The Complexity of Finding Tangles

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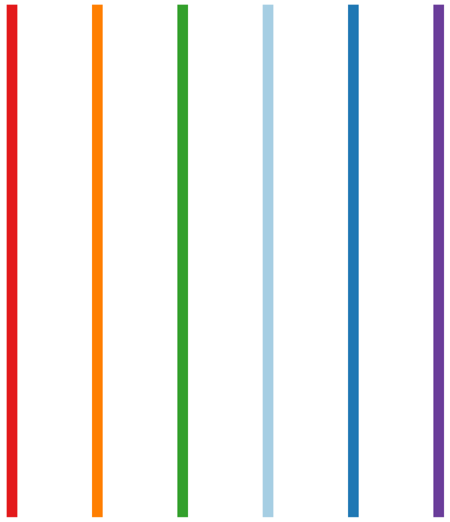
Philipp Kindermann

Universität Trier,
Germany

Introduction

Given an ordered set of n y-monotone wires



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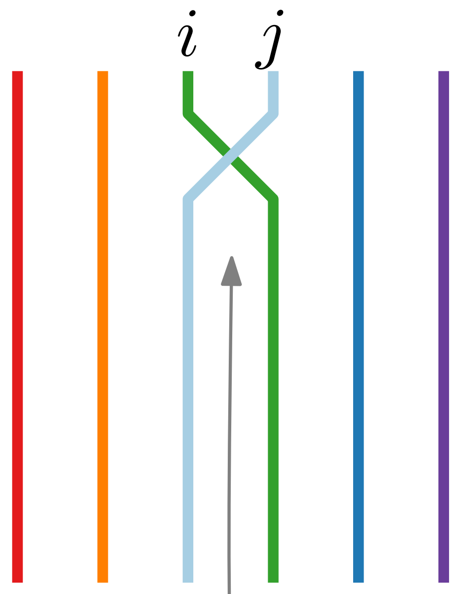

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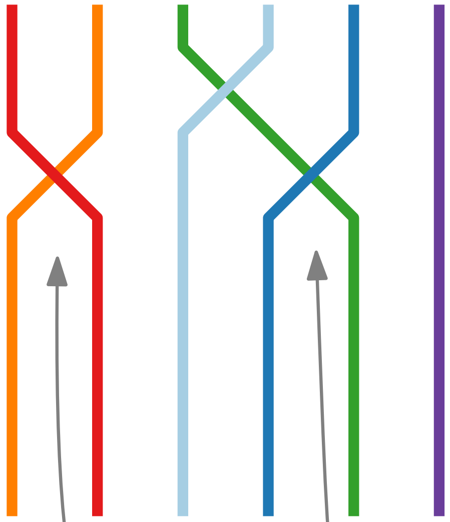

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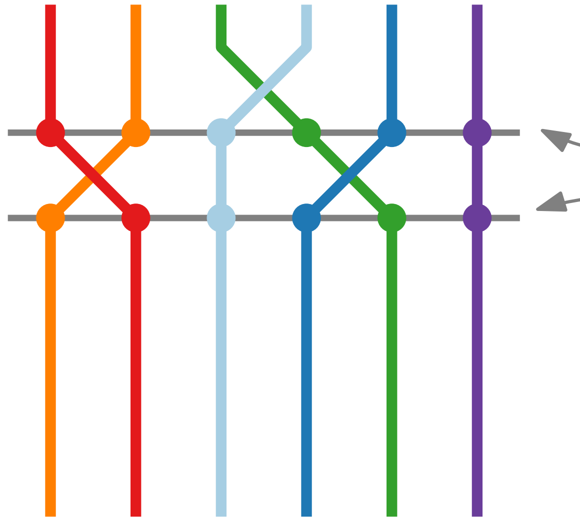

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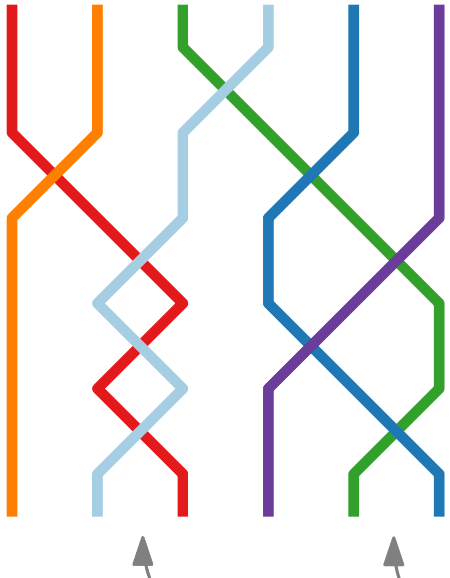

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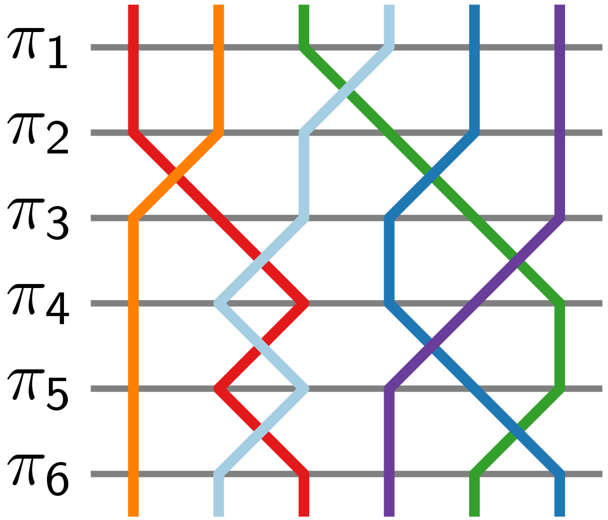

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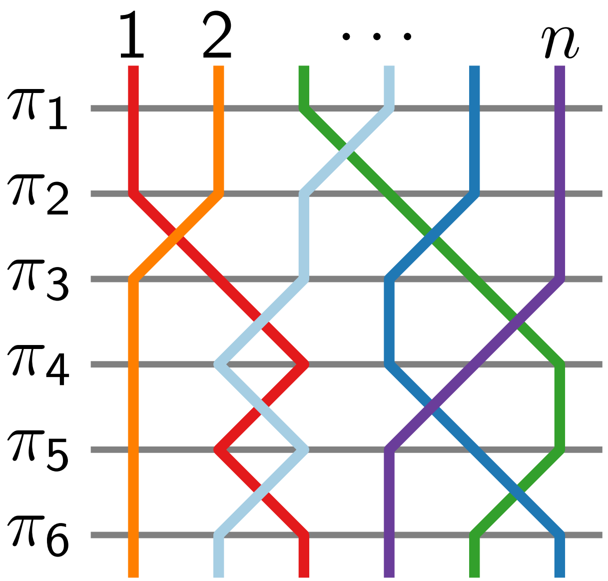

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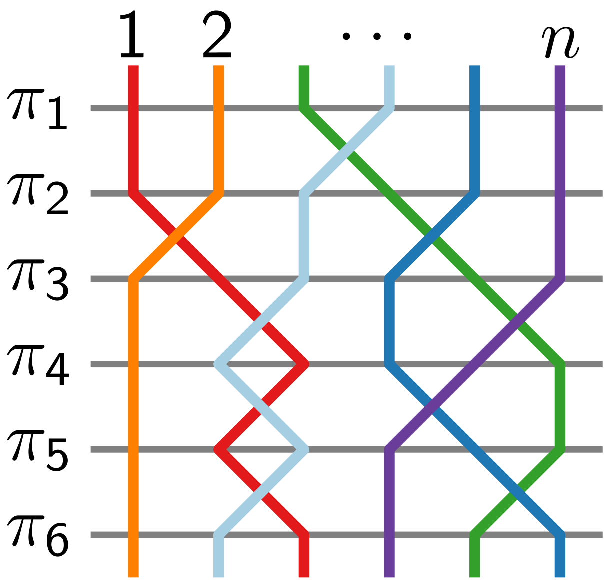

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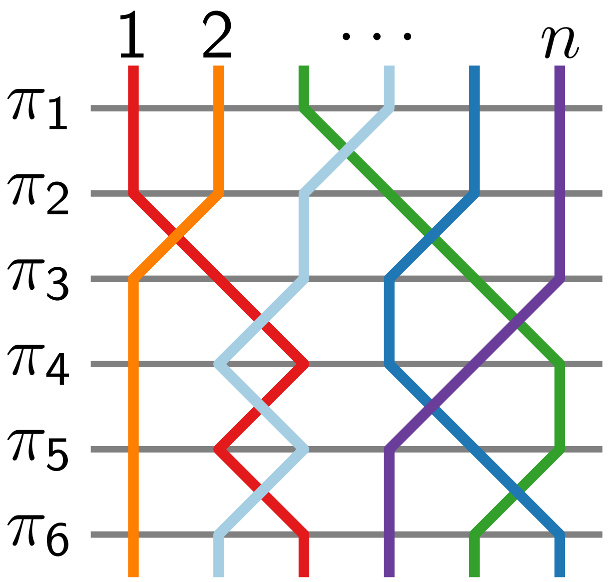

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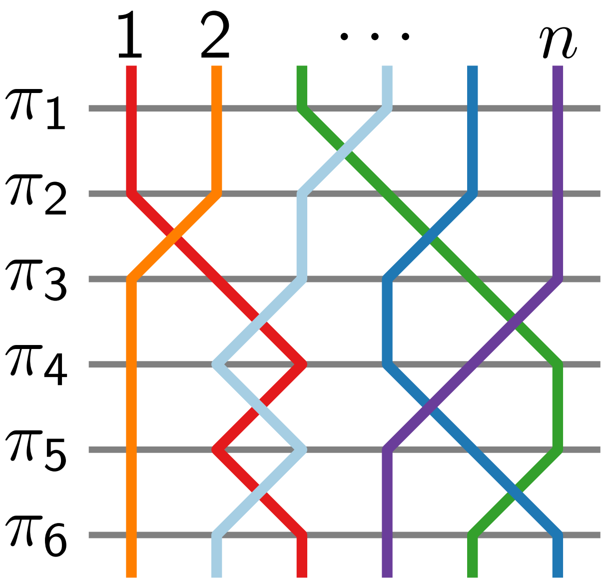

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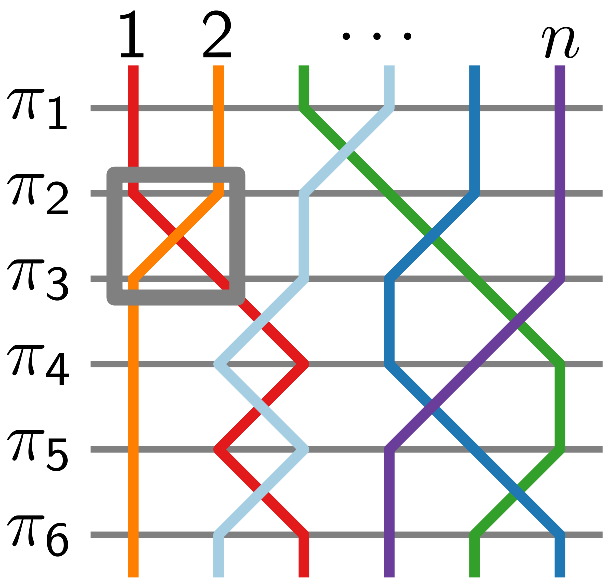

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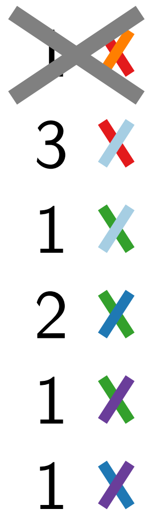

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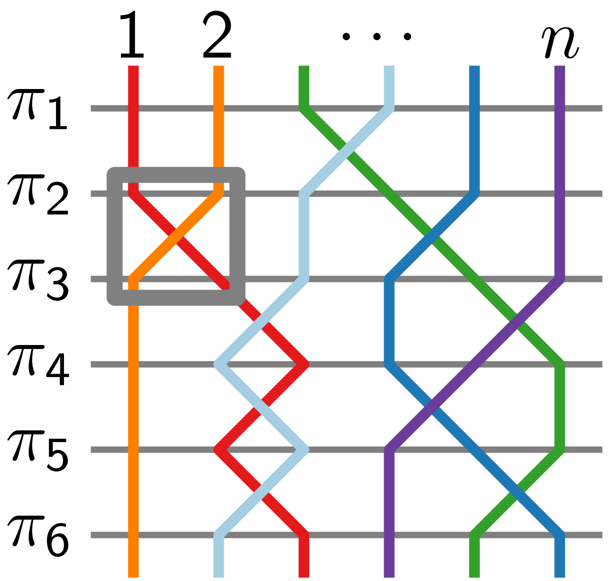

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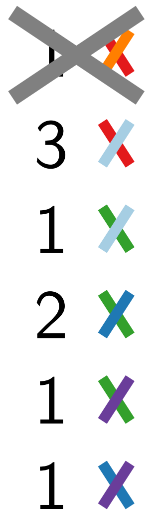

not *feasible*

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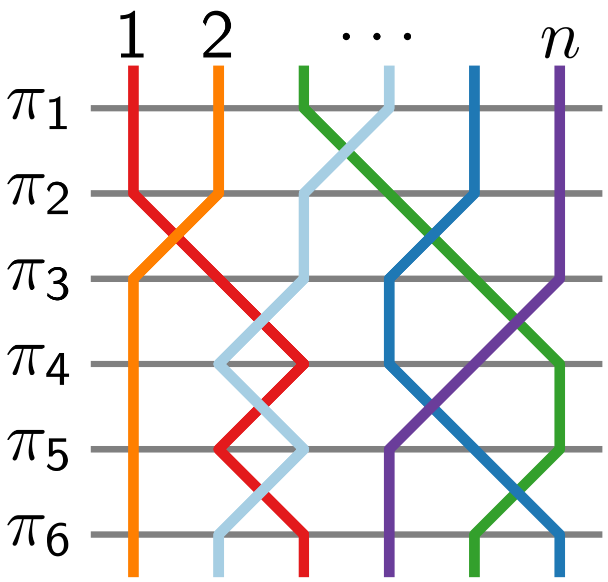

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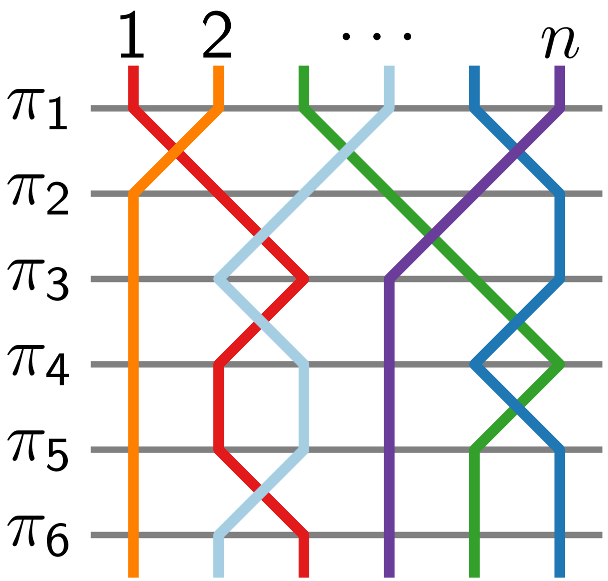

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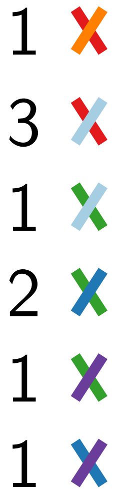

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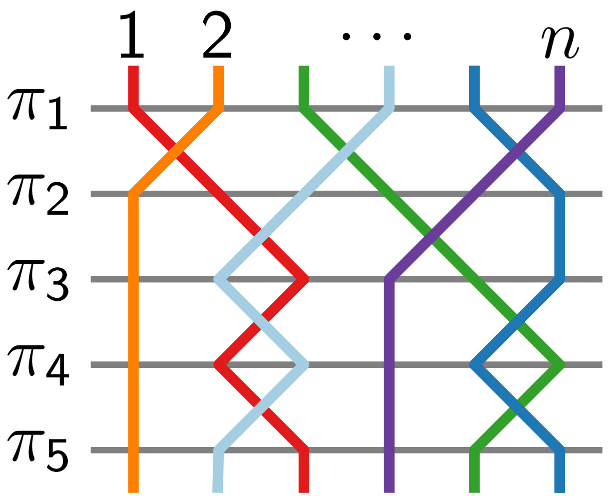

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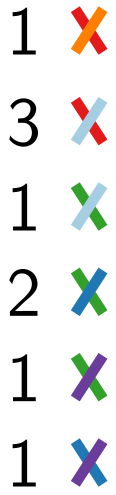

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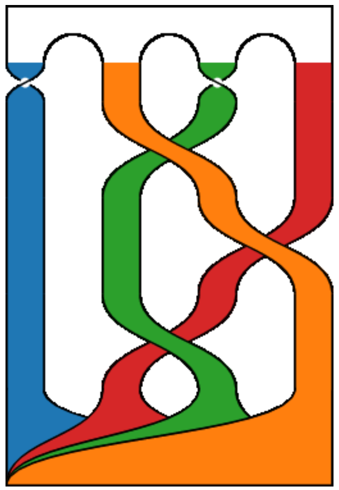

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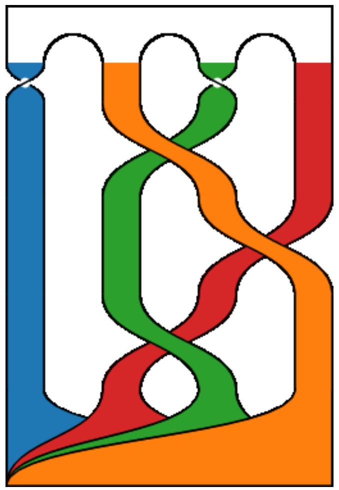

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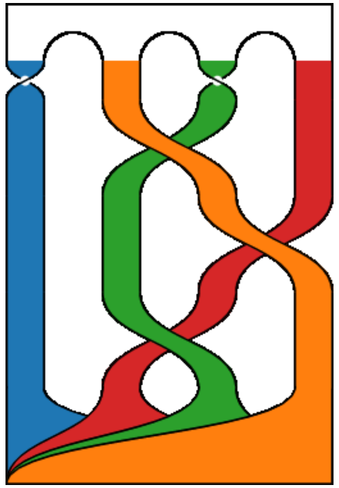

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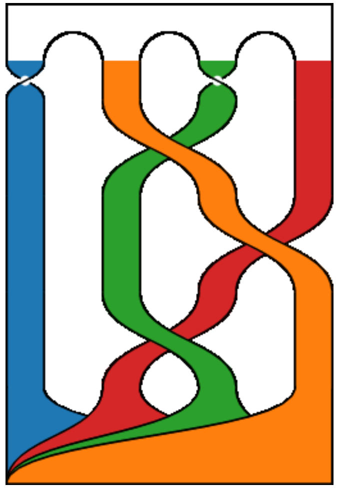

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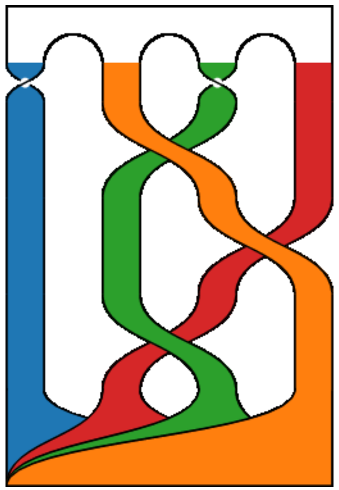

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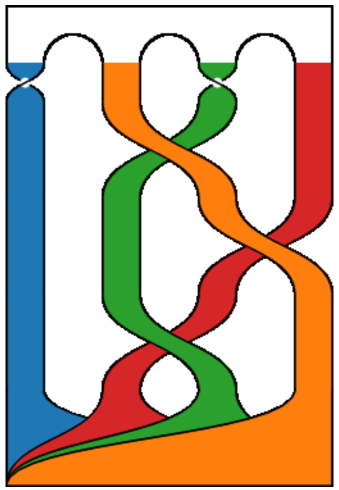
  Algorithms for minimizing the number of *bends*



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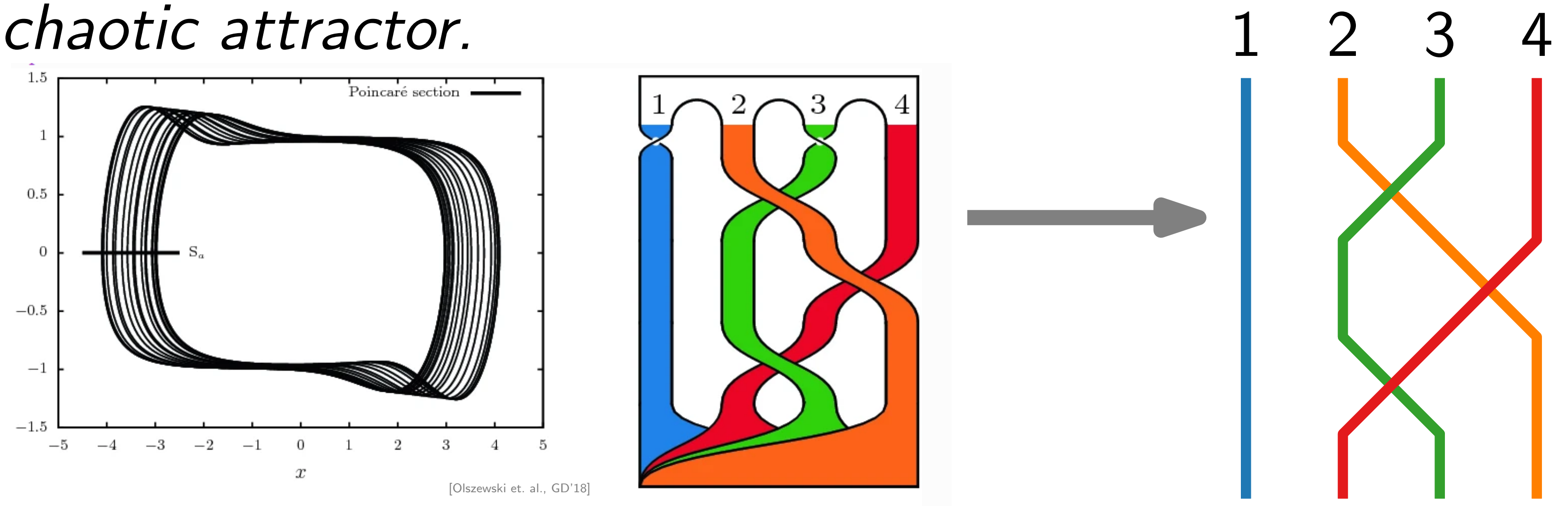

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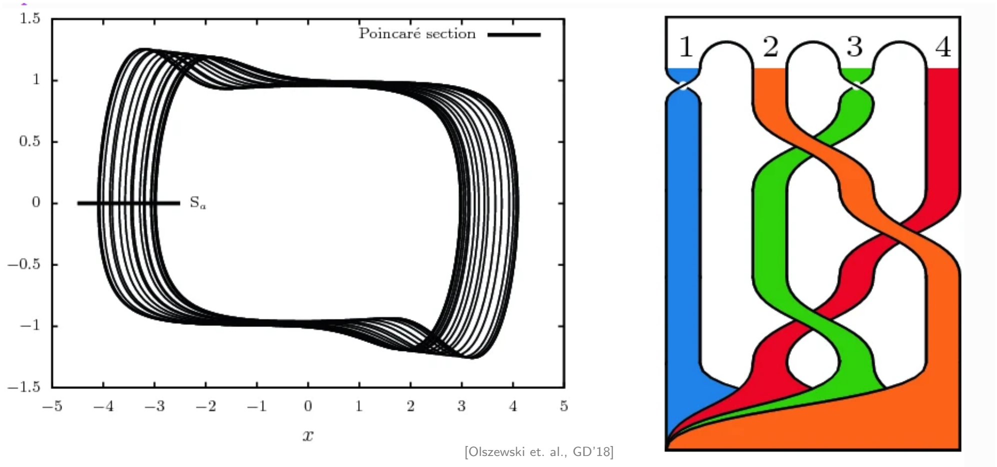

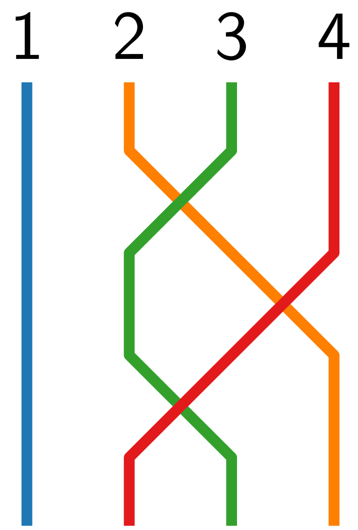

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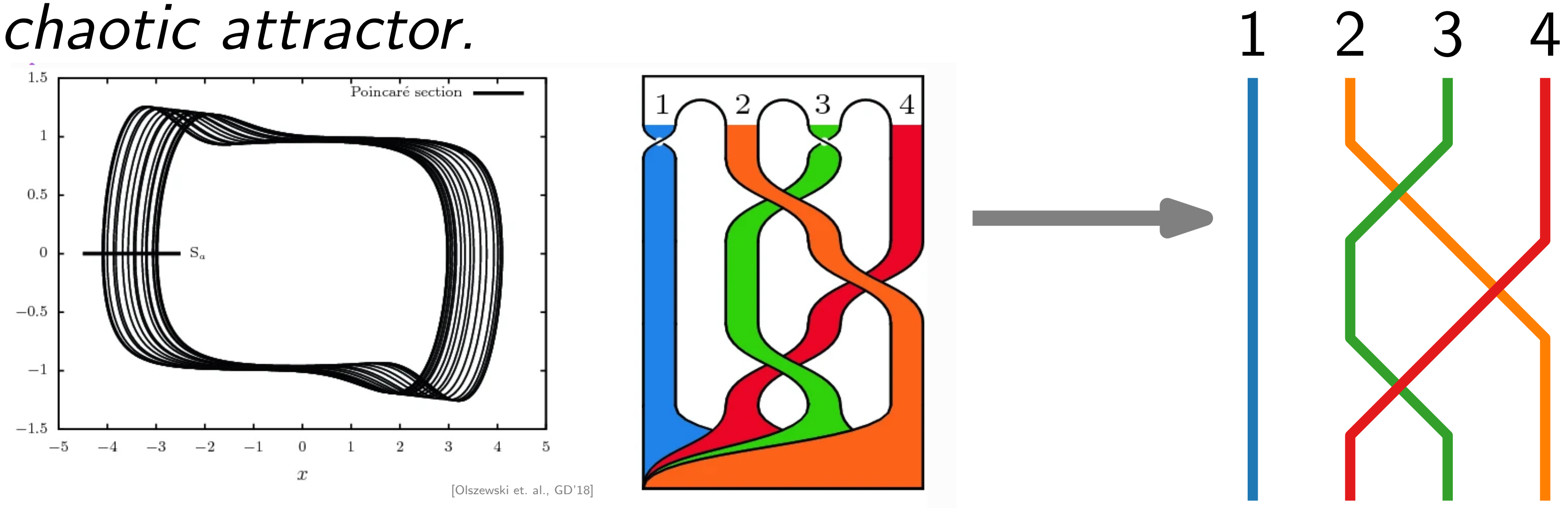

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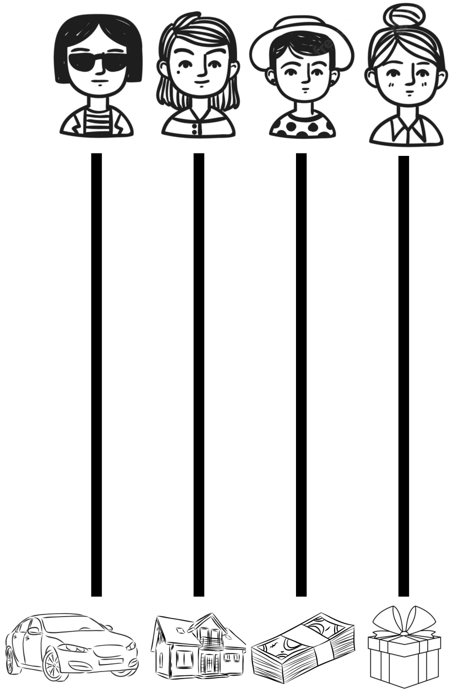

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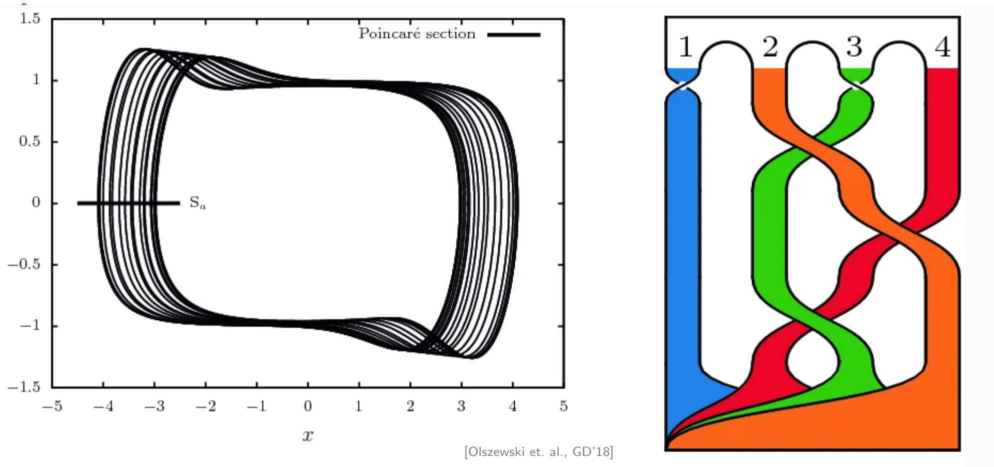

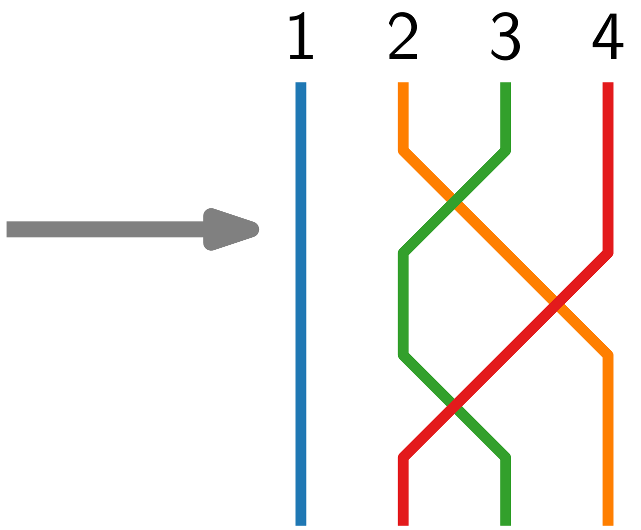

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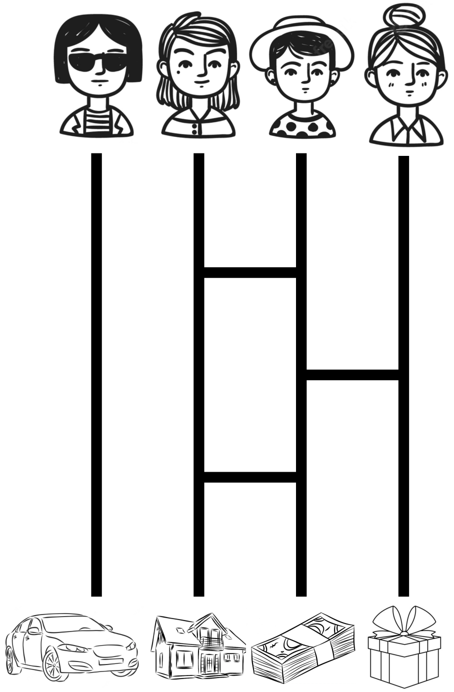

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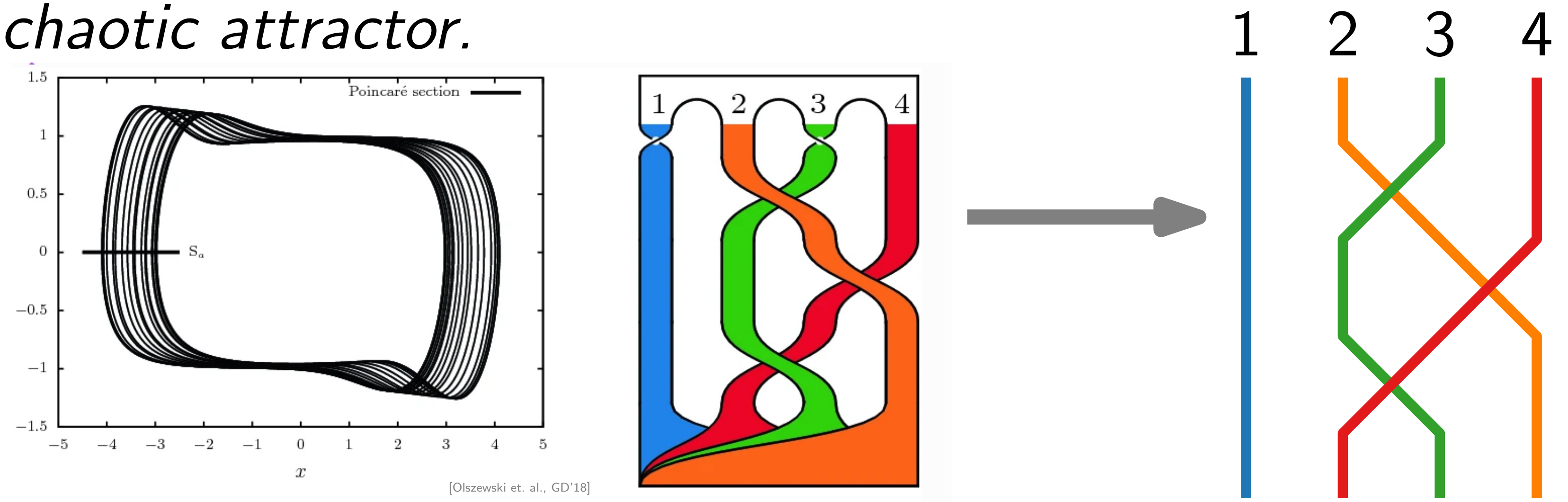

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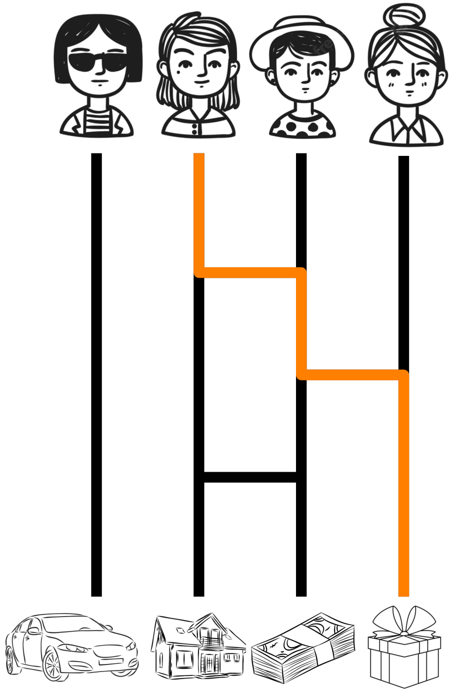

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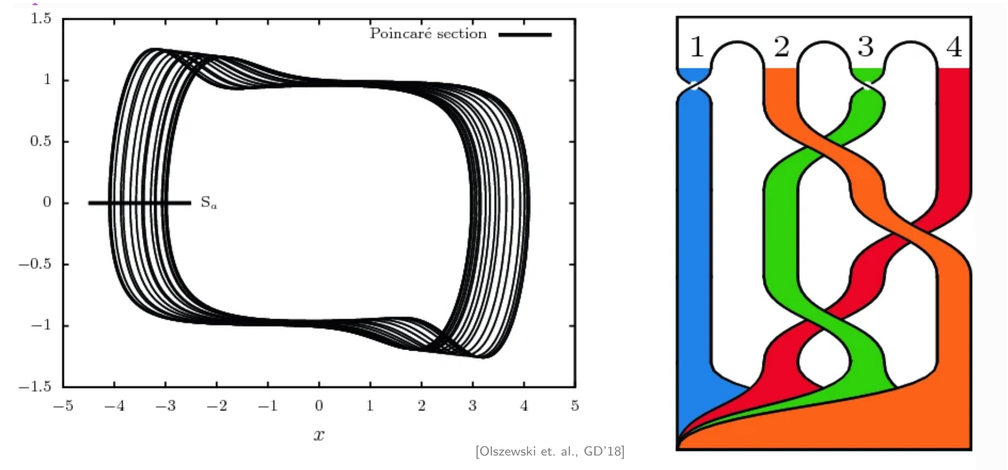

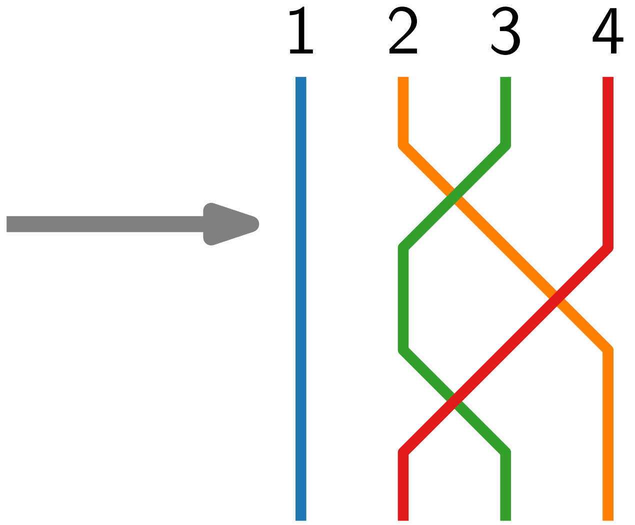

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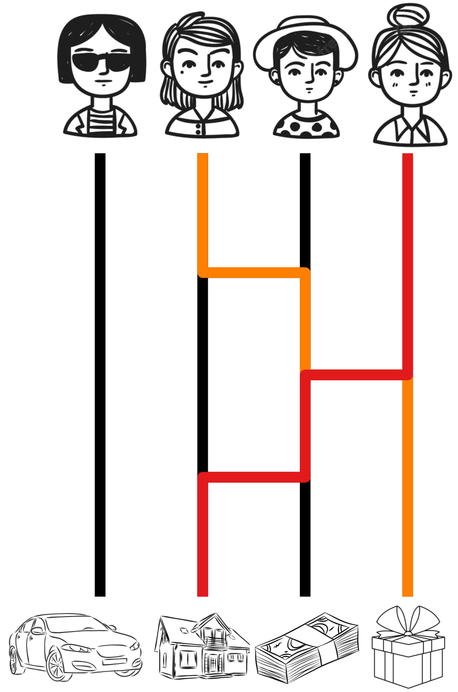

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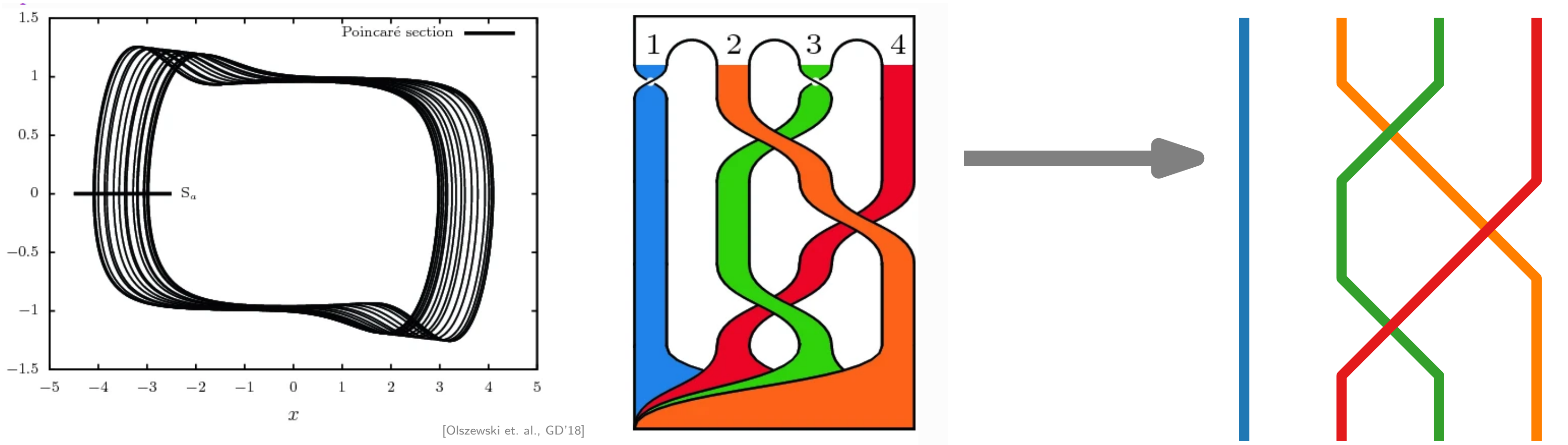

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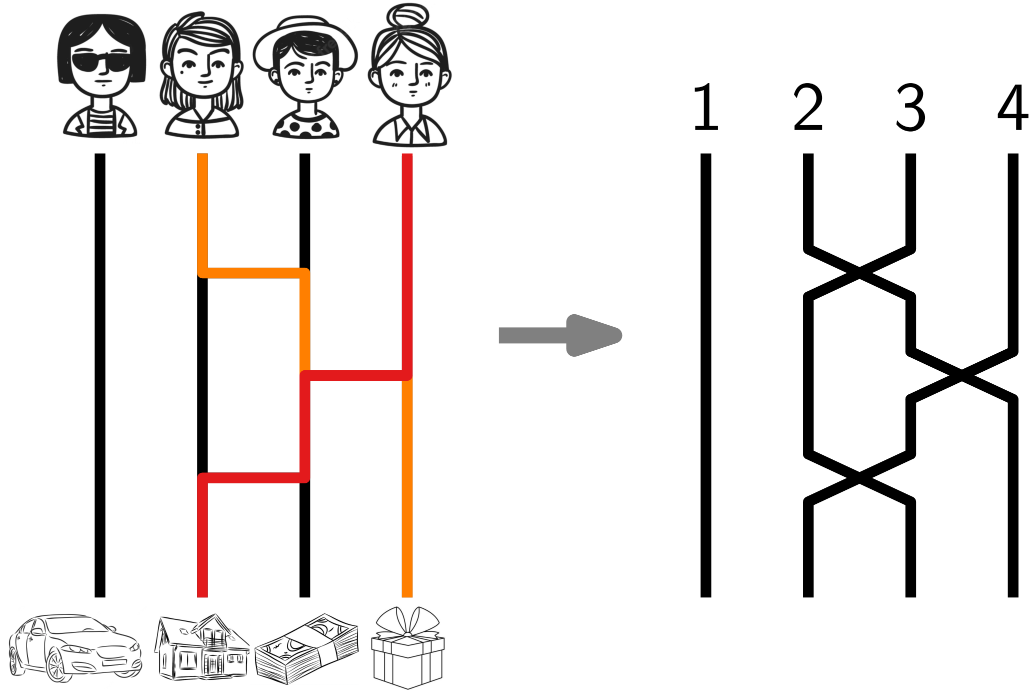

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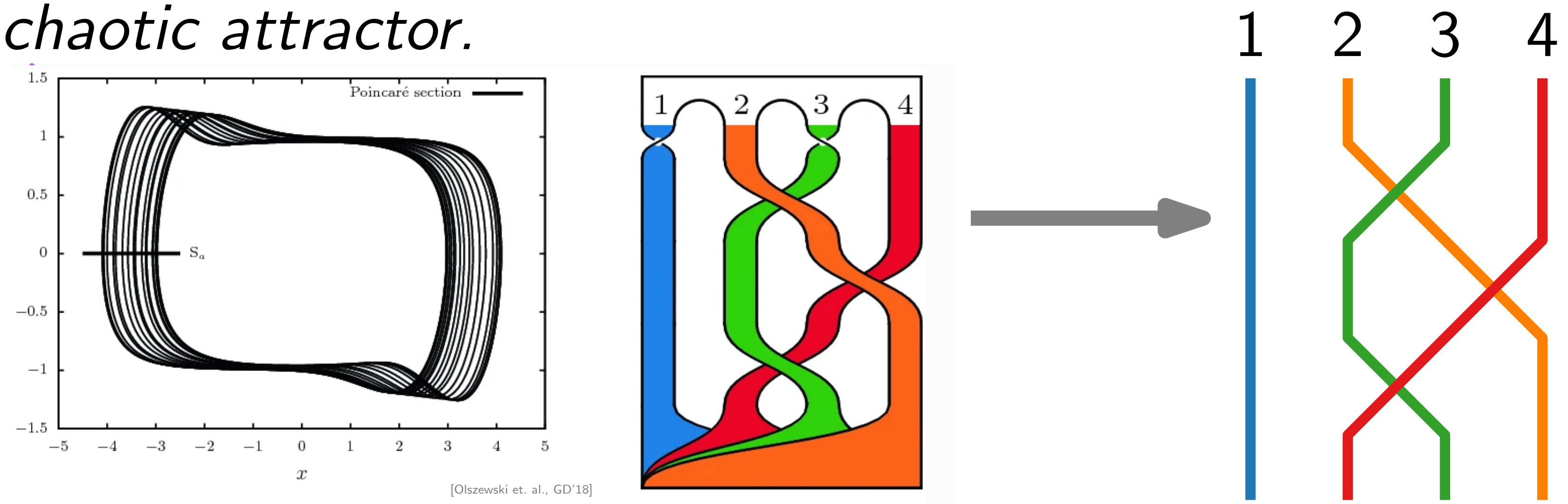

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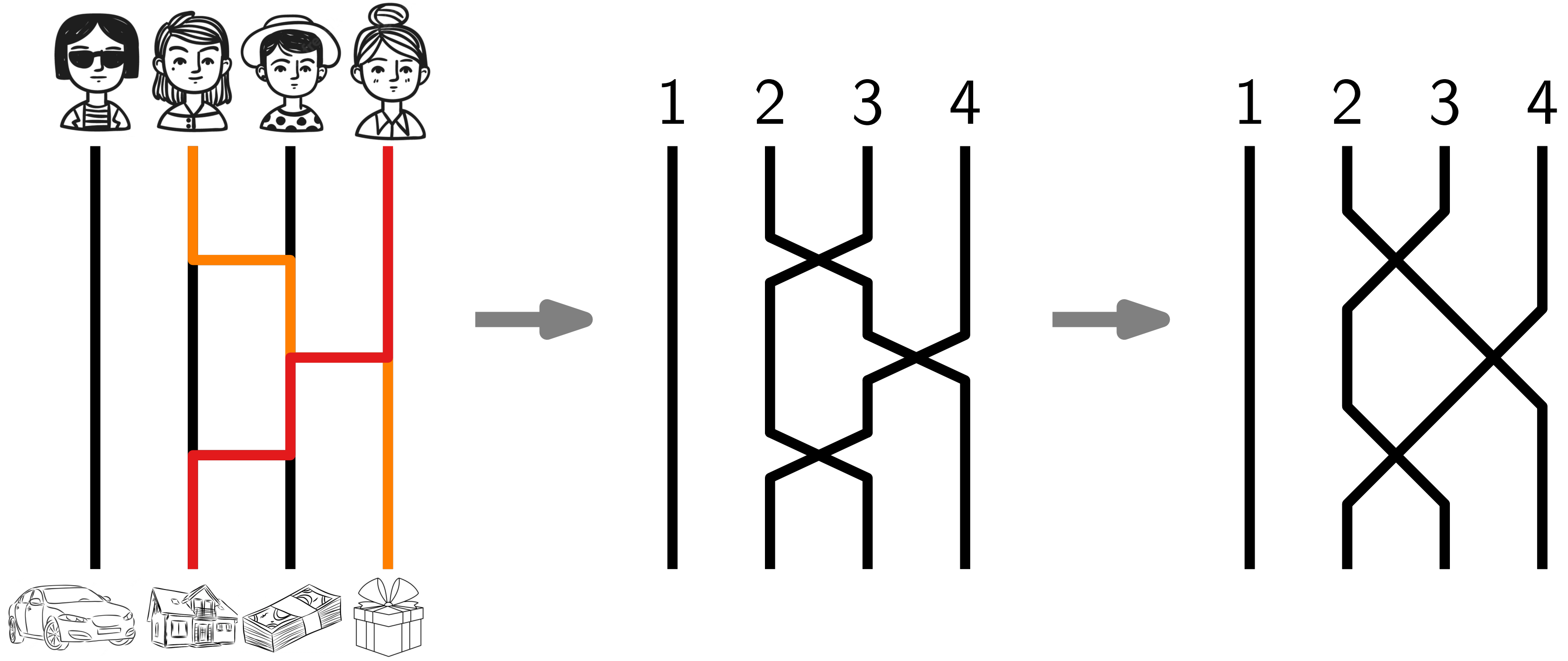

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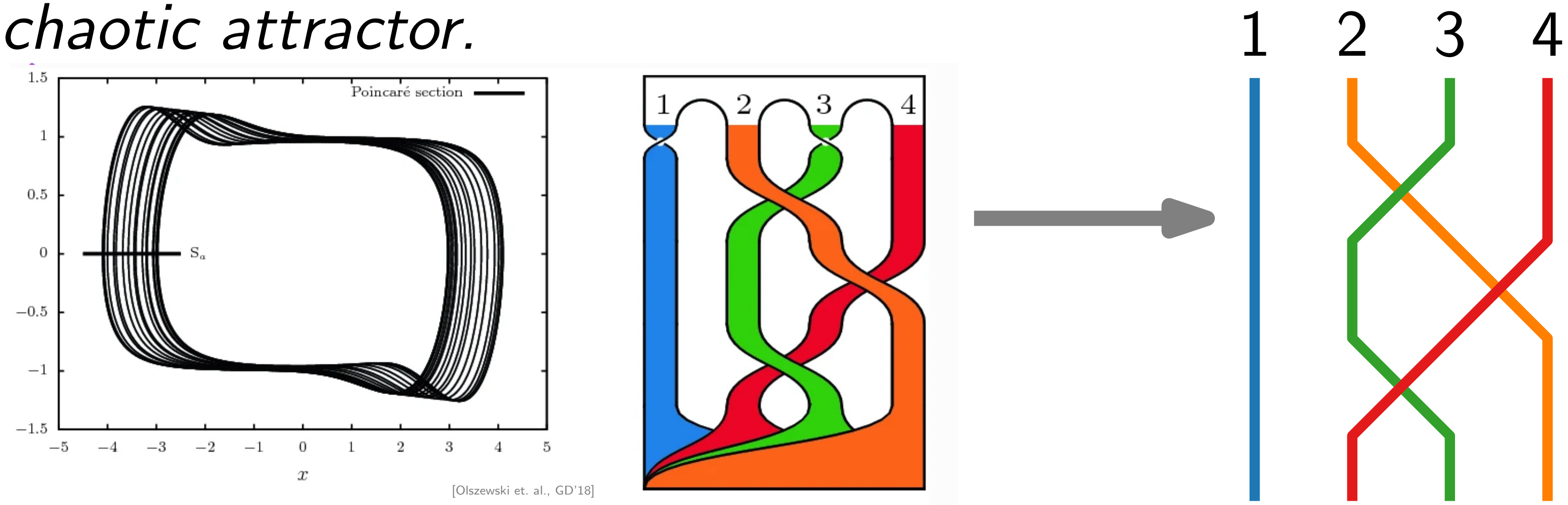

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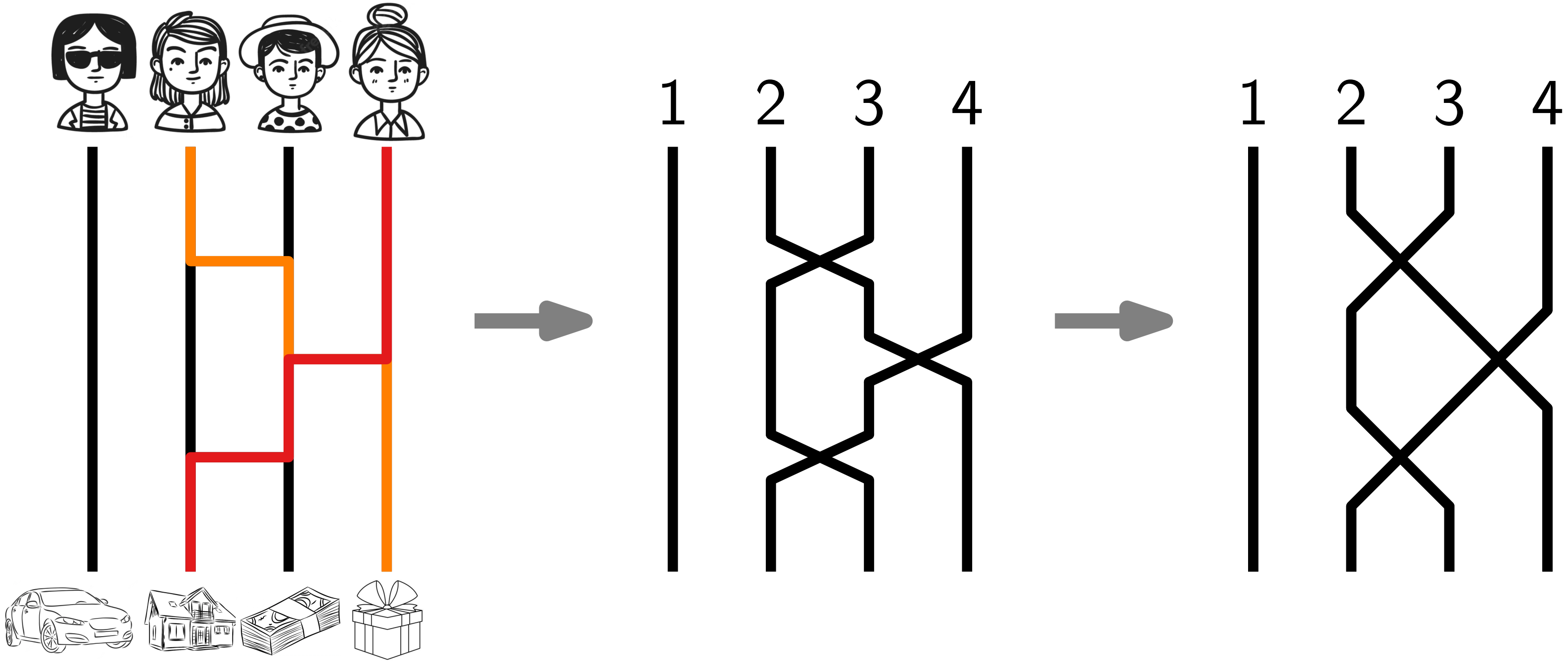

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  Given a list of swaps, it is NP-complete to decide if it is feasible *even if every pair of wires has* $O(1)$ *(eight) swaps*.

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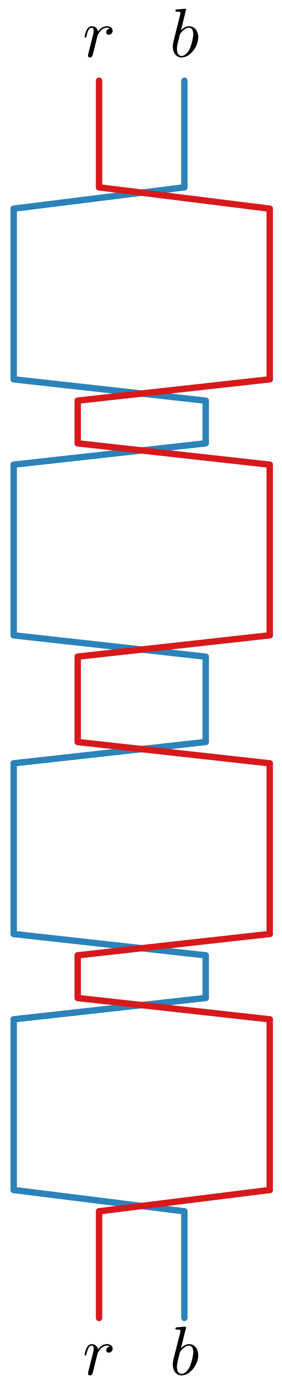

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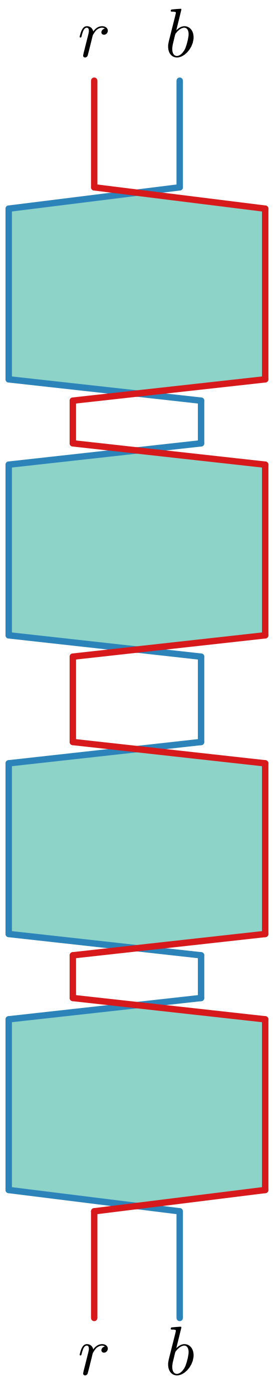

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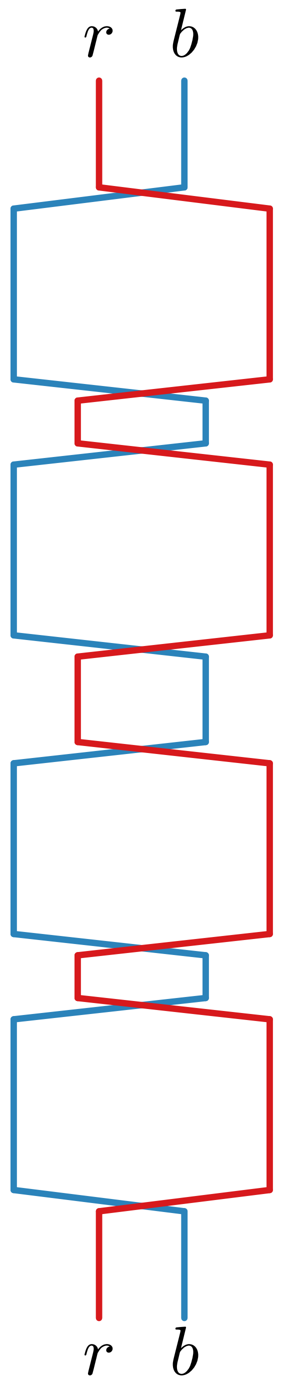

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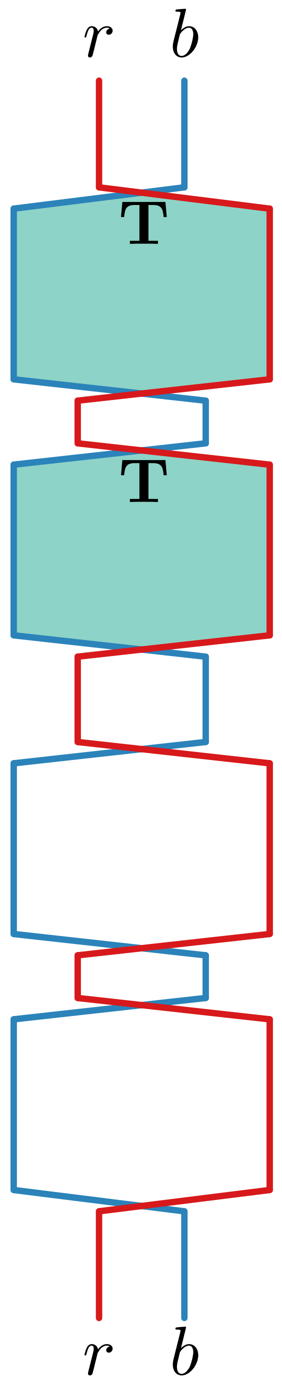

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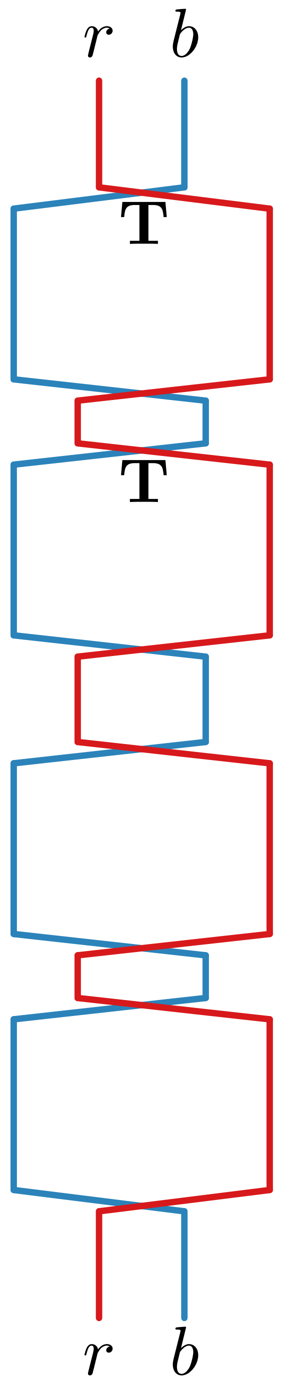

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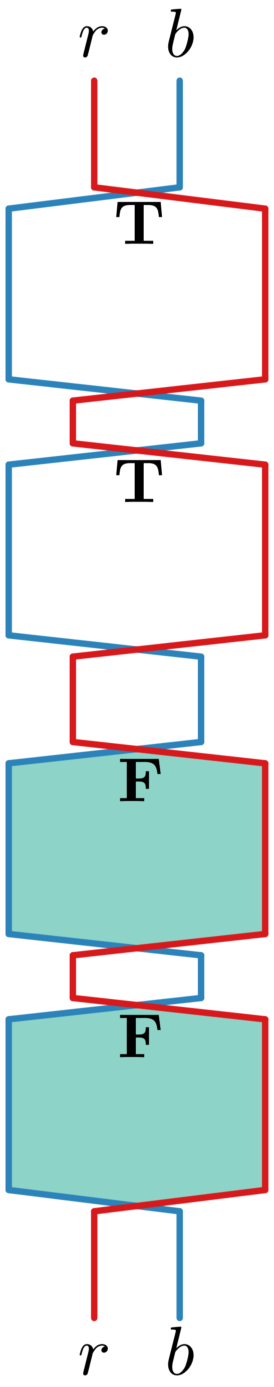

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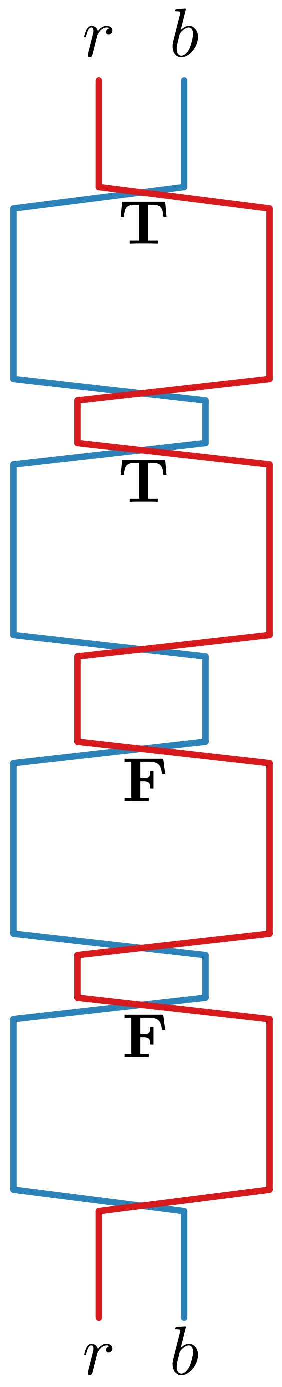

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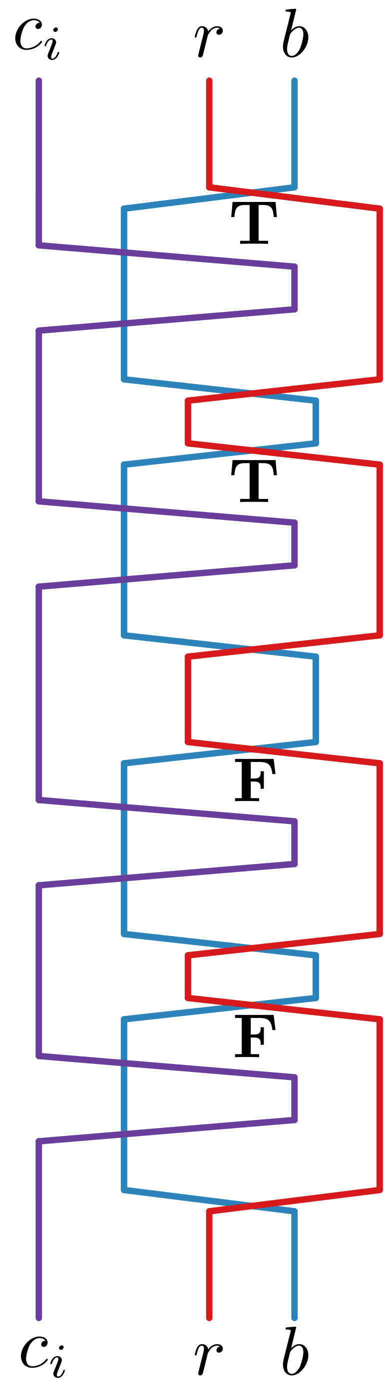

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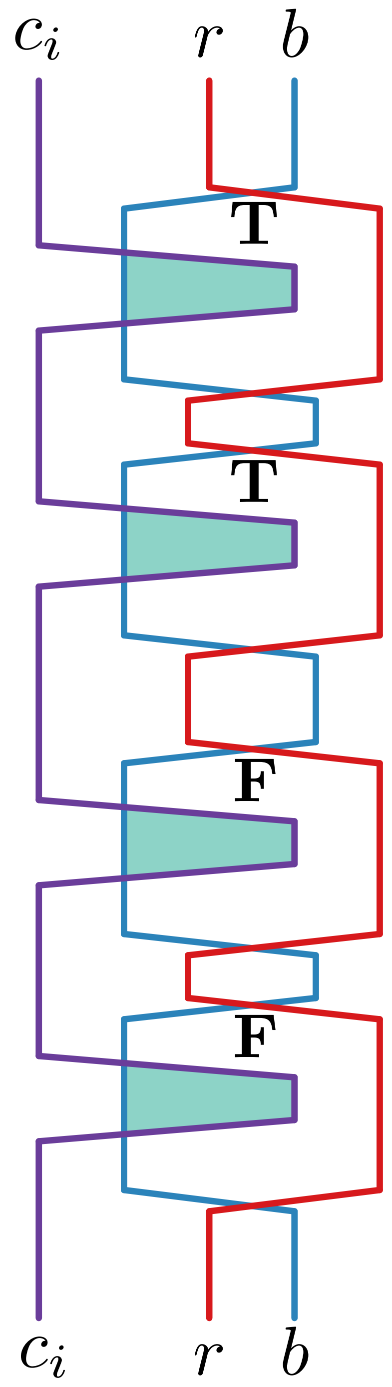

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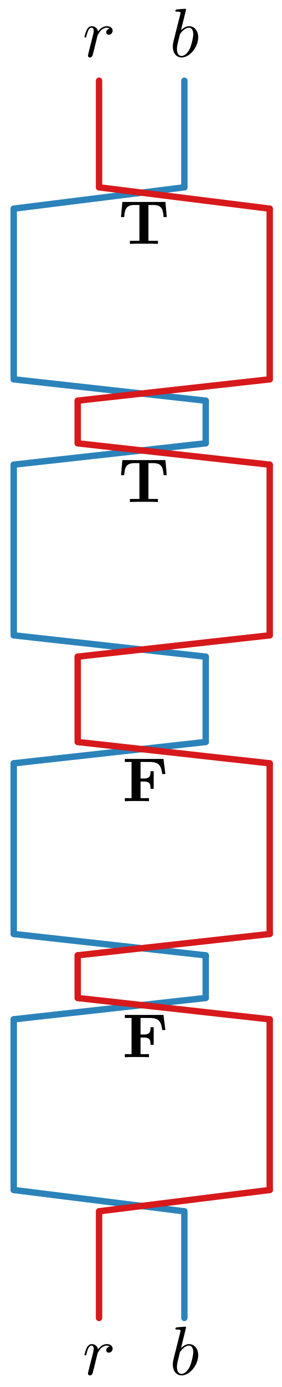

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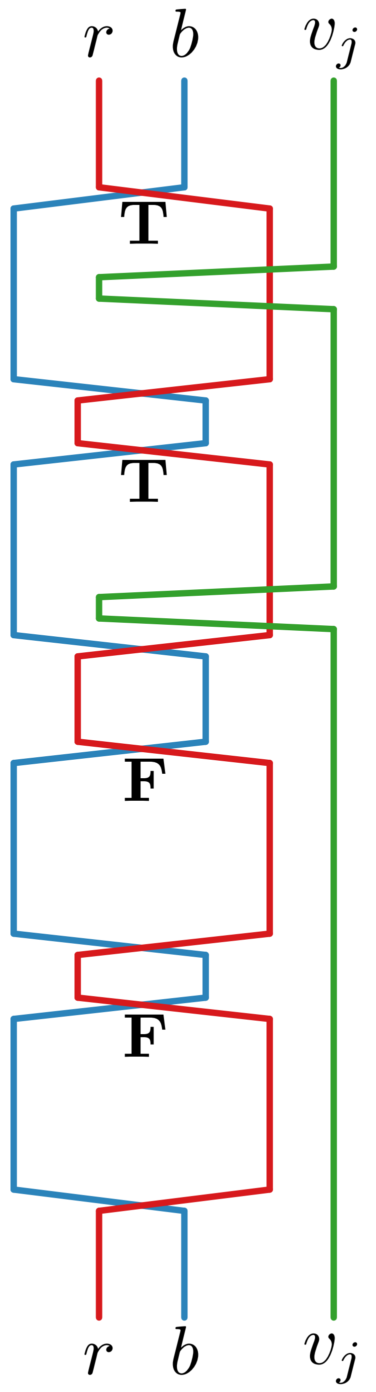

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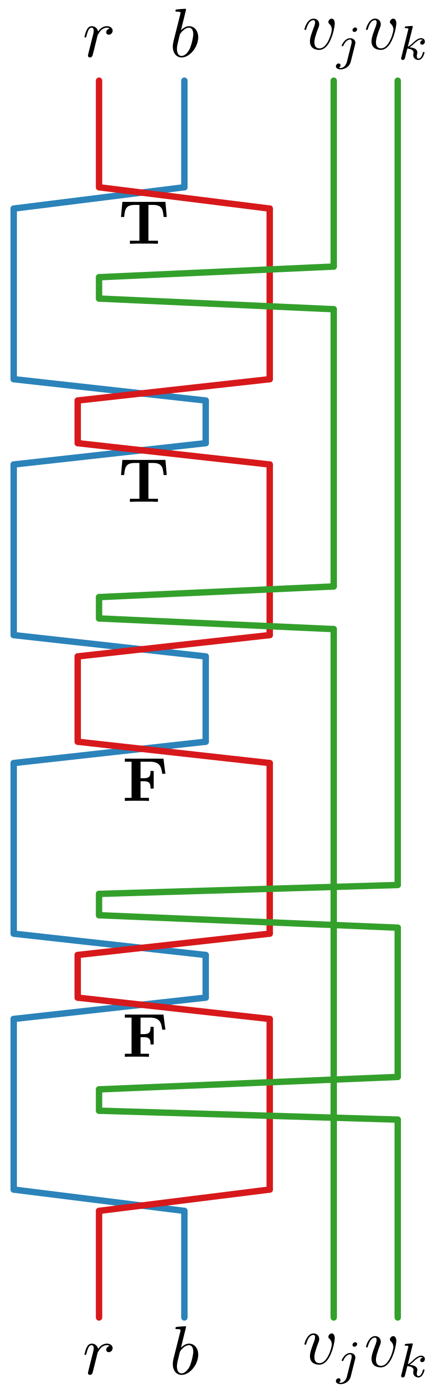

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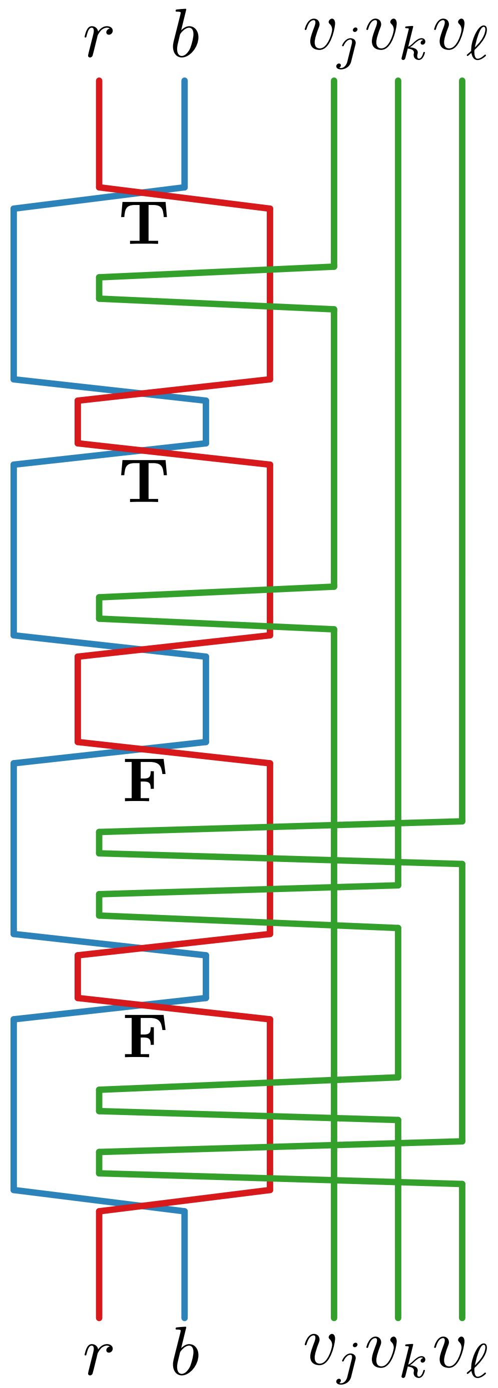

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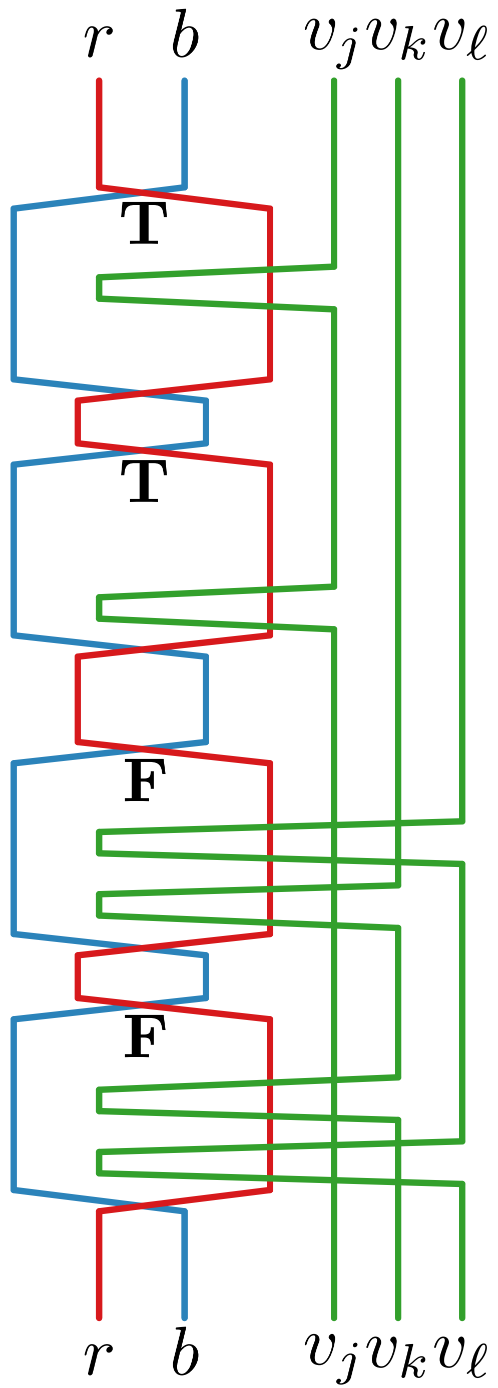

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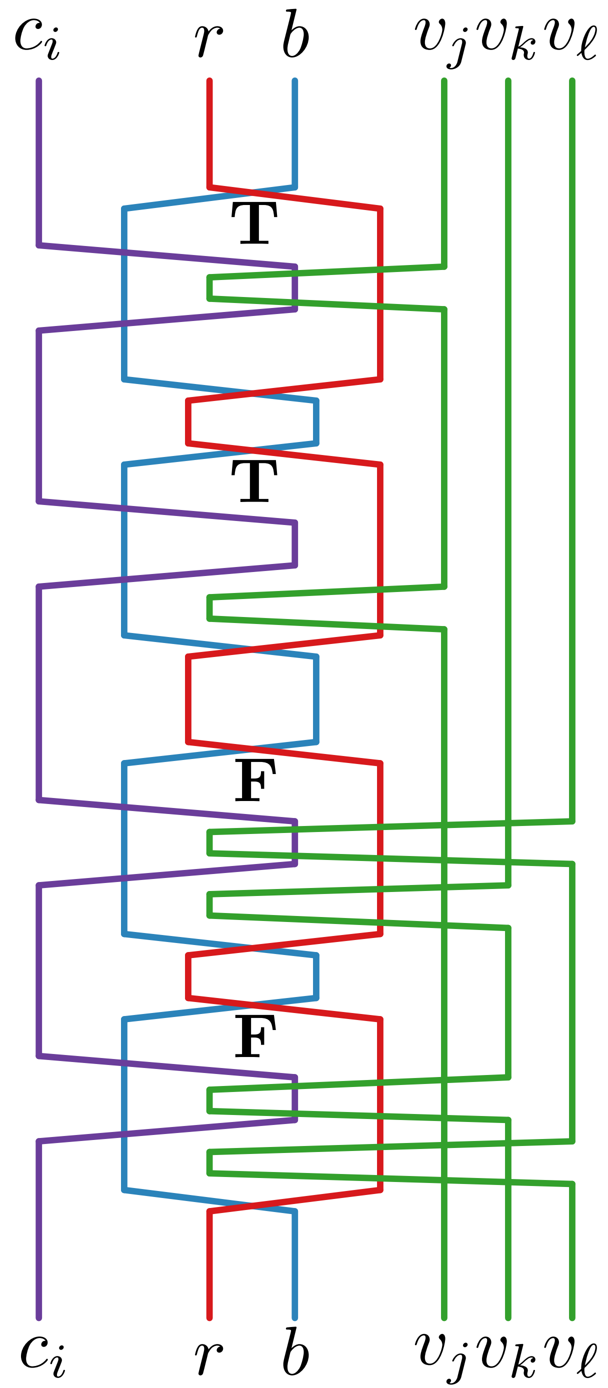

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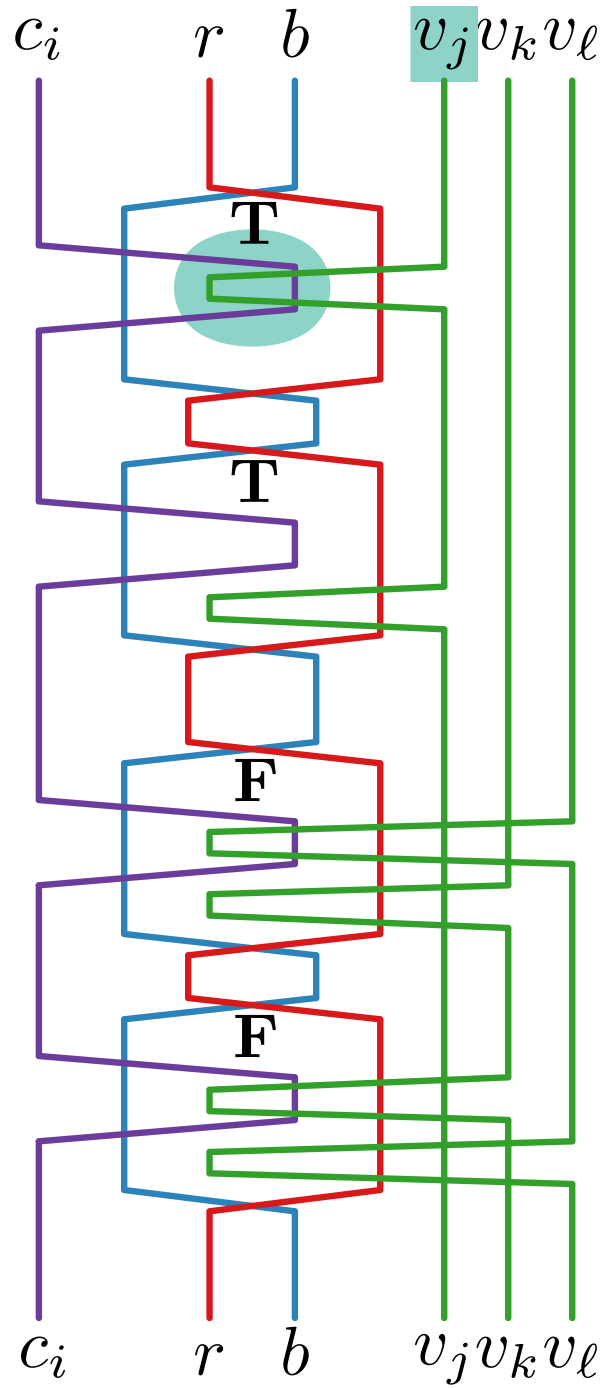

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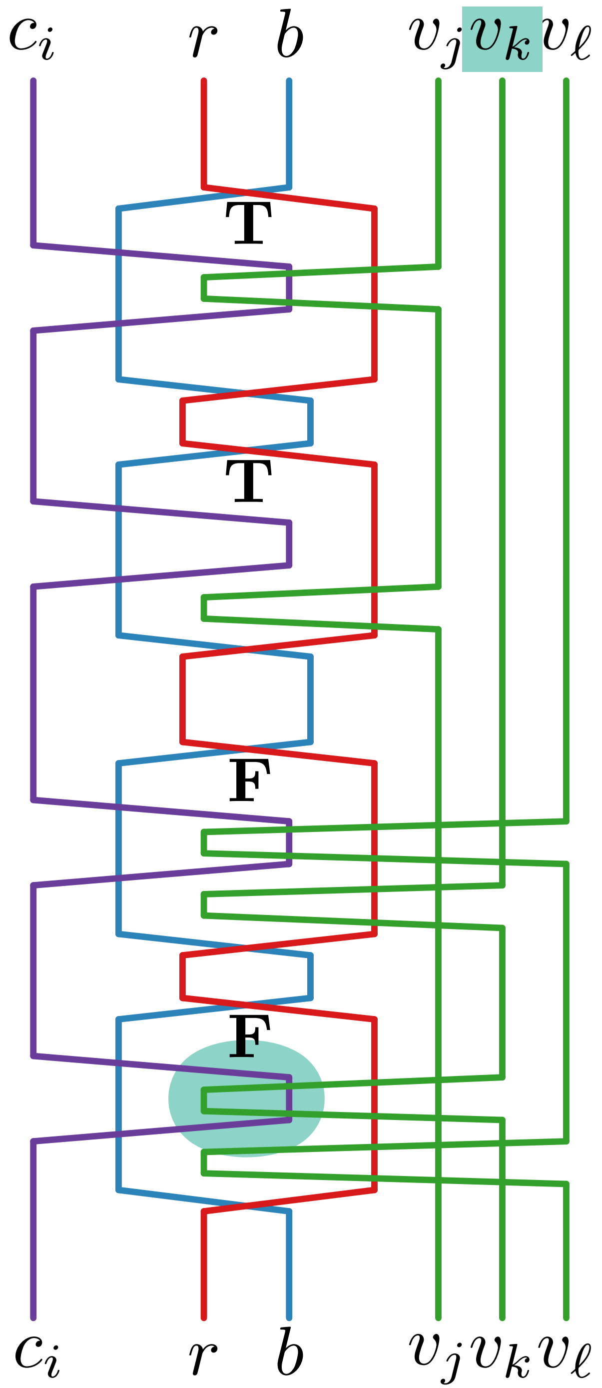

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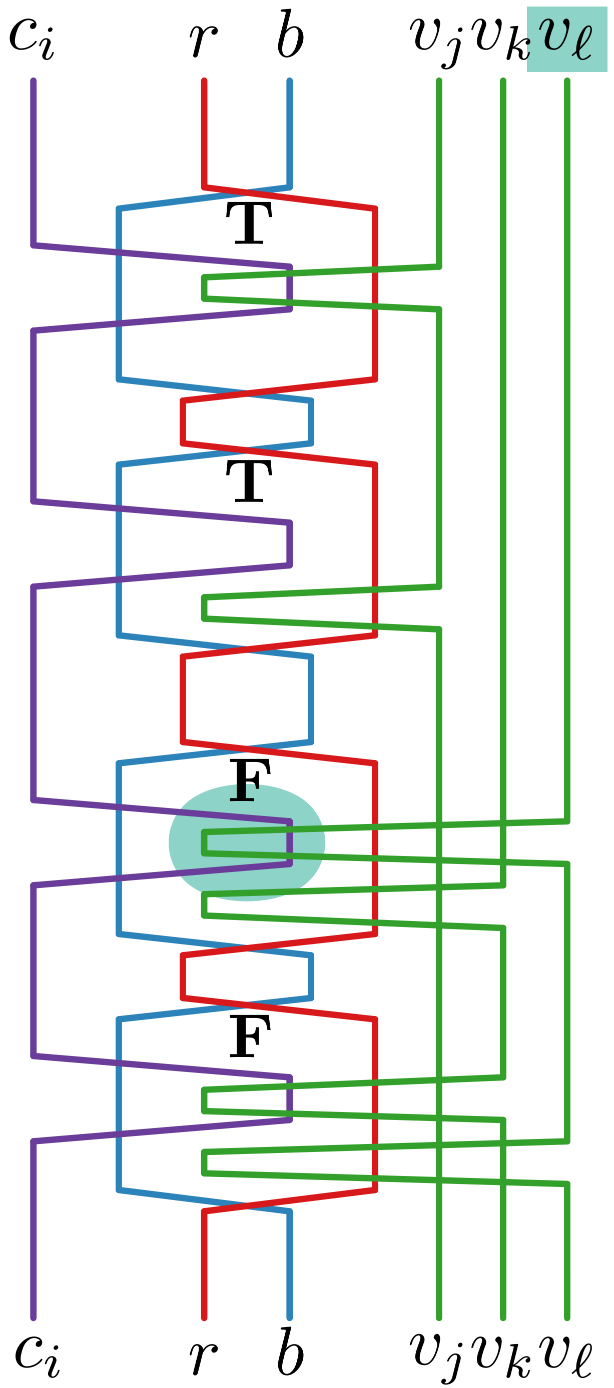

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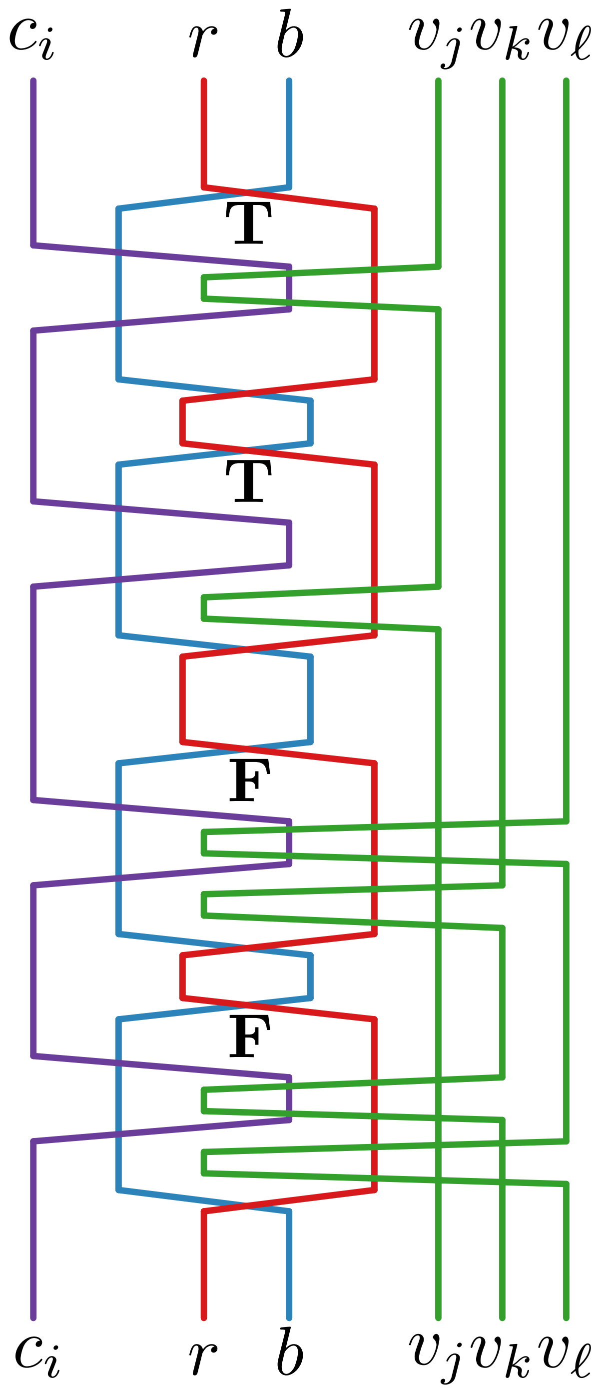

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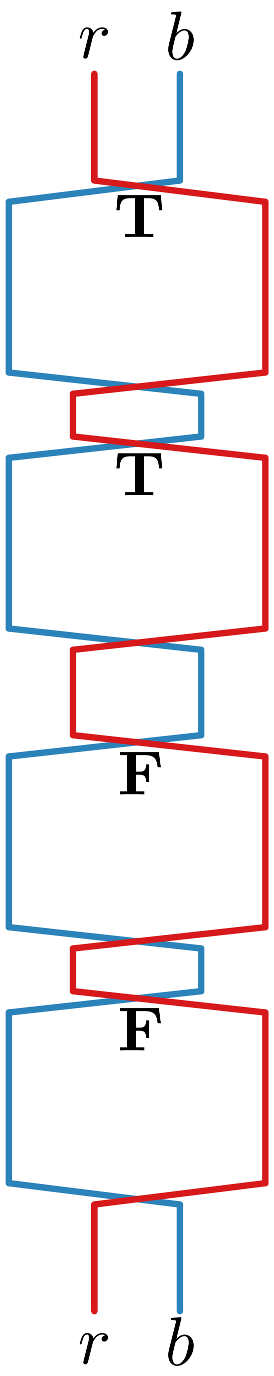

r, b: central loop structure

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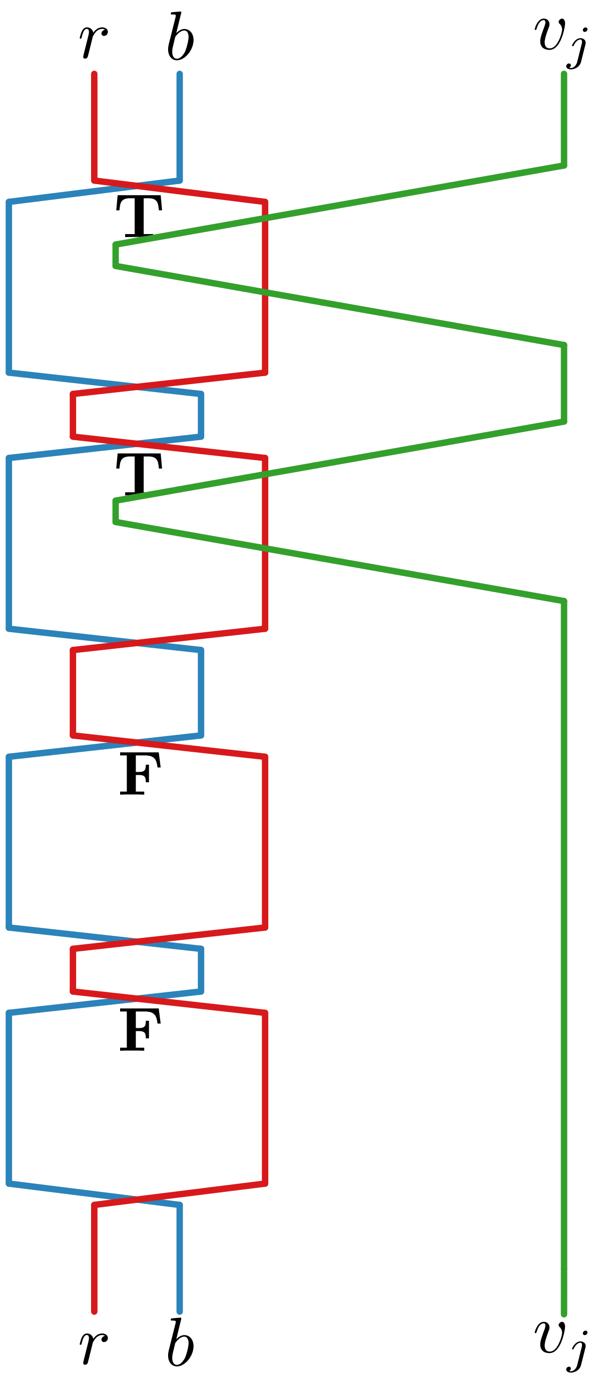

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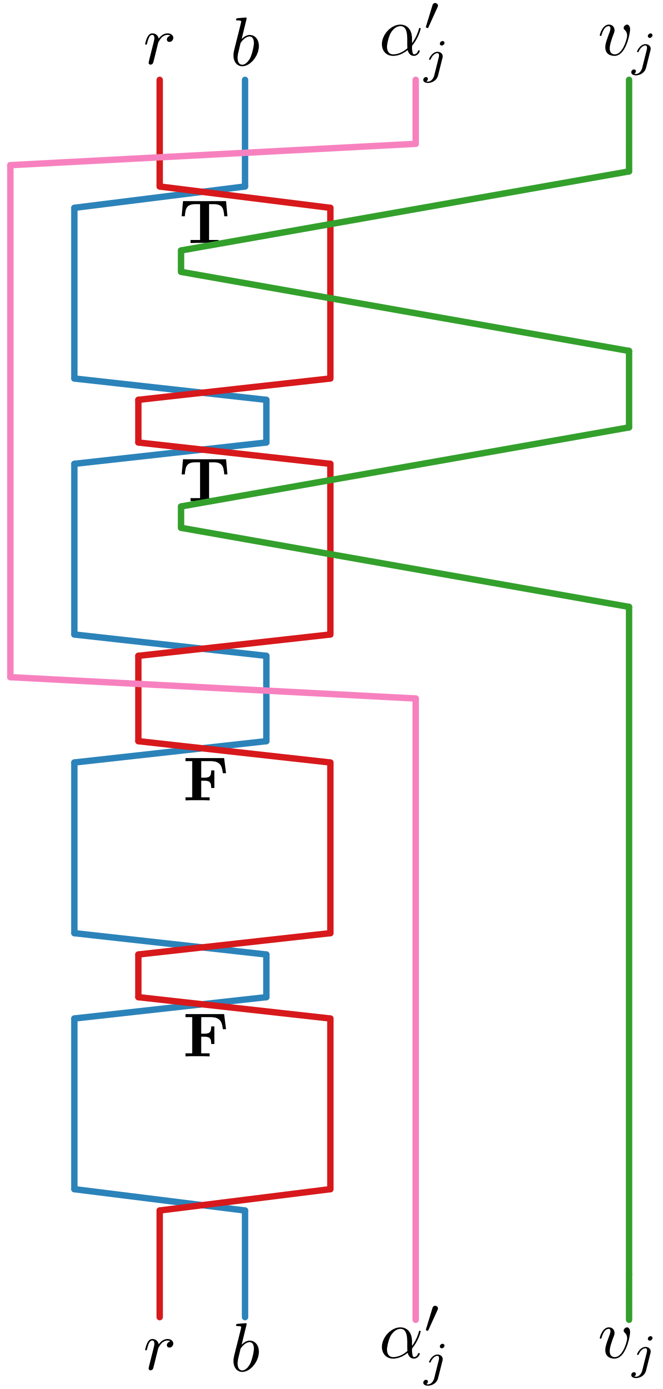

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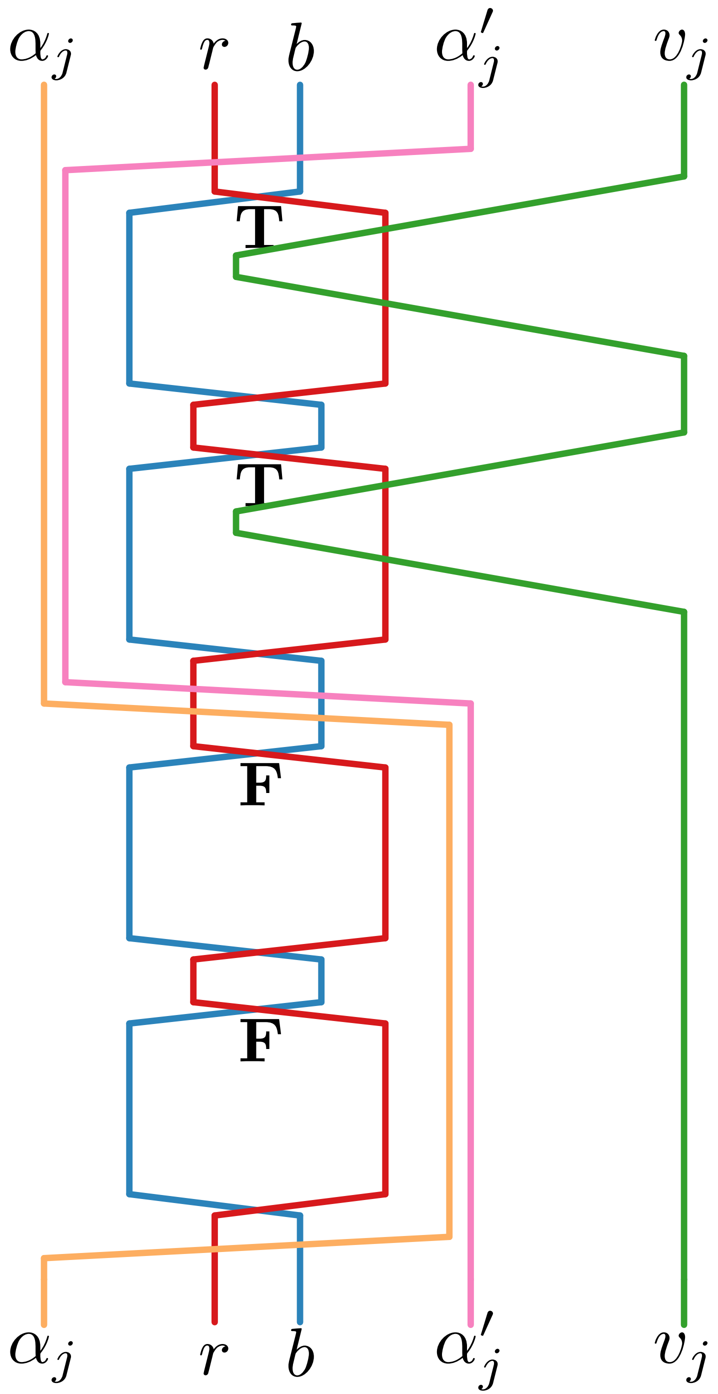

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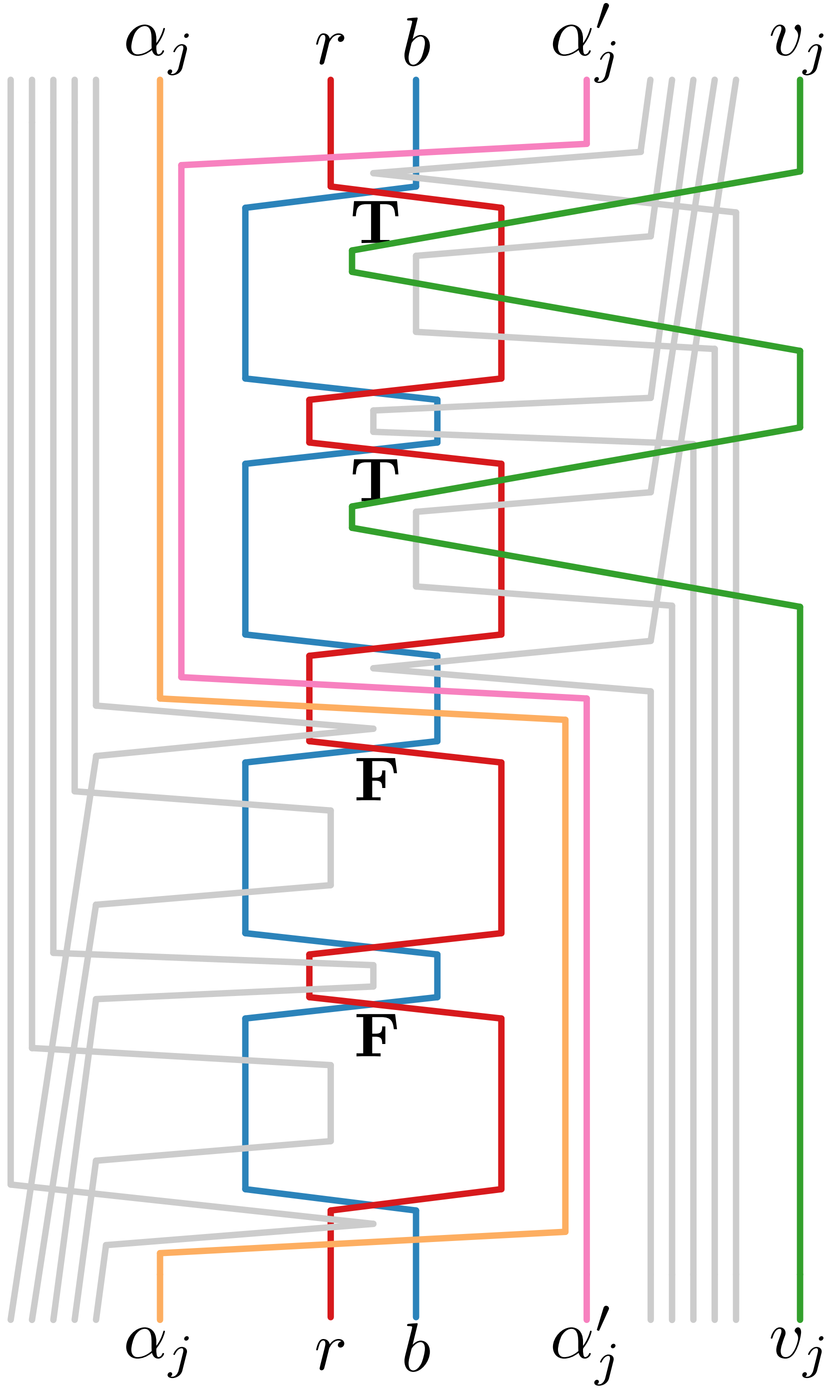

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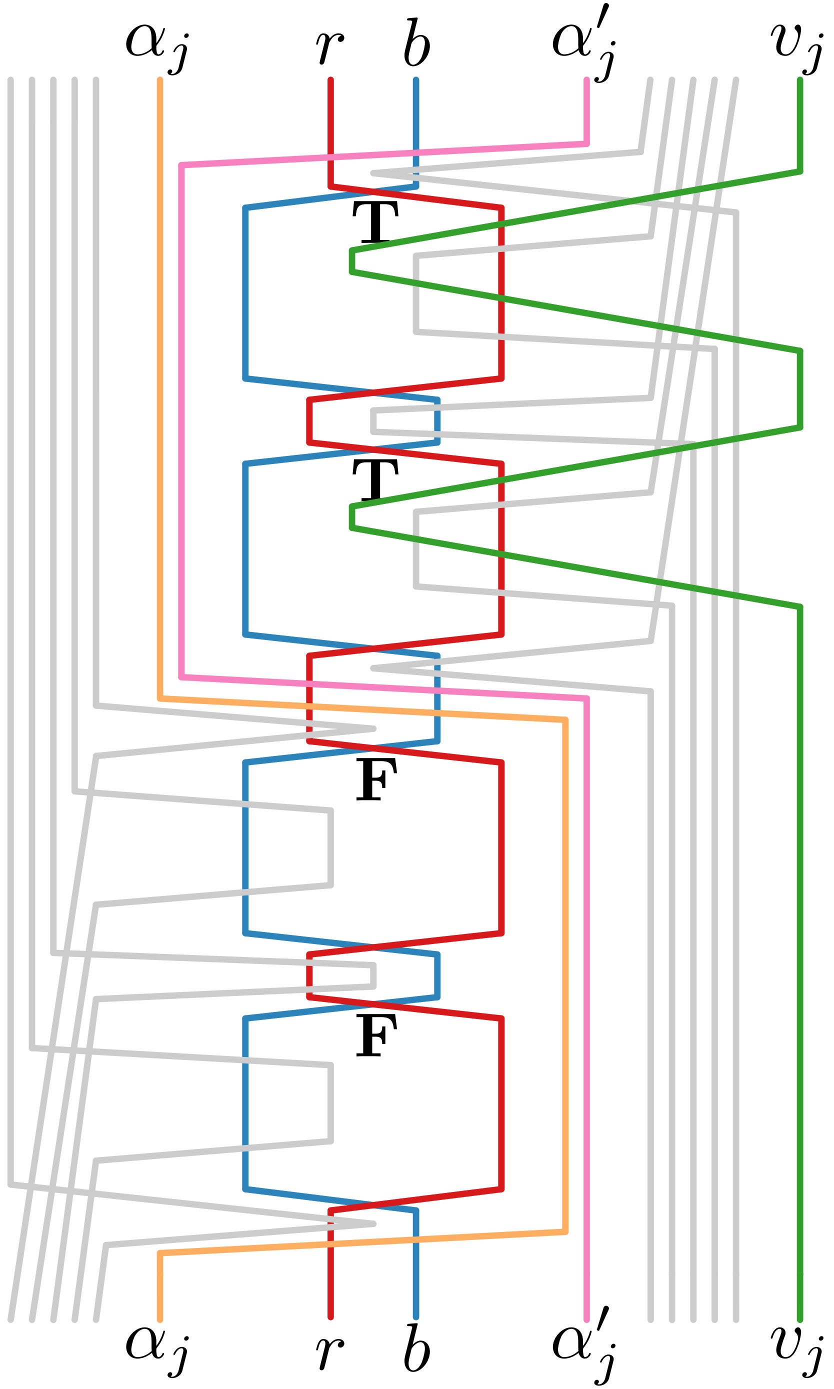

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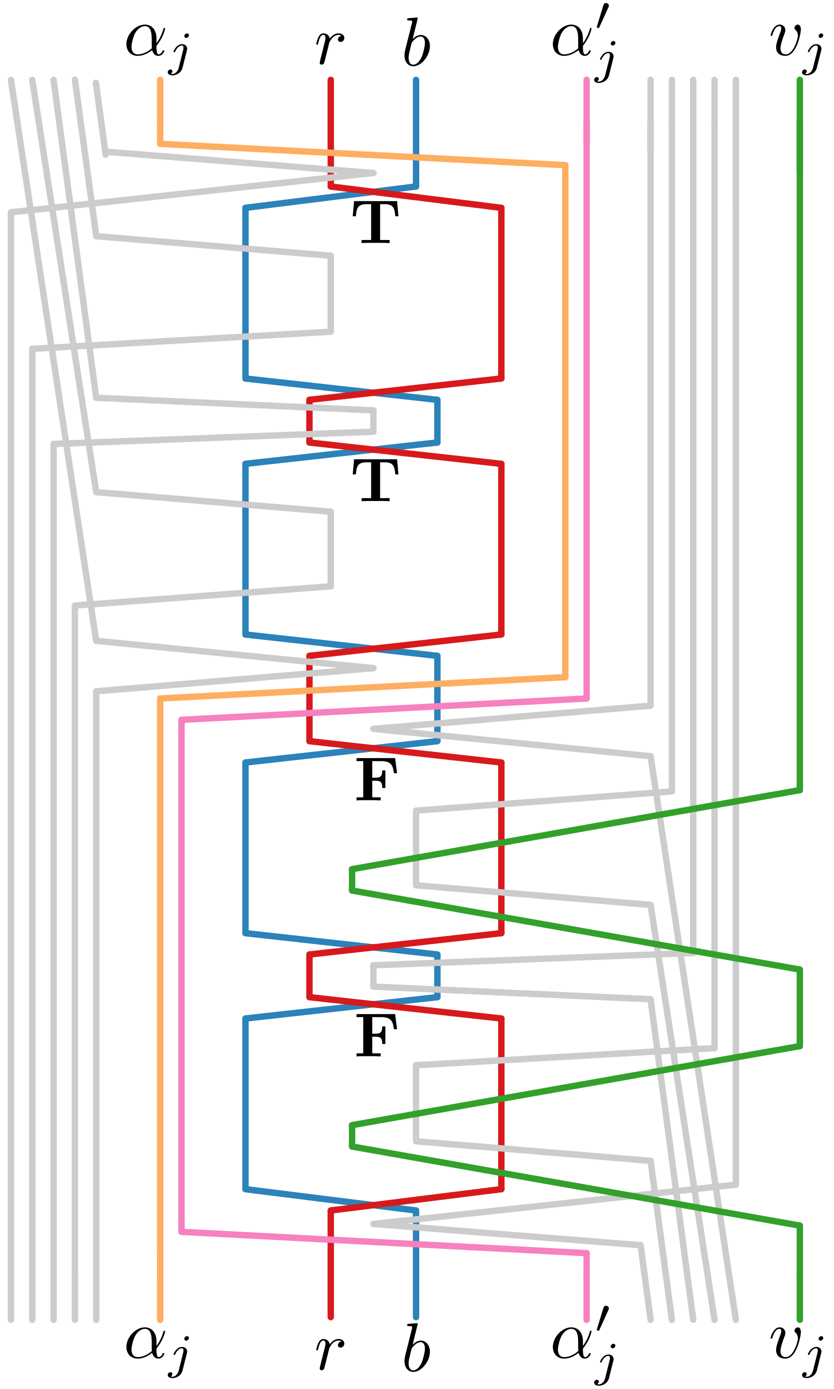

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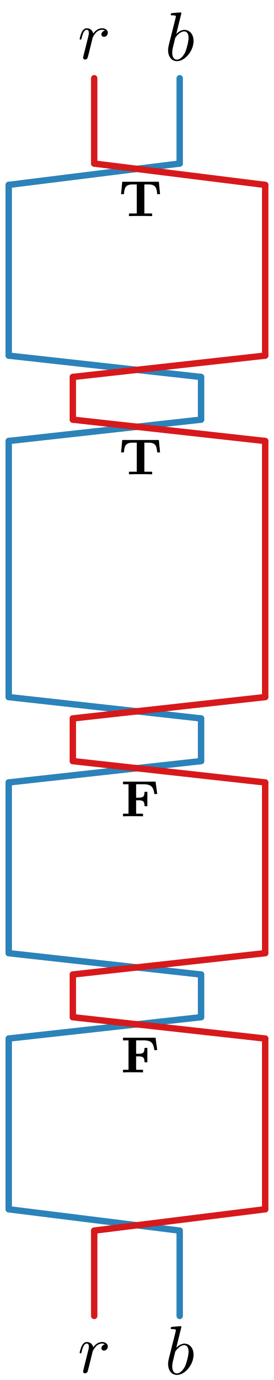

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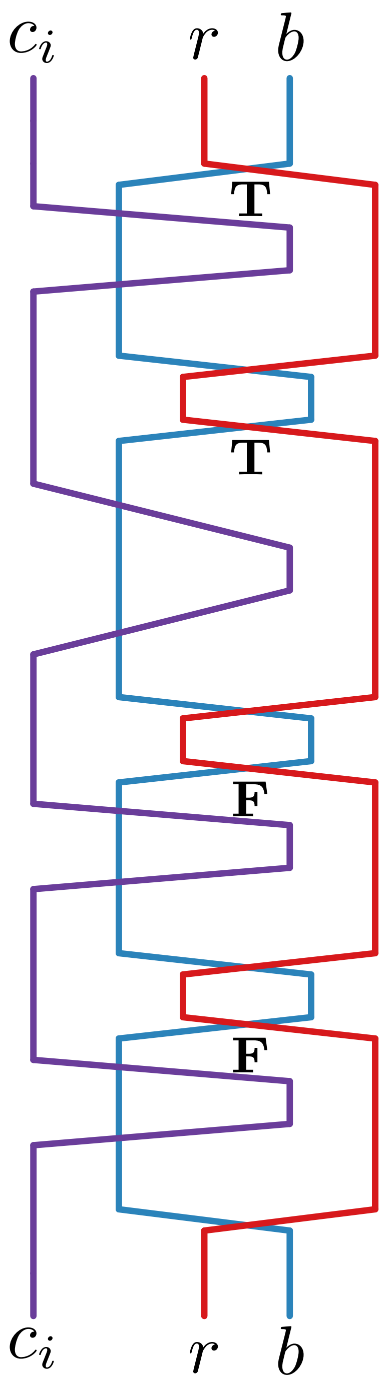

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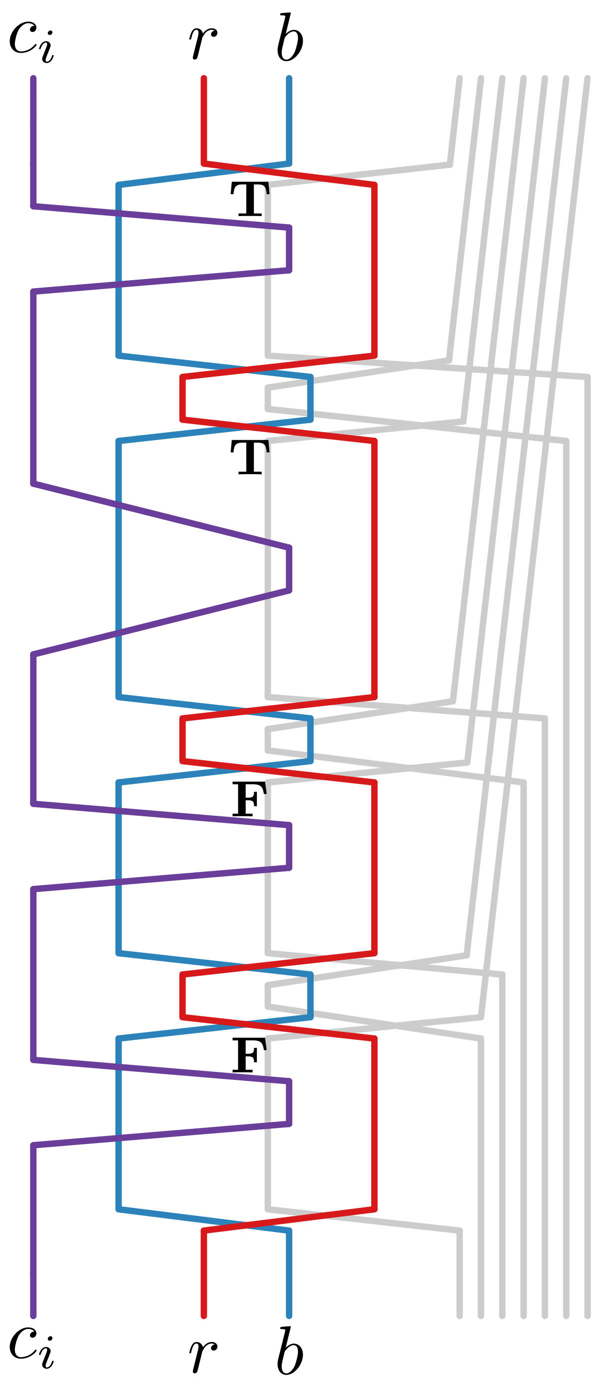

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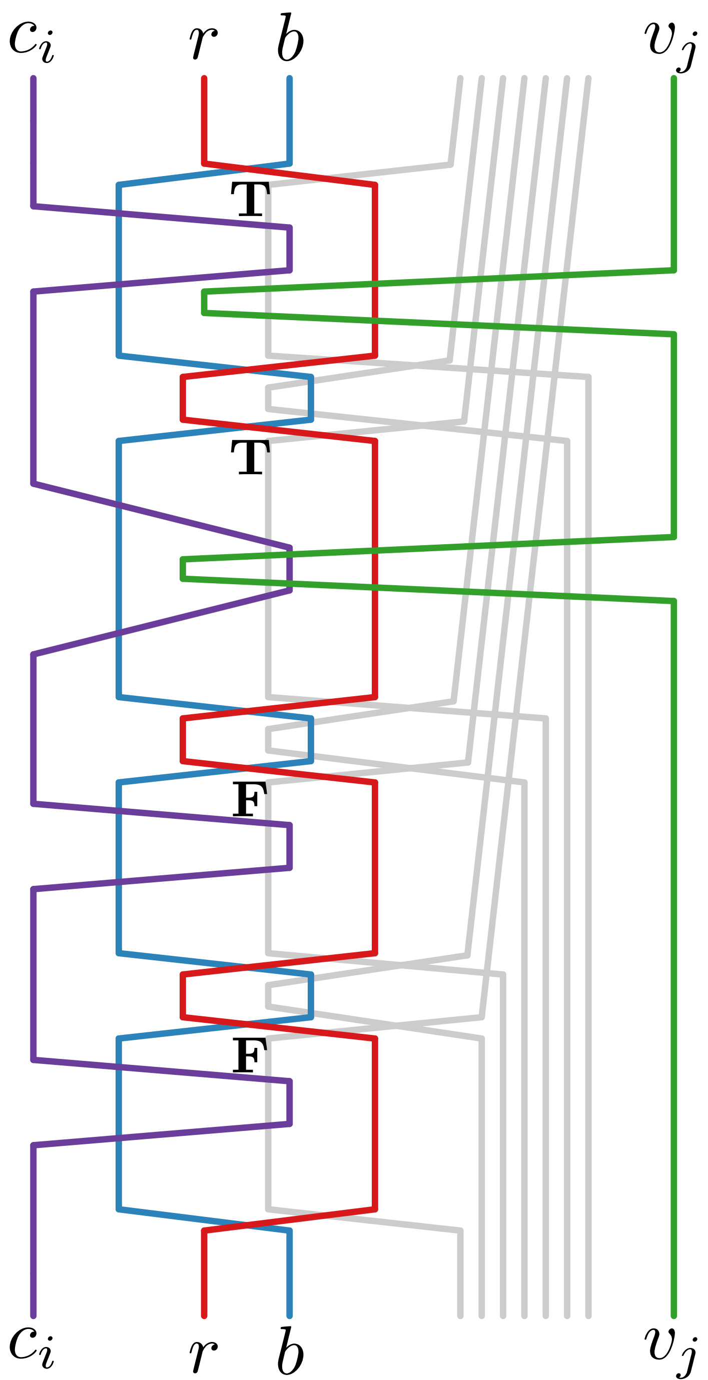

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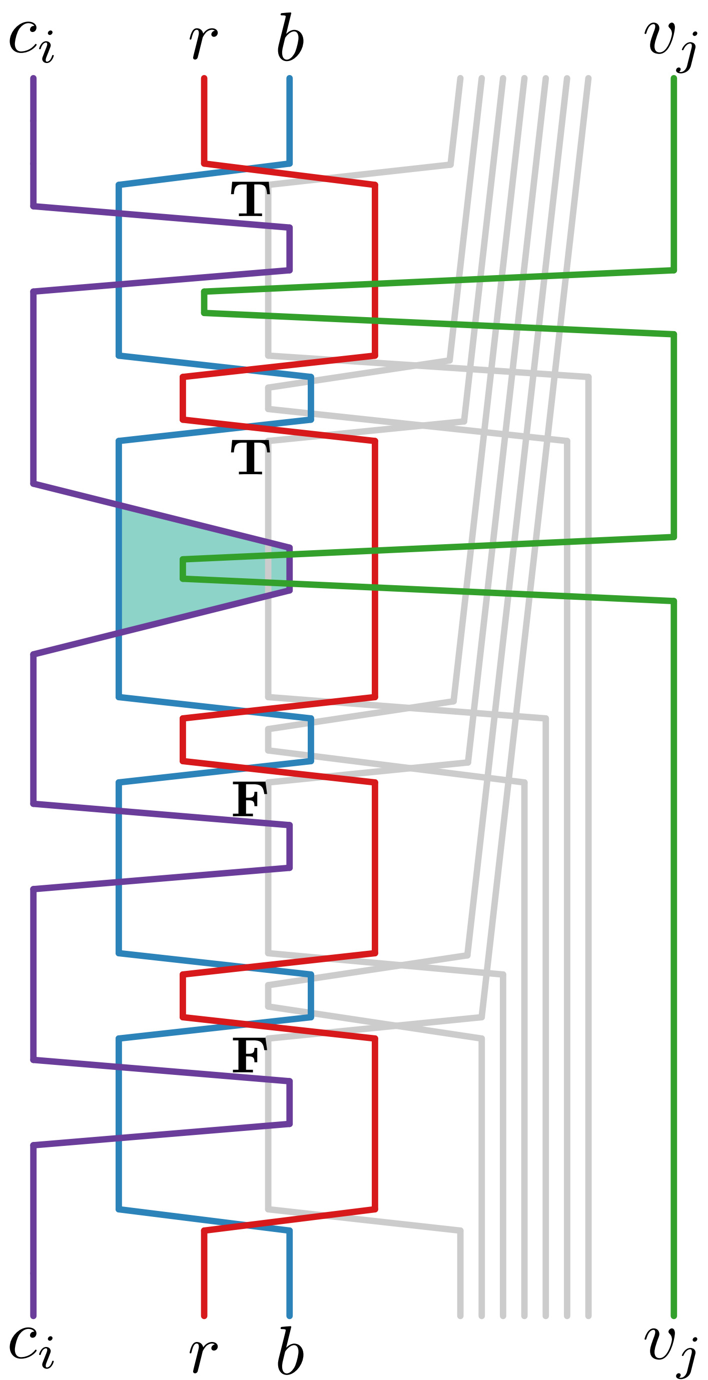

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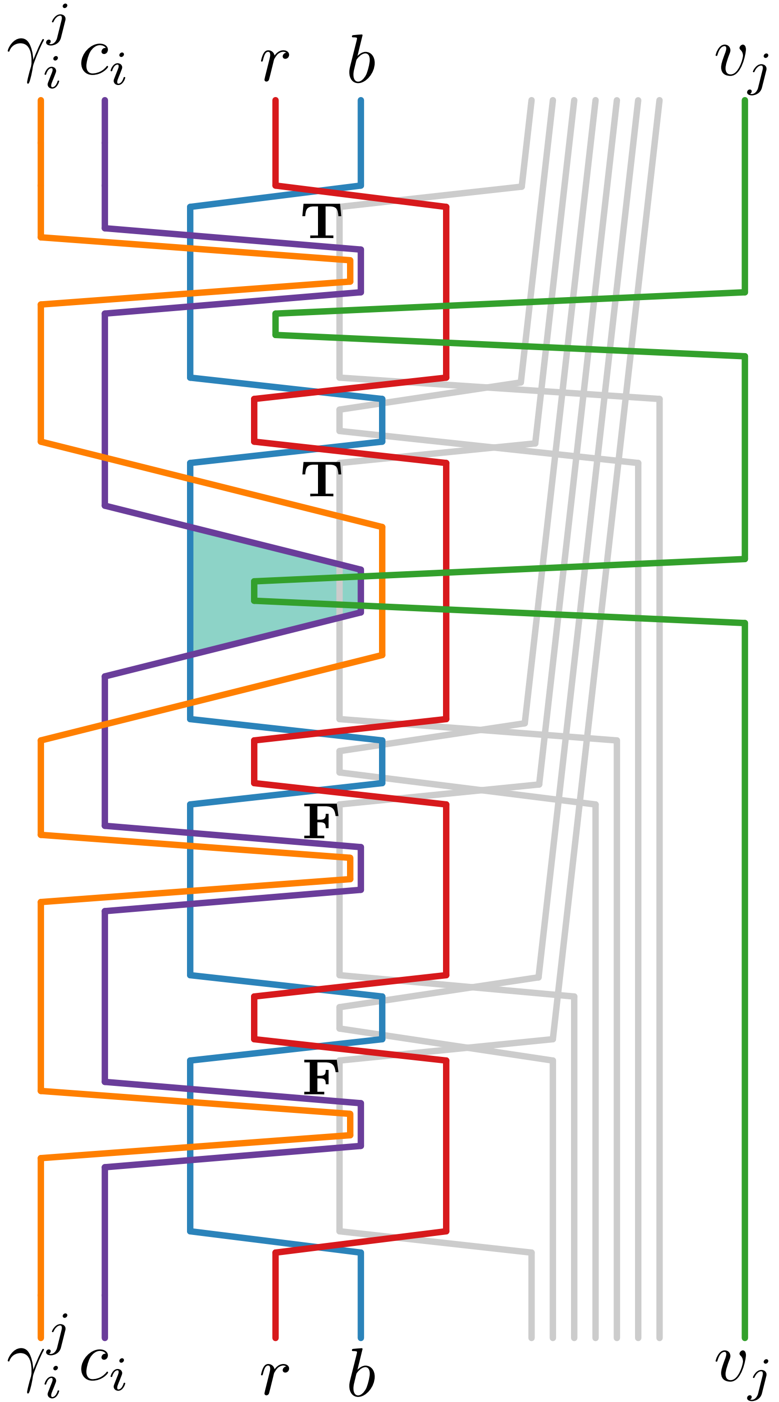

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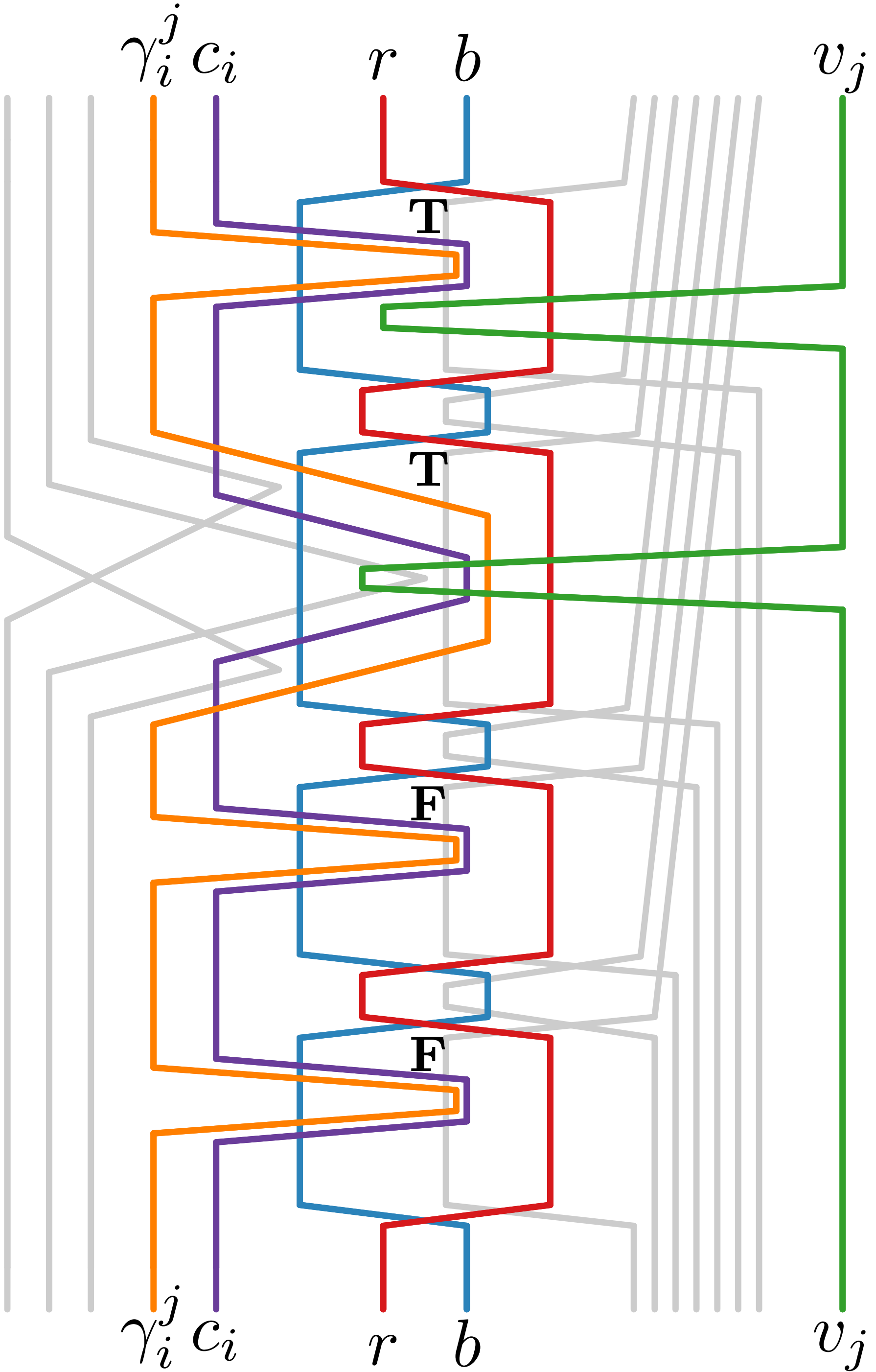

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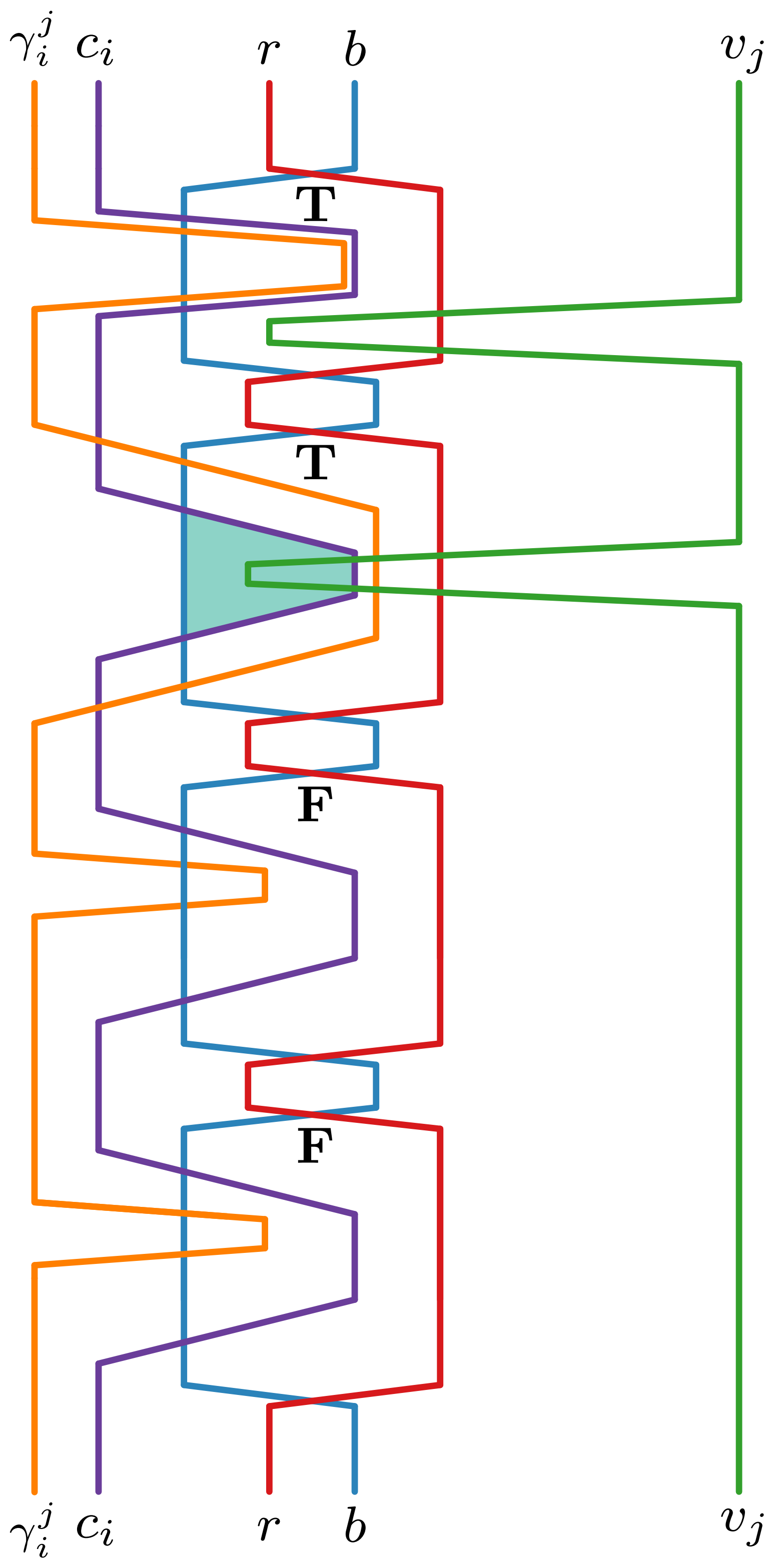

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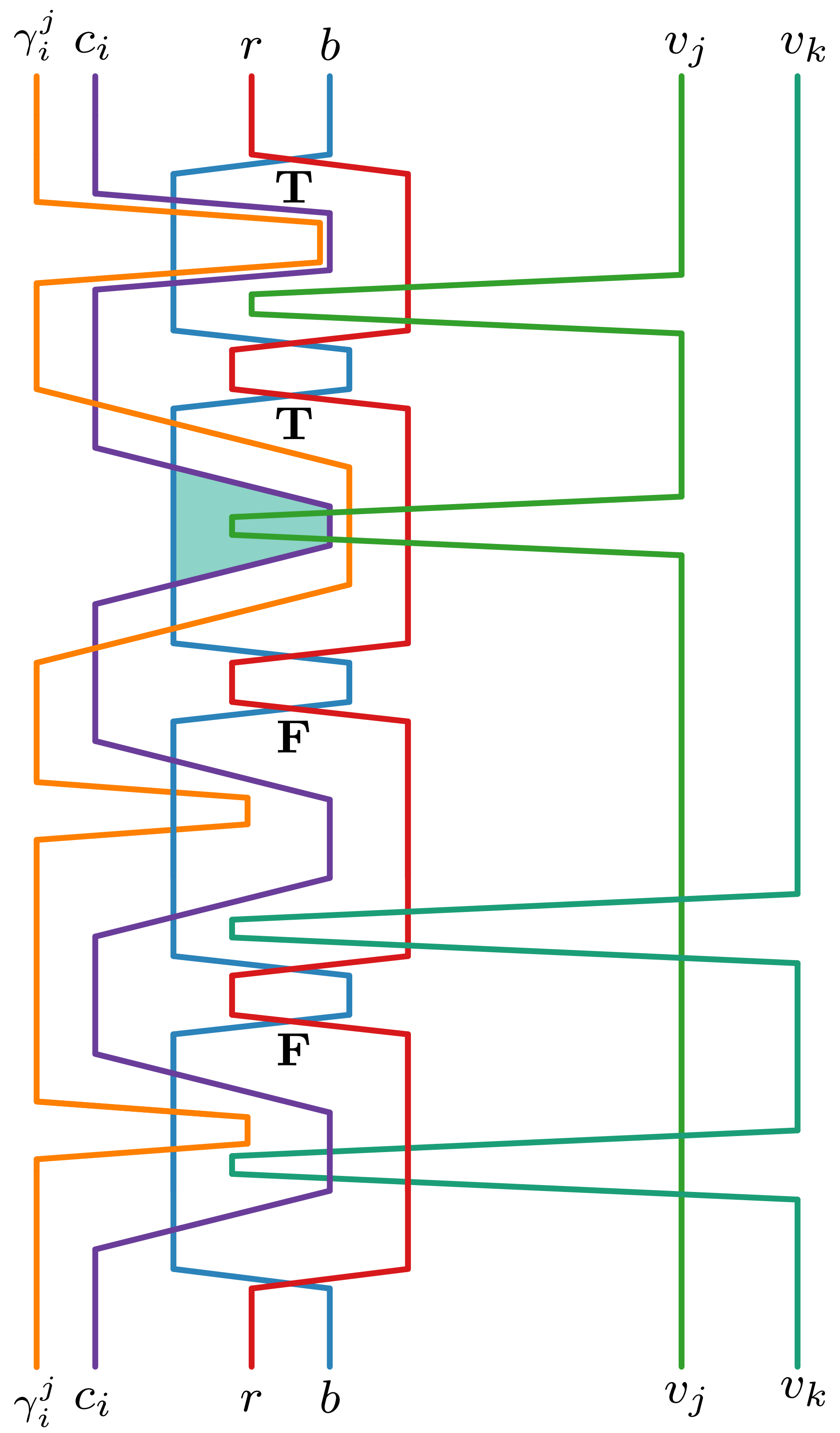

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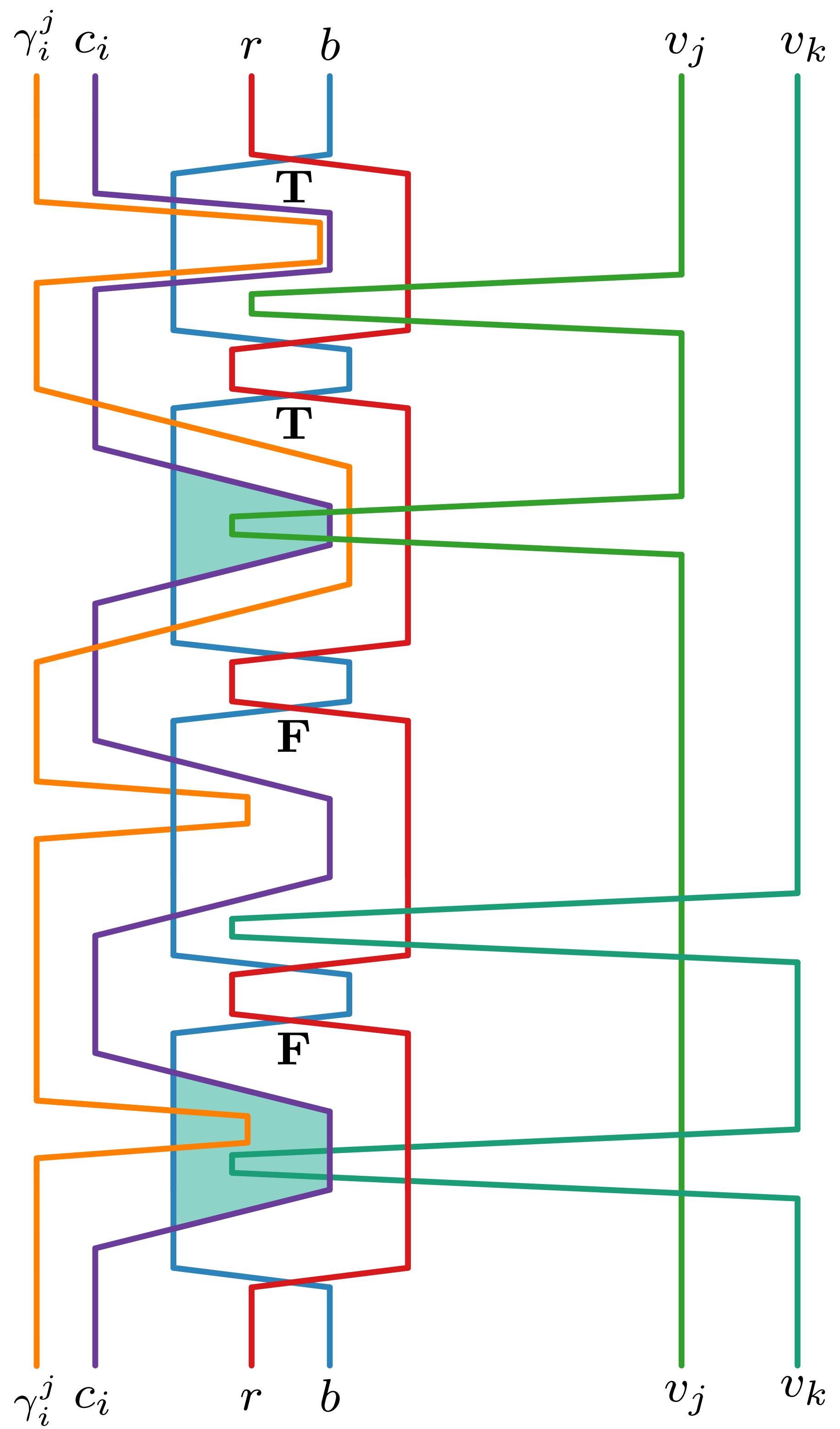

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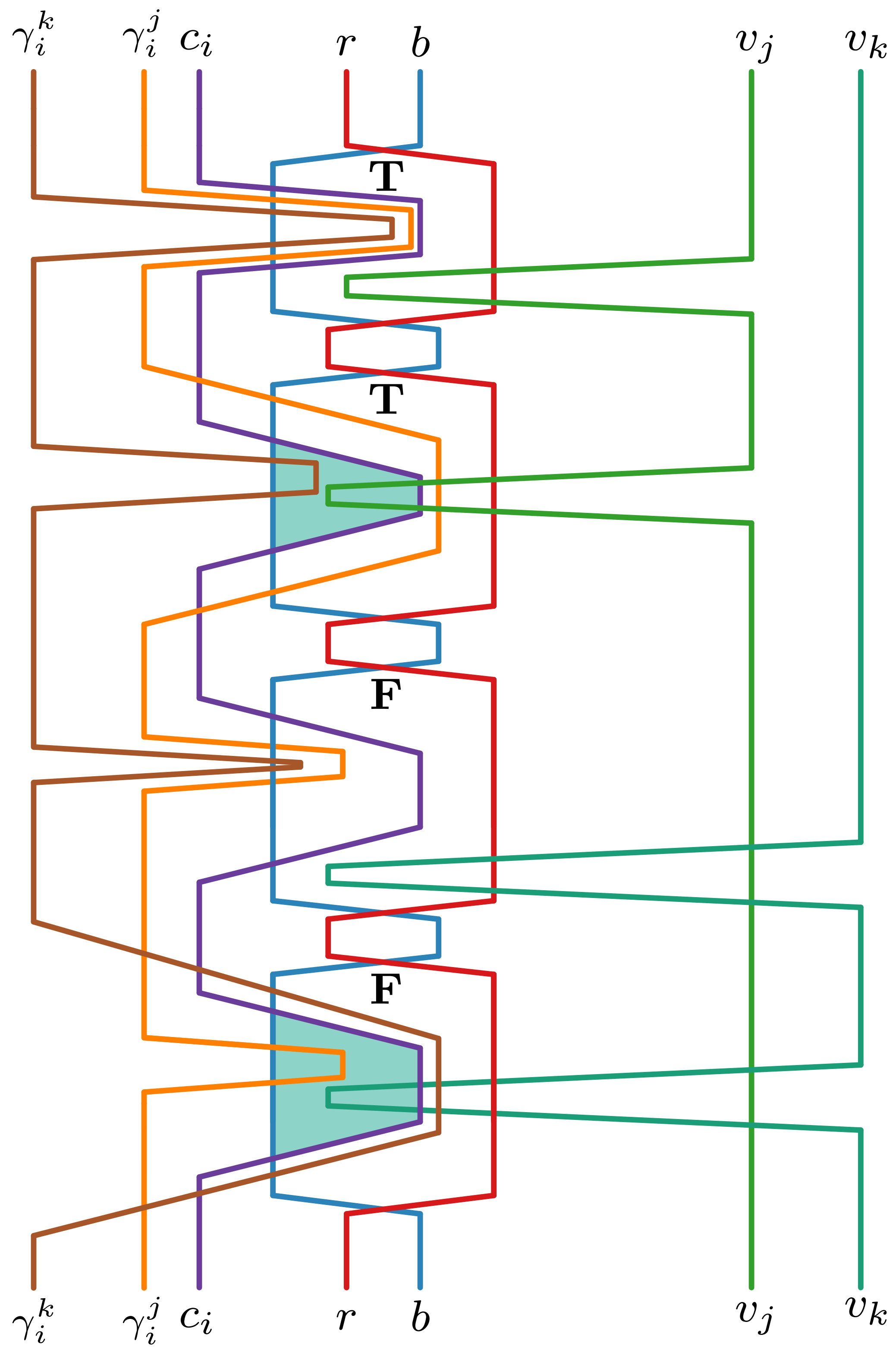

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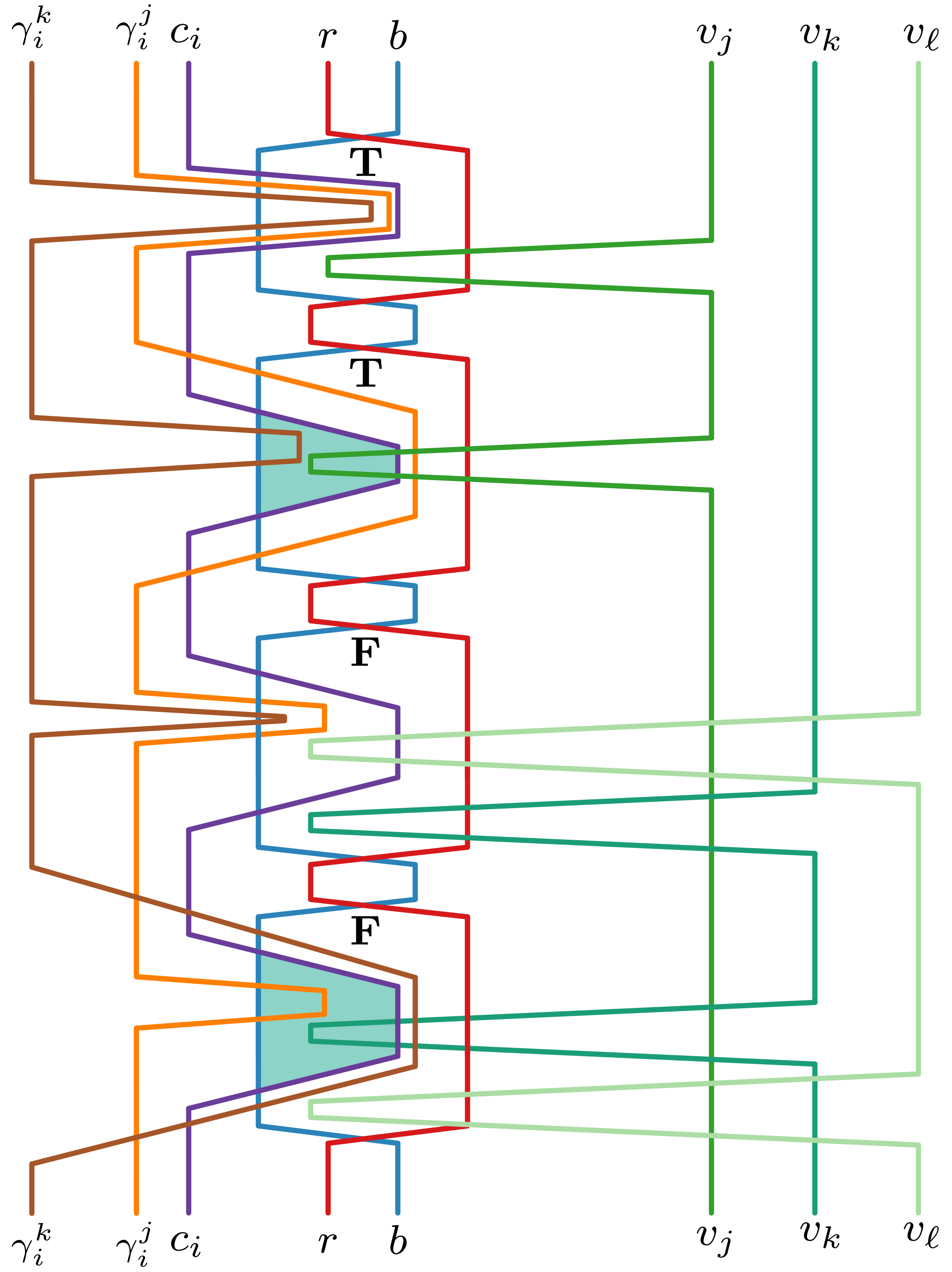

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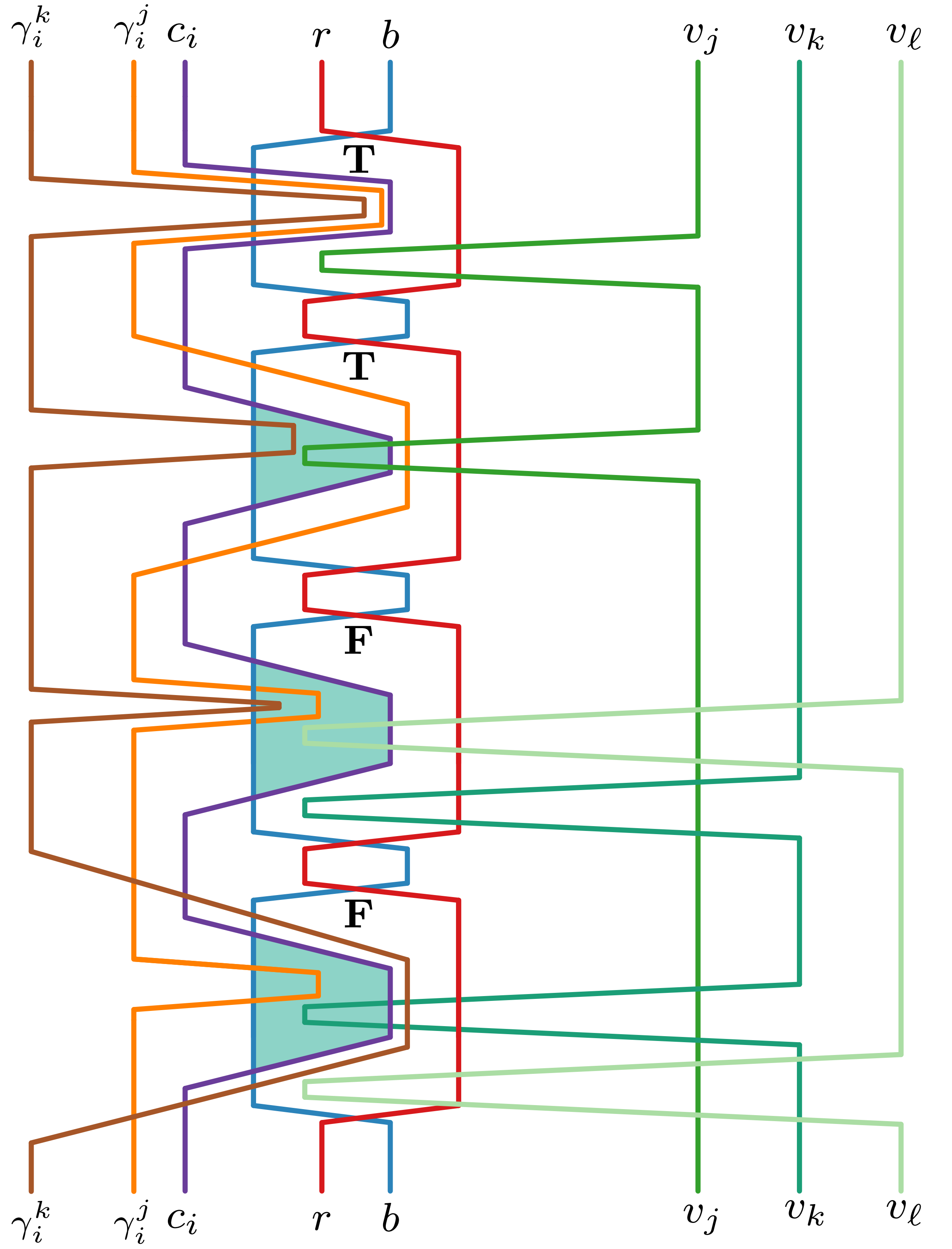

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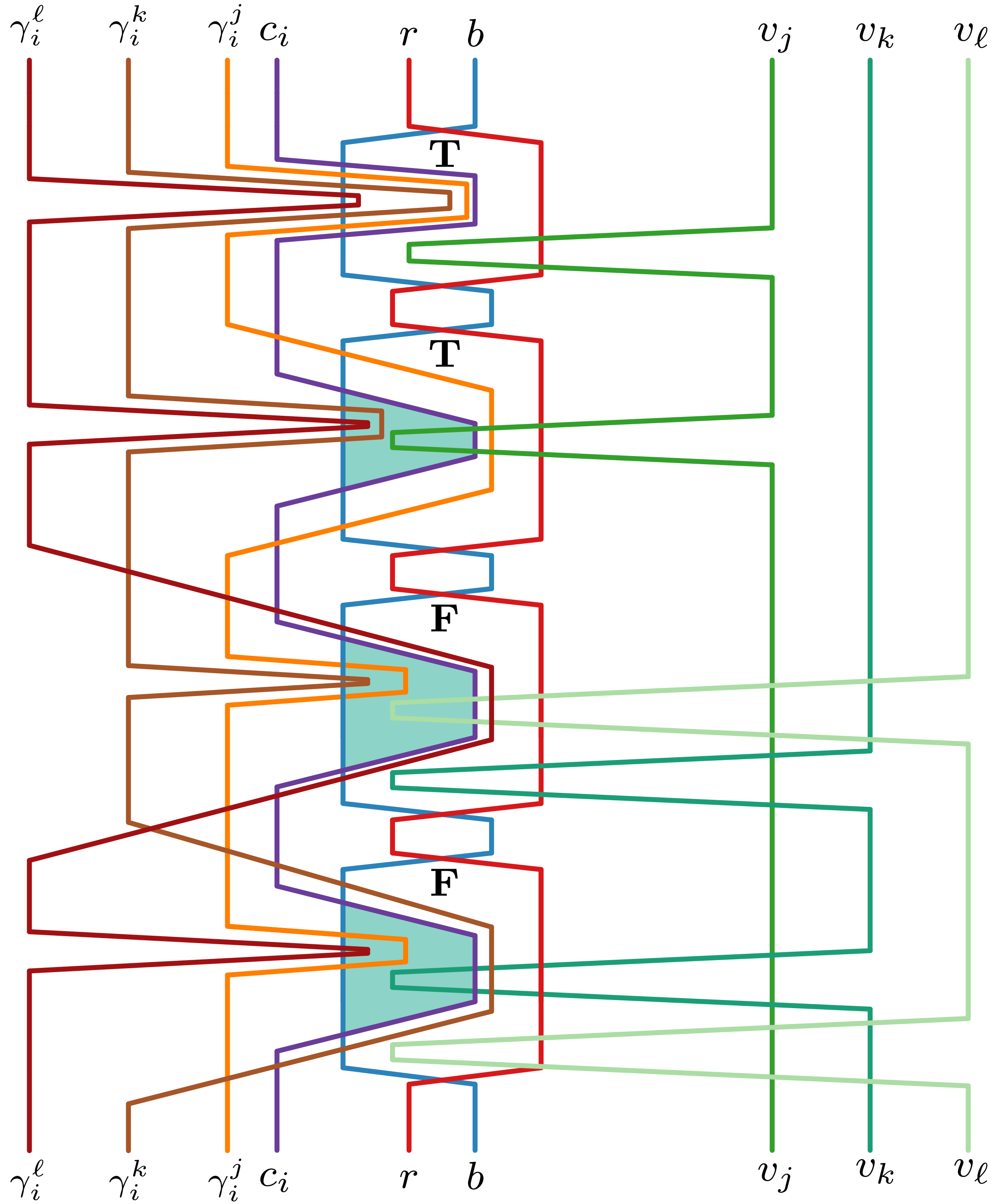

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Hence:

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Positive Not-All-Equal 3-SAT Diff
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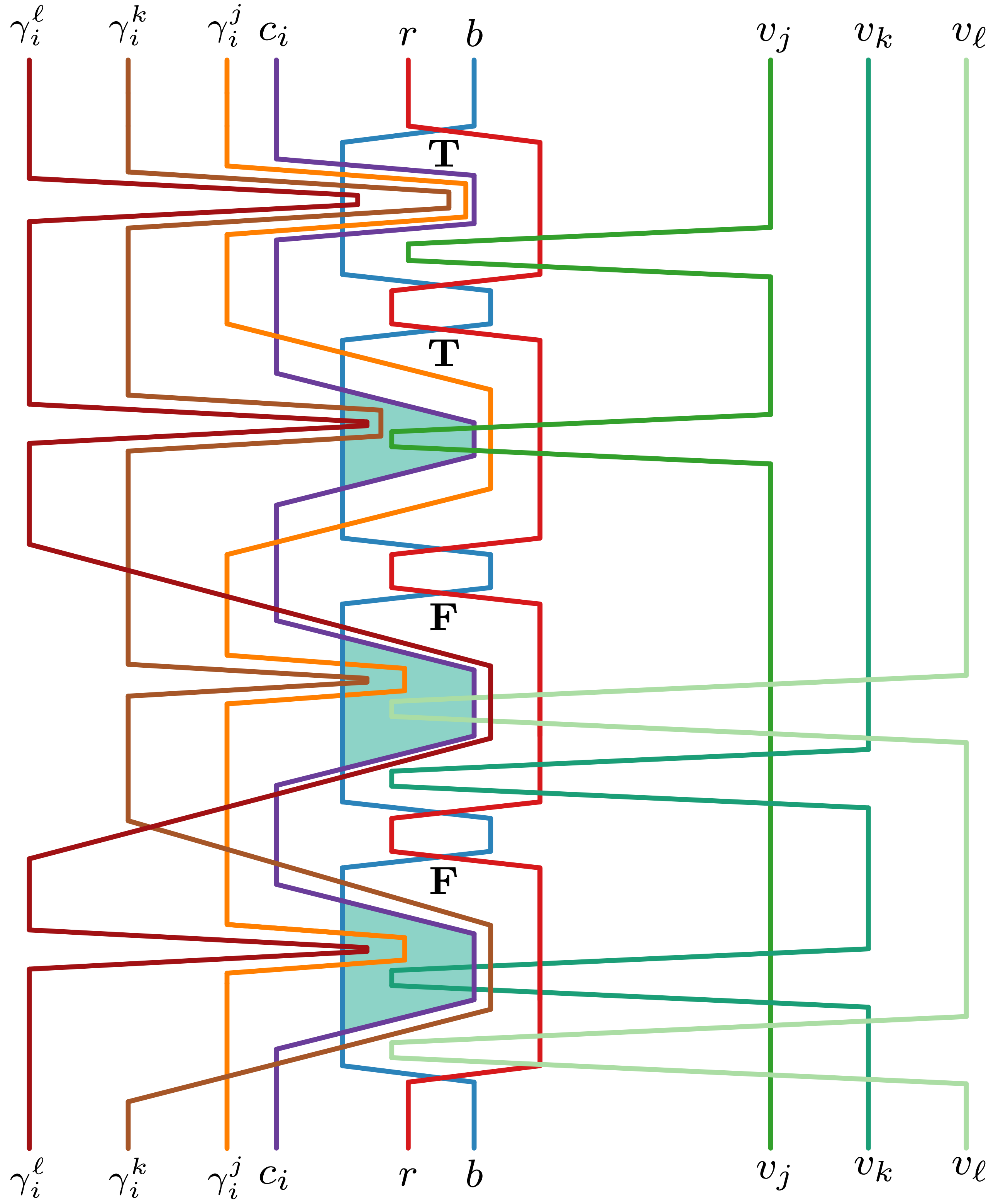

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Dynamic Programming Algorithm

$O\Big(\lambda \varphi^n n \log |L|\Big)$ $\longrightarrow$ $O\Big(\lambda n^3 \log |L|\Big)$

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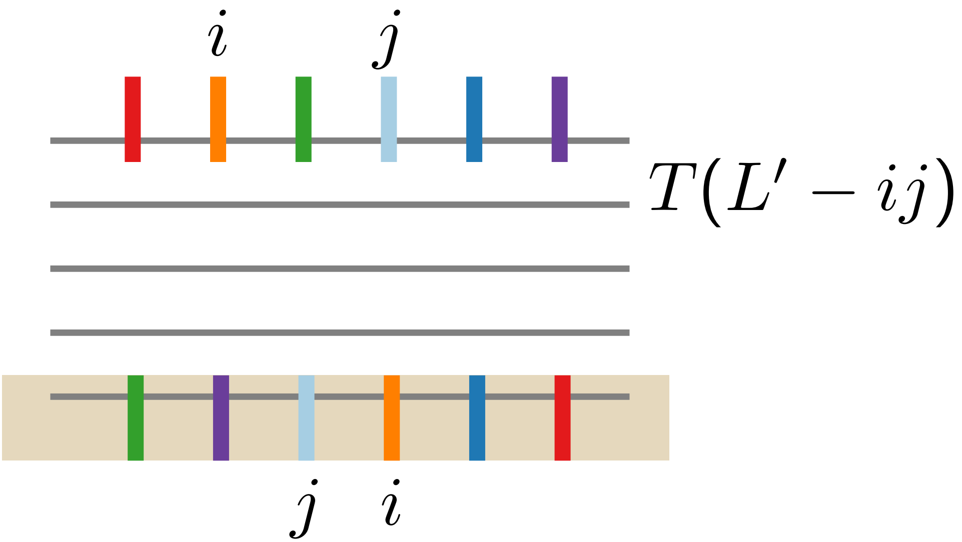

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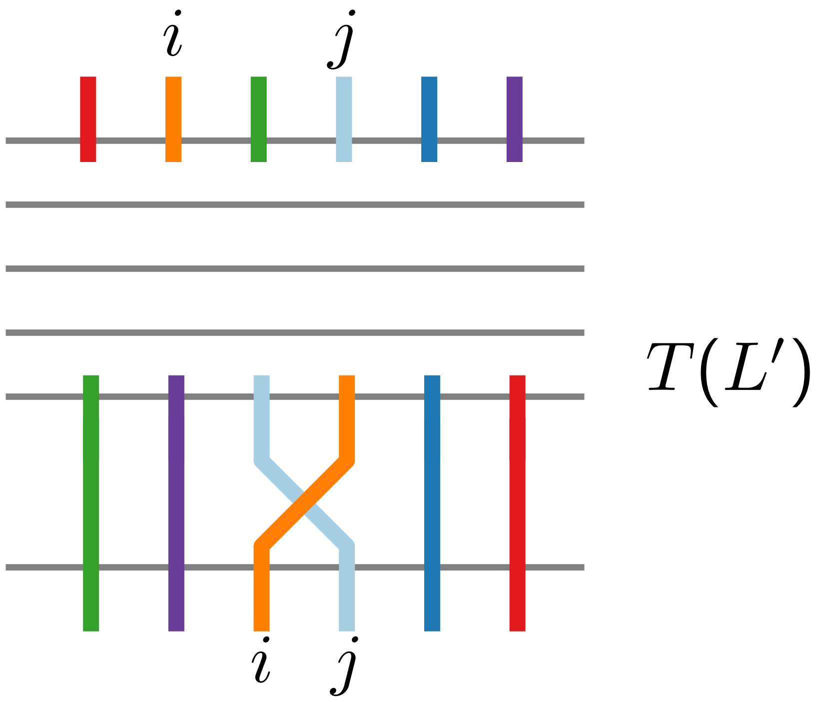

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Running time

$$\lambda = \prod_{i<j} (\ell_{ij} + 1) \leq \left(\frac{2|L|}{n^2} + 1\right)^{n^2/2}$$

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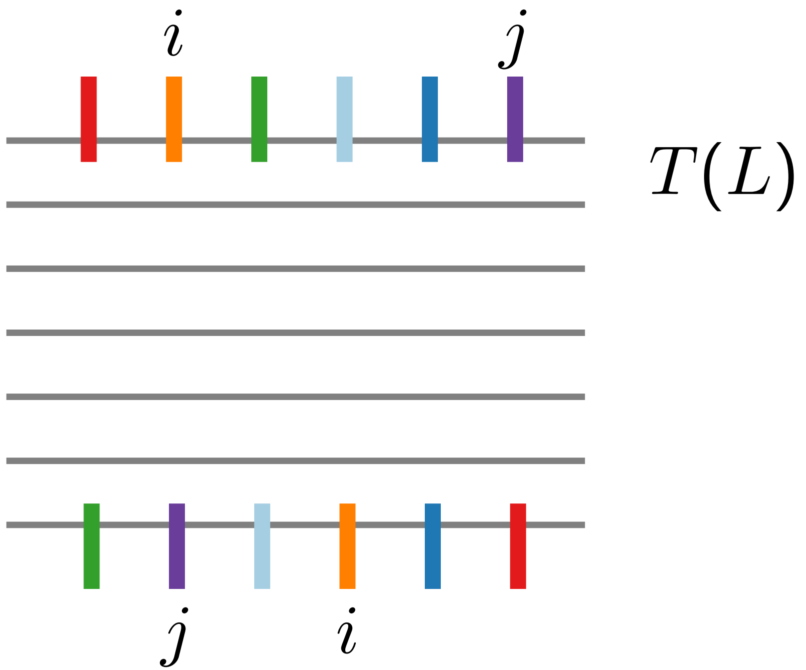

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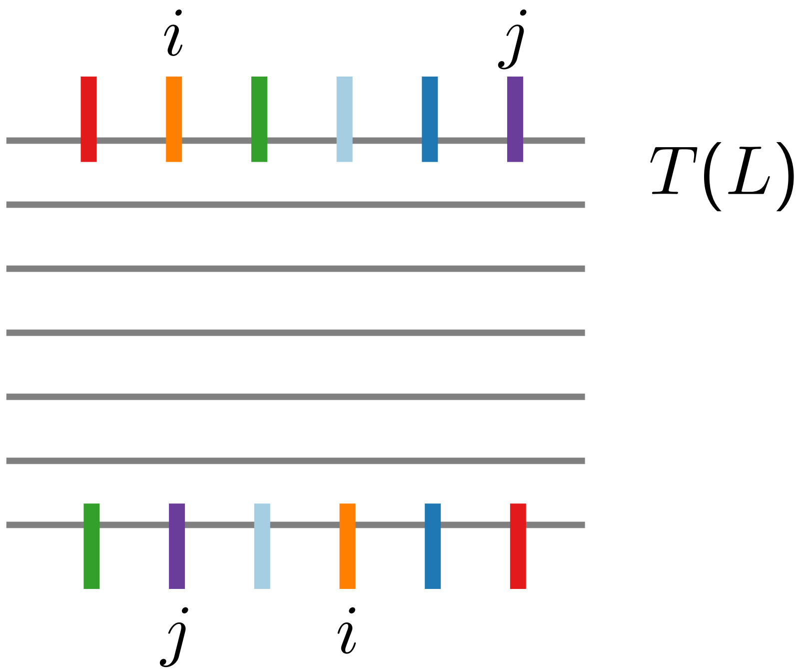

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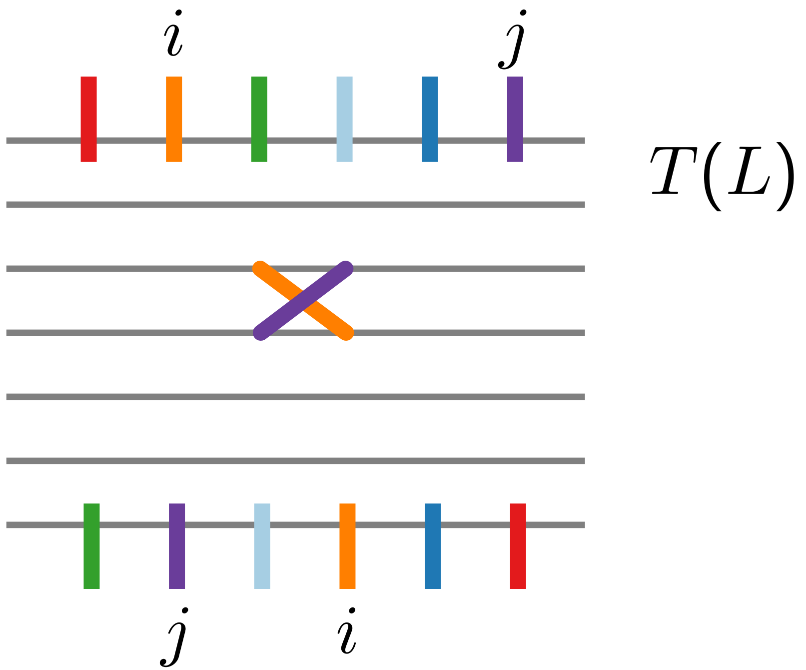

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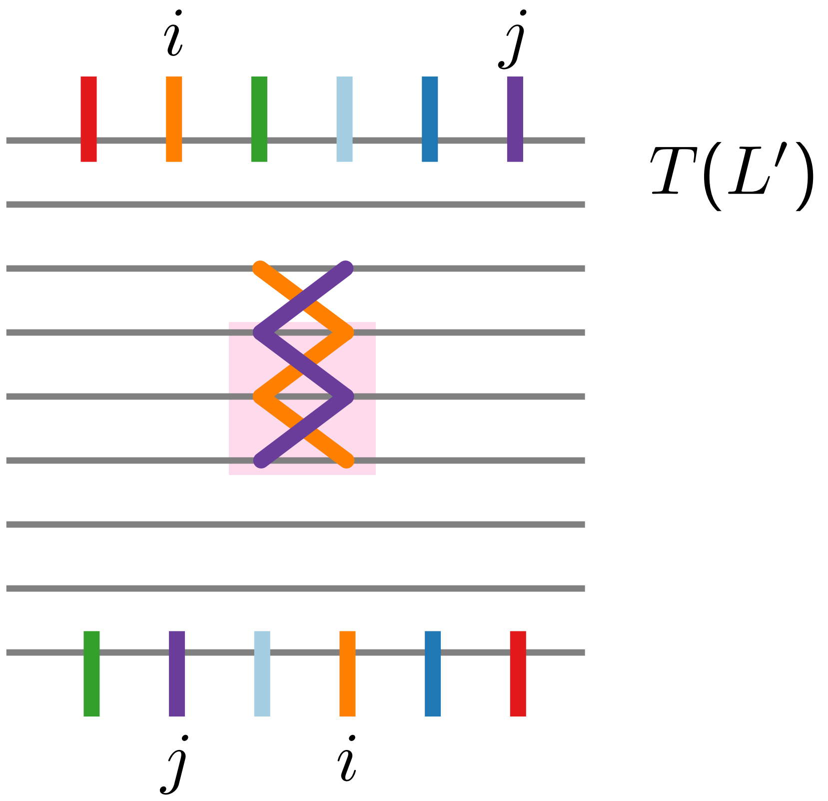

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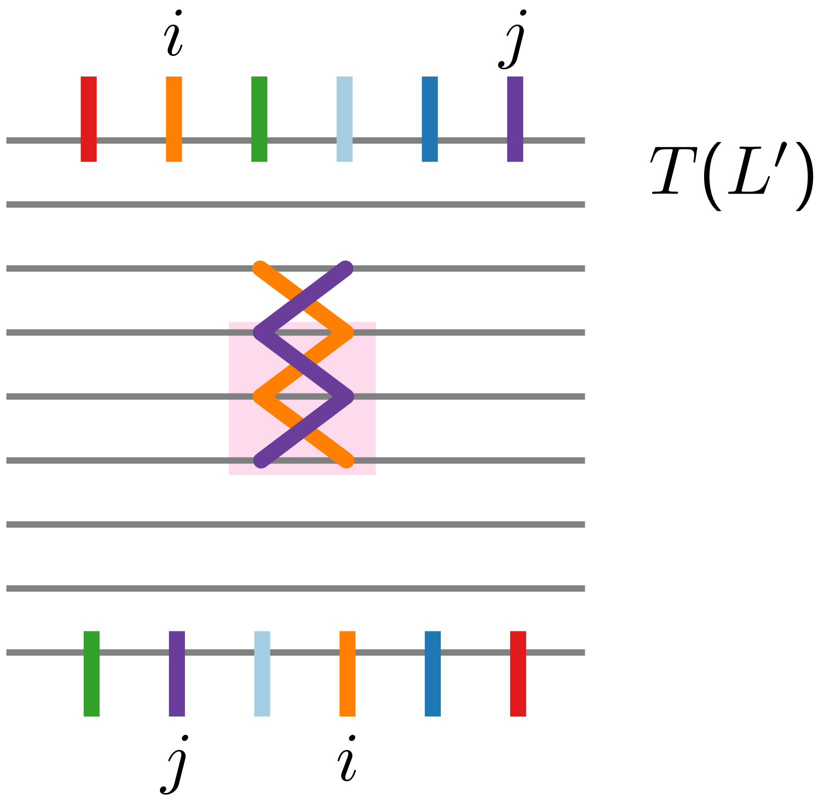
If L contains a swap ij, L' is feasible.



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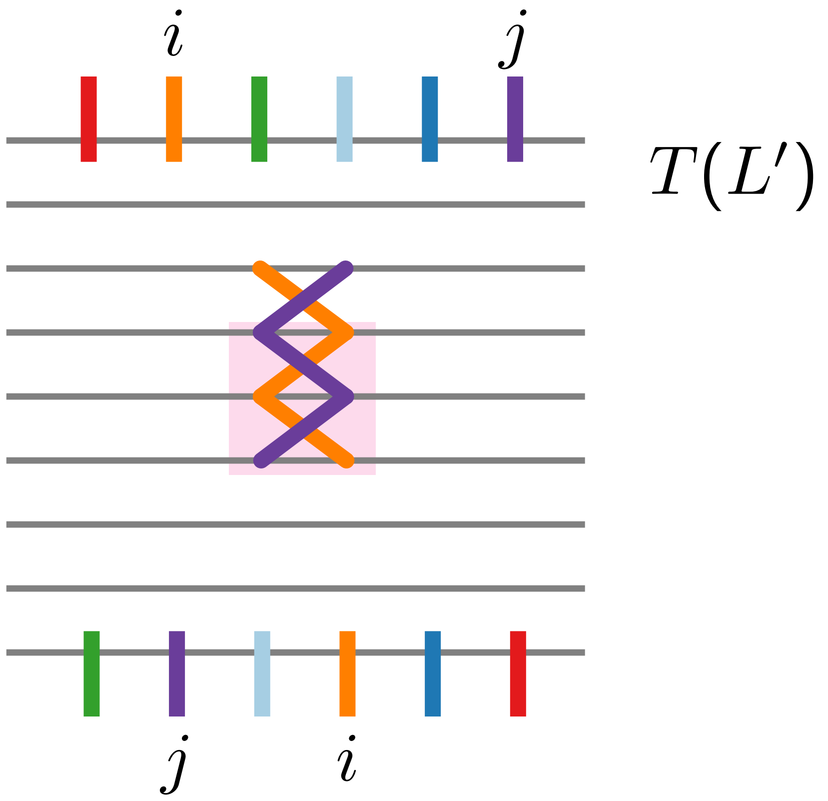
If L contains a swap ij, L' is feasible.

We say that L can be *extended* to L'.



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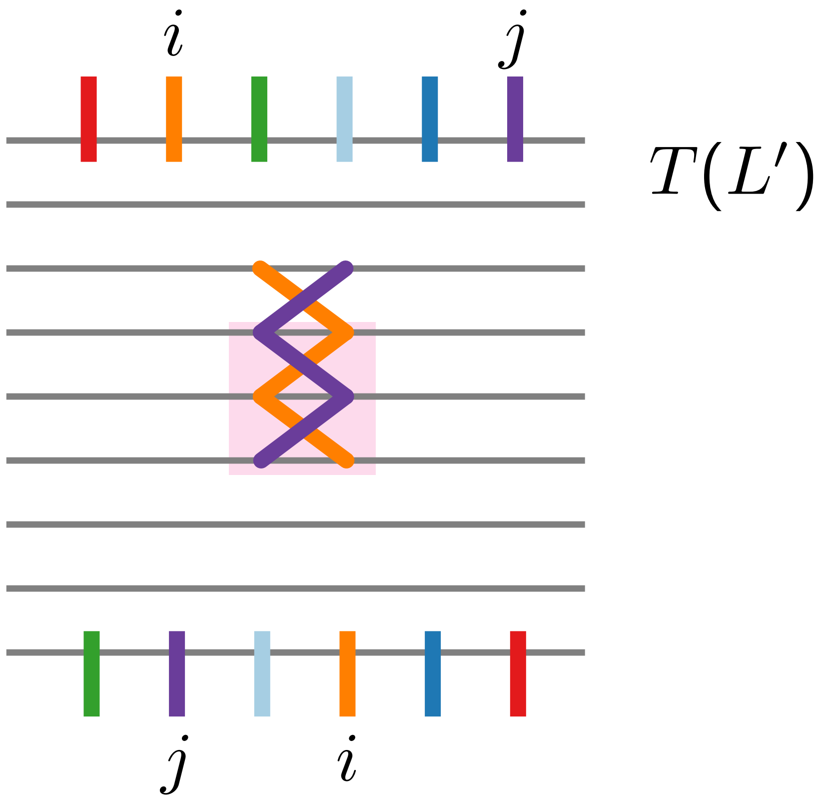

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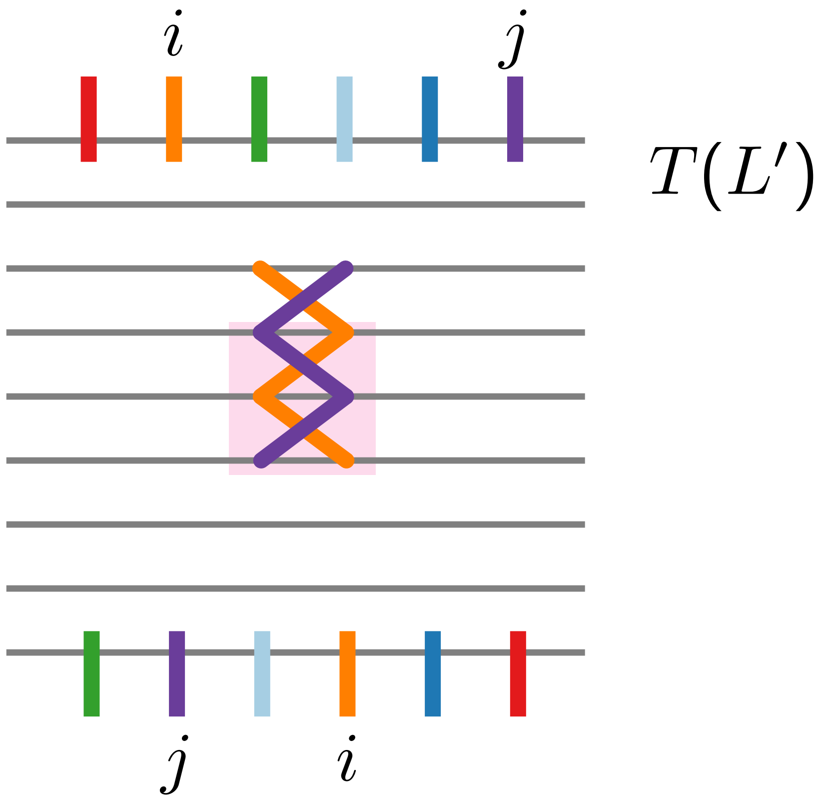

Lemma.
If $L = (l_{ij})$ is a minimal feasible list with n wires,
then $l_{ij} \leq n^2/4 + 1$ for each $i, j \in [n]$.

FPT Algorithm Parametrized by the Number of Wires

Theorem.
There is an FPT algorithm for LIST-FEASIBILITY w.r.t. n.

$$O\Big(\big(\tfrac{2|L|}{n^2}+1\big)^{n^2/2} n^3 \log |L|\Big)$$

$$\downarrow$$

$$O\Big(\big(\tfrac{n}{2}\big)^{n^2} \cdot n^3 \log n + n^2 \log |L|\Big)$$

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$O\Big(\big(\frac{2|L'|}{n^2}+1\big)^{n^2/2} n^3 \log |L'| + n^2 \log |L|\Big)$

FPT Algorithm Parametrized by the Number of Wires

$$O\Big(\big(\tfrac{2|L|}{n^2}+1\big)^{n^2/2} n^3 \log |L|\Big)$$

Theorem.
There is an FPT algorithm for LIST-FEASIBILITY w.r.t. n.

$$O\Big(\big(\tfrac{n}{2}\big)^{n^2} \cdot n^3 \log n + n^2 \log |L|\Big)$$

Proof idea.
Create a list $L' = (l'_{ij})$ from $L = (l_{ij})$ by setting $l'_{ij} = \min\{l_{ij}, n^2/4+1\}$.

Use the algorithm described before for L'.

Check if there is a feasible sublist of L' that can be extended to L.

Running time

$$O\Big(\big(\tfrac{2|L'|}{n^2}+1\big)^{n^2/2} n^3 \log |L'| + n^2 \log |L|\Big)$$

Lemma: $|L'| \le \binom{n}{2}\big(\frac{n^2}{4}+1\big)$

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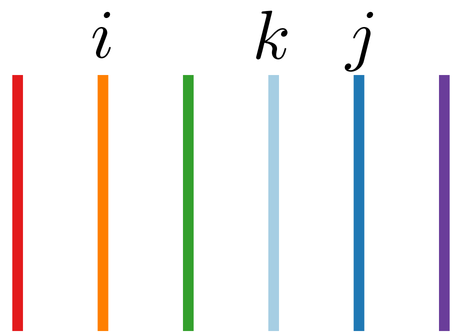

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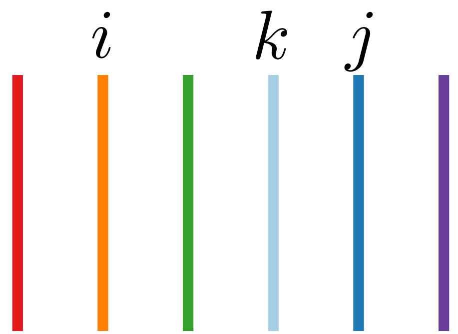

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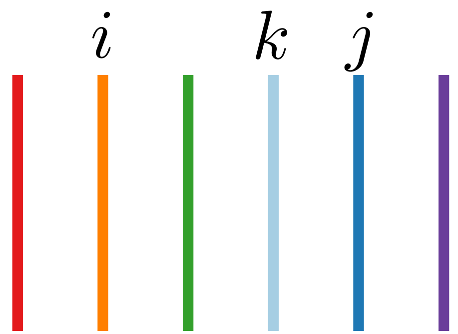

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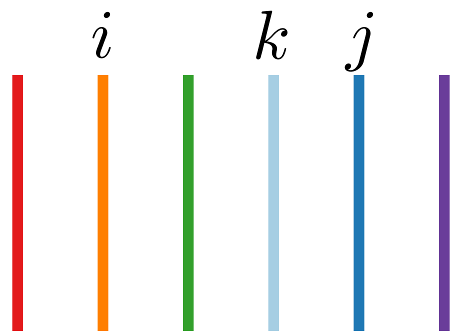

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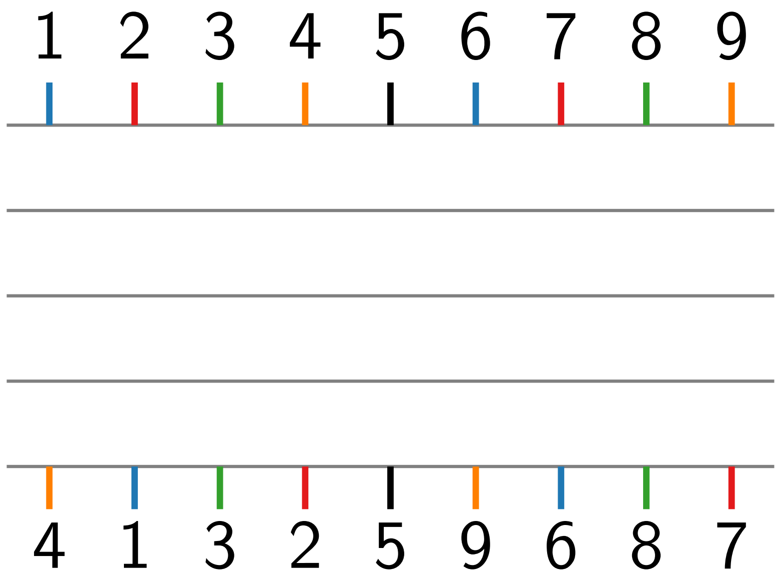

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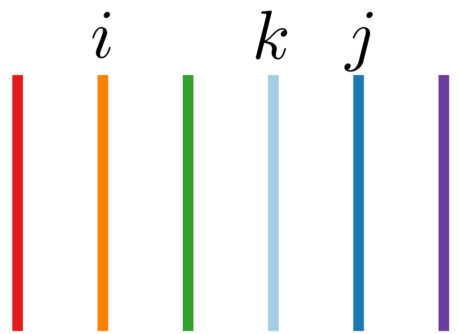

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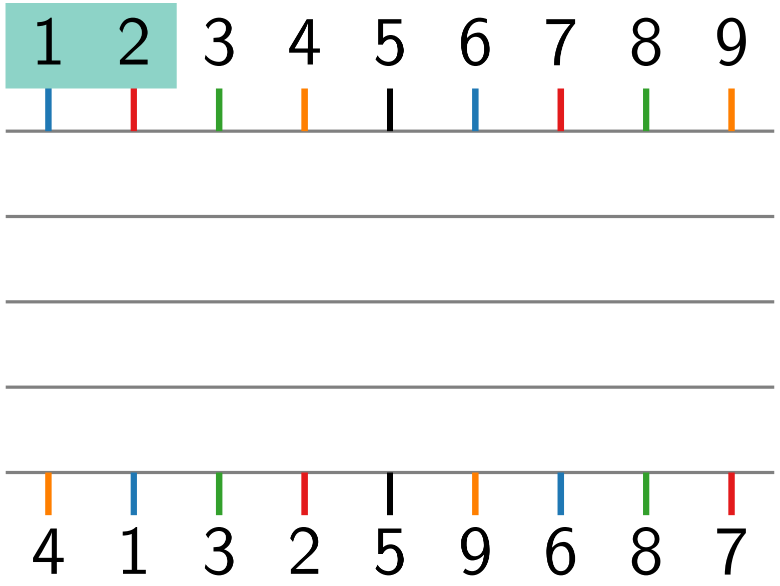

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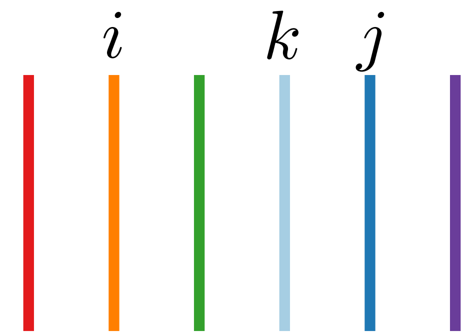

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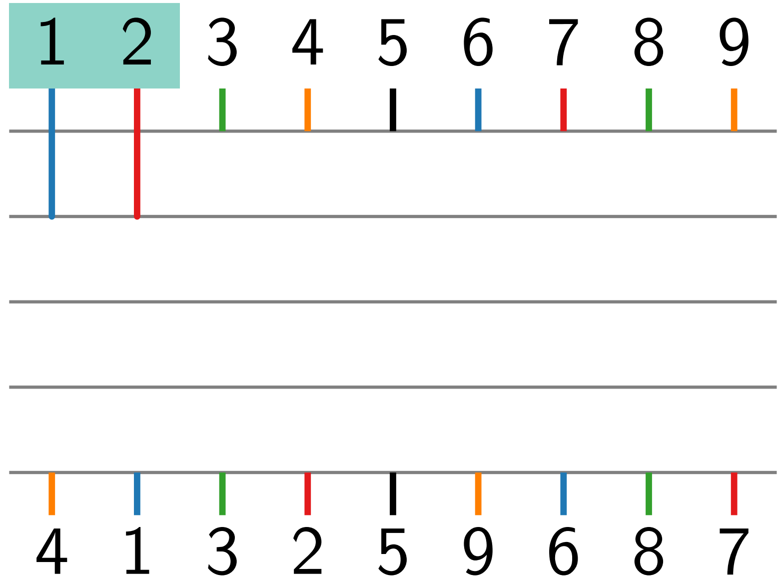

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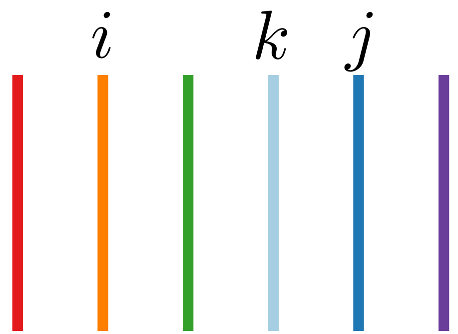

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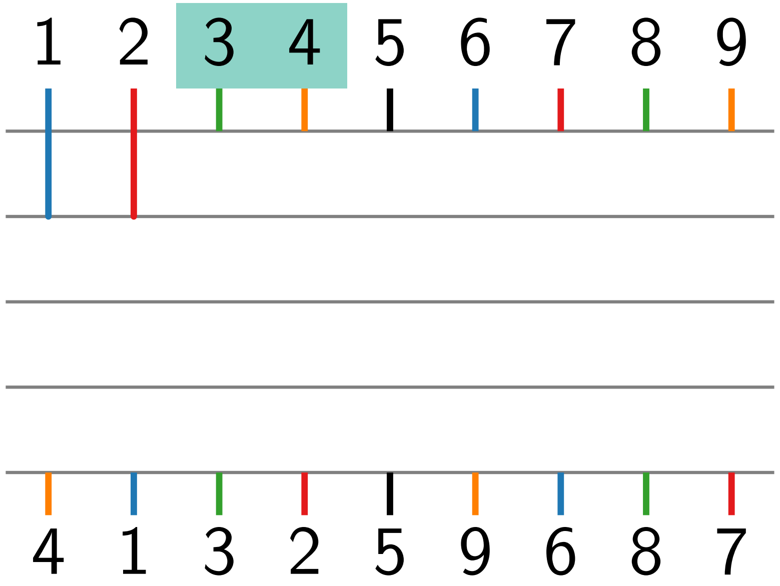

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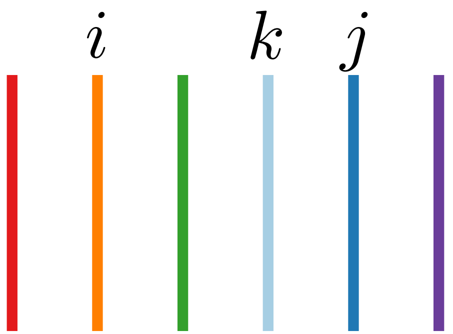

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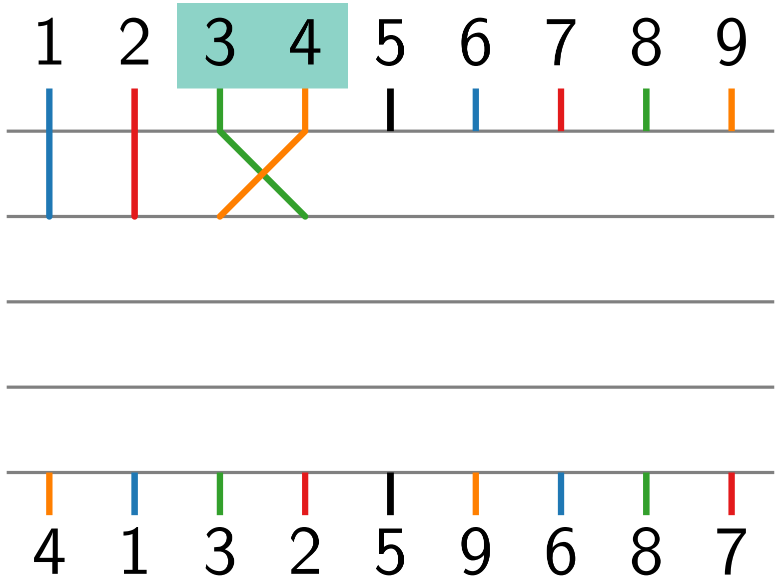

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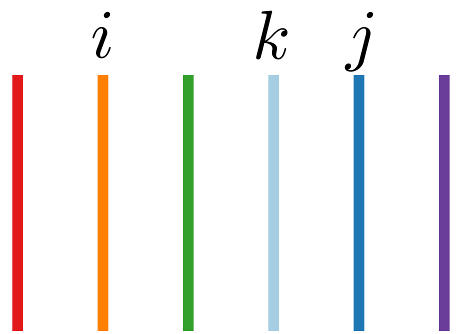

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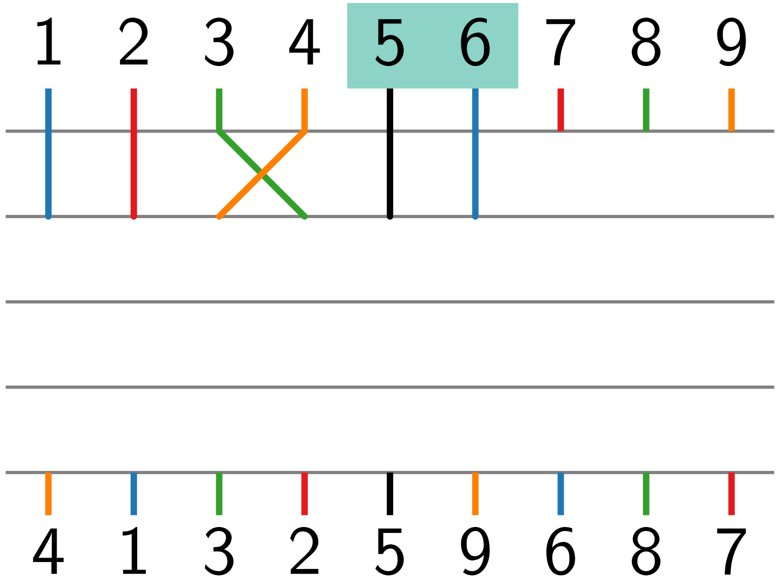

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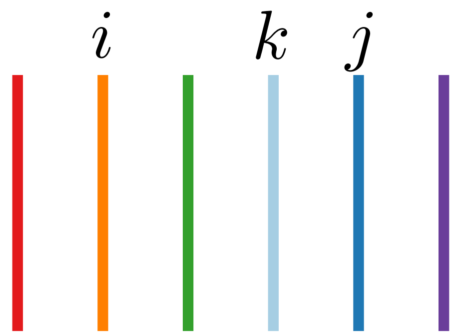

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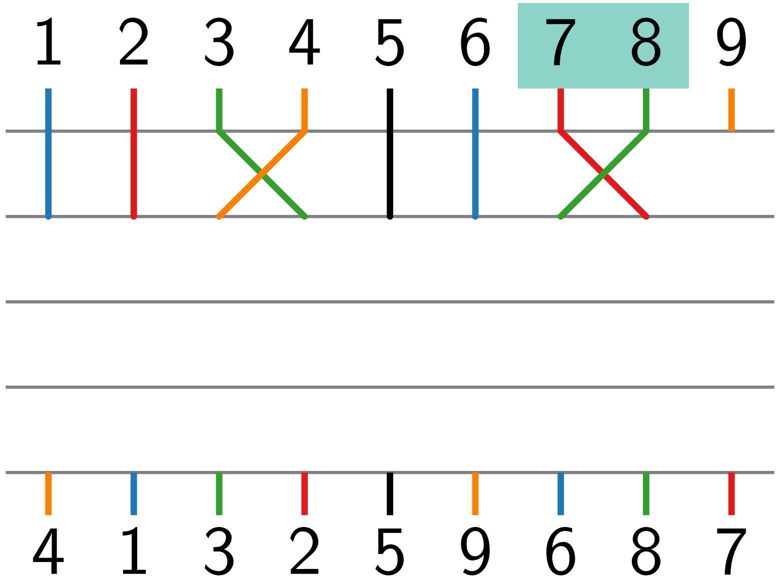

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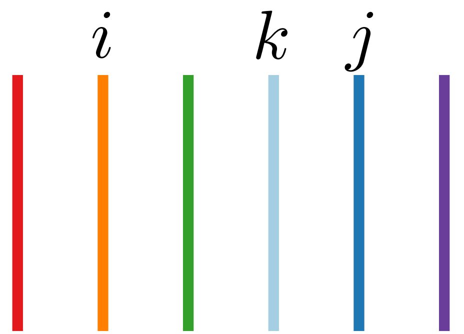

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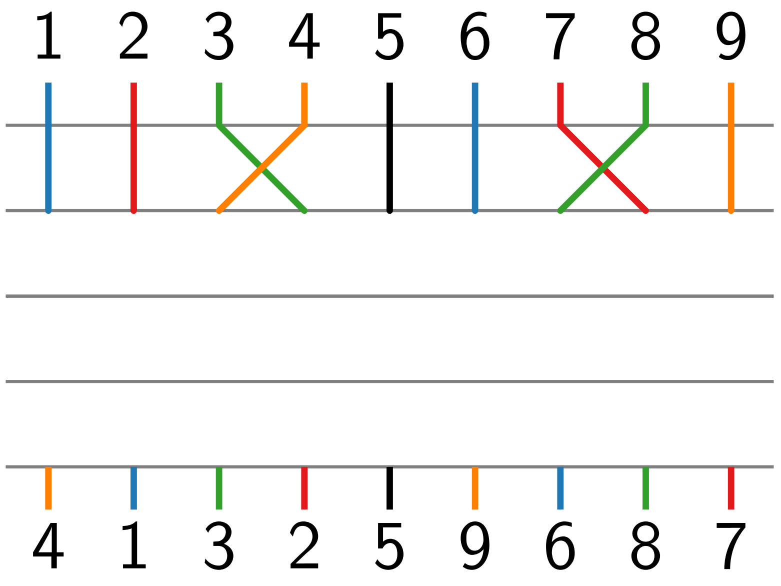

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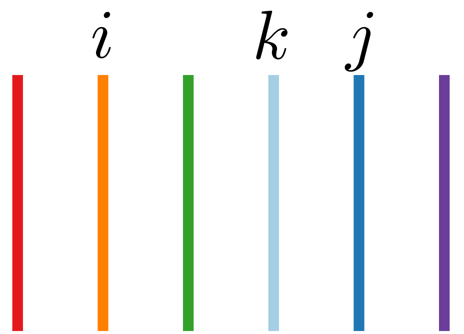

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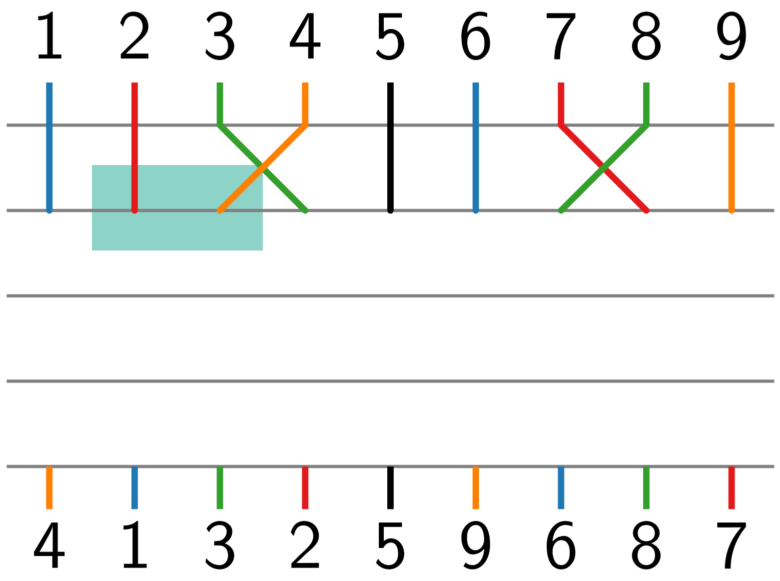

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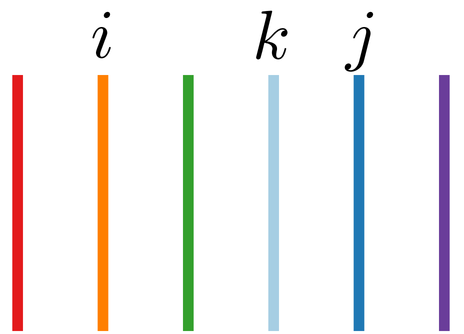

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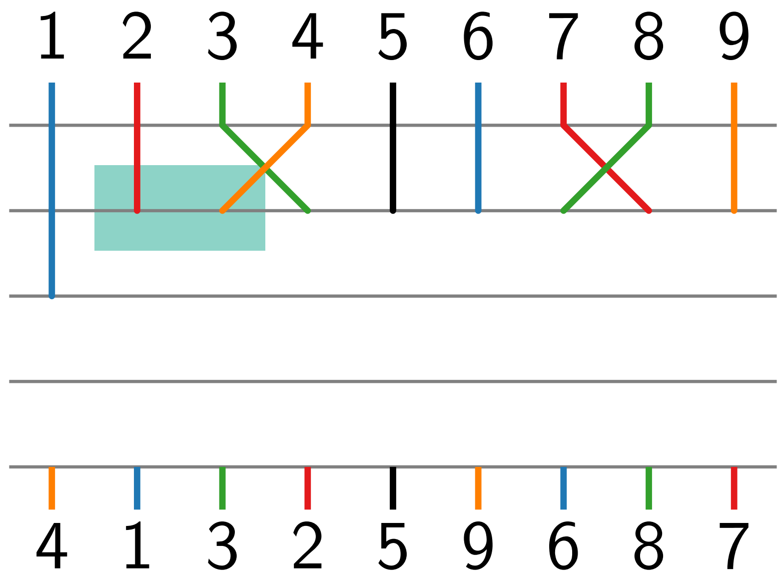

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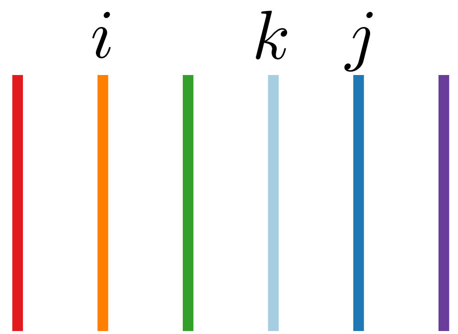

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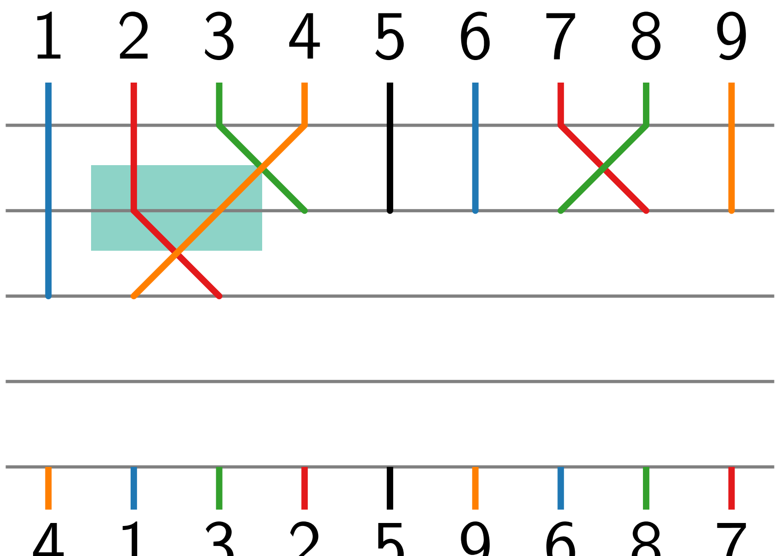

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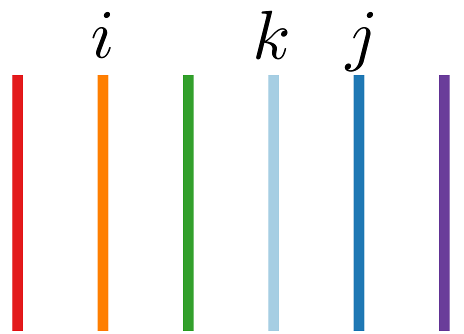

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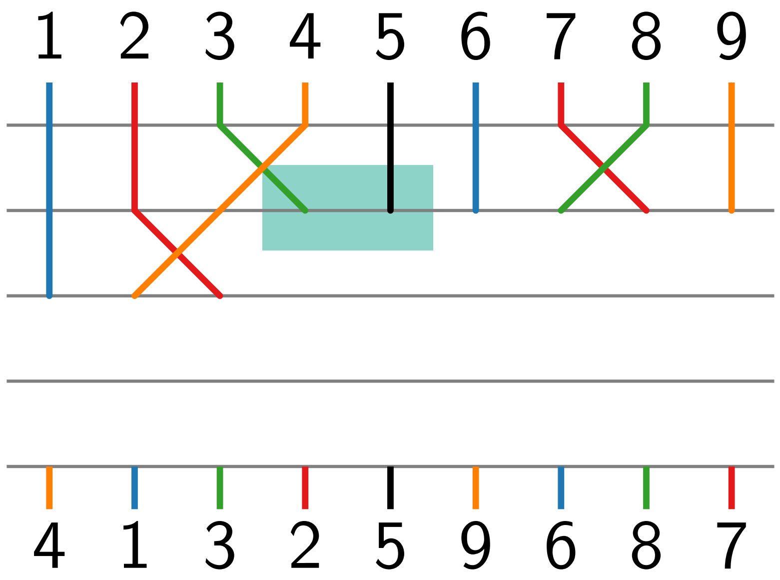

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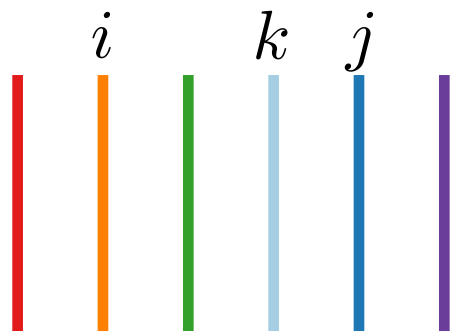

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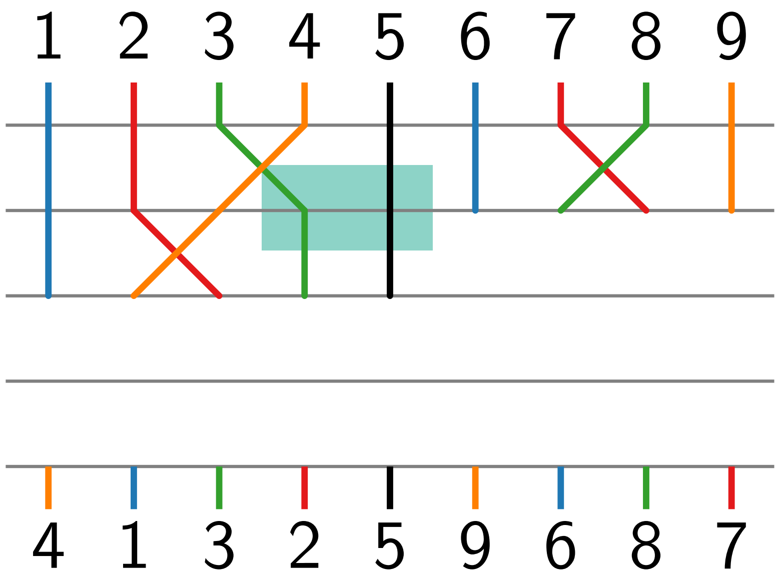

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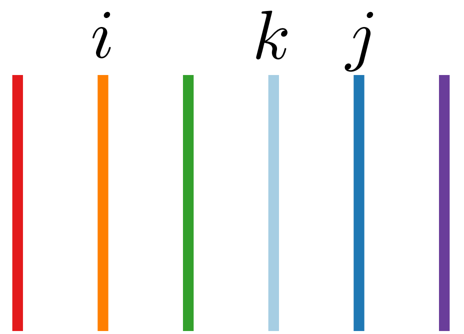

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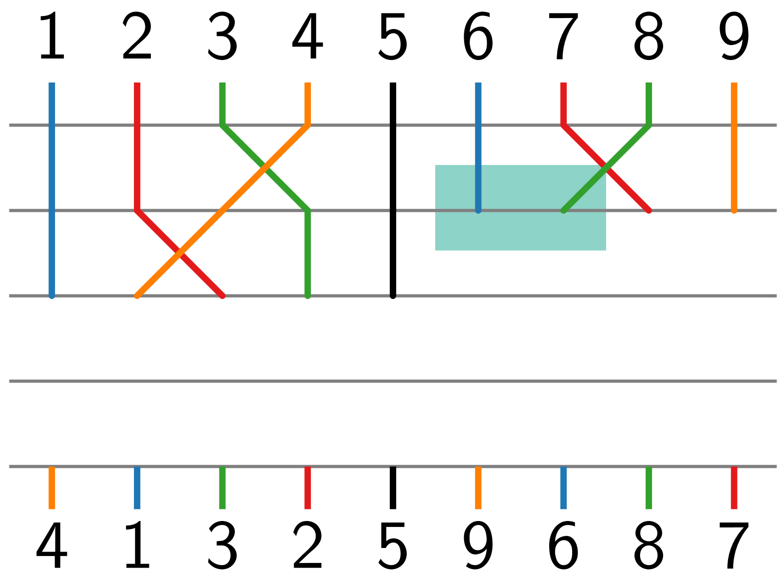

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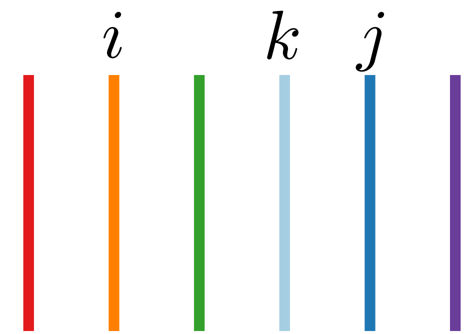

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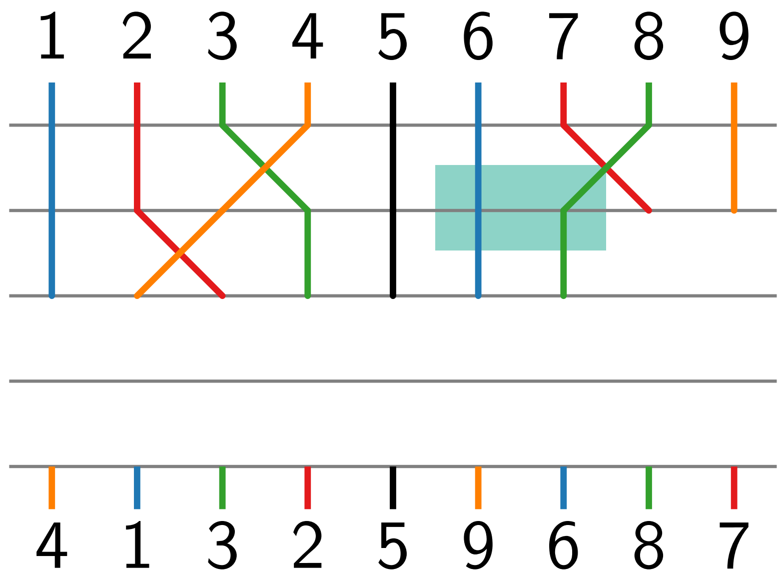

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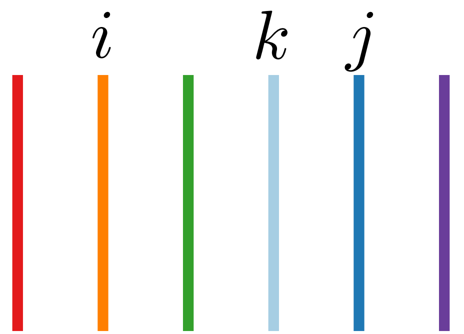

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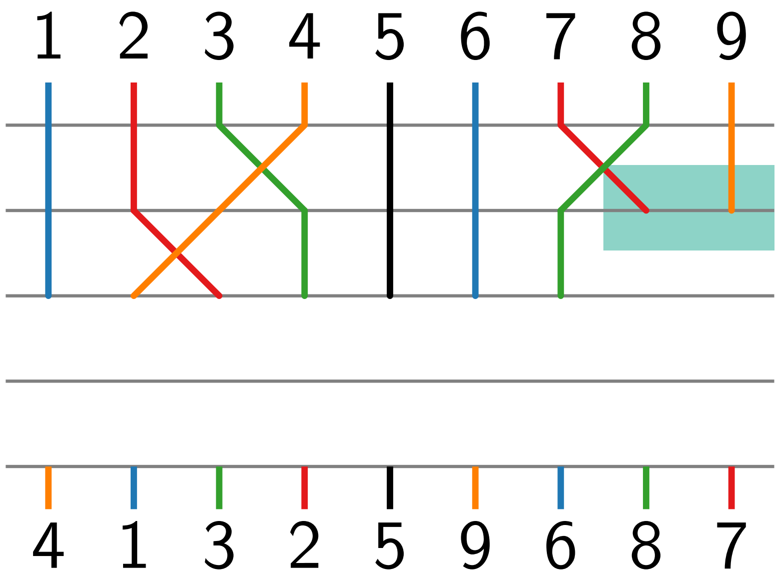

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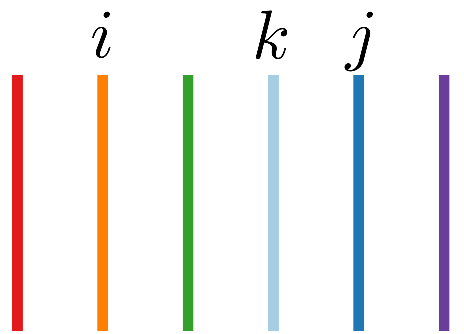

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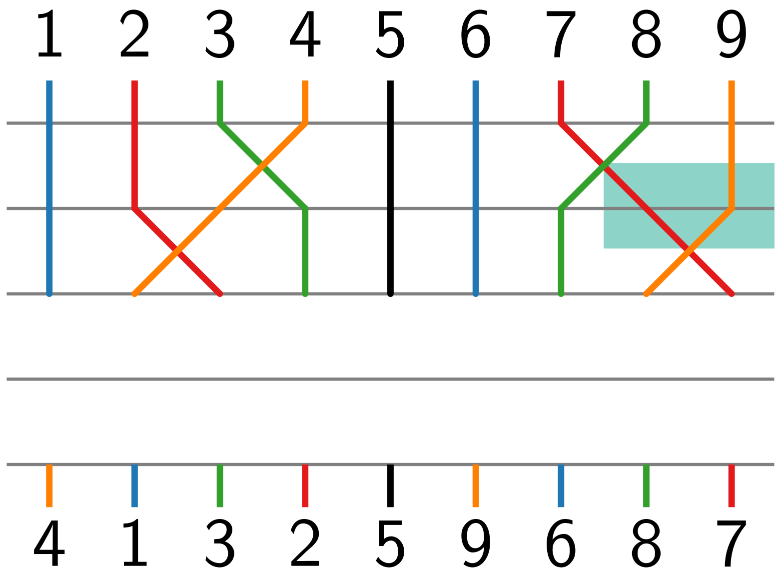

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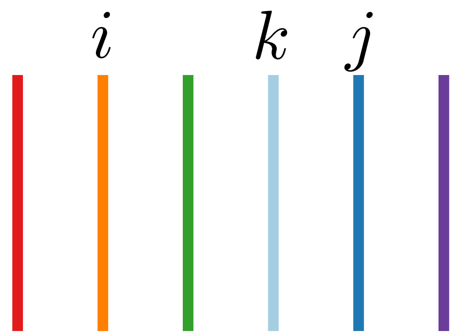

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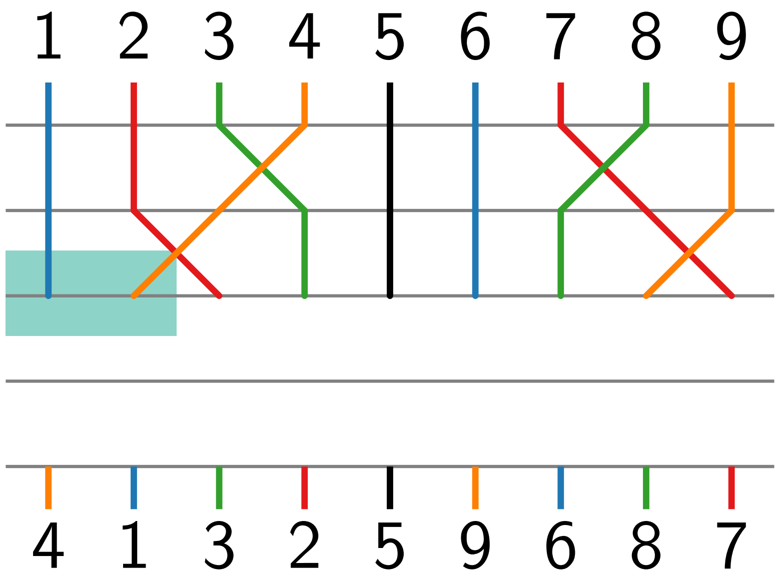

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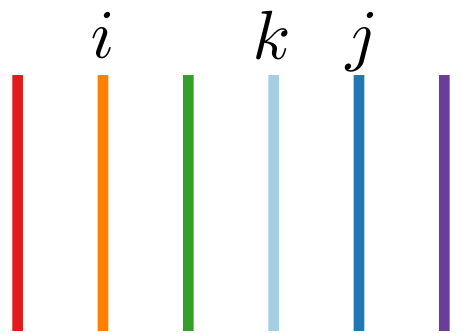

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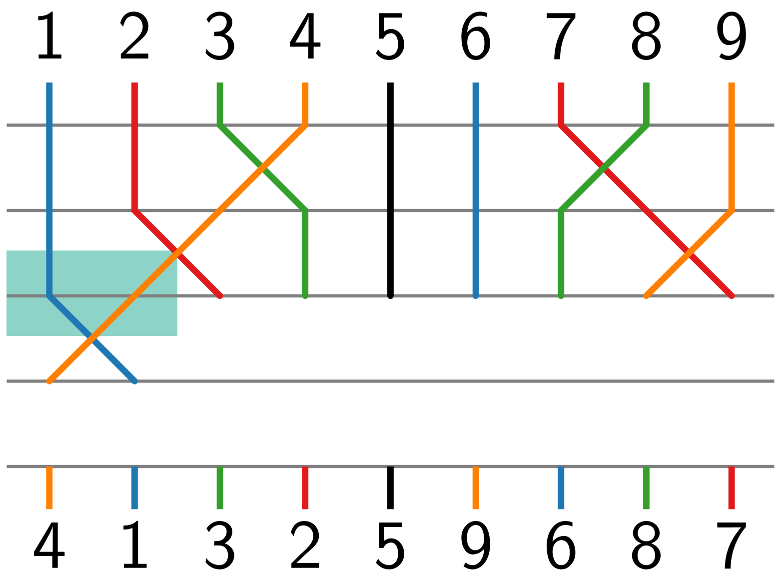

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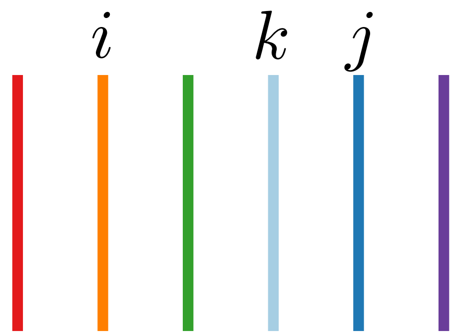

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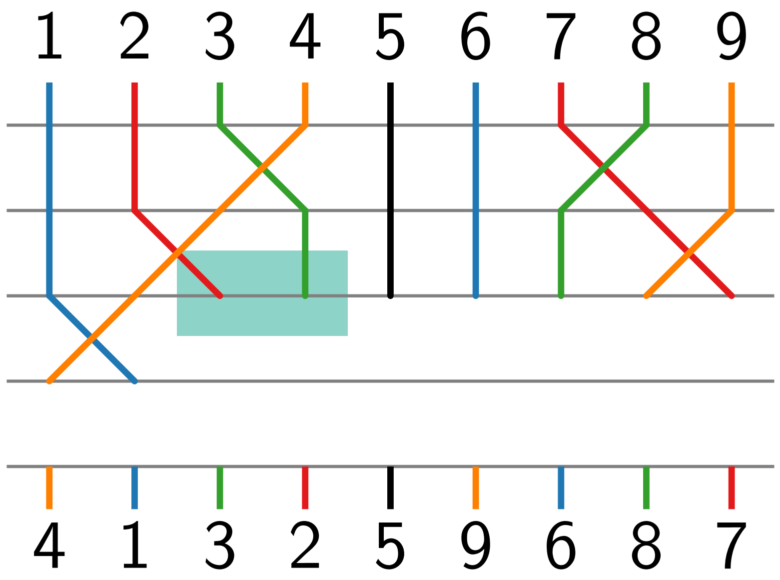

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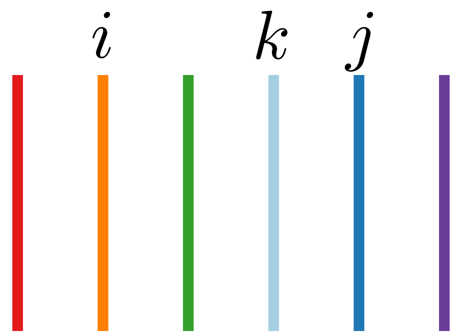

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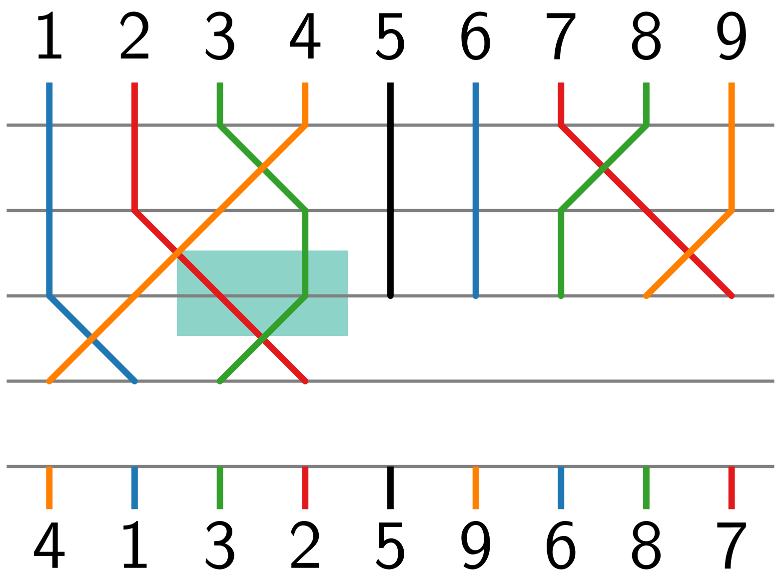

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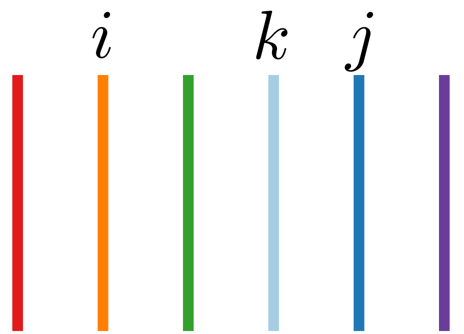

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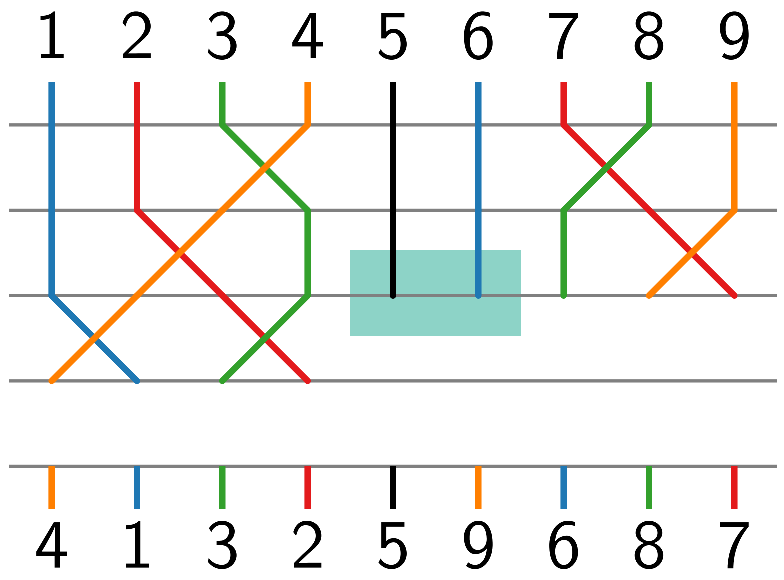

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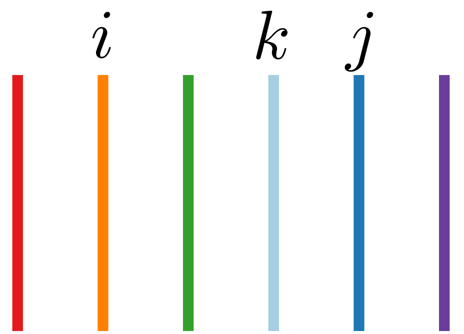

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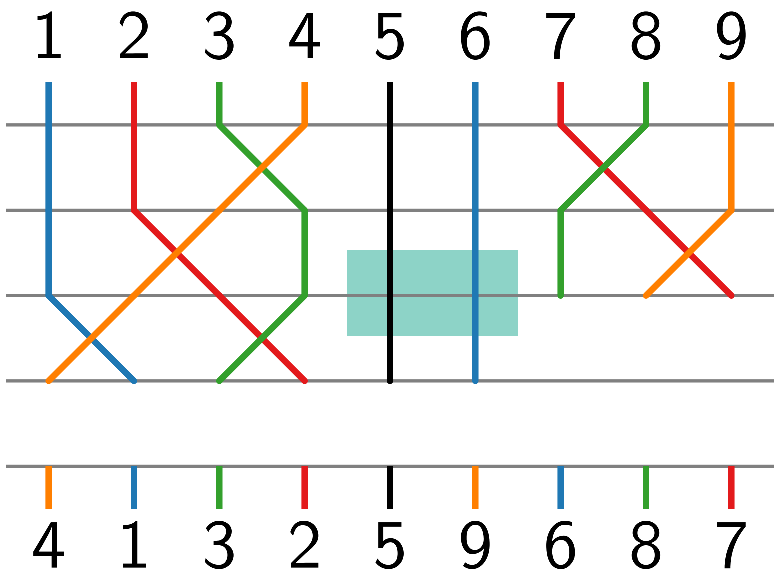

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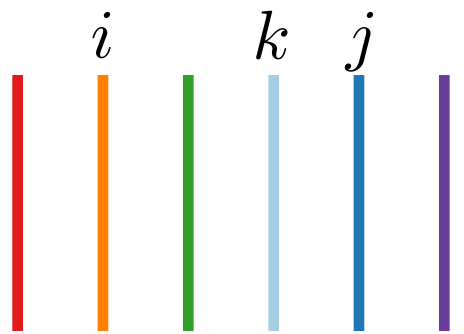

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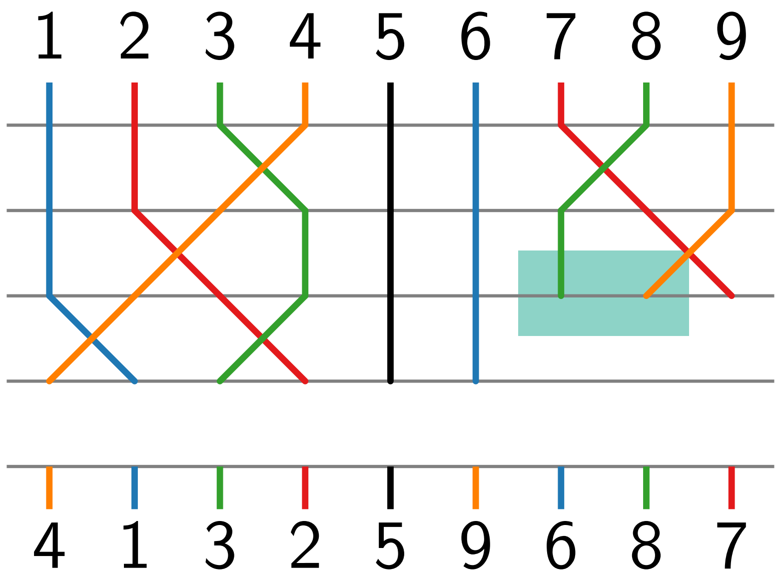

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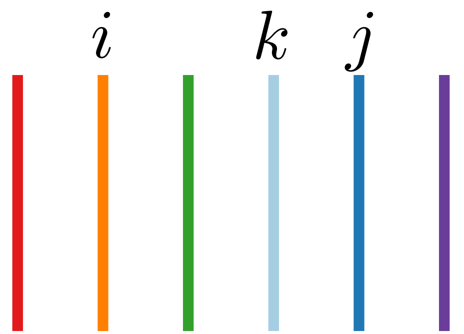

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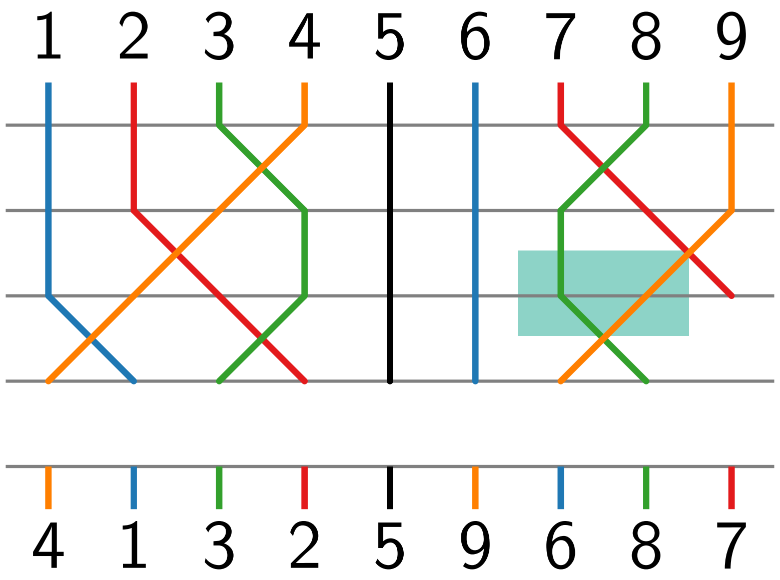

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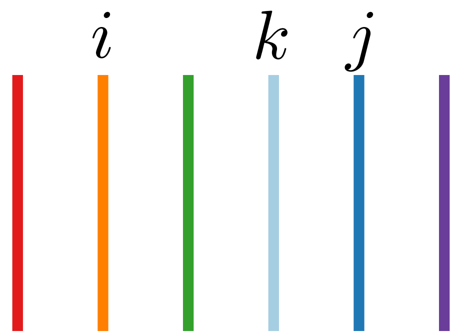

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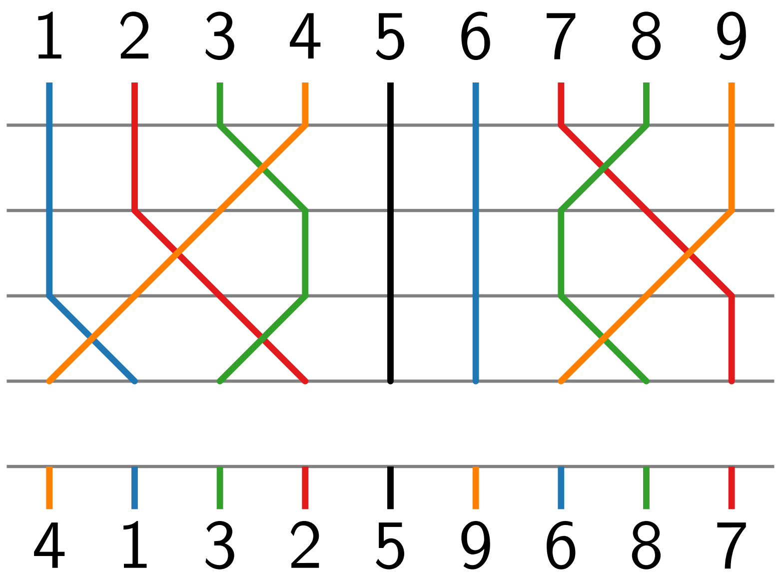

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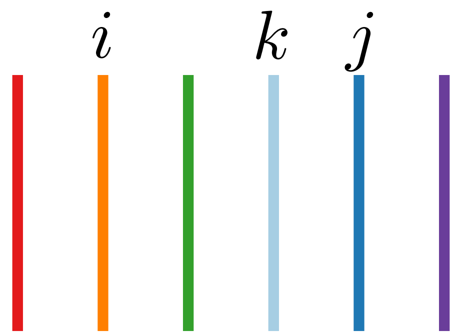

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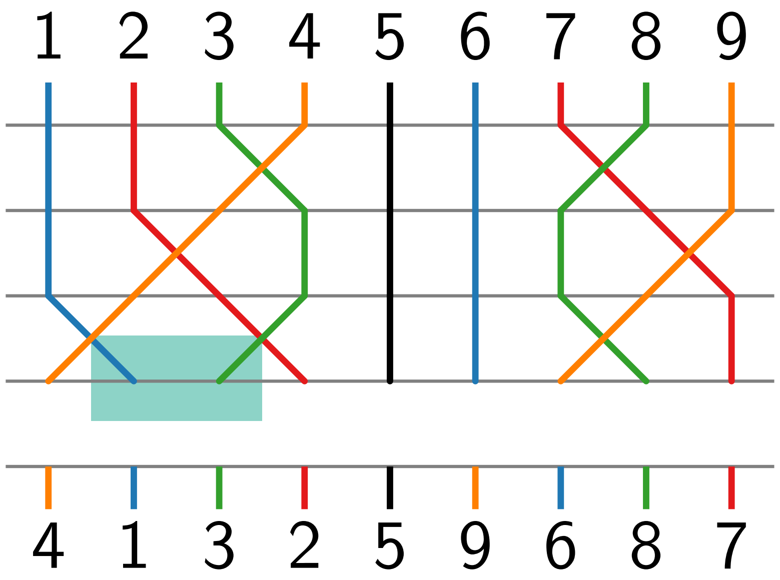

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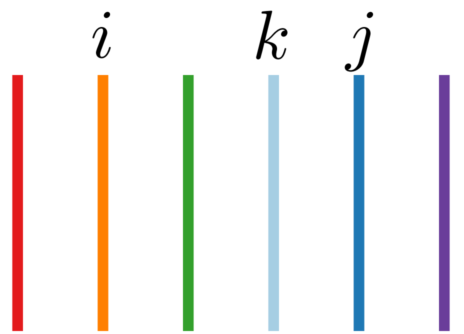

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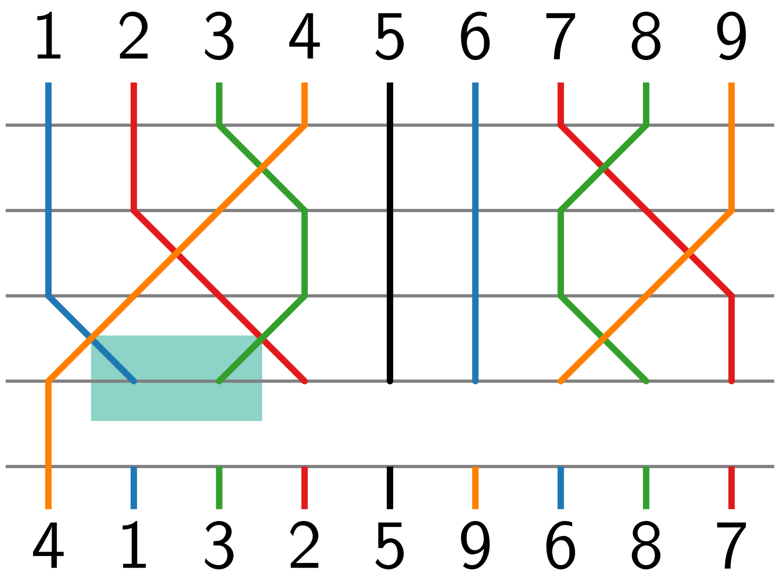

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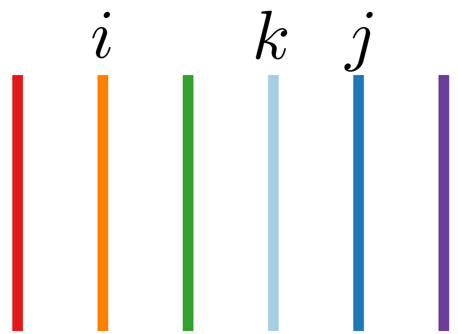

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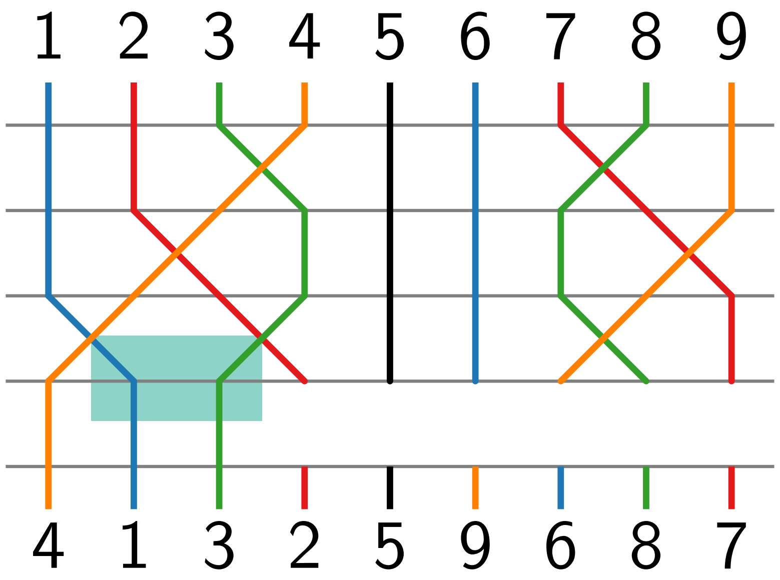

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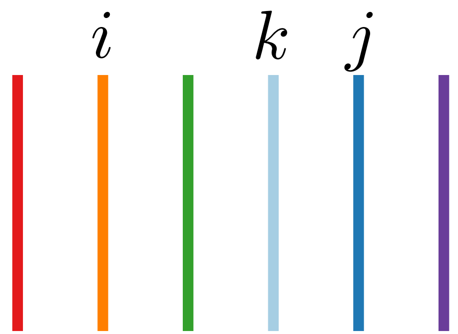

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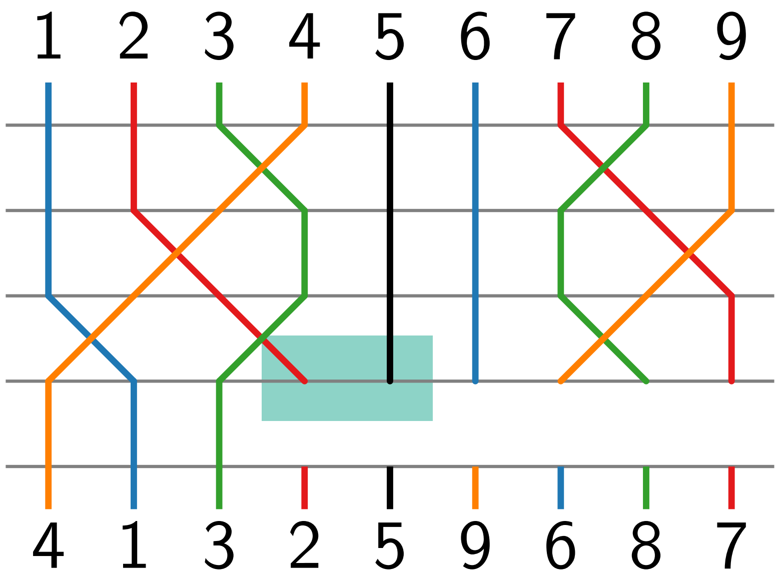

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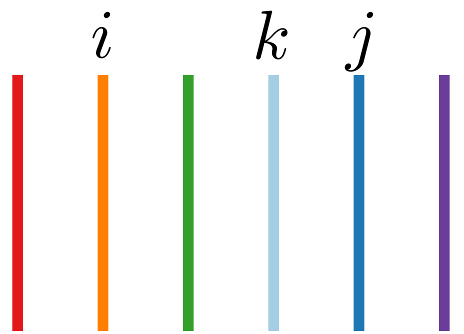

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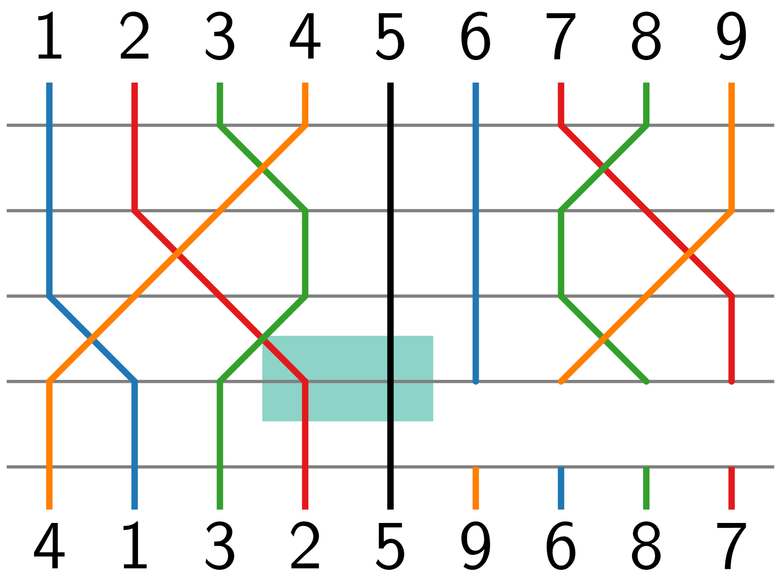

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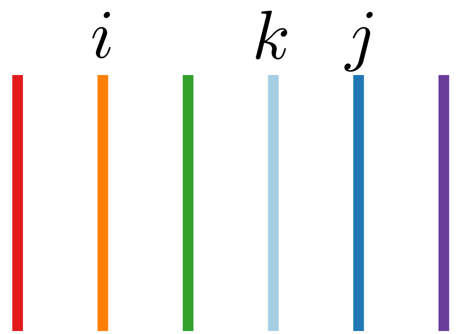

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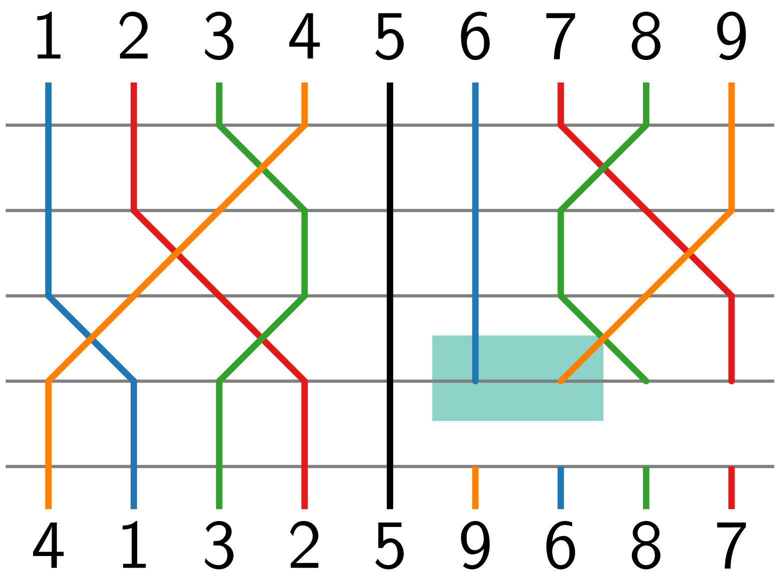

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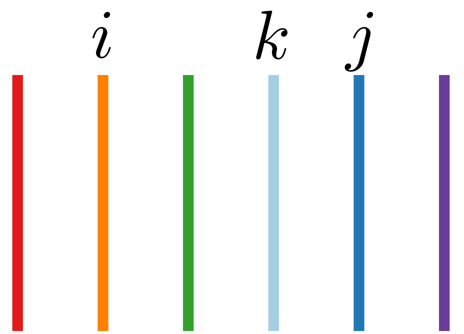

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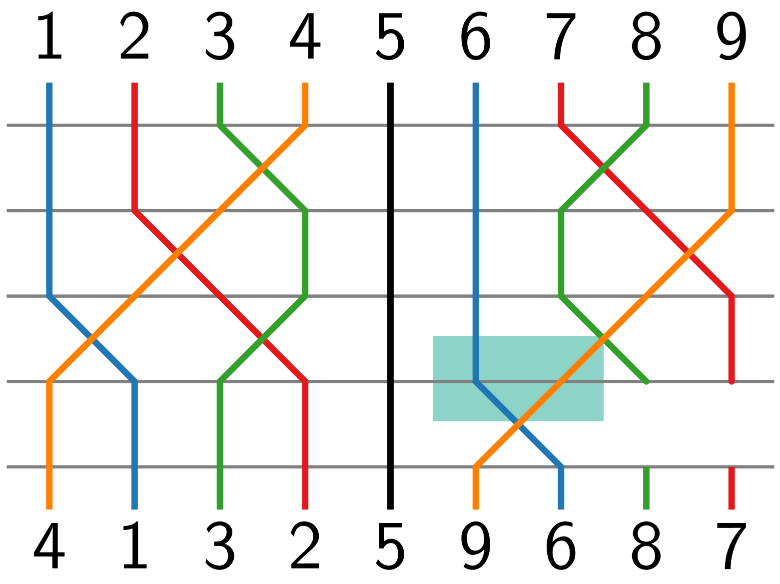

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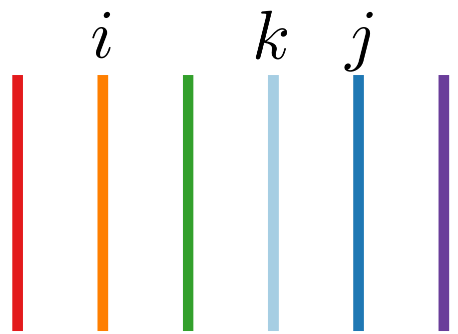

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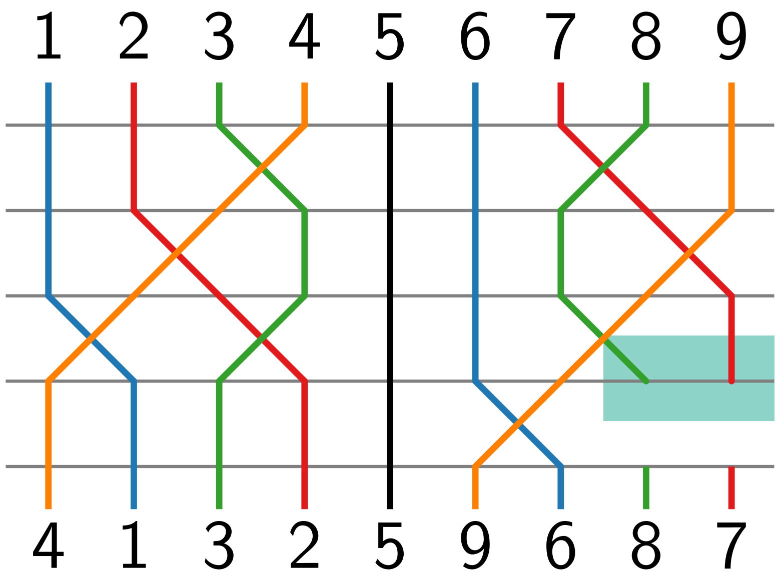

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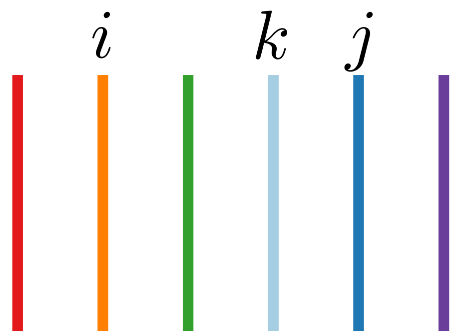

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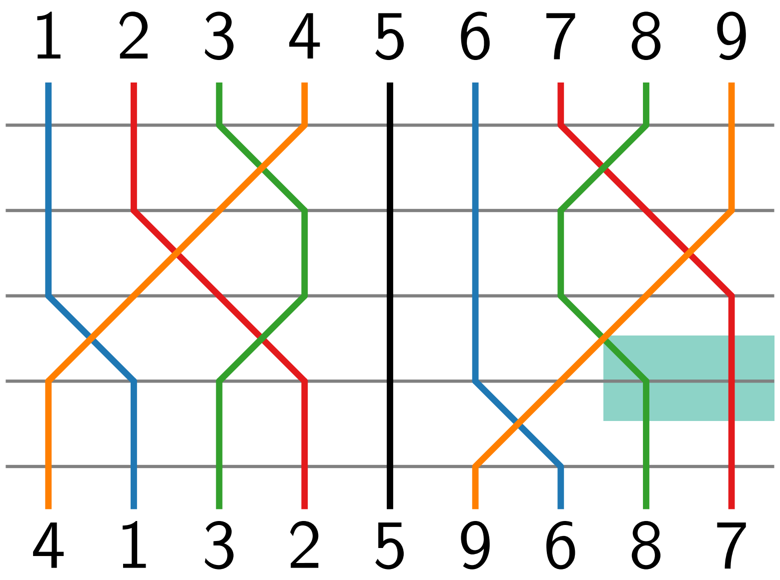
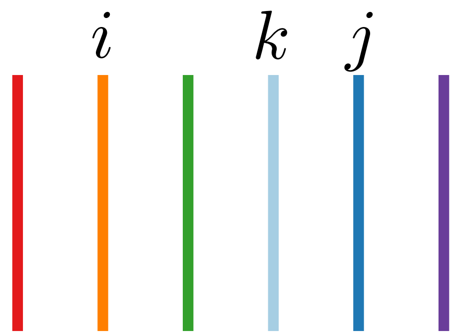
  Can we also always find a tangle of height OPT efficiently?

1–4	6–9
2–3	7–8
2–4	7–9
3–4	8–9



odd-even sort

Open Problems



- Let L be a list where each swap occurs even number of times.
  How difficult is it to check feasibility of L?

Conjecture. [FKWRZ, GD'19]
Every non-separable even list L is feasible.

No!

A list (ℓ_{ij}) is *non-separable*
if $\forall i{<}k{<}j$: $\big(\ell_{ik} = \ell_{kj} = 0$ implies $\ell_{ij} = 0\big)$.

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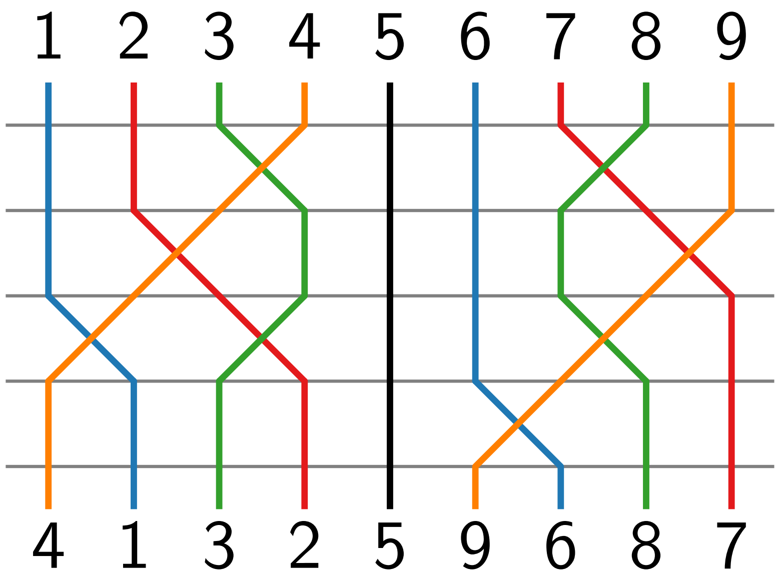

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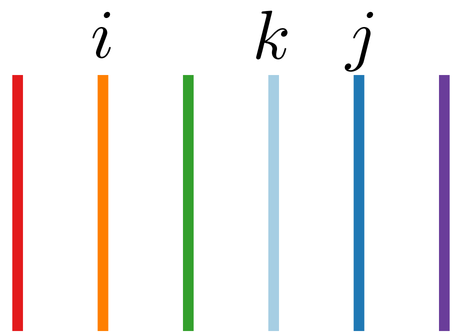

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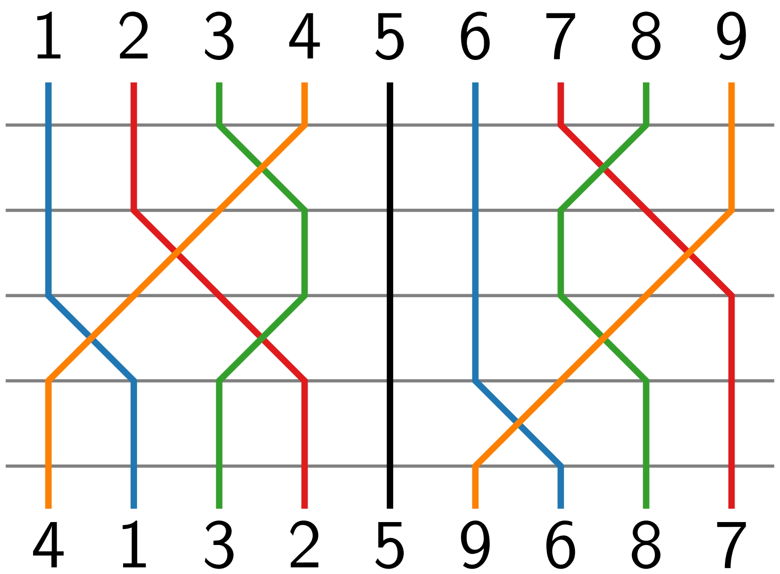
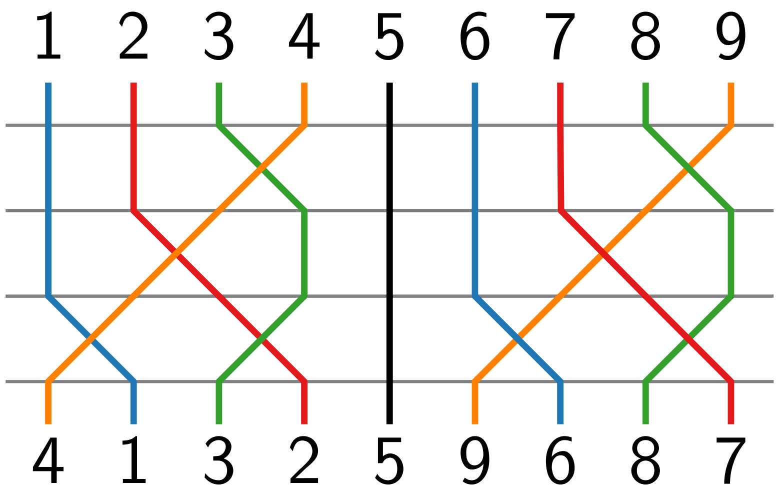


odd-even sort

OPT



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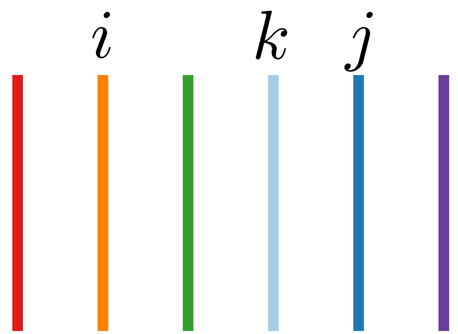

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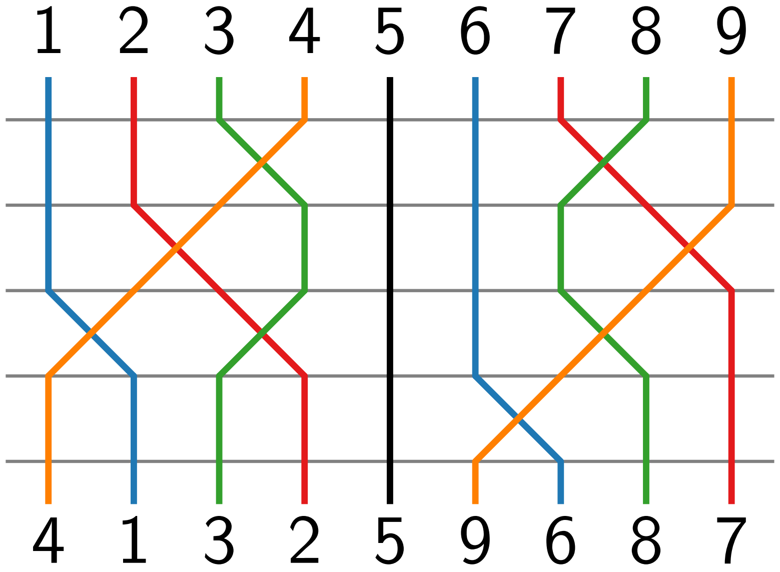

Thank you!

OPT