Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Bachelor Colloquium

Rectangular Representation of Weighted Outerplanar Graphs

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October 01, 2014

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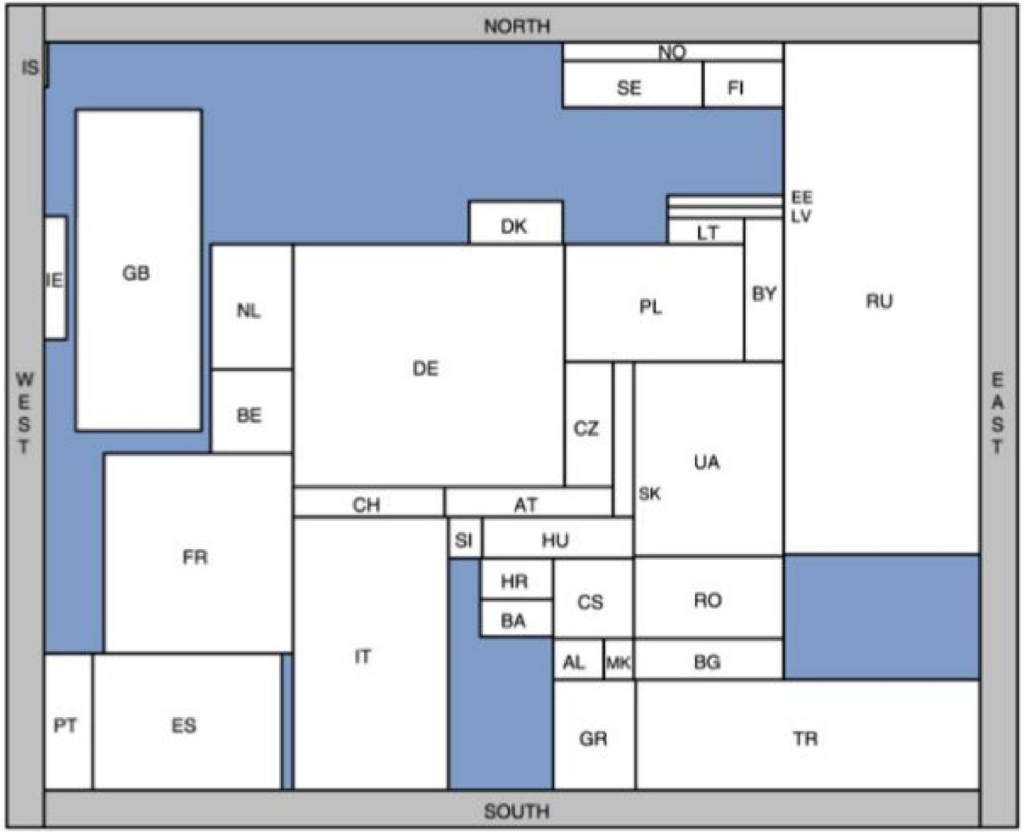
Philipp Kindermann & Alexander Wolff
Chair of Computer Science I
Universität Würzburg

Motivation

Rectangular Cartograms:

Representation of maps where every region is represented by a rectangle.



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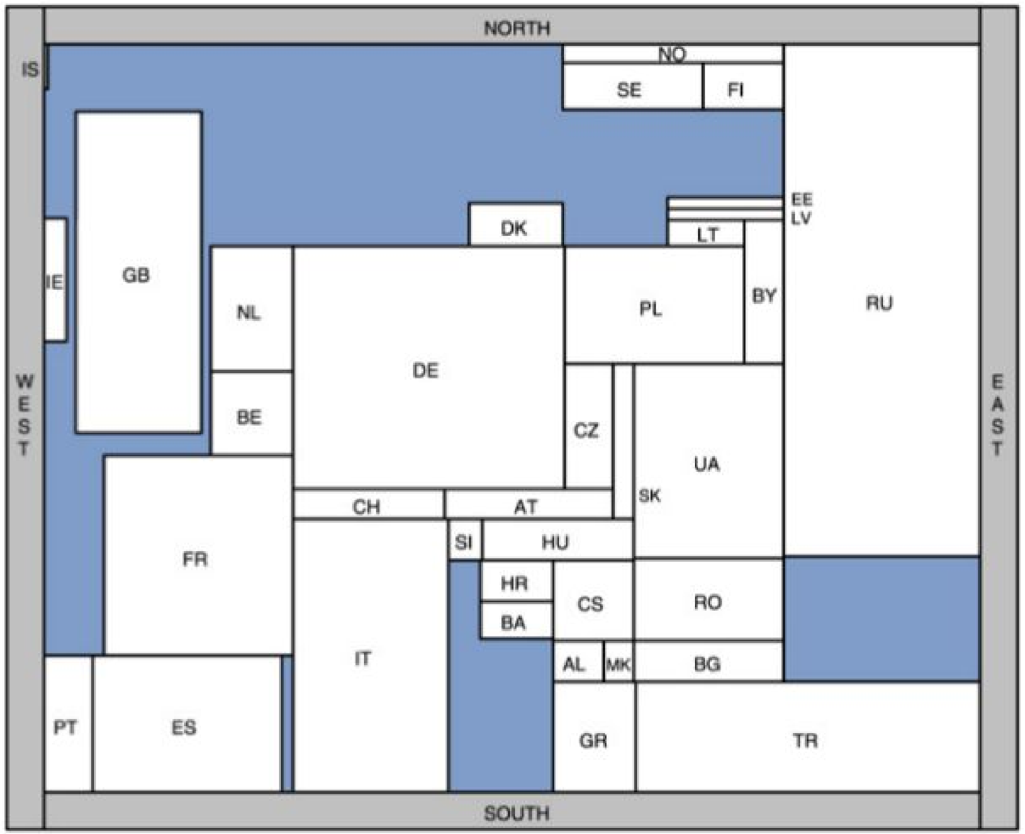

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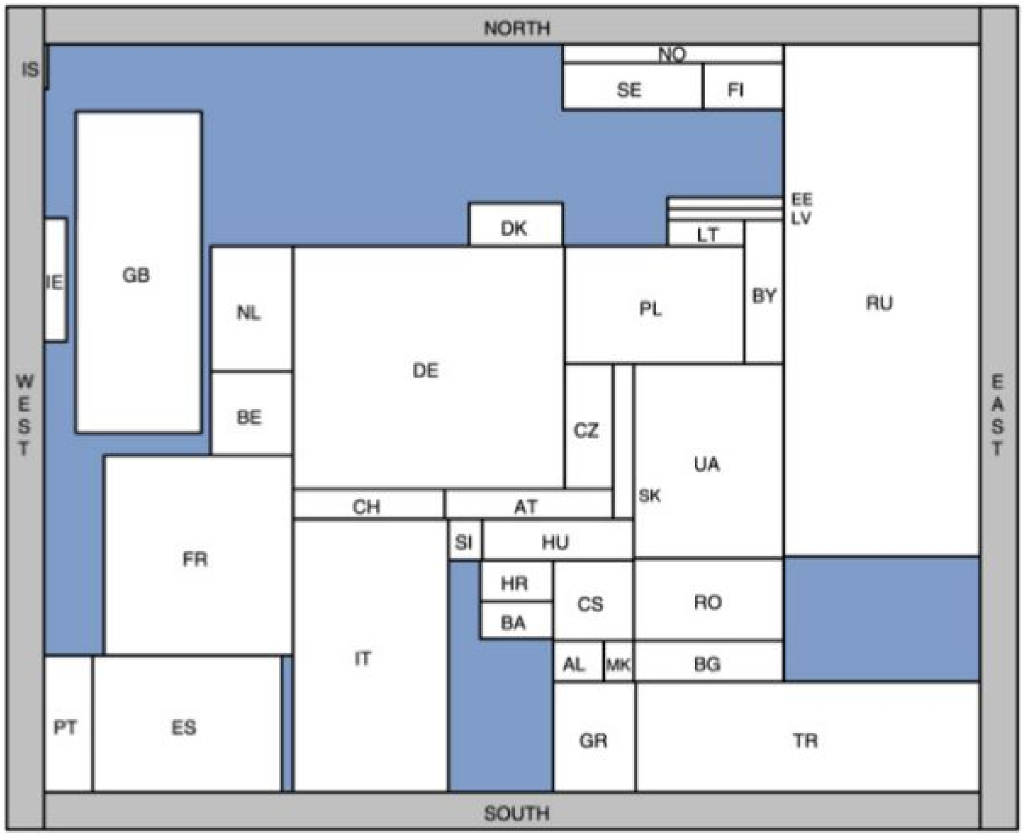

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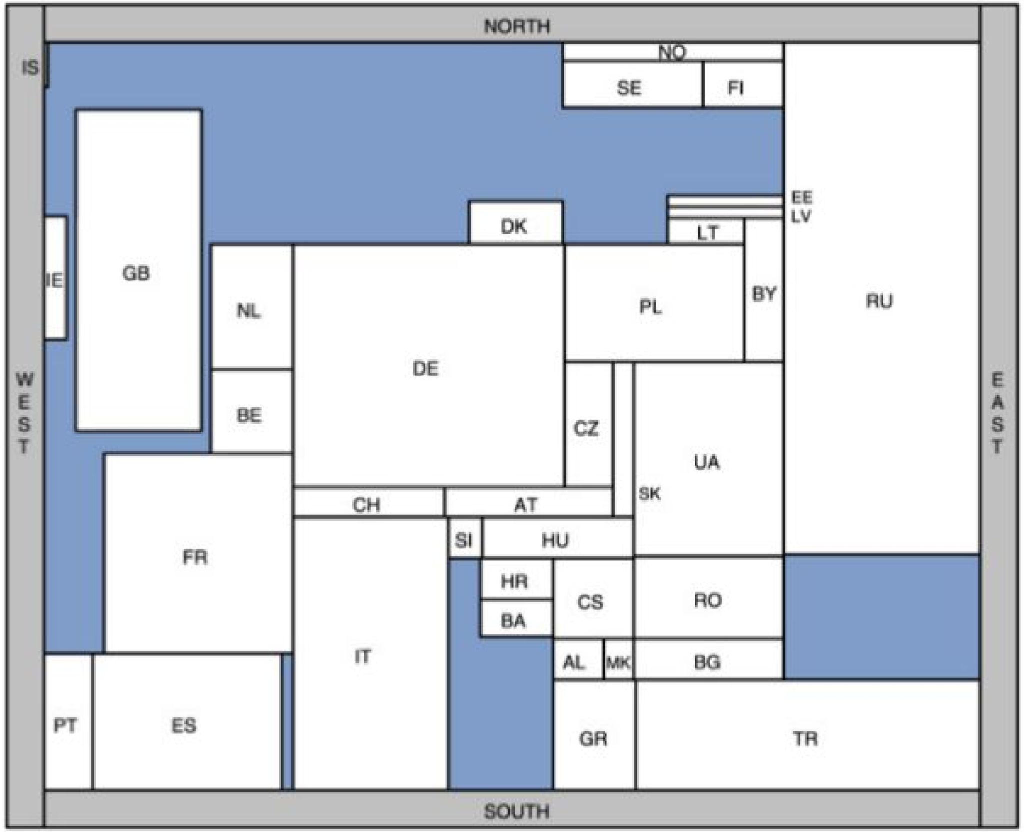

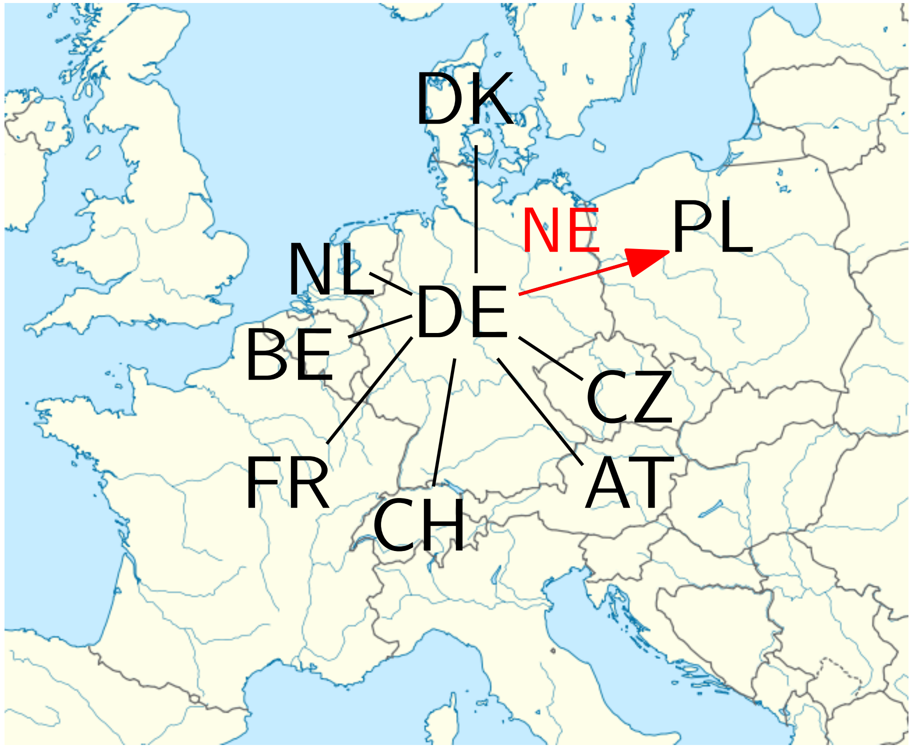

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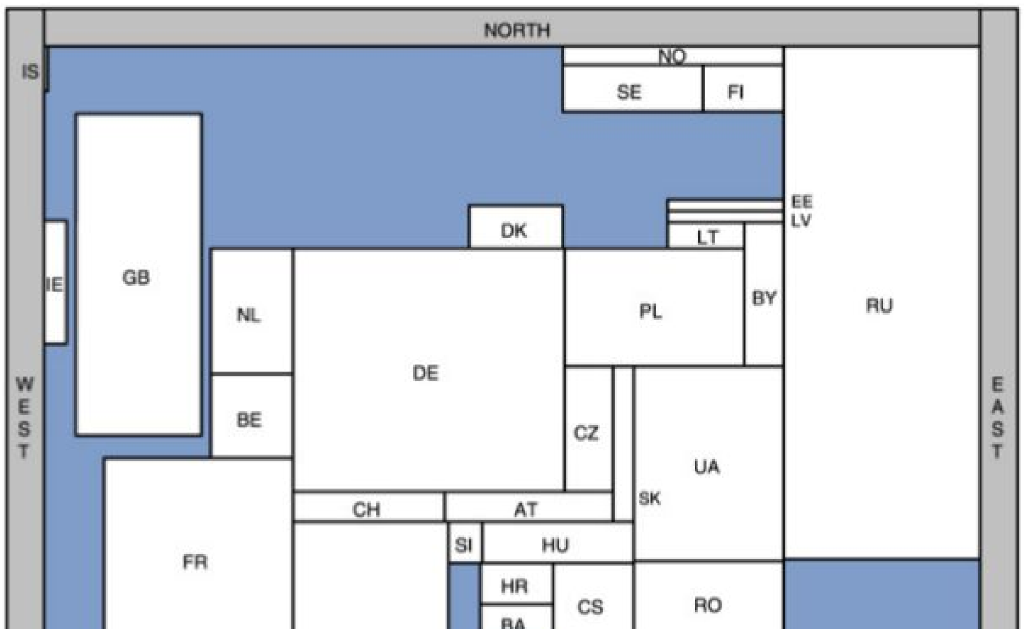

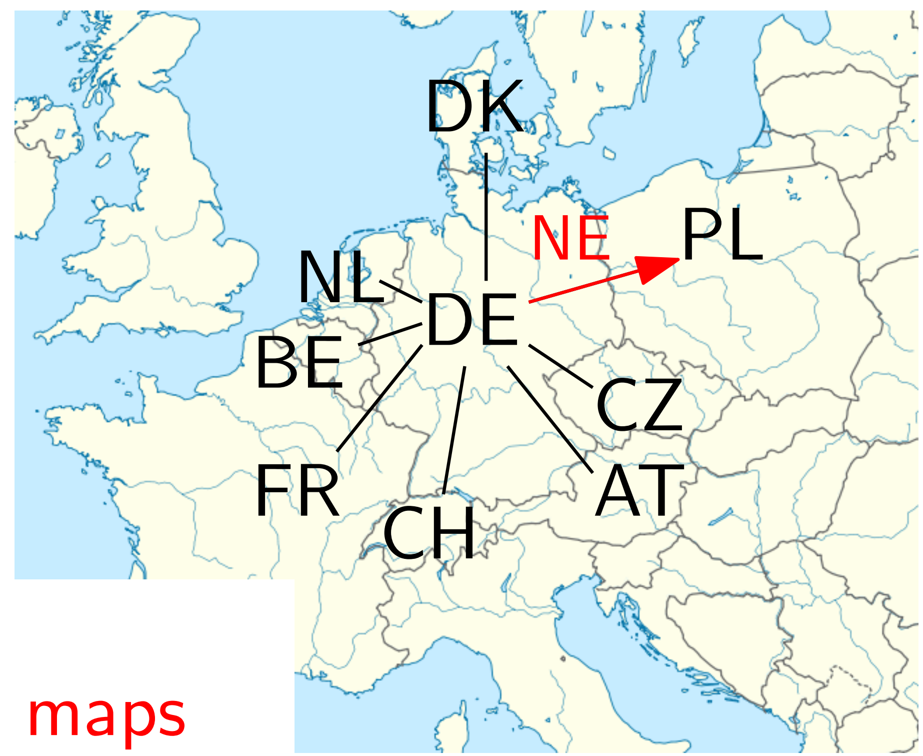

Problems:

- wrong representation of some maps
- graphs without geographic information

Targets

- draw the rectangles with the desired size

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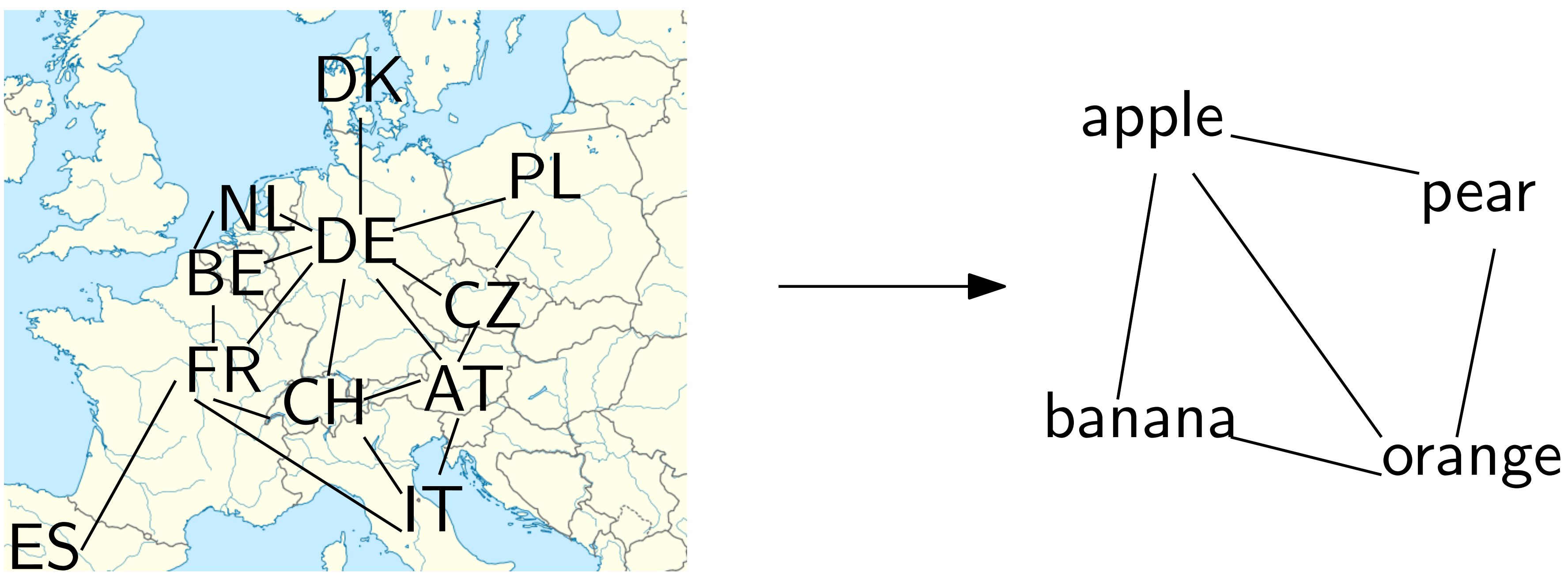
- extension to general graphs



Targets

- draw the rectangles with the desired size
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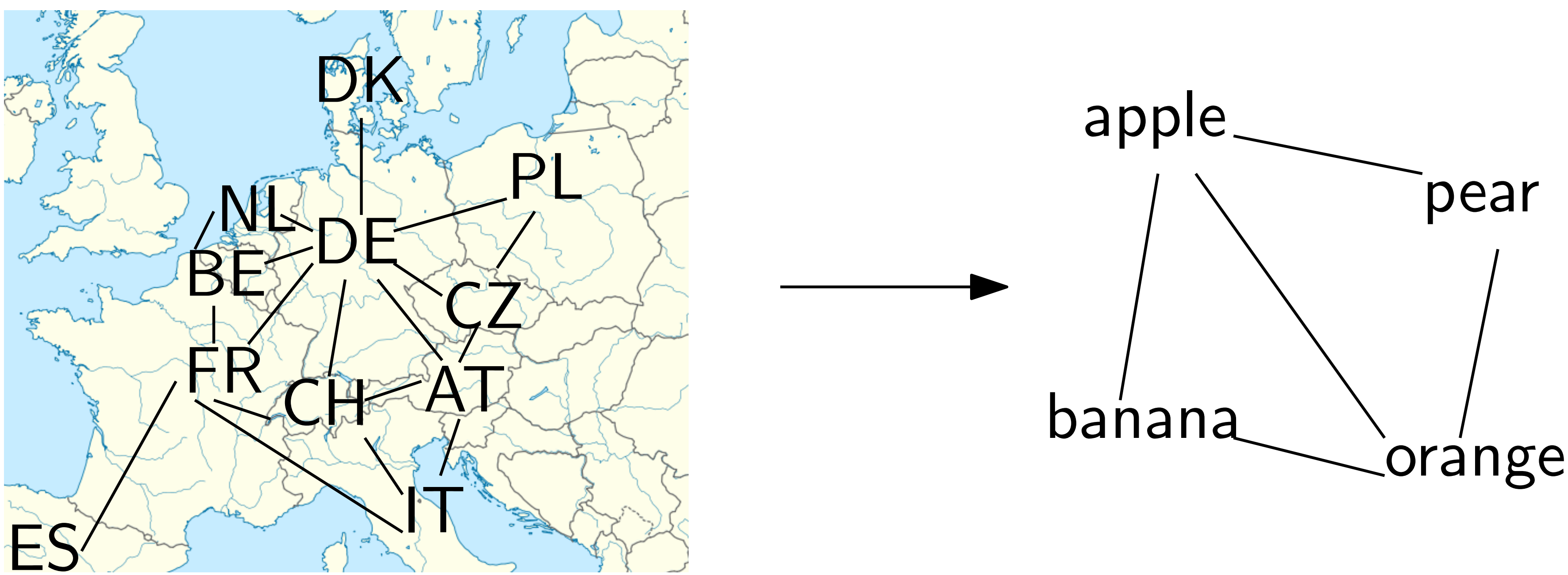
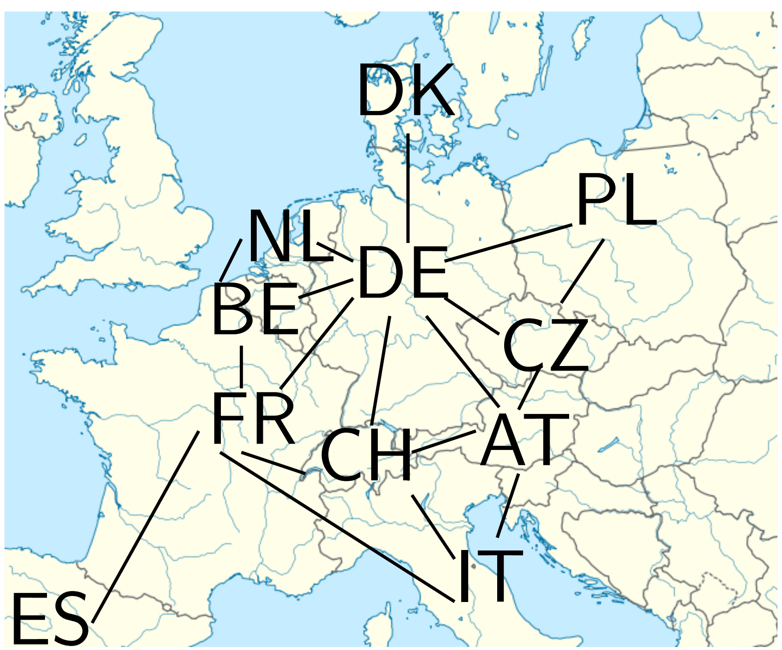
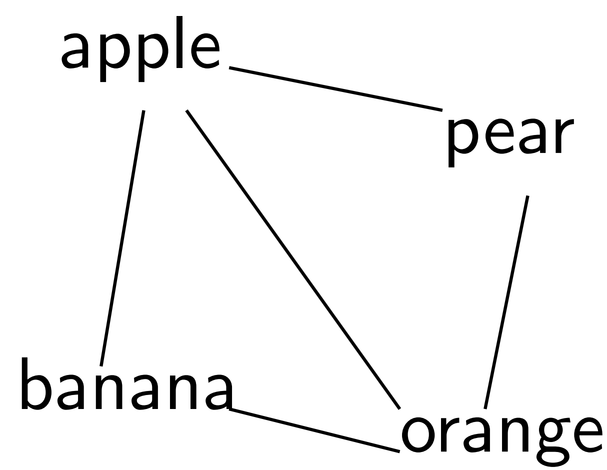
- no need for a surrounding rectangle
- extension to general graphs?



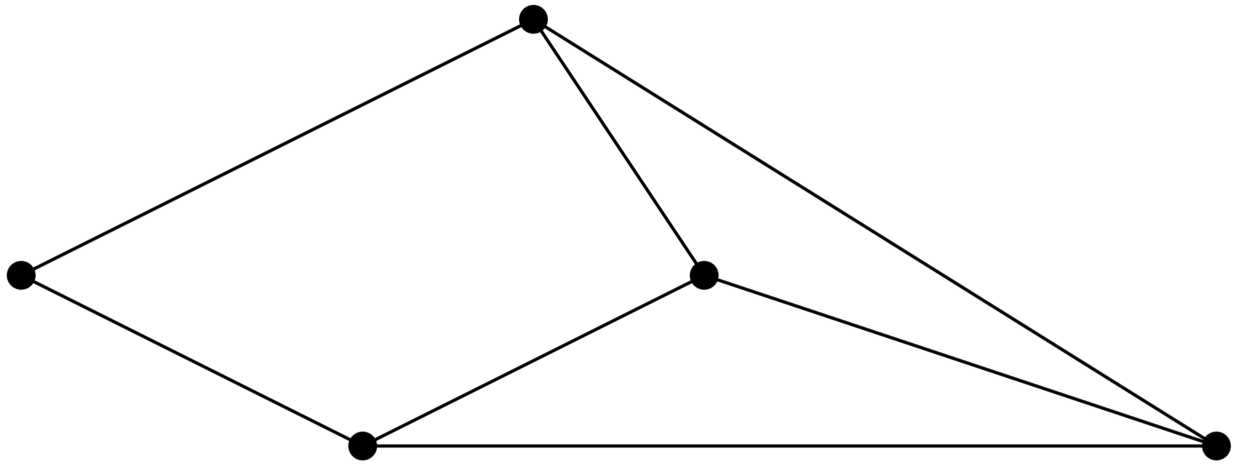
Targets

- draw the rectangles with the desired size
- every rectangular representation should be possible to draw
- no need for a surrounding rectangle
- extension to general graph classes!



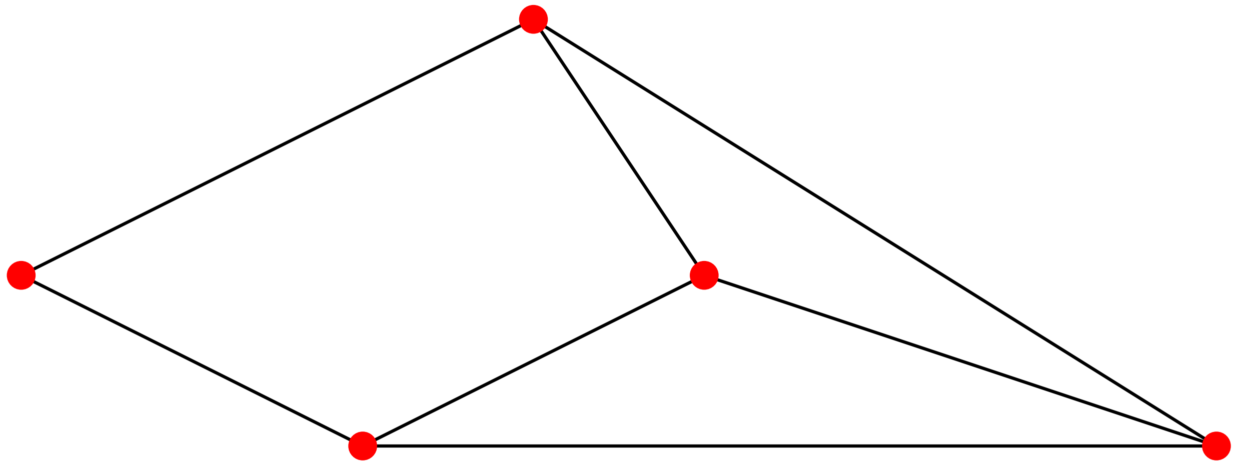


Basic Graphs



Graph $G = (V, E)$

Basic Graphs



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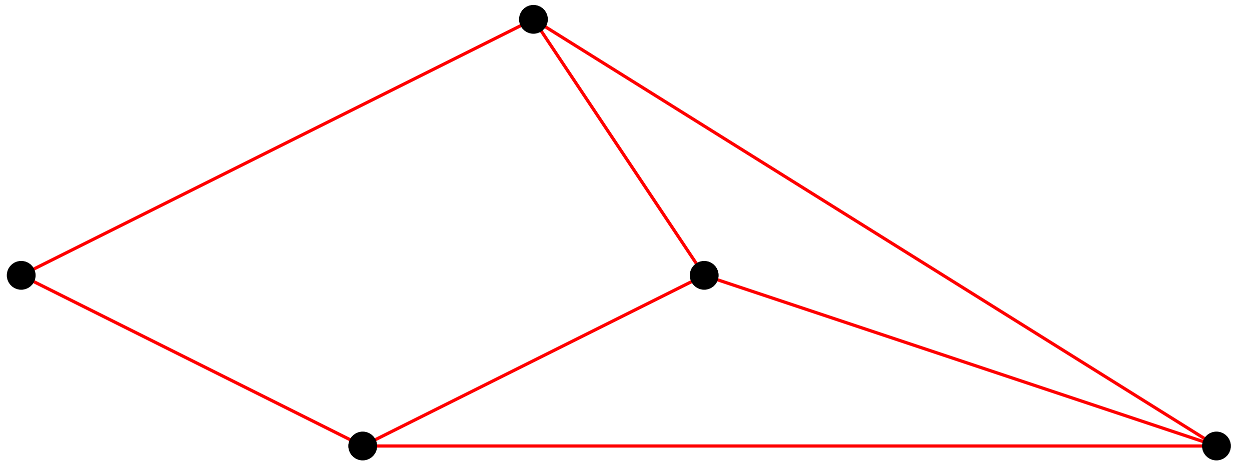

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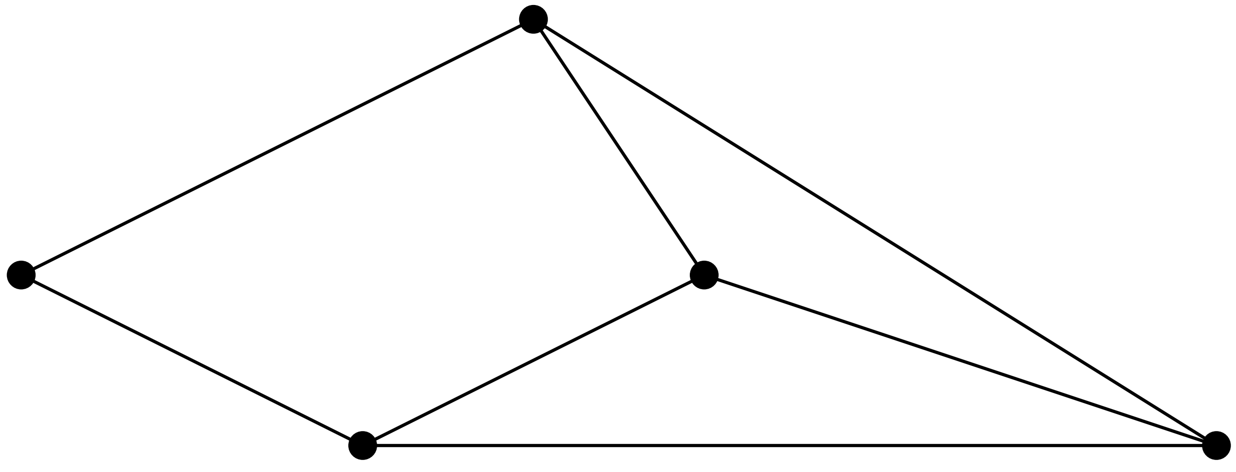
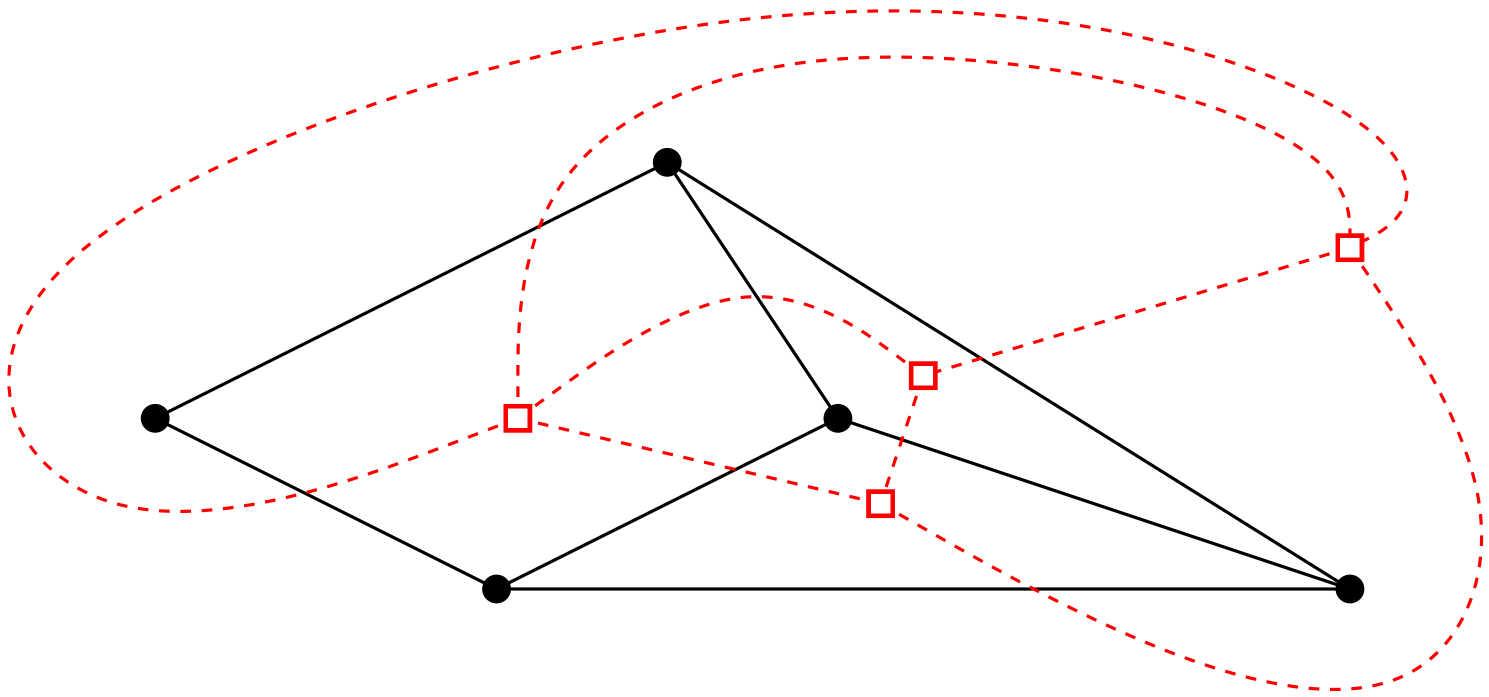


A *Planar Graph* $G = (V, E)$ can be drawn in such a way that no edges cross each other

Basic Graphs

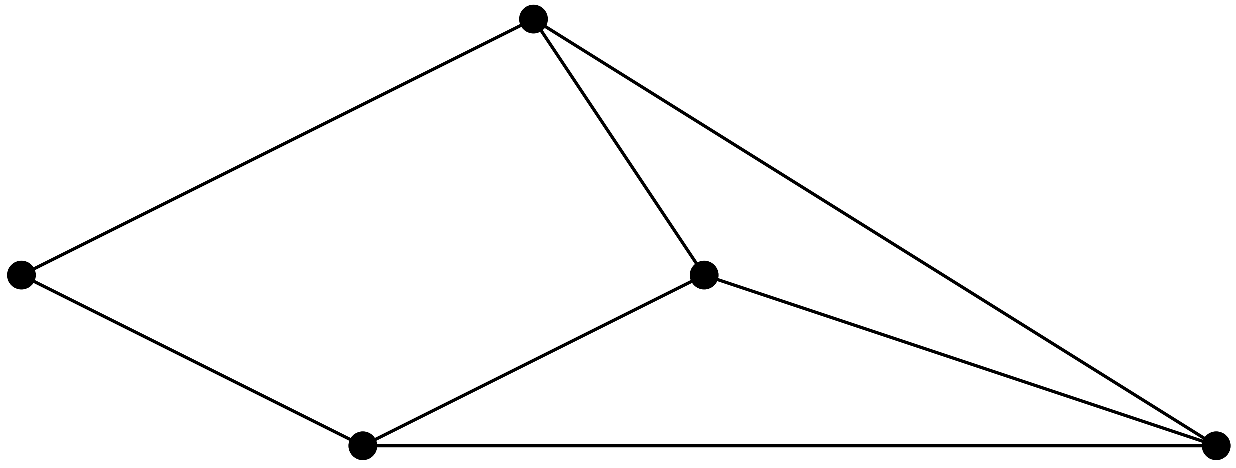


A *Planar Graph* $G = (V, E)$ can be drawn in such a way that no edges cross each other

Dual Graph G*=(V*,E*):

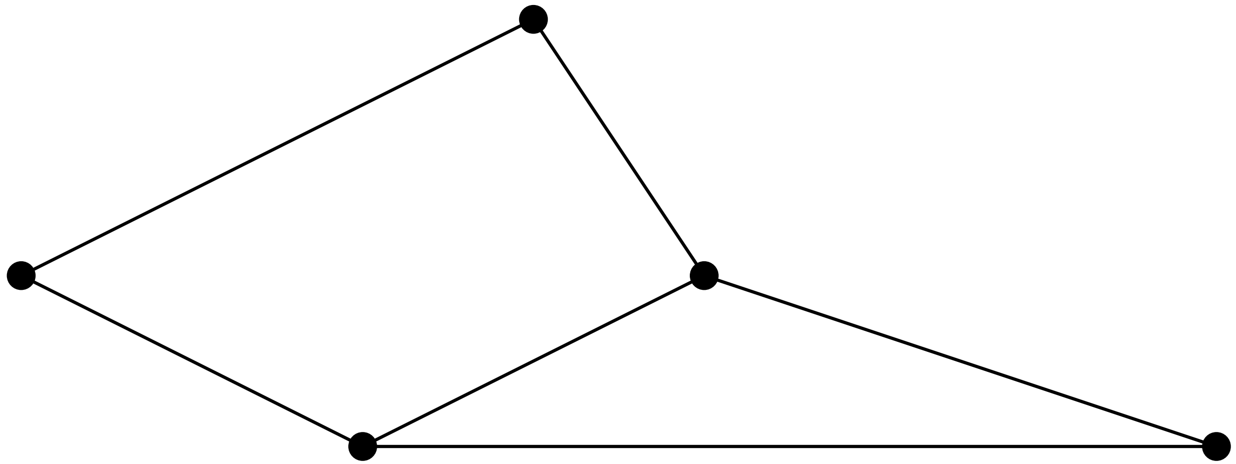
- a vertex corresponding to each face of G
- an edge joining two neighboring faces for each edge in G

Maximal Outerplanar Graphs



Outerplanar graph: all vertices belong to the unbounded face

Maximal Outerplanar Graphs



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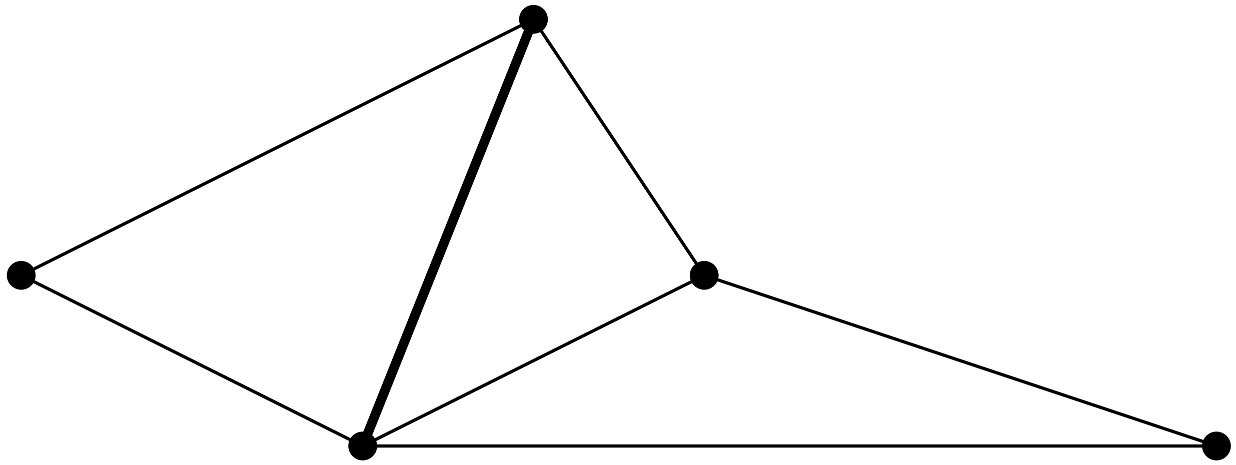
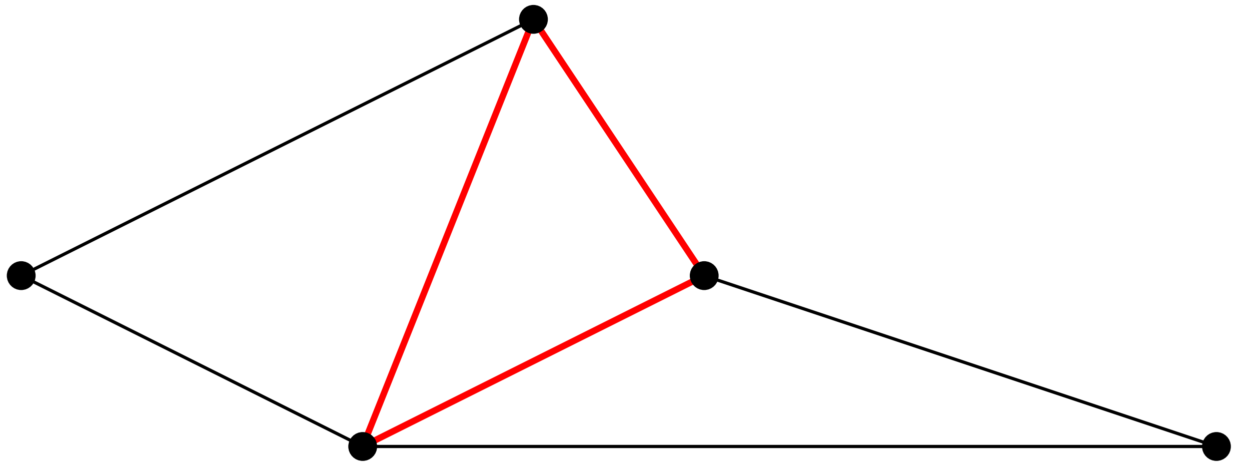


Outerplanar graph: all vertices belong to the unbounded face

Maximal outerplanar Graph: cannot have any additional edges while preserving outerplanarity

Maximal Outerplanar Graphs



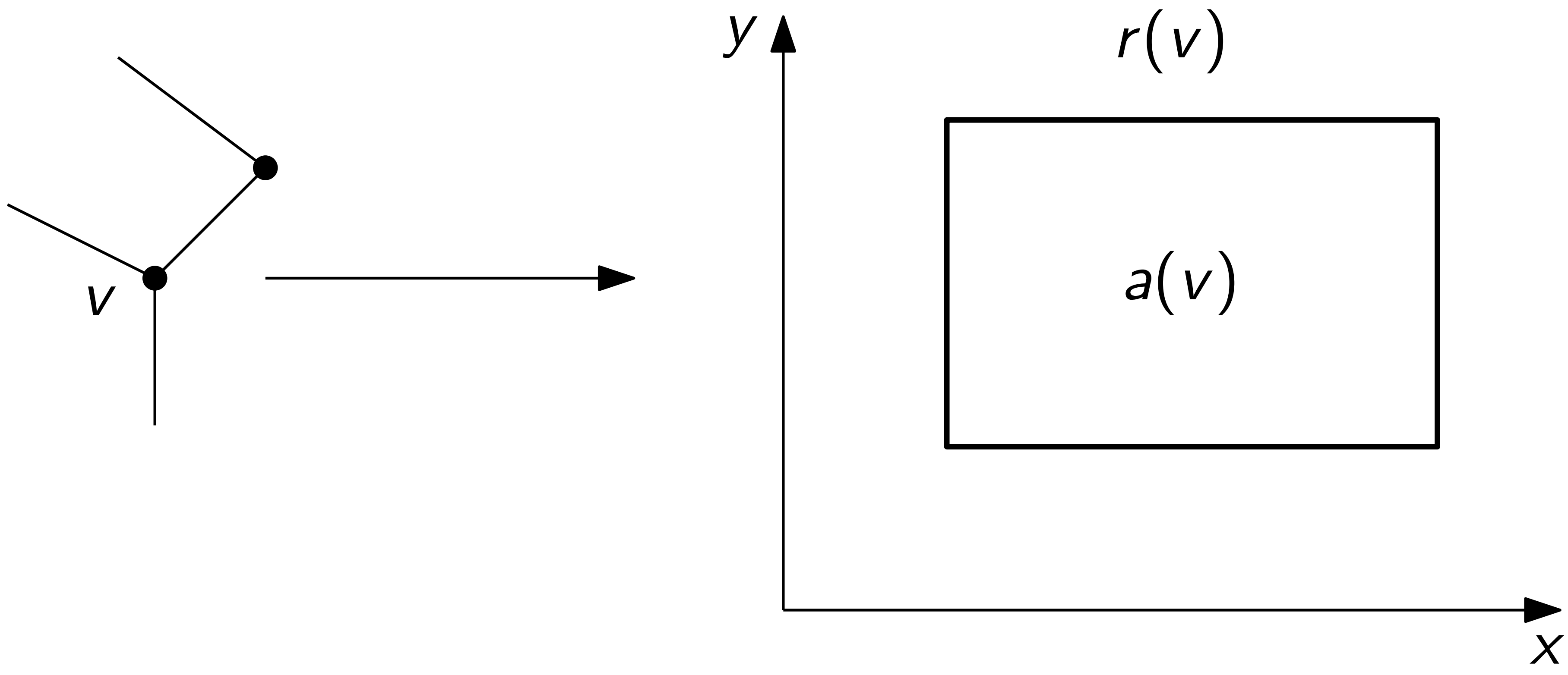
Outerplanar graph: all vertices belong to the unbounded face

Maximal outerplanar Graph: cannot have any additional edges while preserving outerplanarity

Every surface is surrounded by a triangle!

Rectangles

Rectangular representation



A vertex v is corresponding to a rectangle $r(v)$ with size $a(v)$

Rectangles

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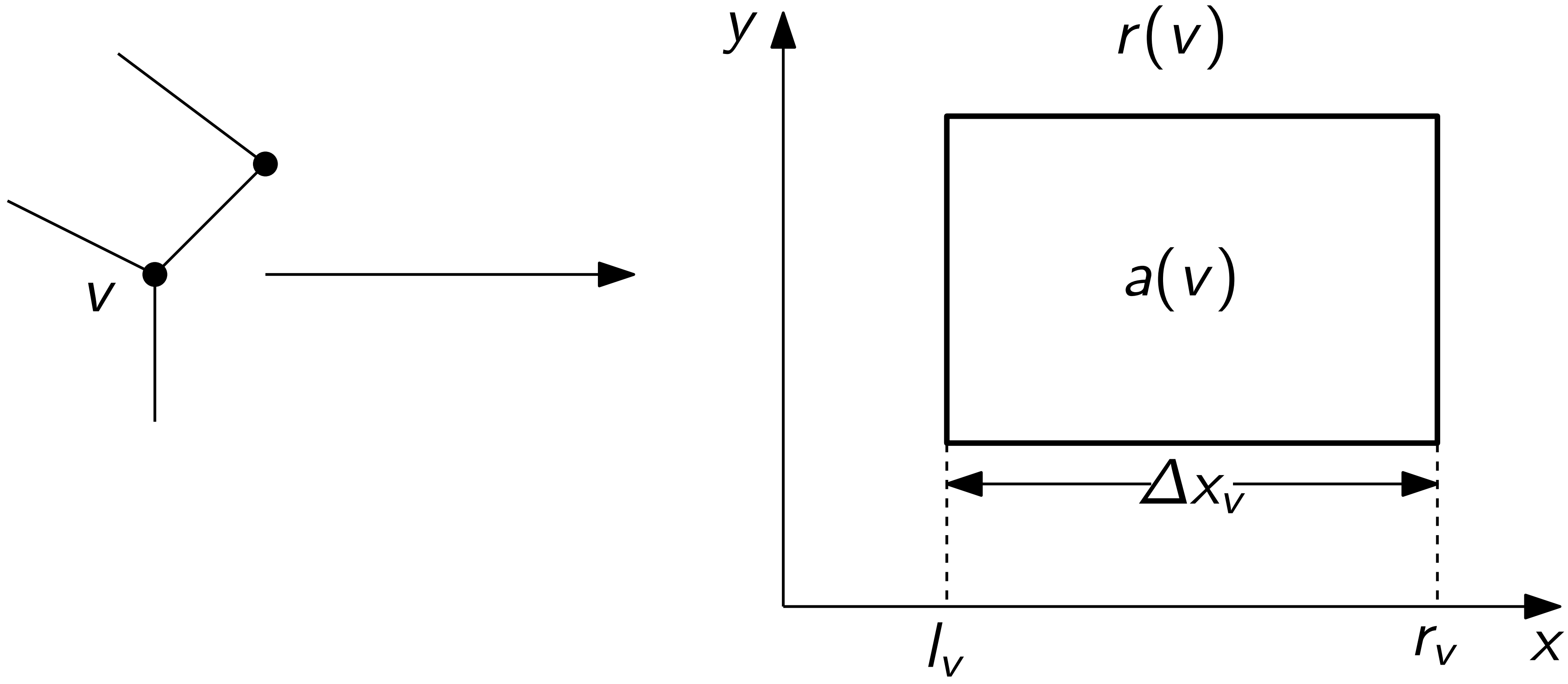

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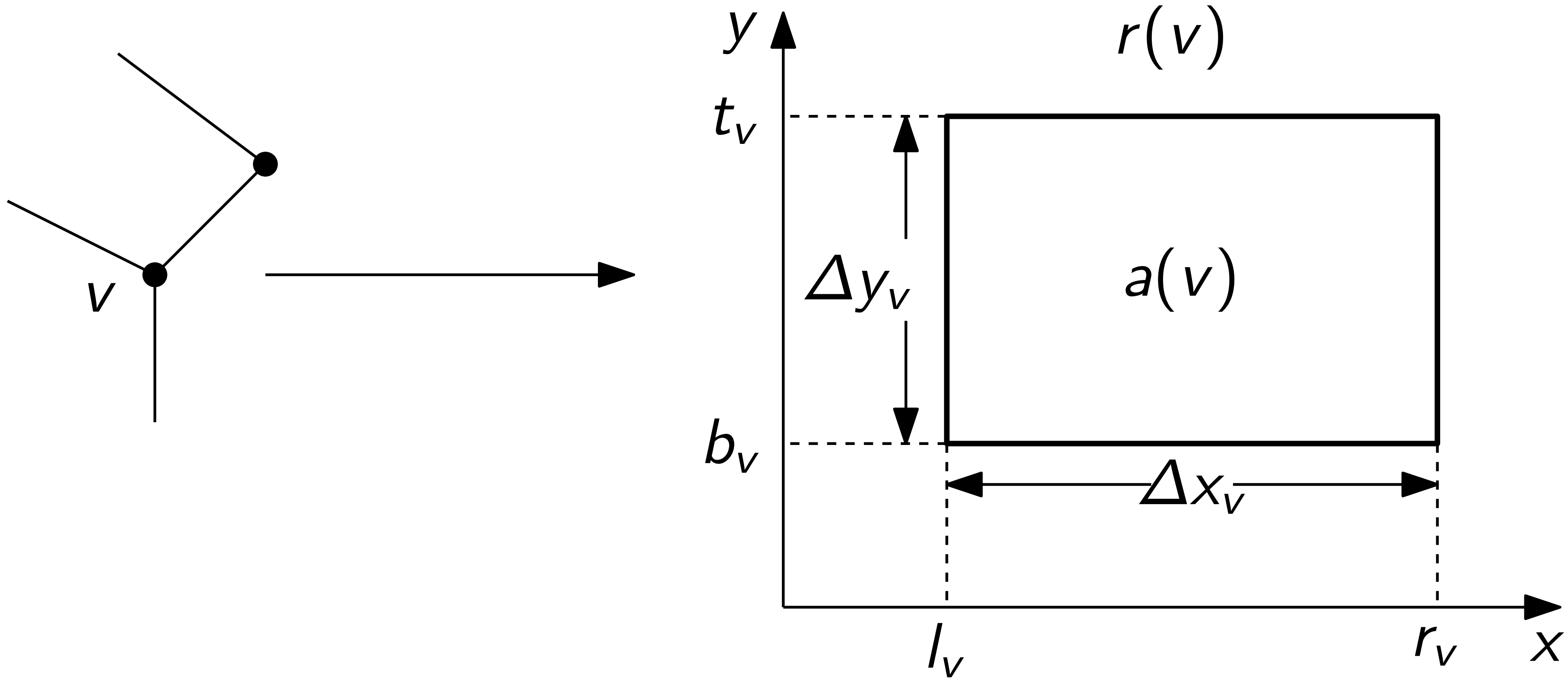

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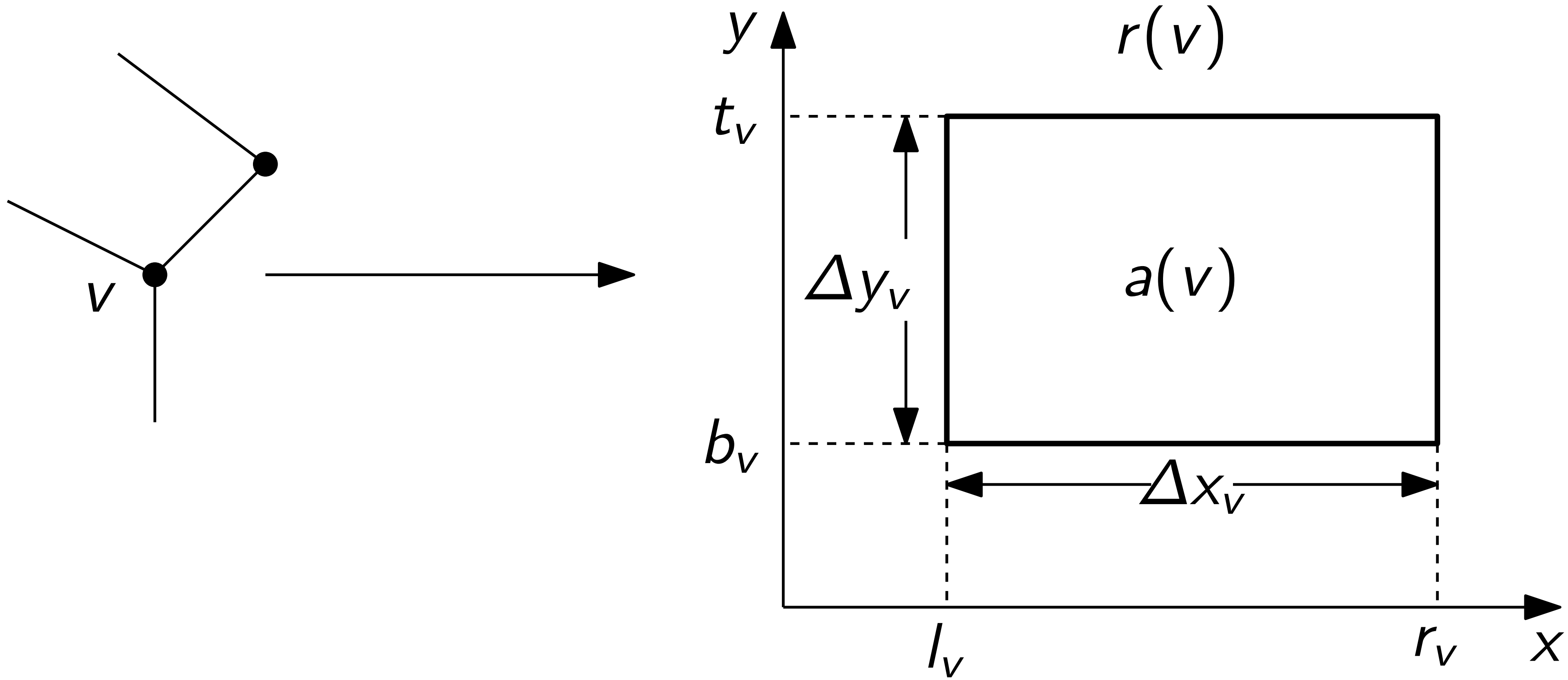
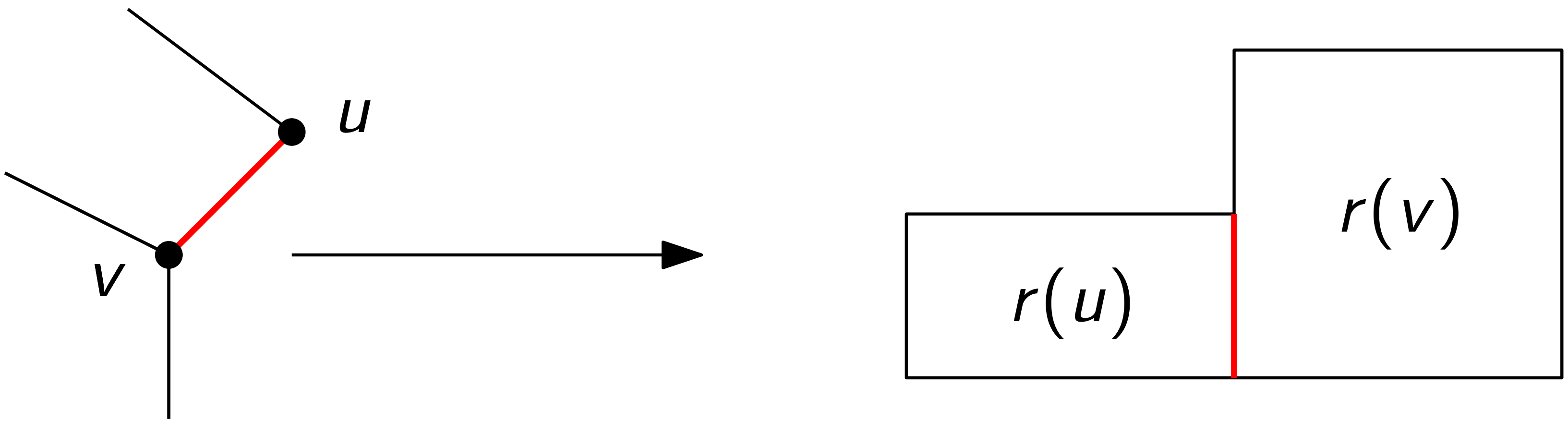


A vertex v is corresponding to a rectangle $r(v)$ with size $a(v)$

$$v \in V \xrightarrow{r : V \to R} r \in R \qquad\qquad v \in V \xrightarrow{a : V \to A} a \in A$$

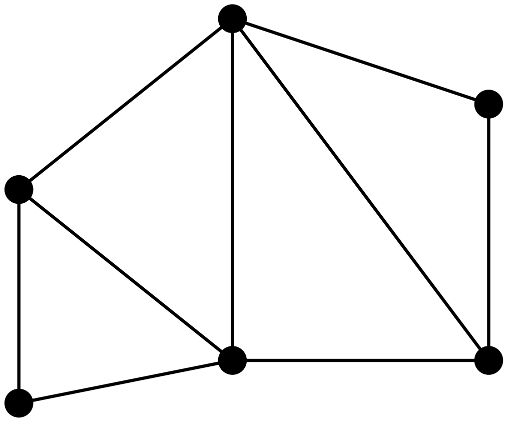
Rectangles

Rectangular representation



$u, v \in V$ and $uv \in E \rightarrow r(u)$ and $r(v)$ are adjacent

Faces and Corners



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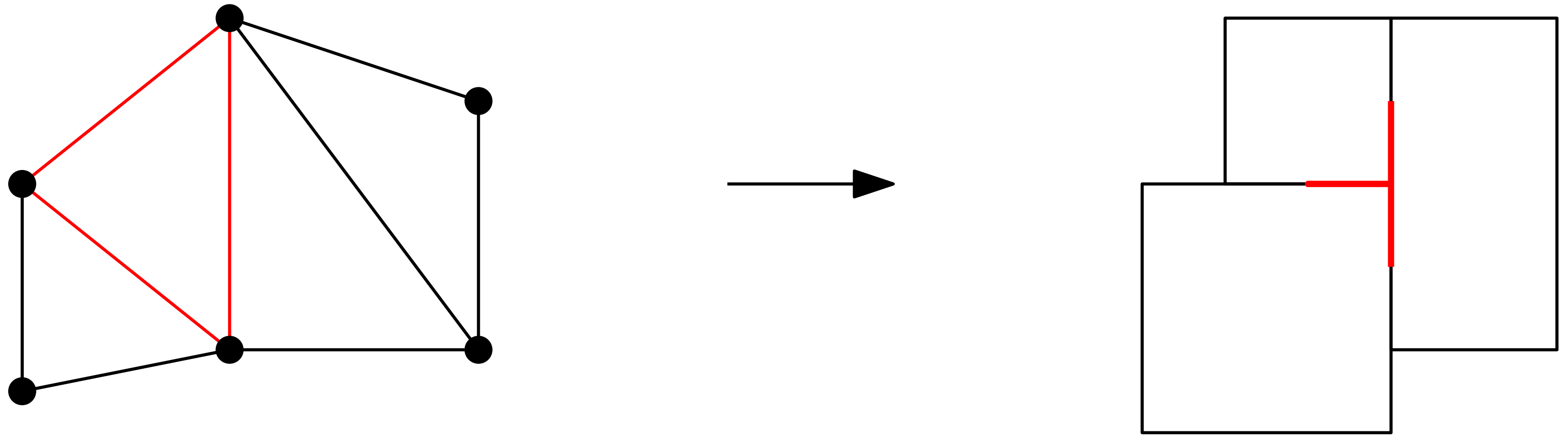

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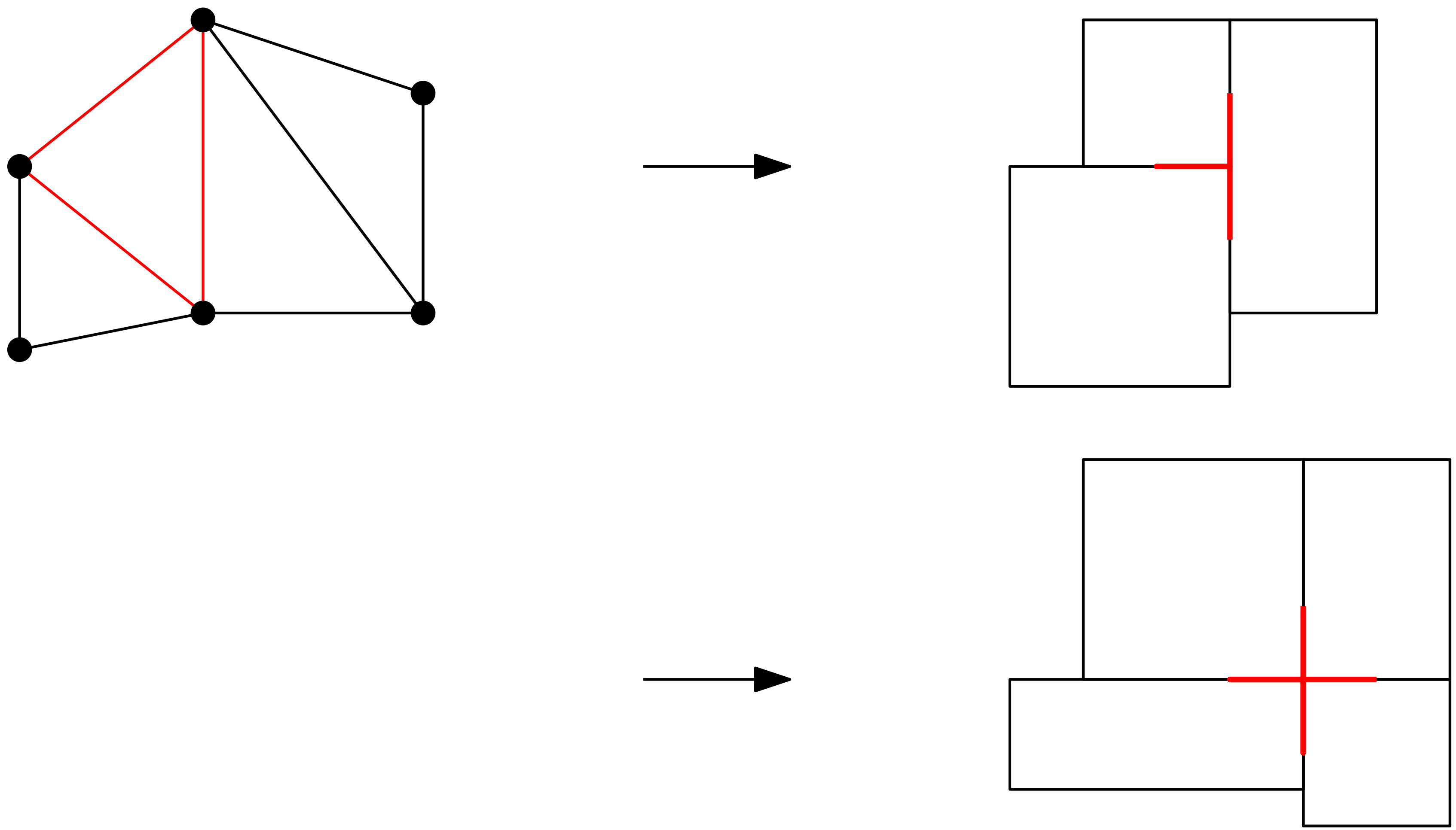

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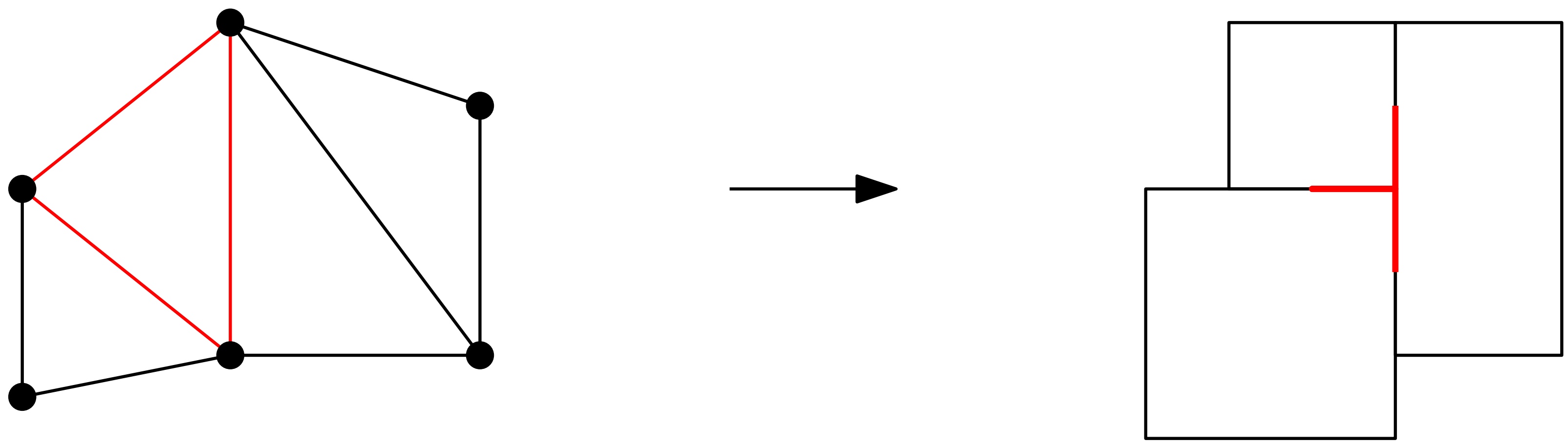
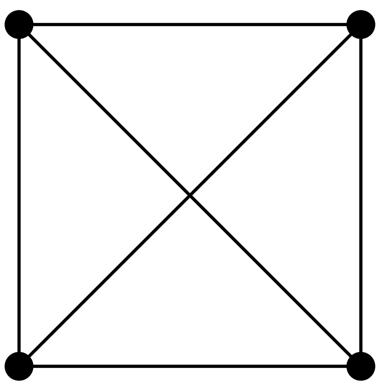
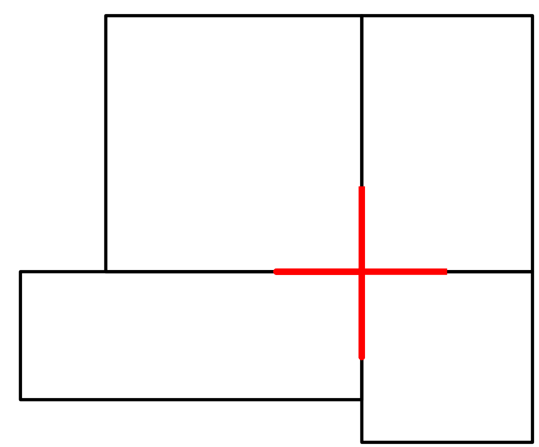




Not planar!



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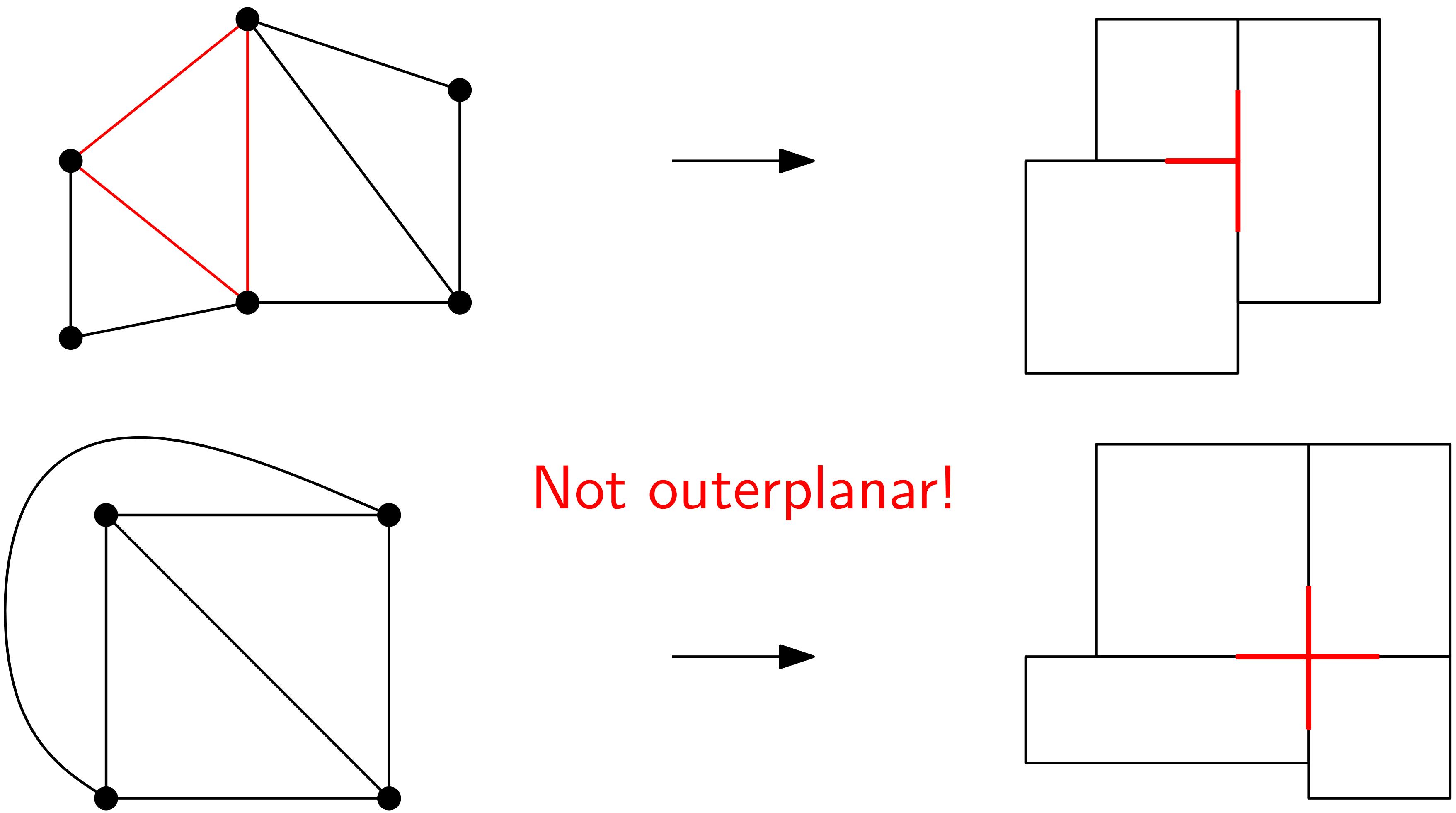

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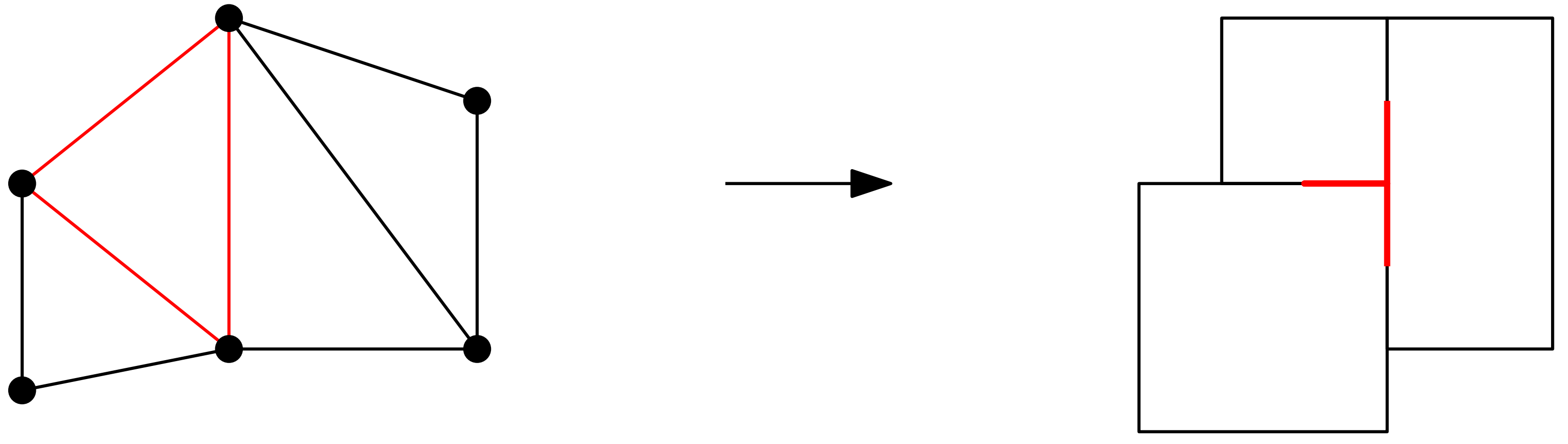
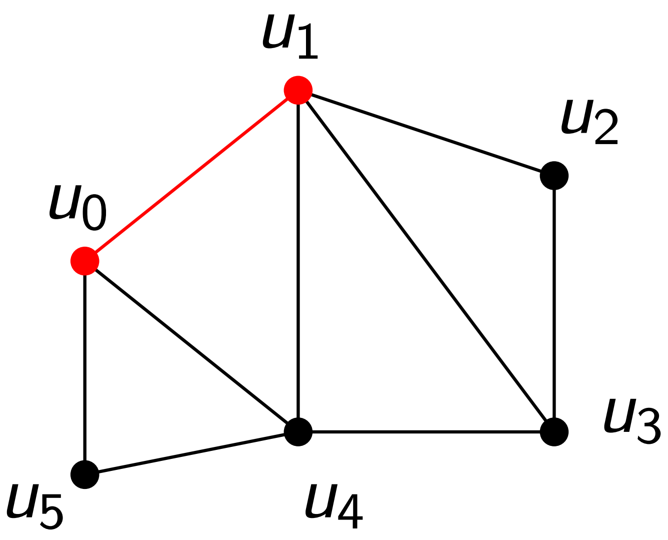


There are only corners with at most three involved rectangles!

Placing the first two rectangles



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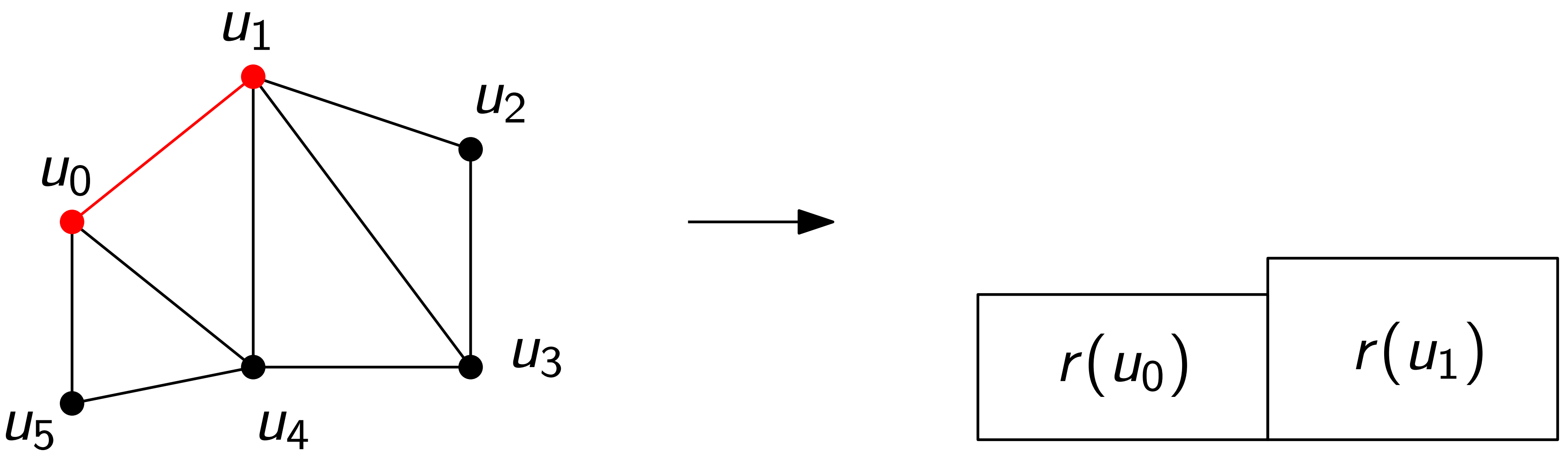

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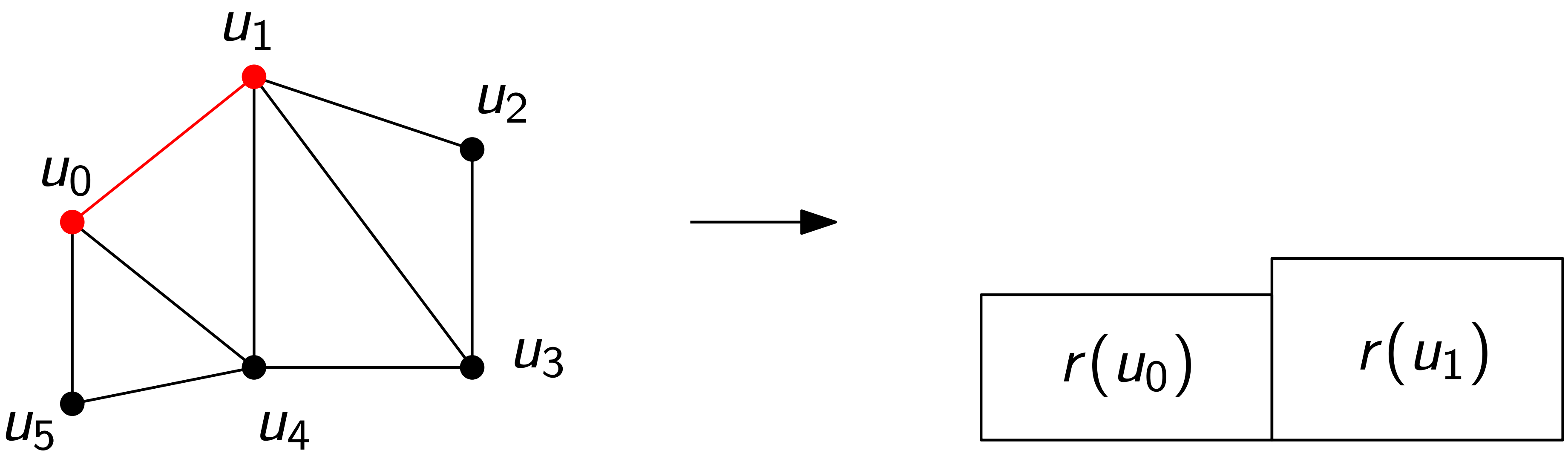
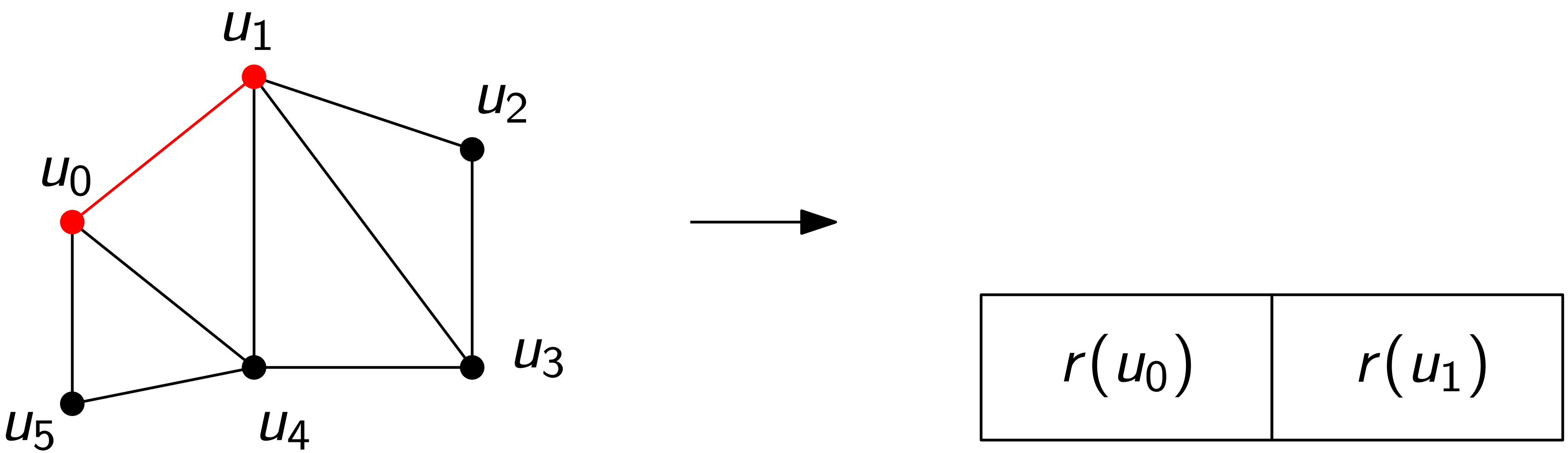


Draw the two rectangles neighboring each other with:

- the same height at the bottom
- and the same width

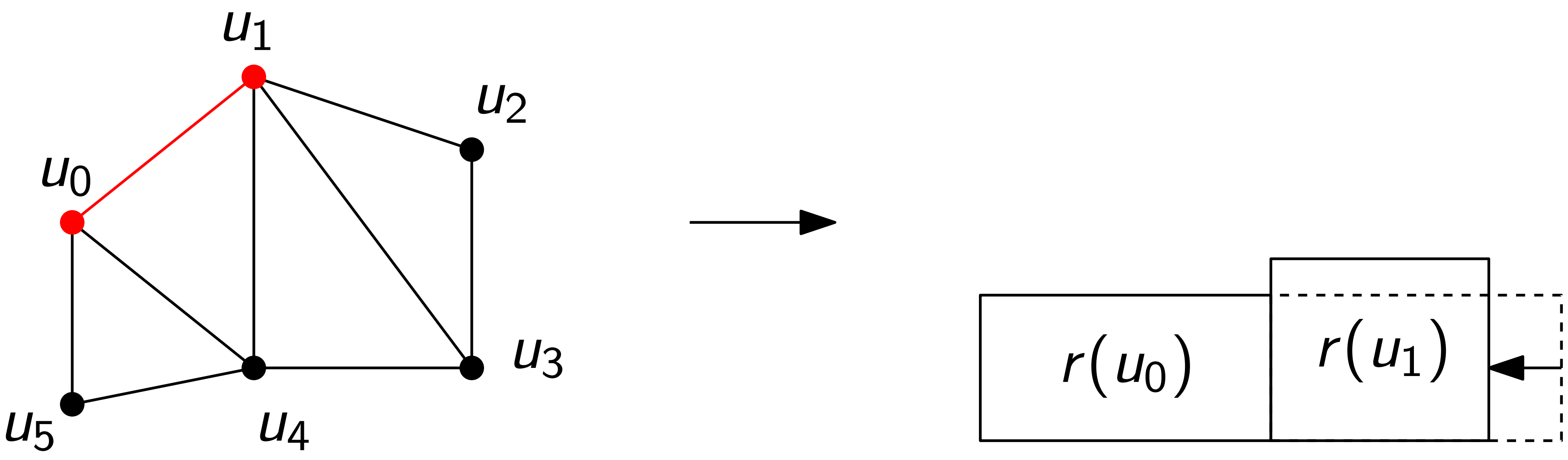
$$b_{u_0} = b_{u_1} \text{ and } \Delta x_{u_0} = \Delta x_{u_1}$$

Placing the first two rectangles



If $a(u_0) = a(u_1)$ we don't get a new edge to place the next rectangle

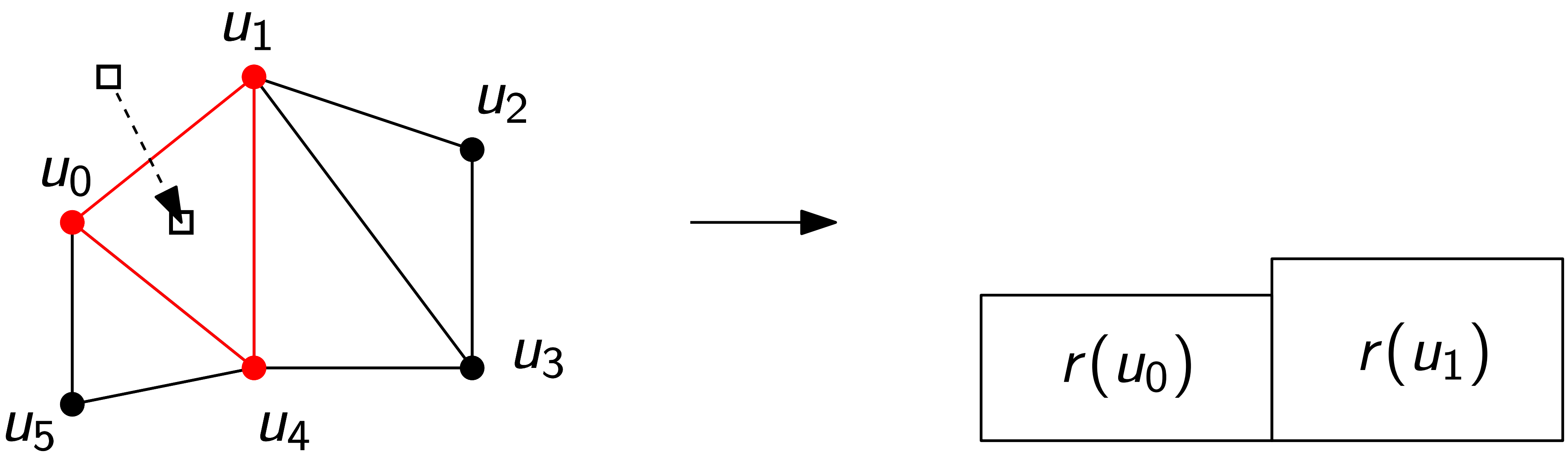
Placing the first two rectangles



If $a(u_0) = a(u_1)$ we don't get a new edge to place the next rectangle

$\rightarrow$ Reduce the width of one rectangle!

Treatment of an inner facet



$r(u_4)$ is adjacent to $r(u_0)$ and $r(u_1)$

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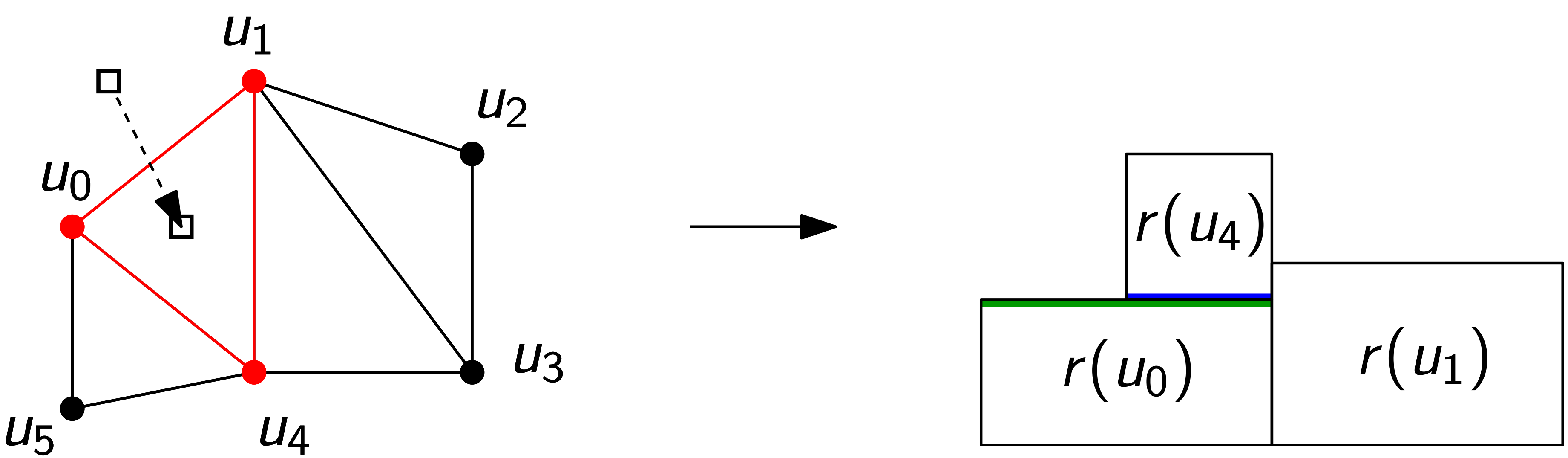
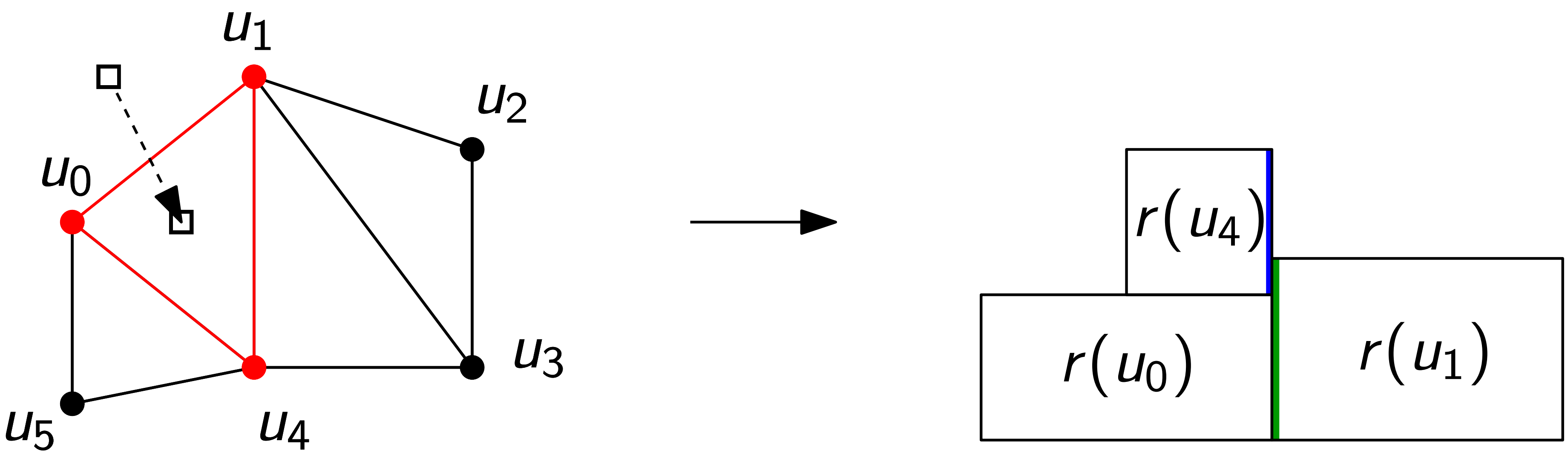


$r(u_4)$ is adjacent to $r(u_0)$ and $r(u_1)$

Place $r(u_4)$ on top of the rectangle with the lower top edge

$$b_{u_4} = t_{u_0}$$

Treatment of an inner facet

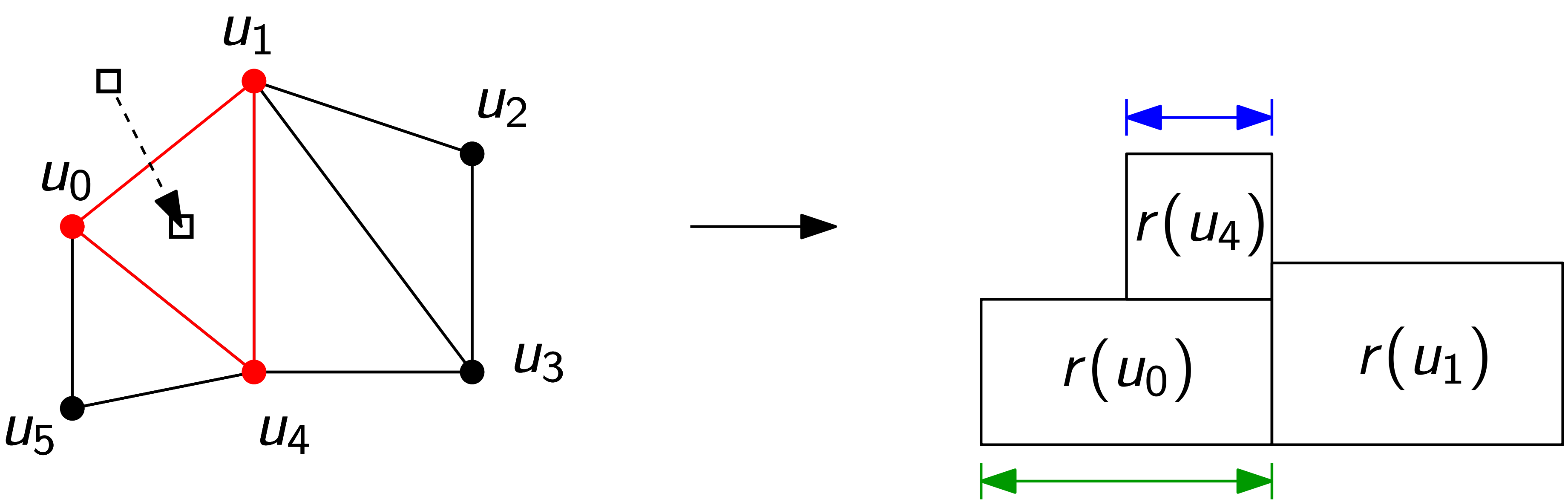


$r(u_4)$ is adjacent to $r(u_0)$ and $r(u_1)$

Place $r(u_4)$ on top of the rectangle with the lower top edge and to the left of the other one

$$b_{u_4} = t_{u_0} \text{ and } r_{u_4} = l_{u_1}$$

Treatment of an inner facet

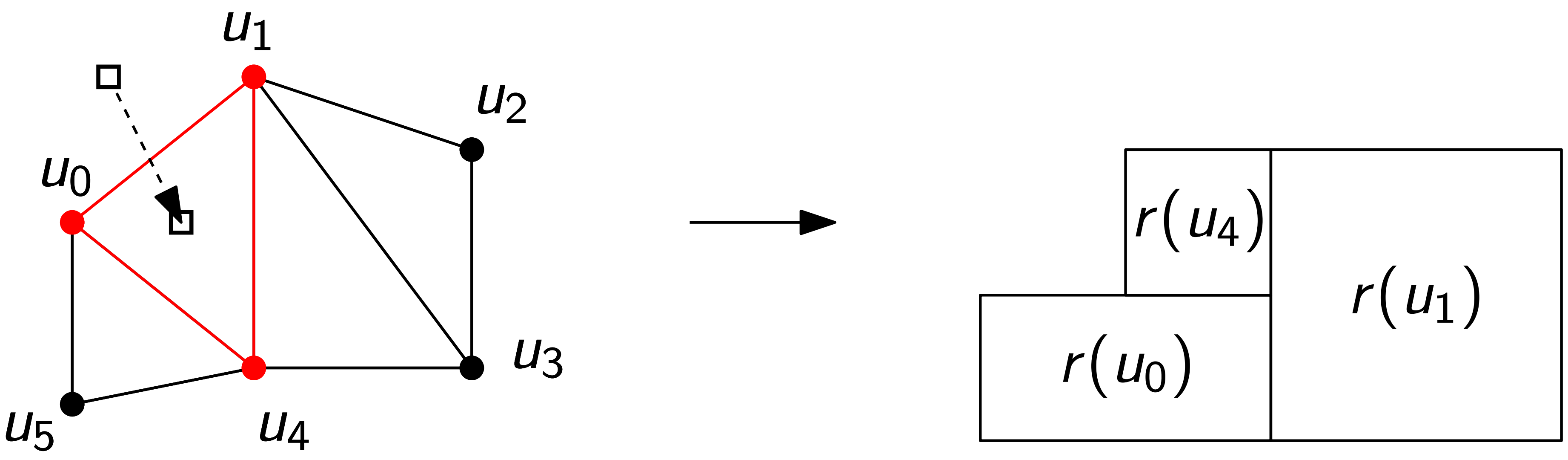


$r(u_4)$ is adjacent to $r(u_0)$ and $r(u_1)$

Leave free space for the rectangles adjacent to $r(u_0)$ and $r(u_4)$ or $r(u_1)$ and $r(u_4)$

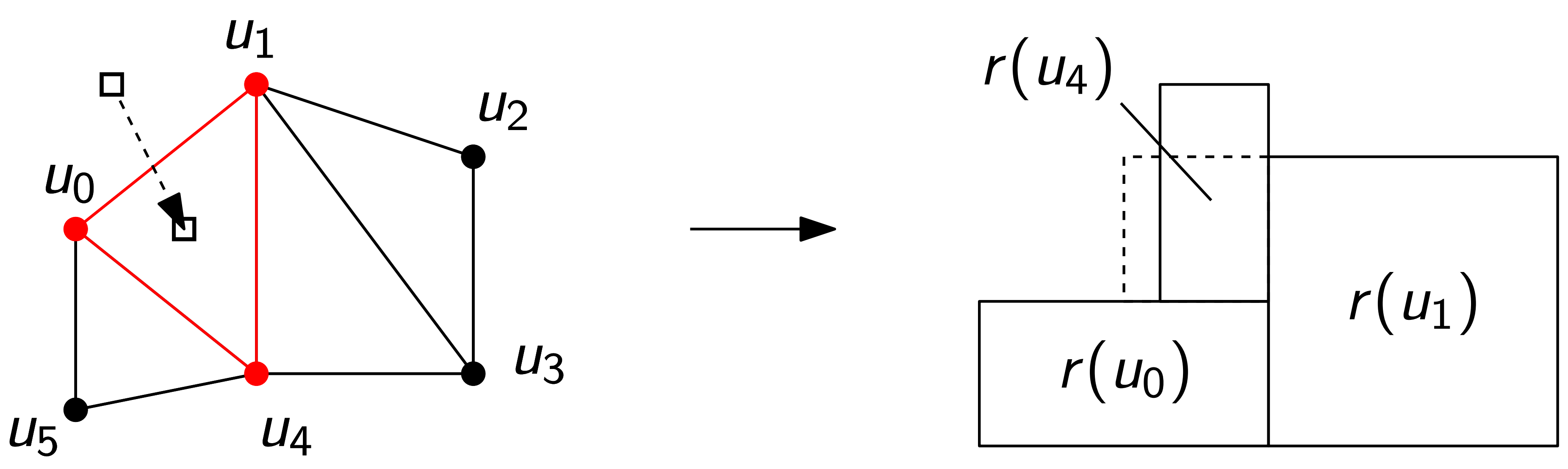
$$\Delta x_{u_4} = 0.5 \cdot Free(u_0)$$

Treatment of an inner facet



What can we do if there is no free space?

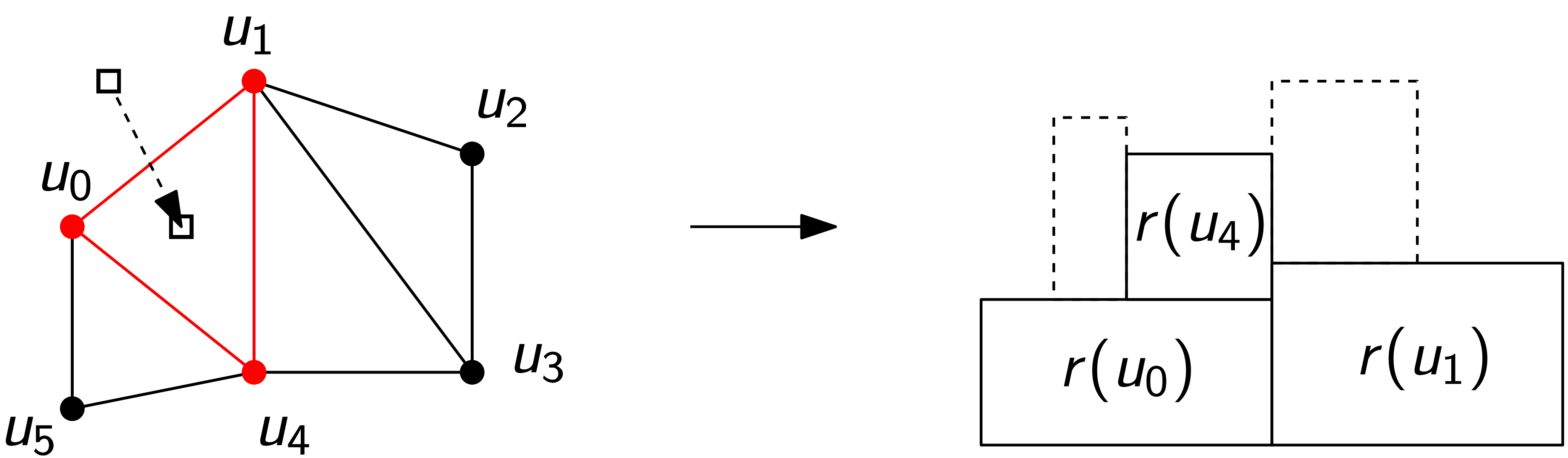
Treatment of an inner facet



What can we do if there is no free space?

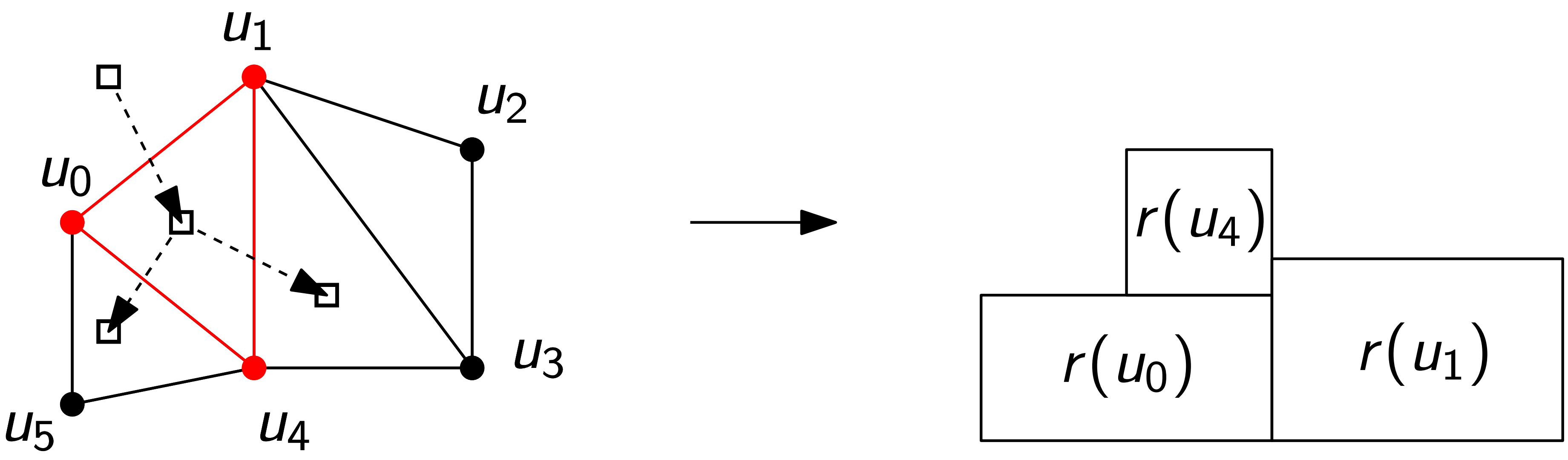
$\rightarrow$ Reduce the width of the new rectangle!

Treatment of an inner facet



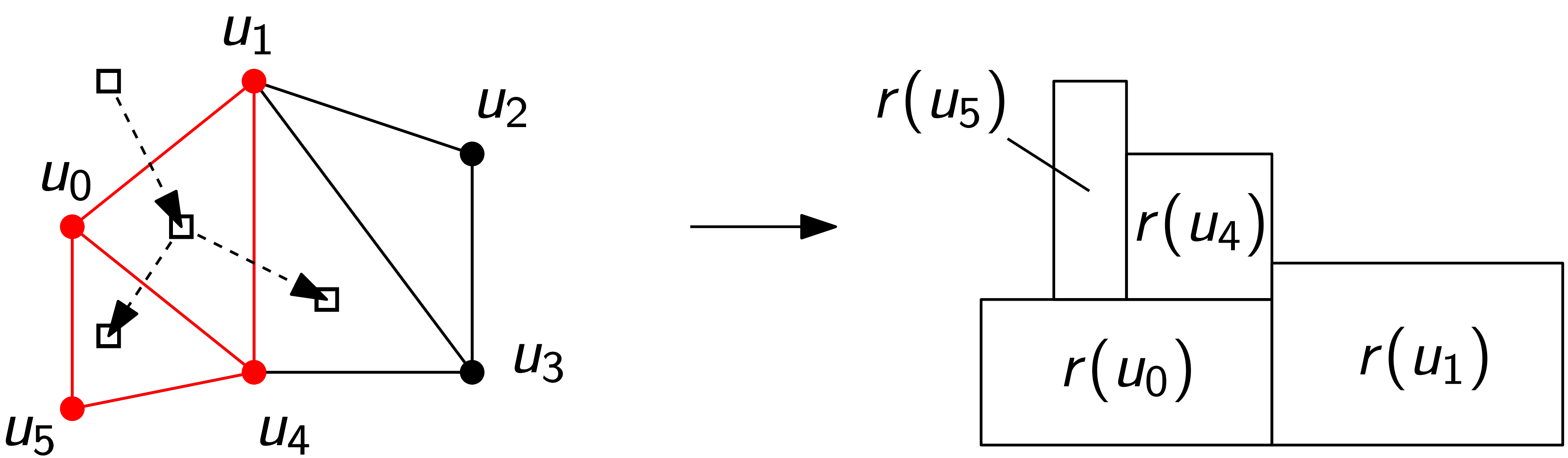
- The new Rectangle $r(u_4)$ is adjacent to $r(u_0)$ and $r(u_1)$
- Possibility to place the two rectangles neighboring $r(u_0)$ and $r(u_4)$ or $r(u_1)$ and $r(u_4)$

Finishing the graph



Choose one direction to work on first

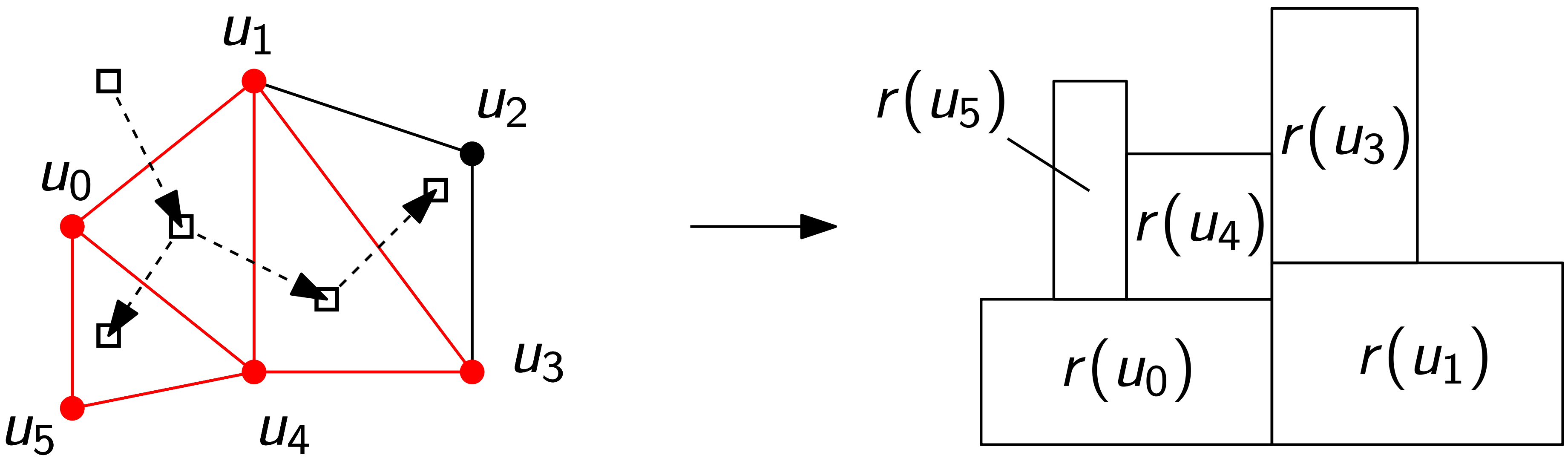
Finishing the graph



Choose one direction to work on first

- Place $r(u_5)$ next to $r(u_0)$ and $r(u_4)$

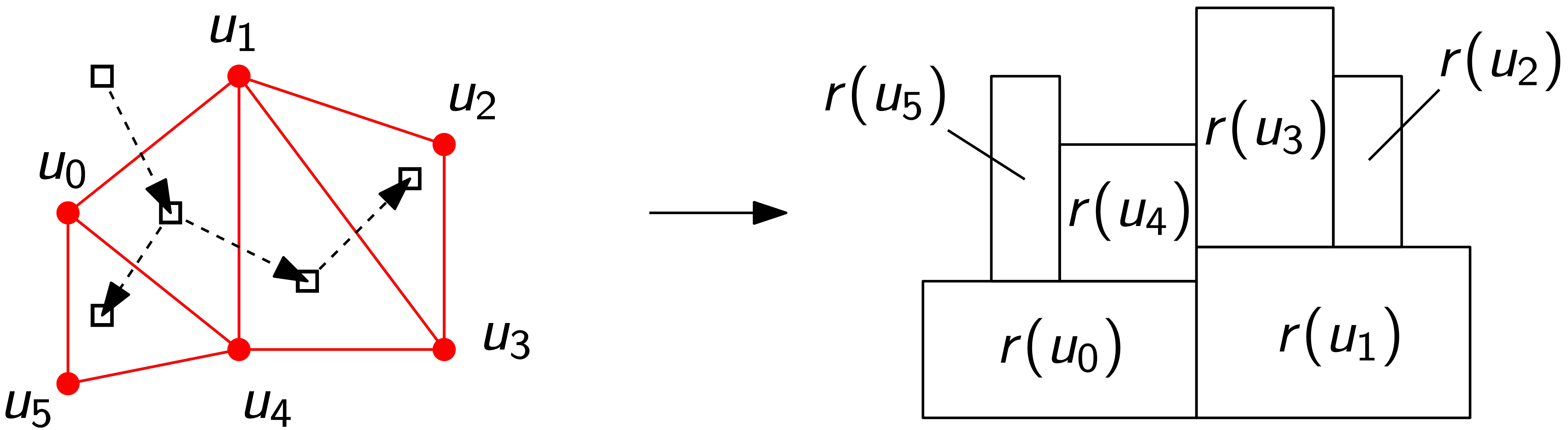
Finishing the graph



Choose one direction to work on first

- Place $r(u_5)$ next to $r(u_0)$ and $r(u_4)$
- Place $r(u_3)$ next to $r(u_1)$ and $r(u_4)$

Finishing the graph



Choose one direction to work on first

- Place $r(u_5)$ next to $r(u_0)$ and $r(u_4)$
- Place $r(u_3)$ next to $r(u_1)$ and $r(u_4)$
- Place $r(u_2)$ next to $r(u_1)$ and $r(u_3)$

Proof of correctness

The correctness is proved by induction on the number of rectangles placed so far.

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Graph $G_k = (V_k, E_k)$ is a subgraph of the graph $G = (V, E)$ to be drawn with

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- $V_k \subseteq V$ and
- $E_k = \{(u, v) \in E \mid u, v \in V_k\}$
- and G_k has a rectangular representation with k placed rectangles.

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Invariant:
There is a free corner for the next rectangle $r(w)$ to be placed with $w \in V \setminus V_k$, in such a way that it is adjacent to its predecessors $r(u)$ and $r(v)$ with $u, v \in V_k$.

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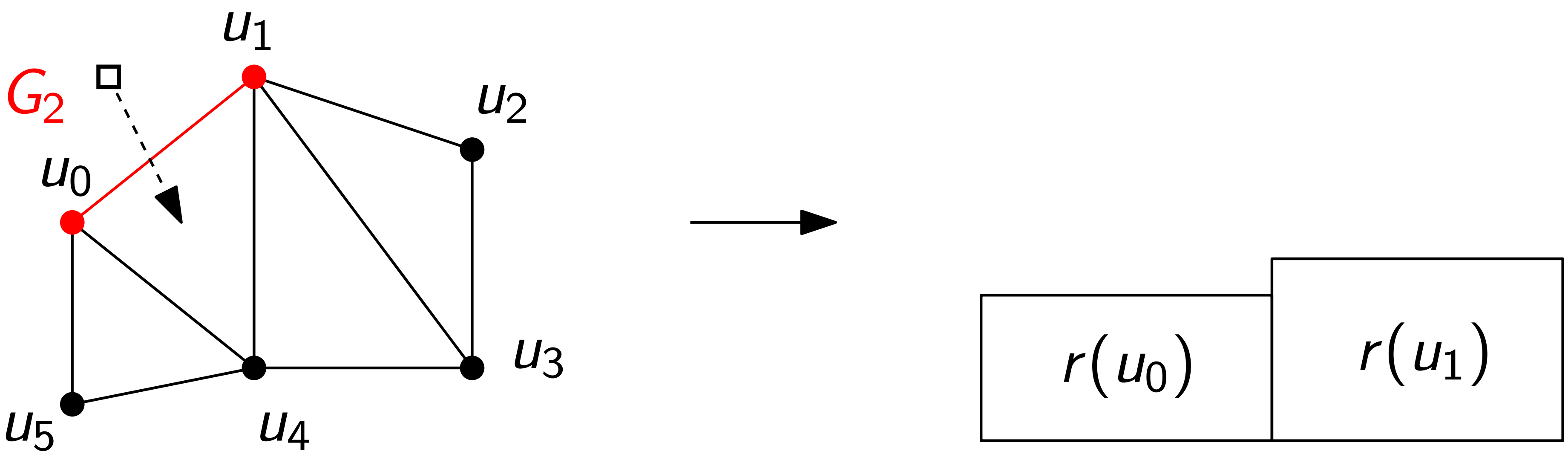

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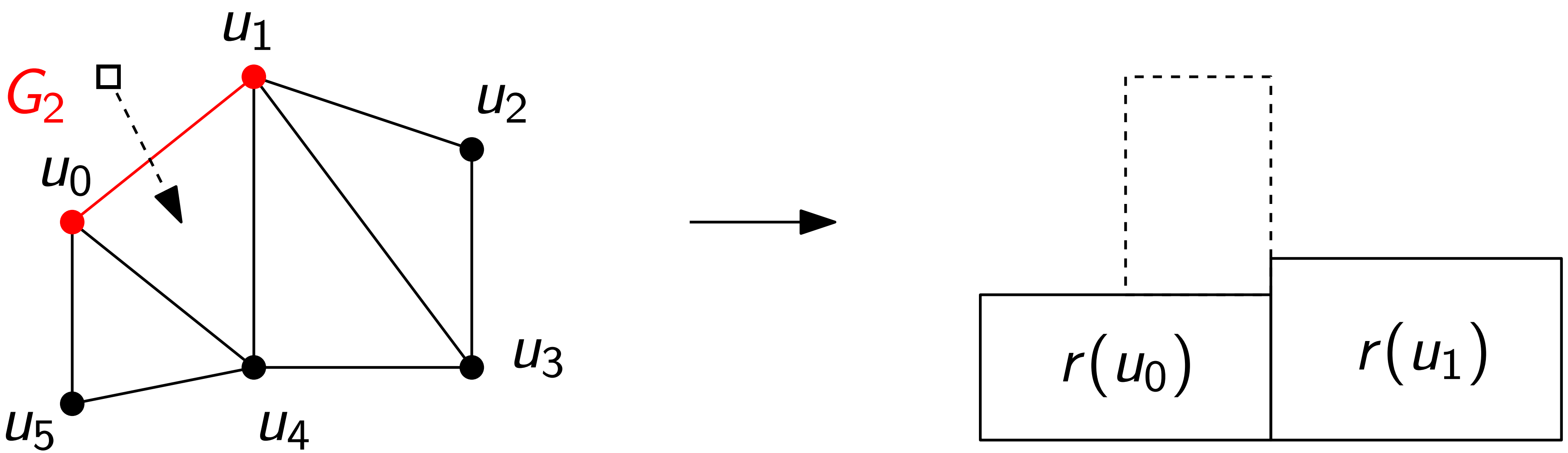

The invariant is fulfilled because we have one corner where $r(u_4)$ can be placed adjacent to its predecessors $r(u_0)$ and $r(u_1)$

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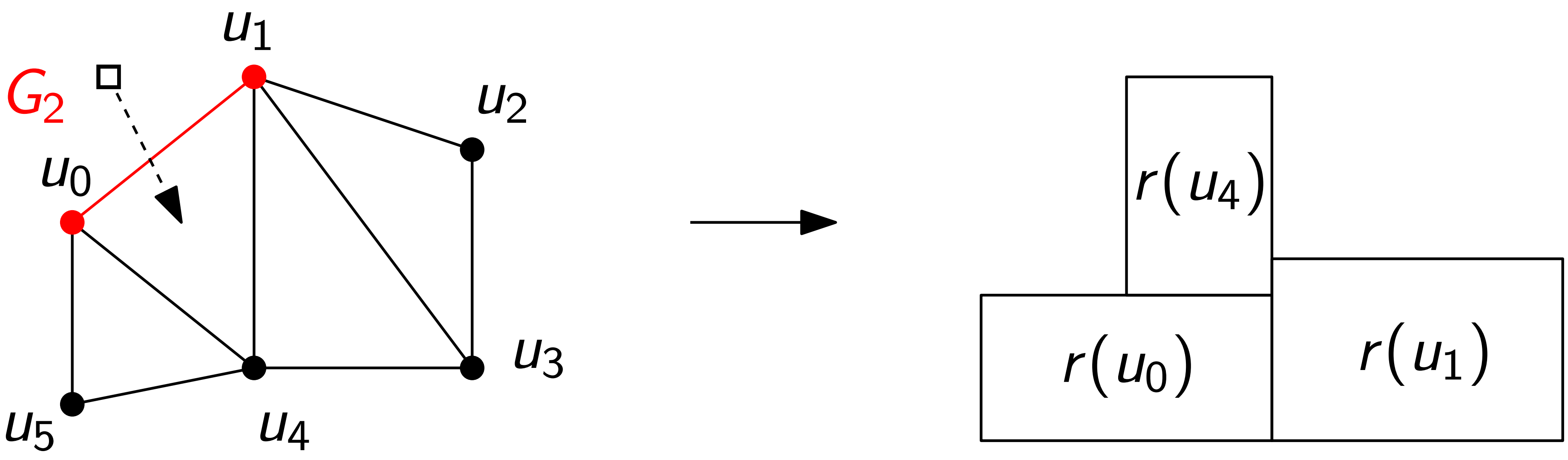

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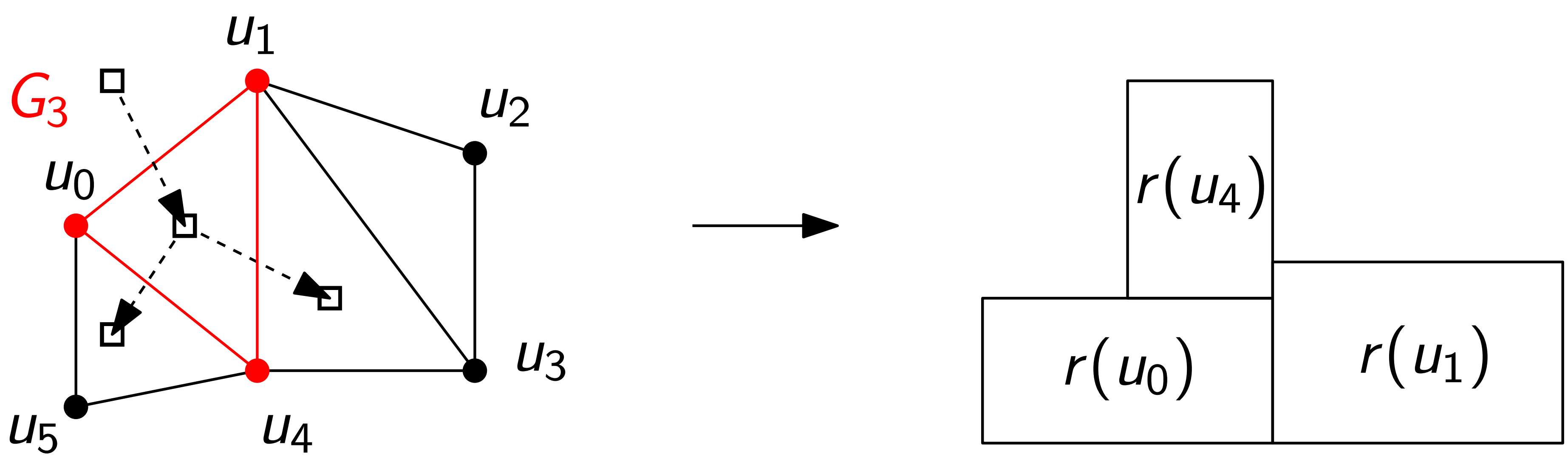

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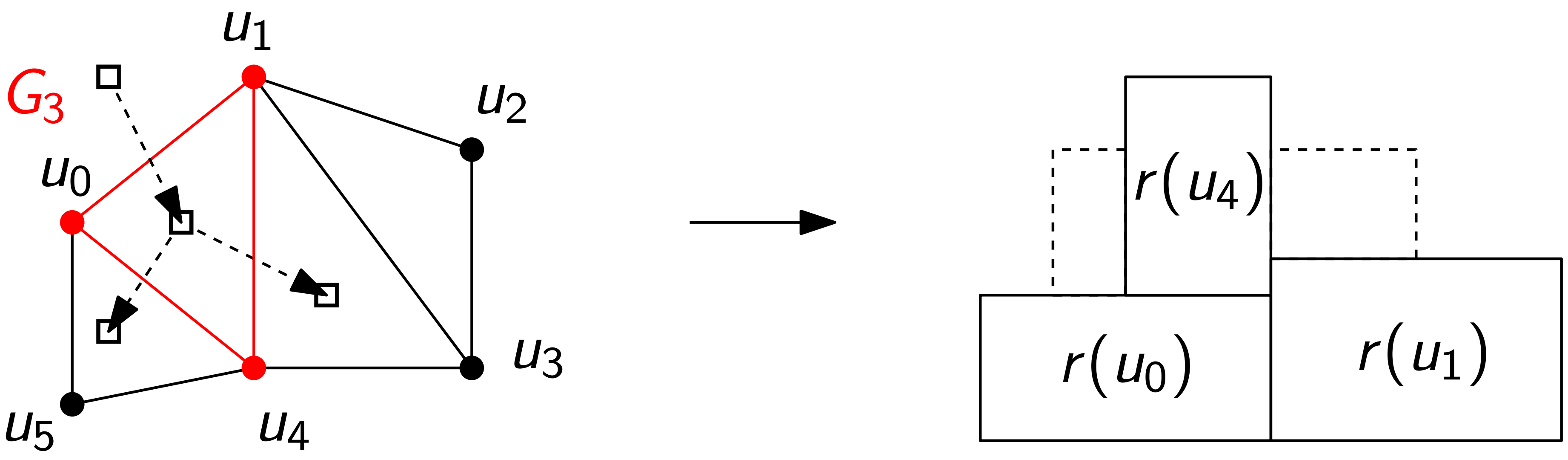

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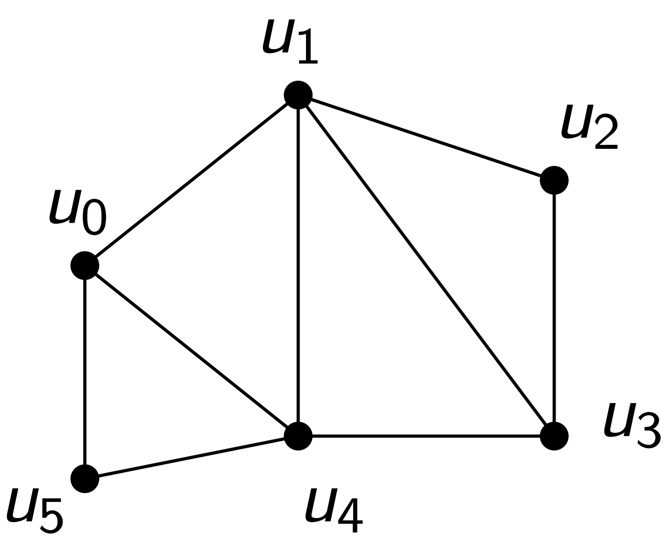
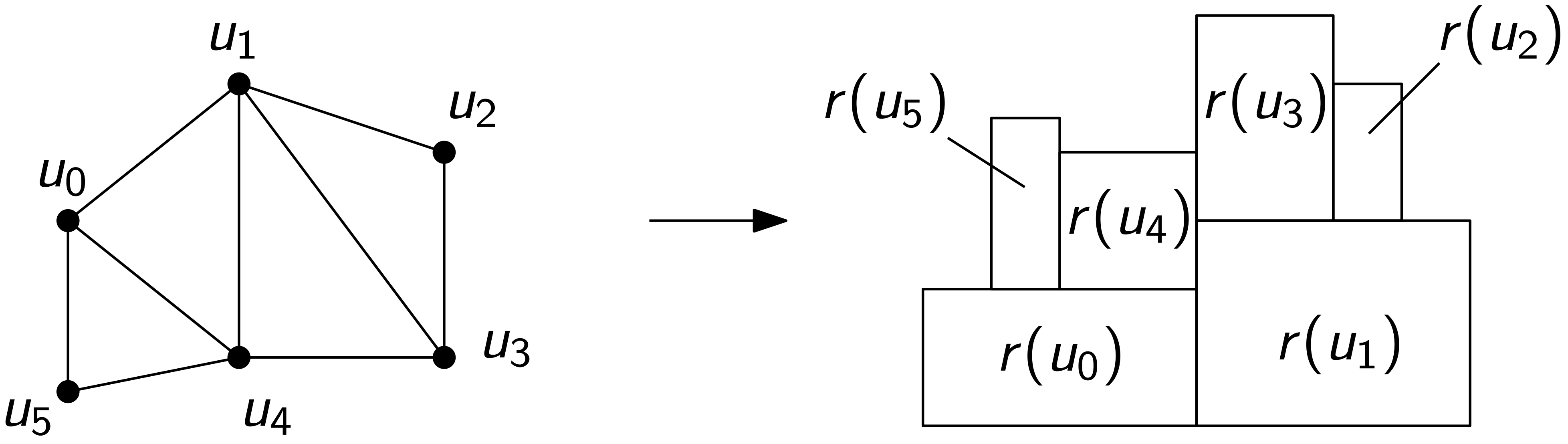
There is a corner for $r(u_5)$ and $r(u_3)$ where they can be placed adjacent to their predecessors.

Outerplanar Graphs



Outerplanar graph: all vertices belong to the unbounded face

Outerplanar Graphs



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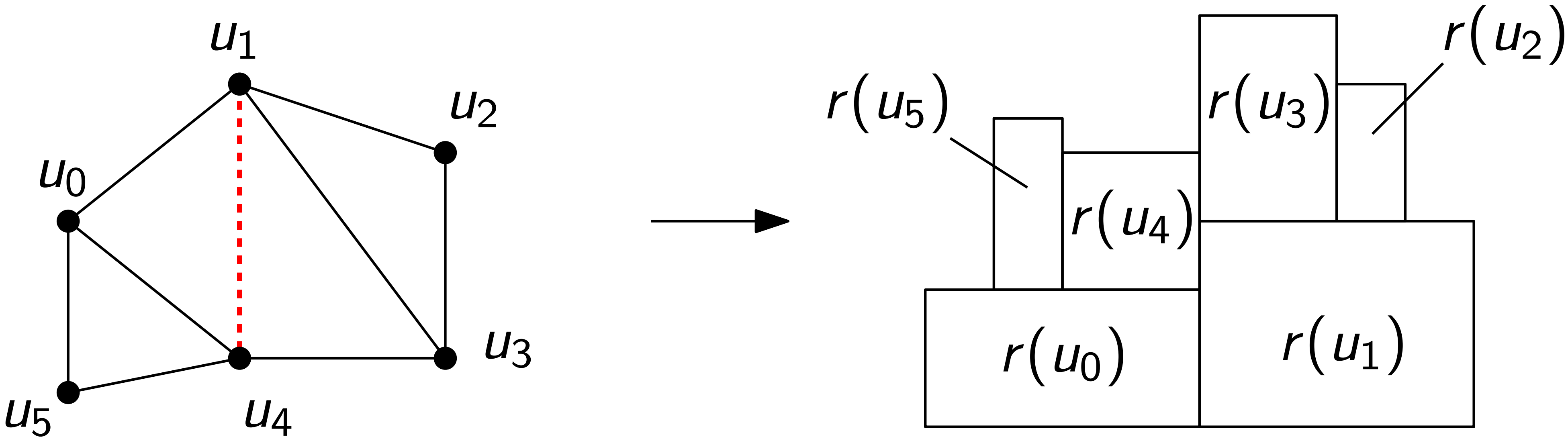
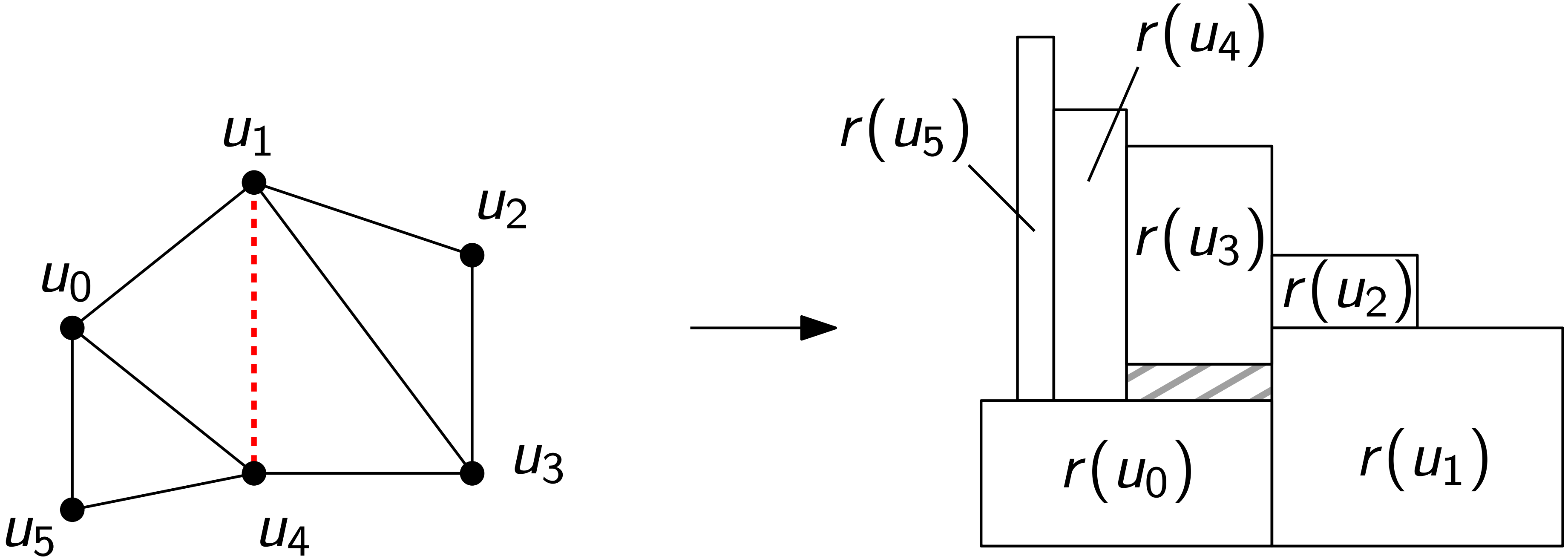


Outerplanar graph: all vertices belong to the unbounded face but not all faces are triangles!

Outerplanar Graphs



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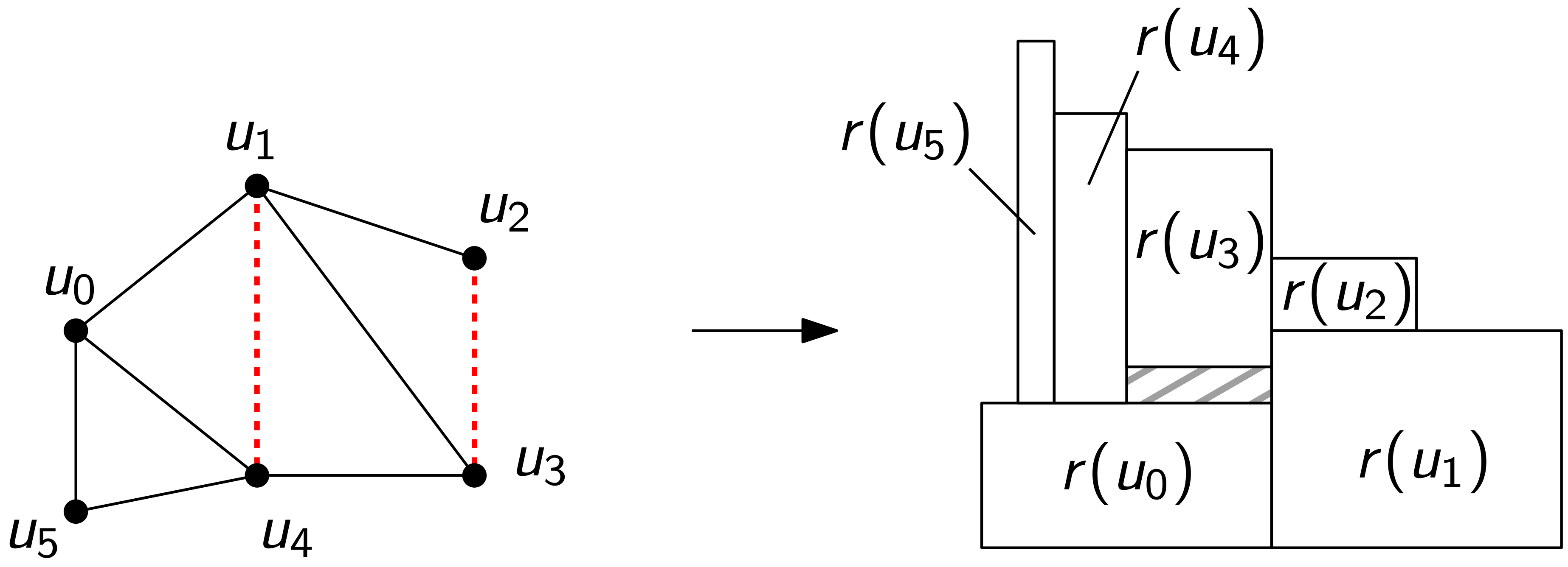

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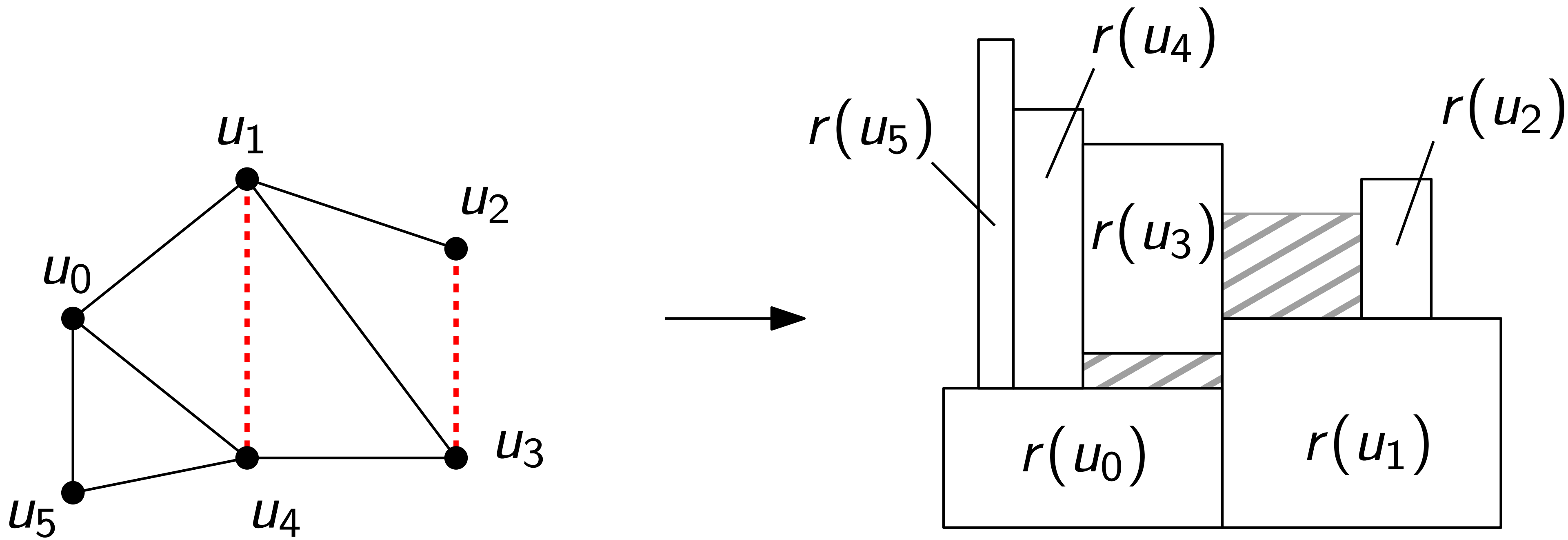

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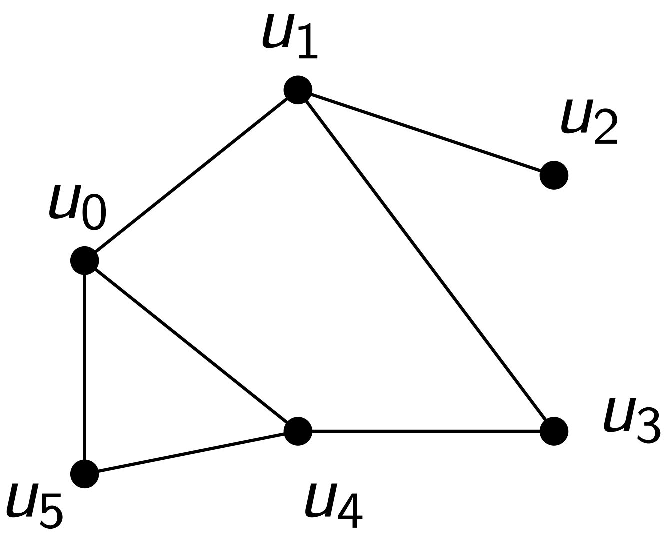
Dummy-nodes



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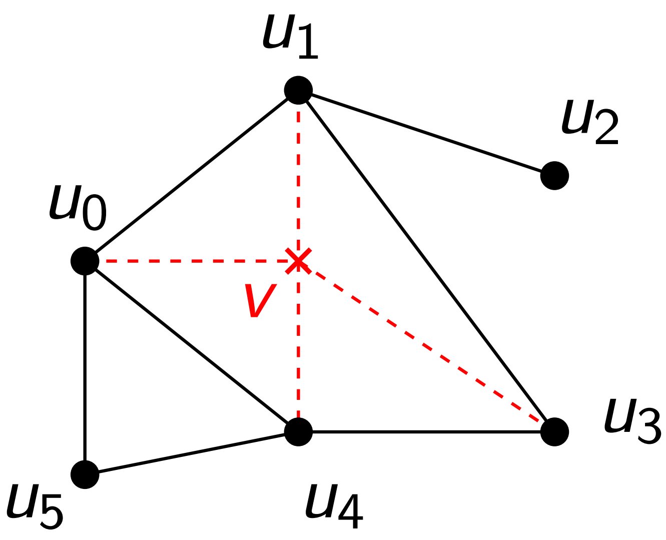

Insert an inner dummy-node $v \in V_i$ for every not triangular inner face and connect it to the surrounding vertices with $vu \in E_i$.

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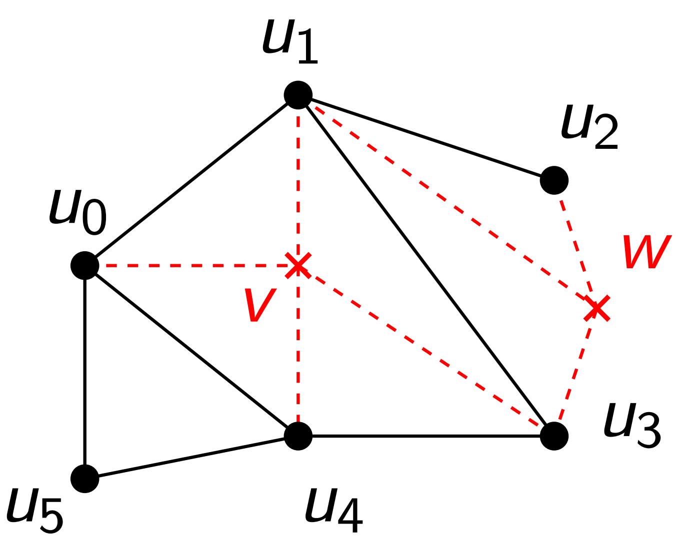

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Insert an outer dummy-node $w \in V_a$ for every separating vertex in the graph and add the edges $wu \in E_a$.

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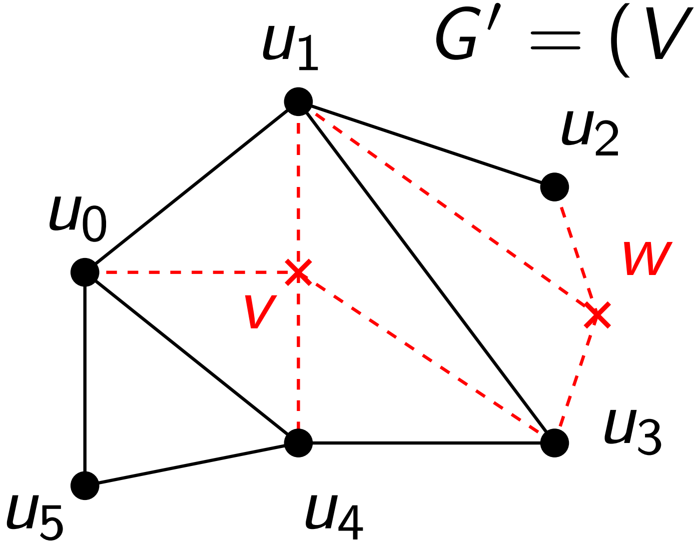

$$G' = (V', E') = (V \cup V_i \cup V_a, E \cup E_i \cup E_a)$$

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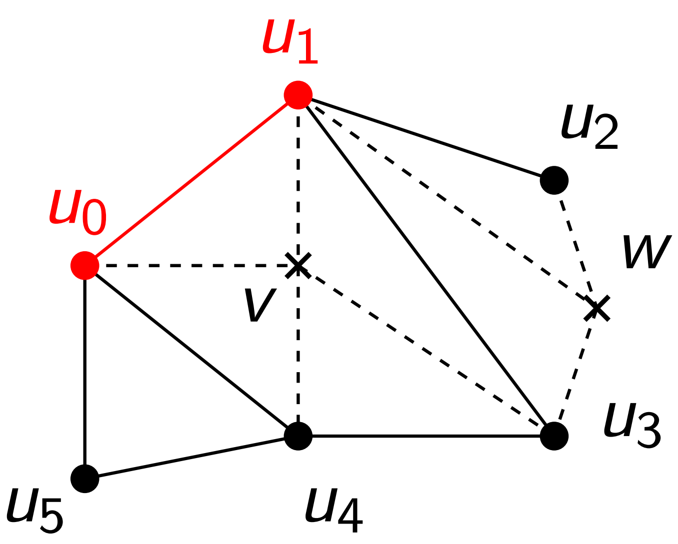
Placing the first two rectangles



Start with an edge in $E = E' \setminus \{E_a \cup E_i\}$

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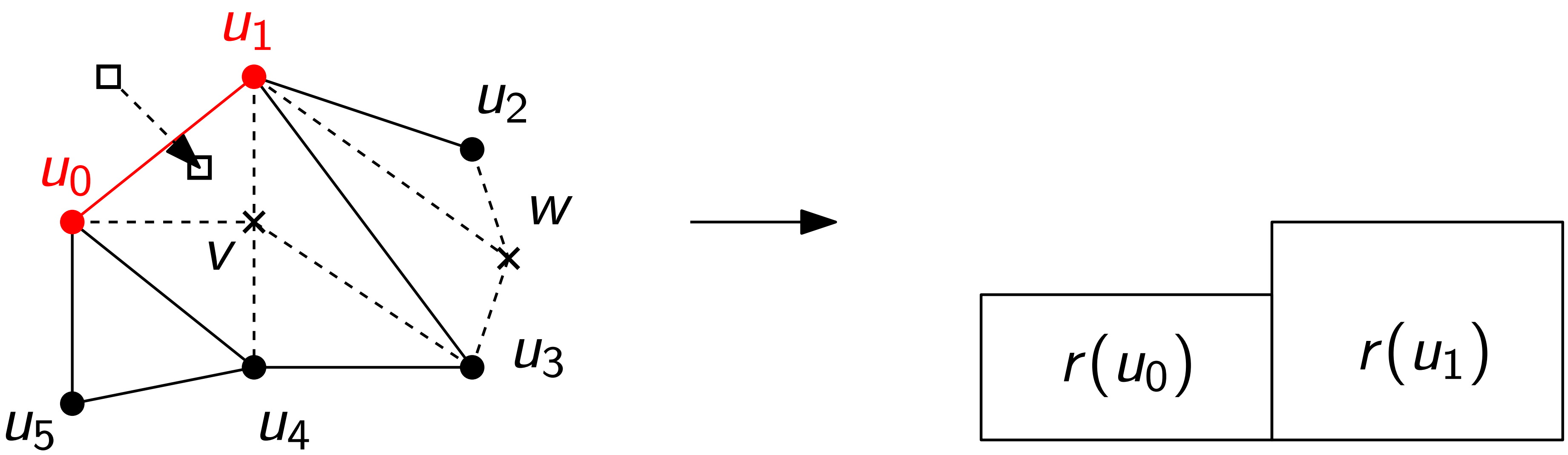

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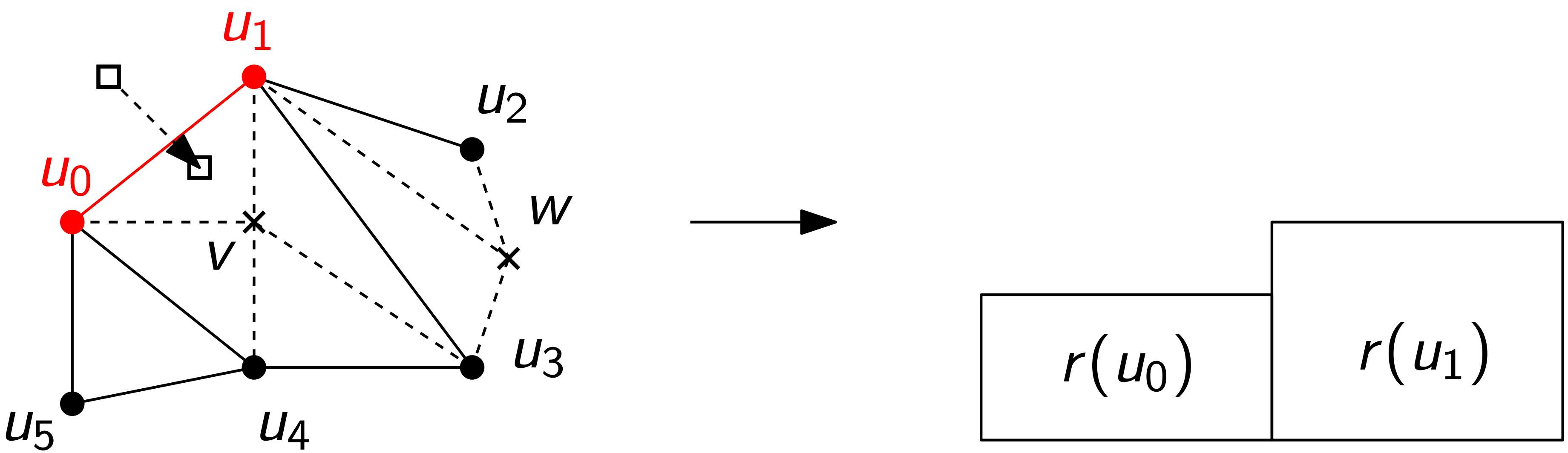

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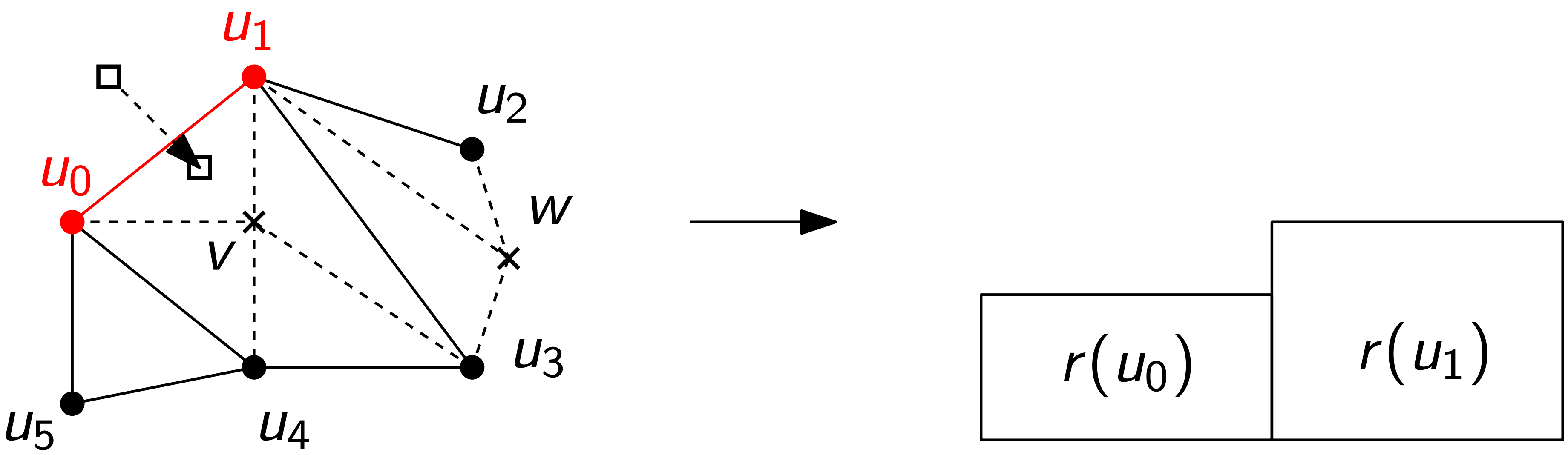
Differentiate between three cases:

- Inner dummy-node $v \in V_i$
- Outer dummy-node $v \in V_a$
- Normal vertex $v \in V = V' \setminus \{V_i \cup V_a\}$

An inner dummy-node



Place the free space corresponding to the node v

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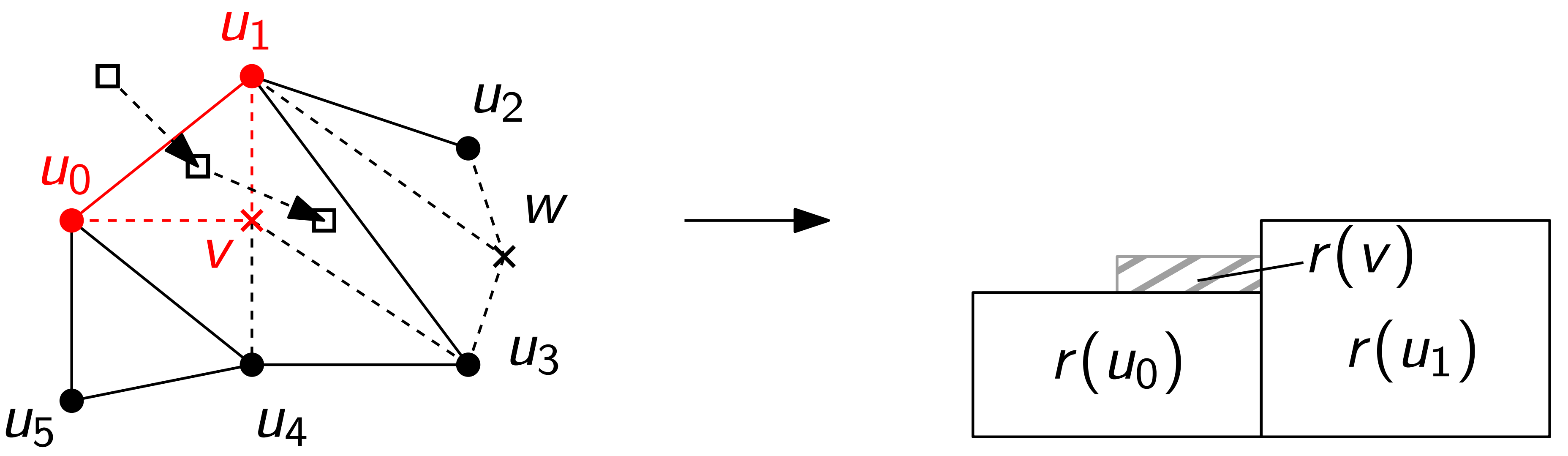

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- Place $r(v)$ on top of the rectangle with the lower top edge

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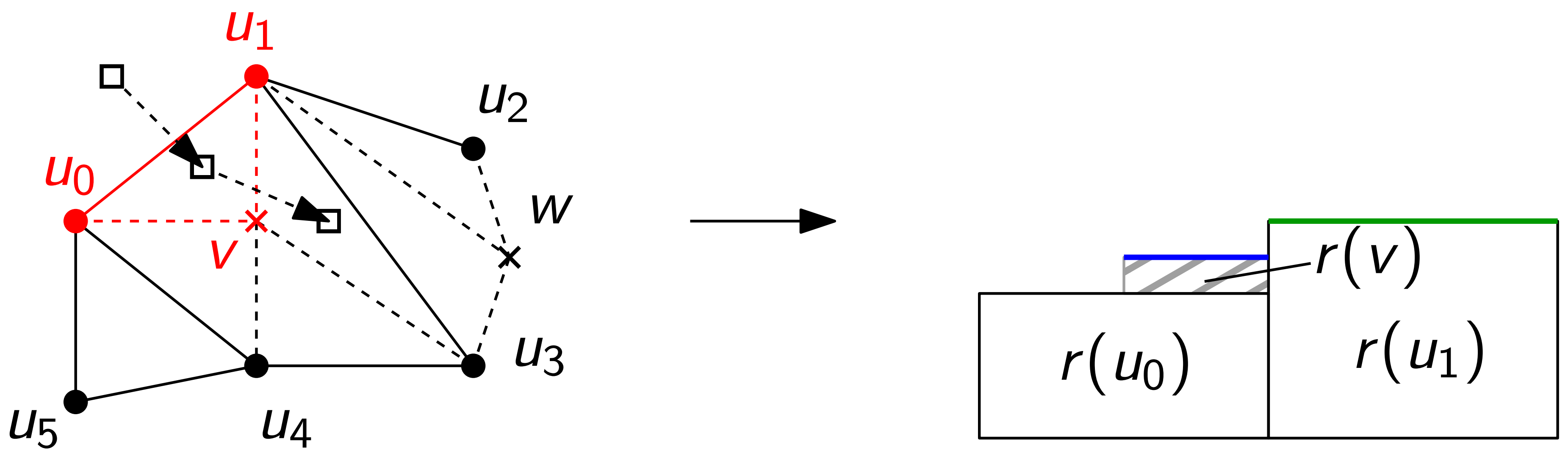

Place the free space corresponding to the node v

- Place $r(v)$ on top of the rectangle with the lower top edge
- The top edge of $r(v)$ must be below the top edge of $r(u_1)$ with $t_v = 1/2 \cdot (t_{u_0} + t_{u_1})$

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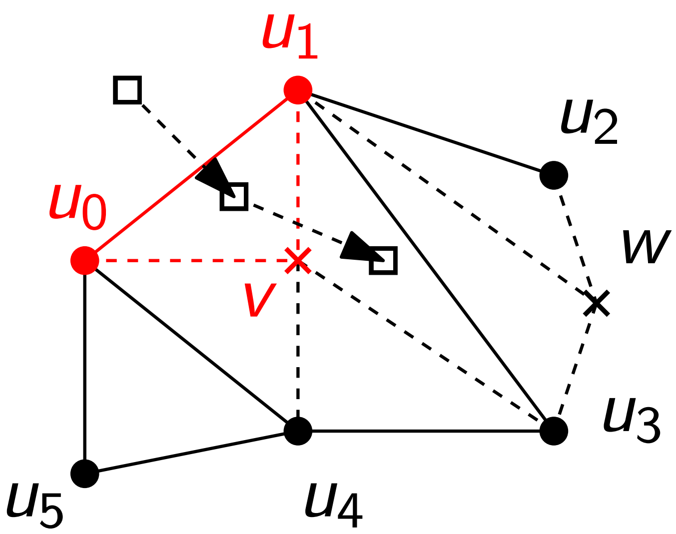

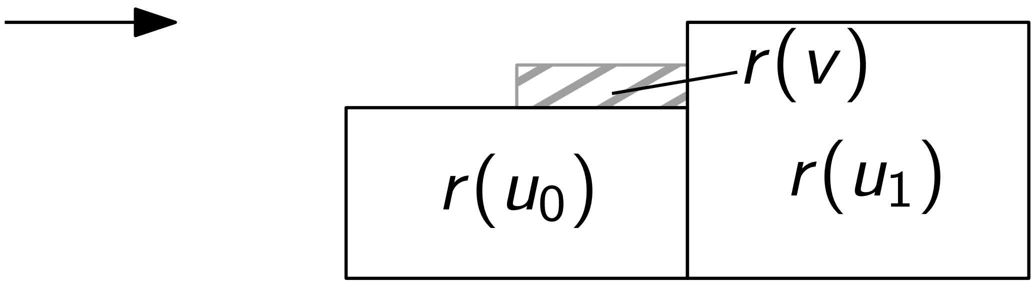

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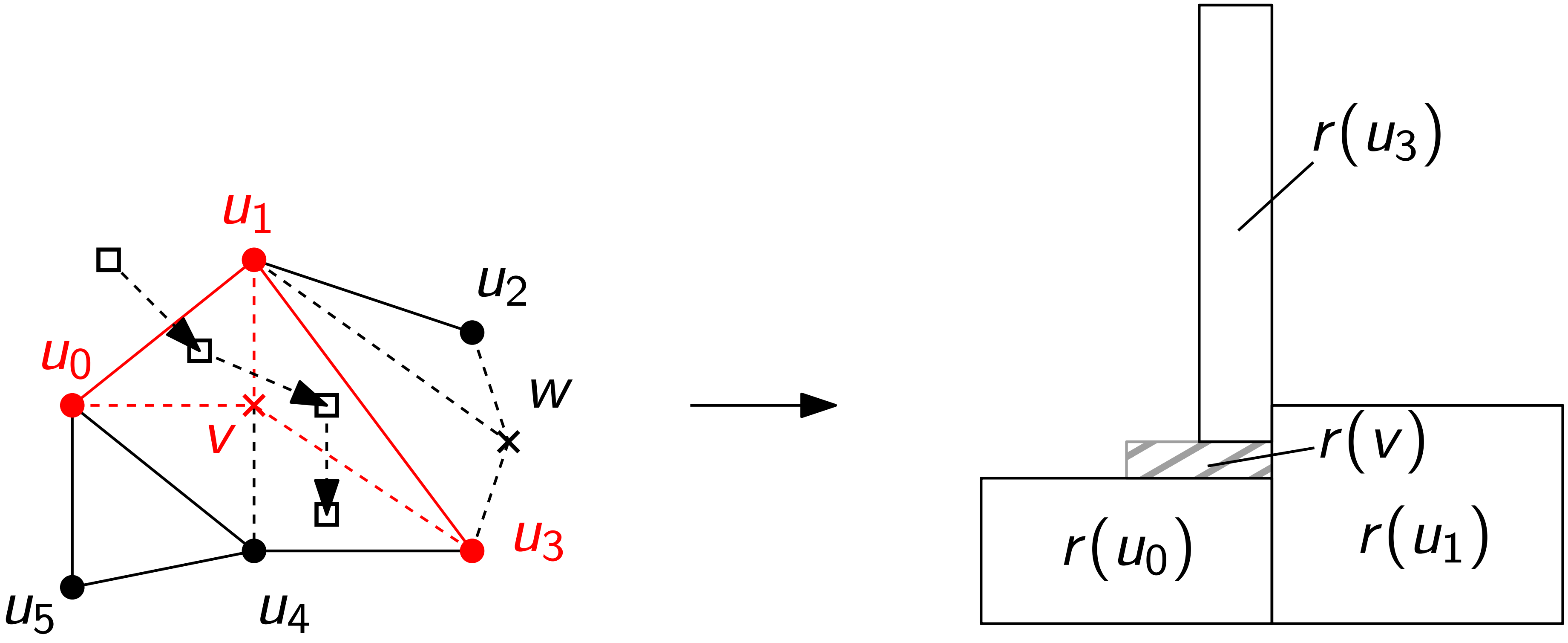
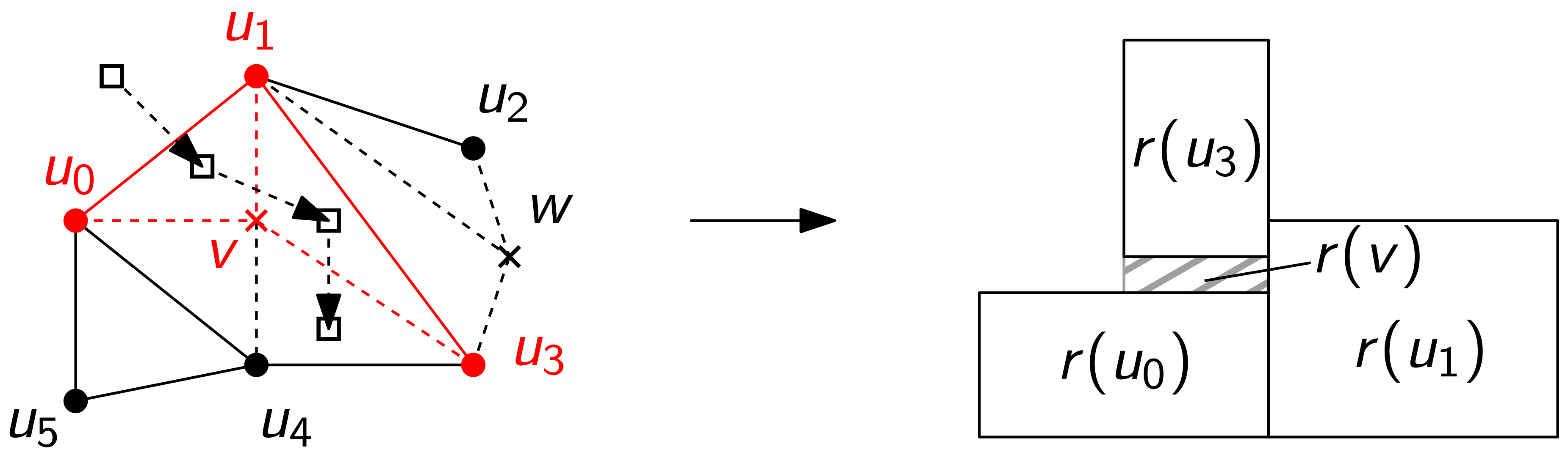
Walk clockwise around the inner dummy-node

An inner dummy-node



Place all following vertices on top of $r(v)$ except the last one

An inner dummy-node



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The last but one rectangle, in this case $r(u_3)$, must fill the remaining free space on top of $r(v)$

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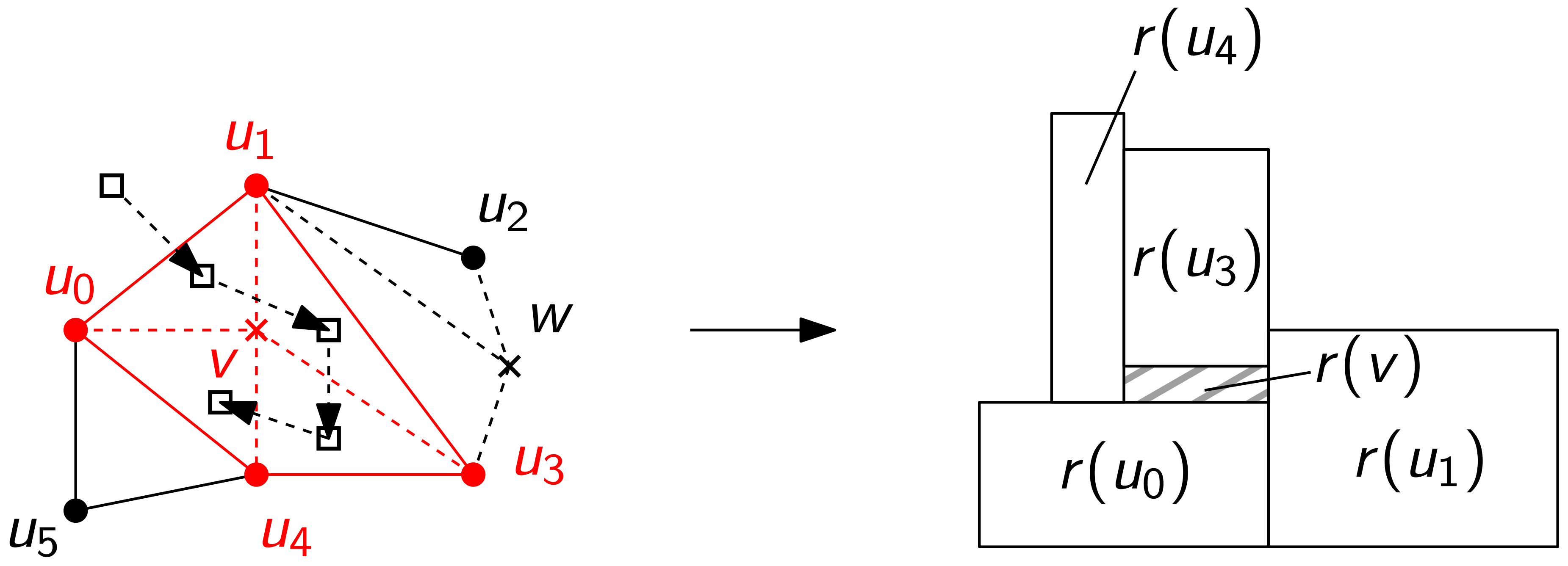

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The last rectangle must connect its predecessor with the first rectangle

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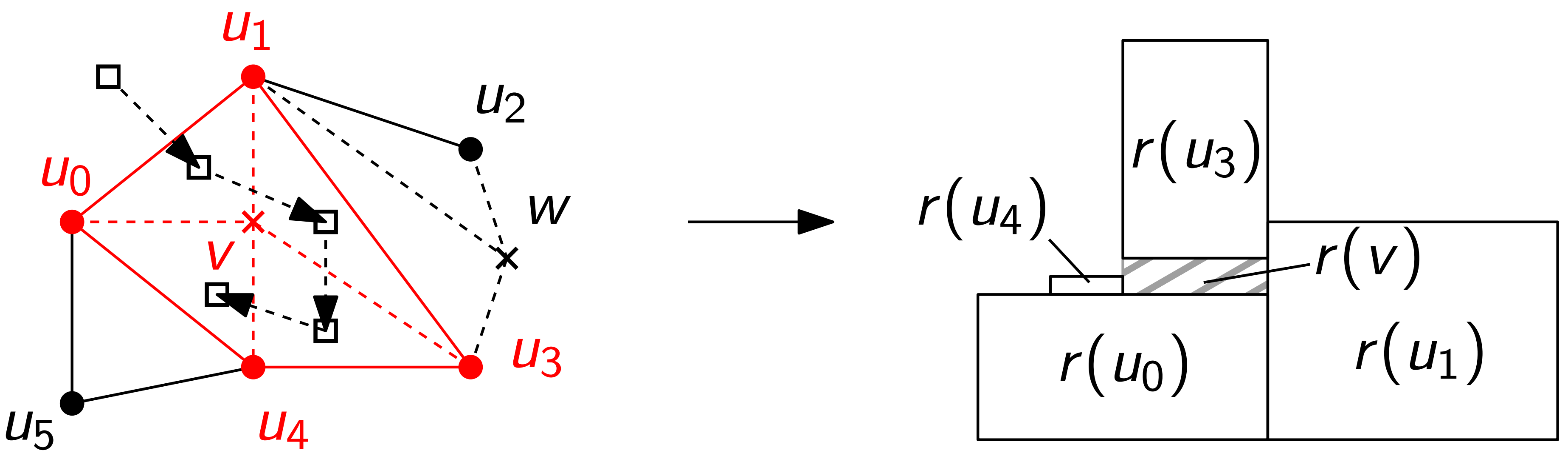

If $r(u_4)$ is too small to connect both rectangles reduce the width to increase its height

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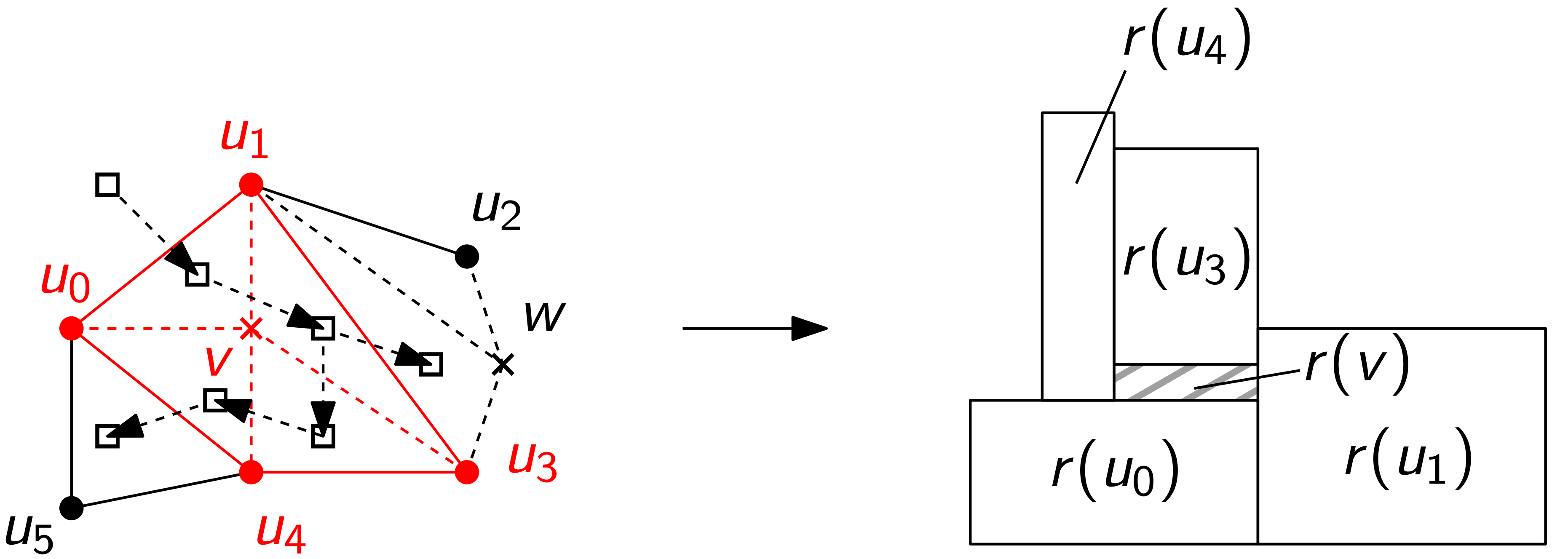
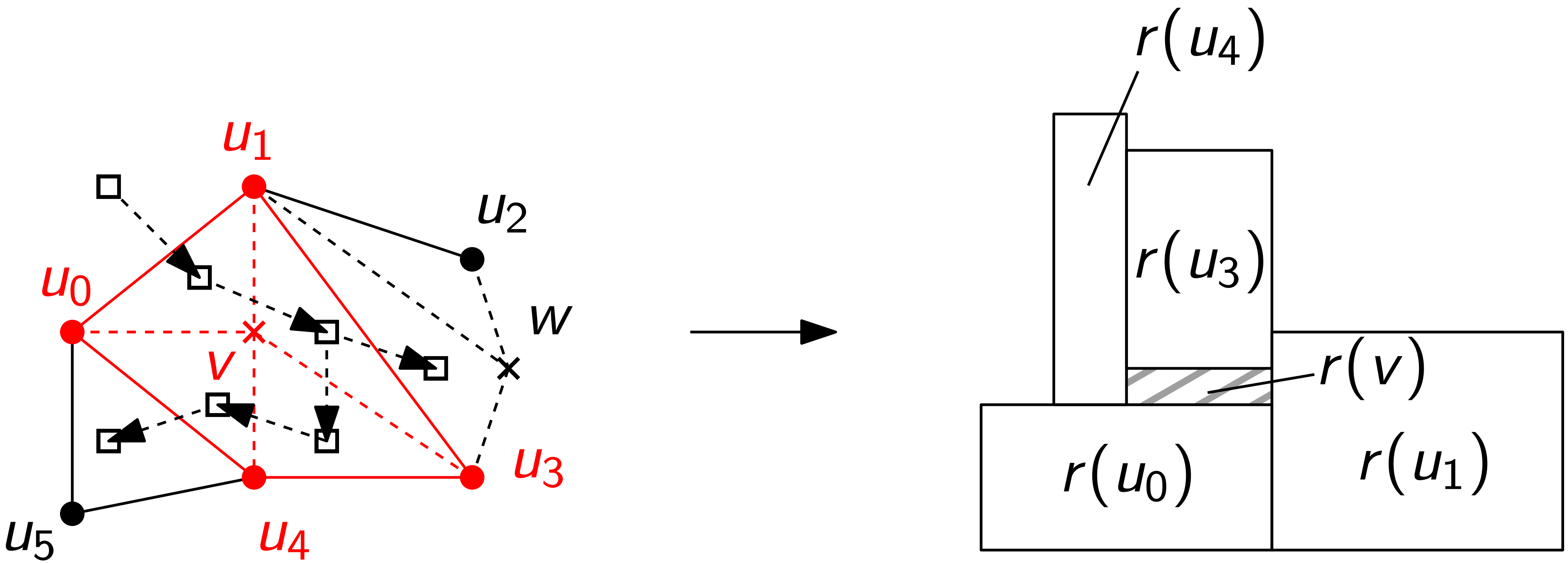


If $r(u_4)$ is too small to connect both rectangles reduce the width to increase its height

All rectangles adjacent to $r(v)$ have been placed correctly
$\rightarrow$ Finally visit all adjacent faces in G'

An outer dummy-node



Insert the outer dummy-node into the rectangular representation with size $a(w) = 1$

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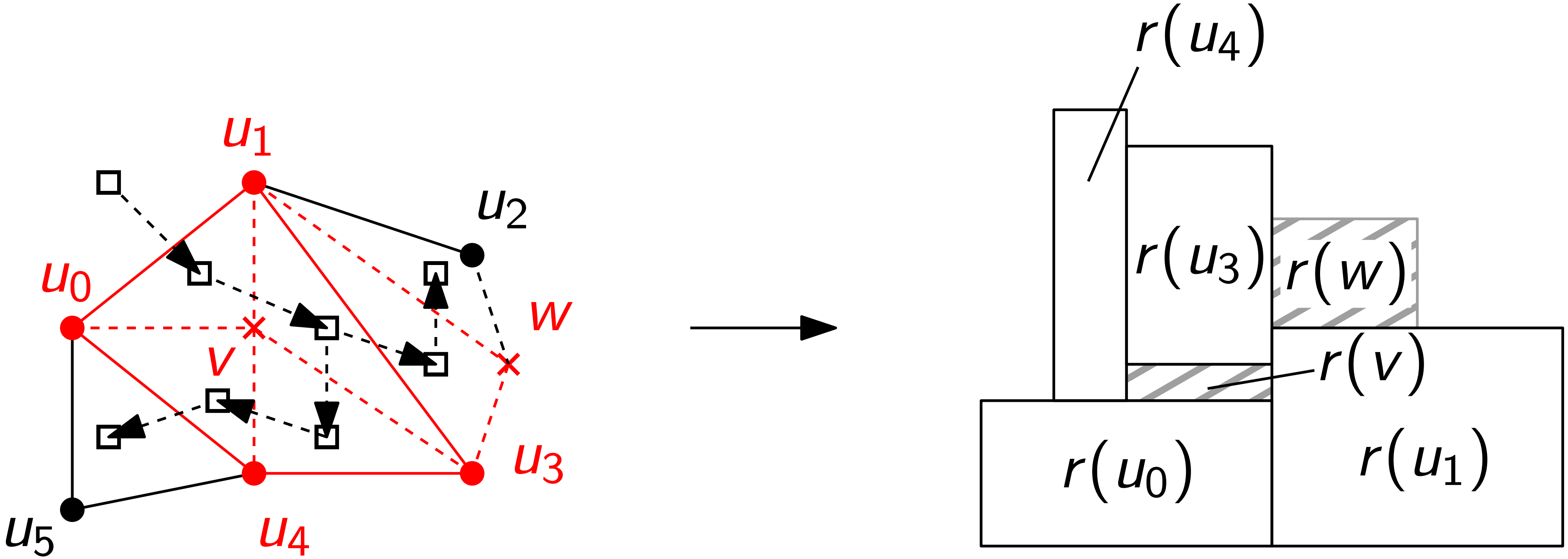

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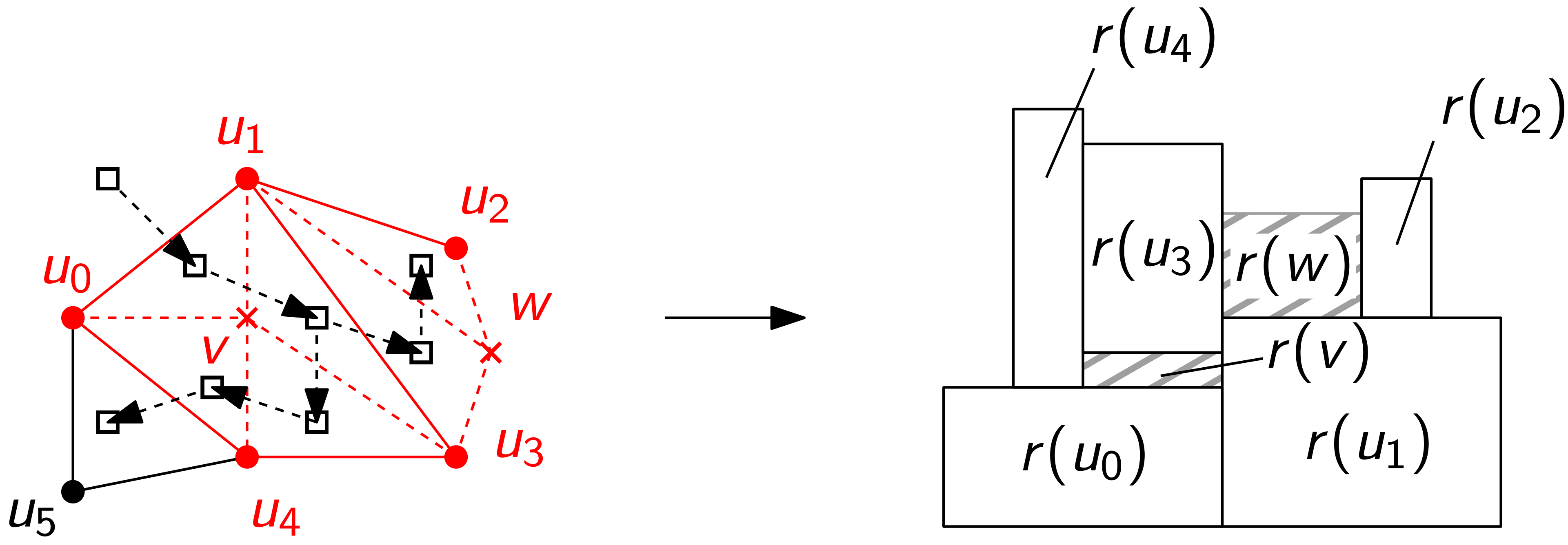

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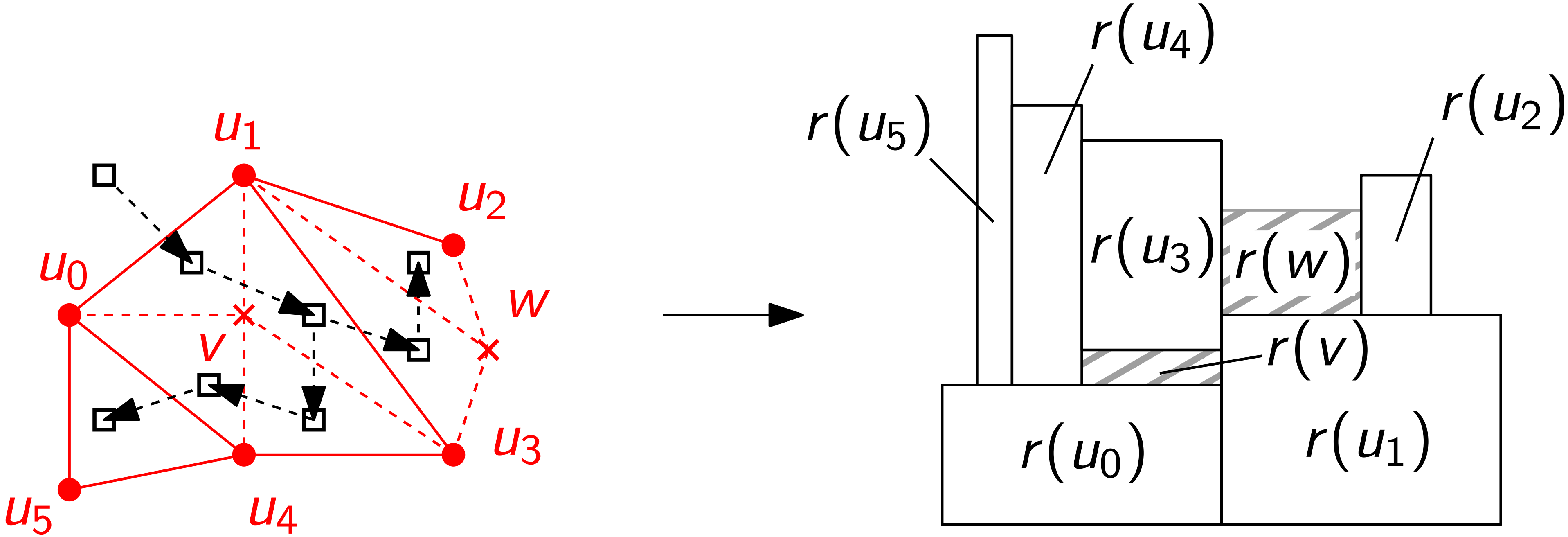

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Proof of correctness for outerplanar graphs

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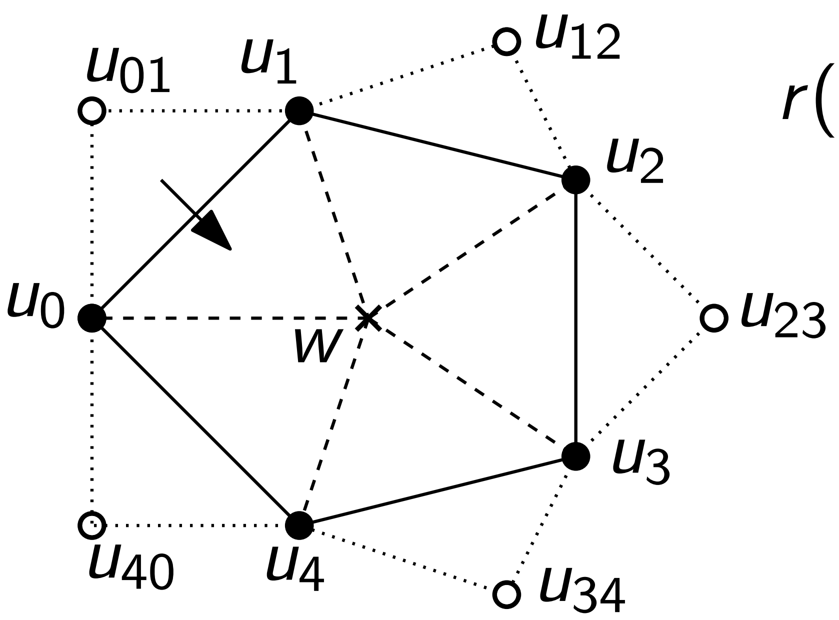
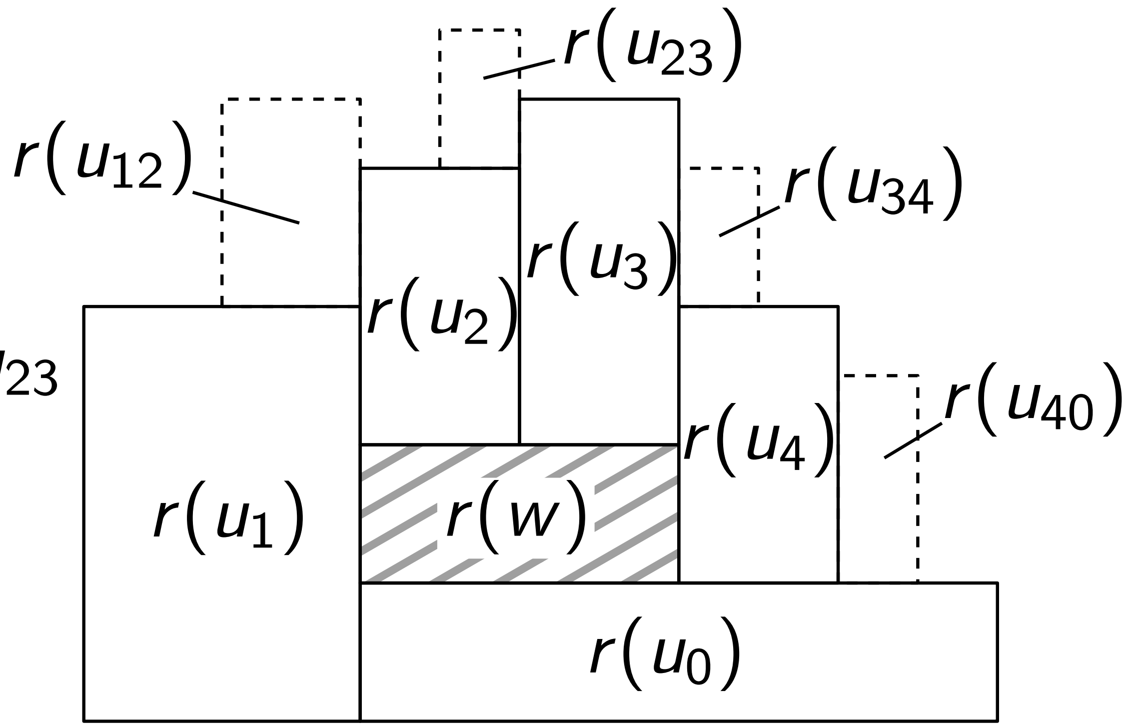
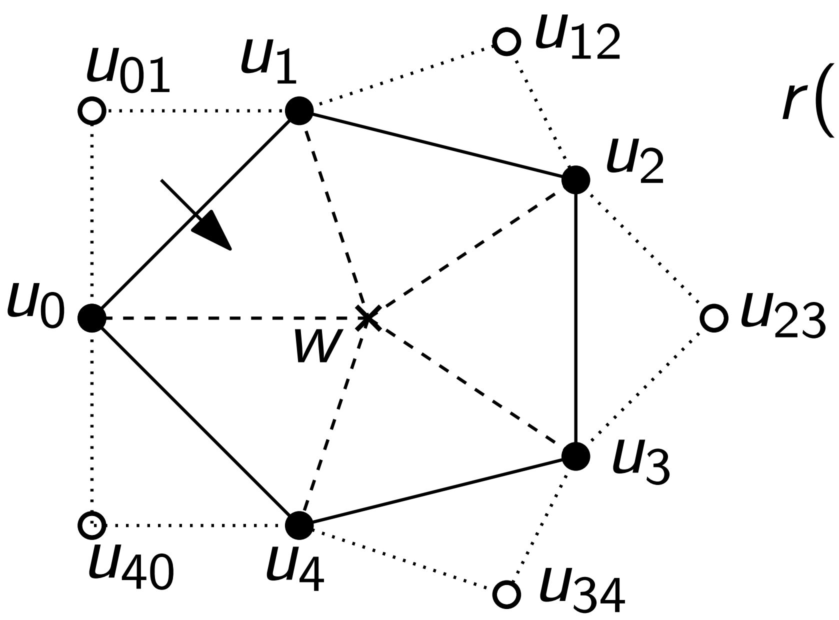


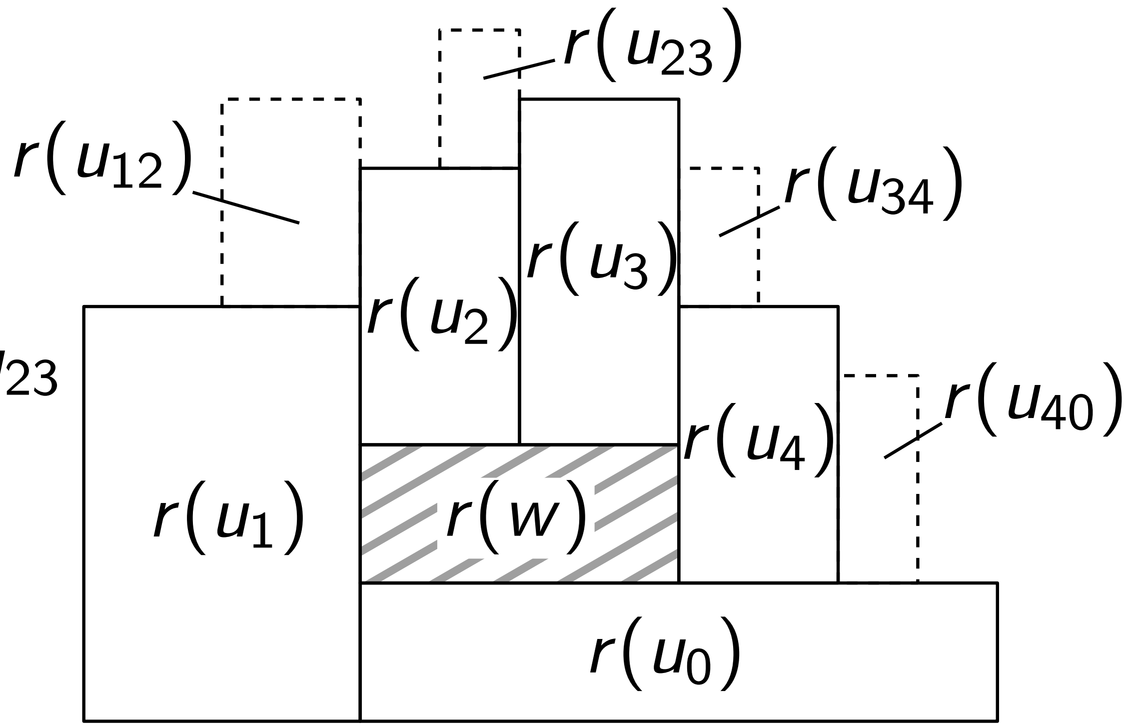


Proof of correctness for outerplanar graphs

Invariant:

There is a *free corner* *for the next rectangle* $r(w)$ *to be placed with* $w \in V \setminus V_k$*, in such a way that it is* *adjacent to its predecessors* $r(u)$ *and* $r(v)$ *with* $u, v \in V_k$*.*





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A graph is k-outerplanar if removing all vertices on the outer face in its embedding results in a $(k-1)$-outerplanar graph

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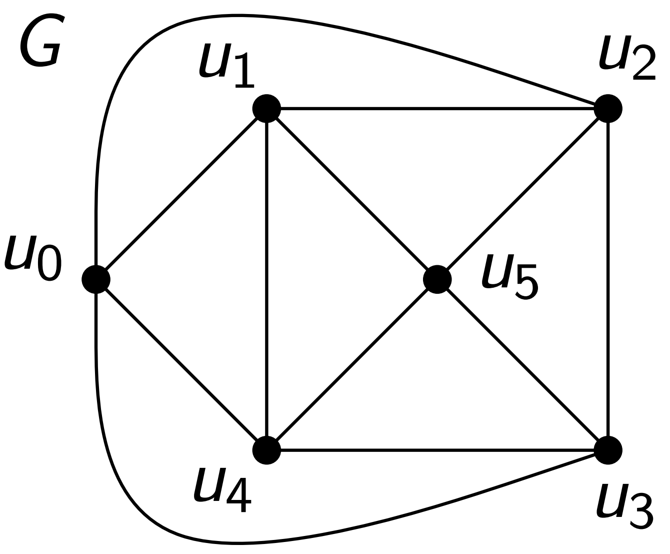

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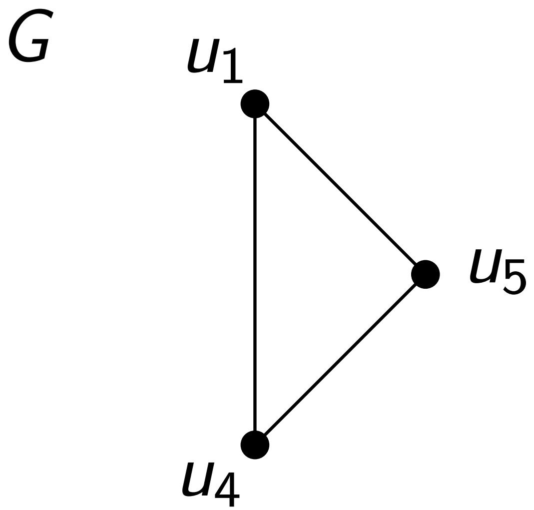

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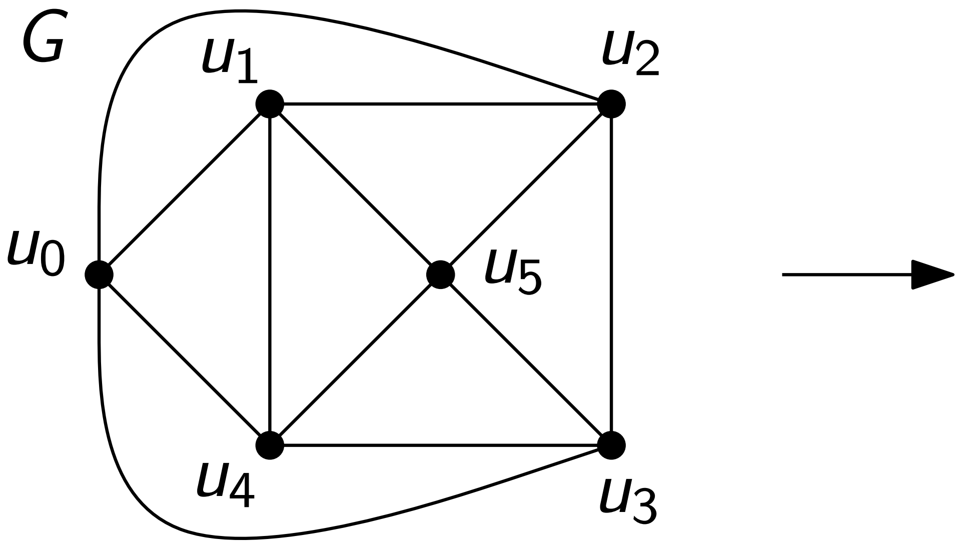

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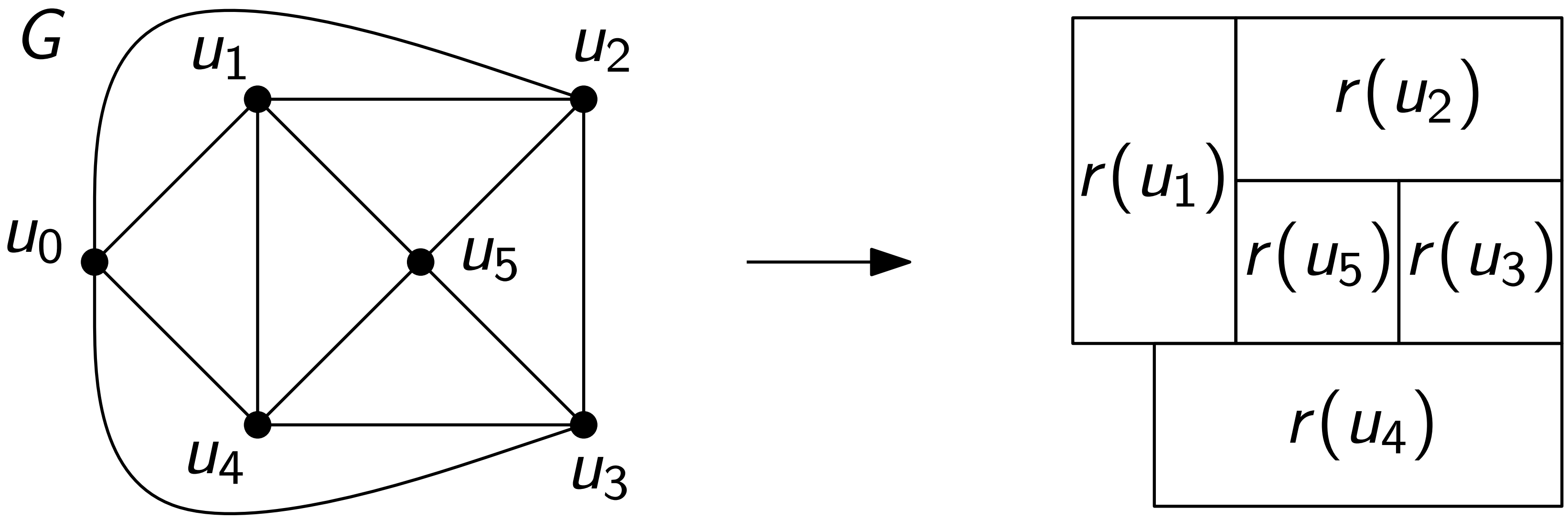

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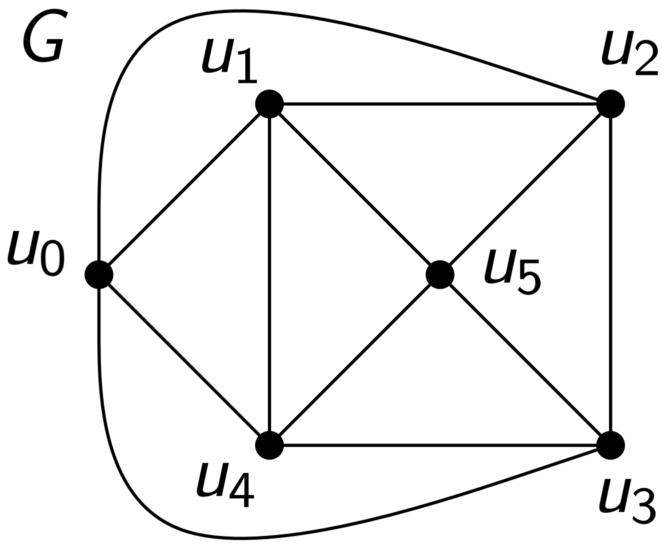

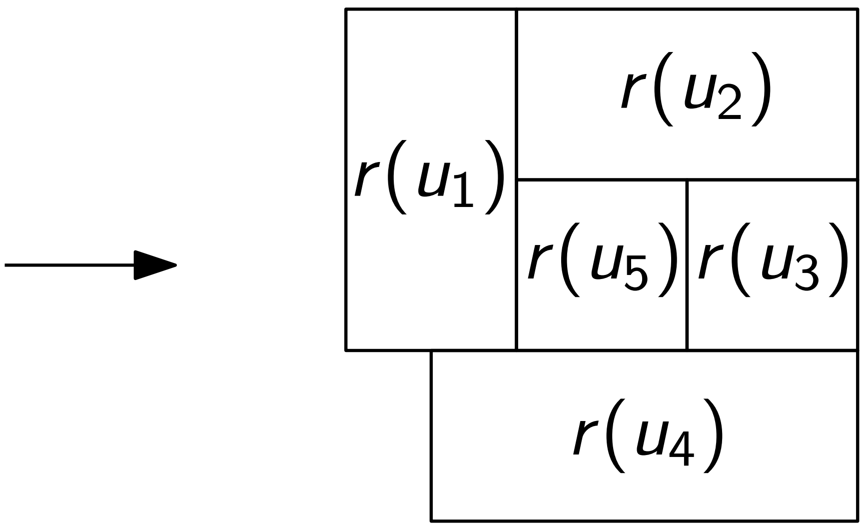

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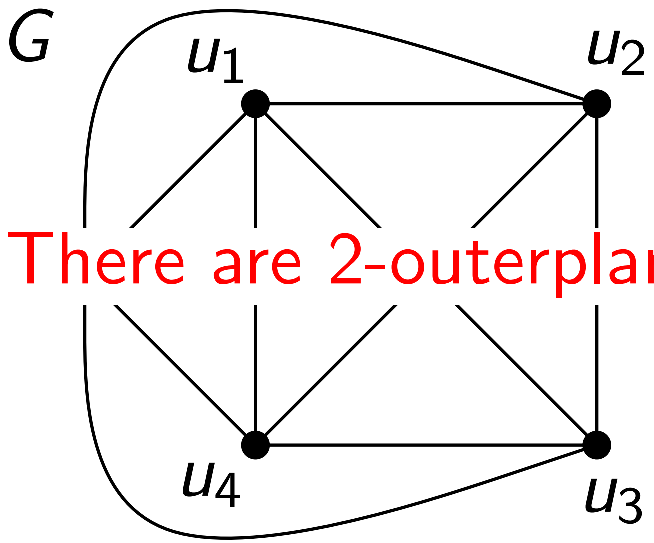

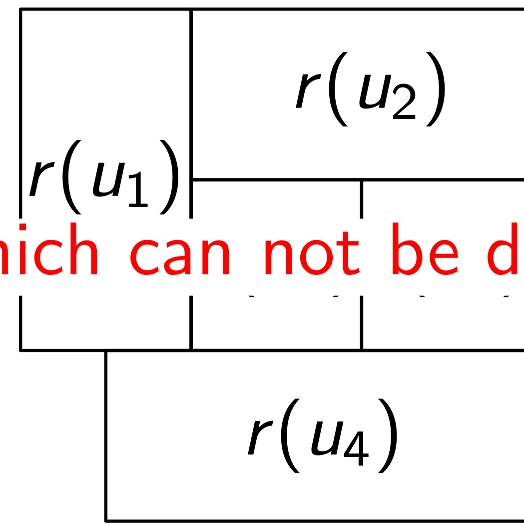

There are 2-outerplanar graphs, which can not be drawn!

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Recapitulation

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Thank you for your attention!
Questions?