Outside-Obstacle Representations with All Vertices on the Outer Face

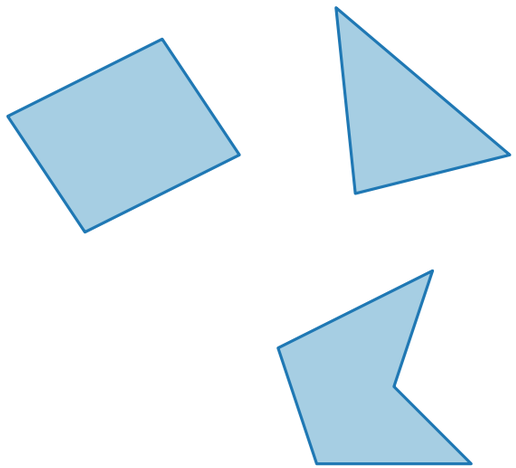
Oksana Firman Philipp Kindermann Jonathan Klawitter
Boris Klemz Felix Klesen Alexander Wolff

Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Universität Trier

Visibility Graph

Given:
- a set of polygons $\mathcal{C}$ (*obstacles*)



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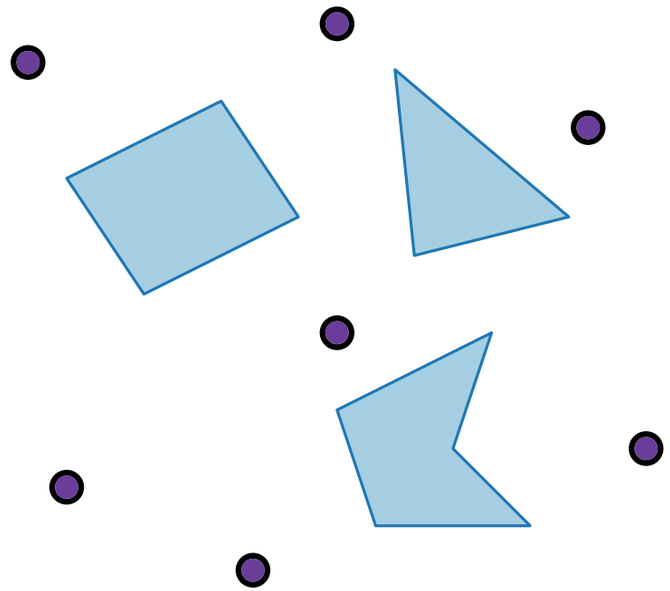

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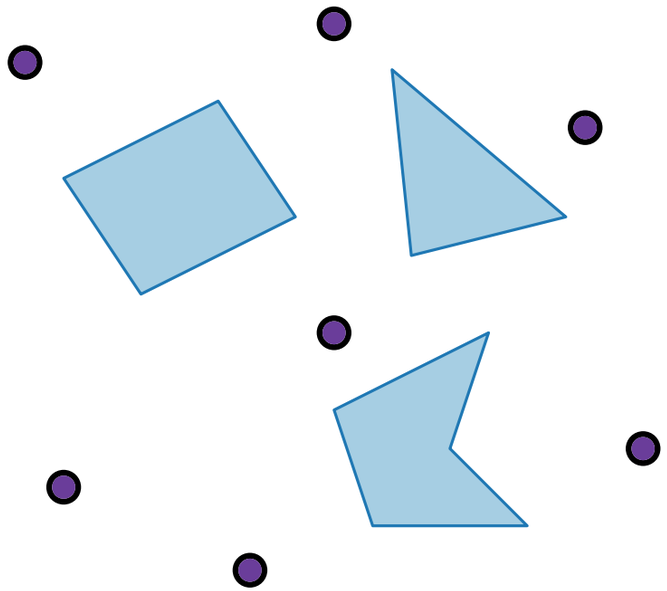

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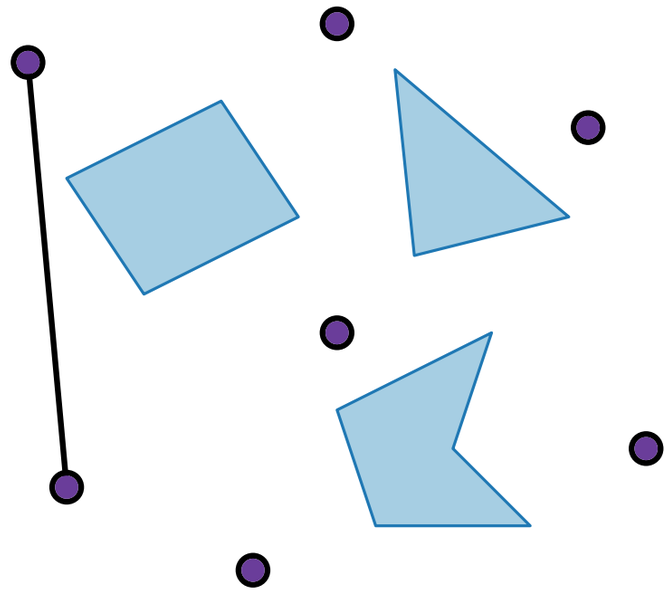

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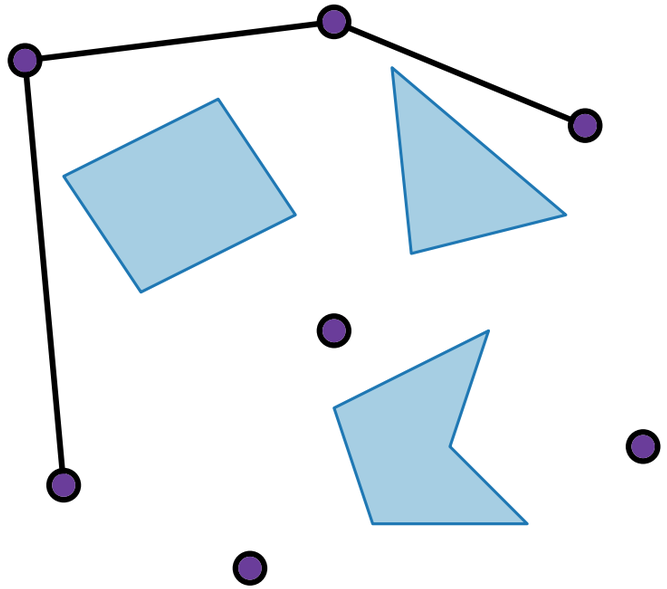

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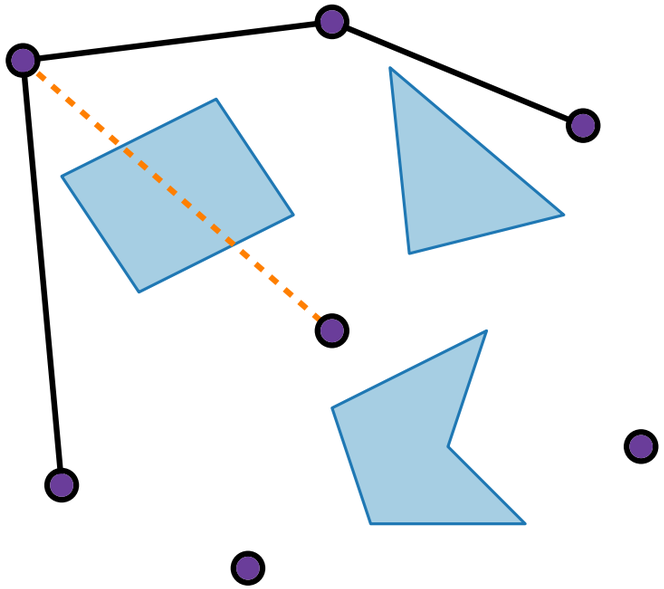

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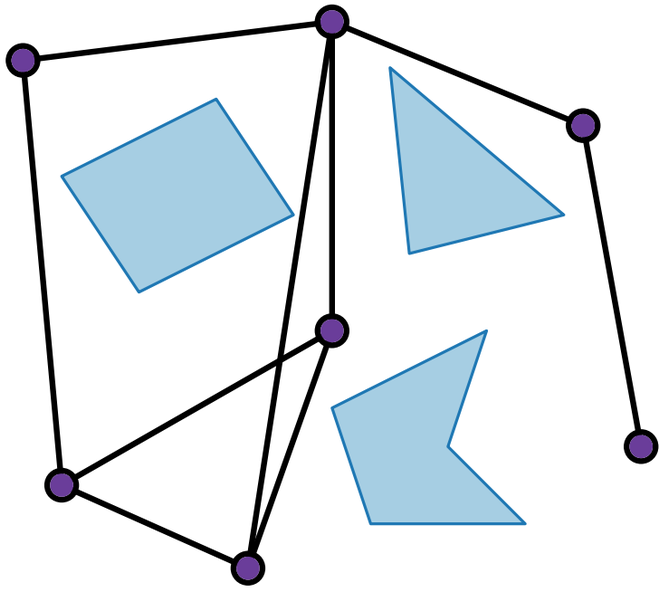

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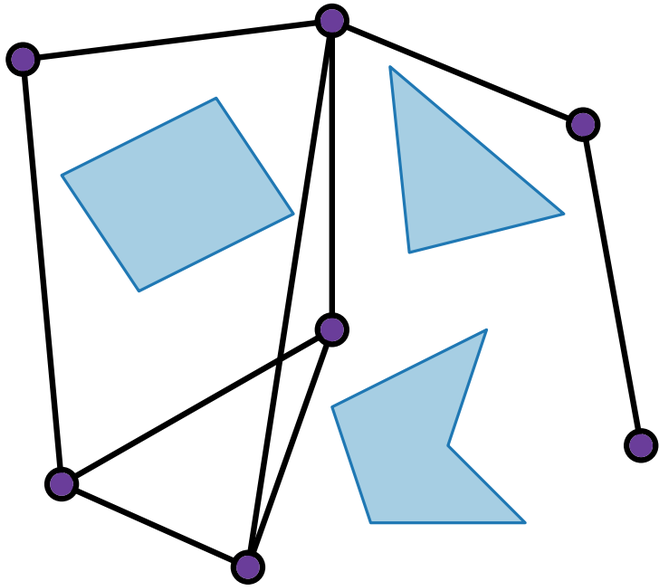

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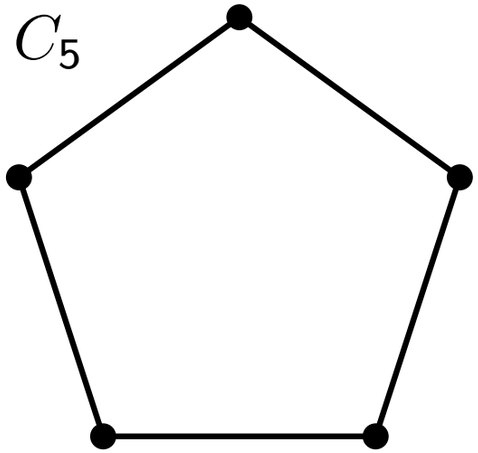

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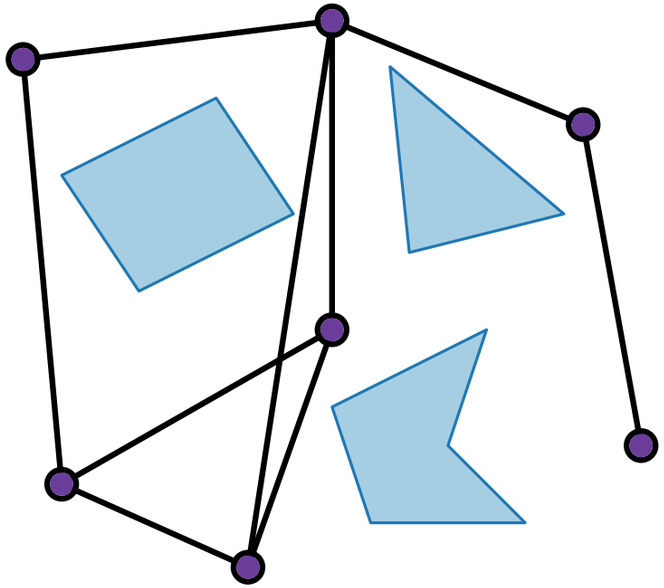

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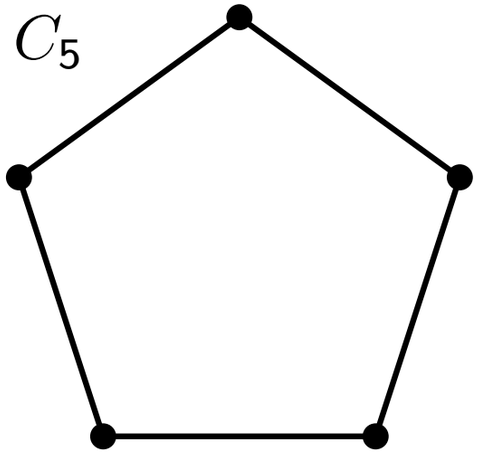

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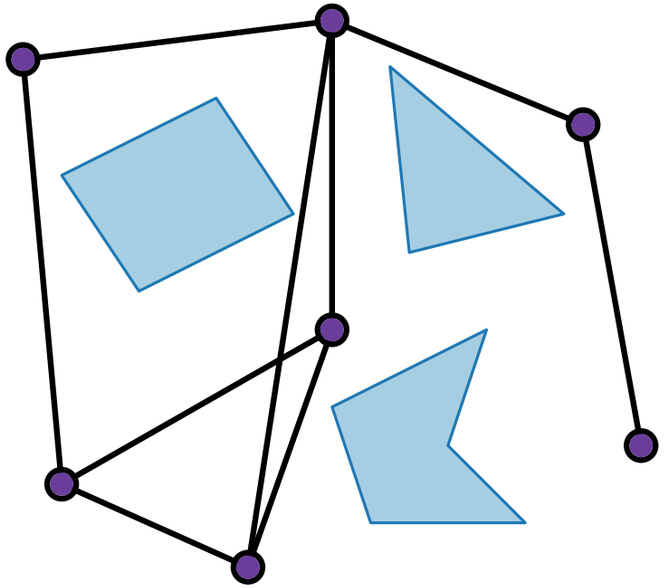

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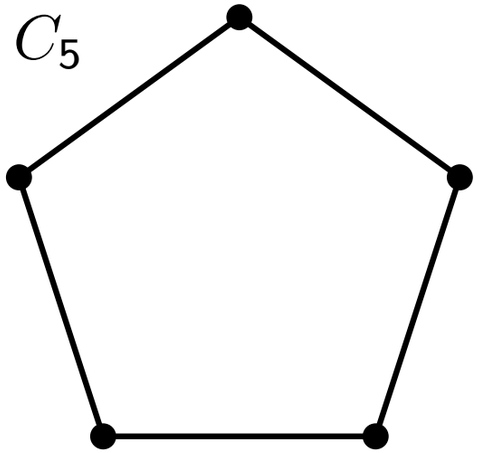

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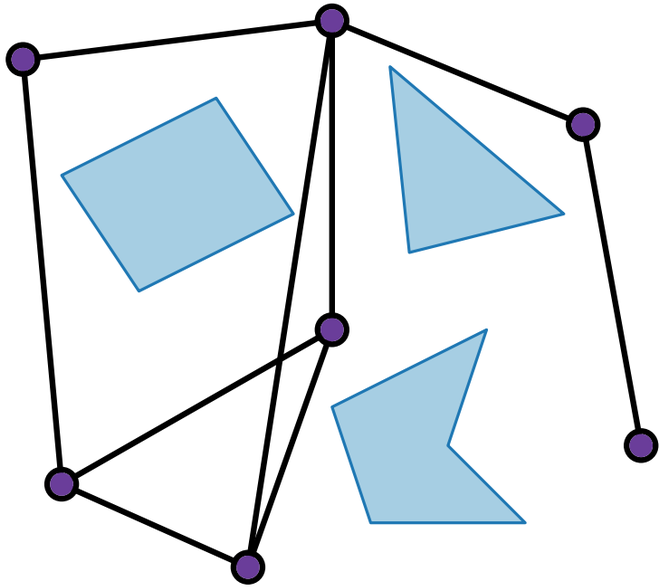

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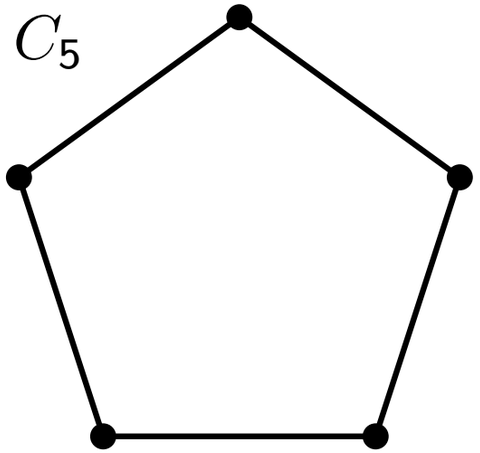

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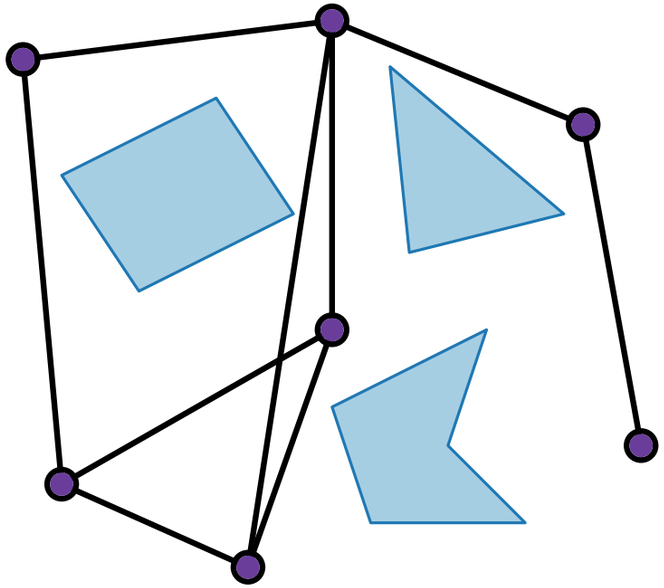

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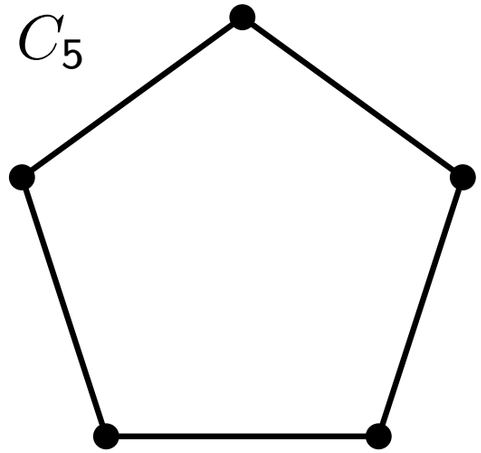
C_5



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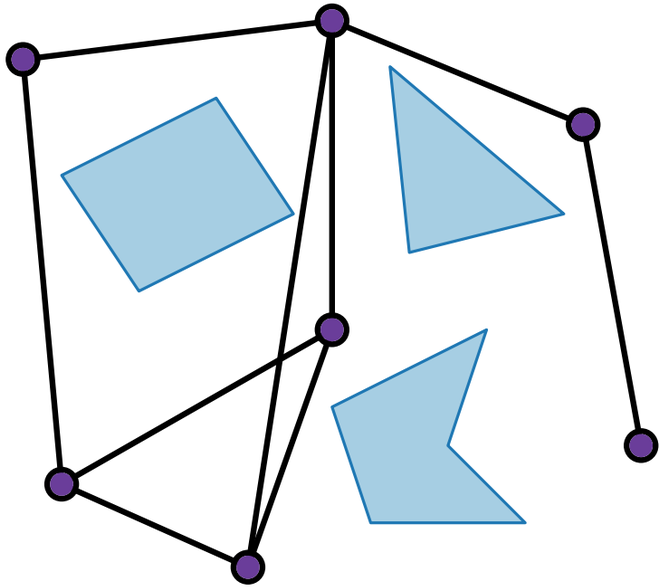

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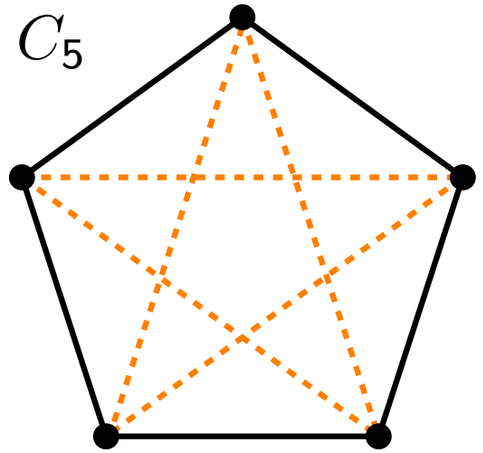

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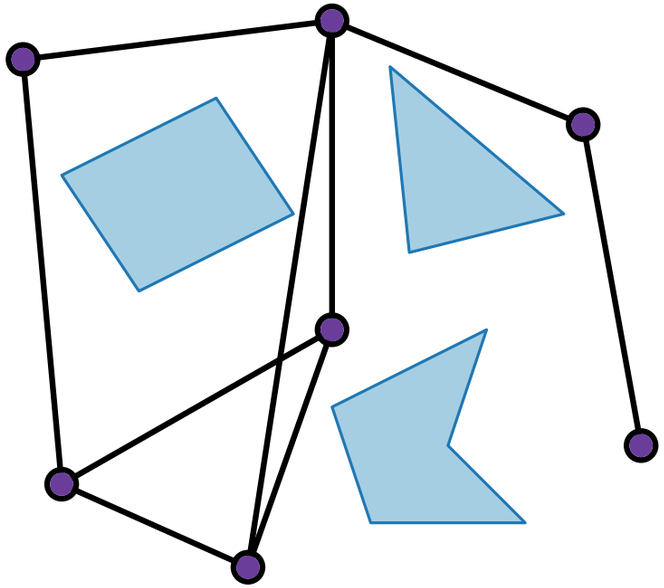

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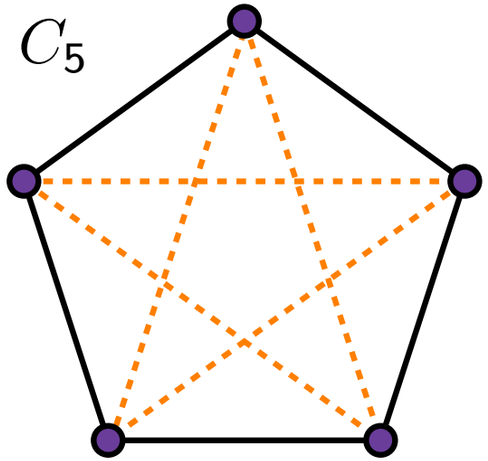

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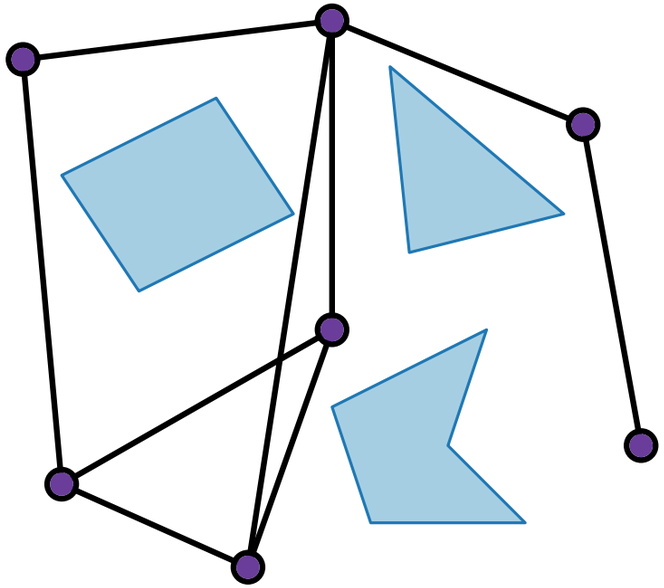

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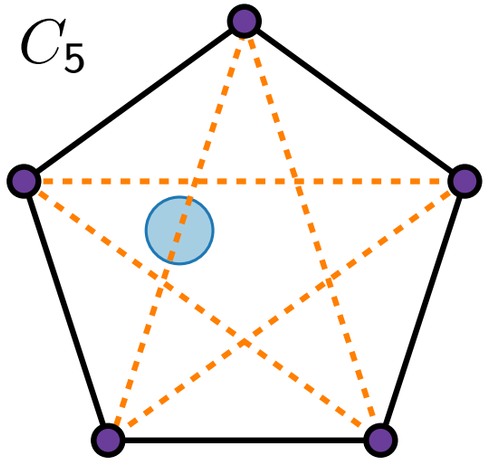

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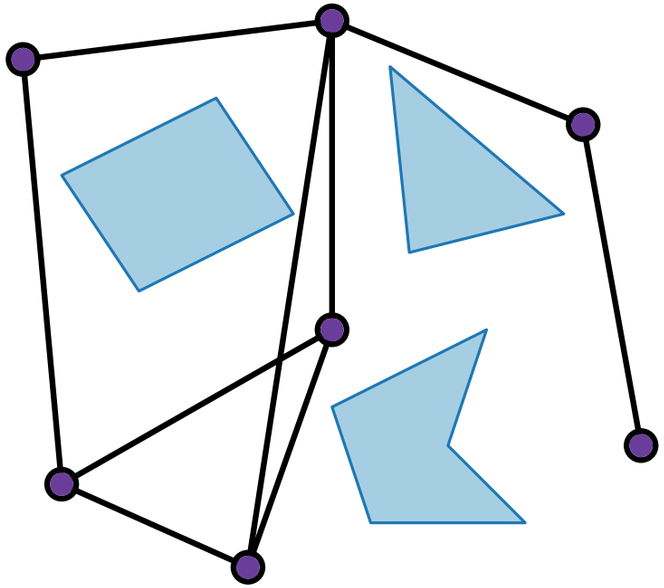

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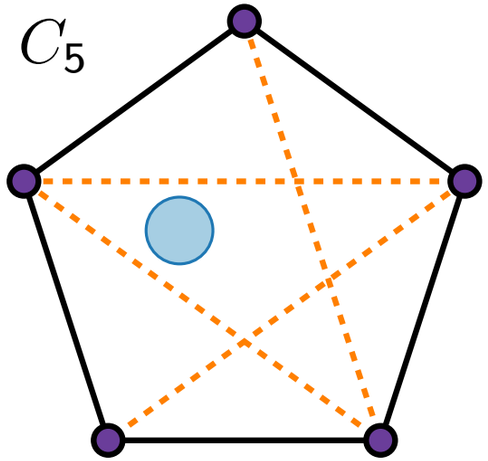

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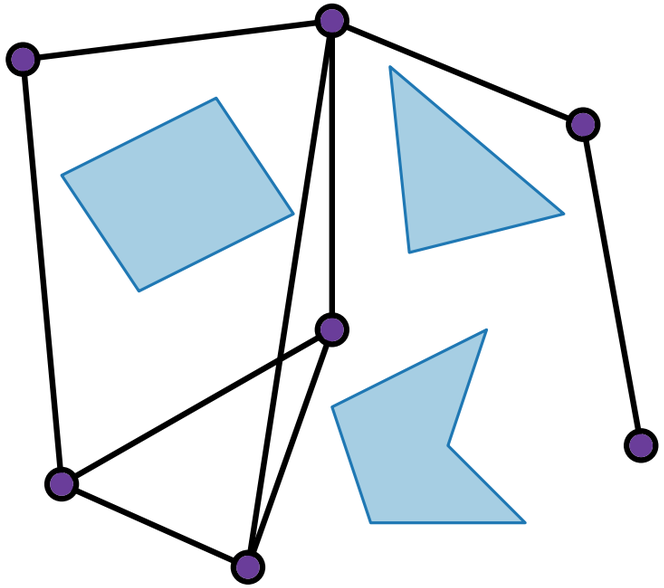

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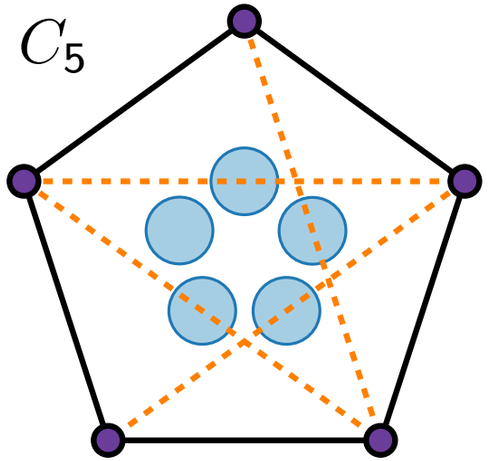

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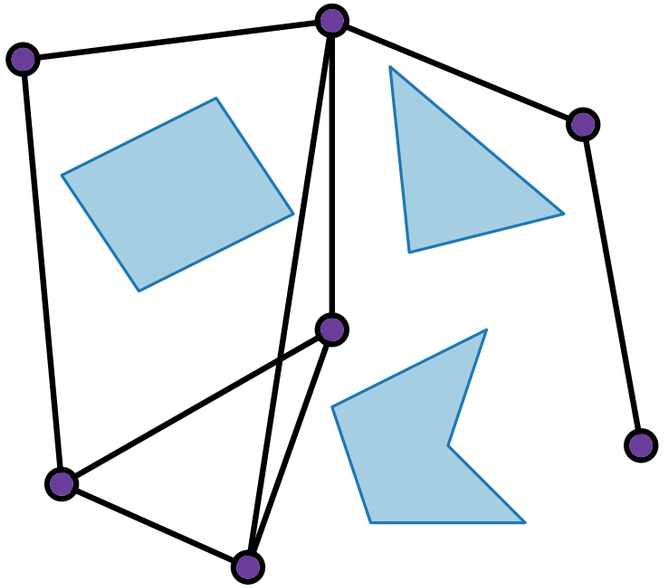

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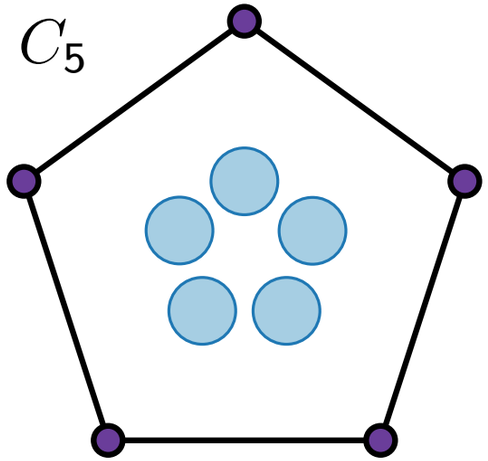

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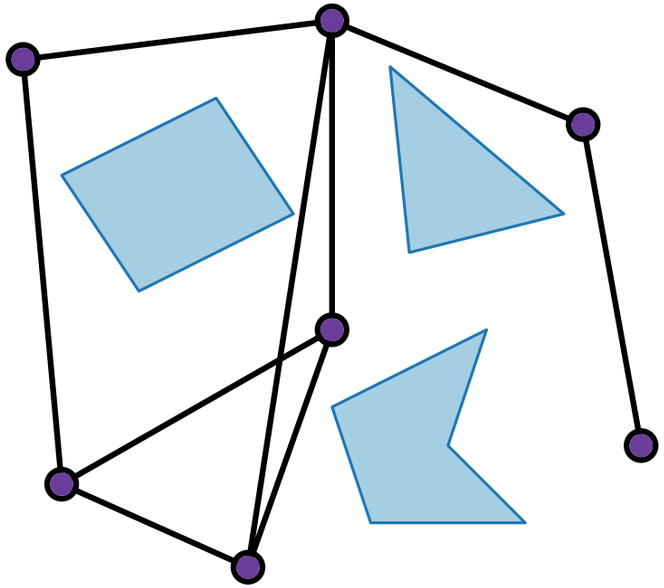

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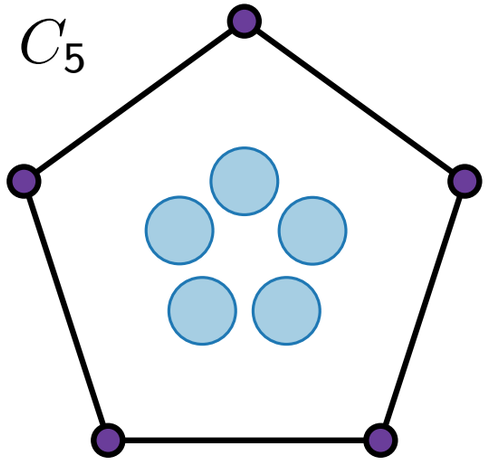
C_5



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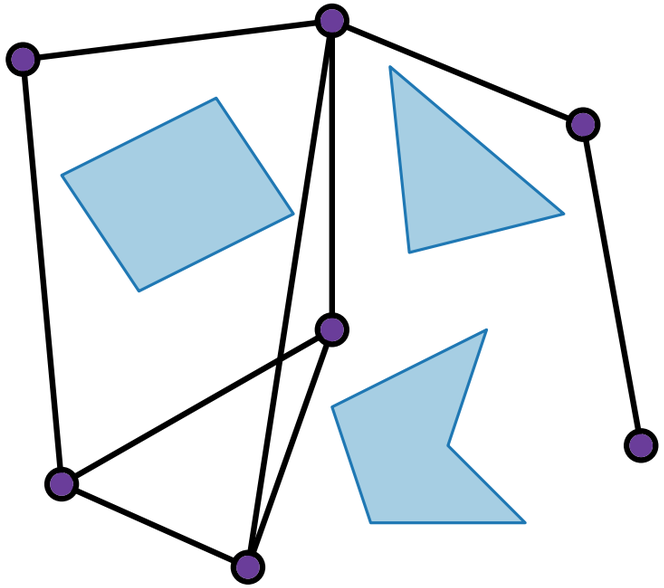

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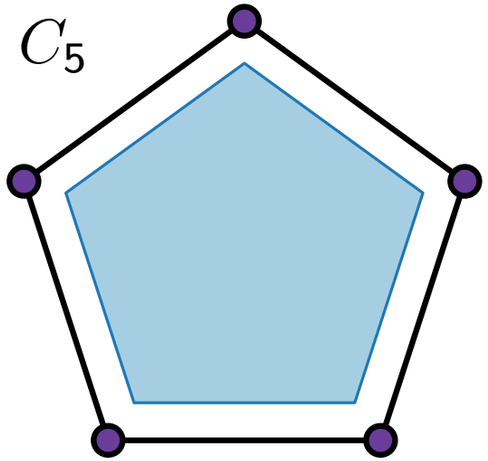
$\Rightarrow$ minimization problem

C_5



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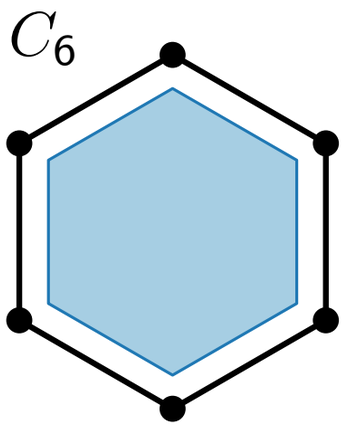
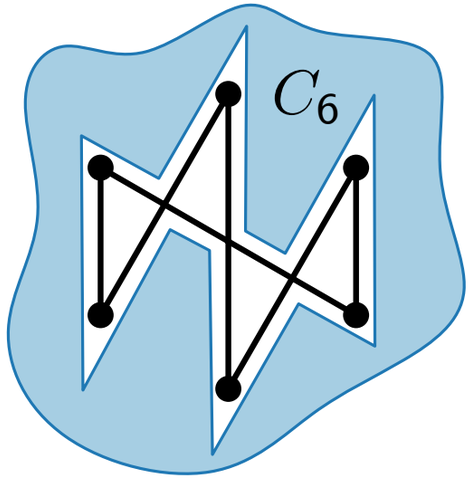


inside obstacle



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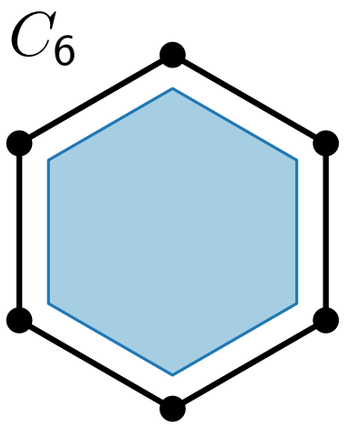

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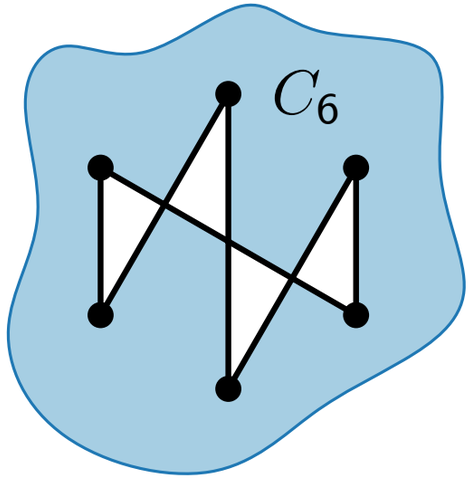

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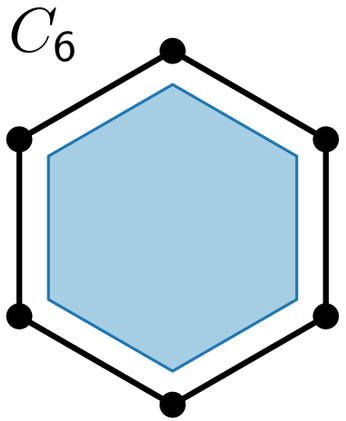

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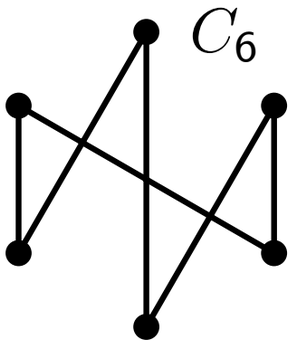

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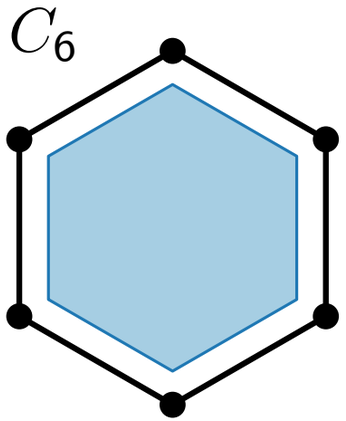
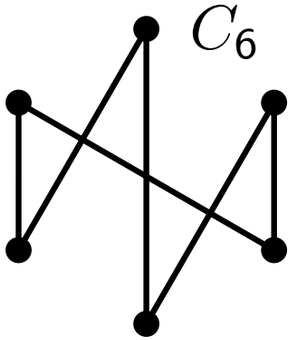


inside obstacle

- single convex obstacle

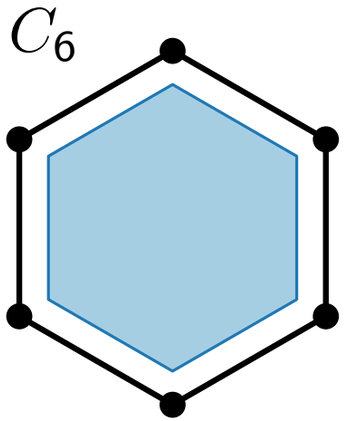
[Alpert et al., *DCG* 2010]



outside obstacle

- every outerplanar graph

Previous Work

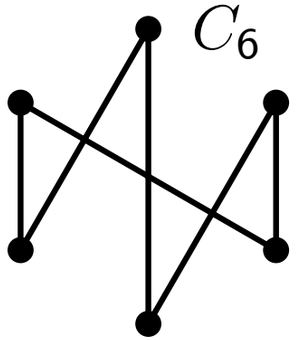


inside obstacle

- single convex obstacle
- necessary conditions to have a single obstacle
- planar graphs (icosahedron needs 2 obstacles)

[Alpert et al., *DCG* 2010]

[Berman et al., *JGAA* 2017]



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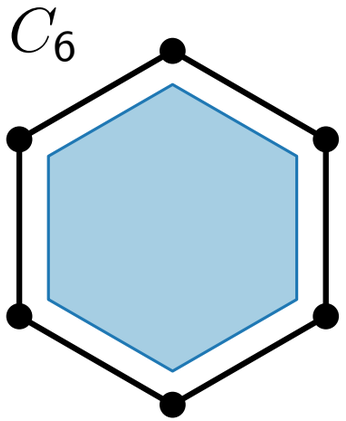

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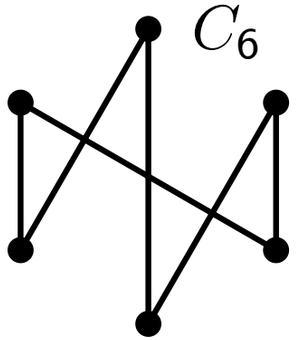

outside obstacle

- single convex obstacle [Alpert et al., *DCG* 2010]
- every outerplanar graph
- necessary conditions to have a single obstacle [Berman et al., *JGAA* 2017]
- planar graphs (icosahedron needs 2 obstacles)
- every graph with ≤ 7 vertices [Chaplick et al., *GD* 2016]
- incomparable (with a single obstacle)

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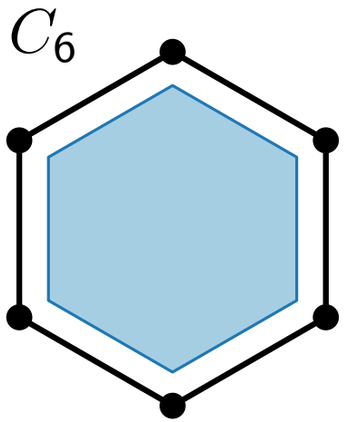

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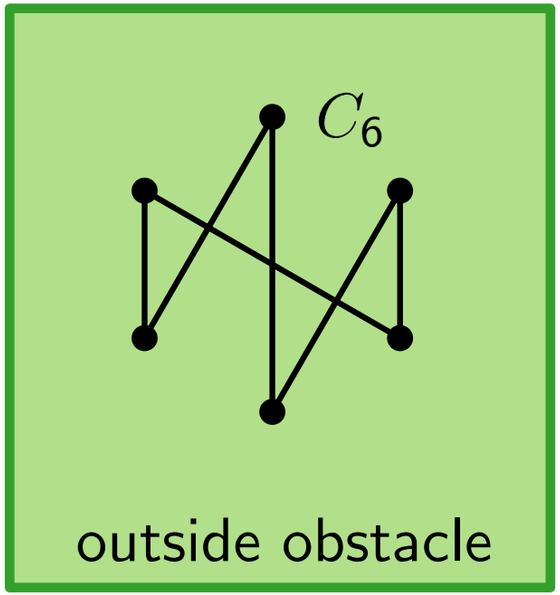
[Berman et al., *JGAA* 2017]

- planar graphs (icosahedron needs 2 obstacles)



outside obstacle

- every outerplanar graph

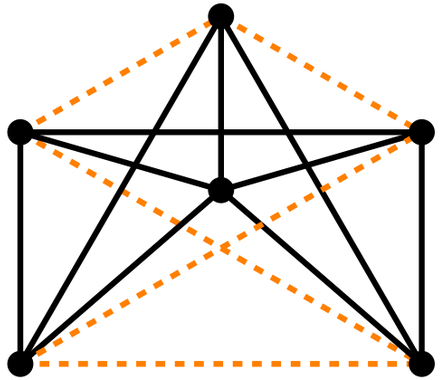
[Chaplick et al., *GD* 2016]

- every graph with ≤ 7 vertices

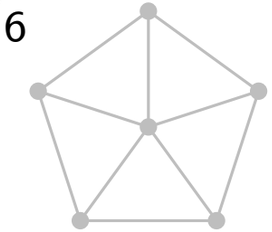
- incomparable (with a single obstacle)

Types of Outside-Obstacle Representation

General



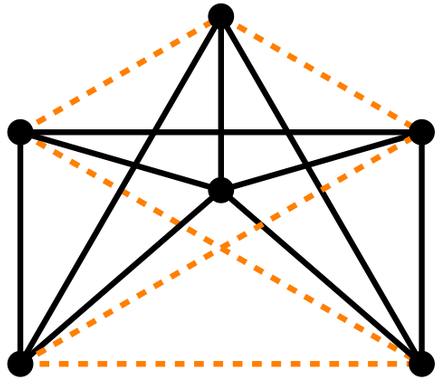
W_6



all vertices on
the outer face

Types of Outside-Obstacle Representation

General



all vertices on
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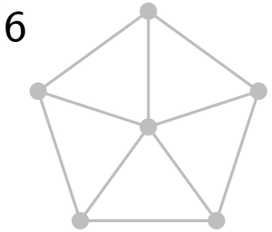
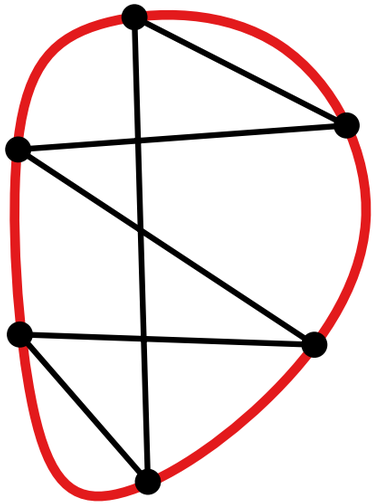
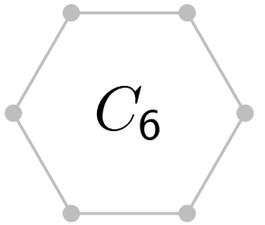
W_6



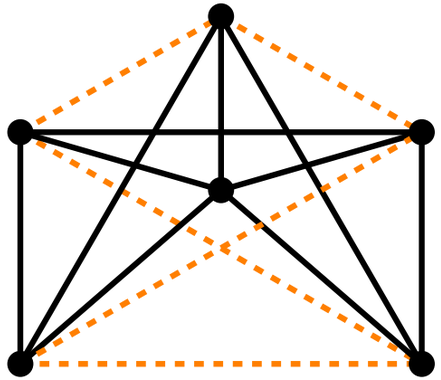
Convex





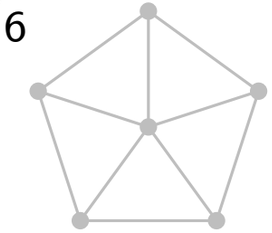
Types of Outside-Obstacle Representation

General

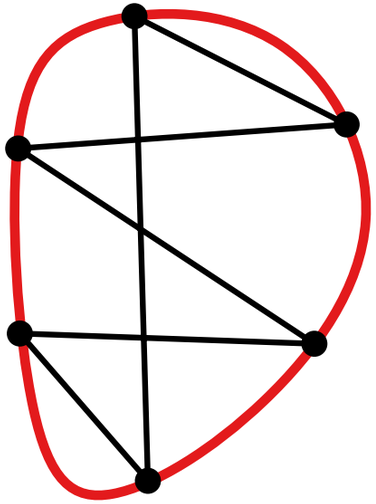


all vertices on
the outer face

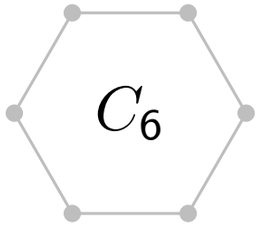
W_6



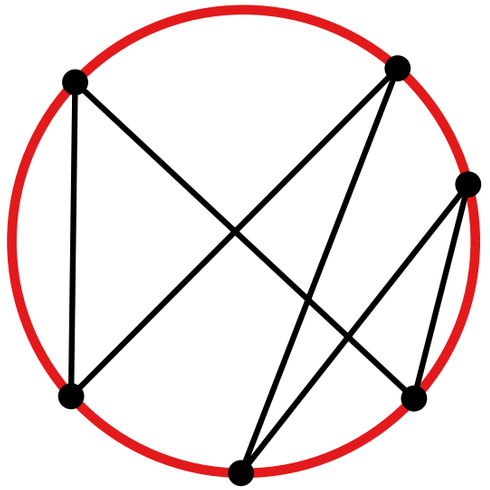
Convex



C_6

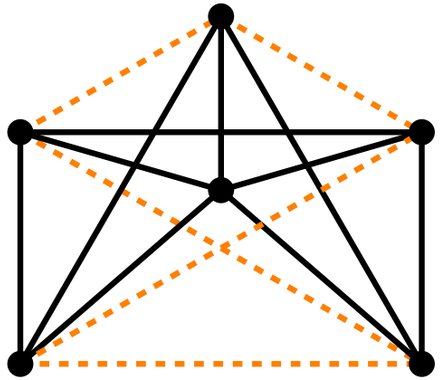


Circular



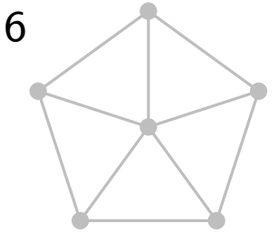
Types of Outside-Obstacle Representation

General

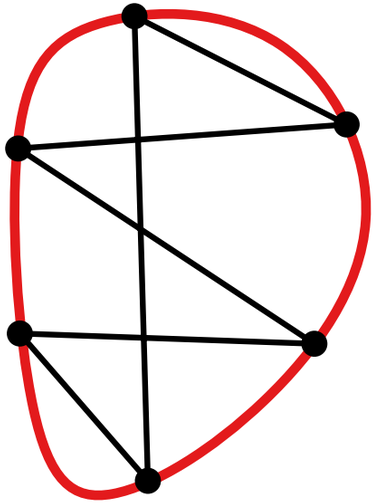


all vertices on
the outer face

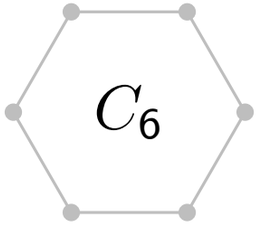
W_6



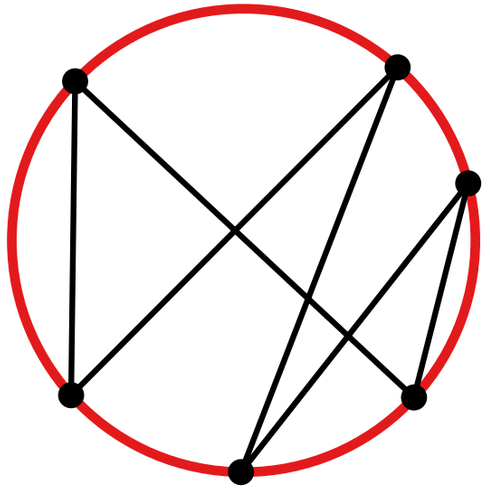
Convex



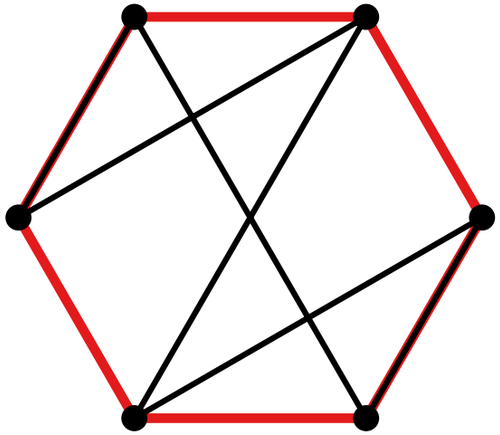
C_6



Circular

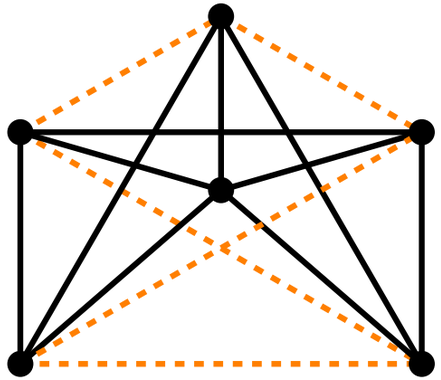


Regular



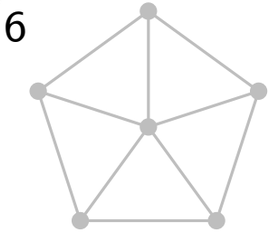
Types of Outside-Obstacle Representation

General

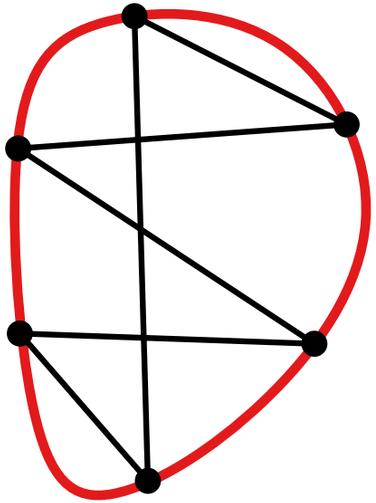


all vertices on
the outer face

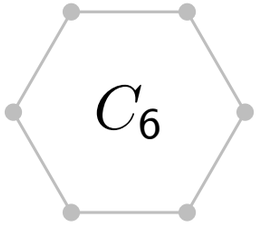
W_6



Convex

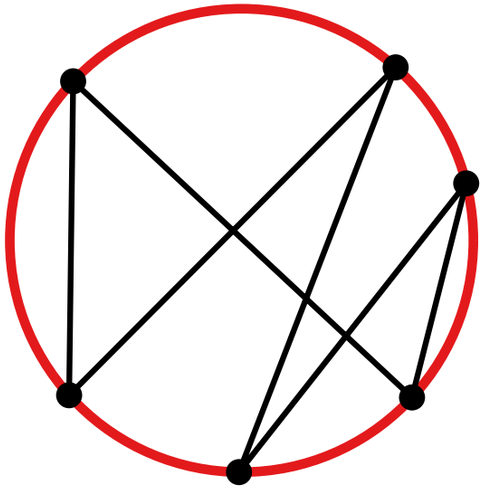


C_6



?

Circular



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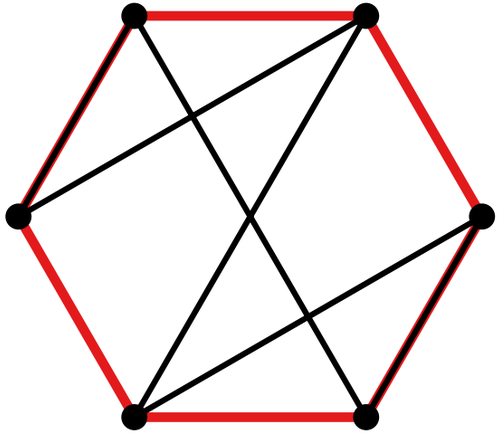
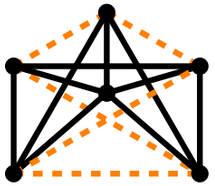


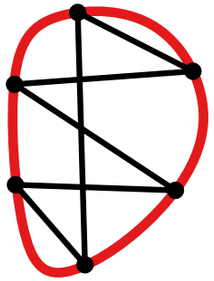
Our Contribution

General

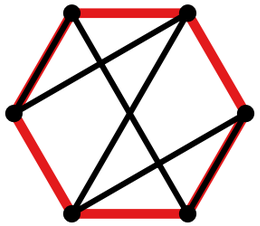


all vertices on
the outer face

Convex



Regular



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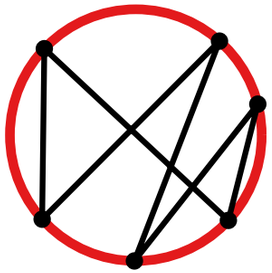
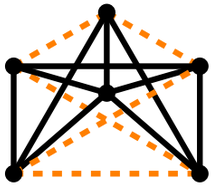


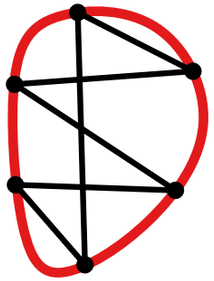
Our Contribution

General

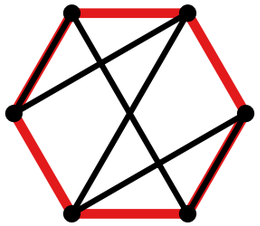


all vertices on
the outer face

Convex

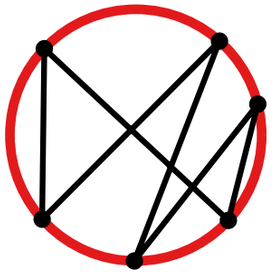


Regular



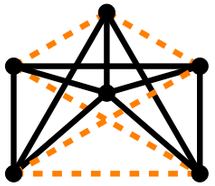
- cactus

Circular



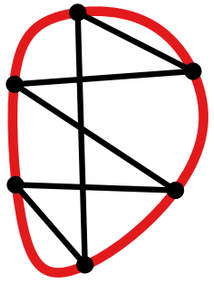
Our Contribution

General

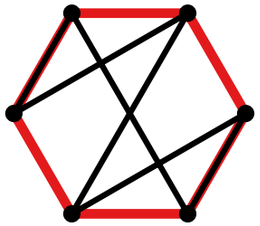


all vertices on
the outer face

Convex

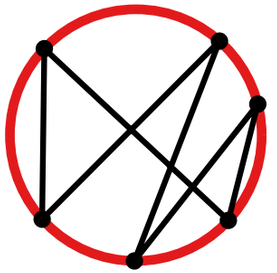


Regular



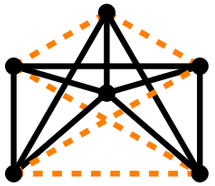
- cactus
- grid

Circular



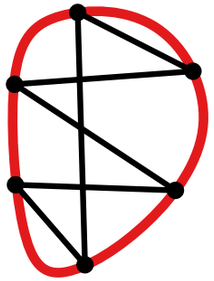
Our Contribution

General



all vertices on
the outer face

Convex



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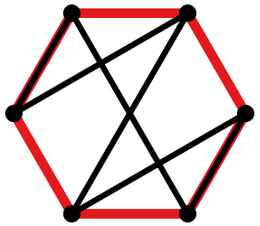
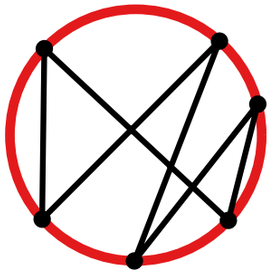


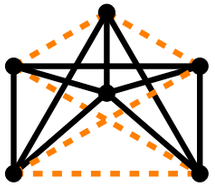
- cactus
- grid
- outerpath

Circular



Our Contribution

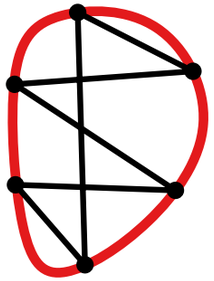
General



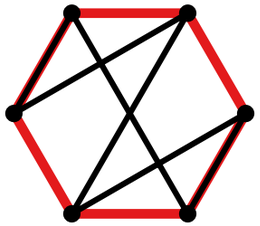
all vertices on
the outer face

- outerplanar graph [Alpert et al., *DCG* 2010]

Convex

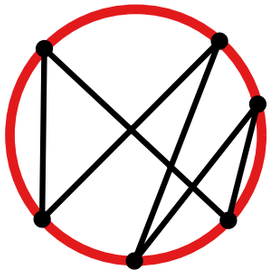


Regular



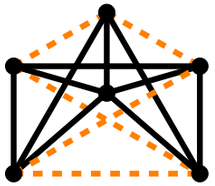
- cactus
- grid
- outerpath

Circular



Our Contribution

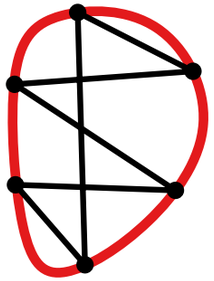
General



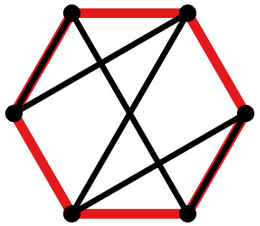
all vertices on
the outer face

- outerplanar graph [Alpert et al., *DCG* 2010]
- 2-tree

Convex

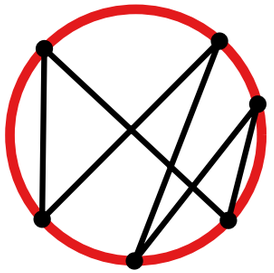


Regular



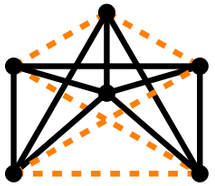
- cactus
- grid
- outerpath

Circular



Our Contribution

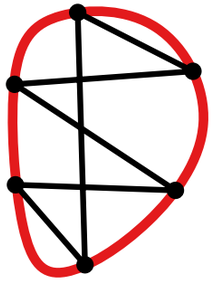
General



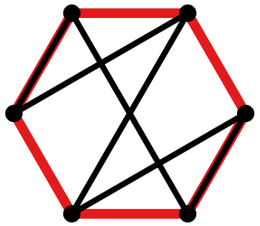
all vertices on the outer face

- outerplanar graph [Alpert et al., *DCG* 2010]
- 2-tree (subgraphs?)

Convex

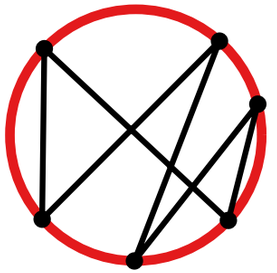


Regular



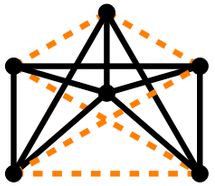
- cactus (subgraphs?)
- grid (subgraphs?)
- outerpath (subgraphs?)

Circular



Our Contribution

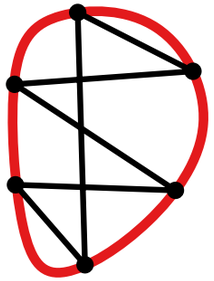
General



all vertices on the outer face

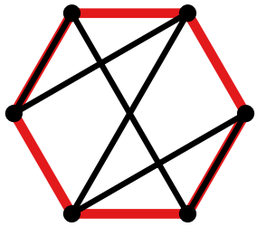
- outerplanar graph [Alpert et al., *DCG* 2010]
- 2-tree (subgraphs?)

Convex



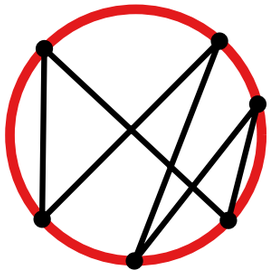
Def. An outside-obstacle representation of a graph is *reducible* if all its edges are incident to the outer face.

Regular



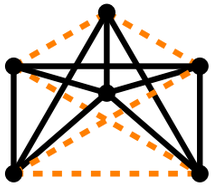
- cactus (subgraphs?)
- grid (subgraphs?)
- outerpath (subgraphs?)

Circular



Our Contribution

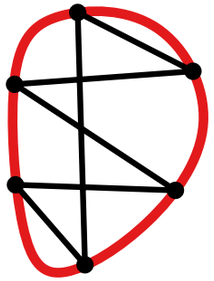
General



all vertices on the outer face

- outerplanar graph [Alpert et al., *DCG* 2010]
- 2-tree (subgraphs ✓)

Convex



Def. An outside-obstacle representation of a graph is *reducible* if all its edges are incident to the outer face.

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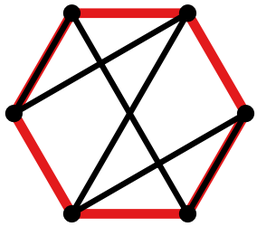

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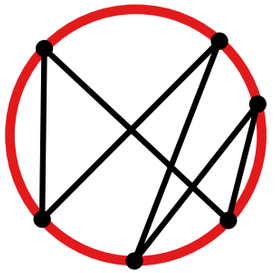

Our Contribution

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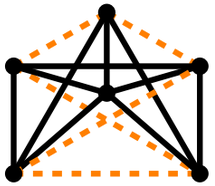

all vertices on the outer face

- outerplanar graph [Alpert et al., *DCG* 2010]
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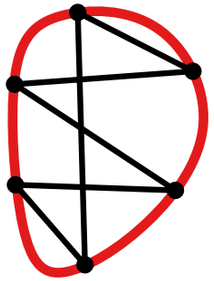

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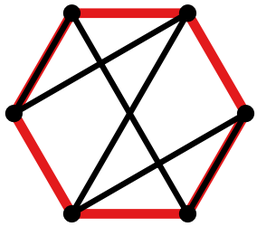

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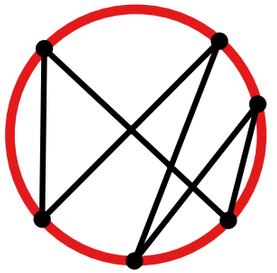

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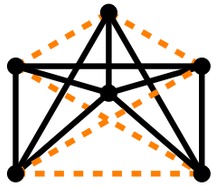

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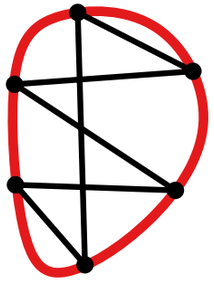

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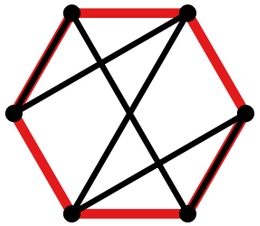

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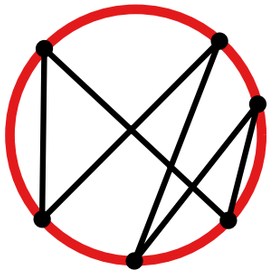

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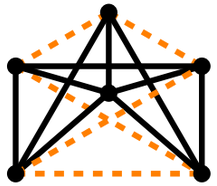

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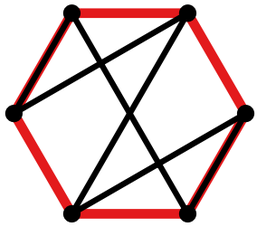

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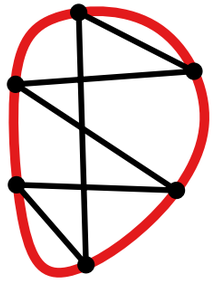

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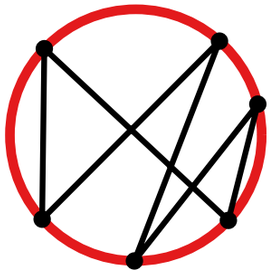

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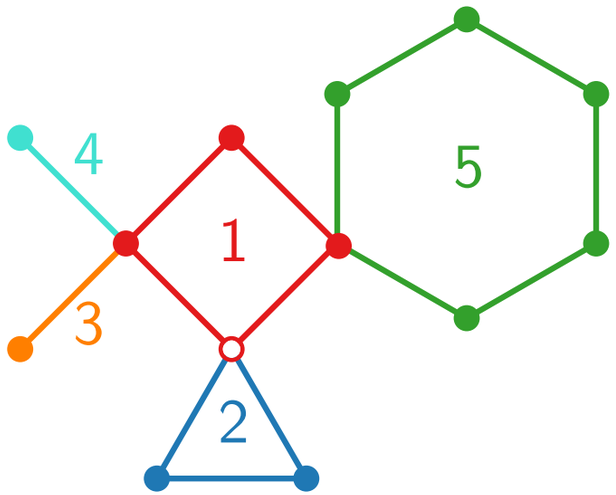

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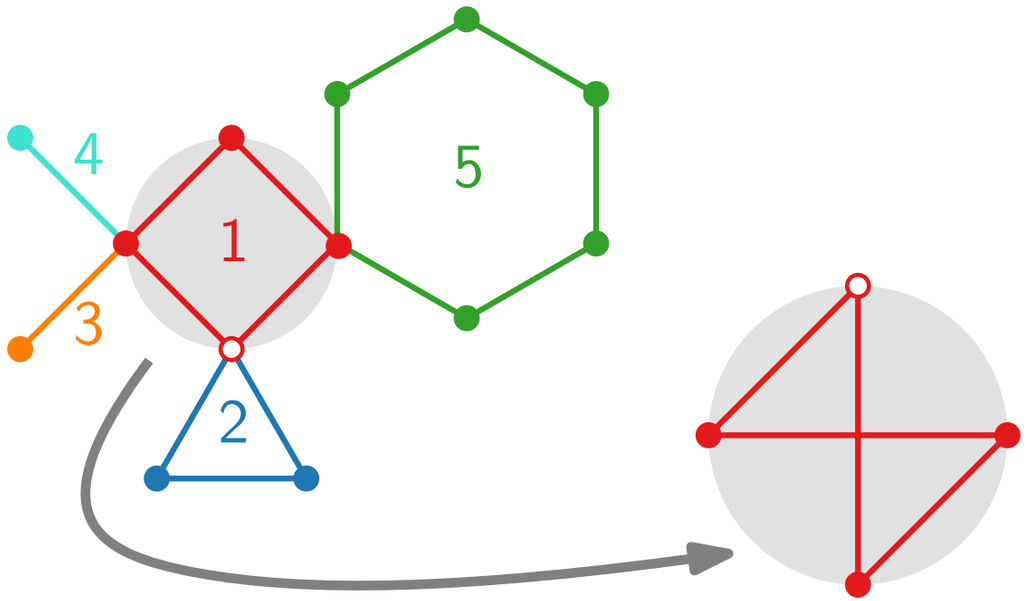

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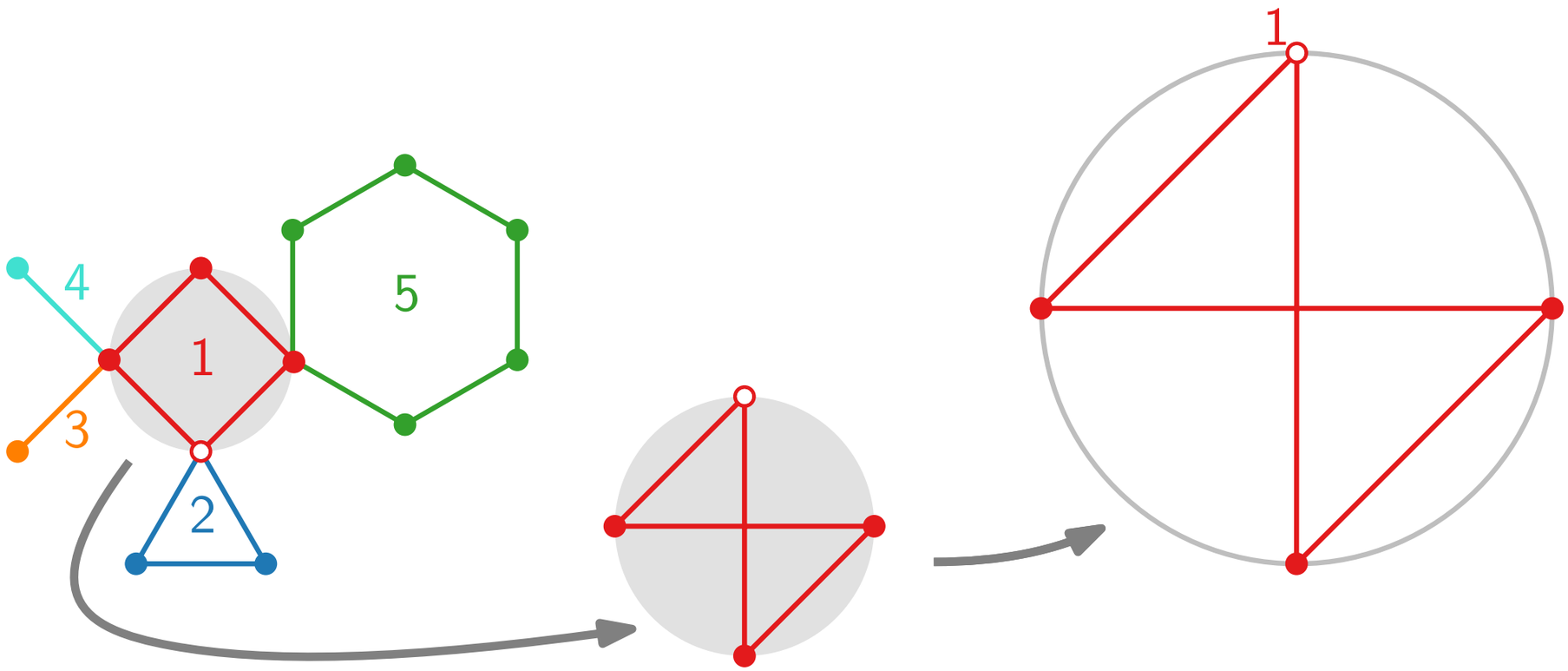

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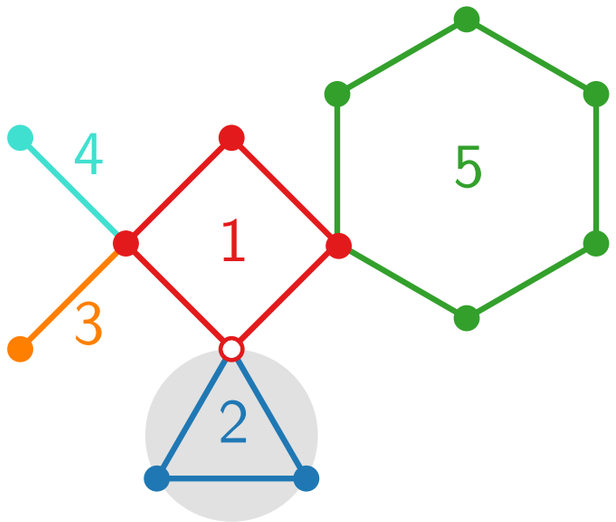

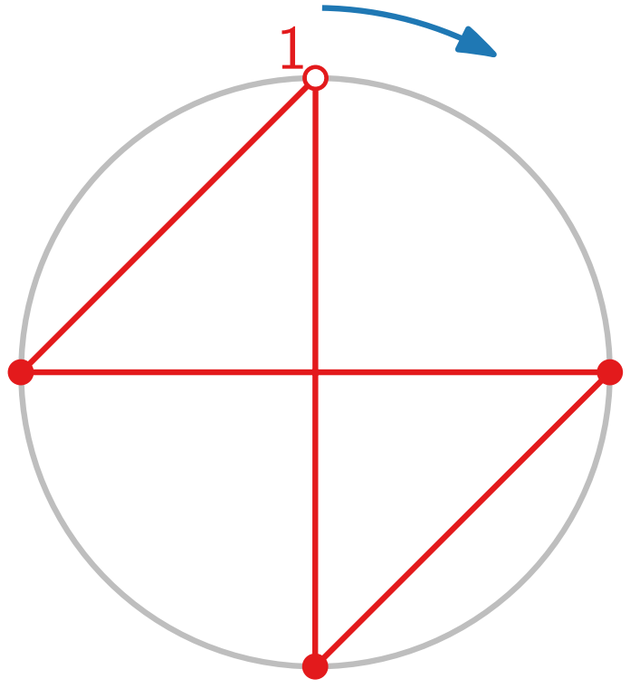

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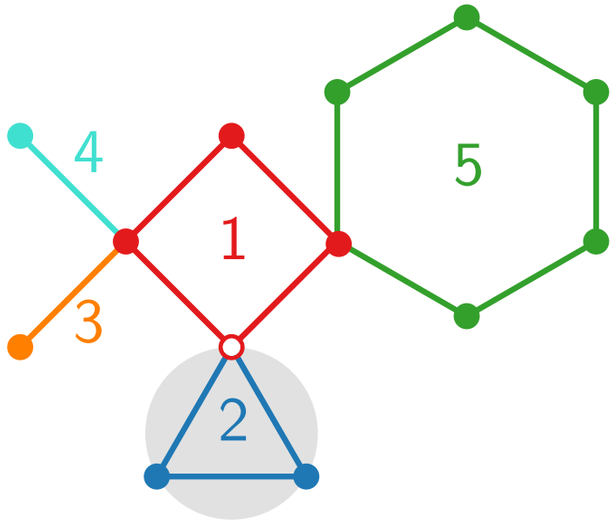

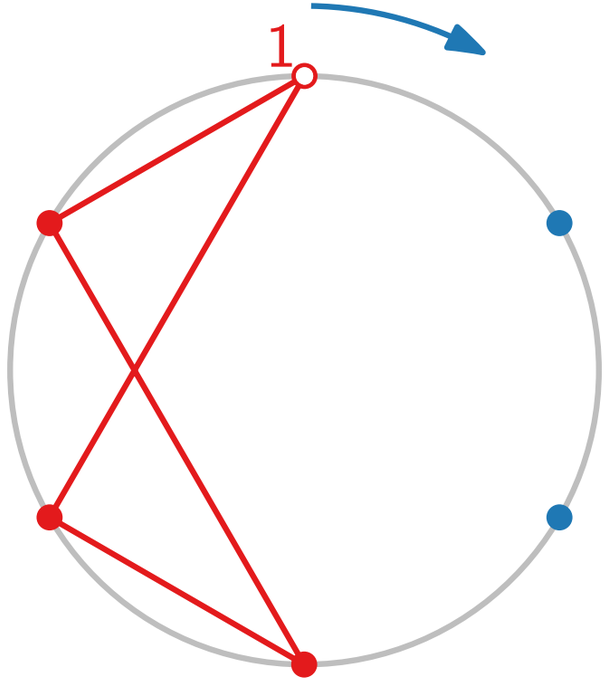

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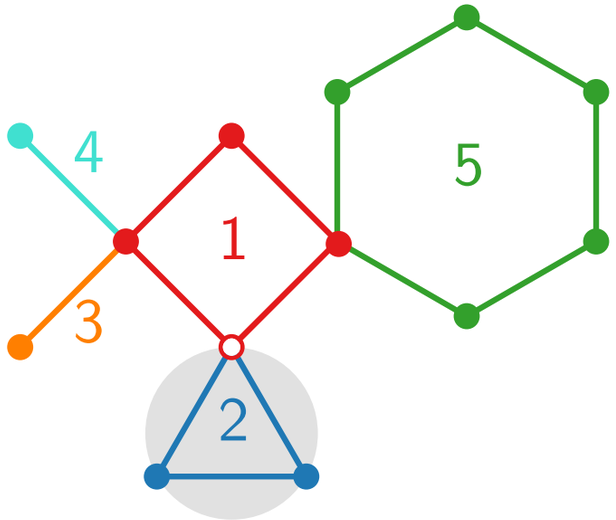

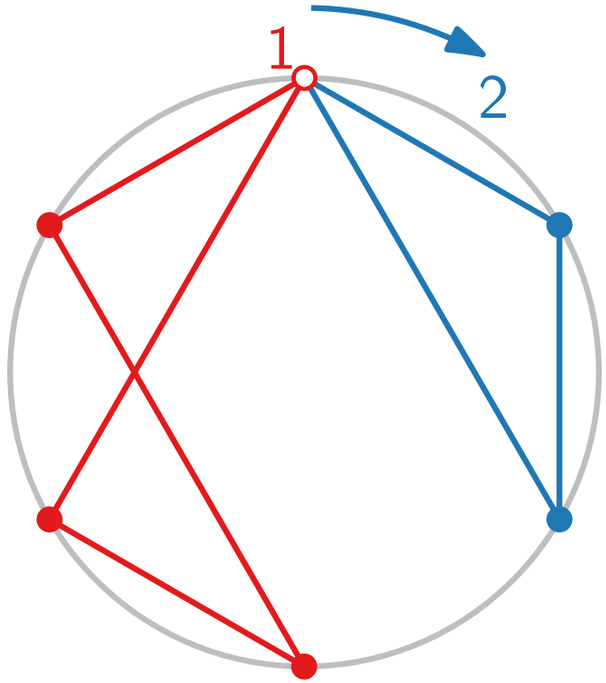

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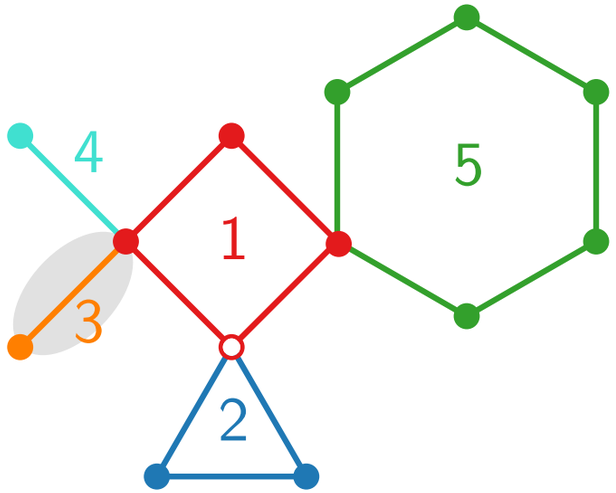

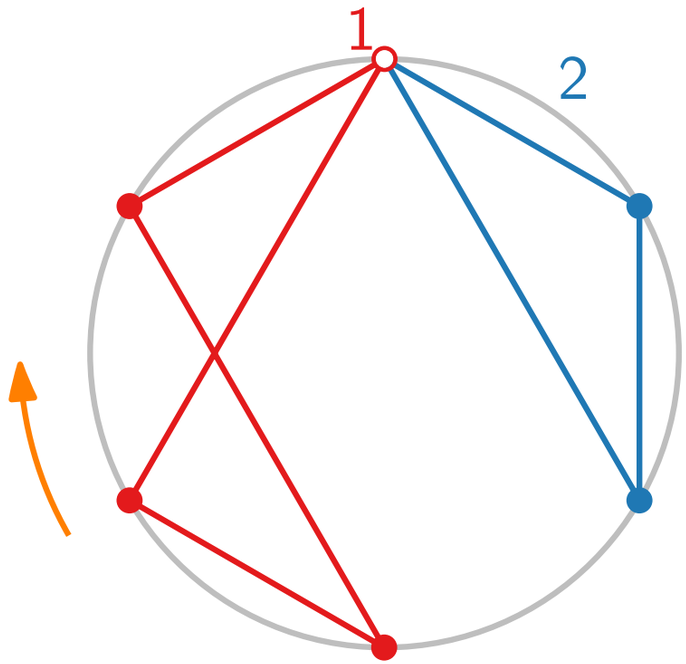

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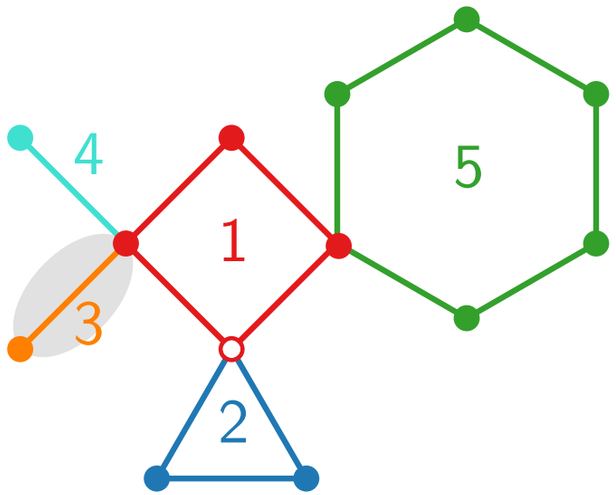

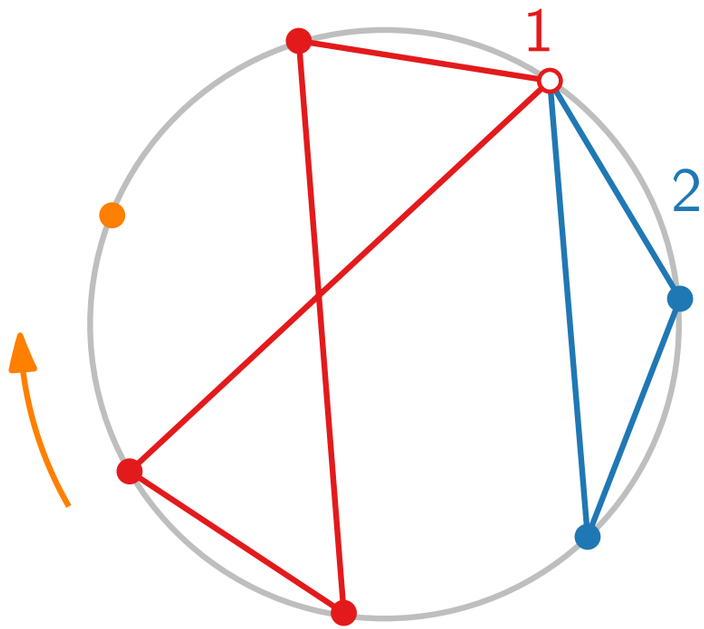

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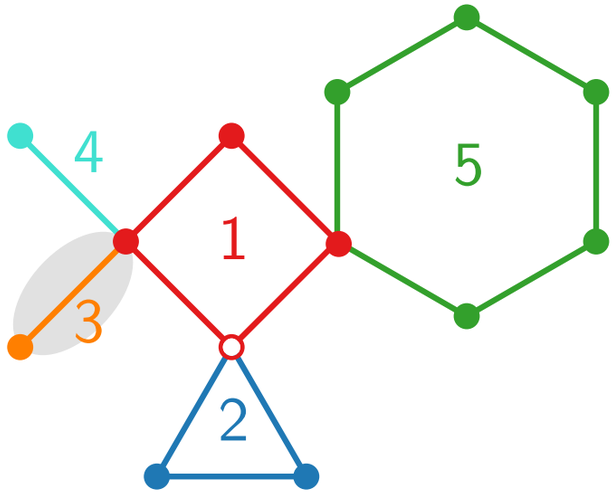

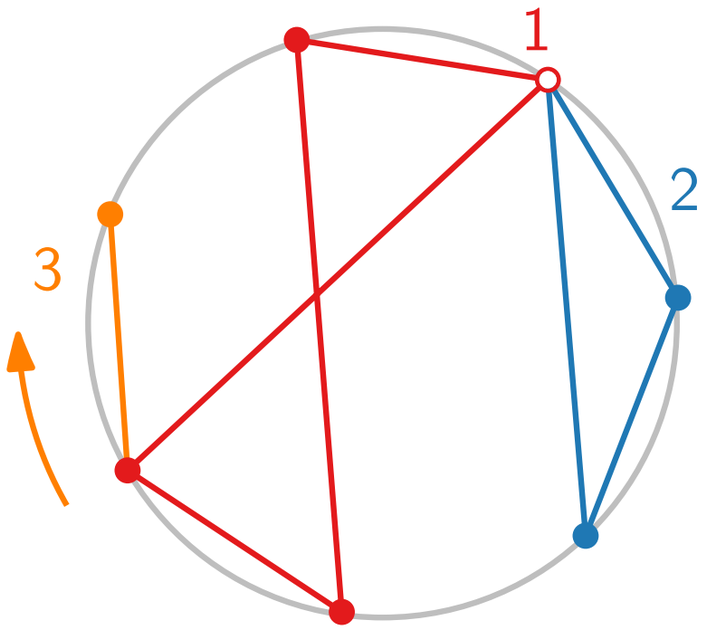

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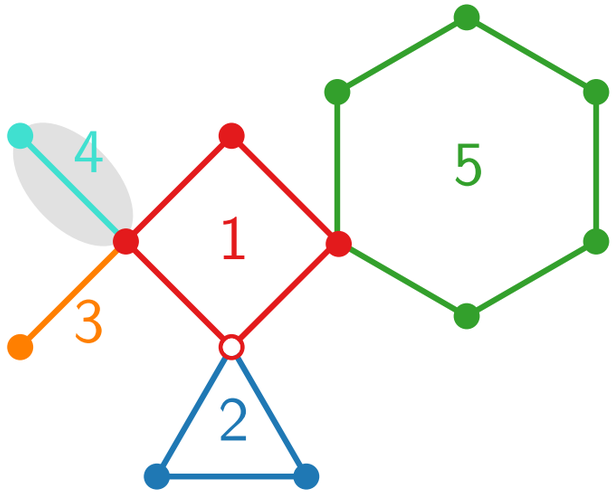

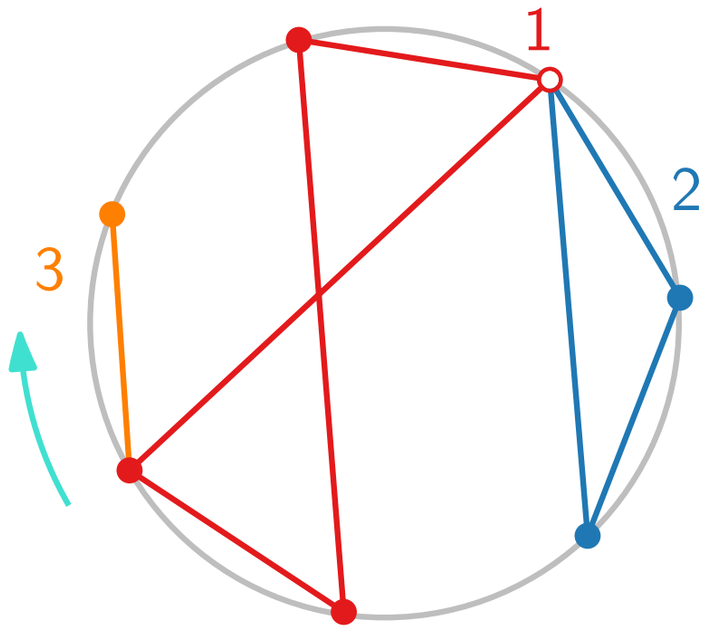

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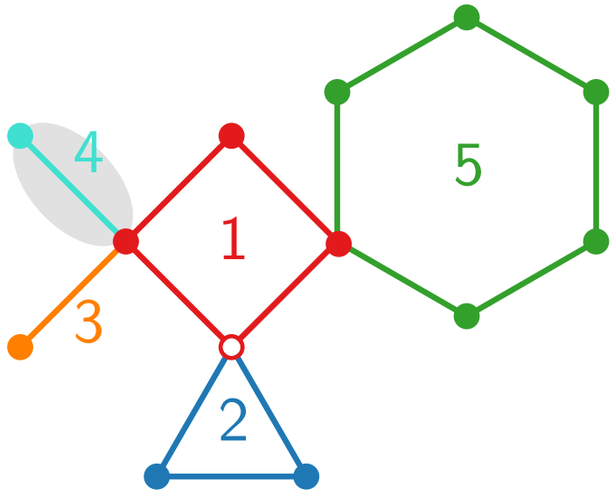

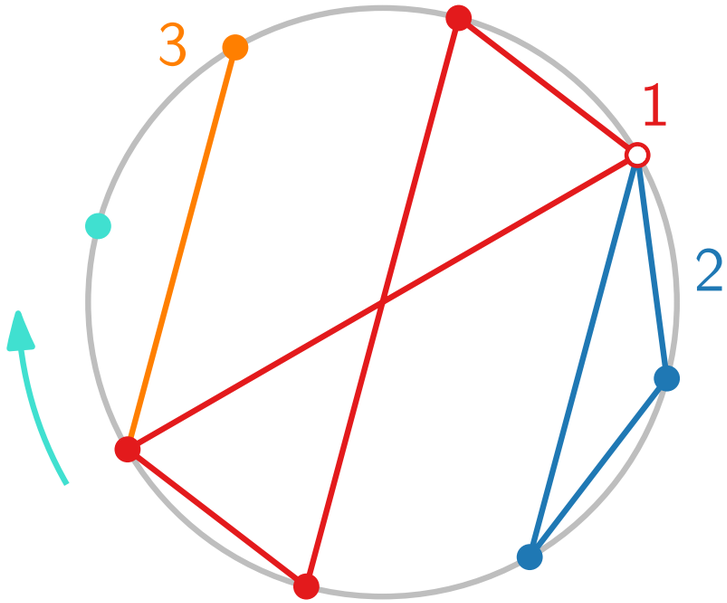

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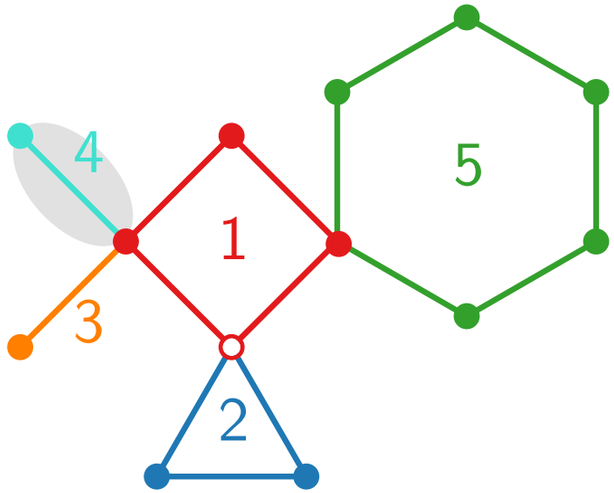

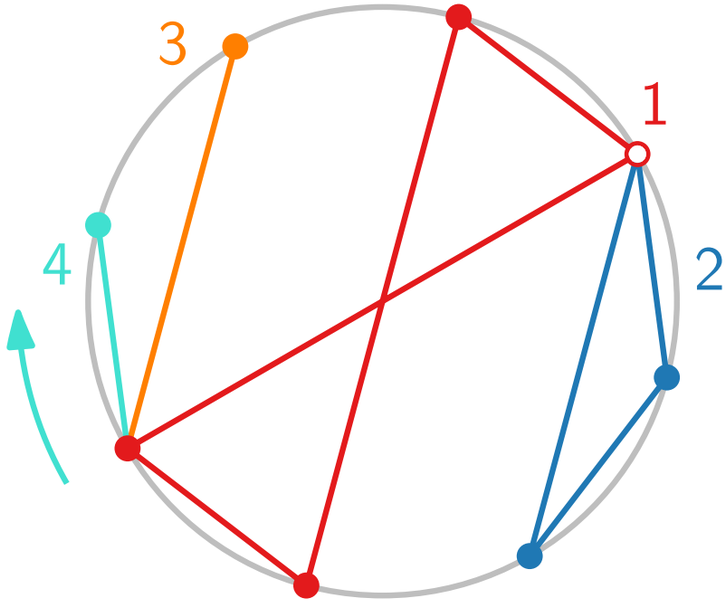

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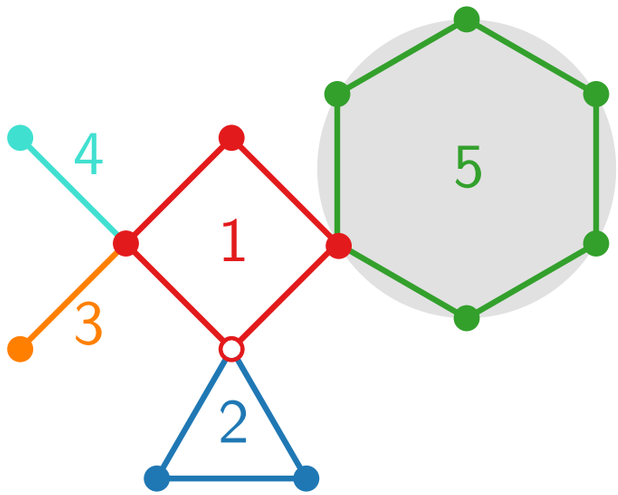

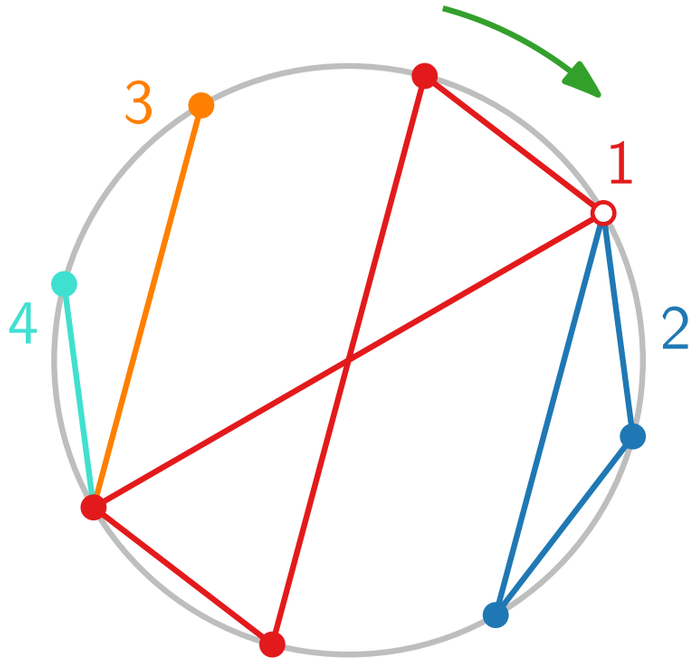

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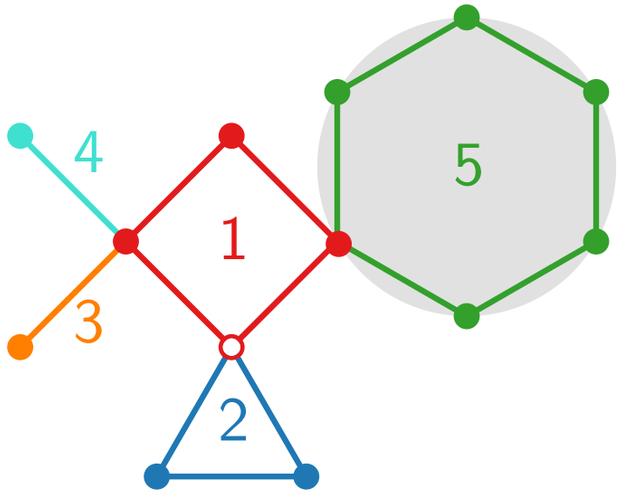

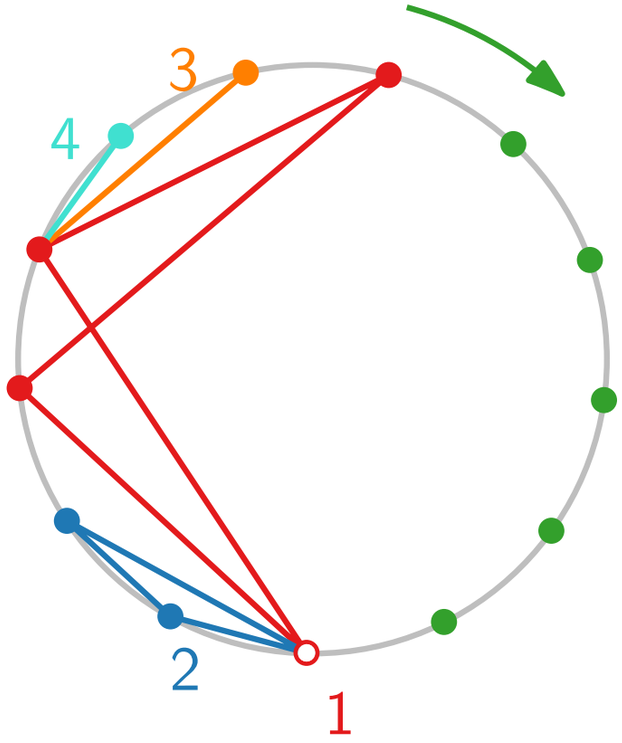

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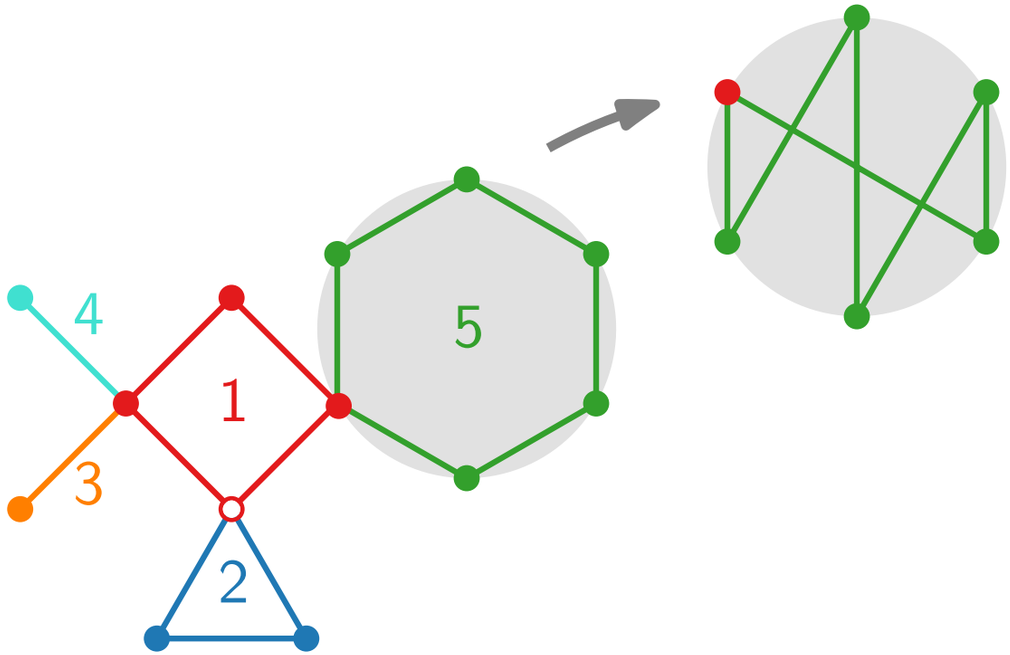

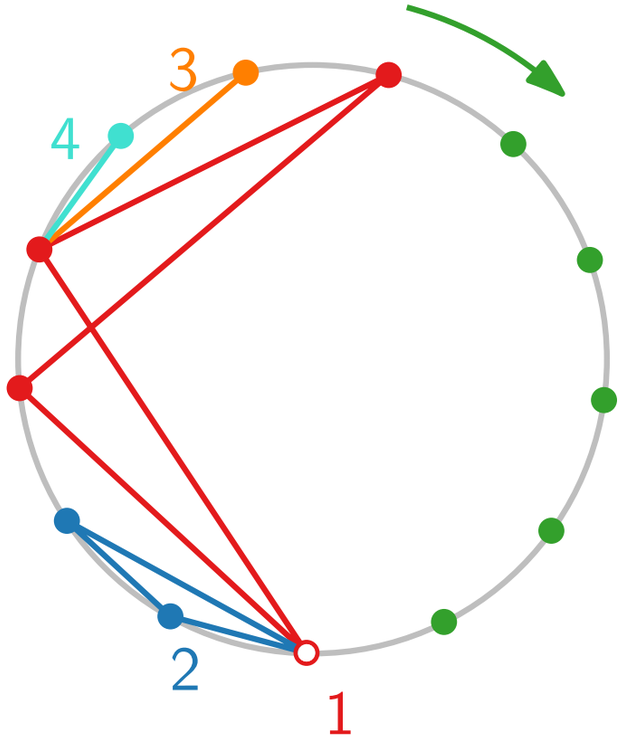

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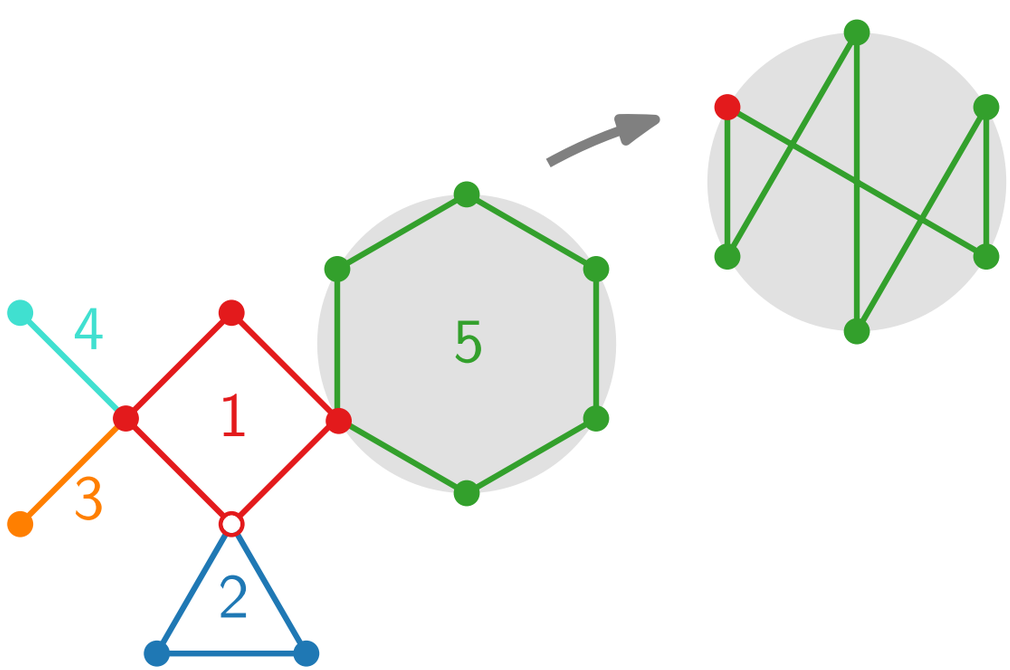

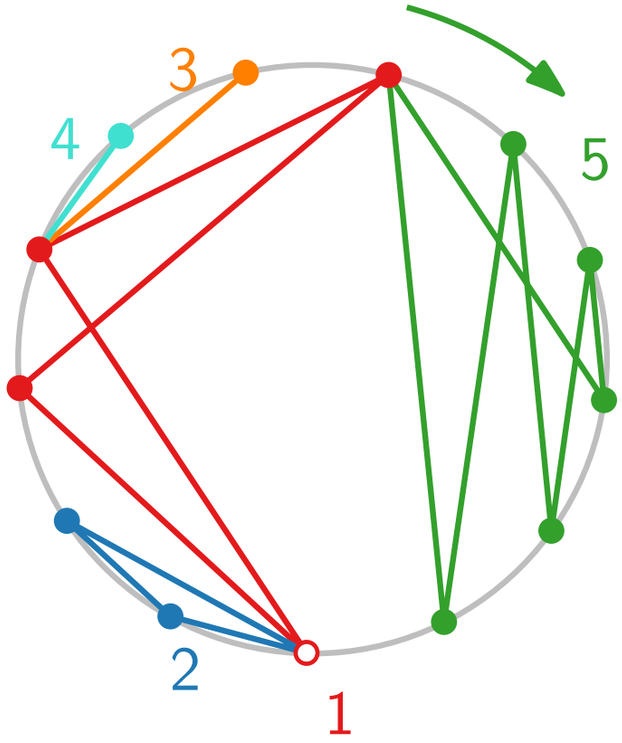

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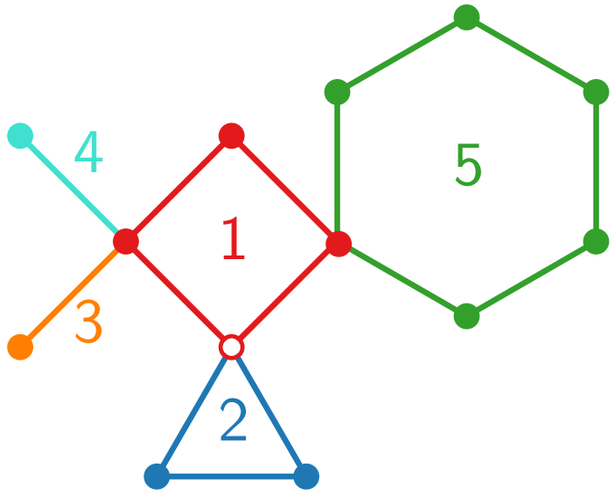

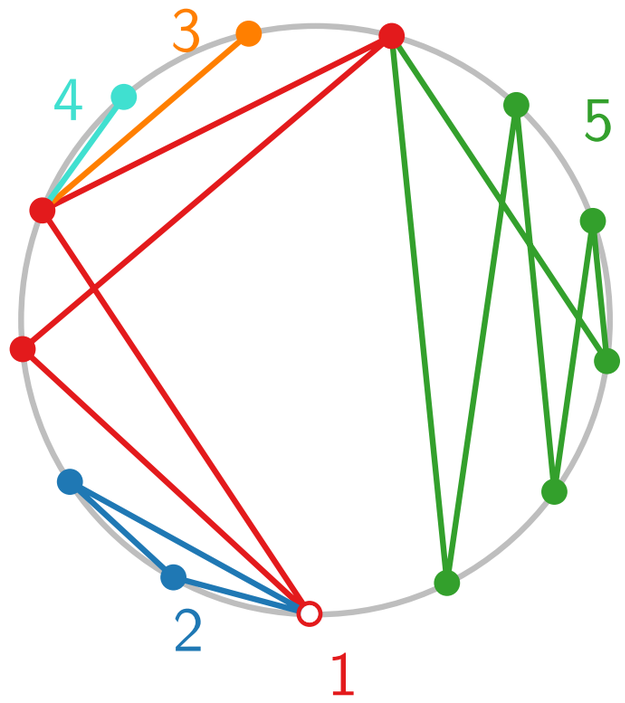

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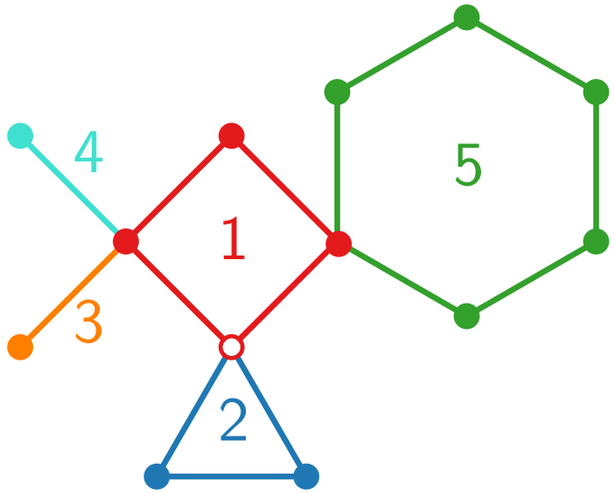

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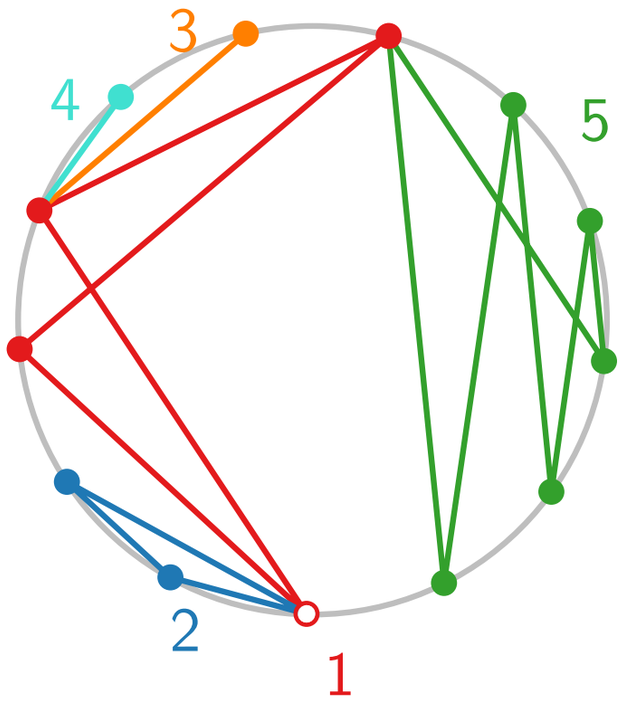

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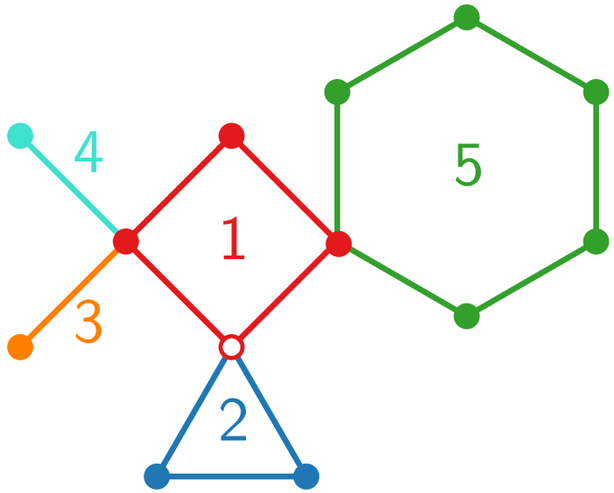

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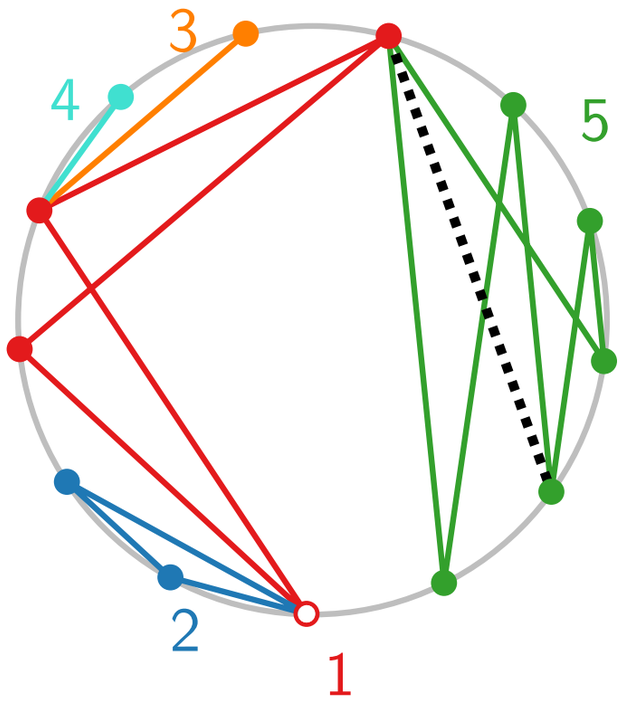

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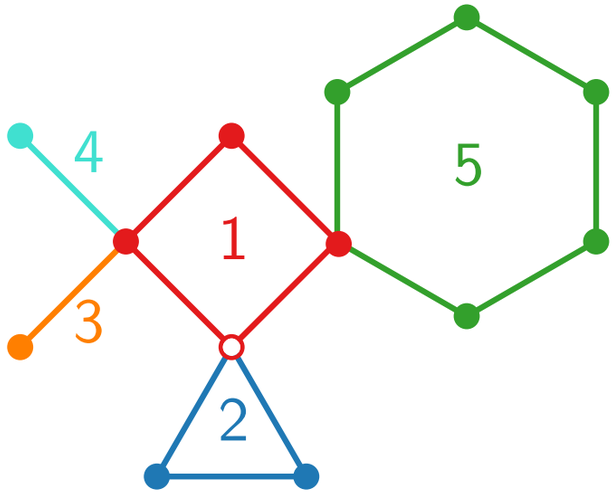

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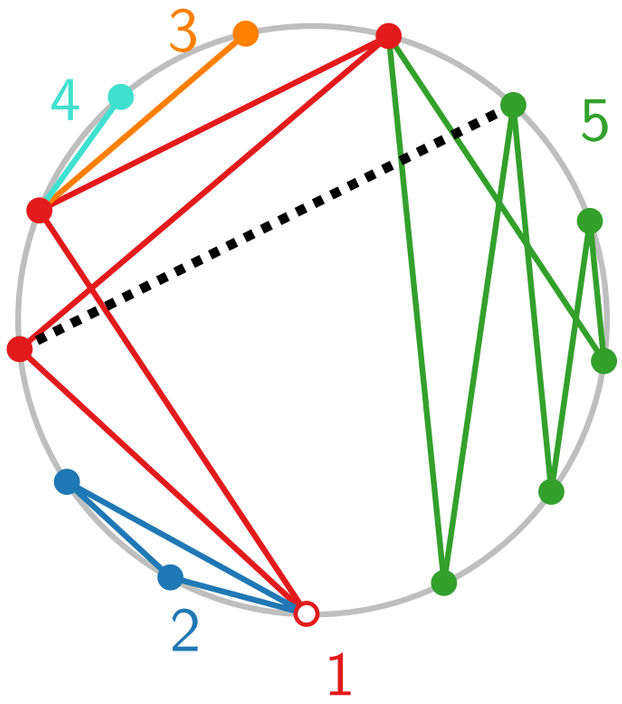
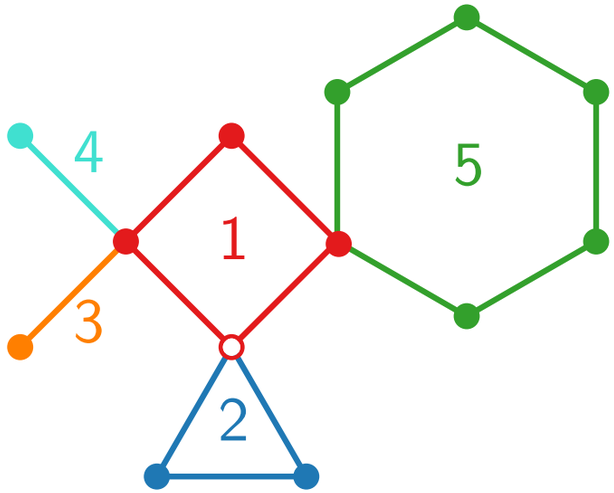


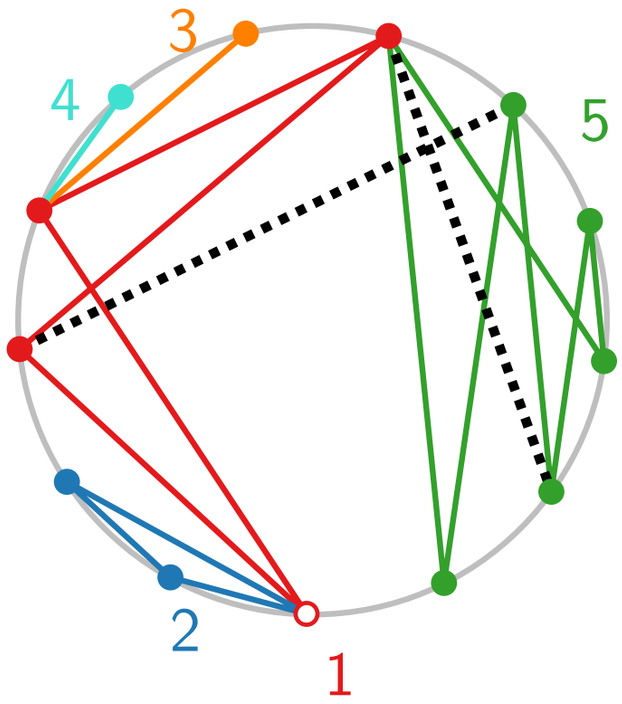
Regular Outside-Obstacle Representation

Theorem.
Every **cactus** admits a reducible regular outside-obstacle representation.

A *cactus* is a connected graph where every edge is contained in at most *one* simple cycle.

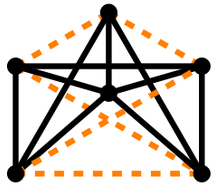


correctness ✓



Our Contribution

General

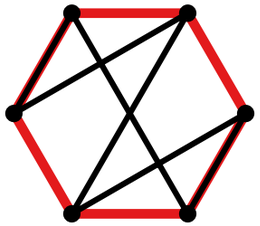


all vertices on the outer face

- outerplanar graph [Alpert et al., *DCG* 2010]
- 2-tree (subgraphs ✓)

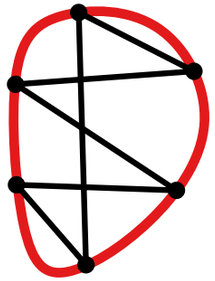
Def. An outside-obstacle representation of a graph is *reducible* if all its edges are incident to the outer face.

Regular



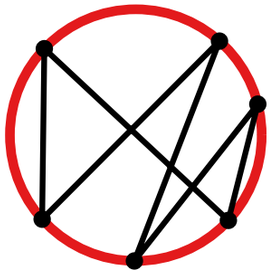
- cactus (subgraphs ✓)
- grid (subgraphs ✓)
- outerpath (subgraphs ✓)

Convex



- consecutive-neighbors property *sufficient*
- gap condition *necessary*

Circular



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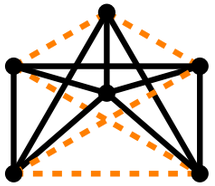

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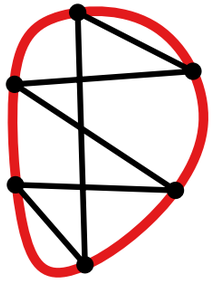

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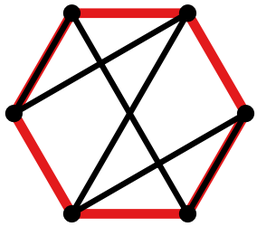

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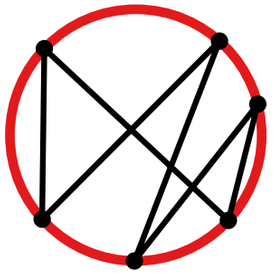

(General) Outside-Obstacle Representation

Theorem.
Every **2-tree** admits a reducible outside-obstacle representation with all vertices on the outer face.

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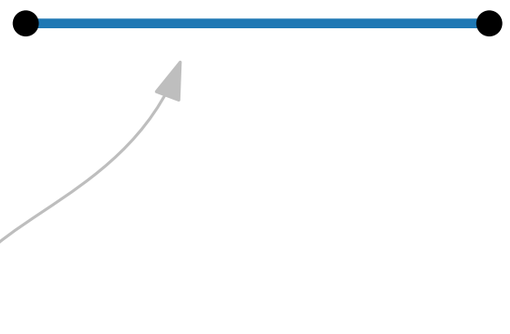

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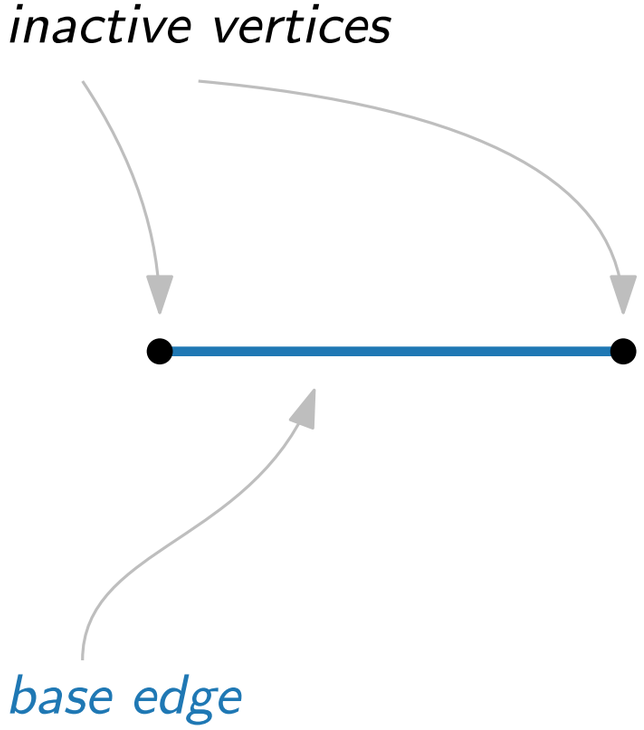

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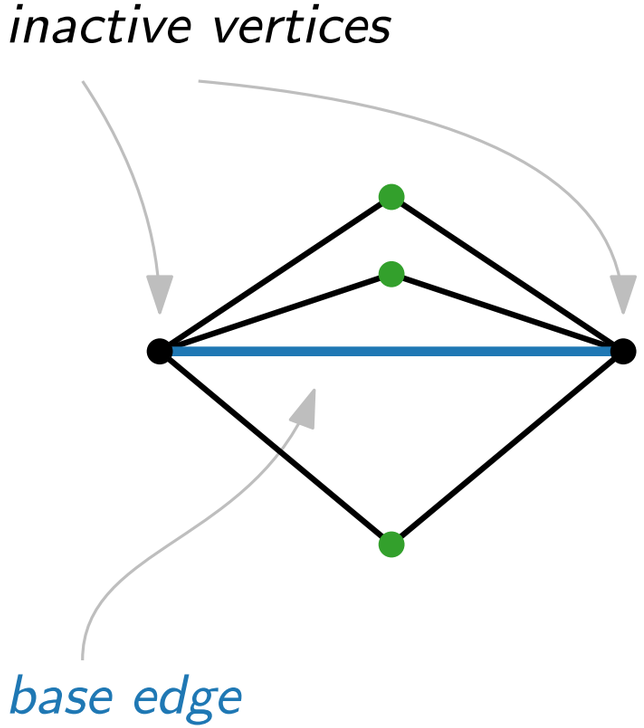

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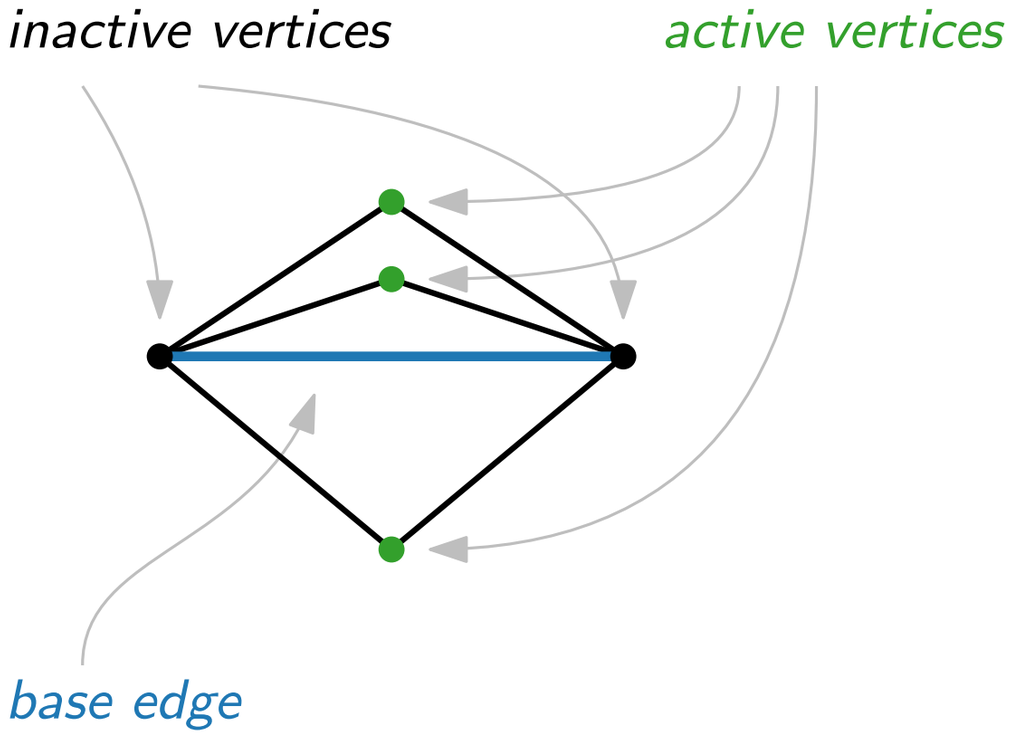

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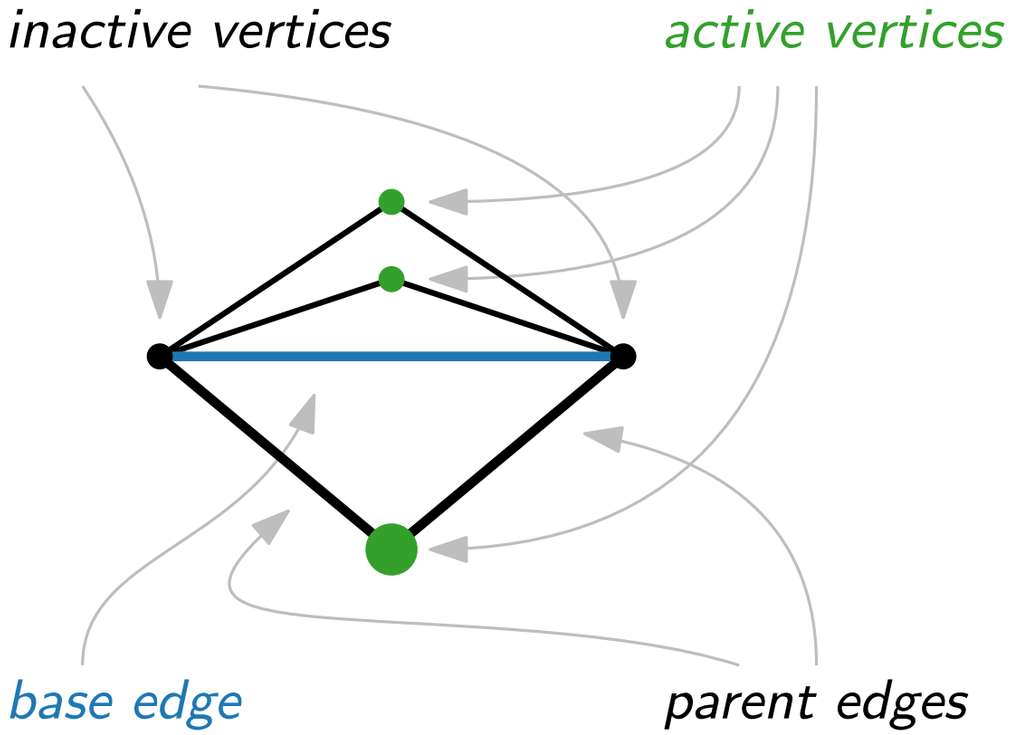

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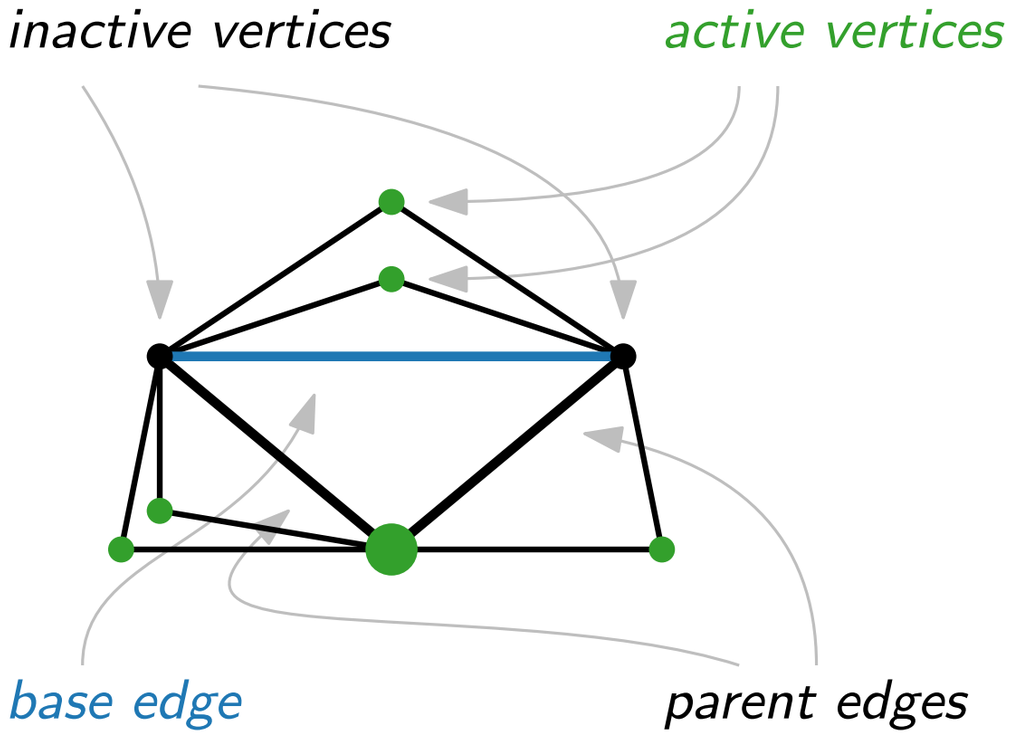

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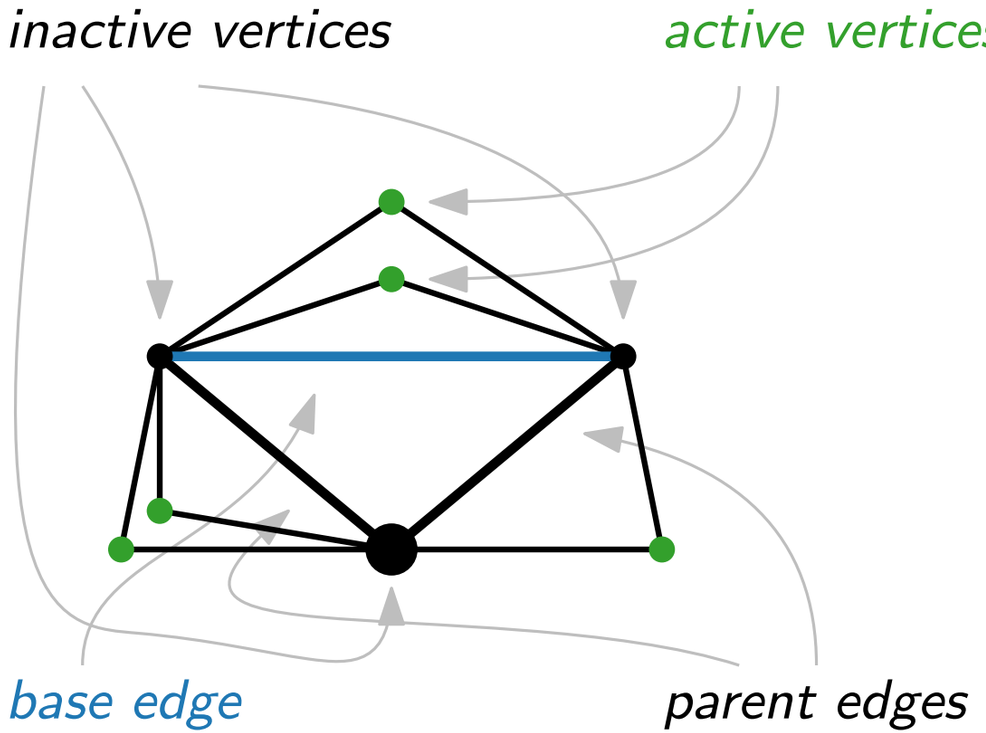

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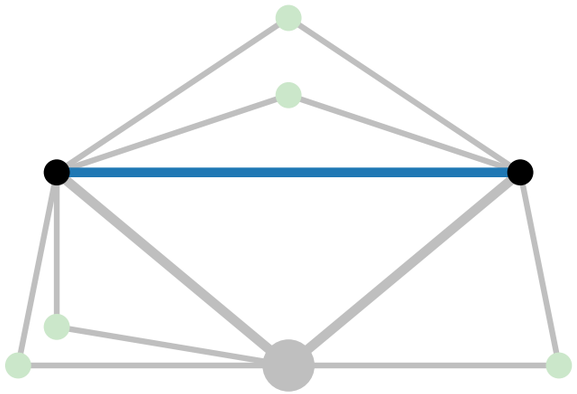

base edge *parent edges*

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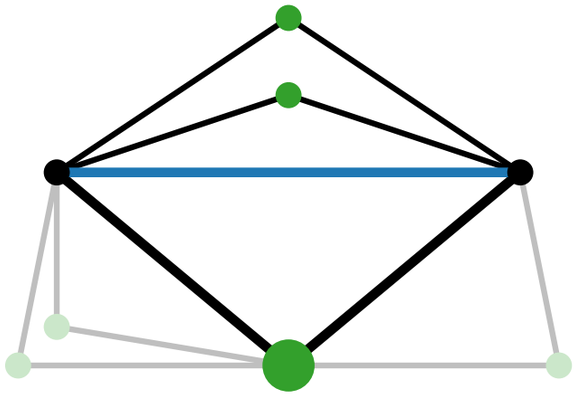

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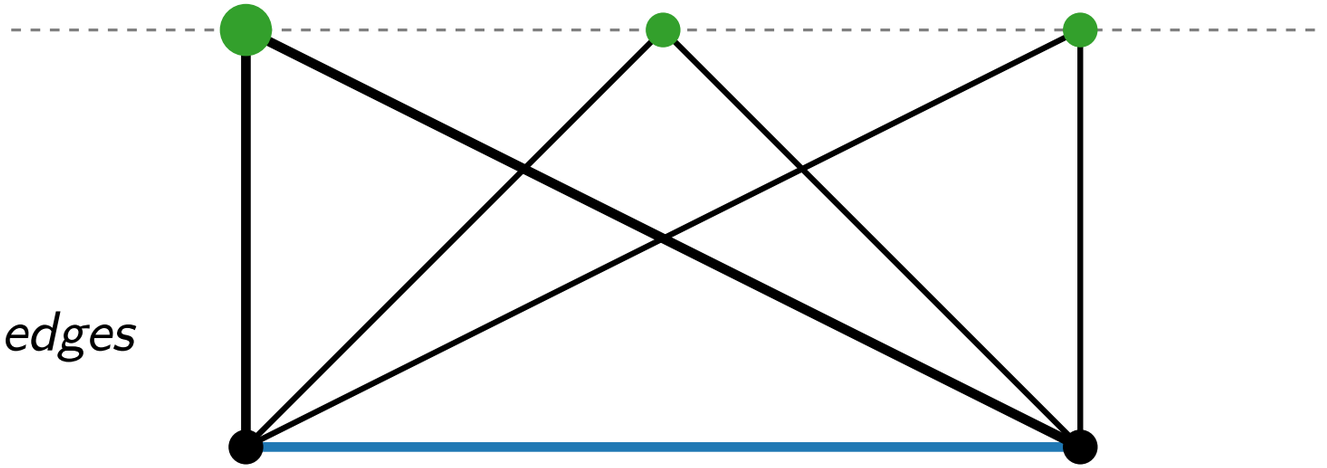

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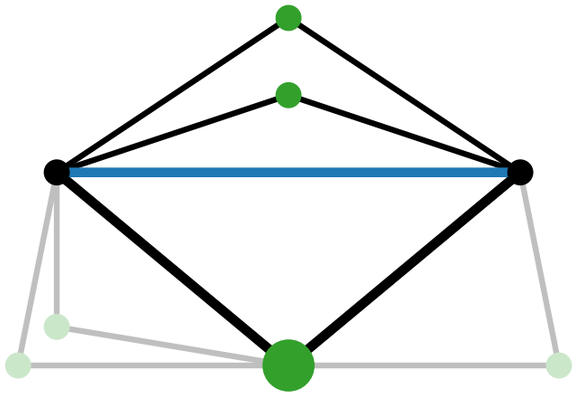

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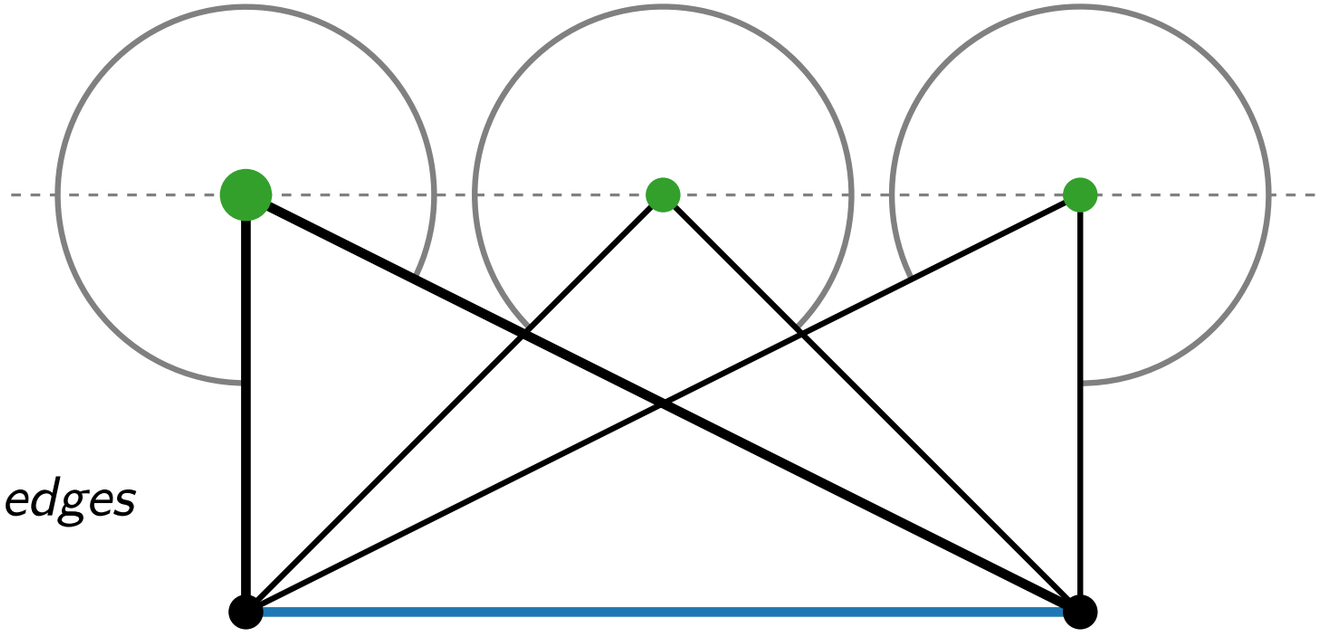

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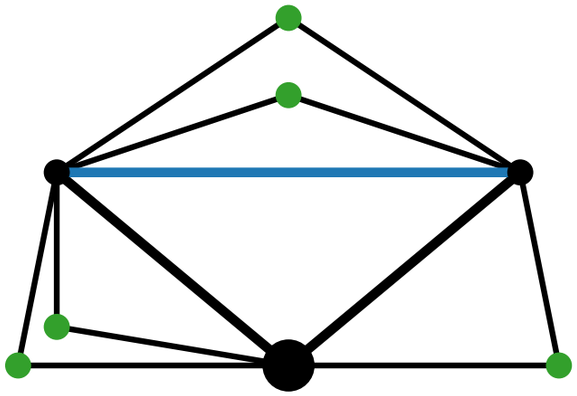

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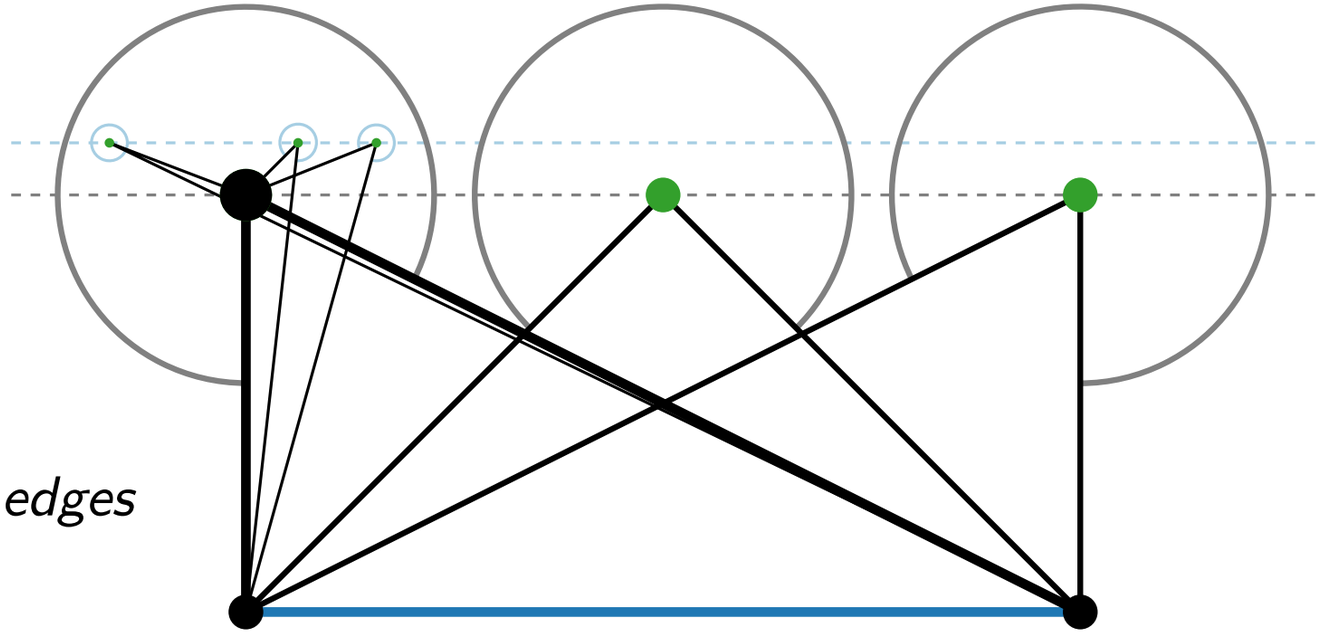

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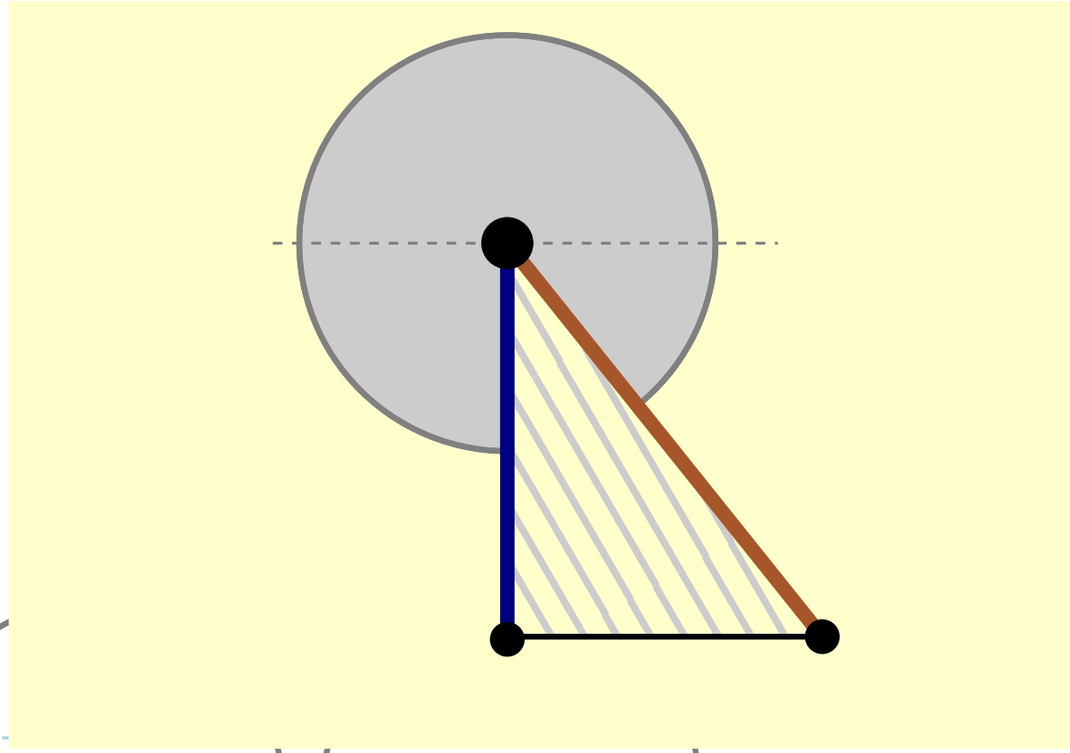

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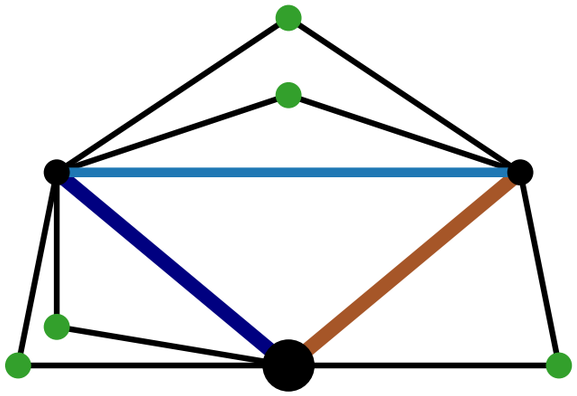

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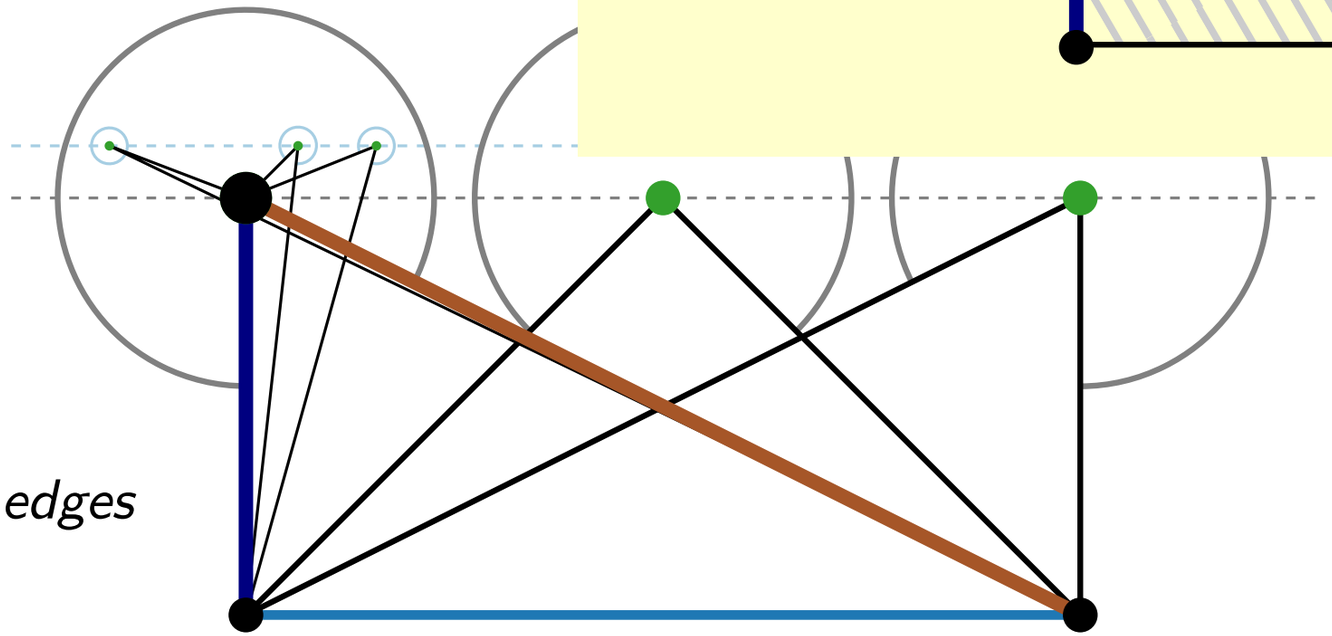

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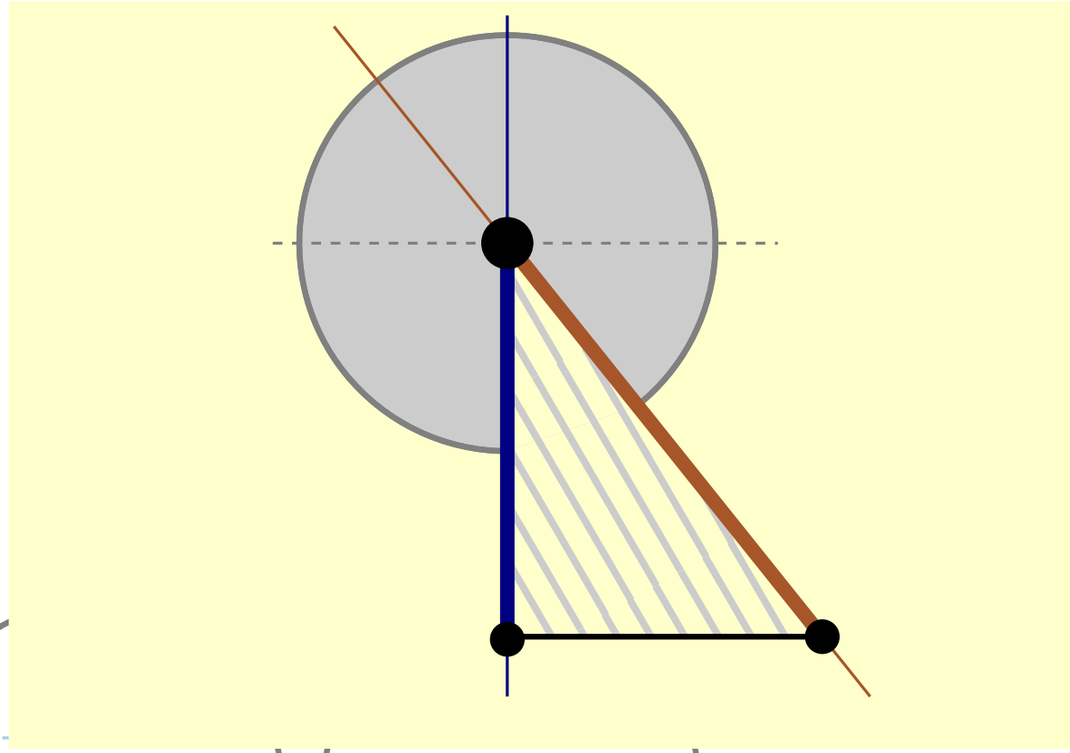

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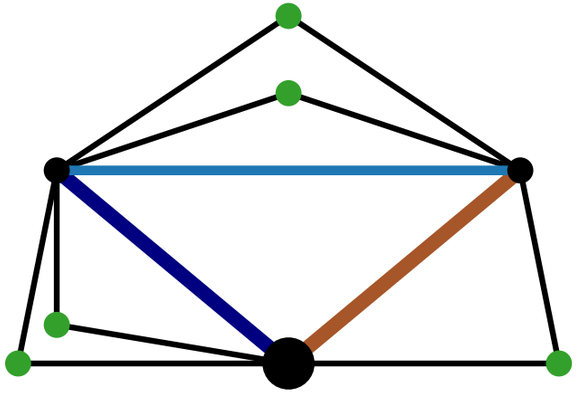

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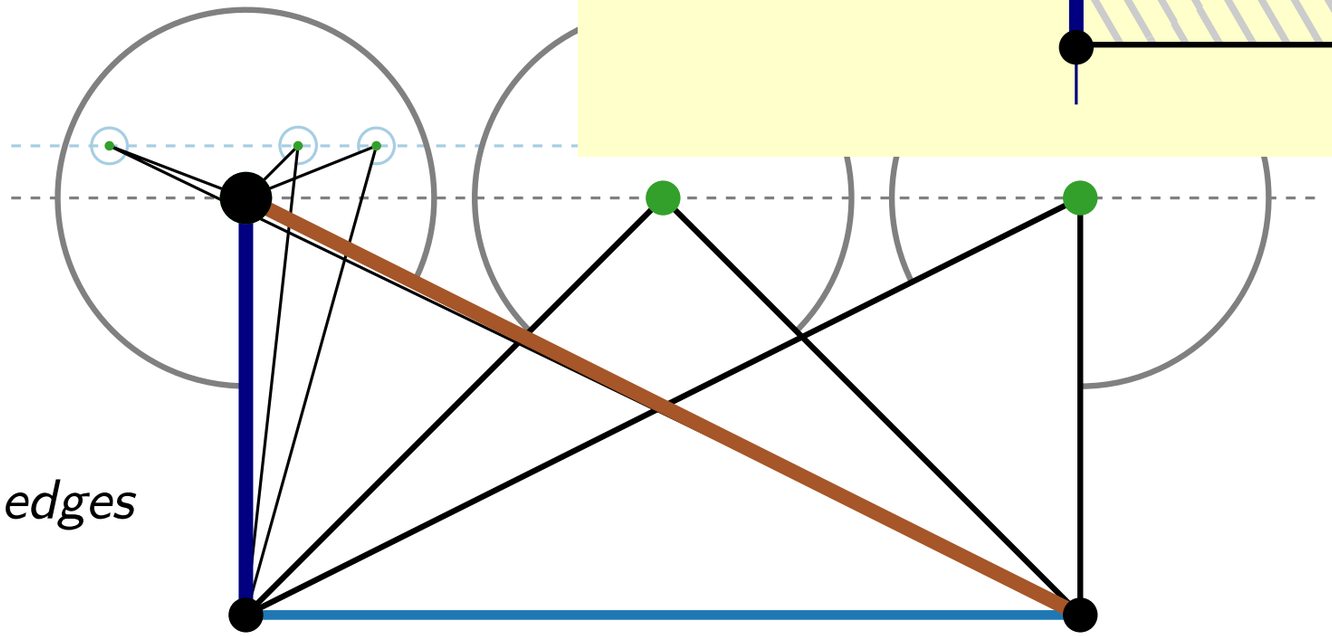

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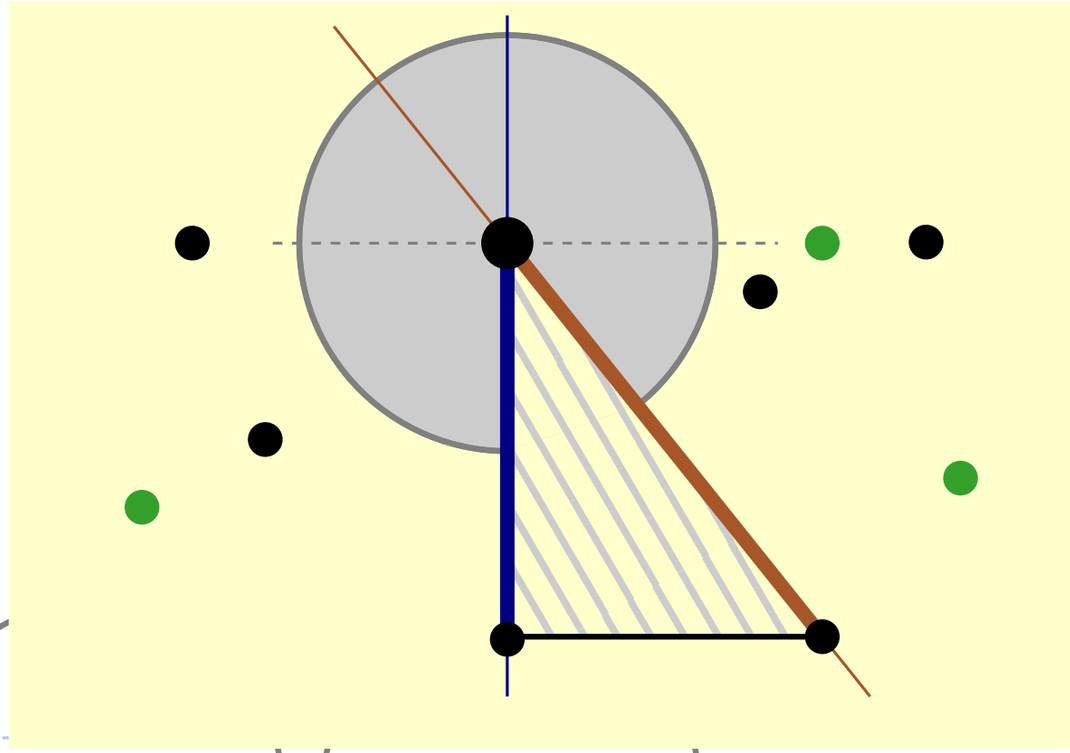

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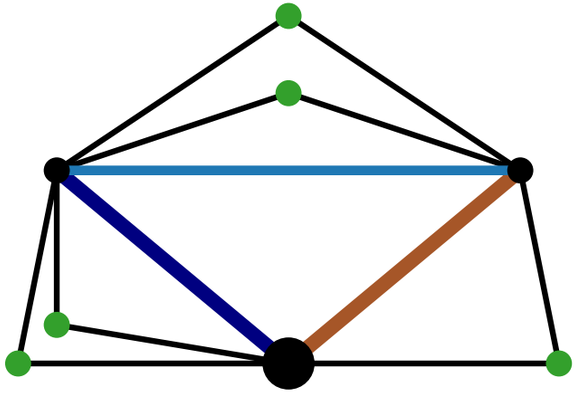

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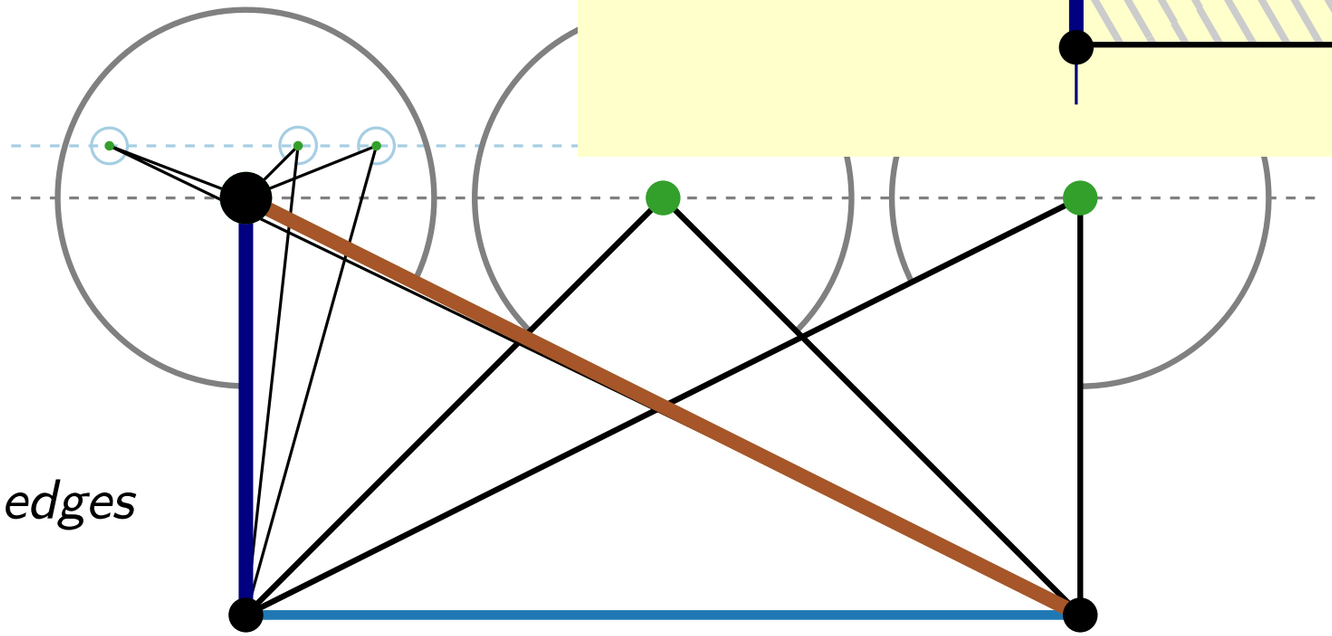

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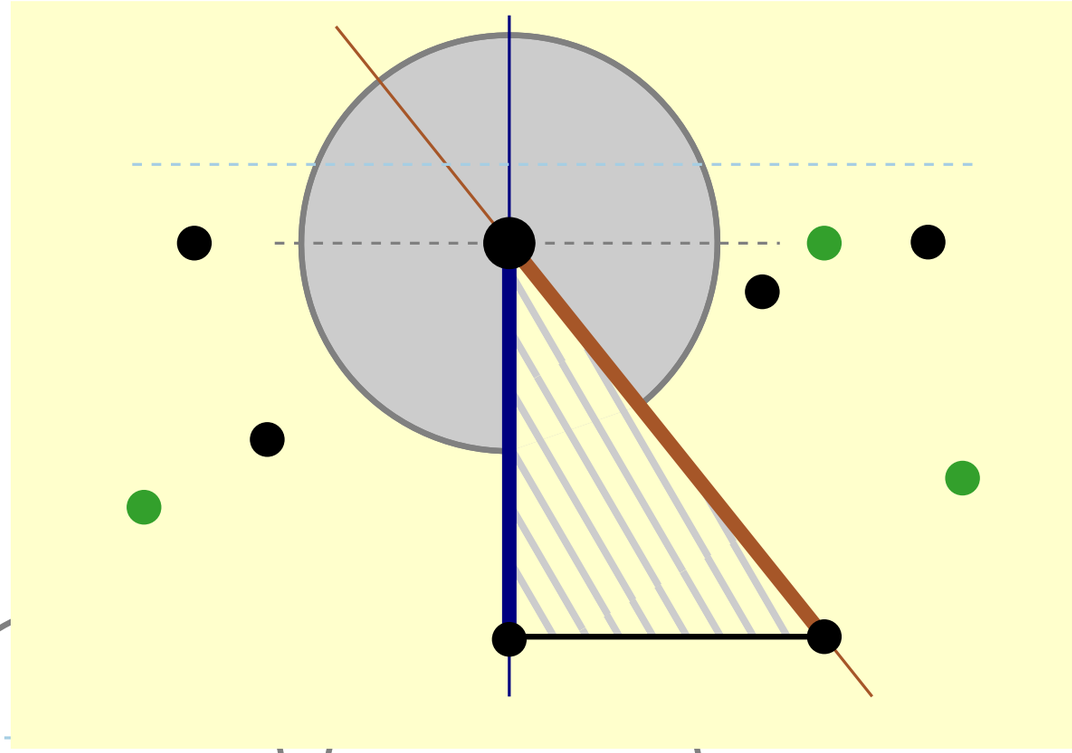

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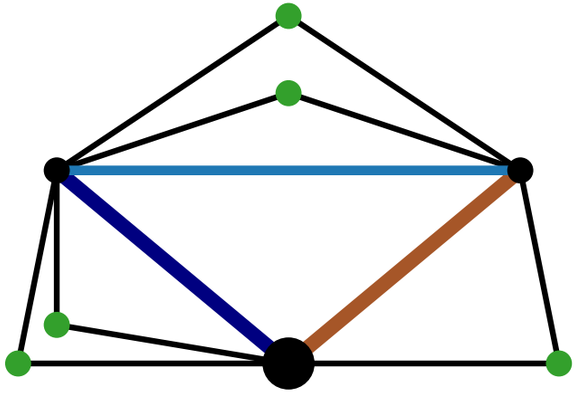

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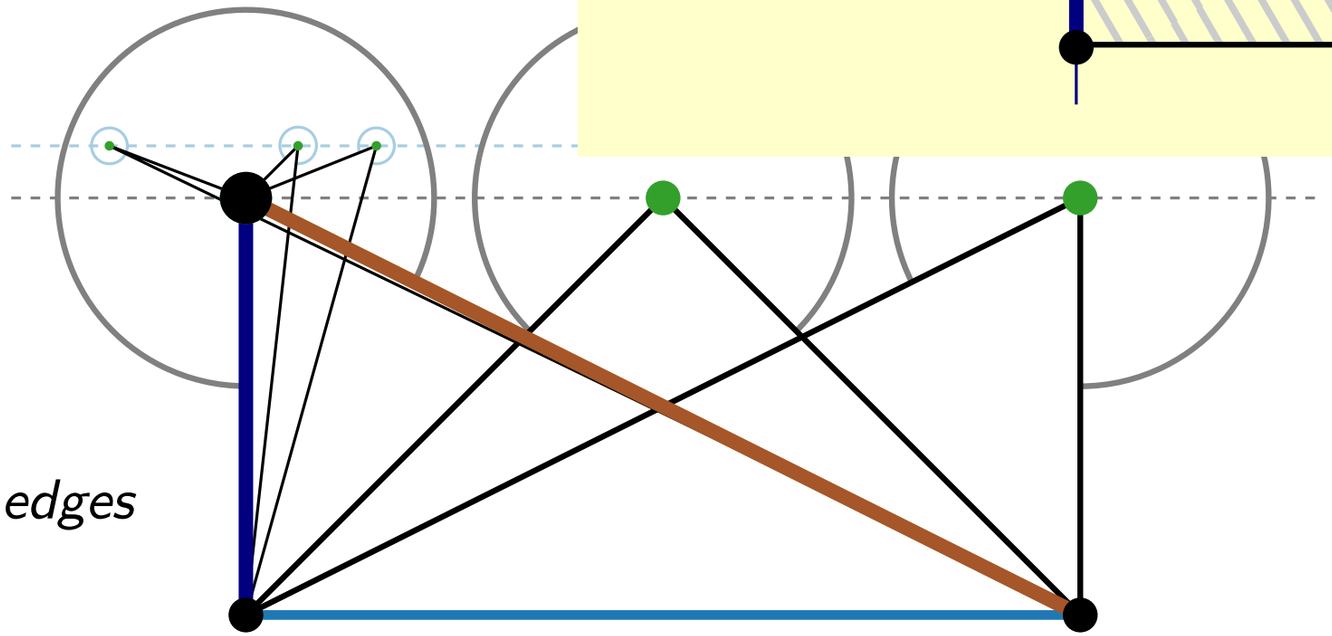

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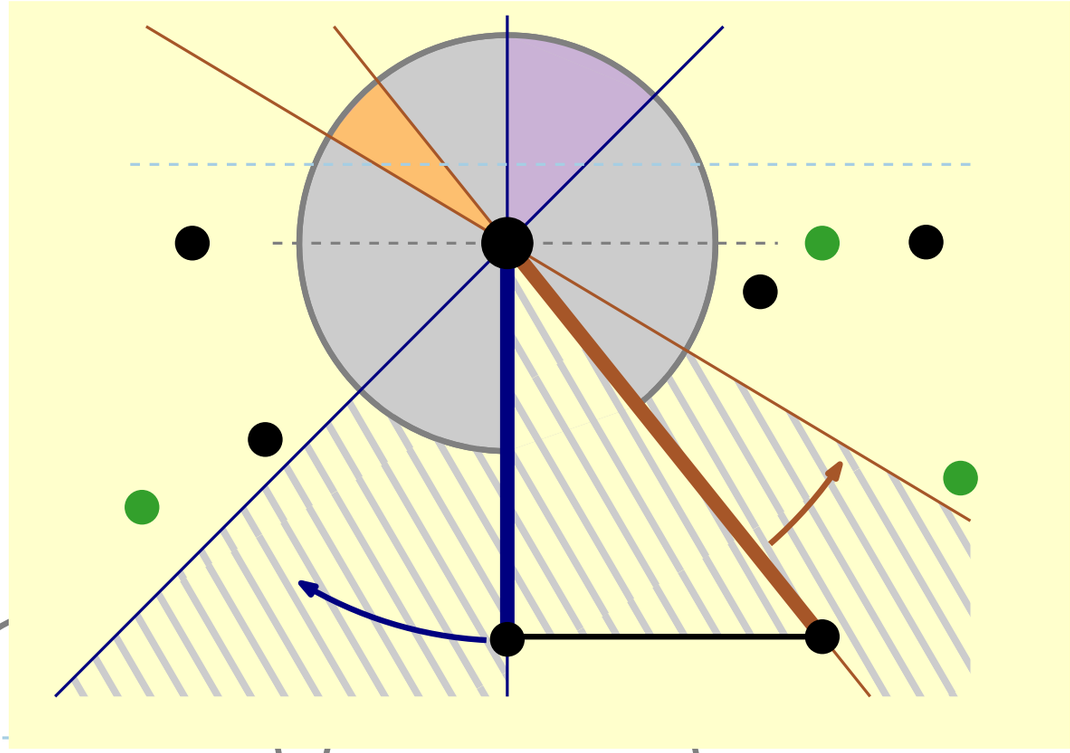

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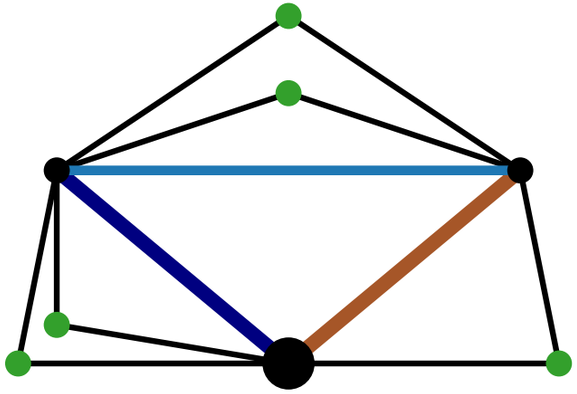

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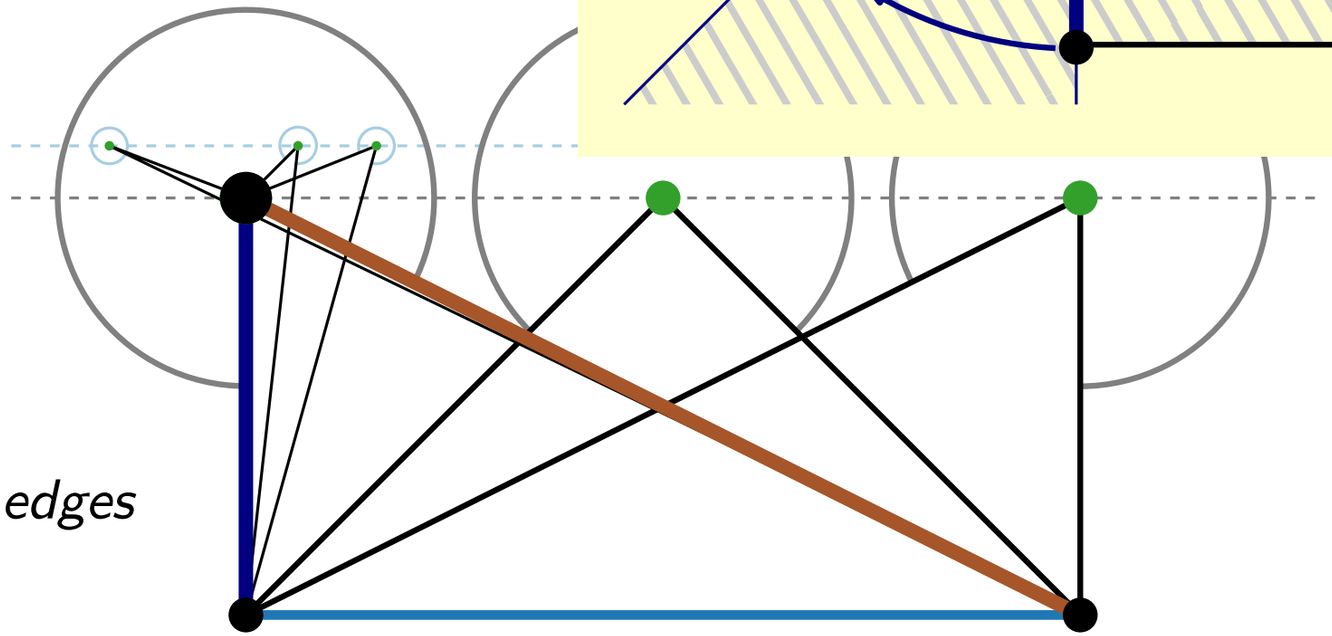

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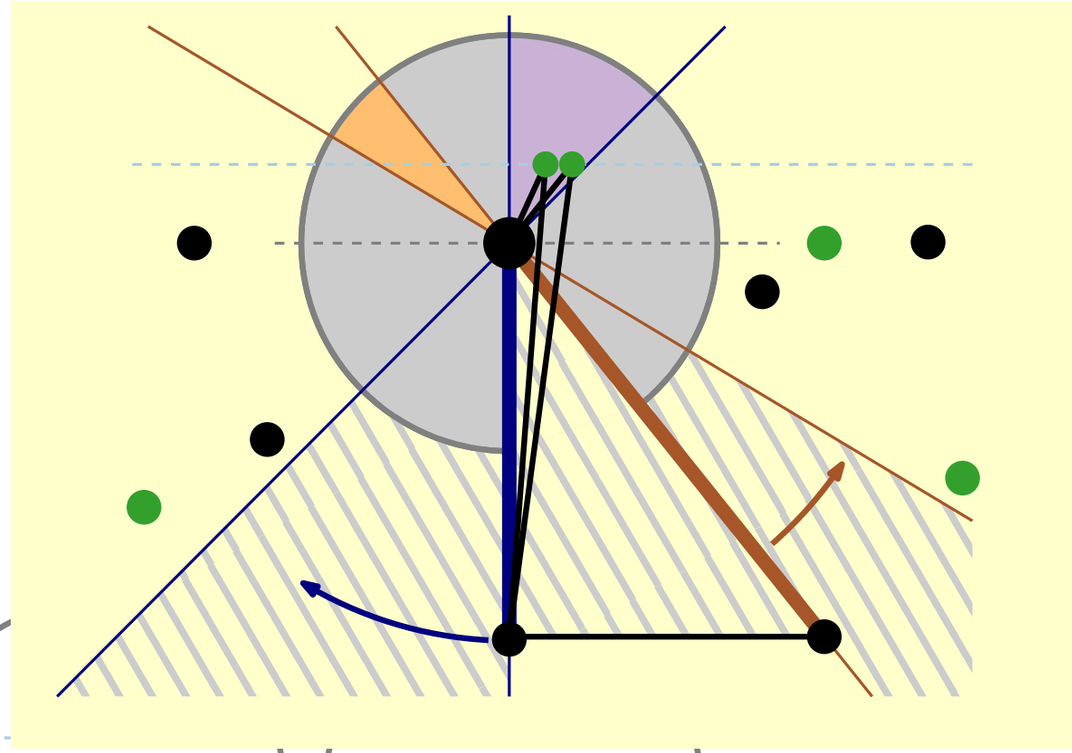

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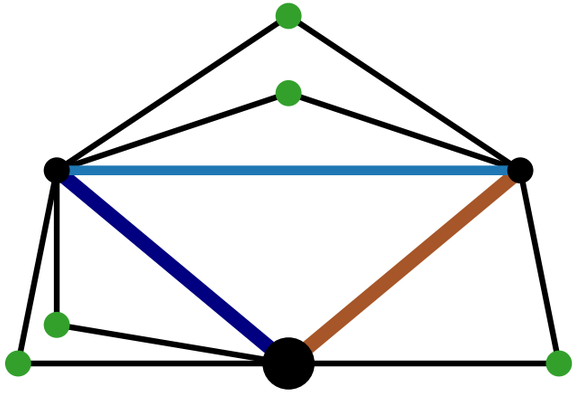

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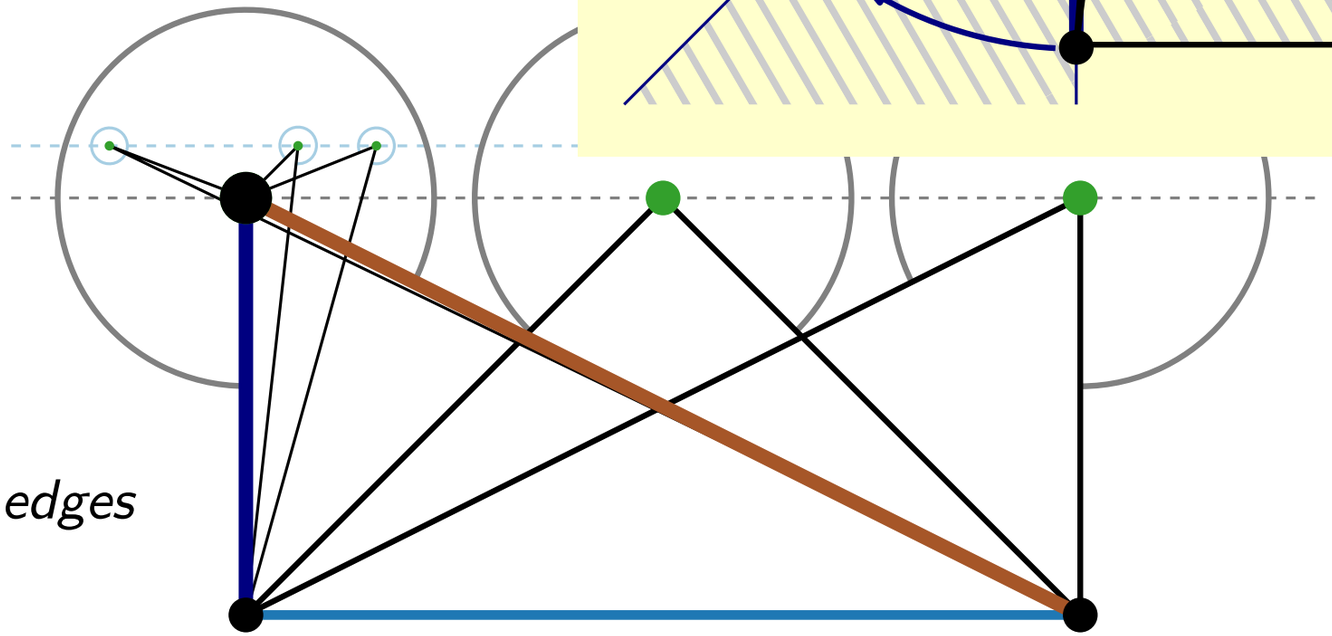

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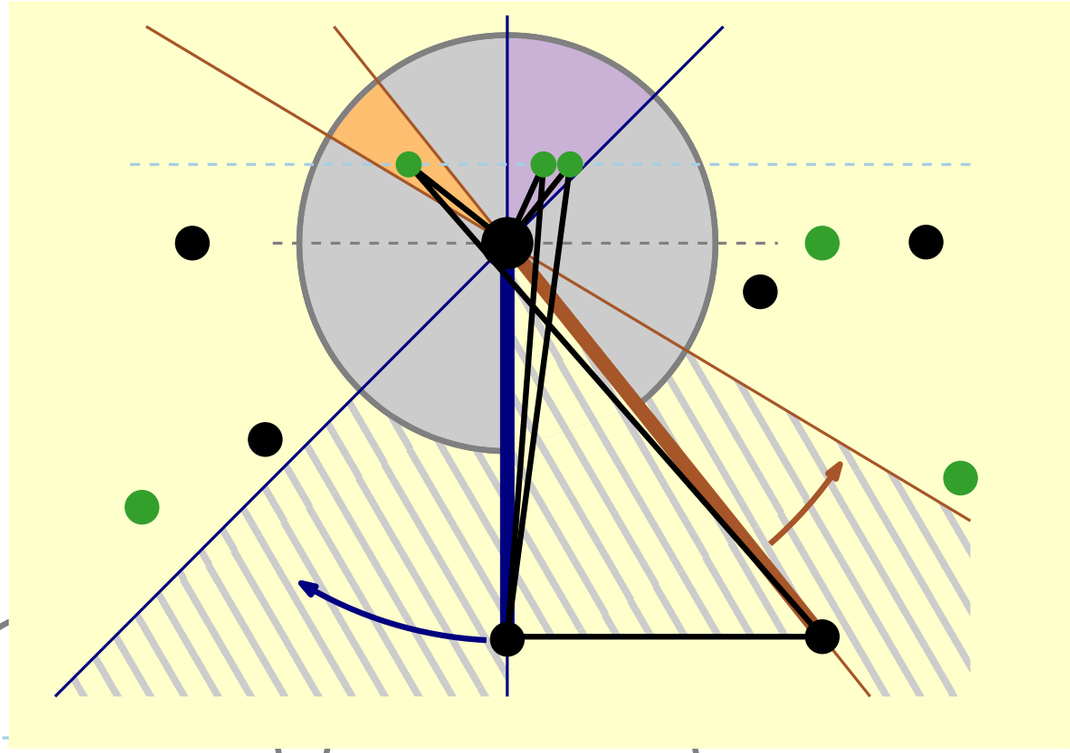

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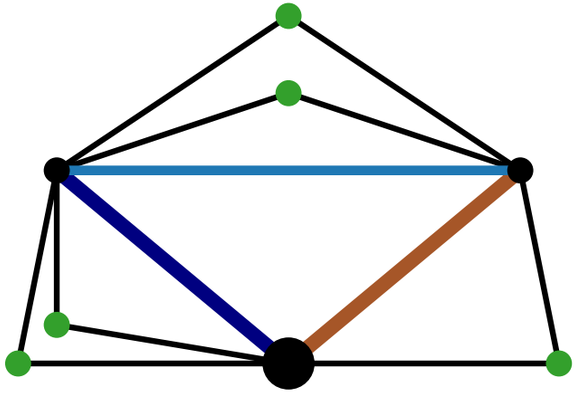

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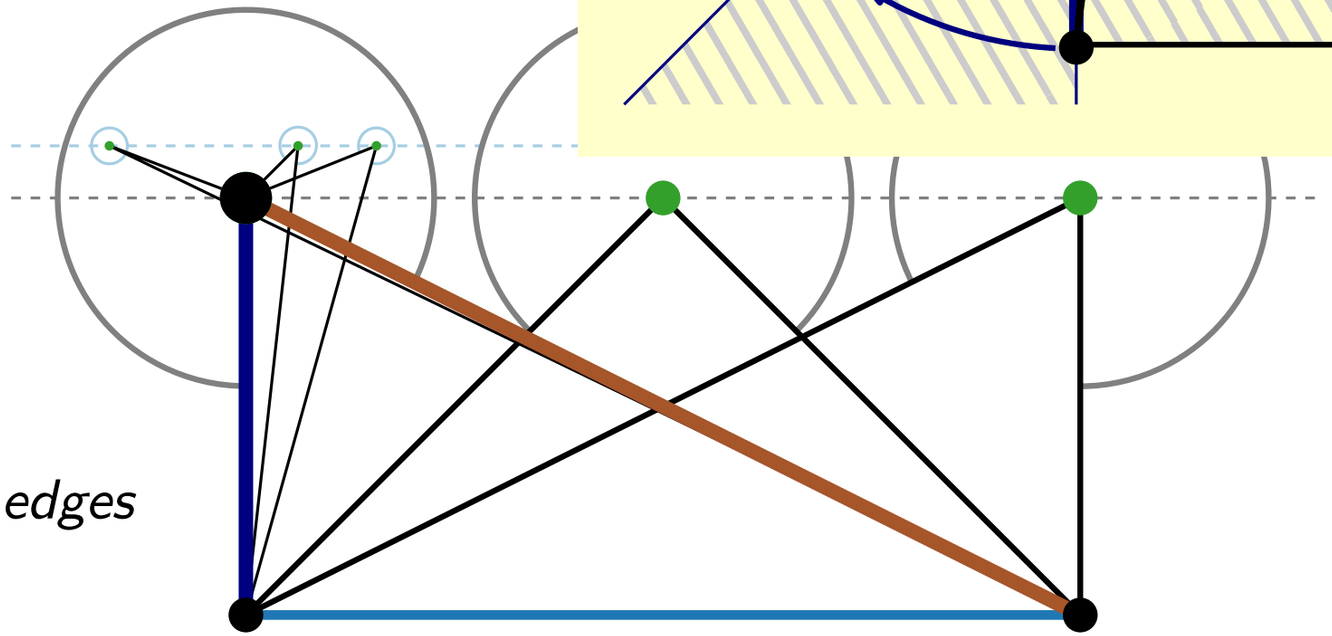

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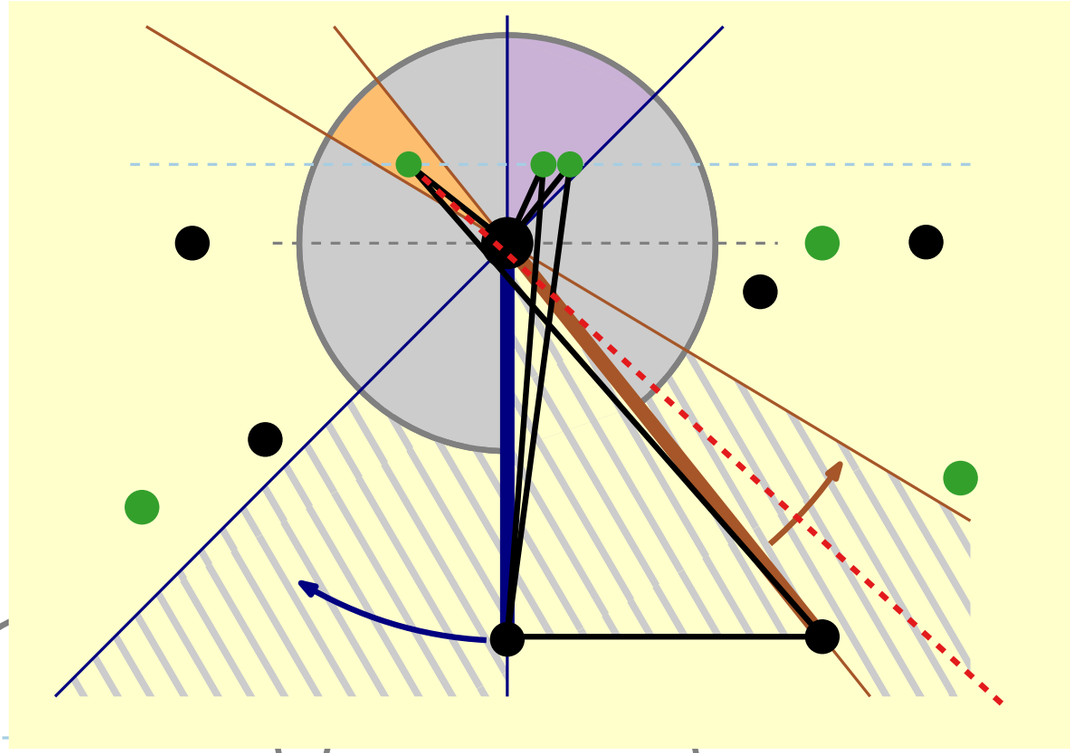

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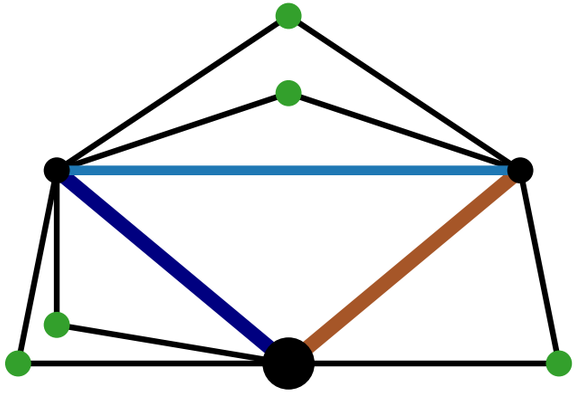

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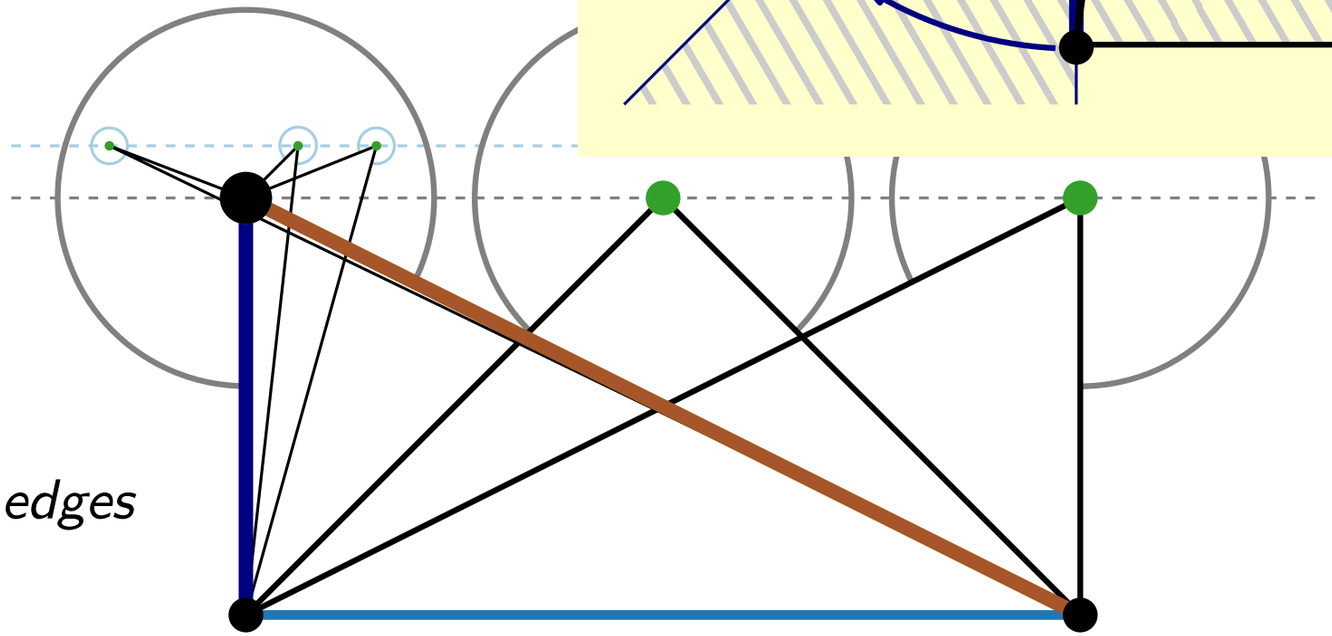

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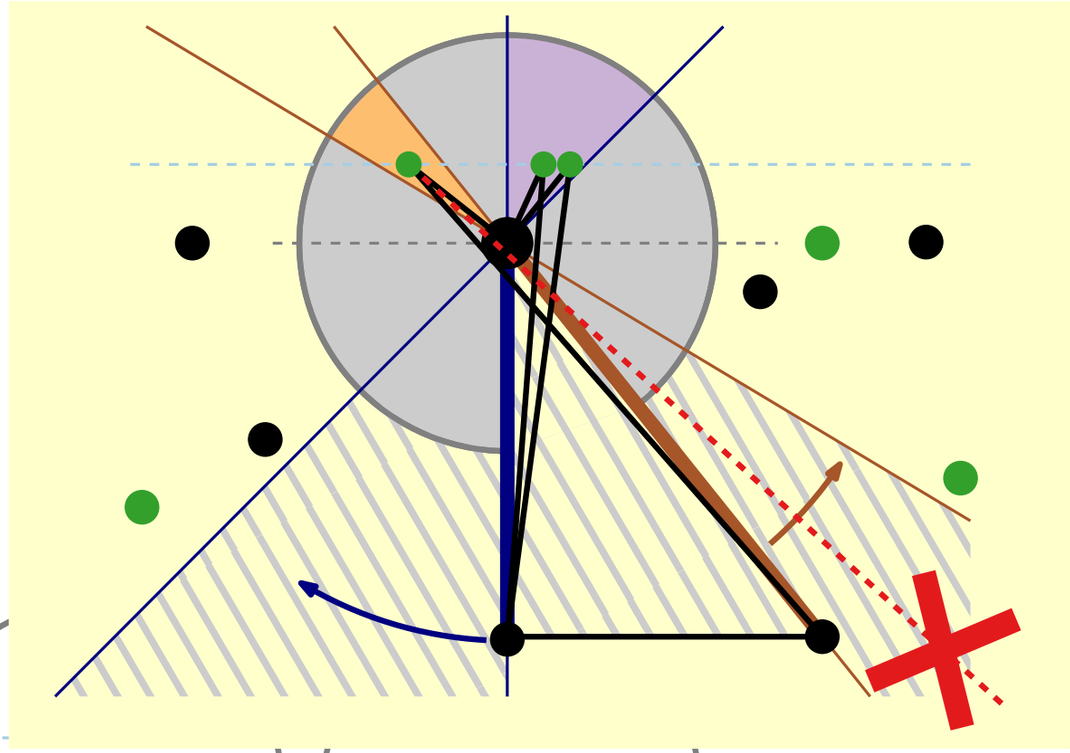

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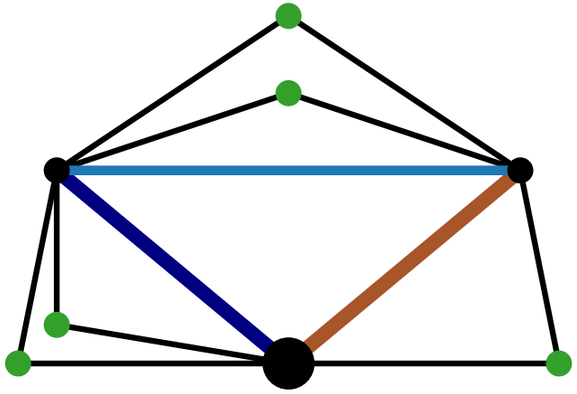

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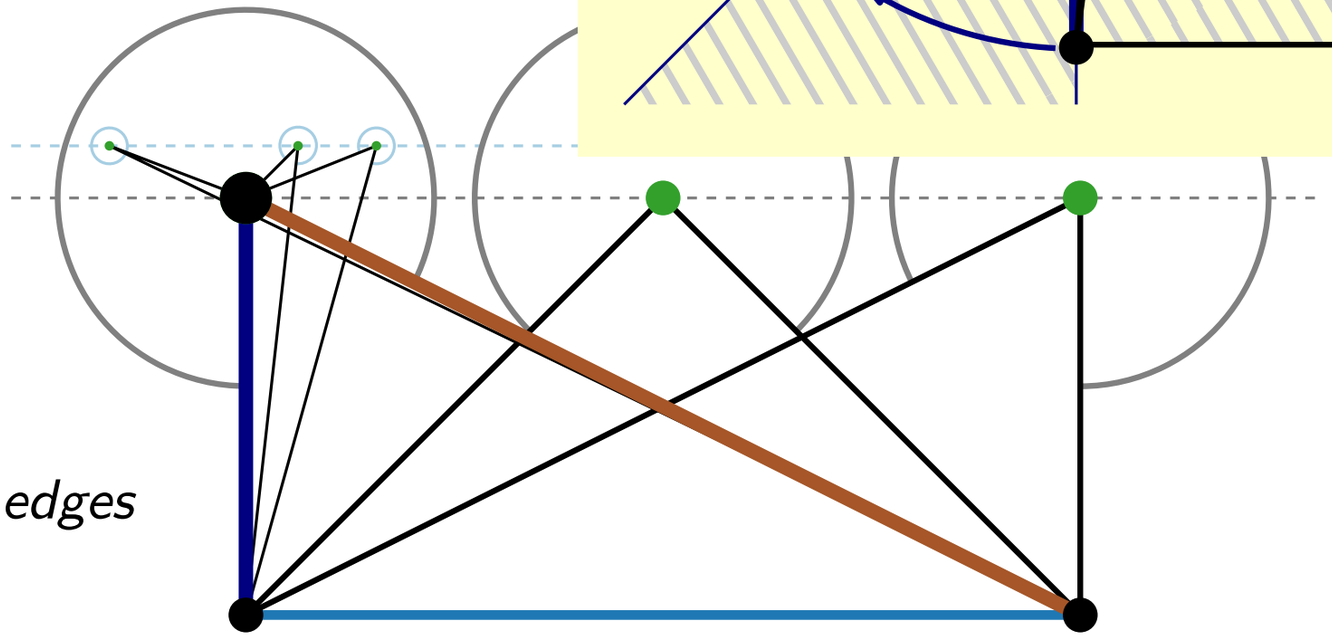

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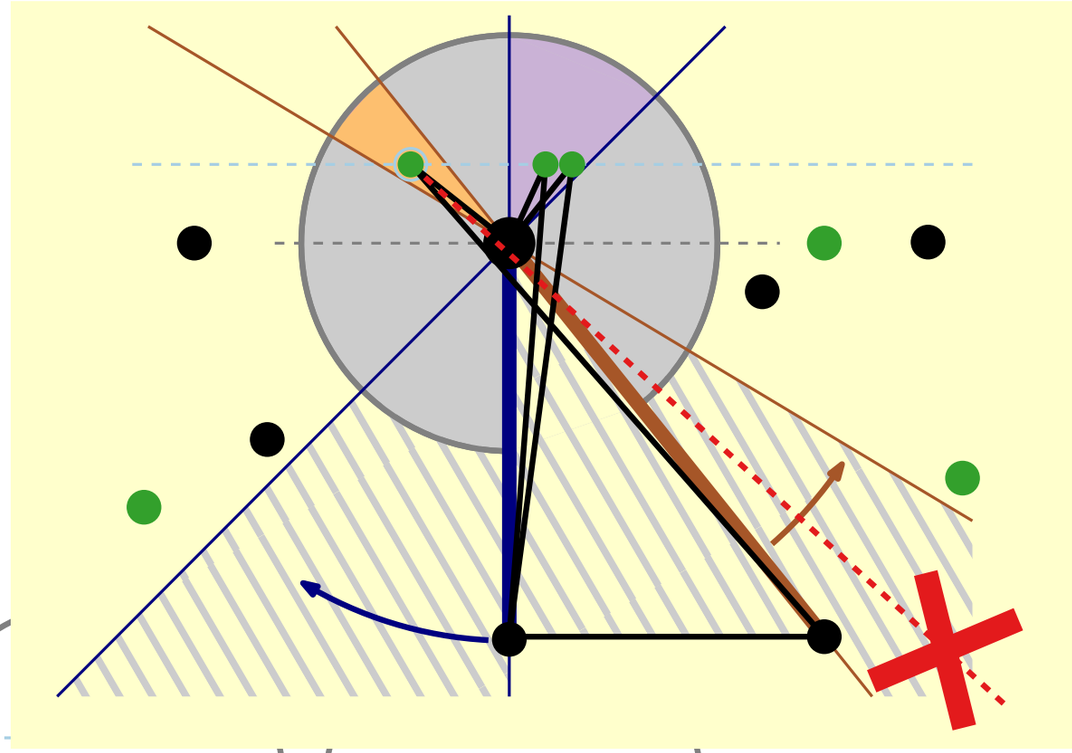
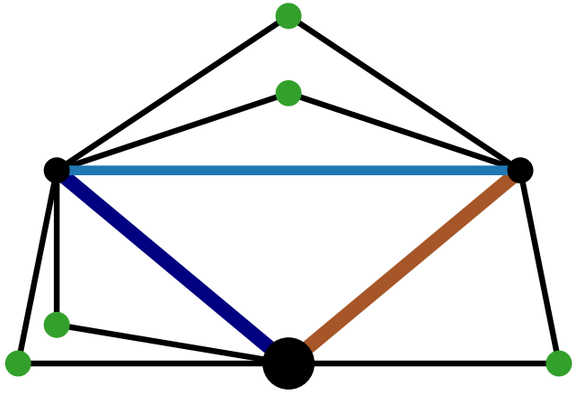
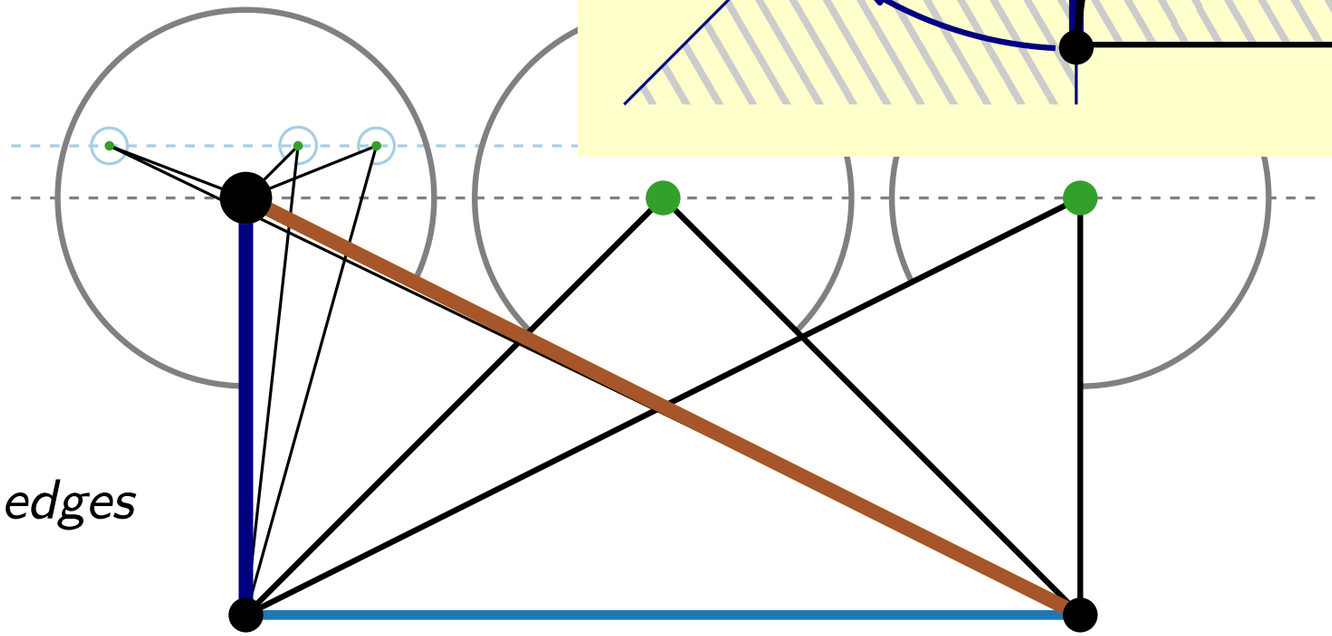


inactive vertices *active vertices*



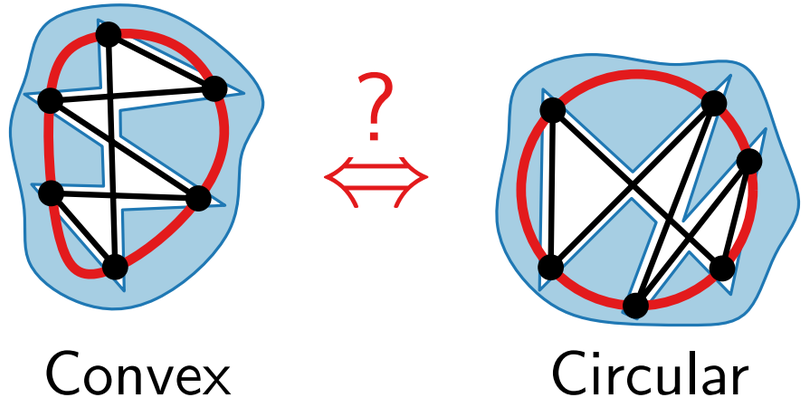
base edge

parent edges



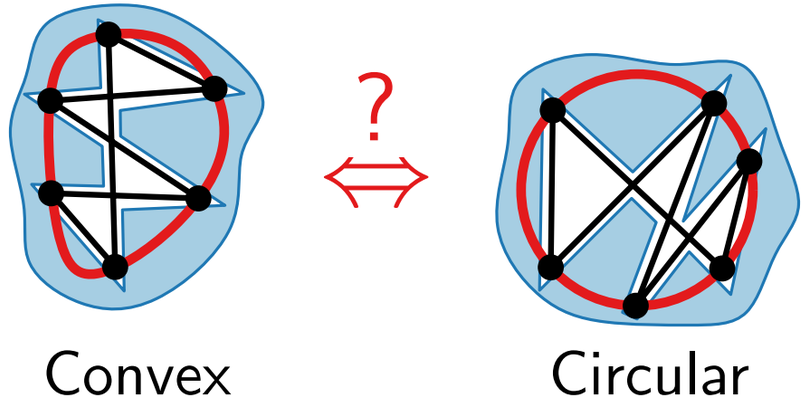
Open Problems

- Does every graph that admits a **convex** outside-obstacle representation also admit a **circular** outside-obstacle representation?



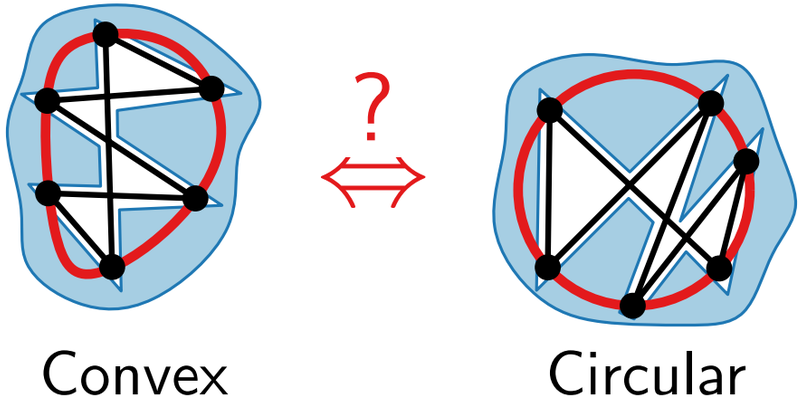
Open Problems

- Does every graph that admits a **convex** outside-obstacle representation also admit a **circular** outside-obstacle representation?
- Does every **outerplanar graph** admit a (reducible) convex outside-obstacle representation?



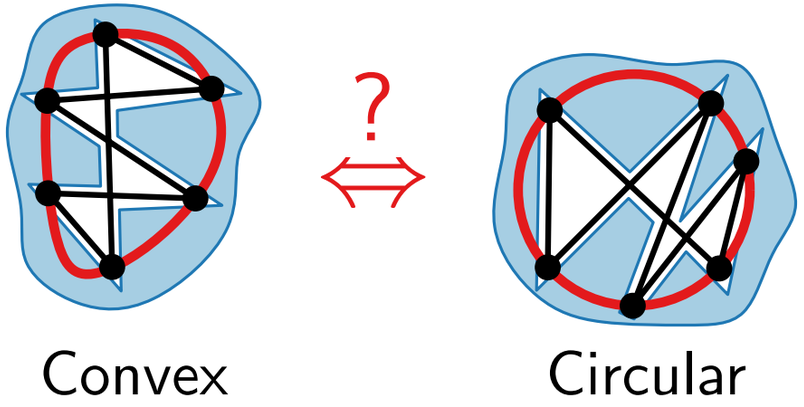
Open Problems

- Does every graph that admits a **convex** outside-obstacle representation also admit a **circular** outside-obstacle representation?
- Does every **outerplanar graph** admit a (reducible) convex outside-obstacle representation?
- What is the **complexity** of deciding whether a given graph admits an outside-obstacle representation?



Open Problems

- Does every graph that admits a **convex** outside-obstacle representation also admit a **circular** outside-obstacle representation?
- Does every **outerplanar graph** admit a (reducible) convex outside-obstacle representation?
- What is the **complexity** of deciding whether a given graph admits an outside-obstacle representation?
- Which **other classes** of graphs admit regular or circular outside-obstacle representations?



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Thank you!

Stand with Ukraine!

Fight for truth!