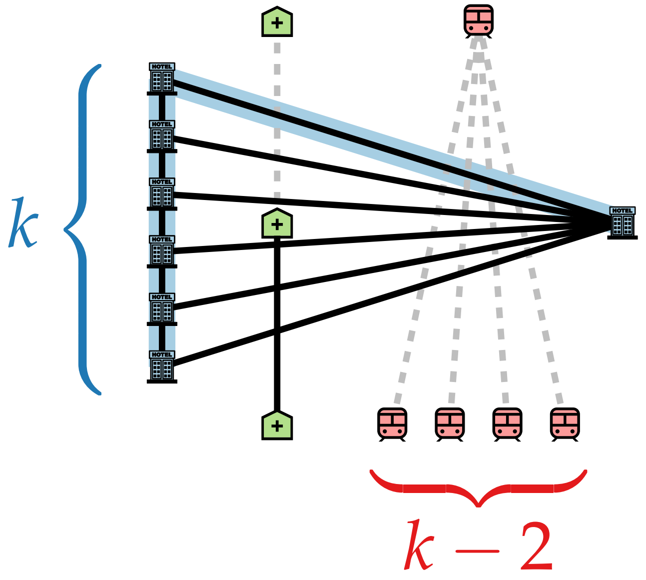
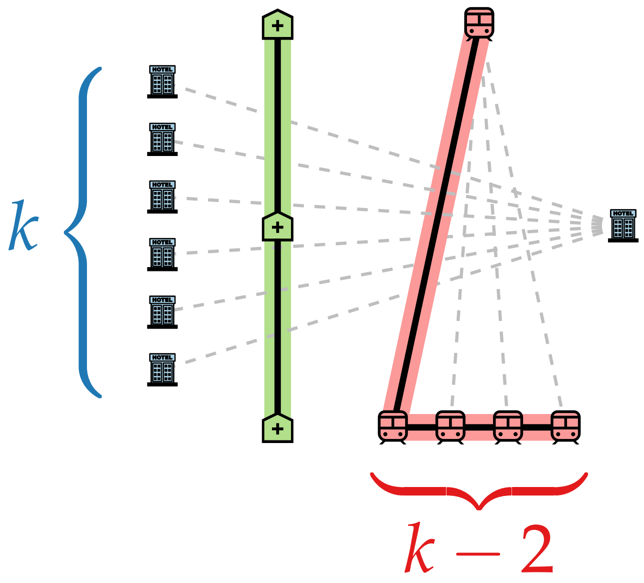


Bad Examples

Greedy:



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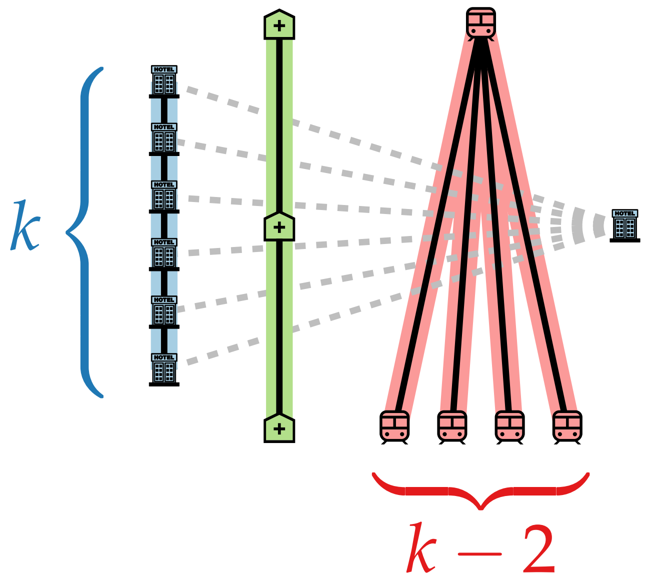

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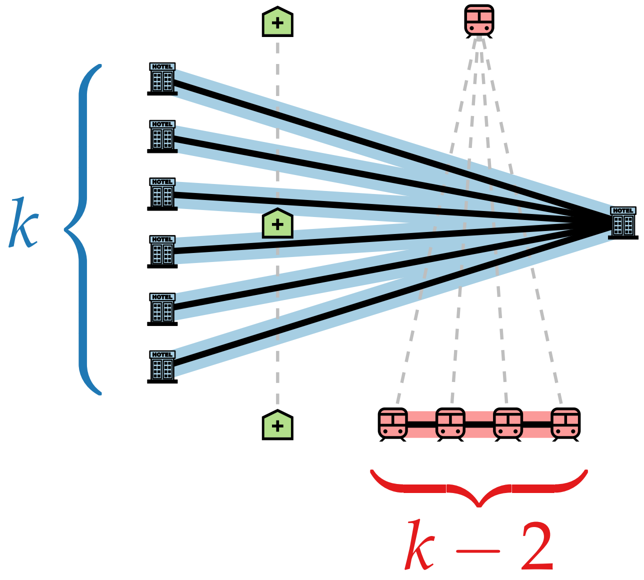

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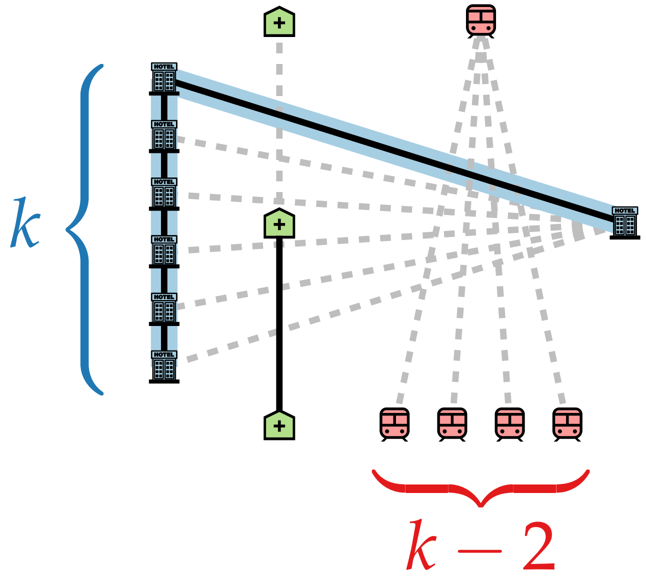
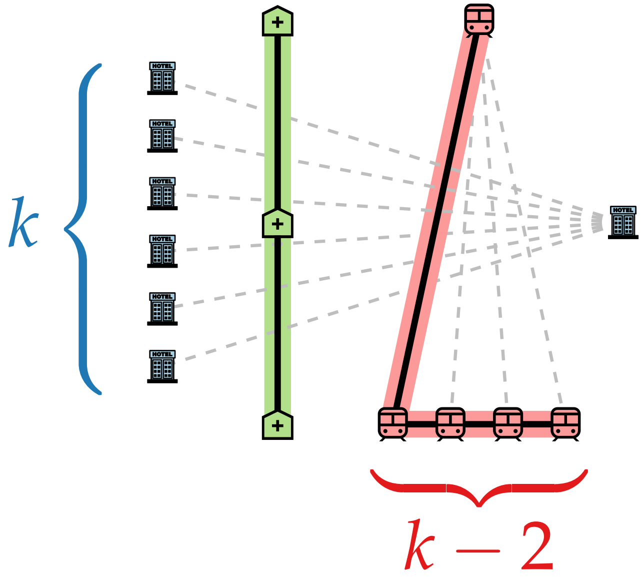


Bad Examples

Greedy:



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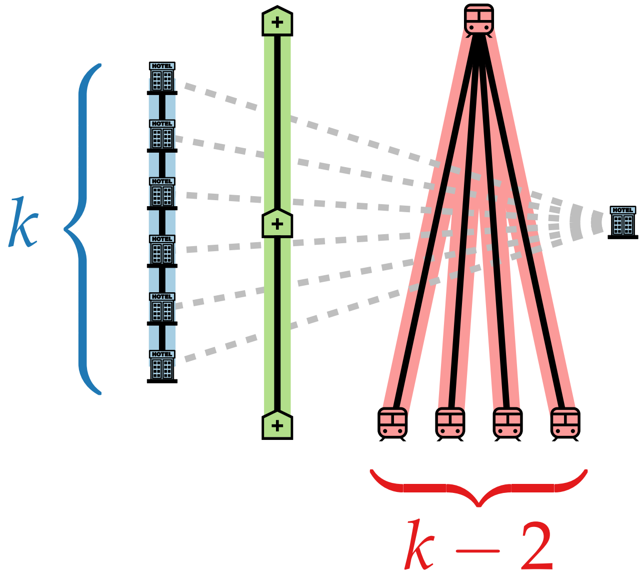
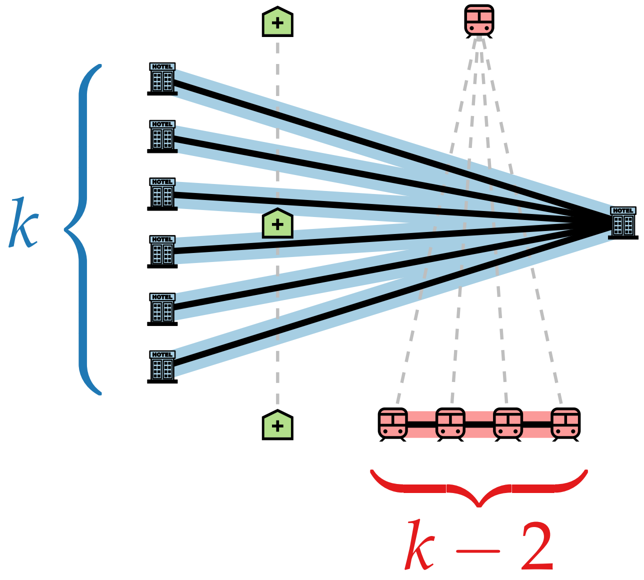
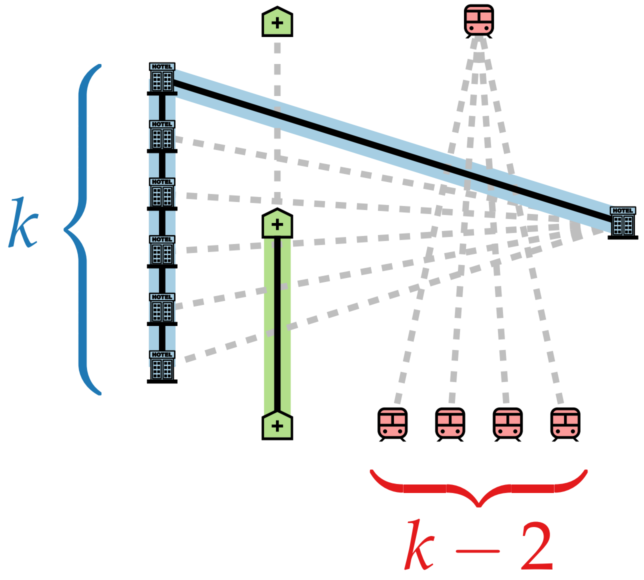


ReverseGreedy:



ReverseGreedy:



Planar Spanning Forest

- Greedy Heuristic

Planar Spanning Forest

- Greedy Heuristic
- ReverseGreedy Heuristic

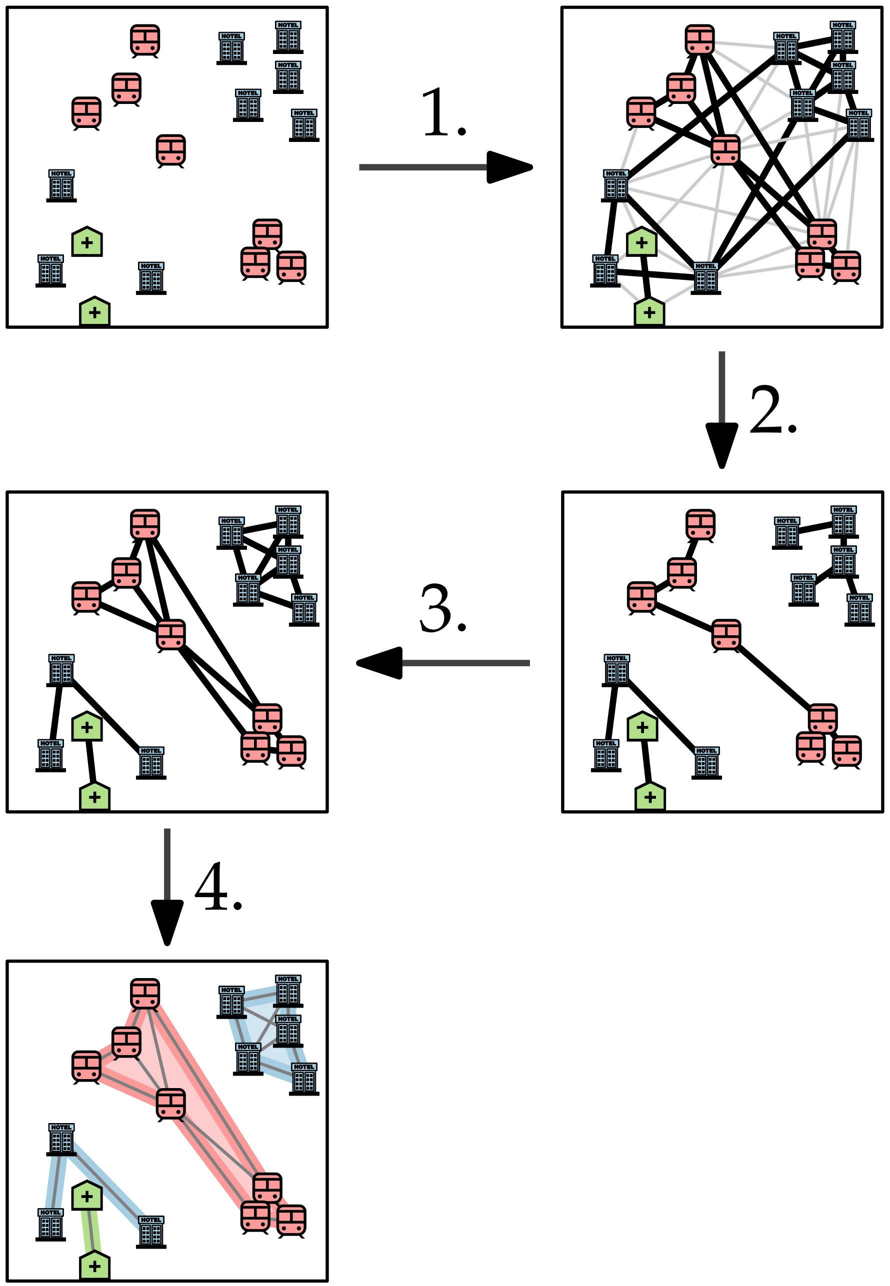
Planar Spanning Forest

- Greedy Heuristic
- ReverseGreedy Heuristic
- Exact ILP Formulation

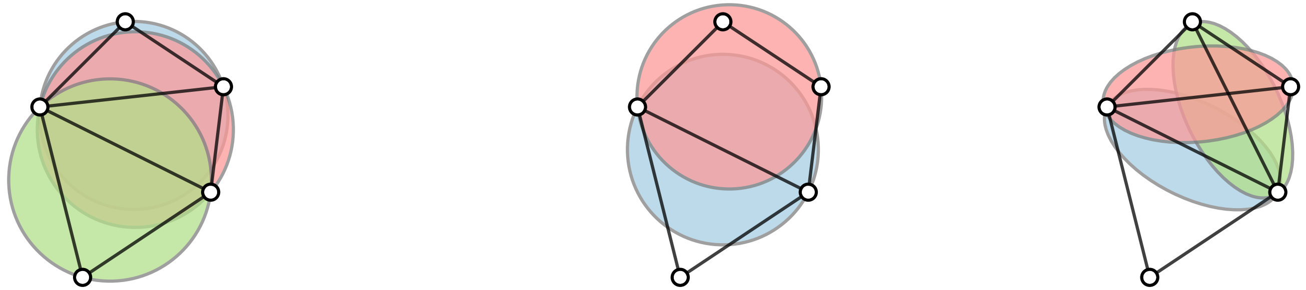
Planar Spanning Forest

- Greedy Heuristic
- ReverseGreedy Heuristic
- Exact ILP Formulation
- NP-hard even for one category [Jansen & Woeginger '93]

Pipeline



1. **Proximity Graph**

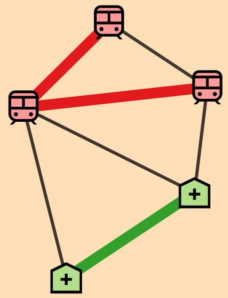


Delaunay Triangulation

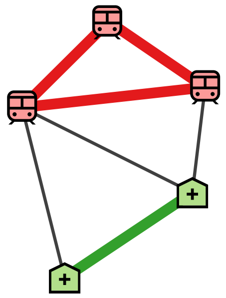
Gabriel Graph

β-Skeleton

2. **Planar Spanning Forest**

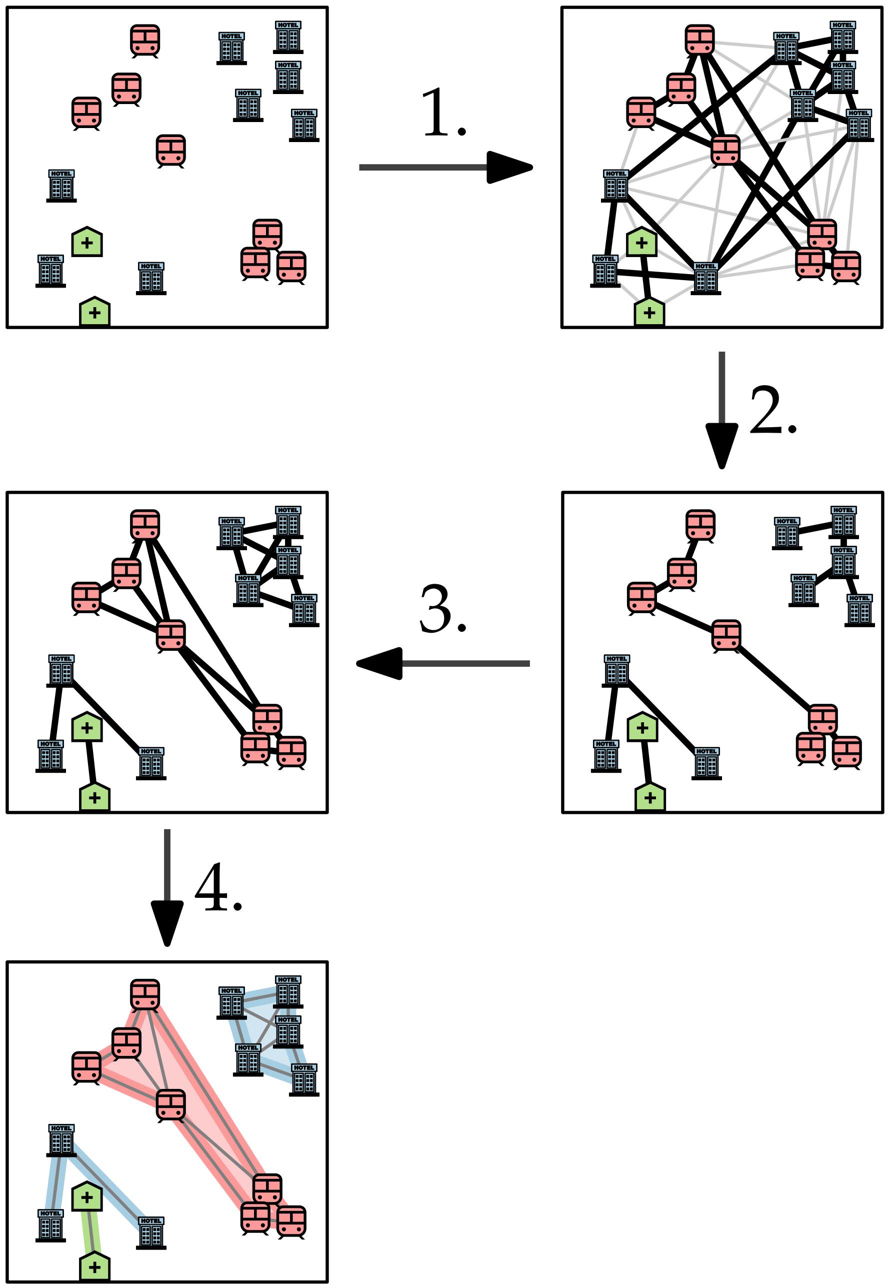


3. **Edge Augmentation**

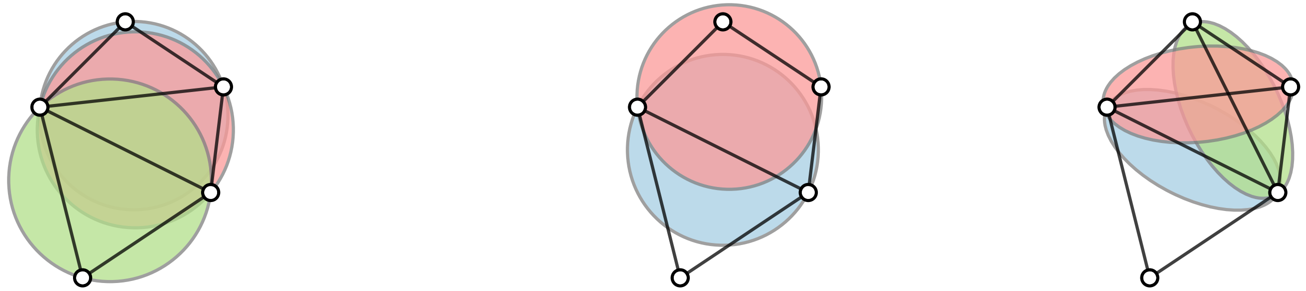


4. **Rendering**
   - Line Voronoi Diagram
   - Tree Representation
   - Polygon Representation

Pipeline



1. Proximity Graph

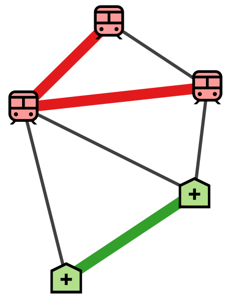


Delaunay Triangulation

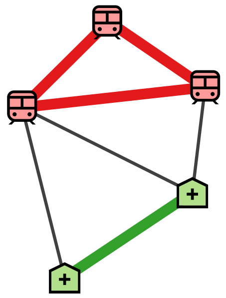
Gabriel Graph

β-Skeleton

2. Planar Spanning Forest



3. Edge Augmentation

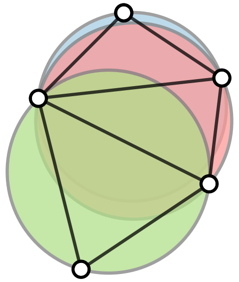


4. Rendering

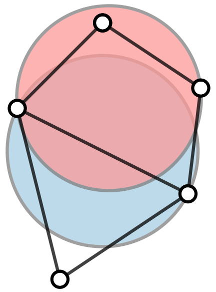
- Line Voronoi Diagram
- Tree Representation
- Polygon Representation

Proximity Graph

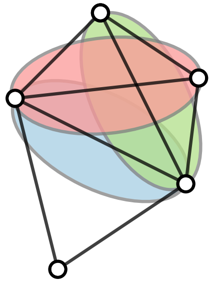
1. Proximity Graph



Delaunay Triangulation



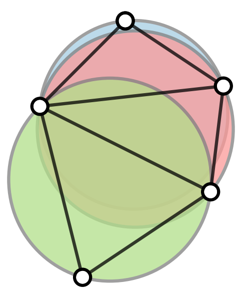
Gabriel Graph



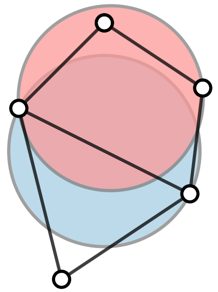
β-Skeleton

Proximity Graph

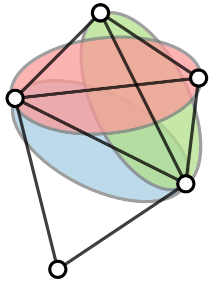
1. Proximity Graph



Delaunay Triangulation $\supseteq$ Gabriel Graph

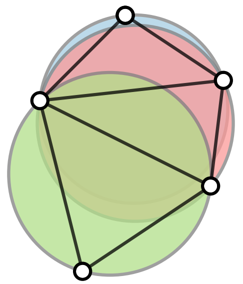


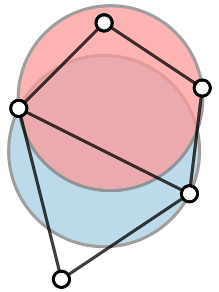
β-Skeleton

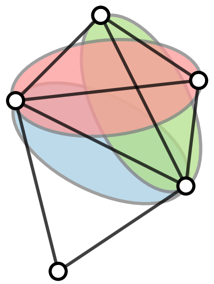


Proximity Graph

1. **Proximity Graph**





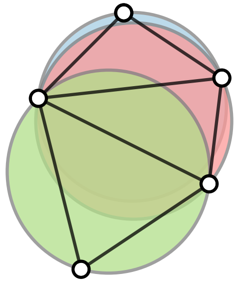


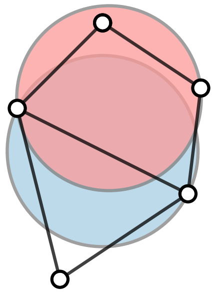
planar → Delaunay Triangulation $\supseteq$ Gabriel Graph

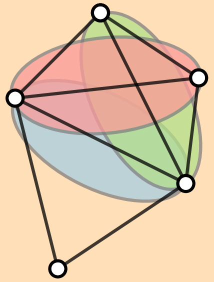
β-Skeleton

Proximity Graph

1. Proximity Graph





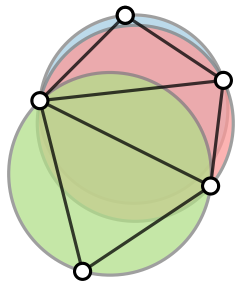


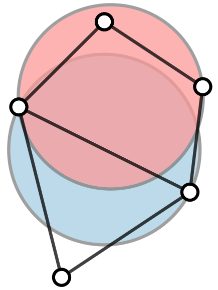
planar → Delaunay Triangulation ⊇ Gabriel Graph

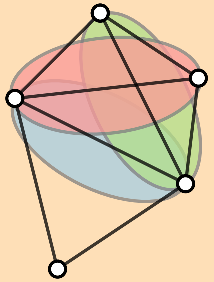
β-Skeleton

Proximity Graph

1. **Proximity Graph**

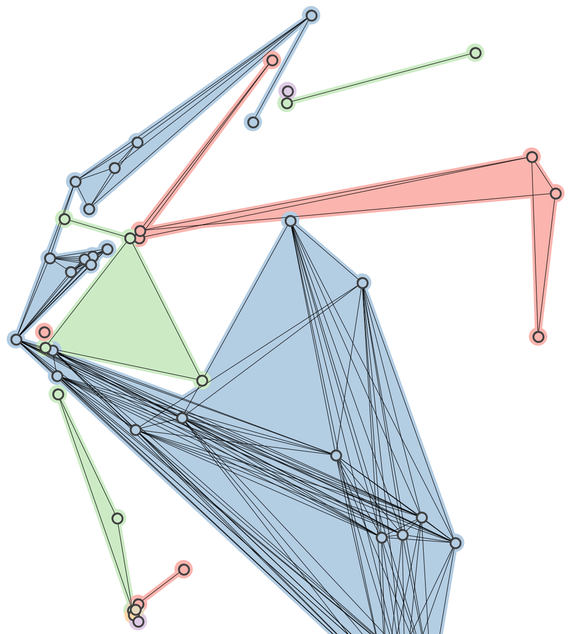






planar → Delaunay Triangulation ⊇ Gabriel Graph

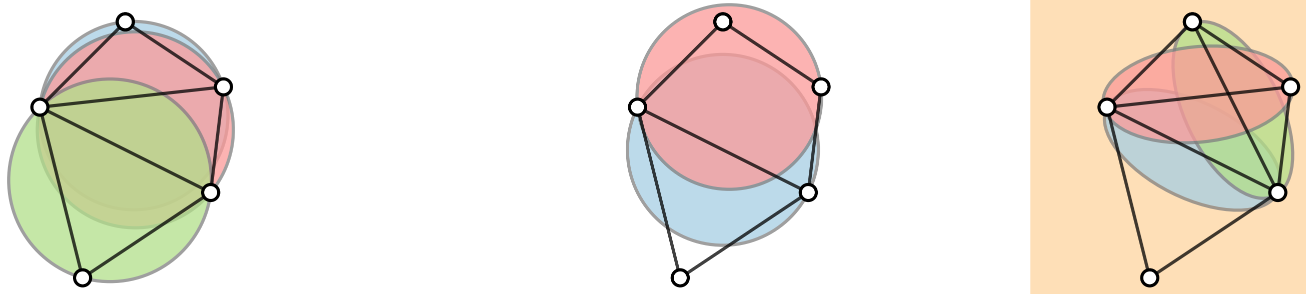
β-Skeleton



$\beta = 0.0$; 11 clusters

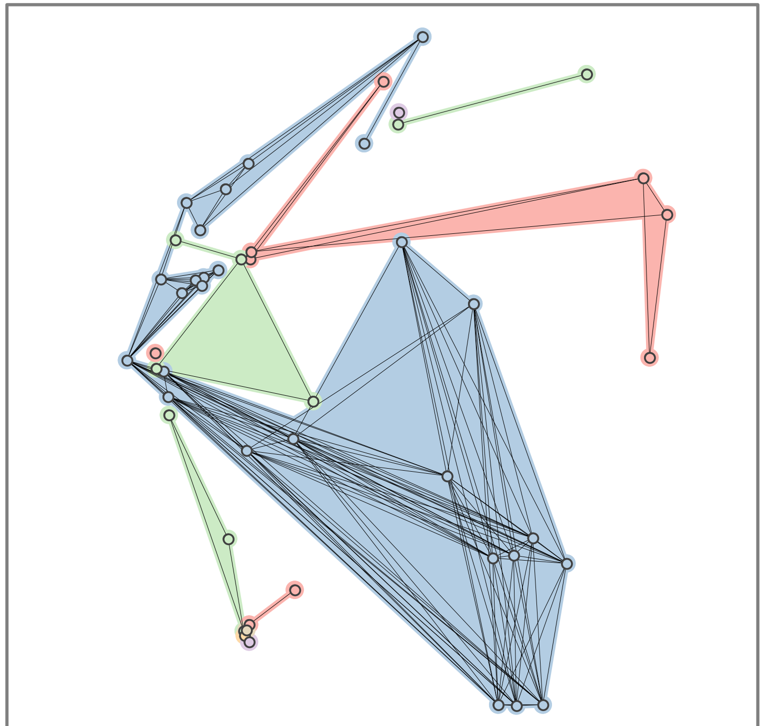
Proximity Graph

1. Proximity Graph

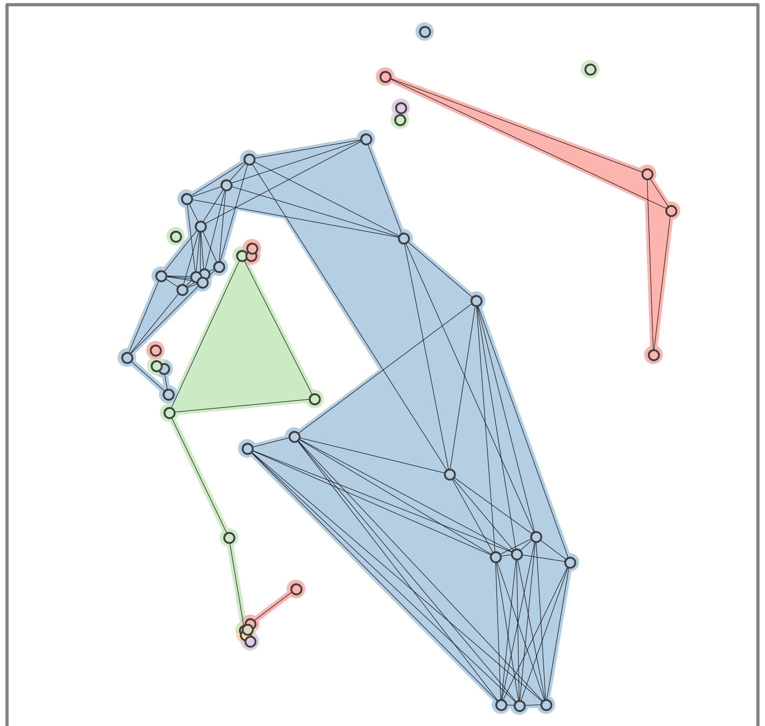


planar → Delaunay Triangulation ⊇ Gabriel Graph

β-Skeleton



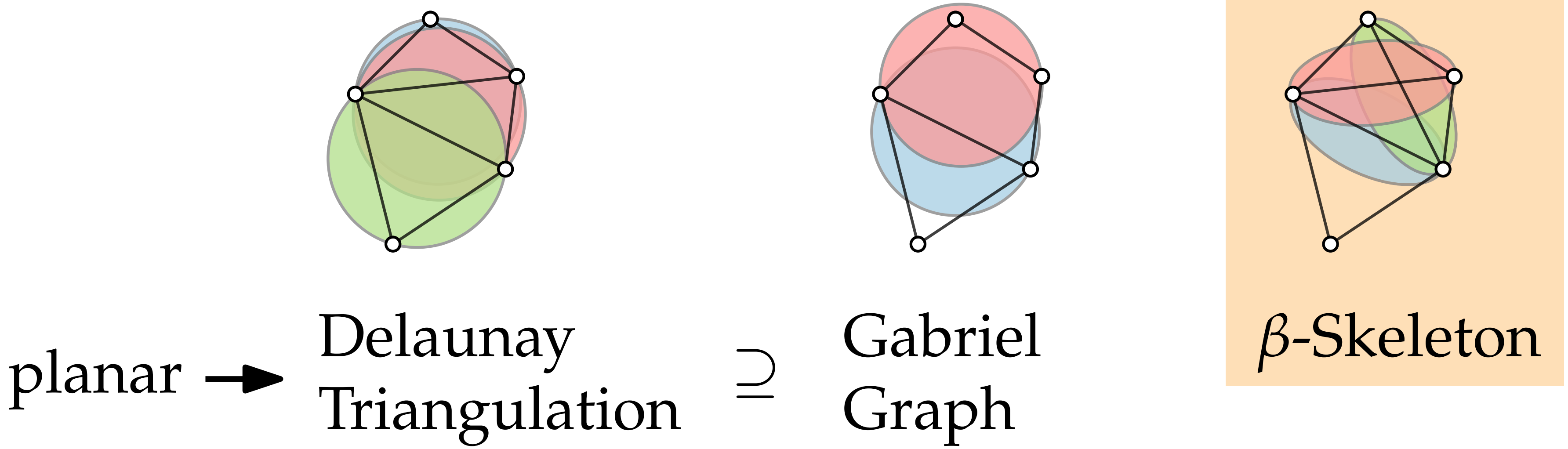
$\beta = 0.0$; 11 clusters

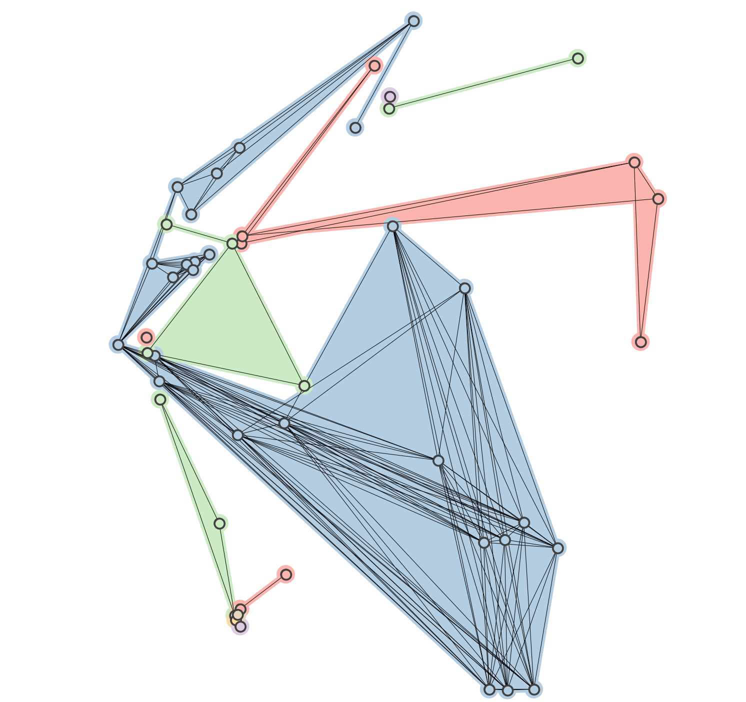


$\beta = 0.5$; 15 clusters

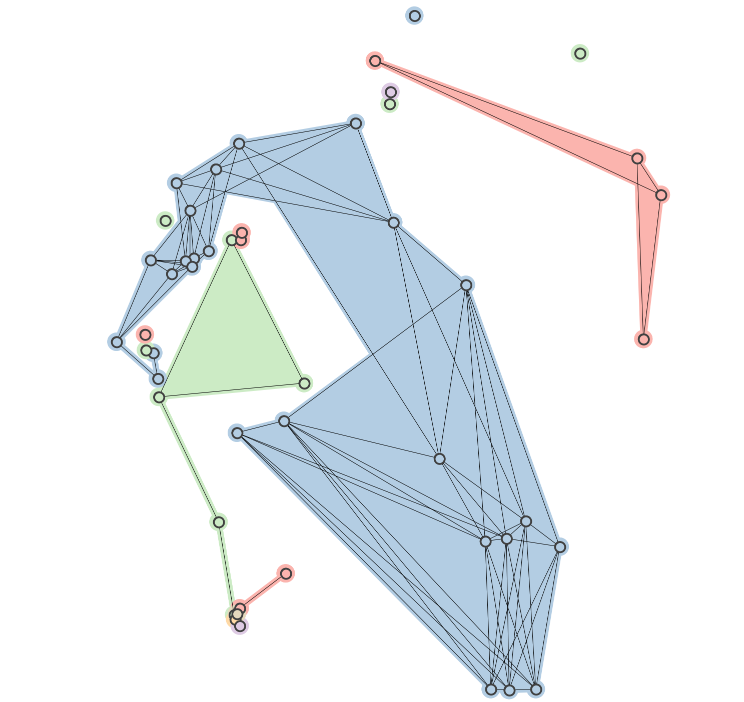
Proximity Graph

1. **Proximity Graph**

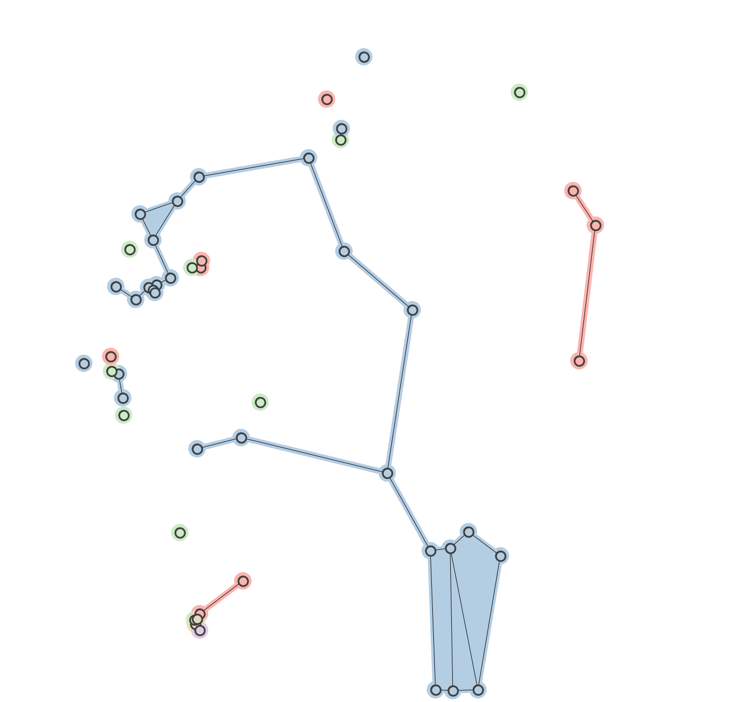




$\beta = 0.0$; 11 clusters

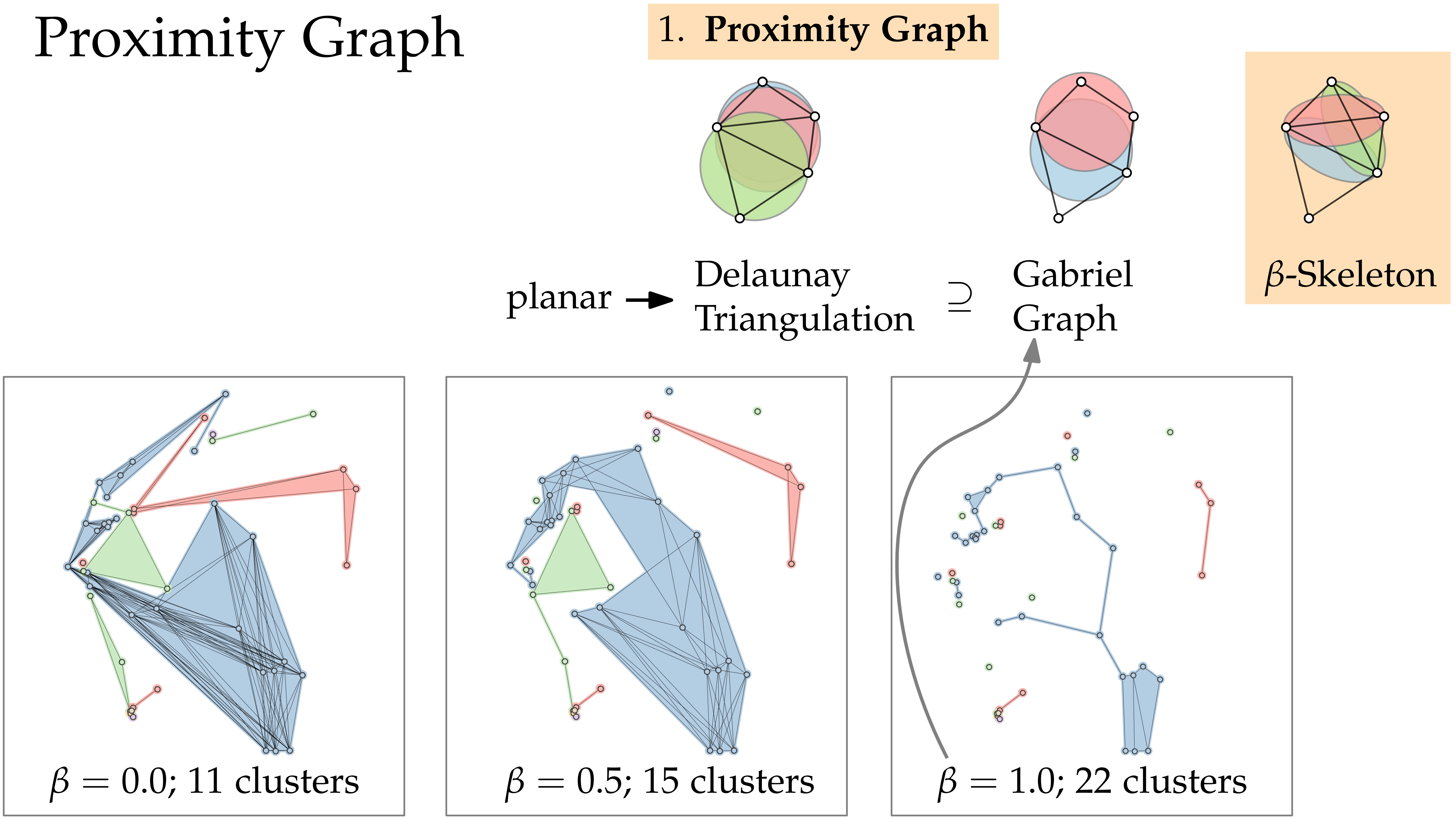


$\beta = 0.5$; 15 clusters

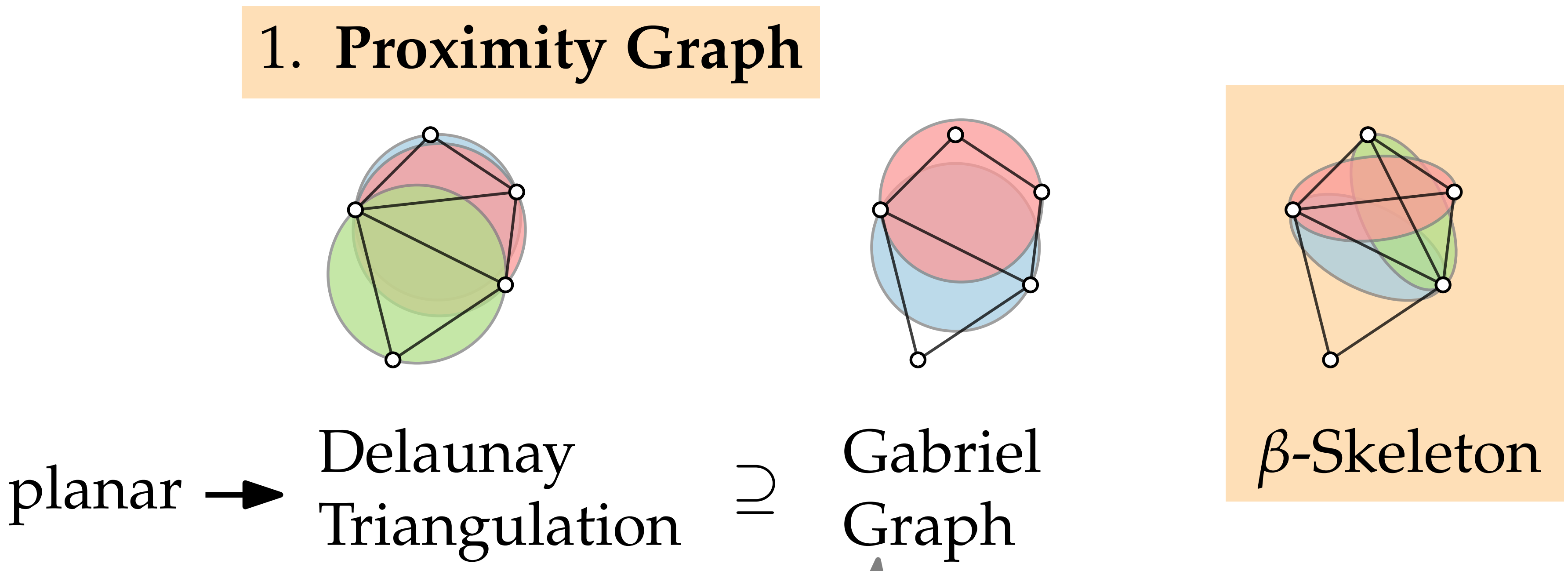


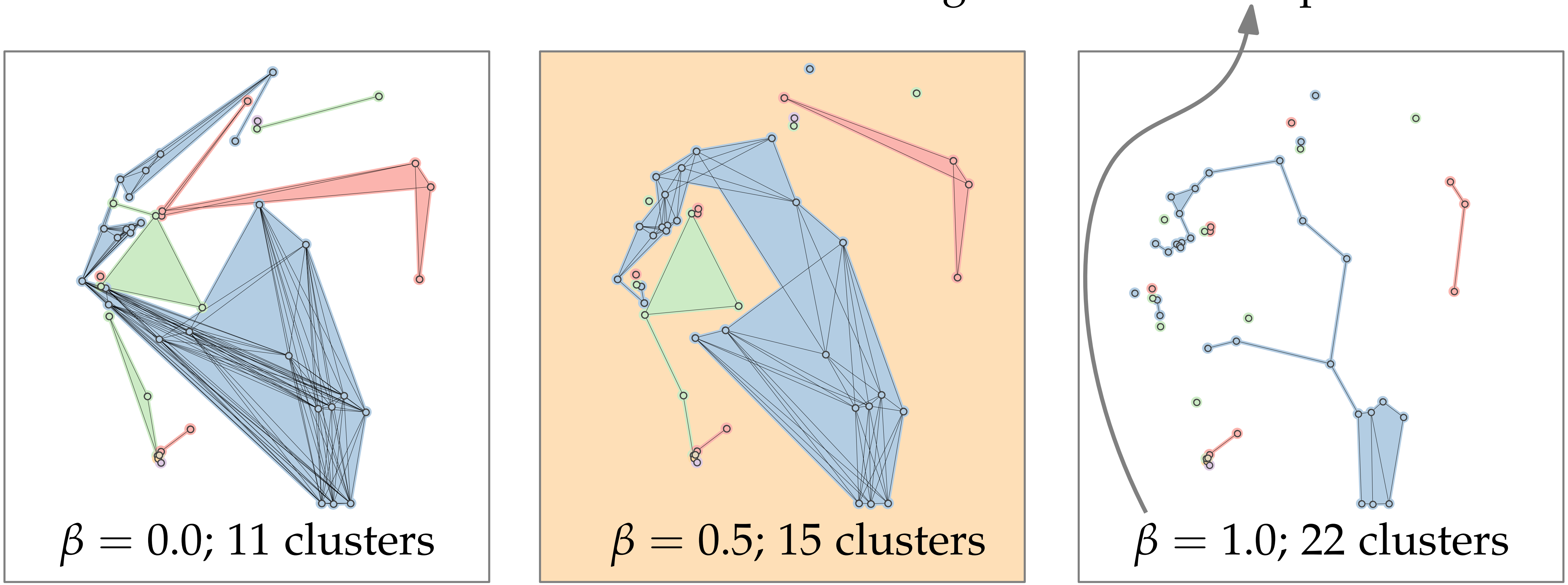
$\beta = 1.0$; 22 clusters

Proximity Graph



Proximity Graph





Case Study

Case Study

UBN: University of Bonn, 78 points

Case Study

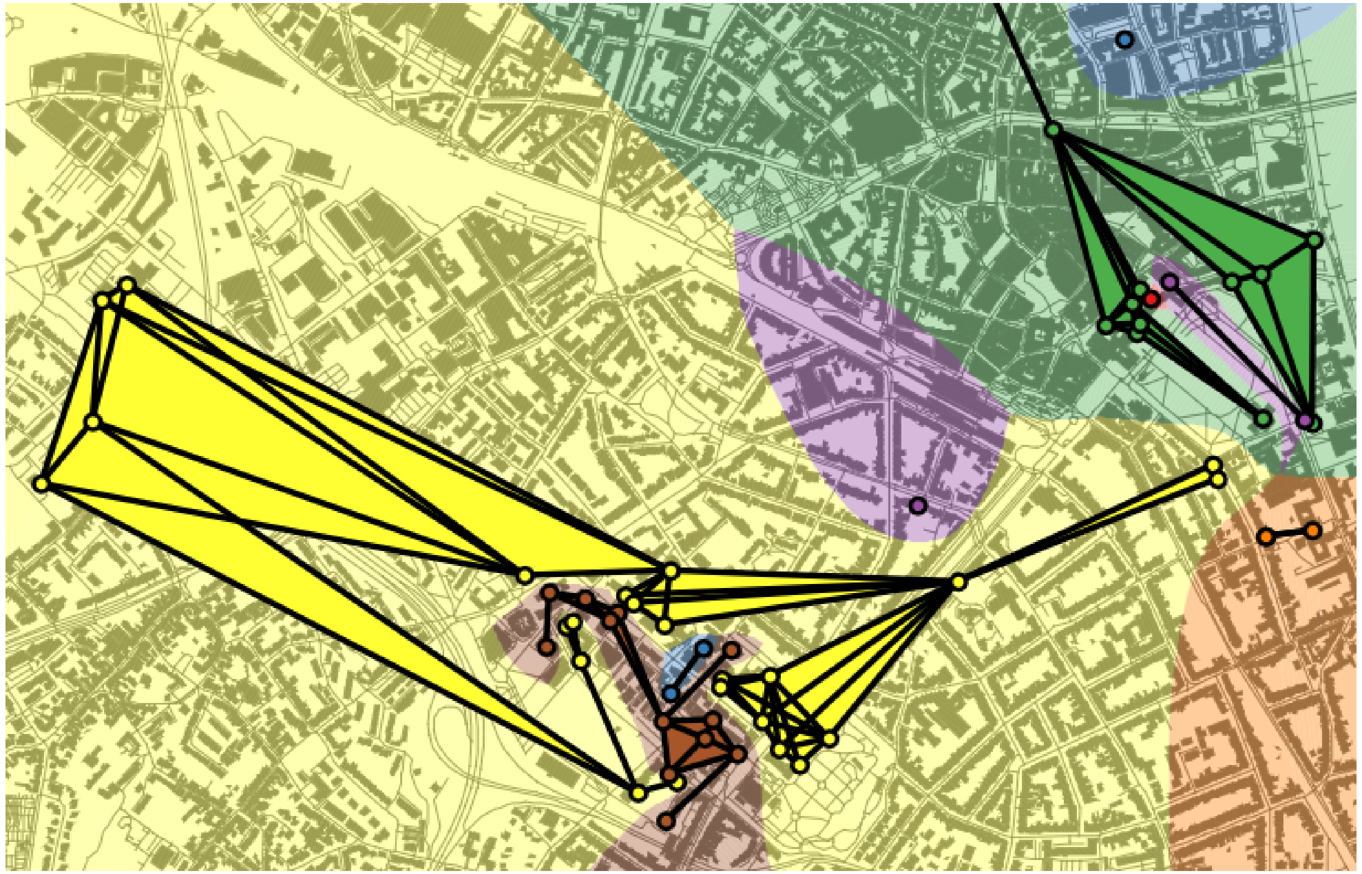
UBN: University of Bonn, 78 points

Voronoi Diagram

Case Study

UBN: University of Bonn, 78 points

Voronoi Diagram



Greedy Algorithm
0.5-skeleton
Line Voronoi Diagram
+ Polygon Representation

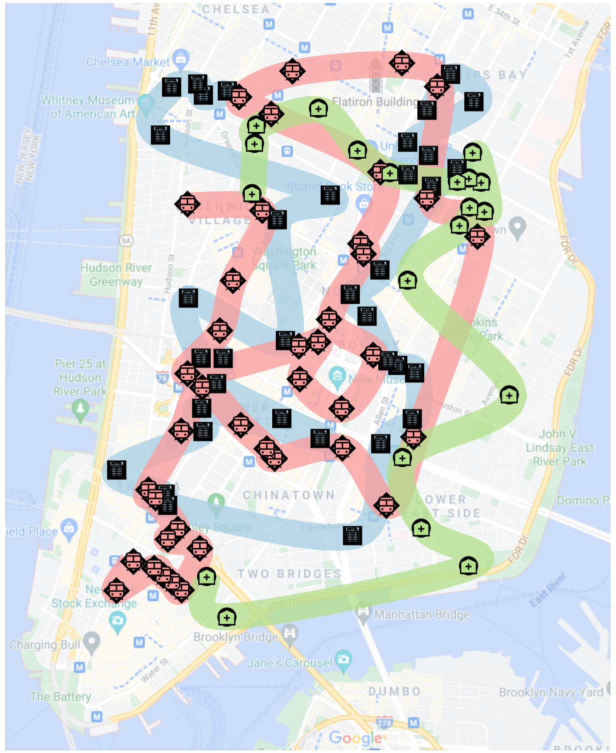
Case Study

NYC: Manhattan, New York, 96 points

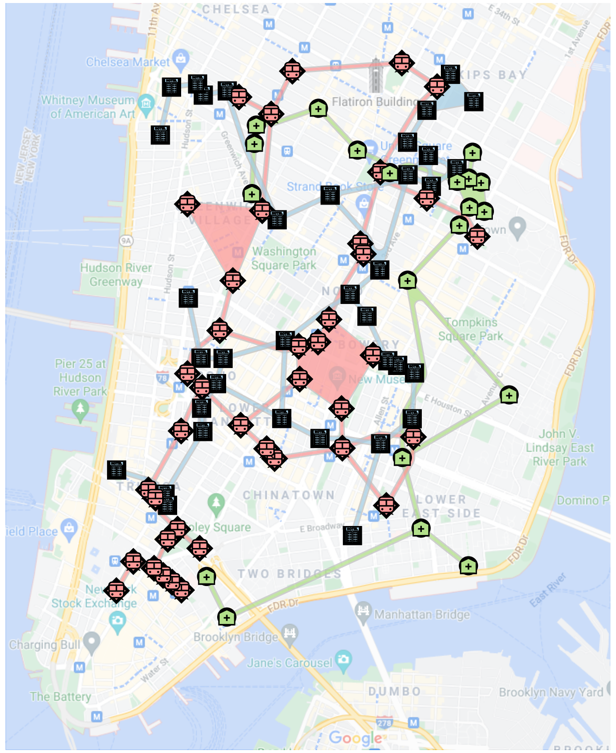
Case Study

NYC: Manhattan, New York, 96 points

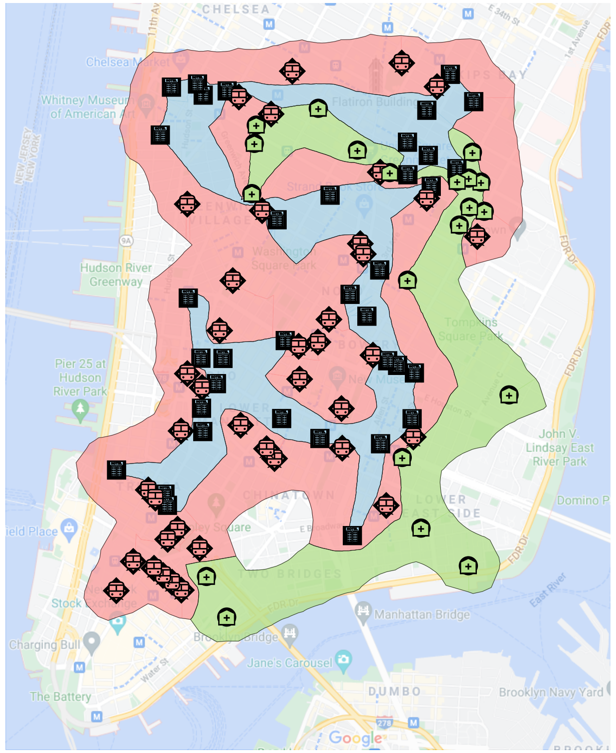
Bubble Sets



LineSets

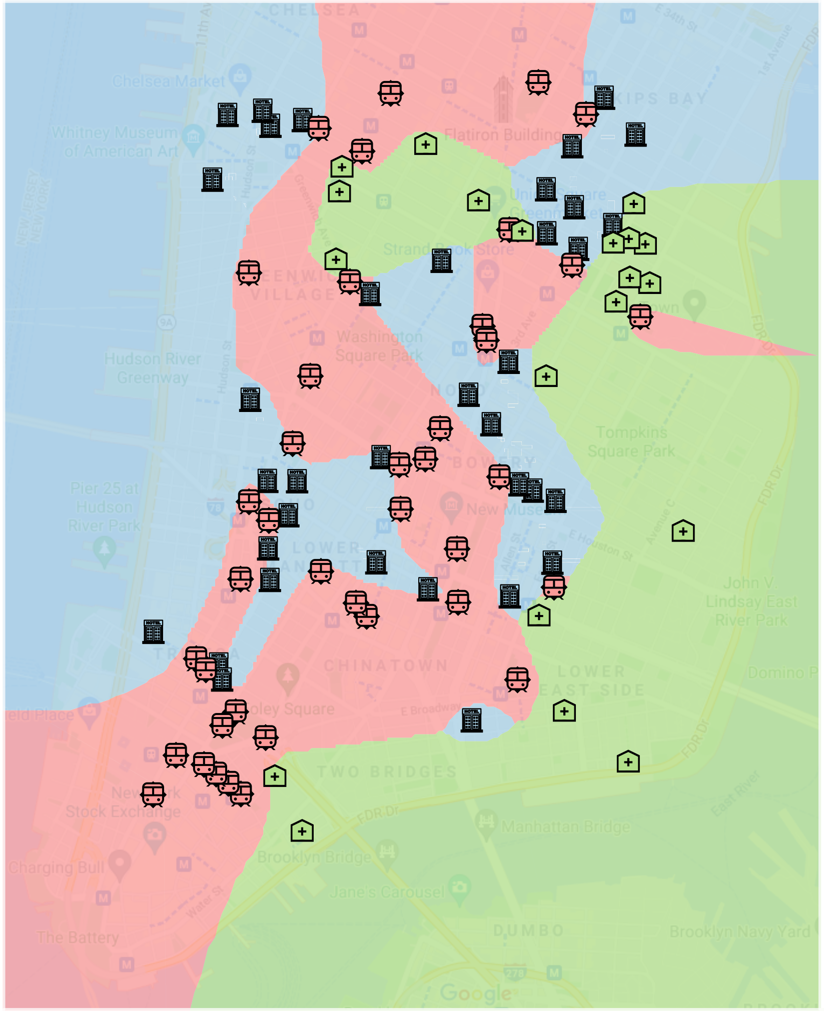


KelpFusion

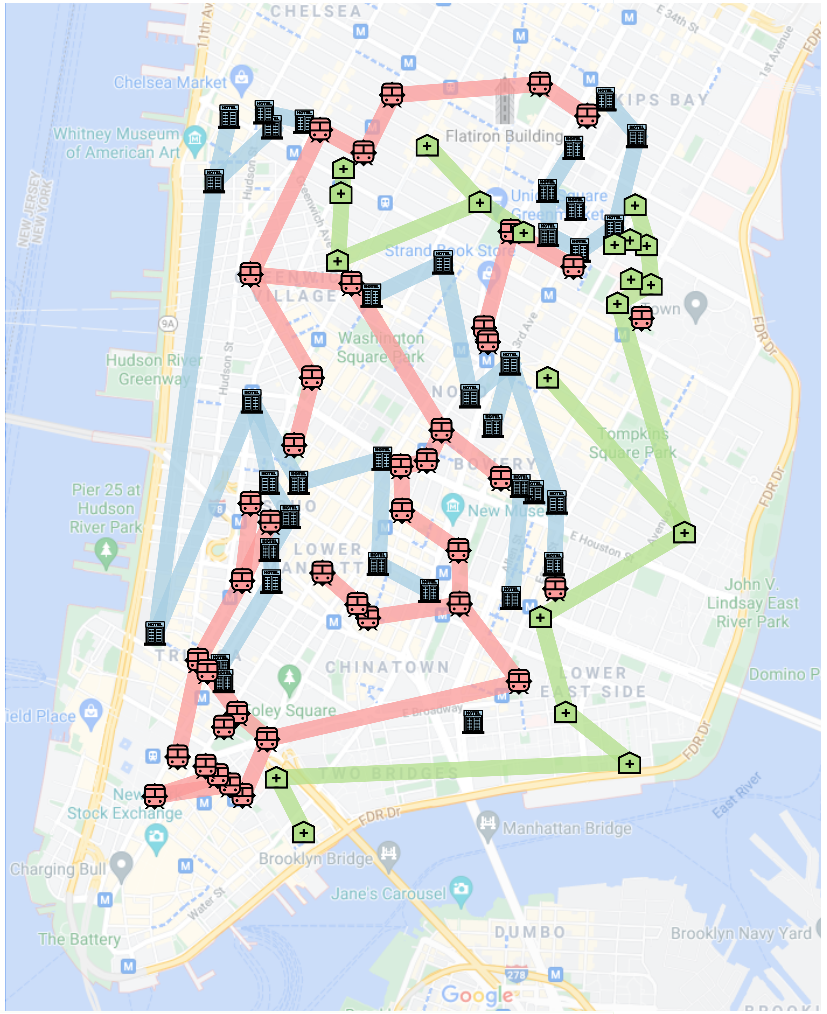


MapSets

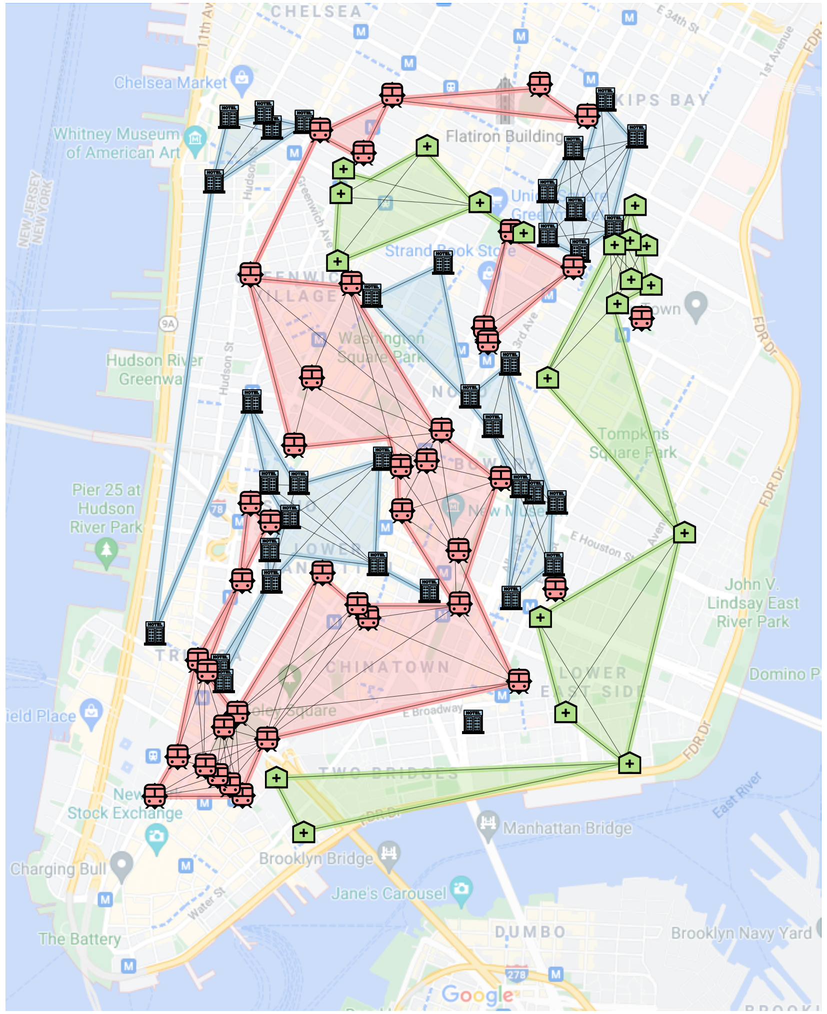
NYC: Manhattan, New York, 96 points



ClusterSets:
Line-Voronoi



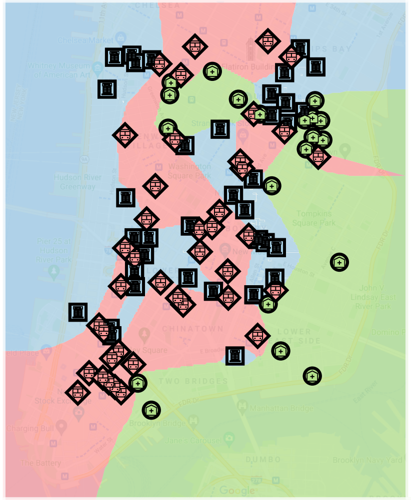
ClusterSets:
Tree Repr.



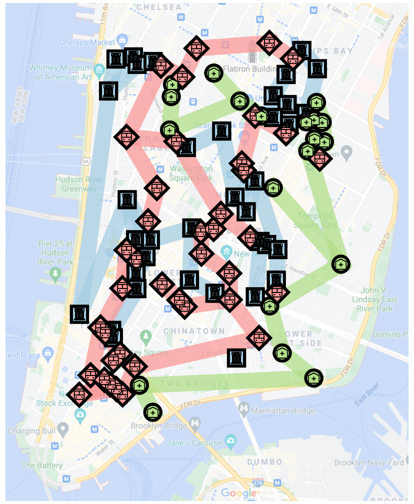
ClusterSets:
Polygon Repr.

Case Study

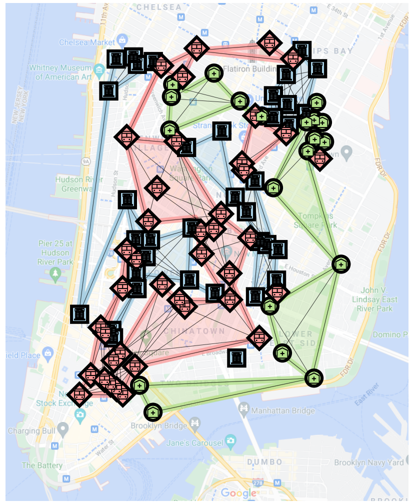
NYC: Manhattan, New York, 96 points



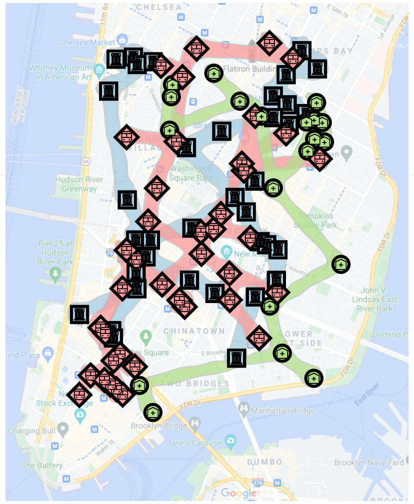
ClusterSets:
Line-Voronoi



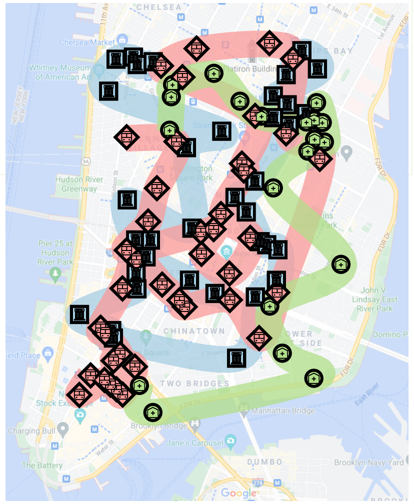
ClusterSets:
Tree Repr.



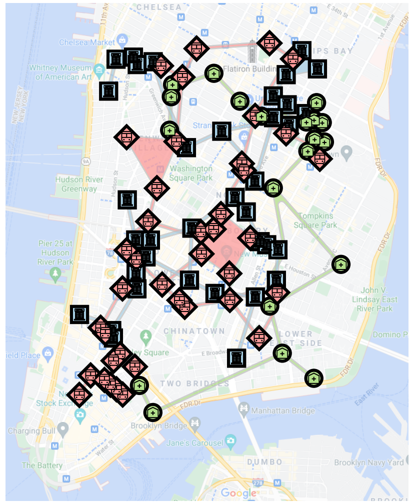
ClusterSets:
Polygon Repr.



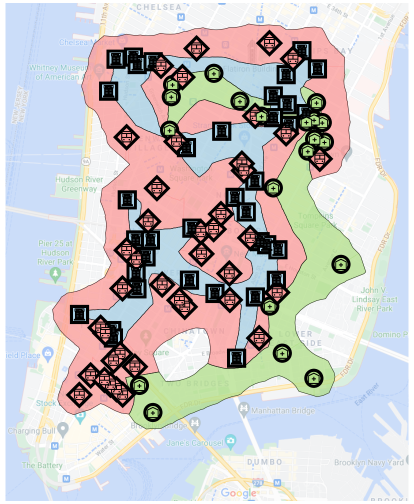
Bubble Sets



LineSets



KelpFusion



MapSets

Case Study

OSM: City of Bonn, 16320 points (OpenStreetMap)

Case Study

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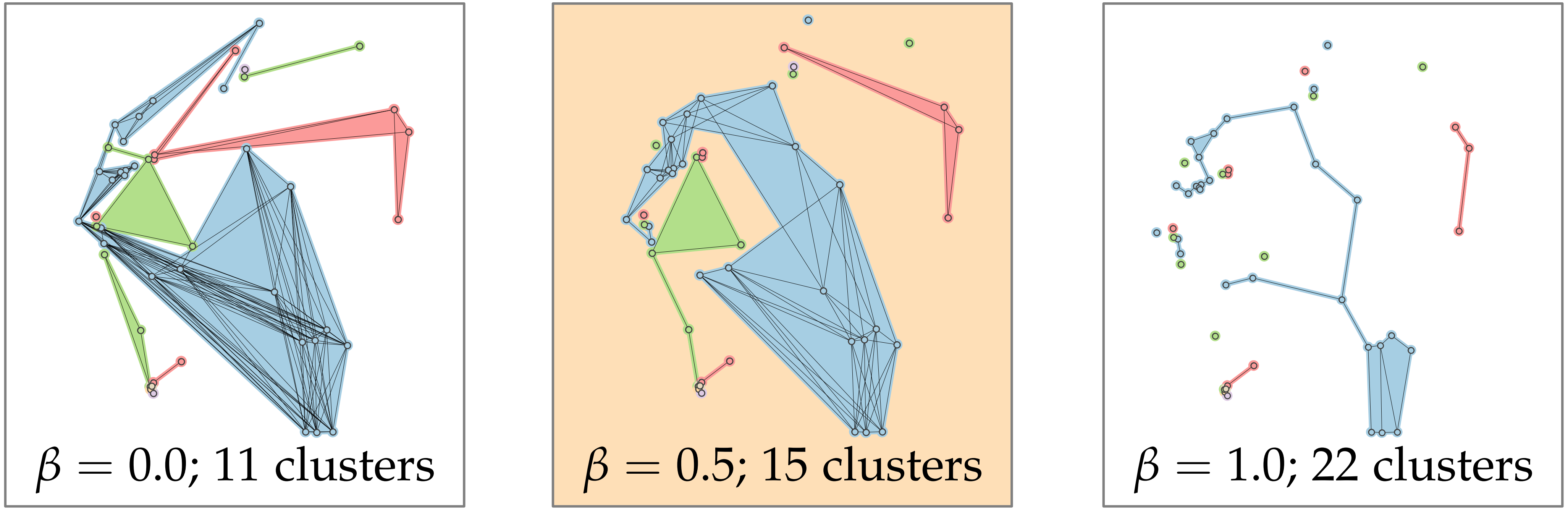
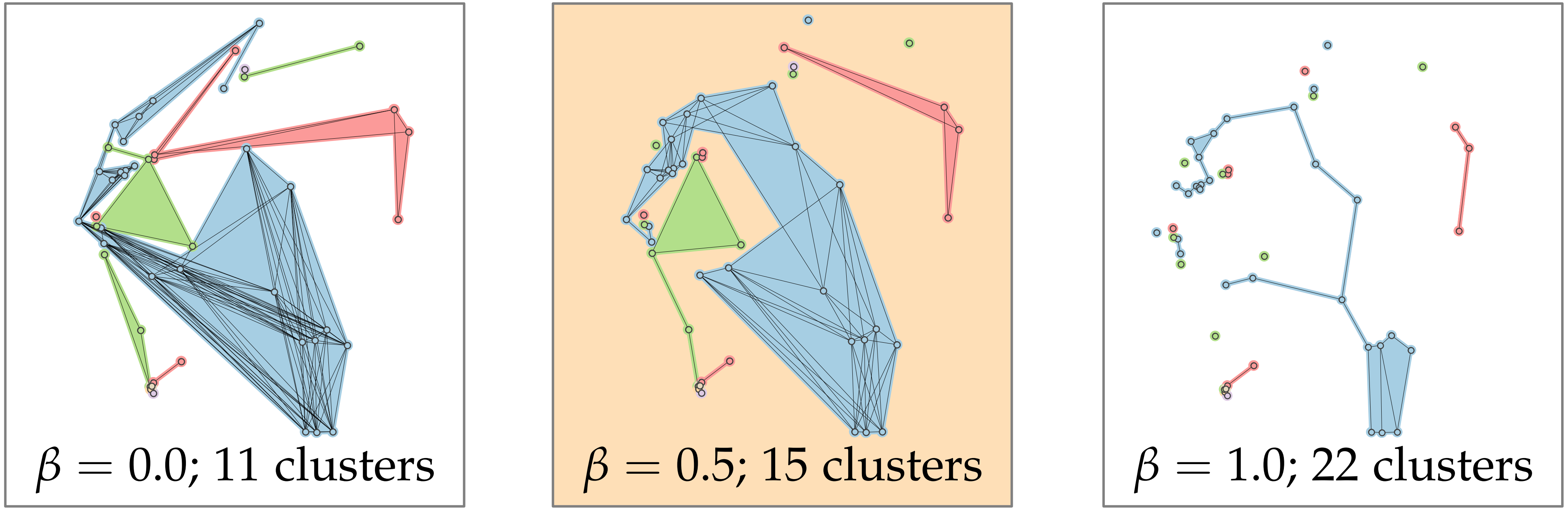
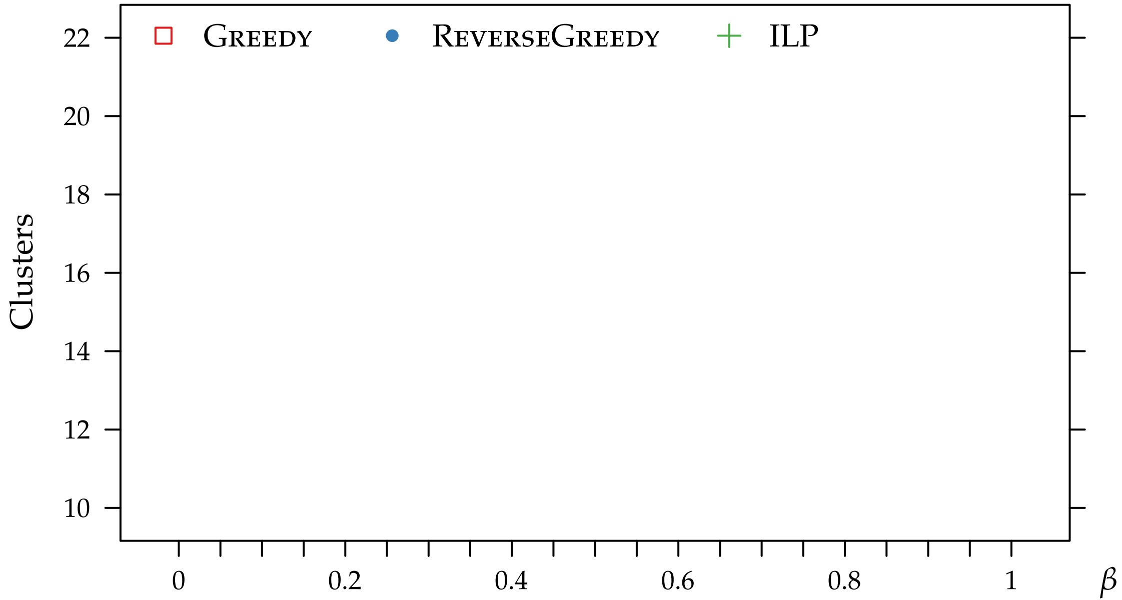


Case Study

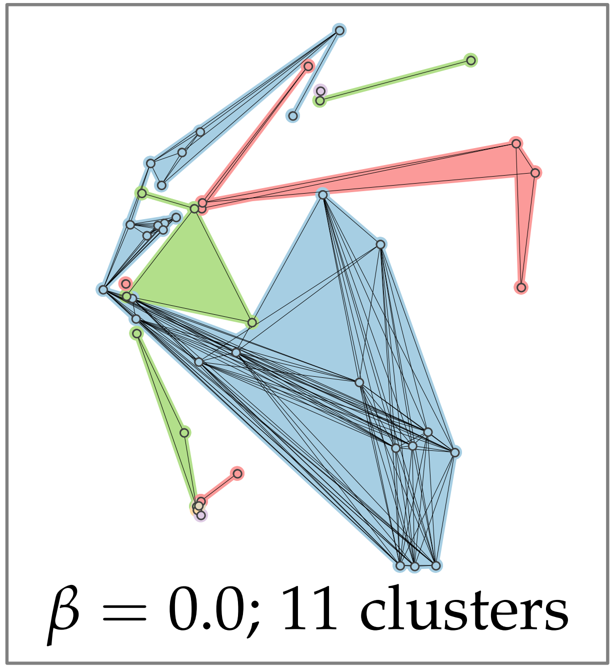
OSM: City of Bonn, 16320 points (OpenStreetMap)

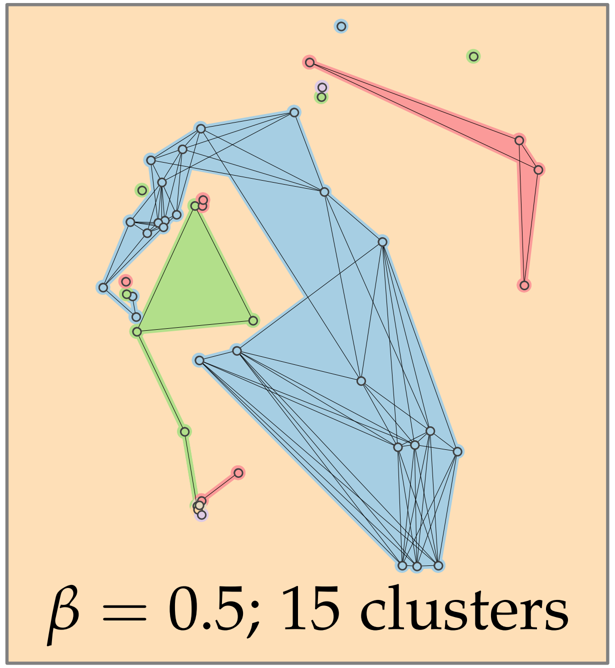


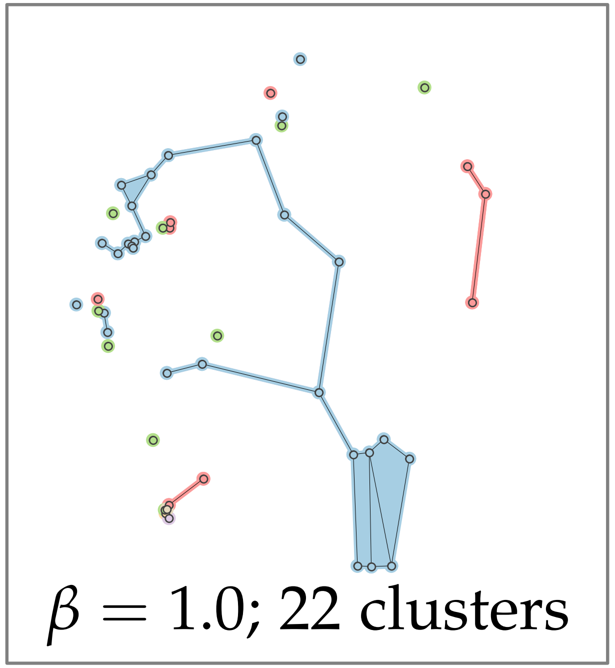


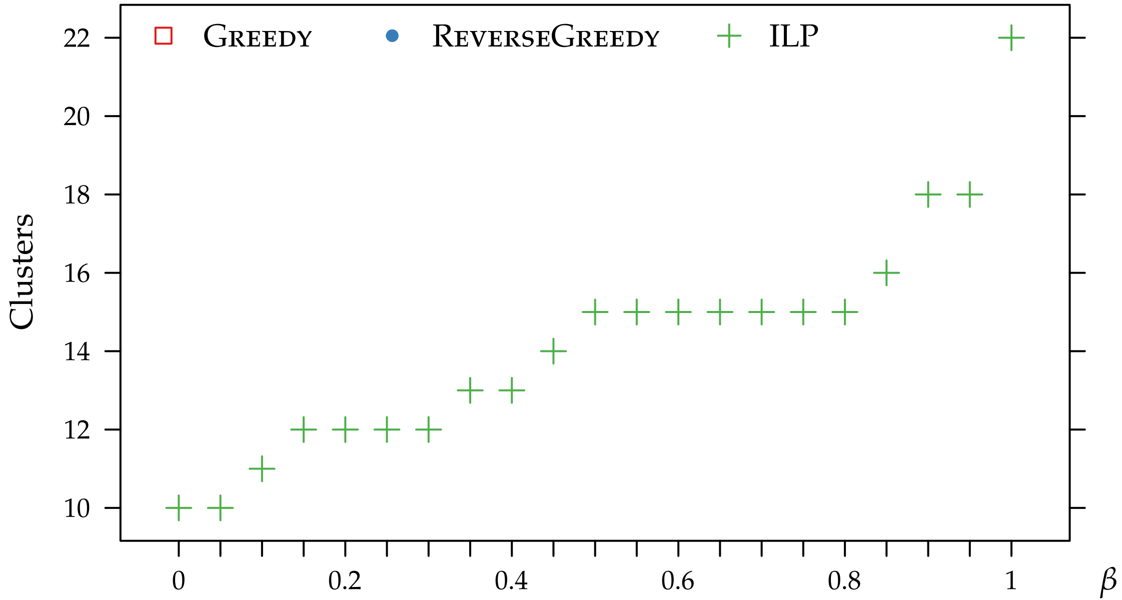
Case Study

OSM: City of Bonn, 16320 points (OpenStreetMap)



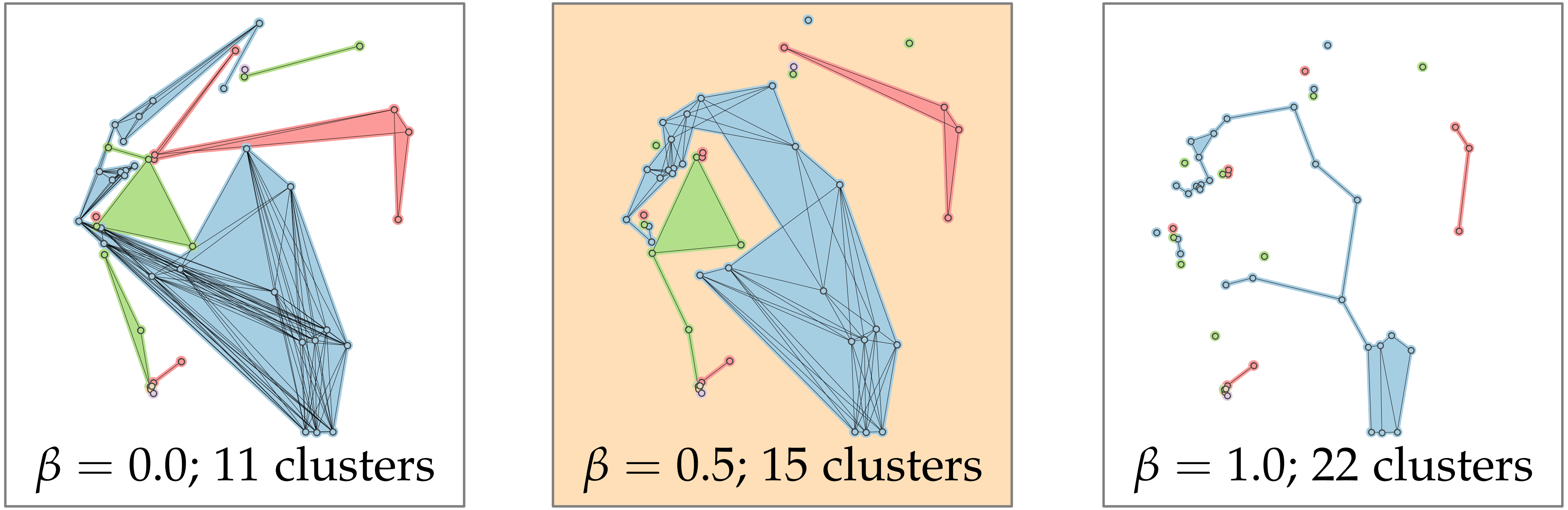


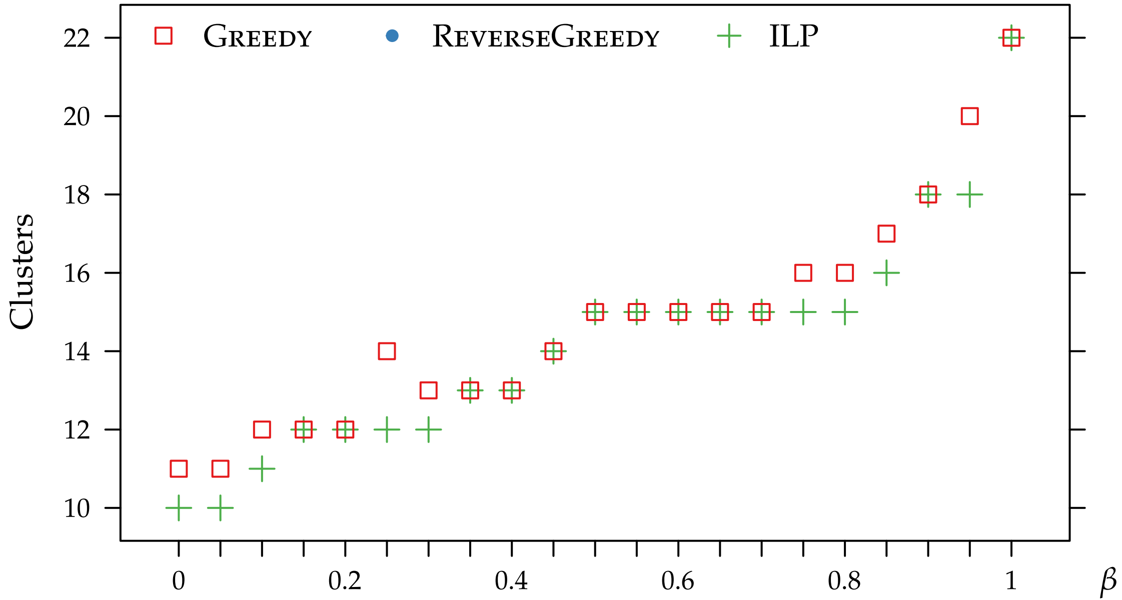




Case Study

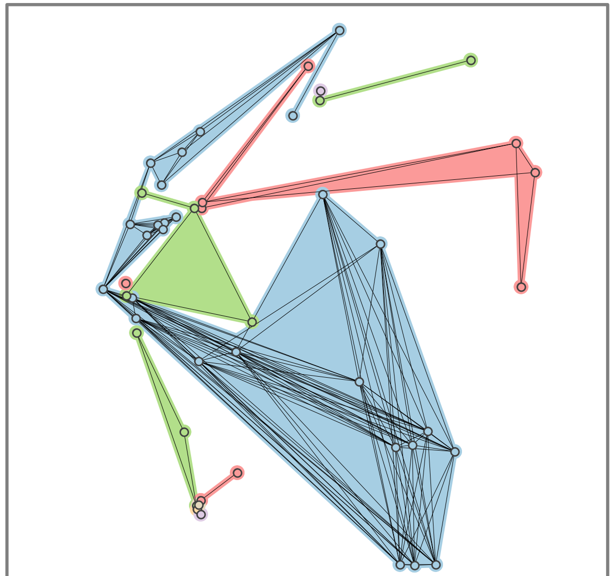
OSM: City of Bonn, 16320 points (OpenStreetMap)



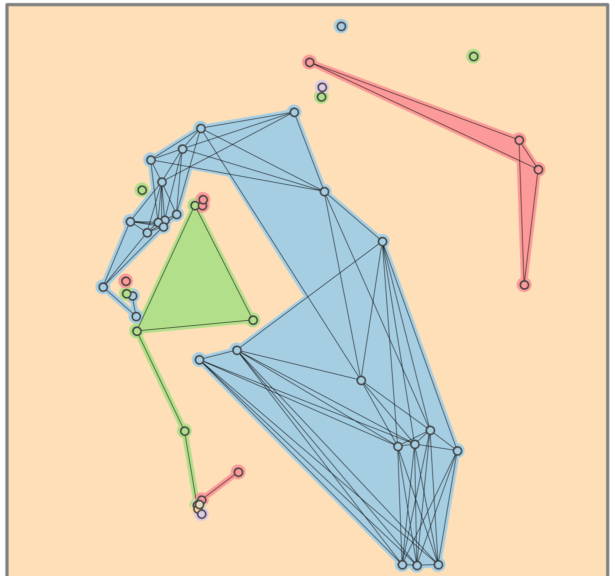


Case Study

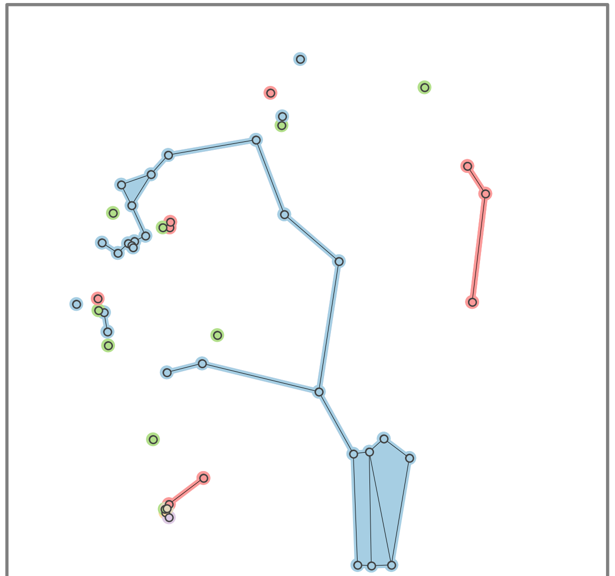
OSM: City of Bonn, 16320 points (OpenStreetMap)



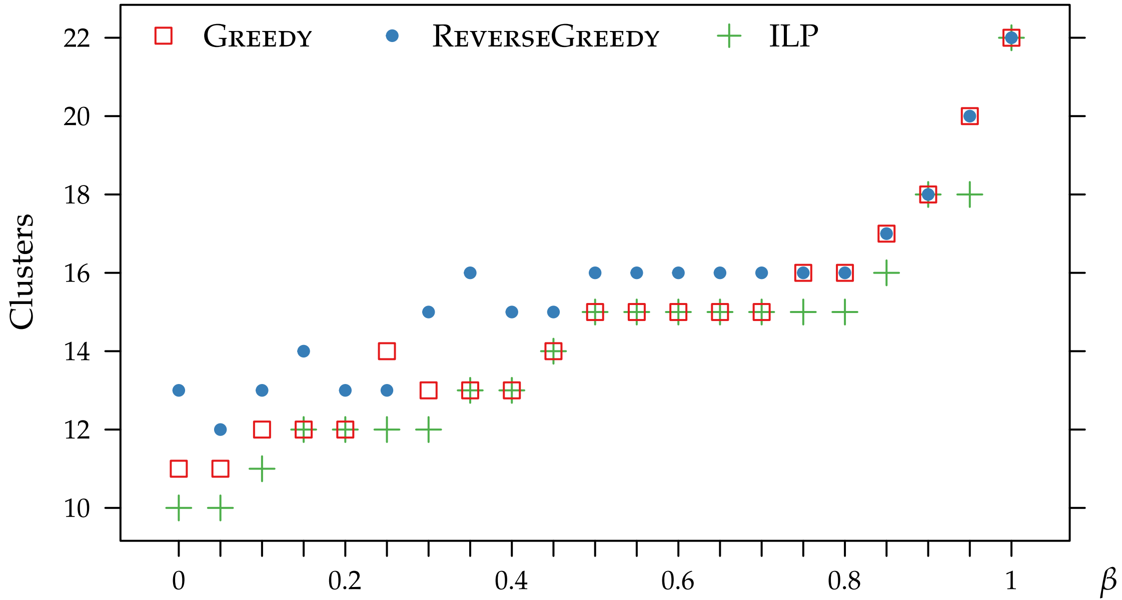
$\beta = 0.0$; 11 clusters



$\beta = 0.5$; 15 clusters



$\beta = 1.0$; 22 clusters



Quantitative Experiments

Quantitative Experiments

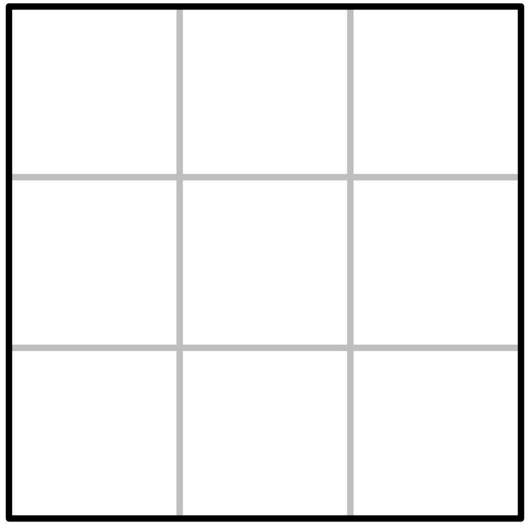
- Based on **OSM**

Quantitative Experiments

- Based on **OSM**

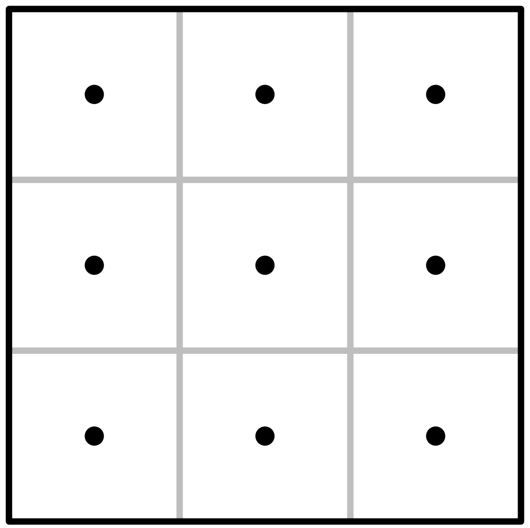
Quantitative Experiments

- Based on **OSM**

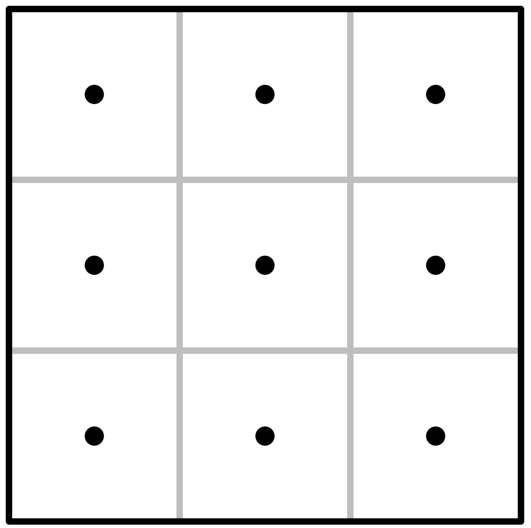


Quantitative Experiments

- Based on **OSM**

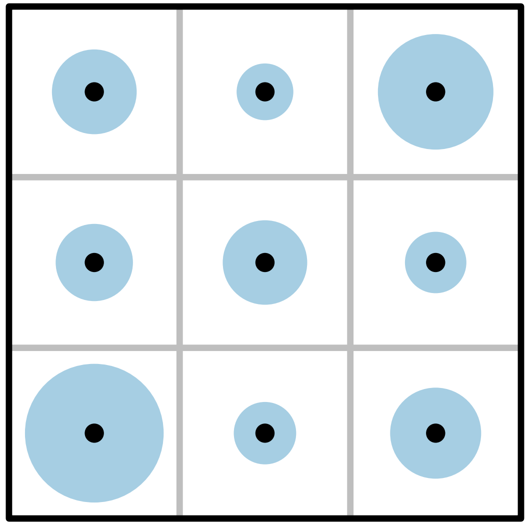


Quantitative Experiments



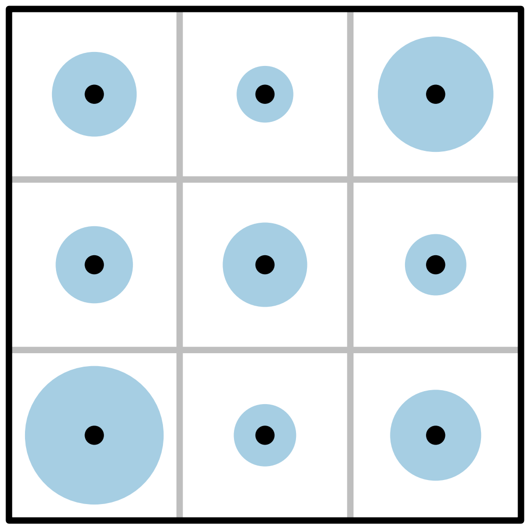
- Based on **OSM**
- $n = 50, 100, 150, 200, 250$

Quantitative Experiments



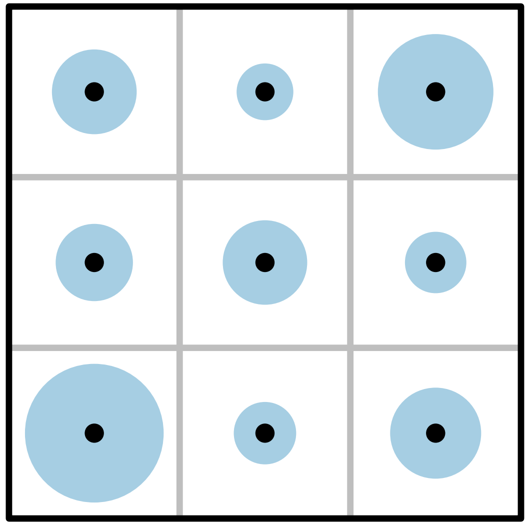
- Based on **OSM**
- $n = 50, 100, 150, 200, 250$

Quantitative Experiments



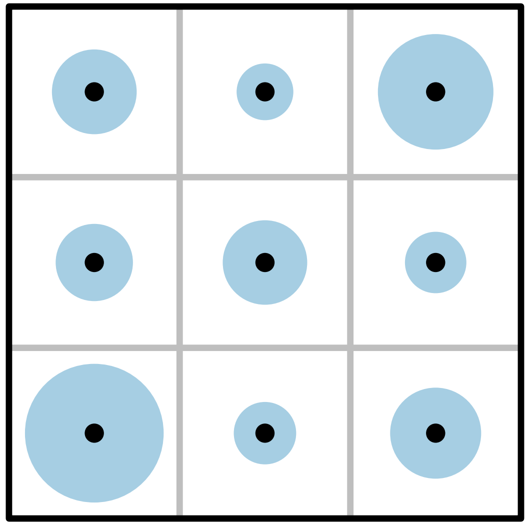
- Based on **OSM**
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Quantitative Experiments



- Based on **OSM** `https://github.com/JakobGeiger/ClusterSets`
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Quantitative Experiments



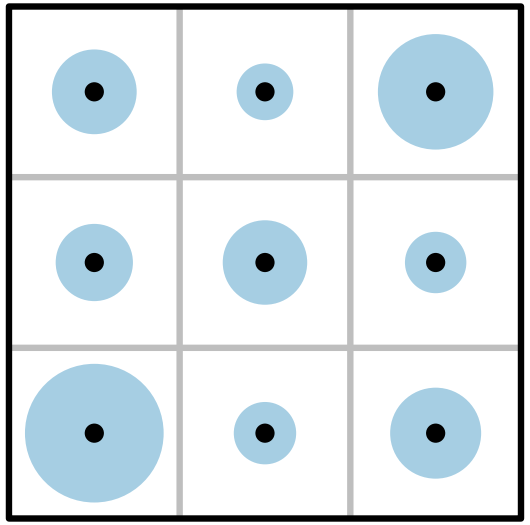
- Based on **OSM**
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

`https://github.com/JakobGeiger/ClusterSets`

Clusters

Running Time

Quantitative Experiments



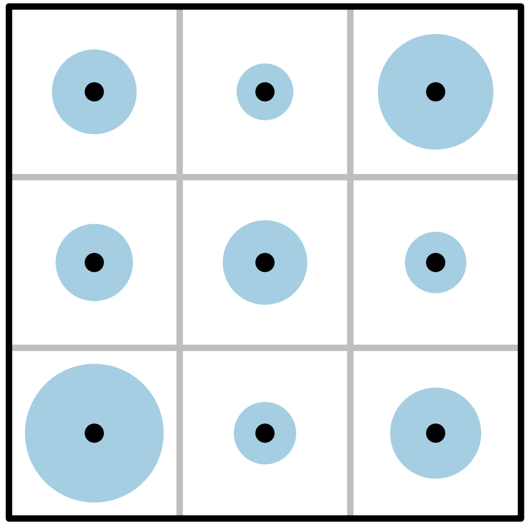
- Based on **OSM** `https://github.com/JakobGeiger/ClusterSets`
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Clusters

Running Time

GREEDY < 50 ms

Quantitative Experiments



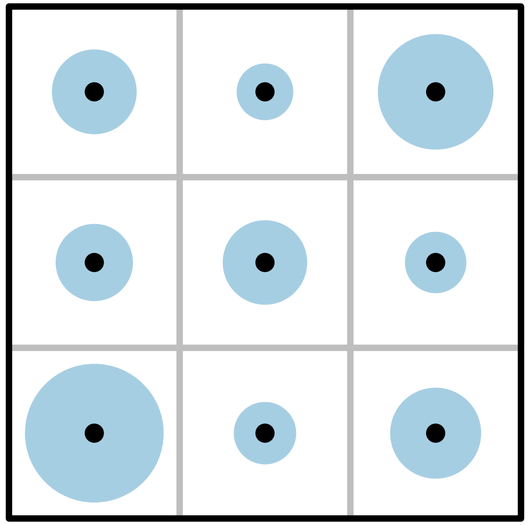
- Based on **OSM** `https://github.com/JakobGeiger/ClusterSets`
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Clusters

Running Time

GREEDY	< 50 ms
REVERSEGREEDY	< 50 ms

Quantitative Experiments



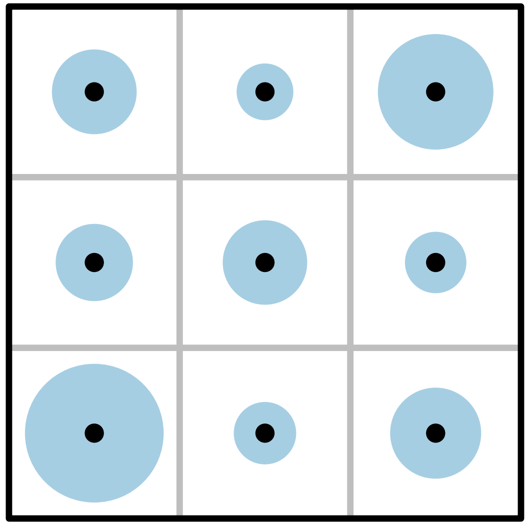
- Based on **OSM** `https://github.com/JakobGeiger/ClusterSets`
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Clusters

Running Time

Greedy	< 50 ms
ReverseGreedy	< 50 ms
ILP	

Quantitative Experiments



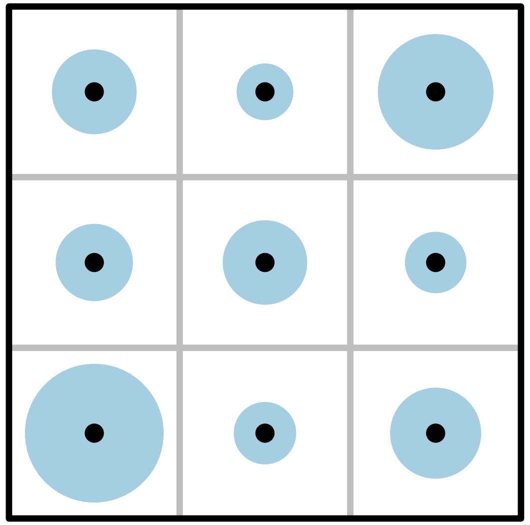
- Based on **OSM** `https://github.com/JakobGeiger/ClusterSets`
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Clusters

Running Time

Greedy		< 50 ms
ReverseGreedy		< 50 ms
ILP	50 points:	≤ 11 s

Quantitative Experiments



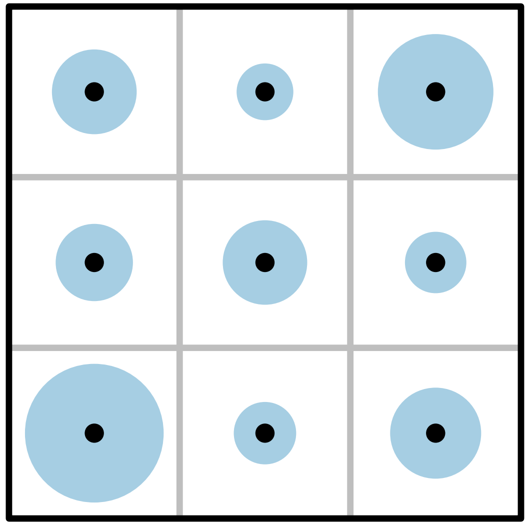
- Based on **OSM** `https://github.com/JakobGeiger/ClusterSets`
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Clusters

Running Time

GREEDY		< 50 ms
REVERSEGREEDY		< 50 ms
ILP	50 points:	≤ 11 s
	100 points:	≤ 1.8 h

Quantitative Experiments



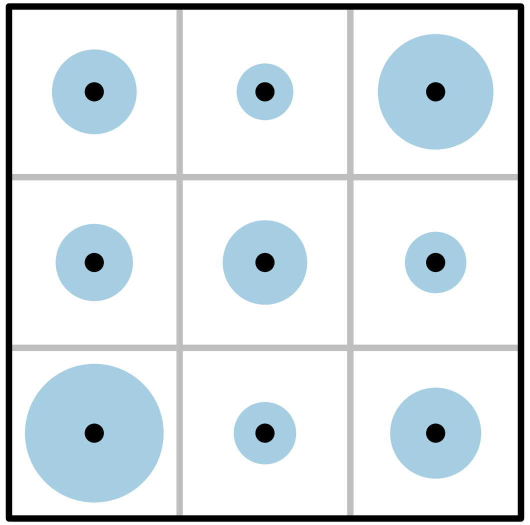
- Based on **OSM** `https://github.com/JakobGeiger/ClusterSets`
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

Clusters

Running Time

GREEDY		< 50 ms
REVERSEGREEDY		< 50 ms
ILP	50 points:	≤ 11 s
	100 points:	≤ 1.8 h
	150 points:	> 2 d

Quantitative Experiments



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- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

$\beta = 0.5$

Clusters

Running Time

GREEDY		< 50 ms
REVERSEGREEDY		< 50 ms
ILP	50 points:	≤ 11 s
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Quantitative Experiments

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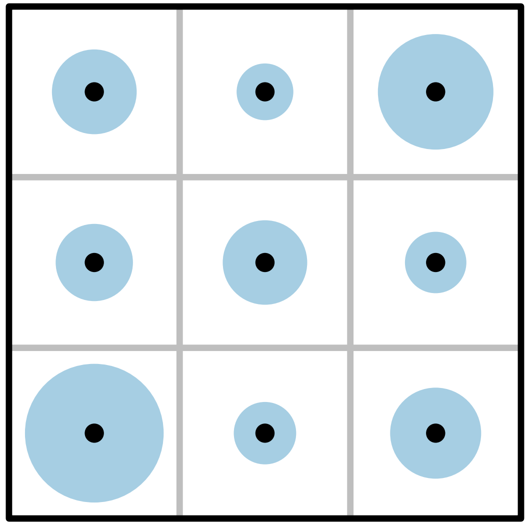
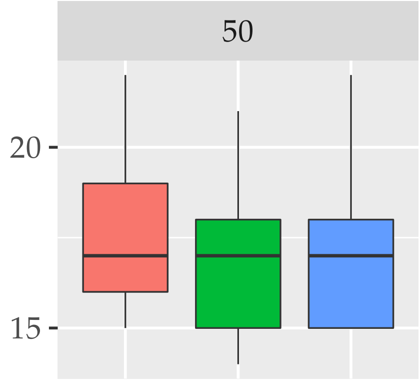
`https://github.com/JakobGeiger/ClusterSets`



$\beta = 0.5$

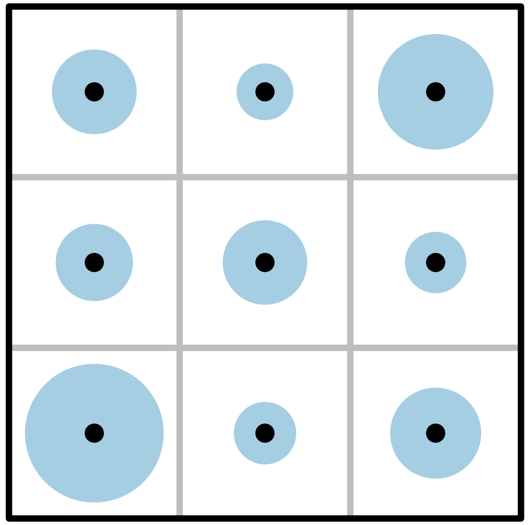
Clusters



Running Time

GREEDY		< 50 ms
REVERSEGREEDY		< 50 ms
ILP	50 points:	≤ 11 s
	100 points:	≤ 1.8 h
	150 points:	> 2 d

Quantitative Experiments



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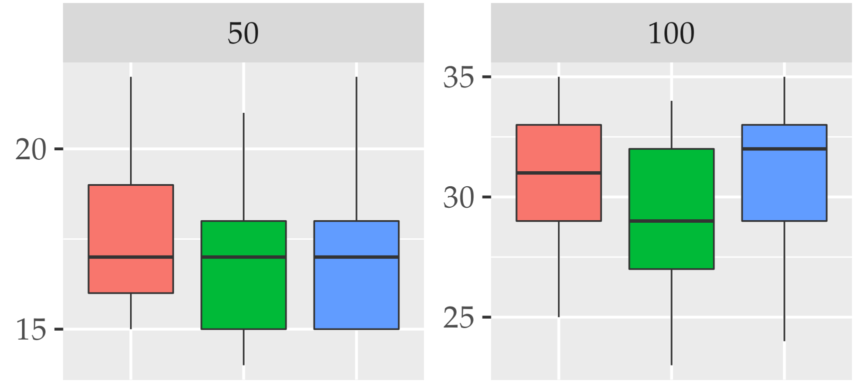
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

$\beta = 0.5$ # Clusters



Running Time

GREEDY		< 50 ms
REVERSEGREEDY		< 50 ms
ILP	50 points:	≤ 11 s
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Quantitative Experiments

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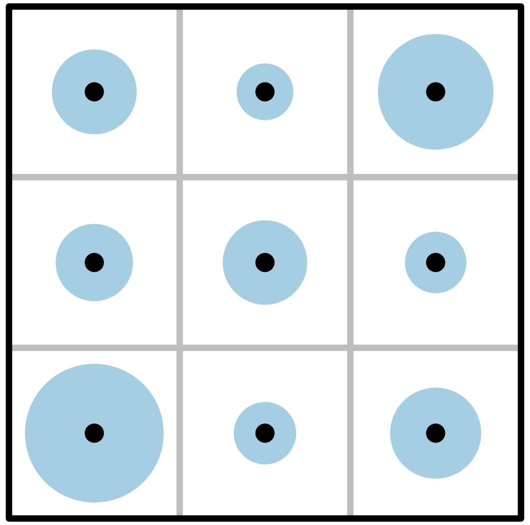
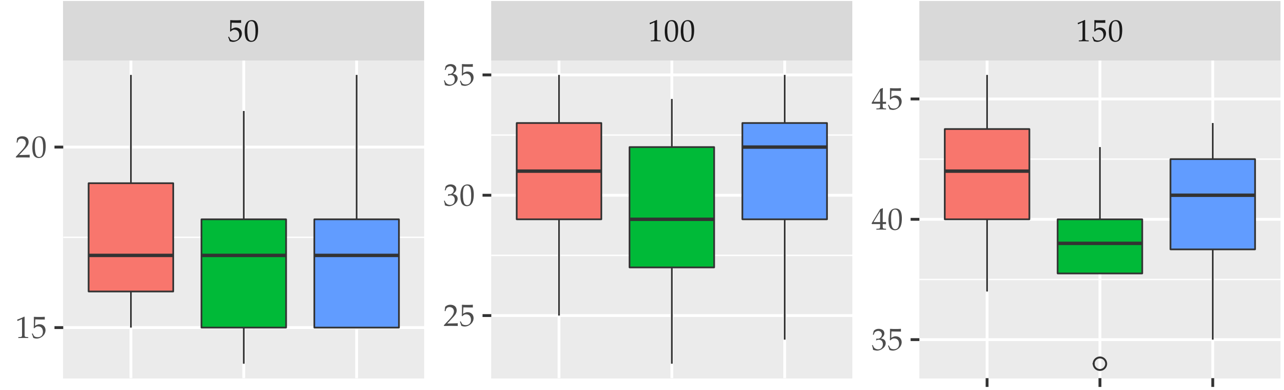


$\beta = 0.5$ # Clusters



Running Time

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ILP	50 points:	≤ 11 s
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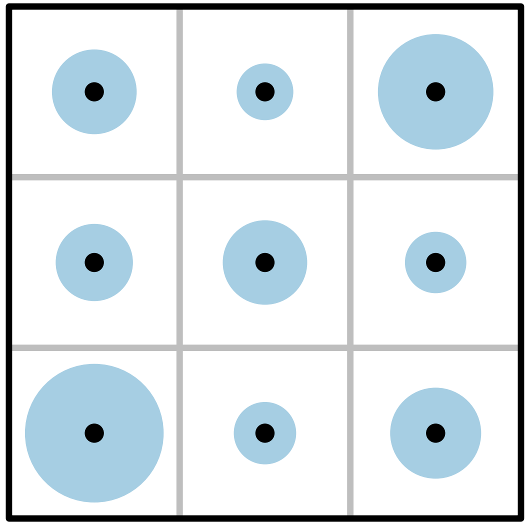
Quantitative Experiments



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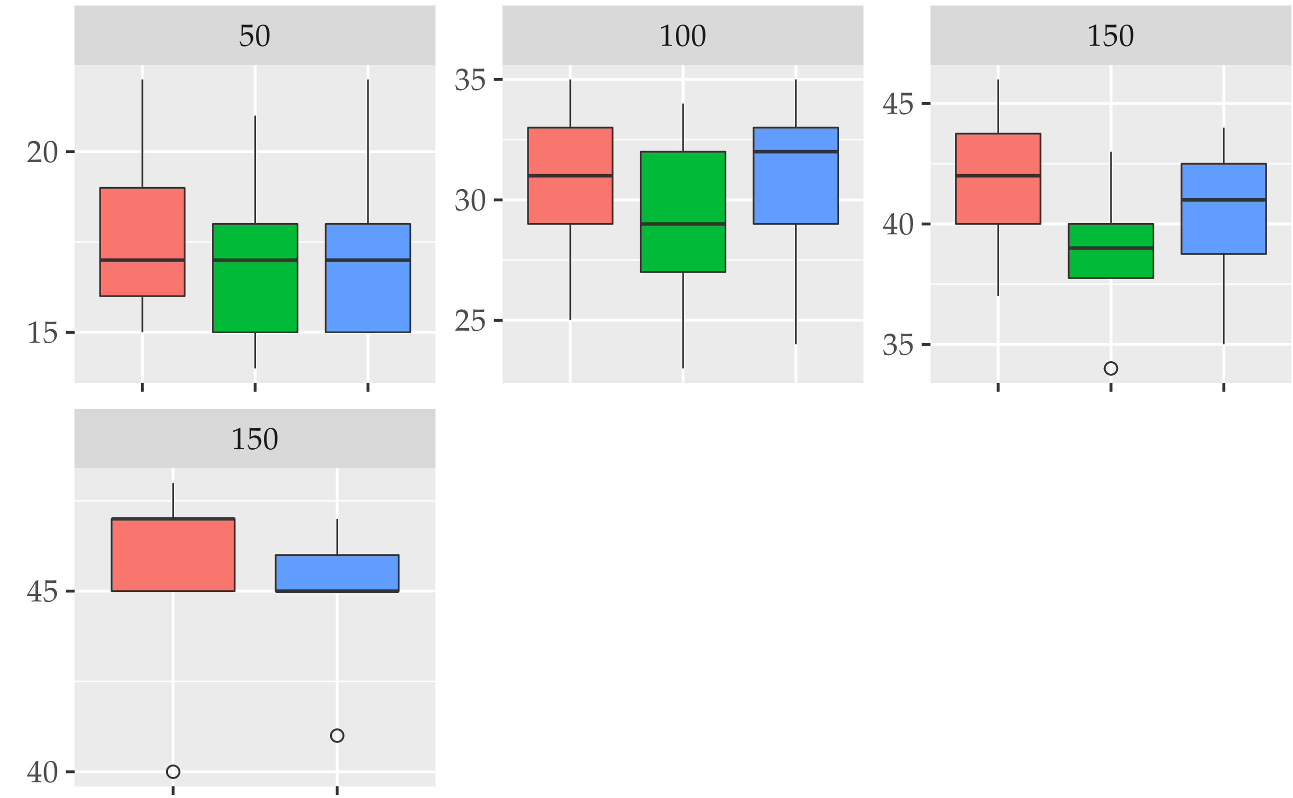
$\beta = 0.5$ # Clusters



Running Time

GREEDY		< 50 ms
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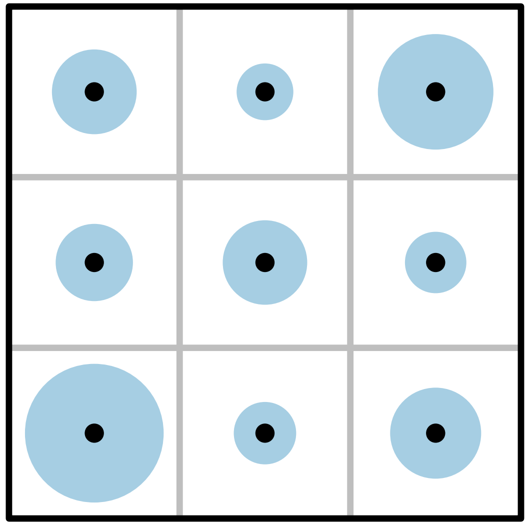
Quantitative Experiments



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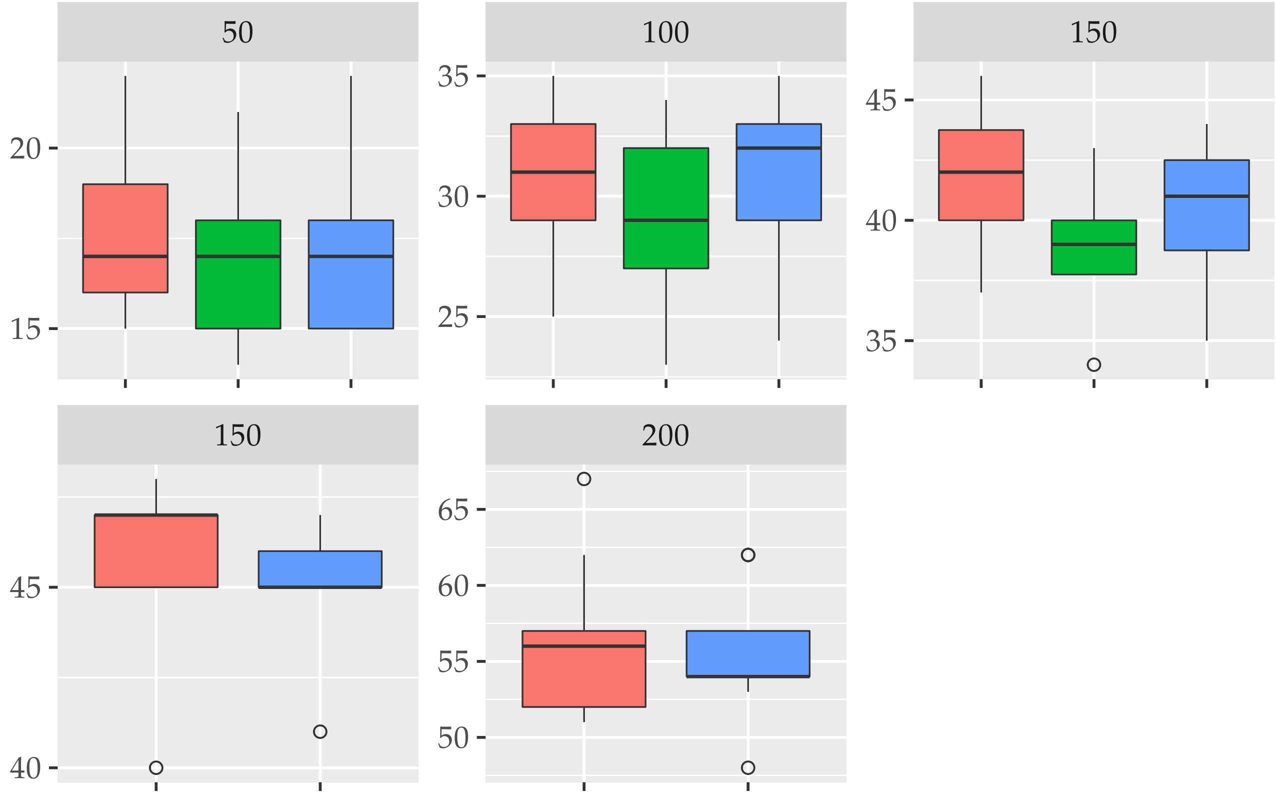

Running Time

GREEDY		< 50 ms
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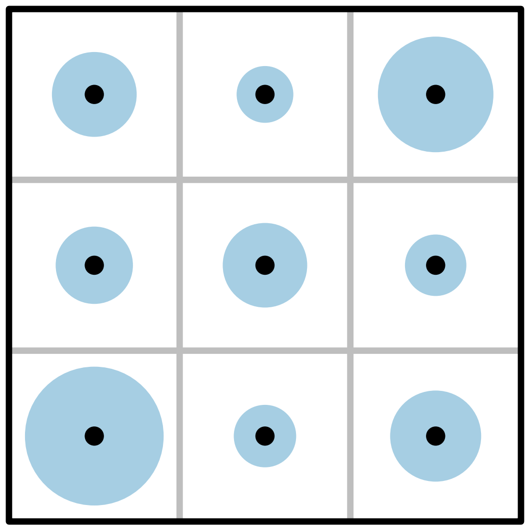
Quantitative Experiments



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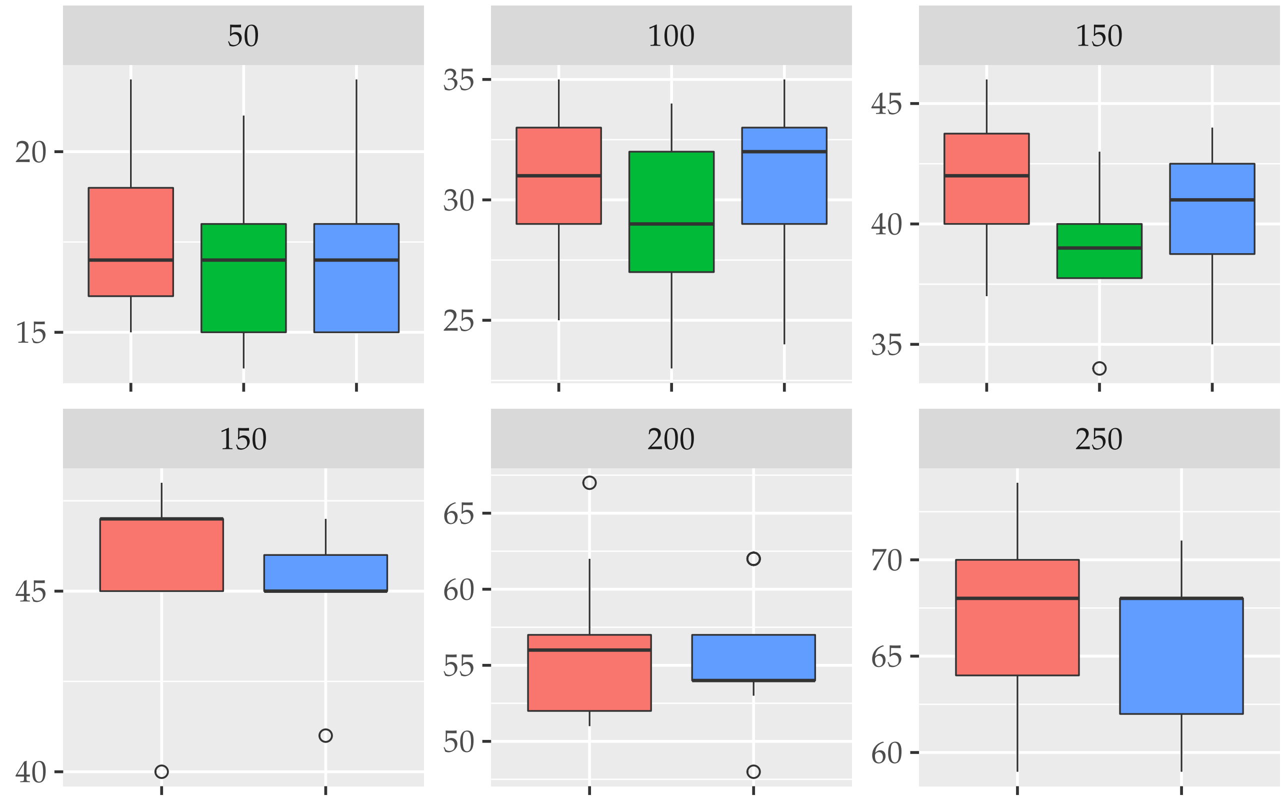
- $\beta = 0.5,\ 0.55,\ \dots,\ 0.9$

$\beta = 0.5$ # Clusters



Running Time

GREEDY		< 50 ms
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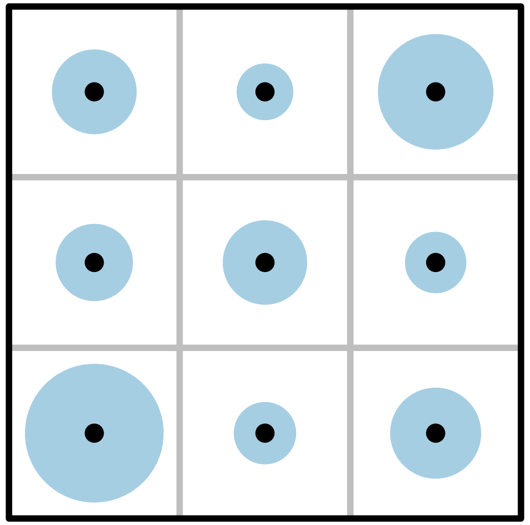
Quantitative Experiments



- Based on **OSM**

`https://github.com/JakobGeiger/ClusterSets`

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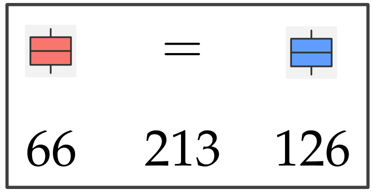
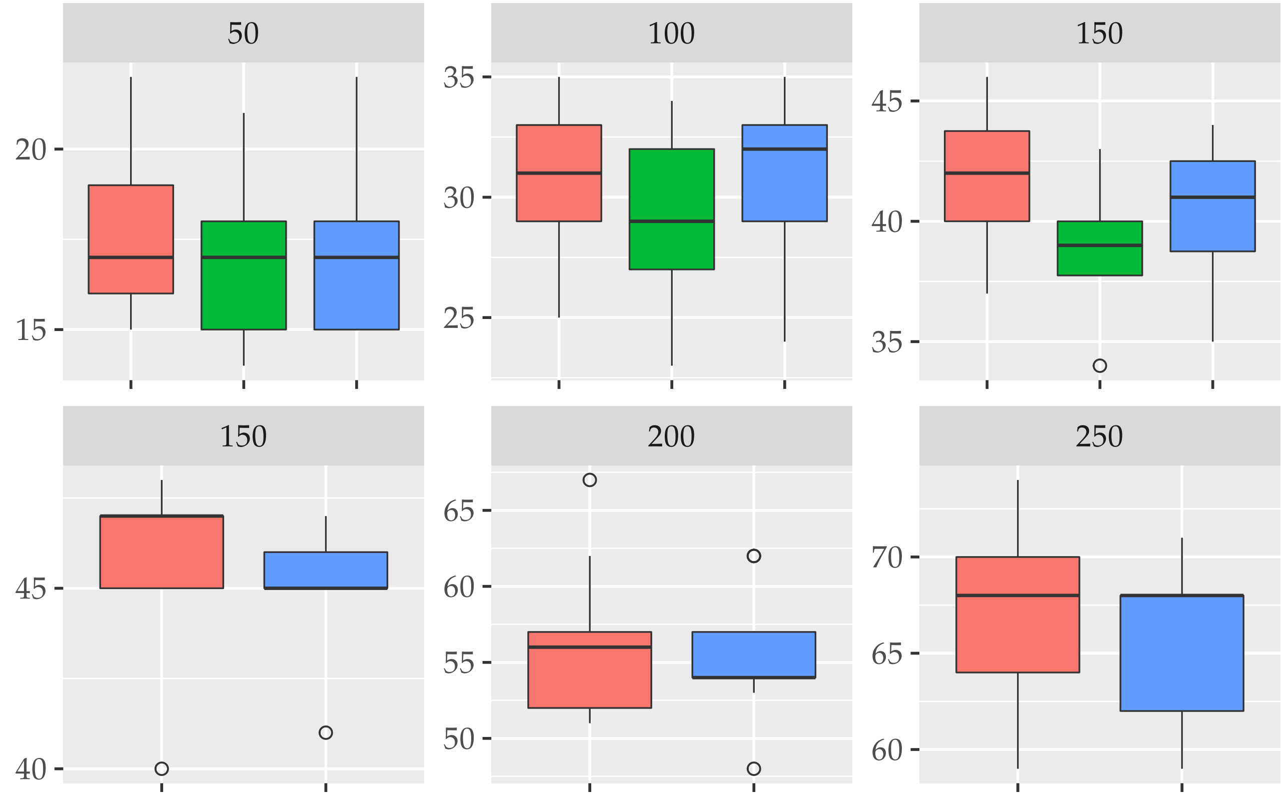
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

better:



$\beta = 0.5$

Clusters



Running Time

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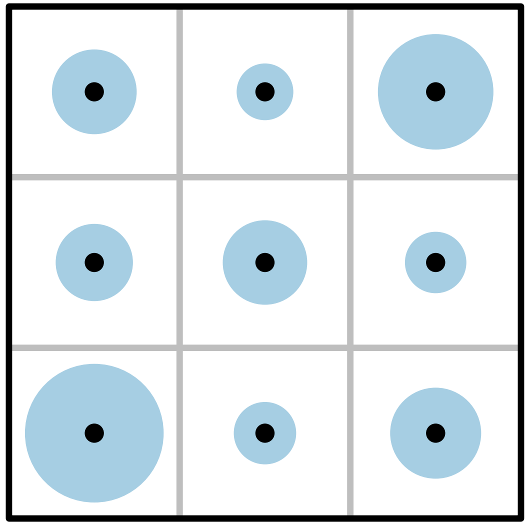
Quantitative Experiments



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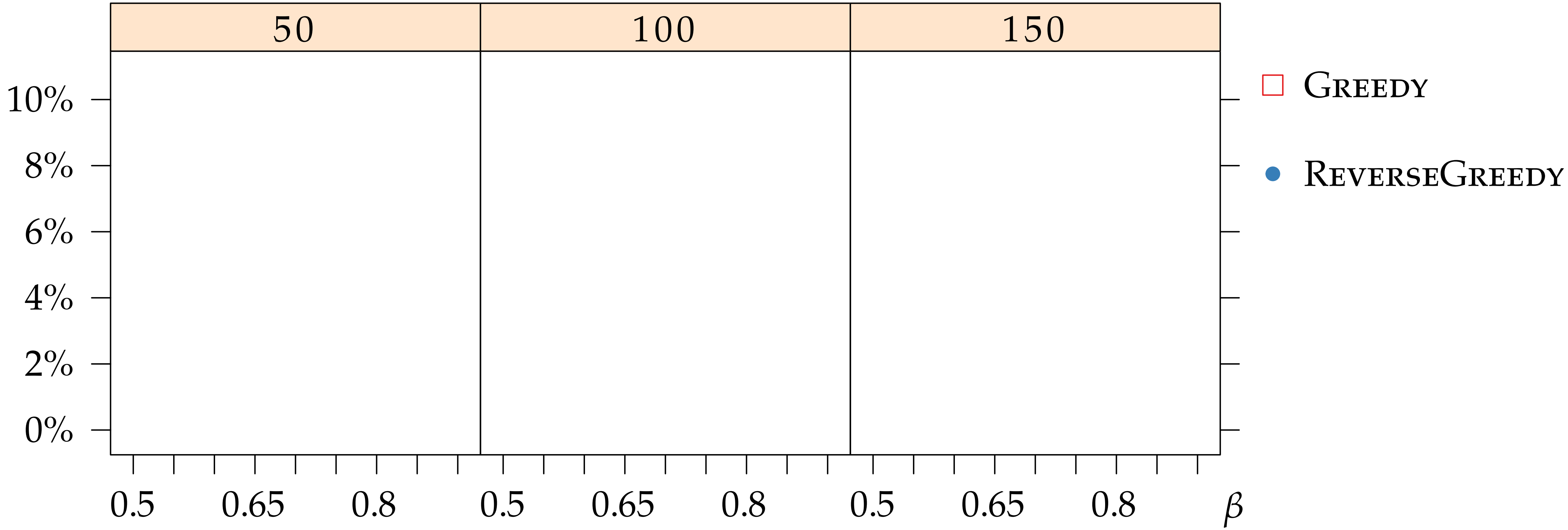
\# Clusters more than optimum



Quantitative Experiments

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- $n = 50, 100, 150, 200, 250$
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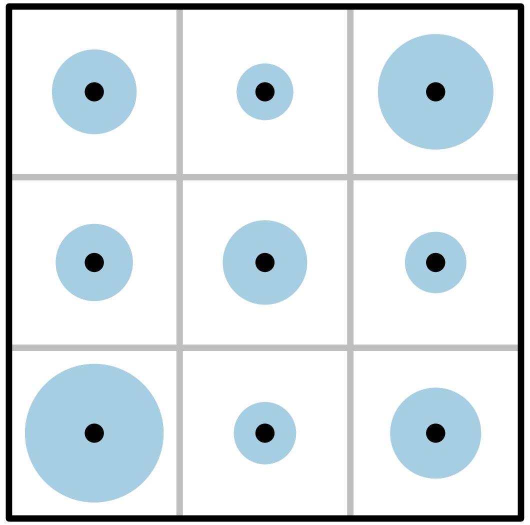
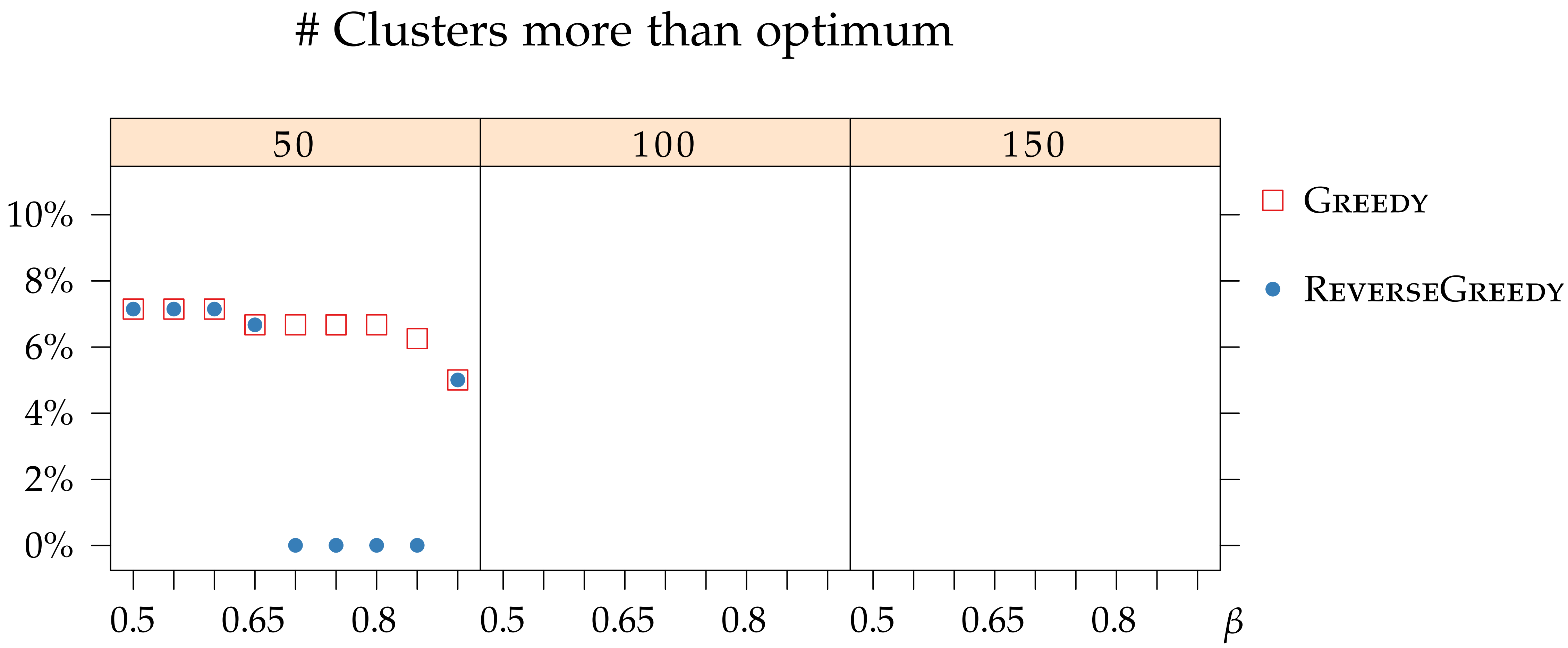
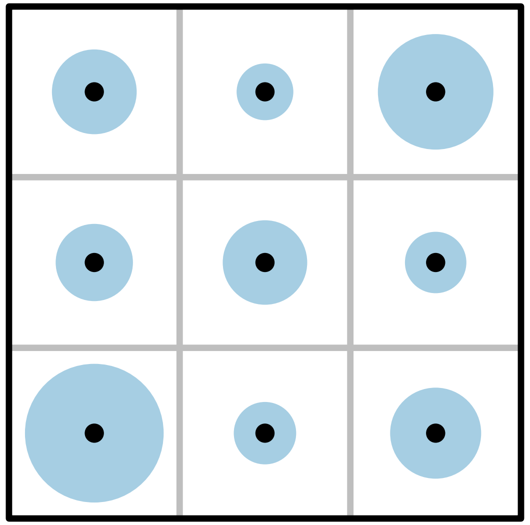




Quantitative Experiments



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- $n = 50, 100, 150, 200, 250$
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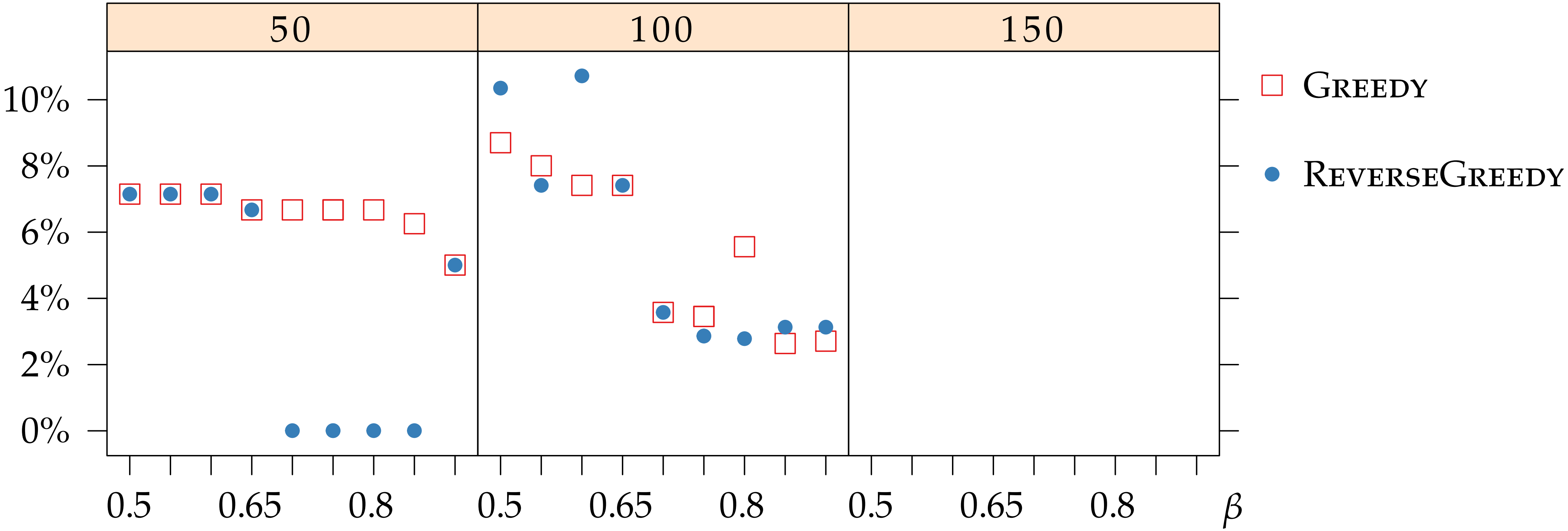
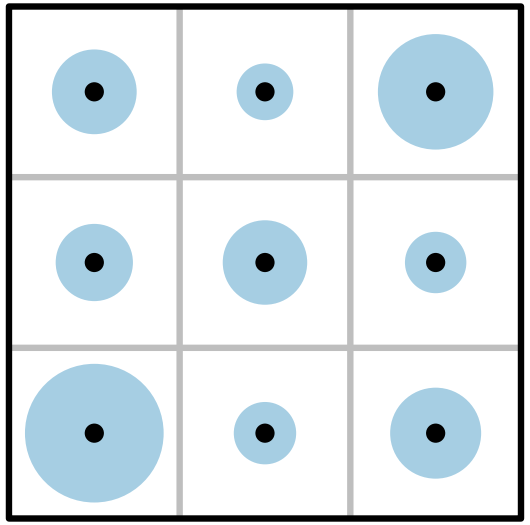
\# Clusters more than optimum



Quantitative Experiments



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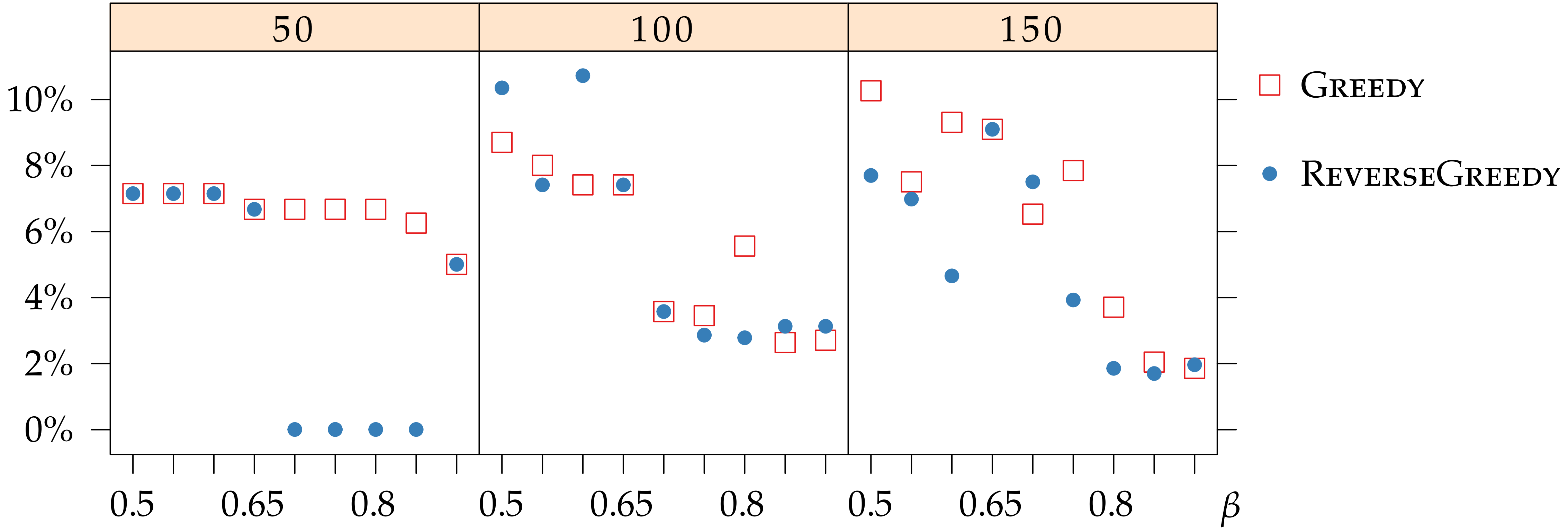
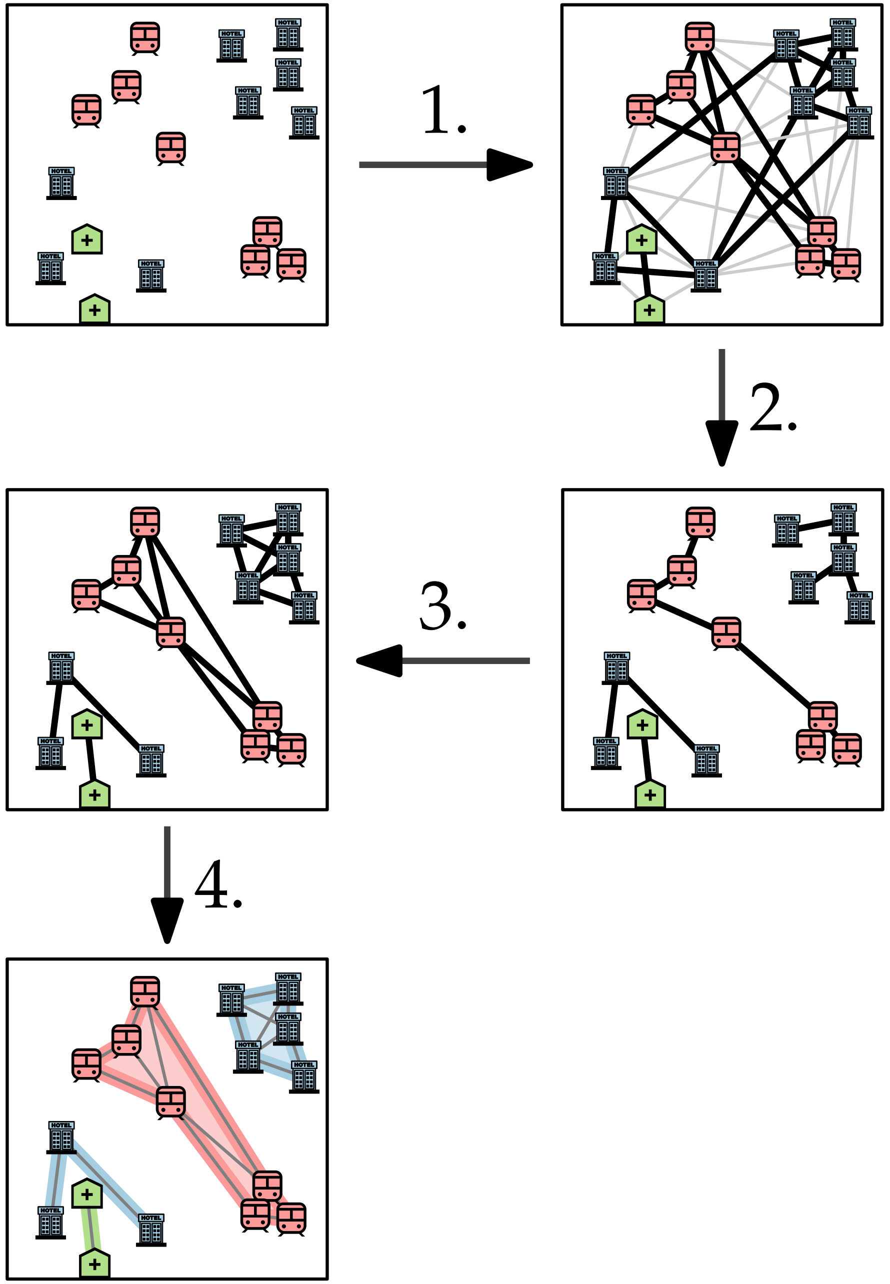
- $n = 50, 100, 150, 200, 250$
- $\beta = 0.5,\ 0.55,\ \ldots,\ 0.9$

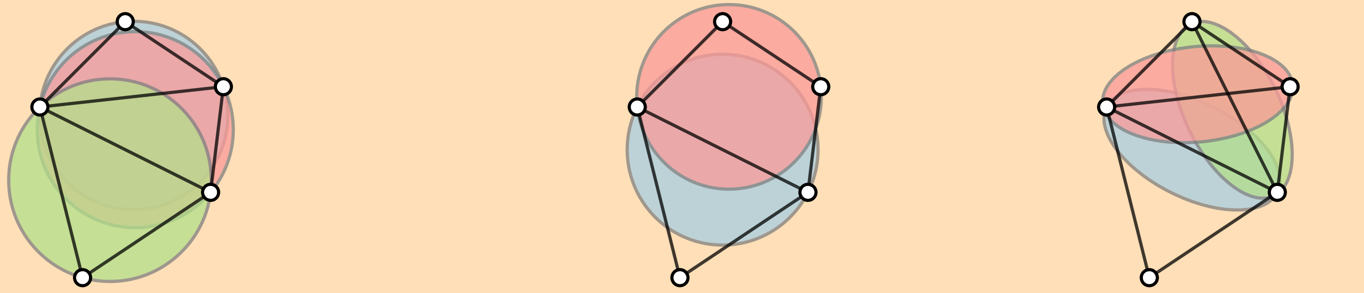
Clusters more than optimum



Conclusion



1. **Proximity Graph**

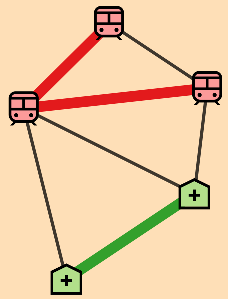


Delaunay Triangulation

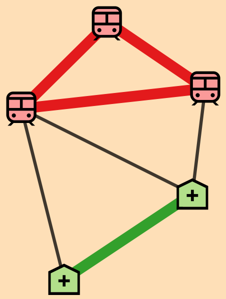
Gabriel Graph

β-Skeleton

2. **Planar Spanning Forest**



3. **Edge Augmentation**



4. **Rendering**
   - Line Voronoi Diagram
   - Tree Representation
   - Polygon Representation