Phylogenetic trees and networks are rooted, binary graphs with labelled leaves used to visualise evolutionary histories.

A tree-based network $N$ is a phylogenetic network that has a spanning tree $T$ which is the subdivision of a phylogenetic tree $[1]$. We call edges not covered by $T$ cross edges.

A tree-based network is used to model reticulate events that extend a phylogenetic tree. Depending on the context, endpoints of cross edges may have the same height, e.g. for horizontal gene transfer $[3]$, or different heights, e.g. for recombination $[4]$.

We consider drawings of a tree-based network $N$ on tree $T$ where

- $T$ is drawn planar,
- leaves are equidistant, and
- heights are preserved.

We assume that heights are the same for leaves, but distinct otherwise, except for the two endpoints of horizontal cross edges.

Our goal is to minimise the number of crossings, which as we note is fully determined by

- the order of the leaves, or equivalently
- the rotations of the vertices of $T$.

We define a drawing as horizontal if it is $x$-monotone with at most two bends.

### Horizontal Drawing Style

- Cross edges are horizontal lines.

**Note:** Rotation of one vertex can effect best rotation of other vertices.

**NP-complete**

**Proof by reduction of MAX-CUT**

- Gadget for edge $\{u, v\}$ adds 1 crossing iff $u_i$ and $v_i$ have “same” rotation.

- Gadget for vertex $u$ adds thick blue bundles fix upper part of tree.

**Example**

- Reaching vertex $u$ of cross edge $\{u, v\}$
  - compute width of left/right subtree for $v$ of $T$ at height of $u$ in $O(n)$, and
  - propagate potential crossings with other subtrees up to lca$(u, v)$ in $O(n)$ $[2]$.

- Reaching vertex $v$ of $T$:
  - decide rotation based on number of potential crossings.

Lastly, extend partial order of nested vertical segments to total order of all. Algorithm runs in $O(nk)$.

### Ears Drawing Style

- Cross edges drawn with two bends
- Vertical segment right of subtree containing endpoints
- Allows labelling with statistical support

**Note:** Best rotation of vertex determined by width of right subtree where cross edges leave the left subtree and vice versa.

**Sweep from leaves to root**

Let $N$ have $n$ vertices and $k$ cross edges.

- Vertices of $T$ store potential crossings from left/right subtree through right/left subtree.

- Reaching $u$ of cross edge $\{u, v\}$:
  - $u$’s forced to have same rotation with purple bundles.

**Snake Drawing Styles**

Cross edges drawn $x$-monotone with/as

- two bends, • curve, • straight line.

Reduces to horizontal style $\Rightarrow$ NP-complete

By propagating potential crossings up to the root, the algorithm also works for a modified ears drawing style, where the vertical segments are to the far right of $T$.

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