Multiobjective Disk Cover Admits a PTAS

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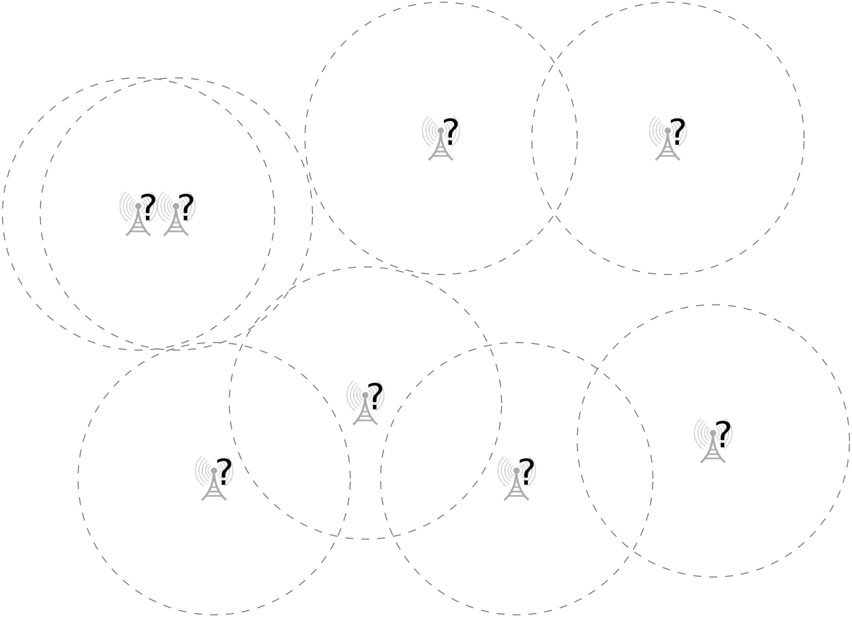
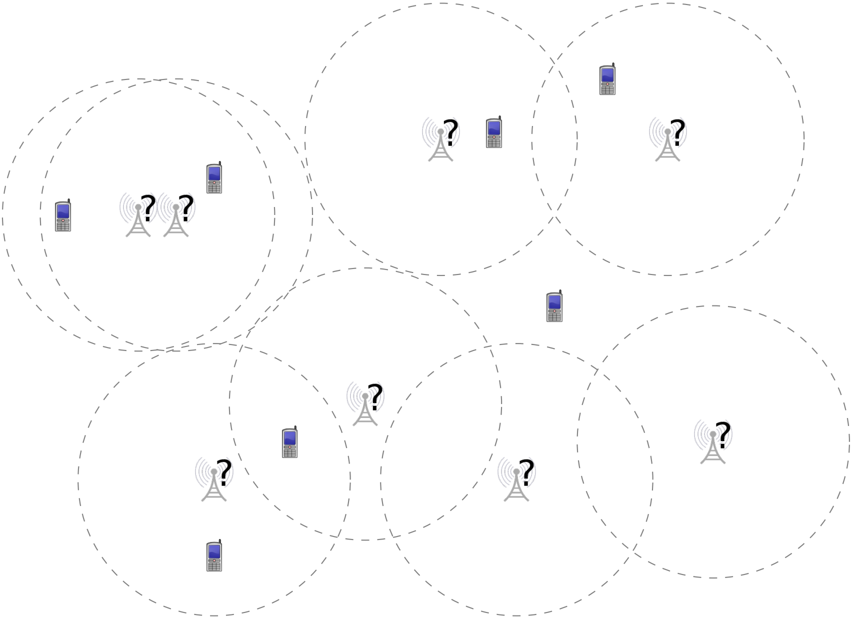
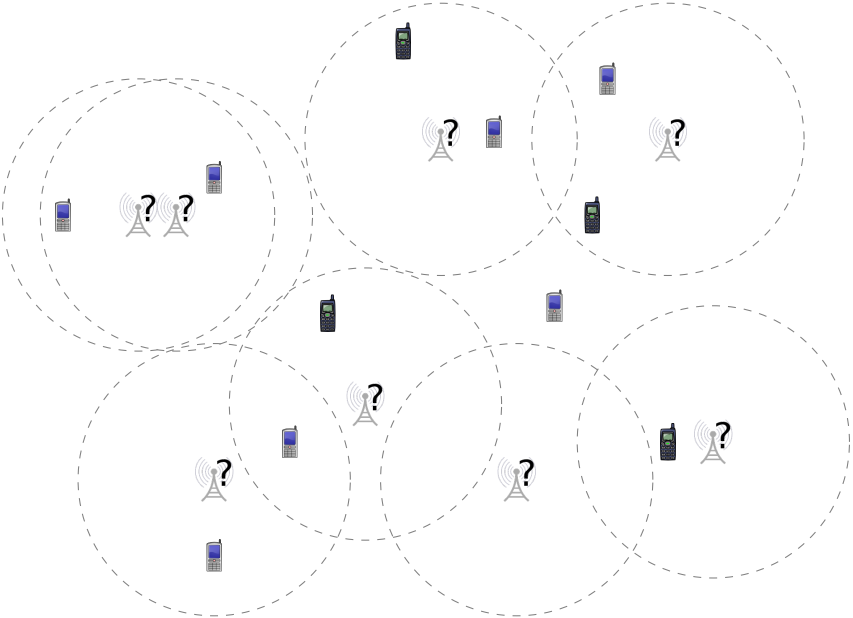
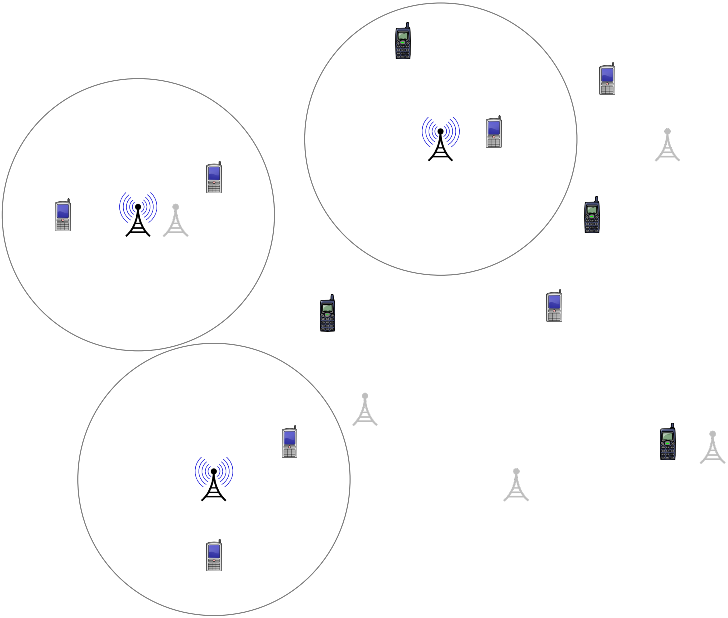
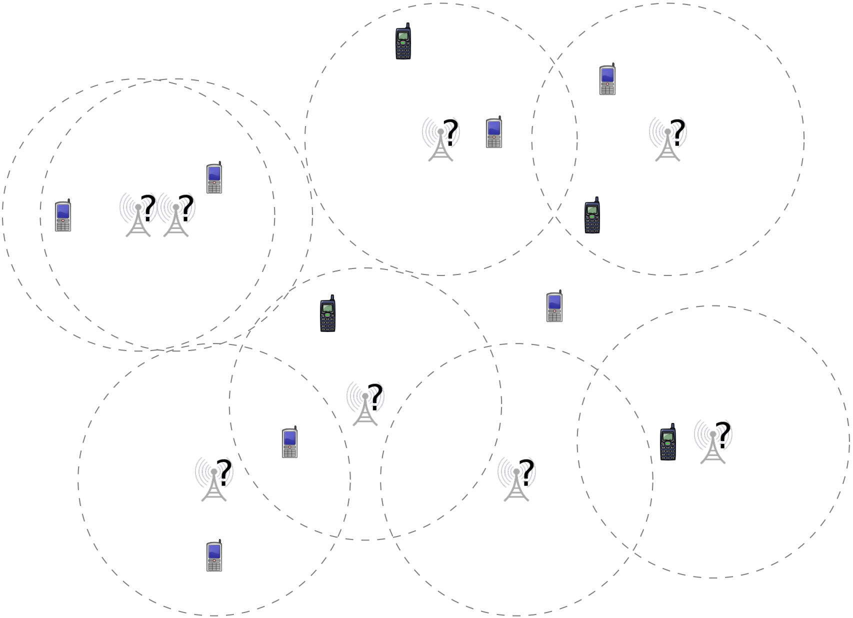
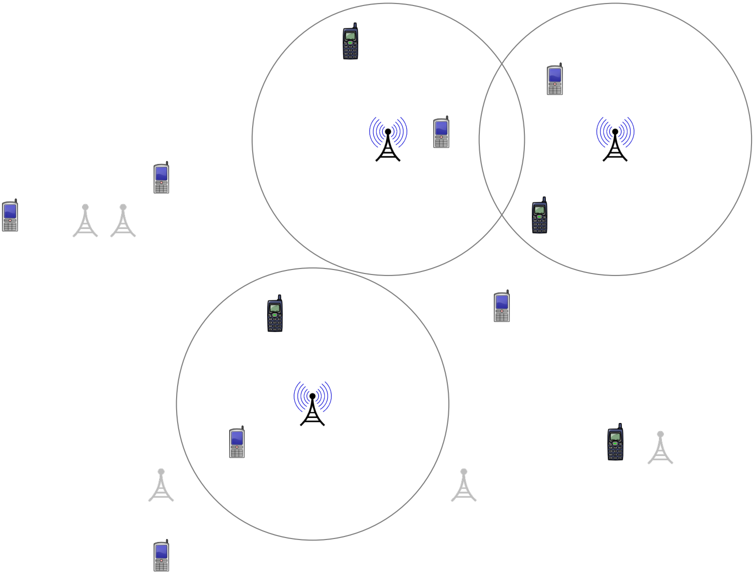






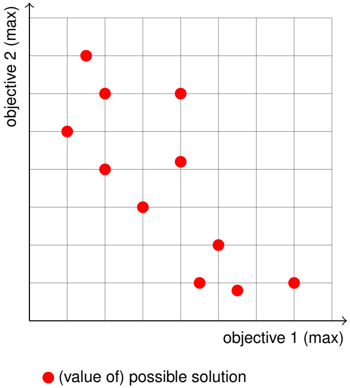






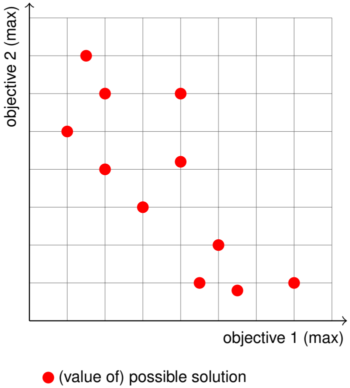
Example

General two-objective optimization problem, both objectives have to be maximized. Values of solutions for one instance are depicted on the right.



Singleobjective Optimization

Constraint Method: Maximize one objective while constraining all others to some arbitrarily-chosen minimal value.



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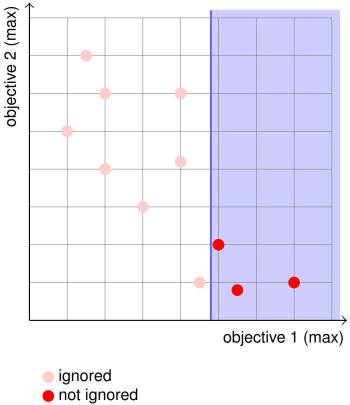

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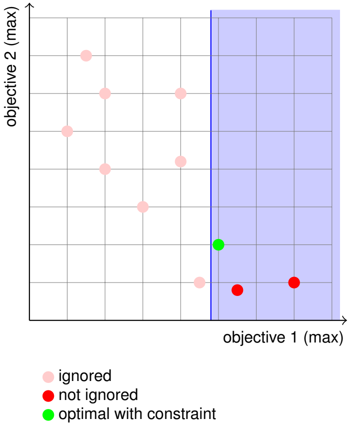
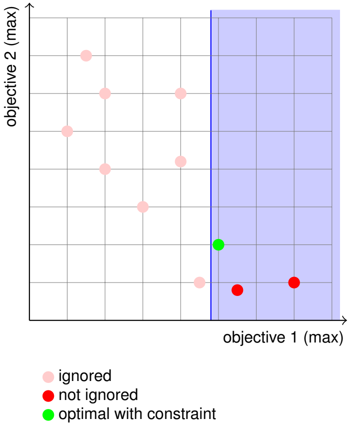


Singleobjective Optimization

Constraint Method: Maximize one objective while constraining all others to some arbitrarily-chosen minimal value.

Observation

Information about the solution set or experience is needed in order to get a good constraint.

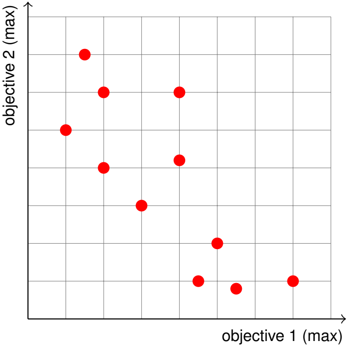


Definition

A solution is Pareto-optimal if there is no other solution that is

- at least as good in every objective and
- better in at least one objective.

The Pareto-set is the collection of all Pareto-optimal solutions.



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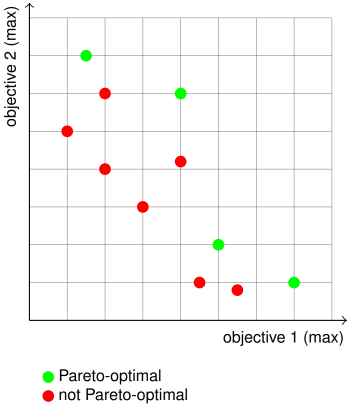
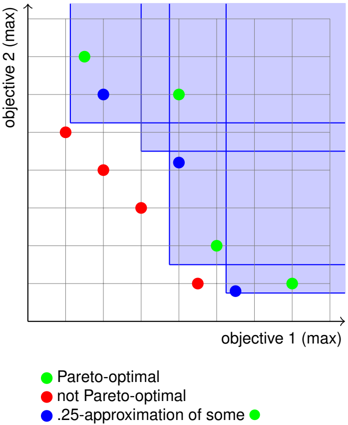


Definition

A set of solutions S is an ε-approximation of the Pareto-set P if for every $s \in P$ there is some $s' \in S$ such that for every objective s' is better or only in a factor of ε worse than s.



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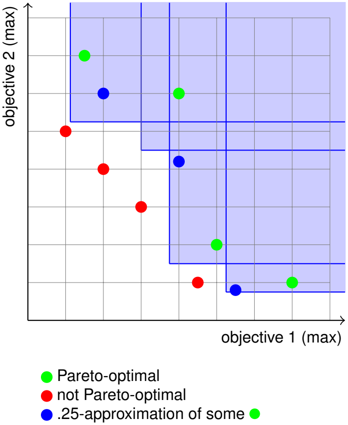
Advantages of ε-Approximation:

1. solutions easier to compute
2. fewer solutions (Pareto set can be exponential)



Model Should be Chosen Wisely

1. demand for minimum distance can improve approximability

Model Should be Chosen Wisely

1. demand for minimum distance can improve approximability

- In general: base stations can be built close to each other
- In practice: this is not the case (think of interference), also holds for other settings.
- $\rightarrow$ base stations in solutions (not possible positions) must have minimum distance of $\rho \cdot r$

Model Should be Chosen Wisely

1. demand for minimum distance can improve approximability
2. formalization using constraint method can dramatically degrade approximability

Fix some $\rho \in (0, \frac{1}{2}]$ and assume $\mathrm{P} \neq \mathrm{NP}$.

- For all numbers of customer types $k \geq 2$, the constraint optimization problem for one type is not in APX.
- For exact cover, this holds additionally for the constraint optimization problem for the number of base stations.

Definition (k-Disk Cover ($k\text{-DC}_\rho$))

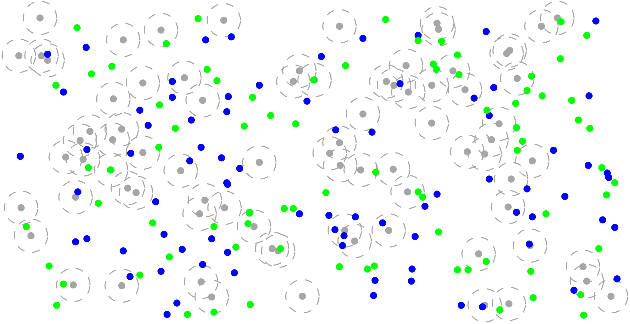
- Instance: finite set of disk positions $D \subseteq \mathbb{Z} \times \mathbb{Z}$, disk radius $r \in \mathbb{N}$, k finite sets of points $P_1, \ldots, P_k \subseteq \mathbb{Z} \times \mathbb{Z}$
- Solution: selection $S \subseteq D$ such that for all different $x, y \in S$, $||x - y||_2 \geq \rho \cdot r$
- Goals: $(\min |S|, \max |C_1|, \ldots, \max |C_k|)$ where $C_i = \{x \in P_i \mid \exists y \in S, ||x - y||_2 \leq r\}$

Remarks

- possible disk positions are fixed
- up to now only studied in singleobjective manner, but multiobjective way is more natural
- we succeeded in finding a PTAS, works also for many variants

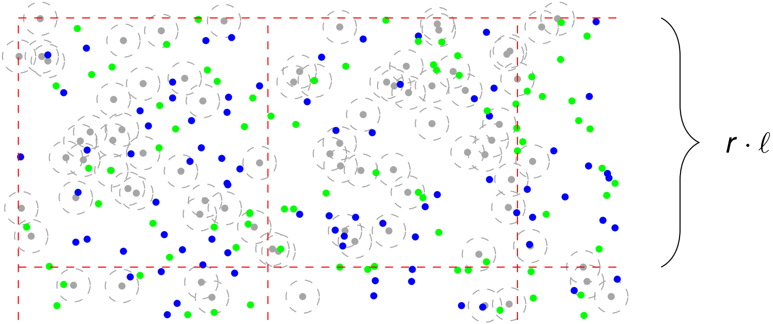
PTAS for k-DC_ρ

Technique: adaption of Hochbaum-Maass shifting strategy.
— fix approximation parameter l ($l \sim 1/\varepsilon$)



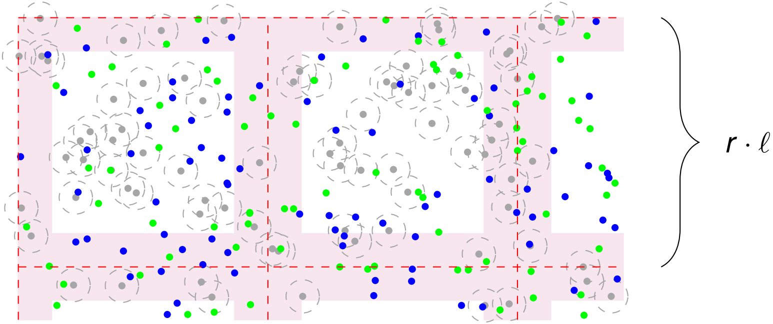
PTAS for k-DC_ρ

Technique: adaption of Hochbaum-Maass shifting strategy.
— subdivide plane in squares of side length $r \cdot l$.



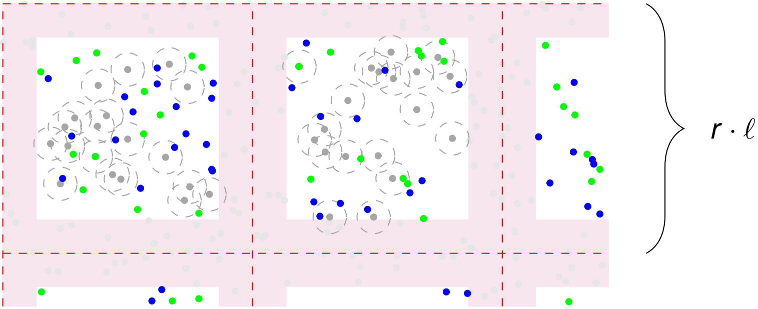
PTAS for k-DC_ρ

Technique: adaption of Hochbaum-Maass shifting strategy.
— determine borders of interference



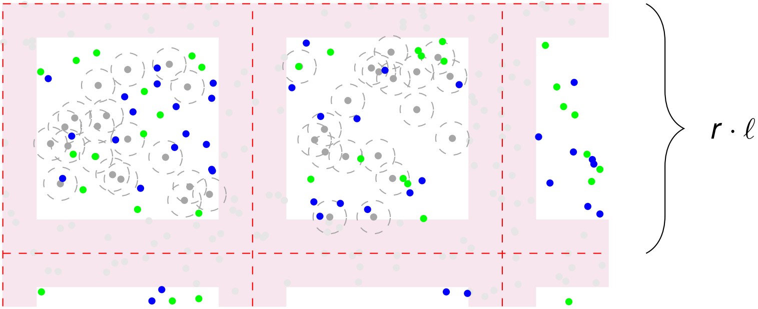
PTAS for k-DC_ρ

Technique: adaption of Hochbaum-Maass shifting strategy.
— remove points on borders (hopefully few)



PTAS for k-DC_ρ

Technique: adaption of Hochbaum-Maass shifting strategy.
— solve squares independently of each other



Solving the Small Squares

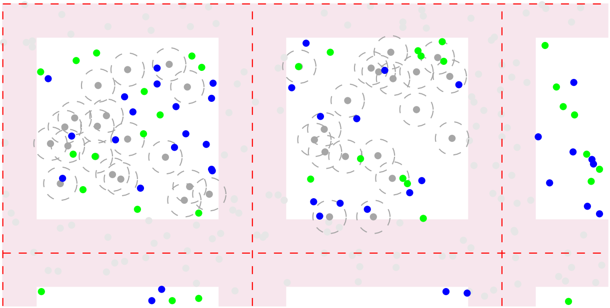
Compute complete (exact) pareto-set using exhaustive search. Possible because demand for minimum distance implies polynomial bound on number of solutions.

Combining the Solutions

Combine pareto-sets of squares using dynamic programming.

PTAS for k-DC_ρ

Technique: adaption of Hochbaum-Maass shifting strategy.
— improve approximation: shift the grid and repeat (use the „best“ solution)



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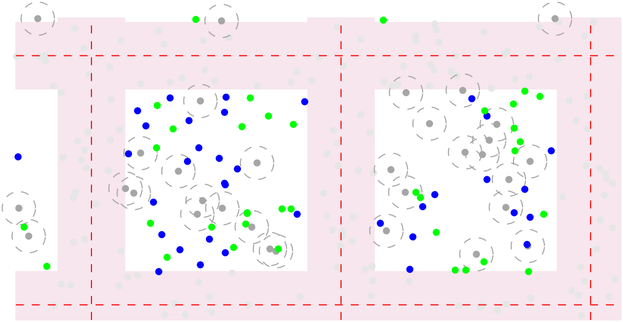

Result

We get a $\frac{16}{l}$-approximation in polynomial time.
Furthermore: algorithm notices if it performed better by counting the number of deleted points.

Conclusion

- multiobjective optimization is more natural than singleobjective optimization
- there is a PTAS for multiobjective disk cover that
  - can be adjusted for many variants of the problem
  - notices if it performs better than the general lower bound

Further Research

- Can the minimum distance demand be removed?
- replace exhaustive search by good heuristic
- replace grid by arbitrary subdivision