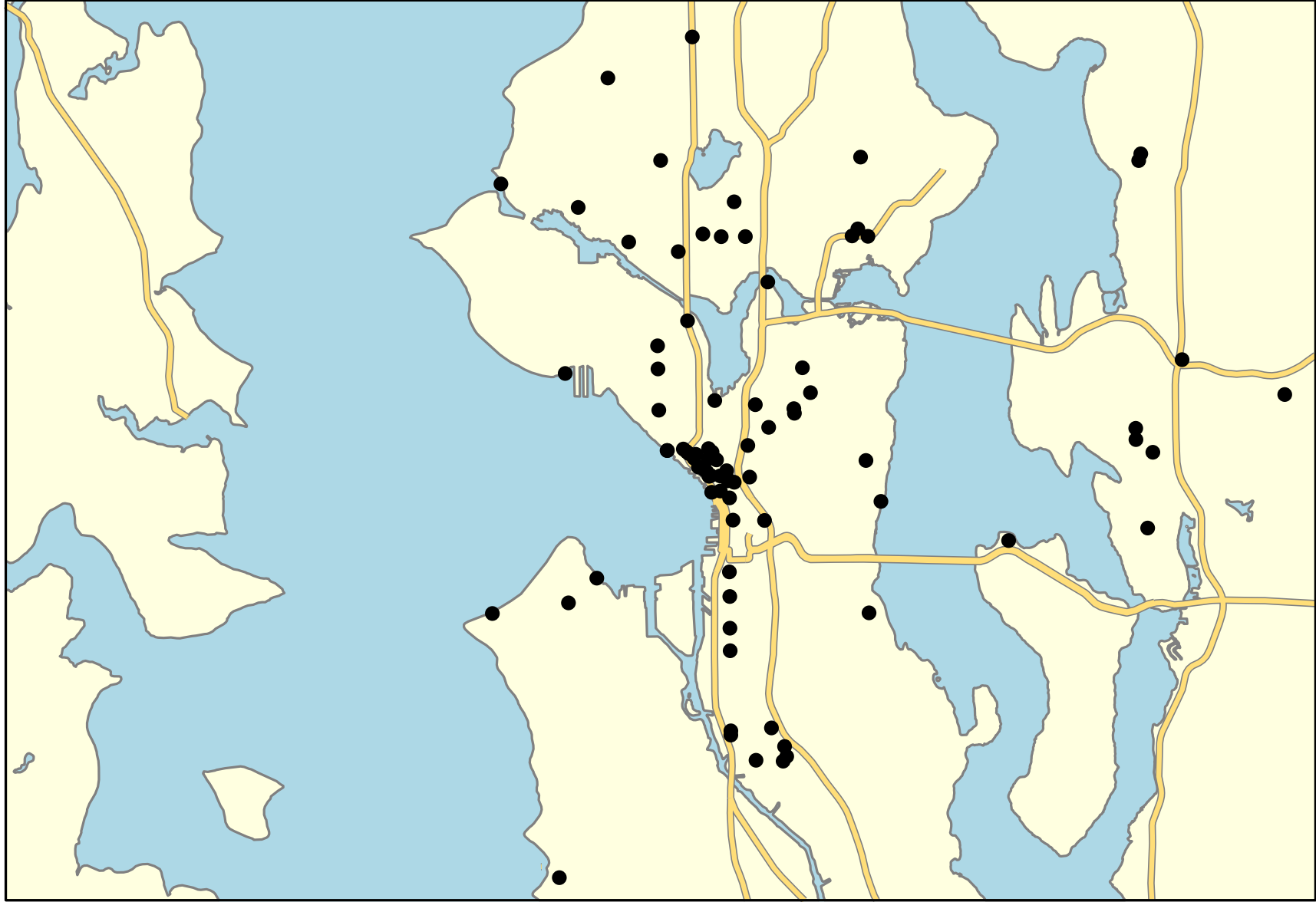
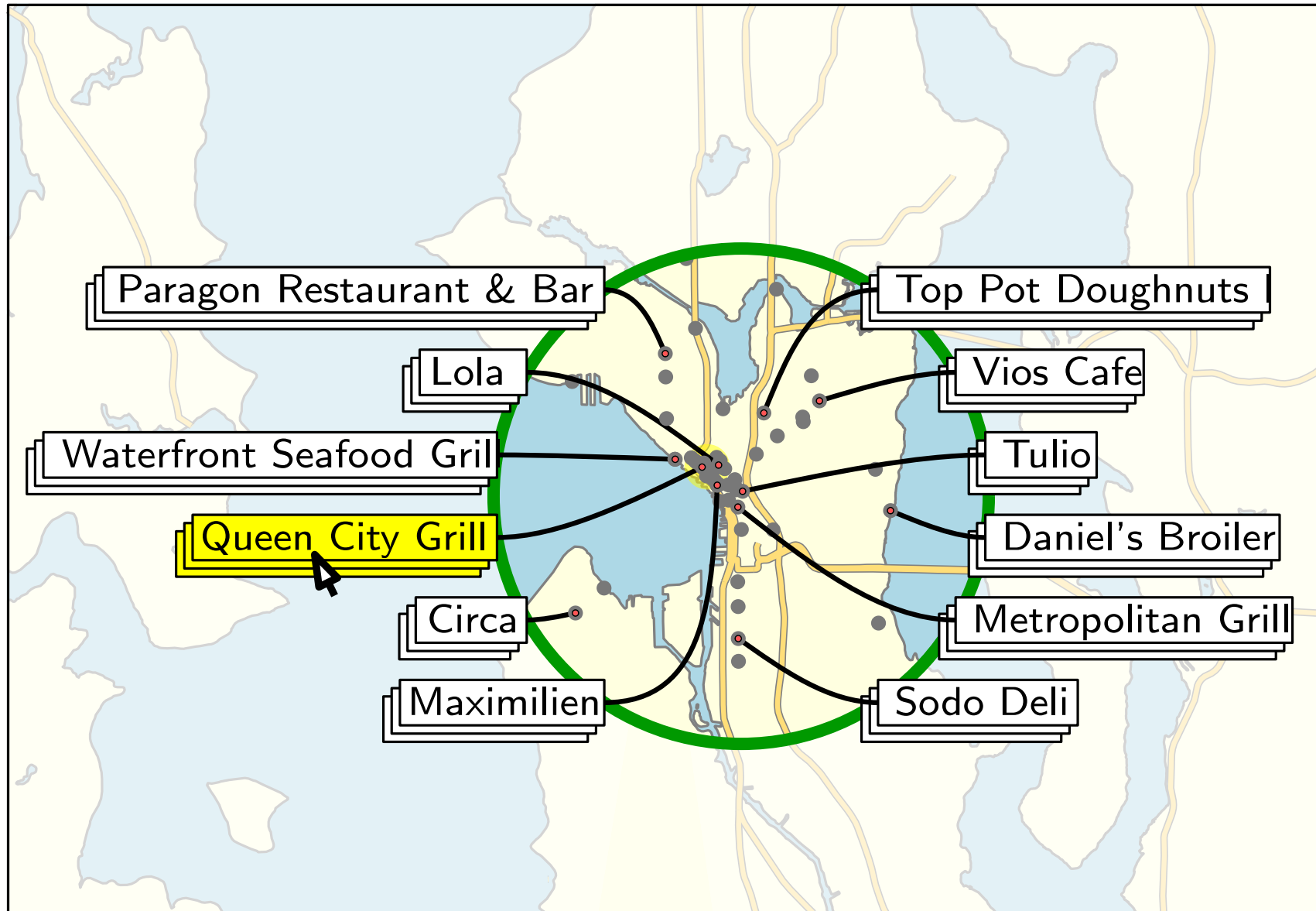


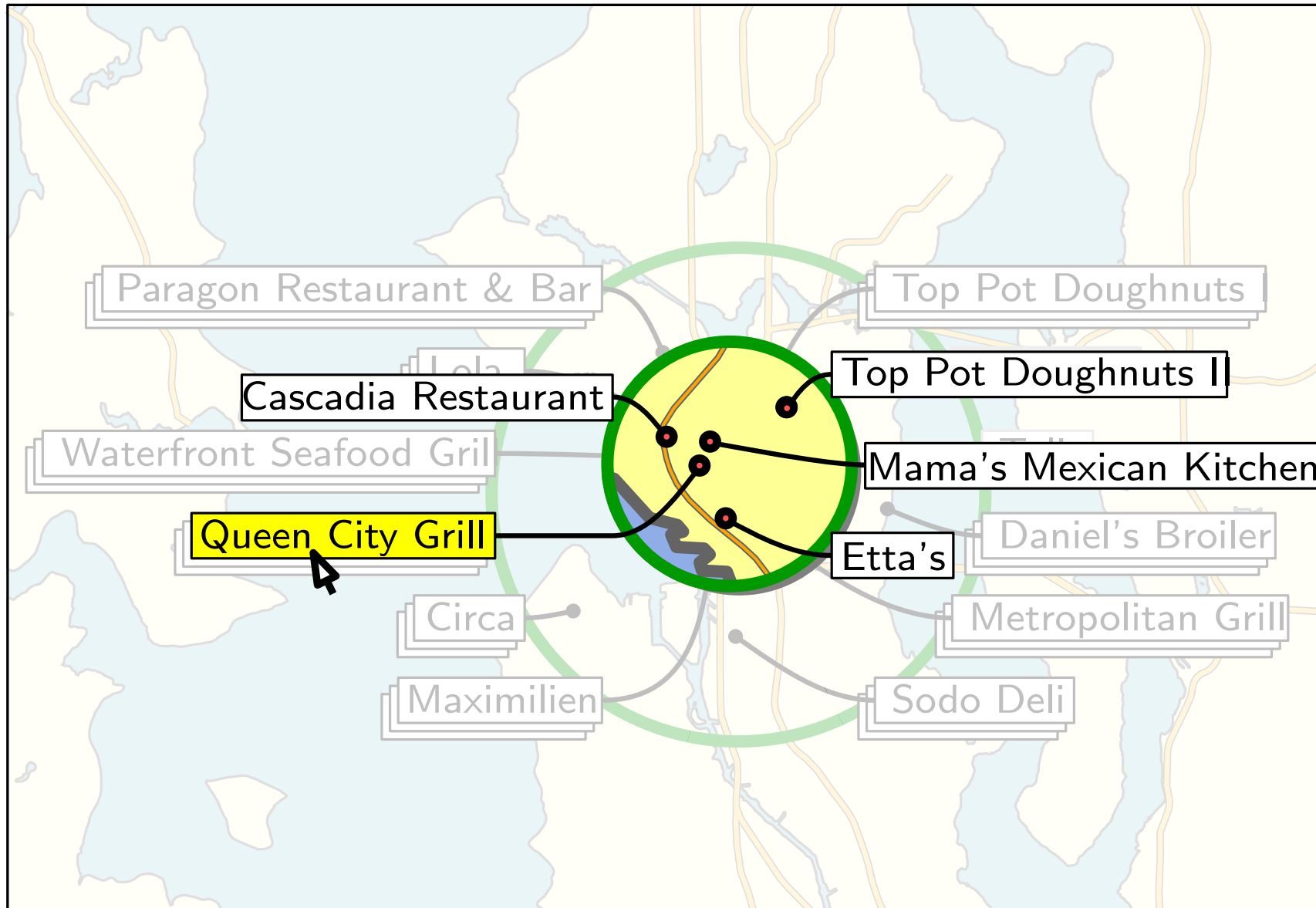
# Algorithms for Labeling Focus Regions

Martin Fink  
Lehrstuhl für Informatik I  
Universität Würzburg

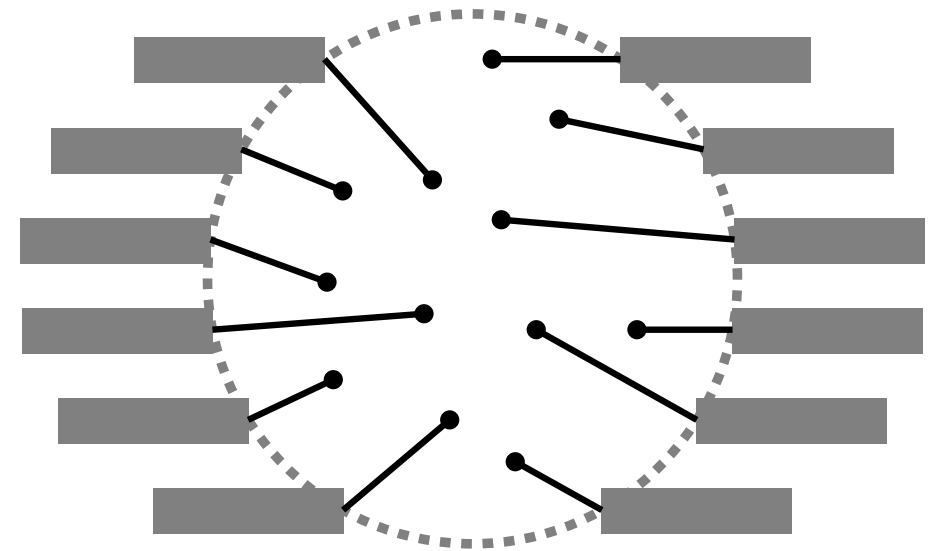
Joint work with  
Jan-Henrik Haunert, André Schulz, Joachim Spoerhase, and Alexander Wolff





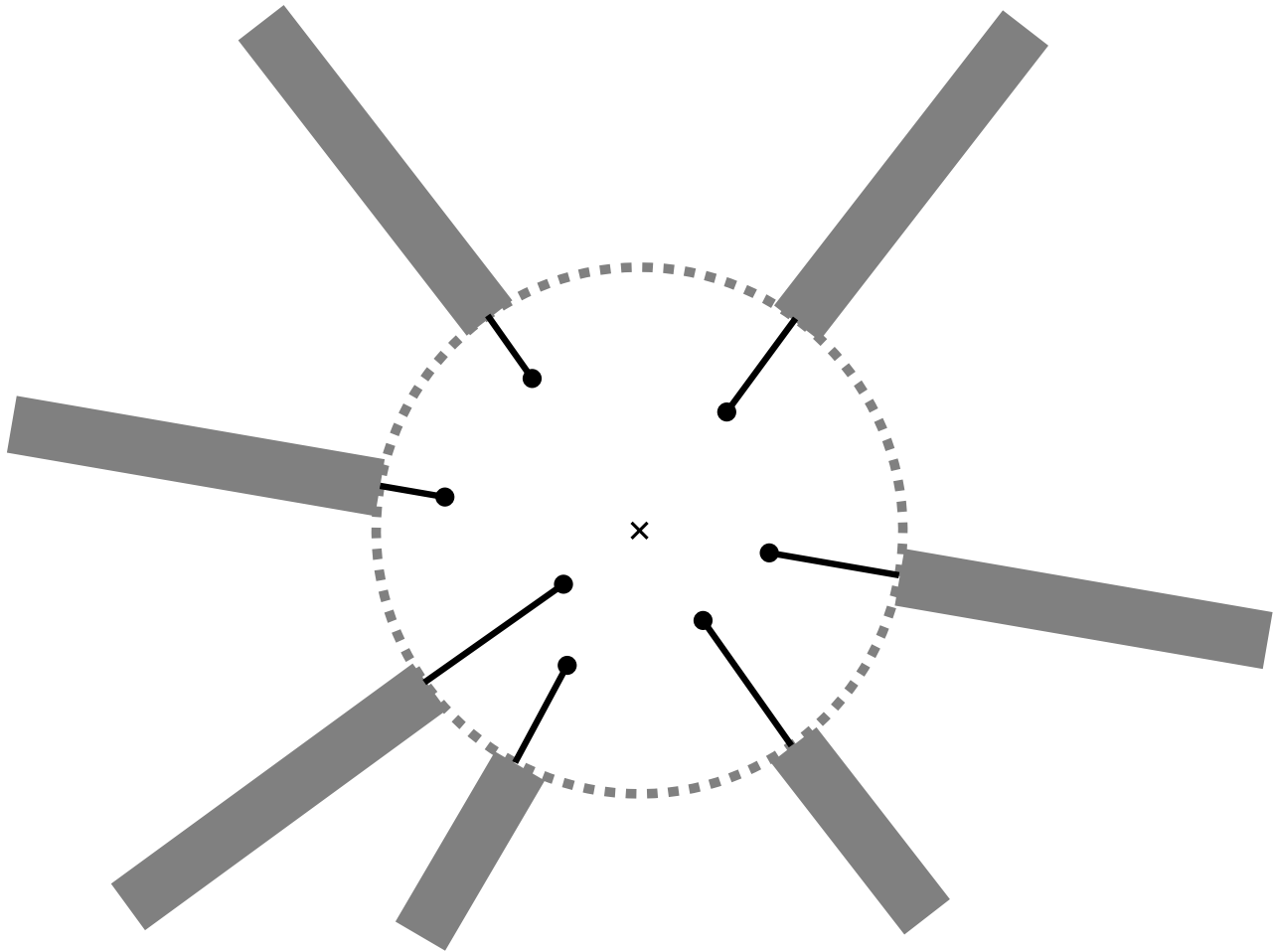


# Our models

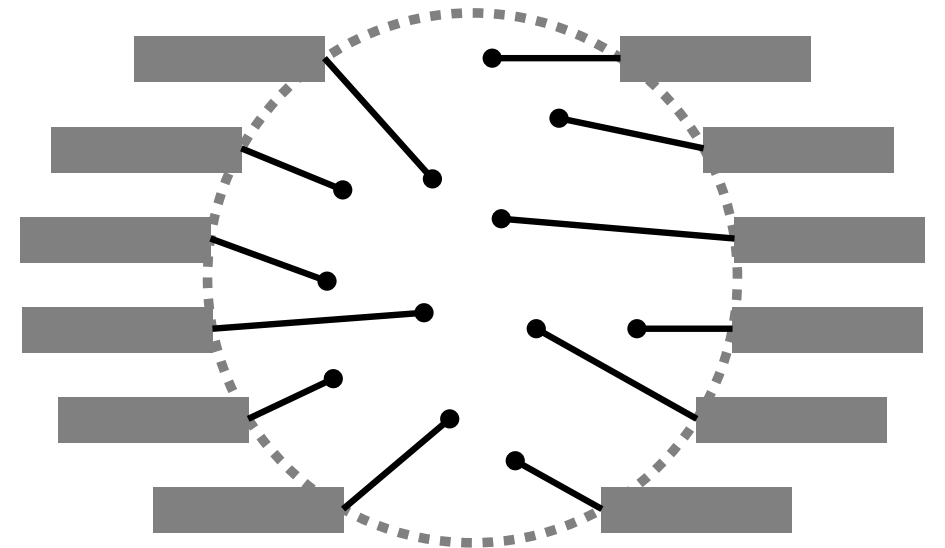


free leaders

# Our models



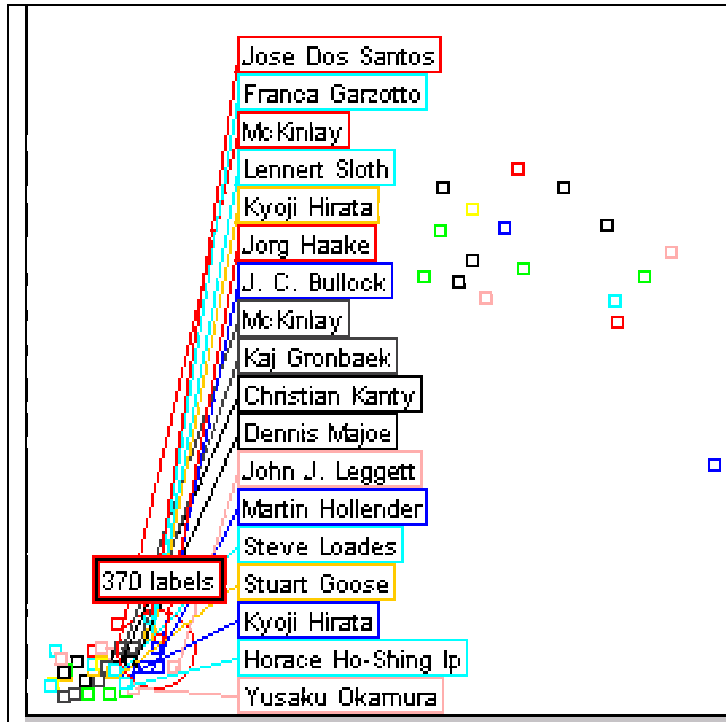
radial leaders



free leaders

# Previous Work

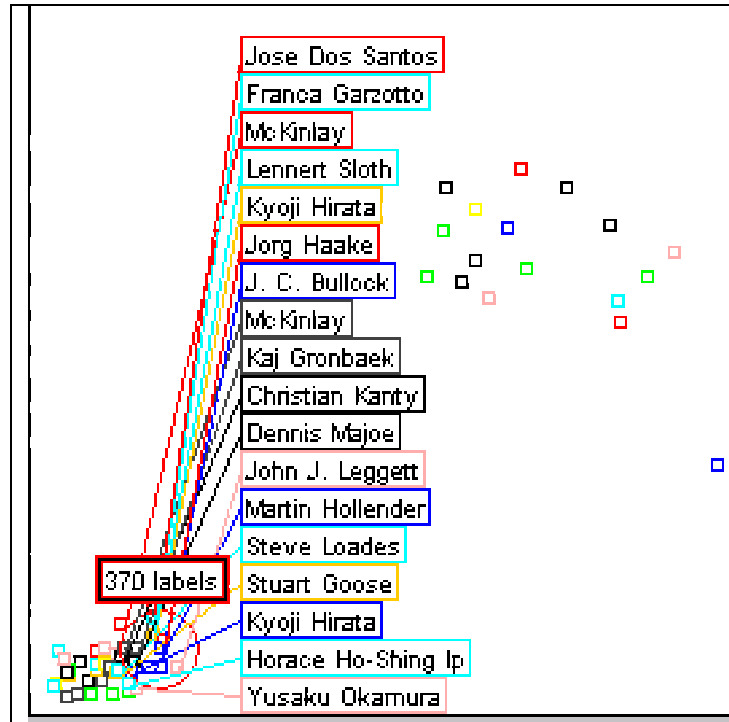
## Excentric Labeling



[Fekete and Plaisant, 1999]

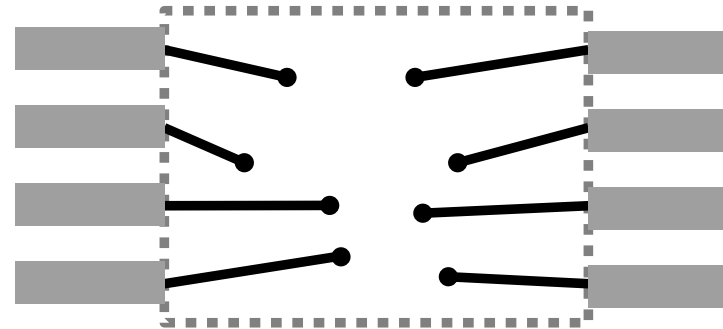
# Previous Work

## Excentric Labeling



[Fekete and Plaisant, 1999]

## Boundary Labeling

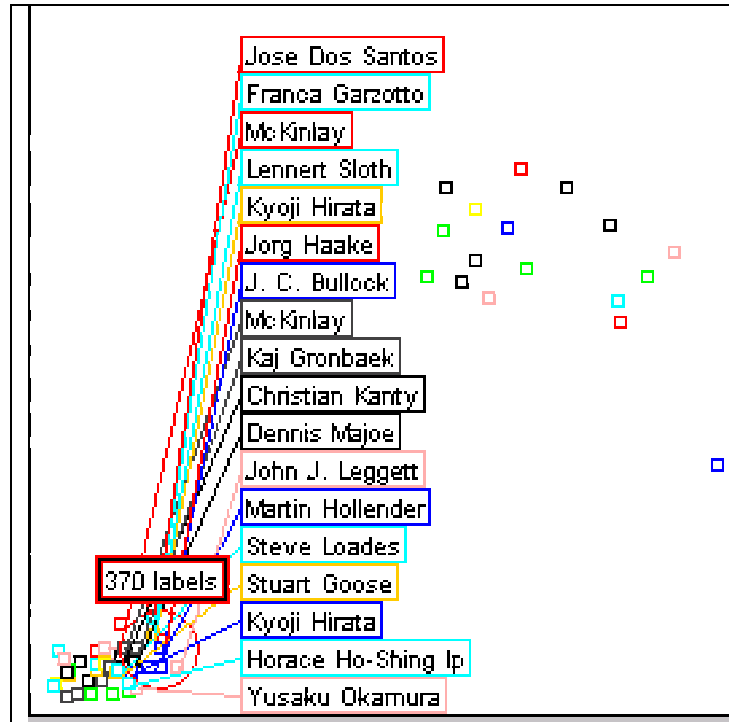


[Bekos et al., 2007]



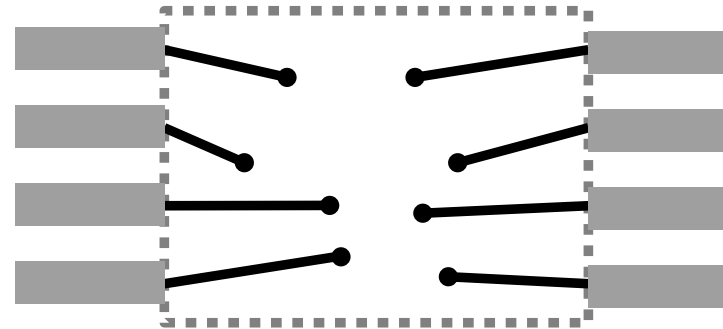
# Previous Work

## Excentric Labeling



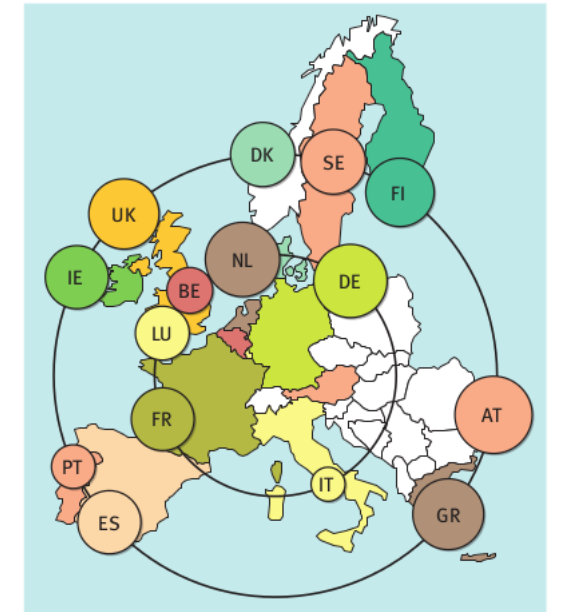
[Fekete and Plaisant, 1999]

## Boundary Labeling



[Bekos et al., 2007]

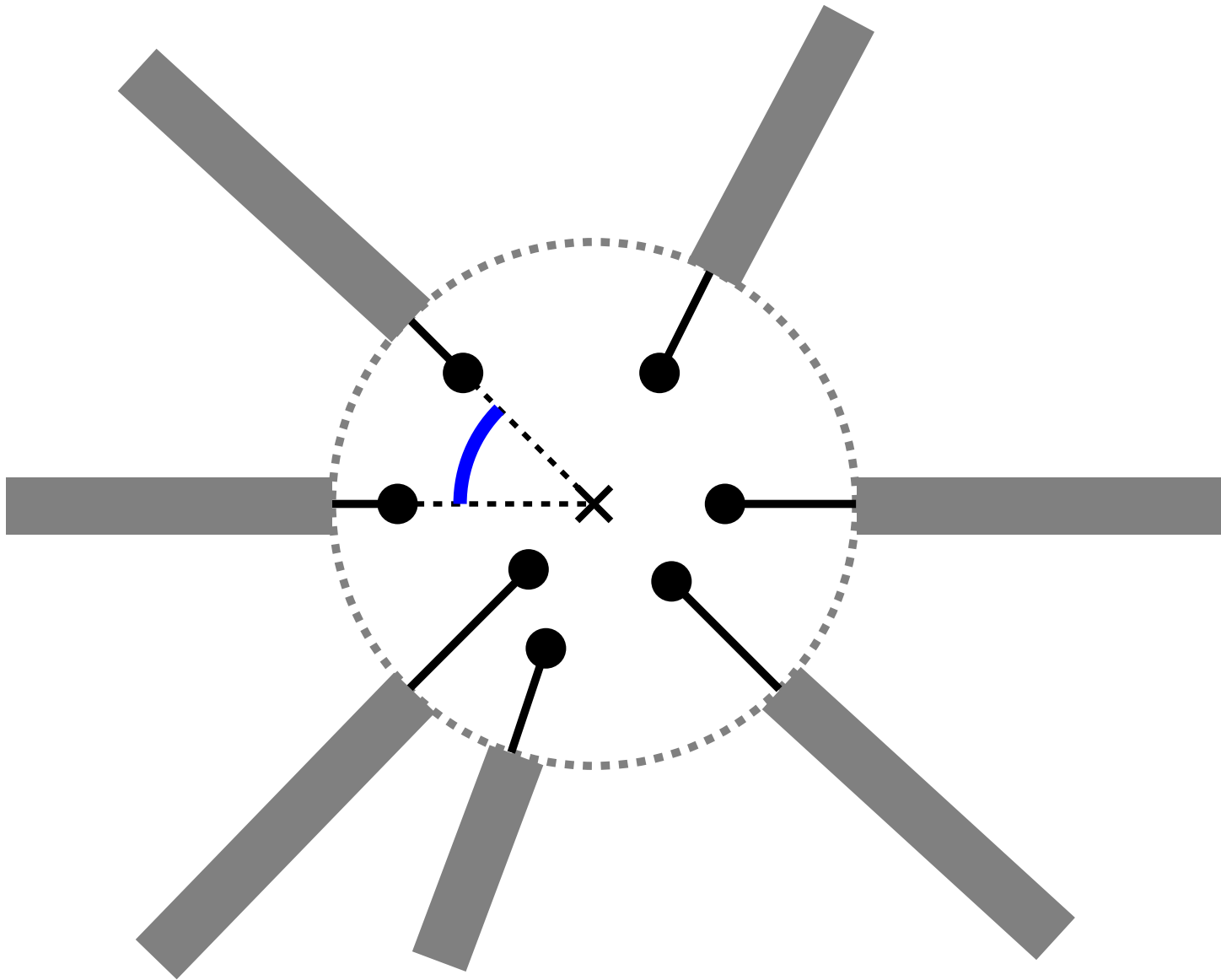
## Necklace Maps



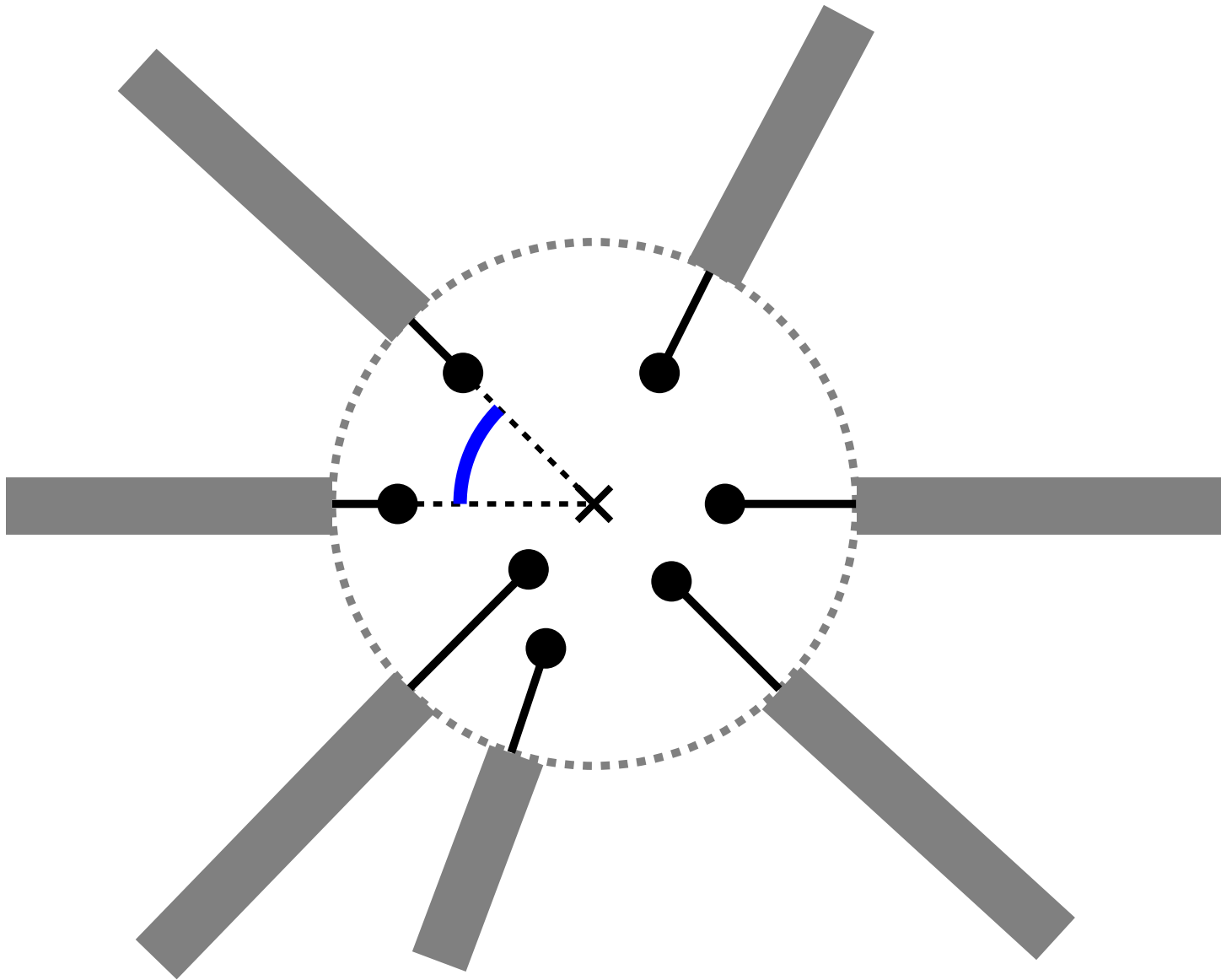
[Speckmann and Verbeek, 2010]

# The Radial Leader Model

- minimum allowed angle to avoid label collisions

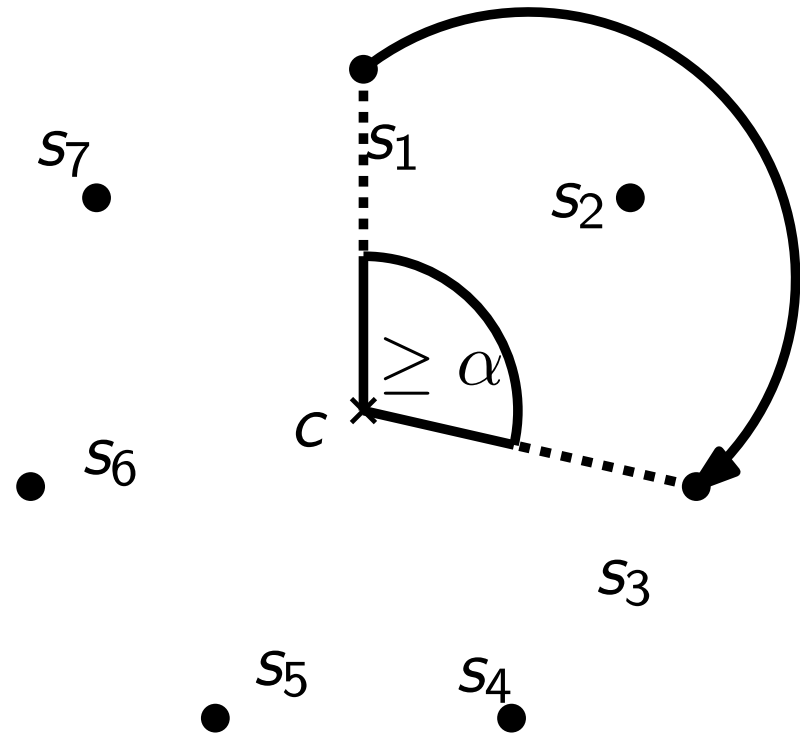


# The Radial Leader Model



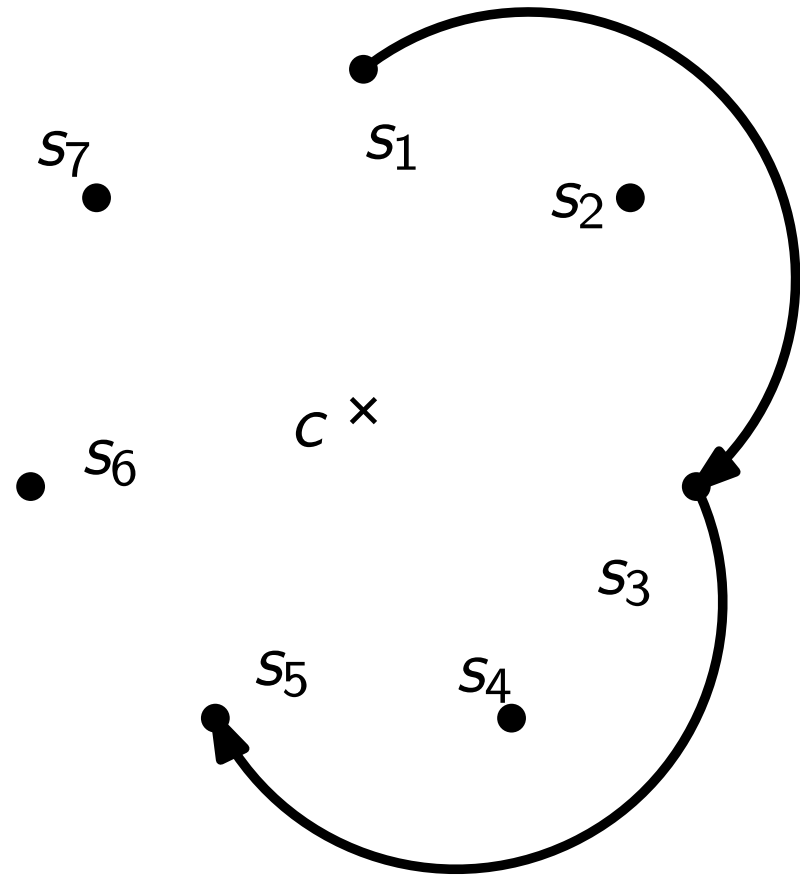
- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program

# The Radial Leader Model



- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program

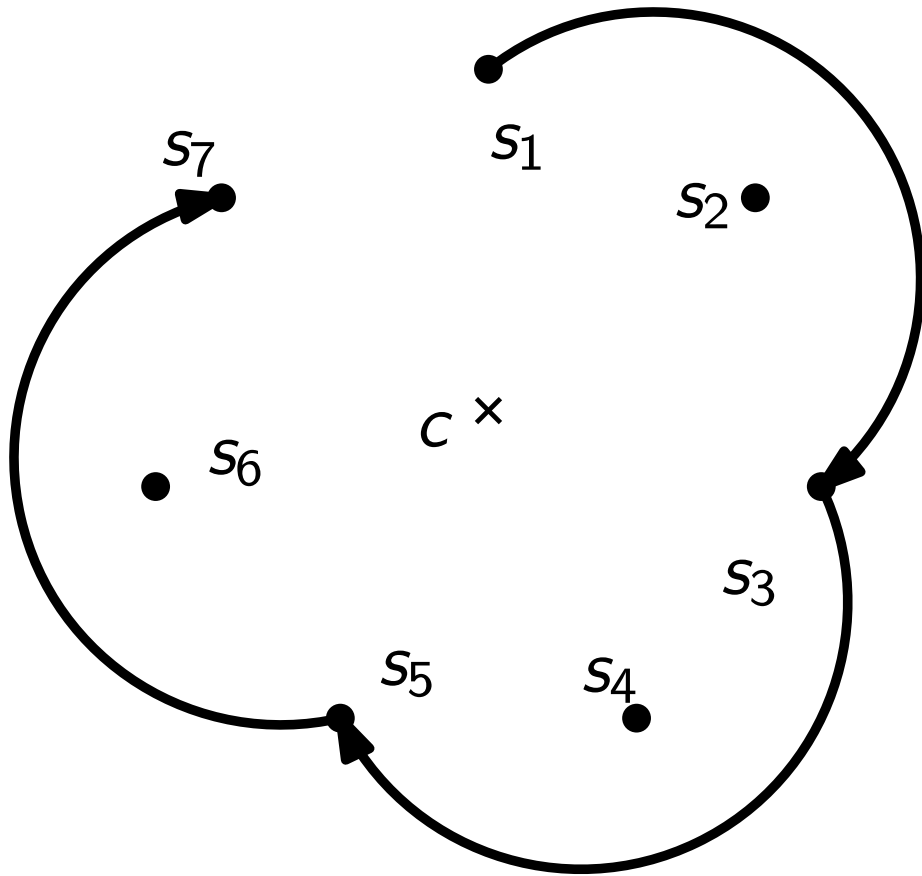
# The Radial Leader Model



- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program

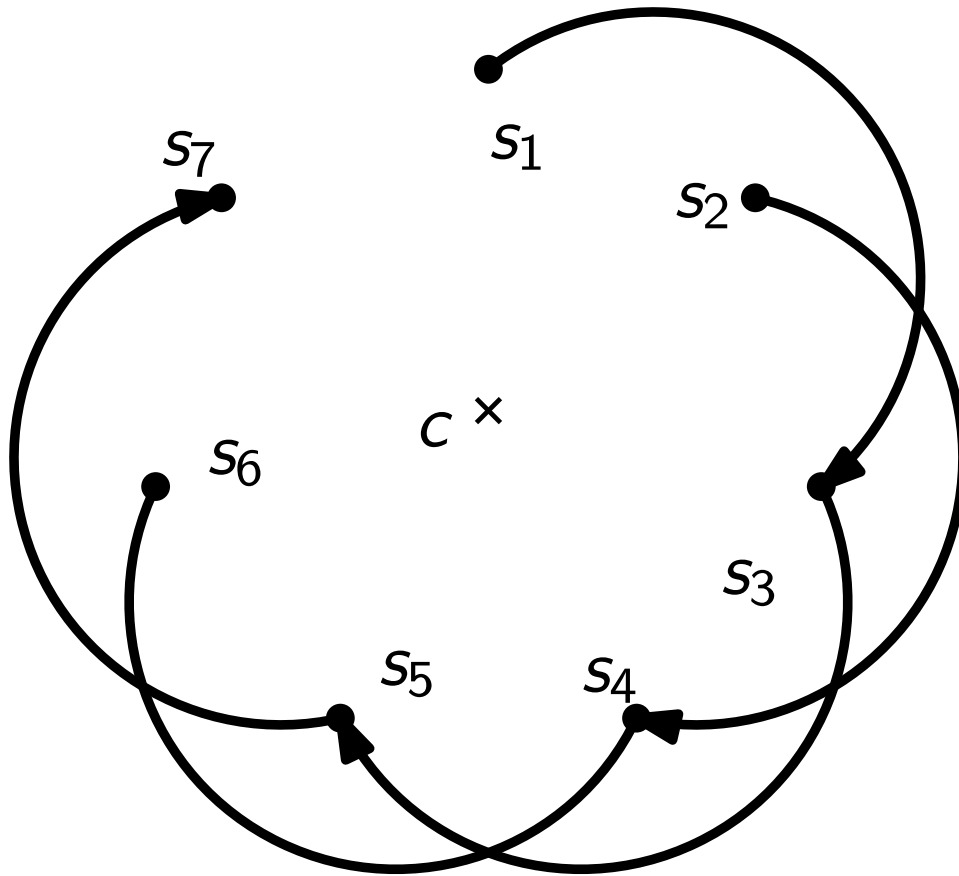
# The Radial Leader Model

- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program



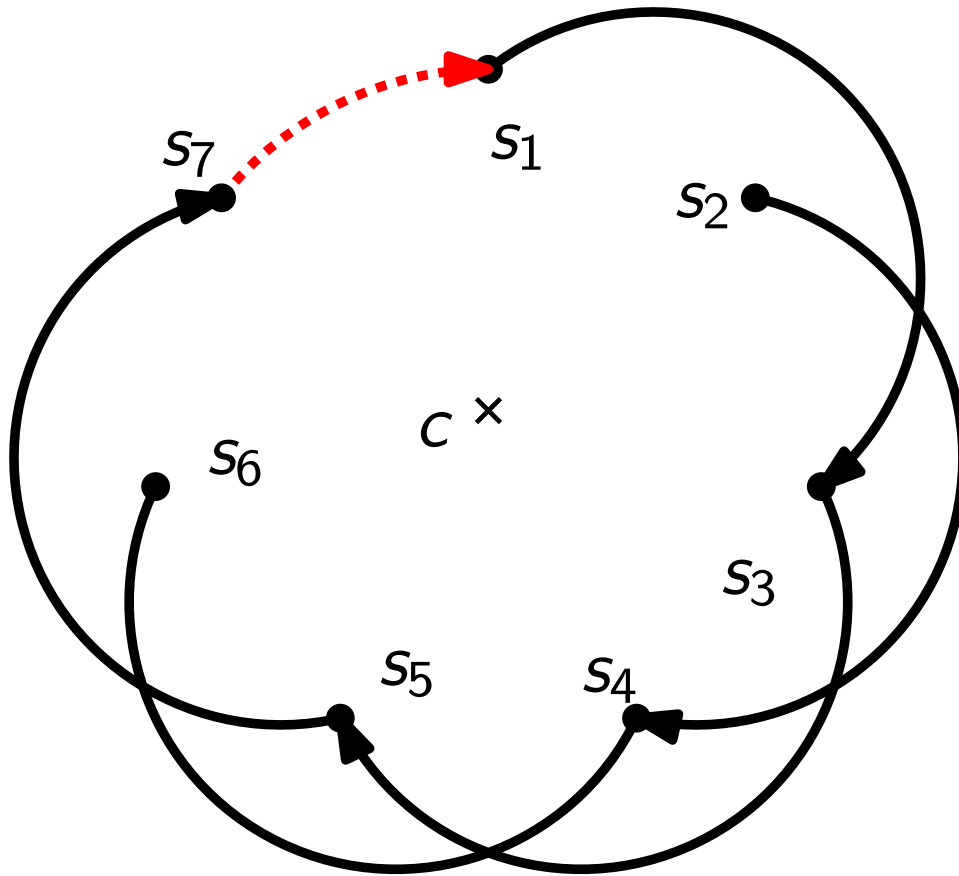
# The Radial Leader Model

- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program



# The Radial Leader Model

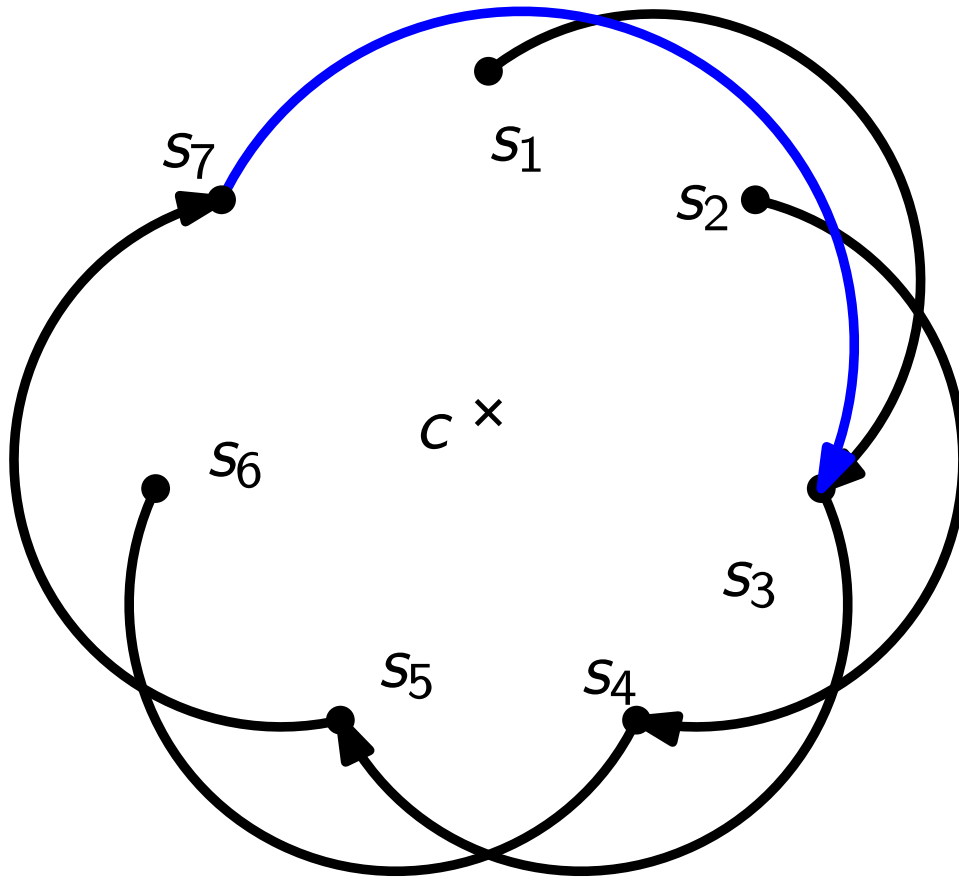
- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program



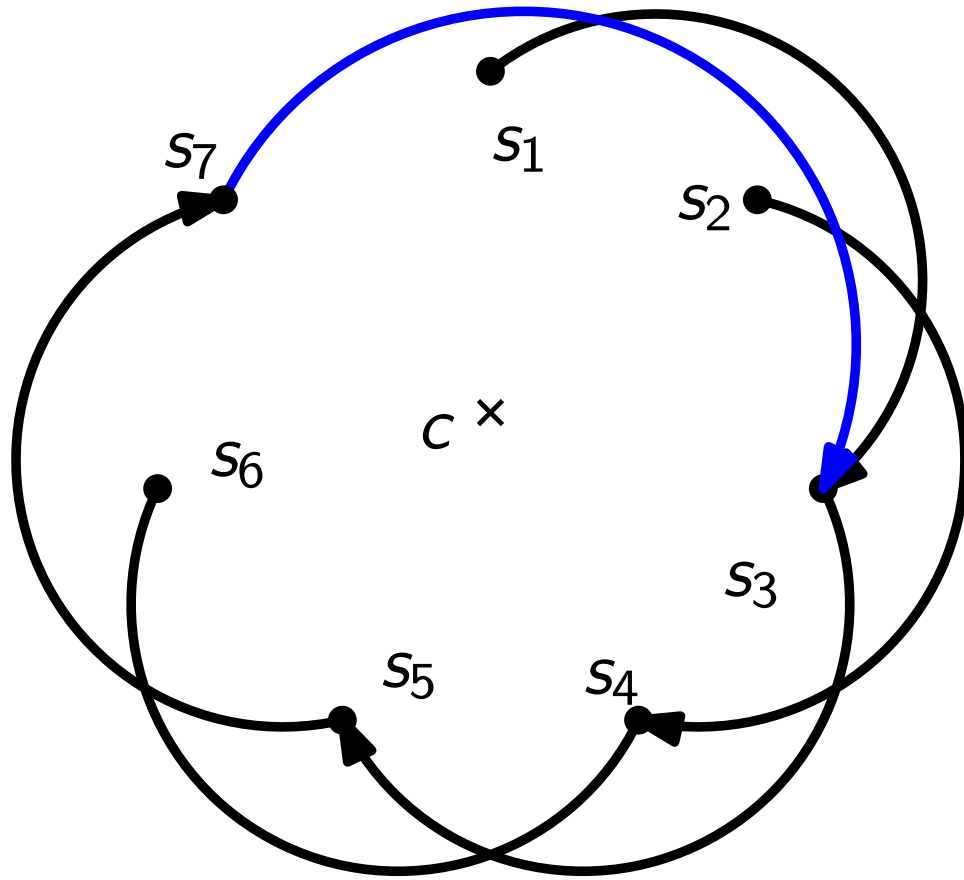


# The Radial Leader Model

- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program



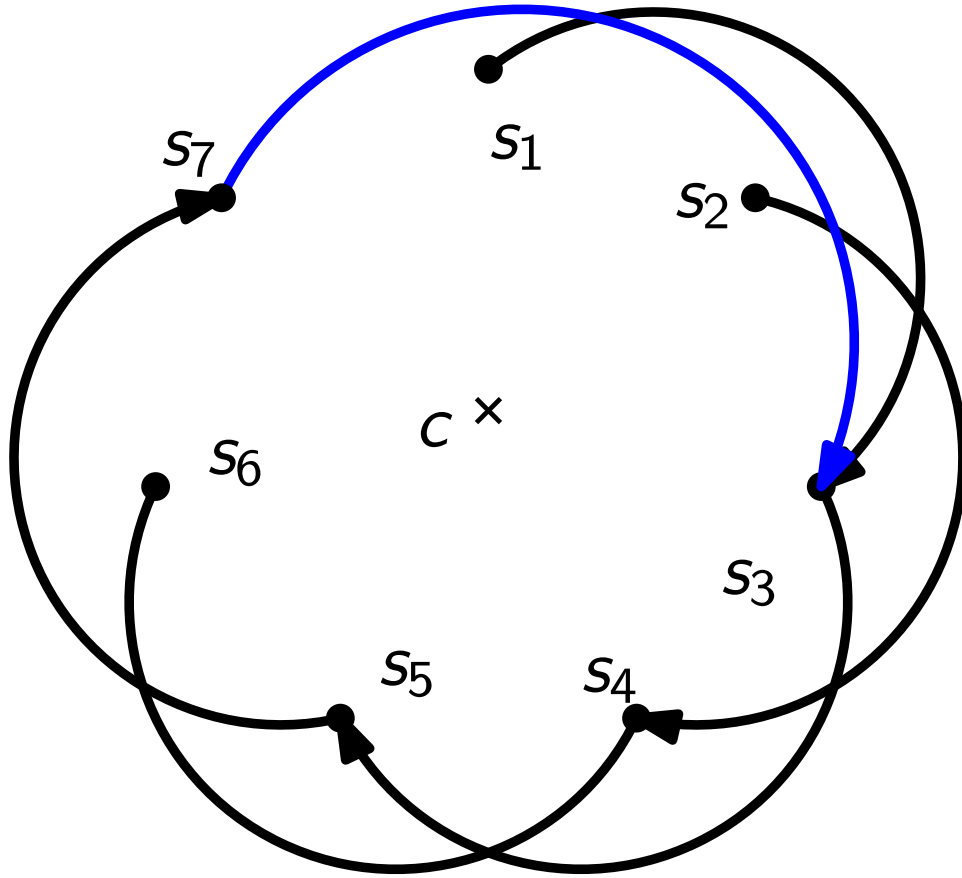
# The Radial Leader Model



- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program

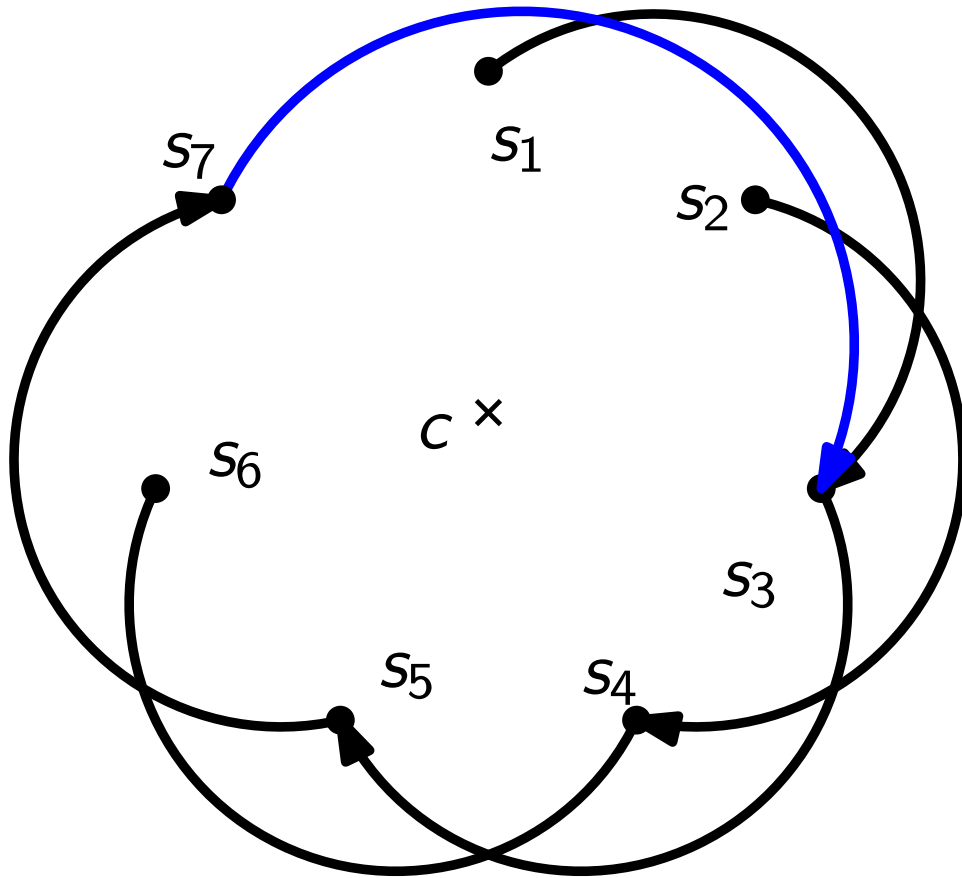
$O(n \log n)$  time

# The Radial Leader Model



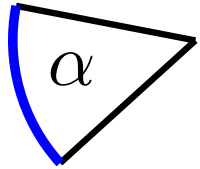
- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program  
 $O(n \log n)$  time
- weighted version: prefer higher rated points

# The Radial Leader Model

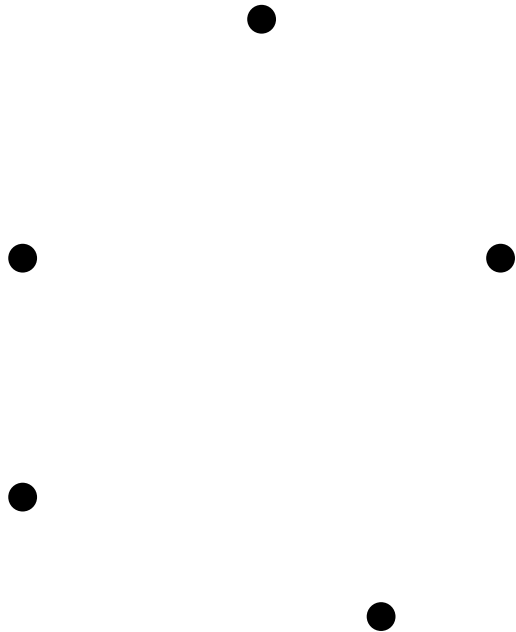


- minimum allowed angle to avoid label collisions
- maximize number of visible labels by a dynamic program  
 $O(n \log n)$  time
- weighted version: prefer higher rated points  
 $O(n^2)$  time

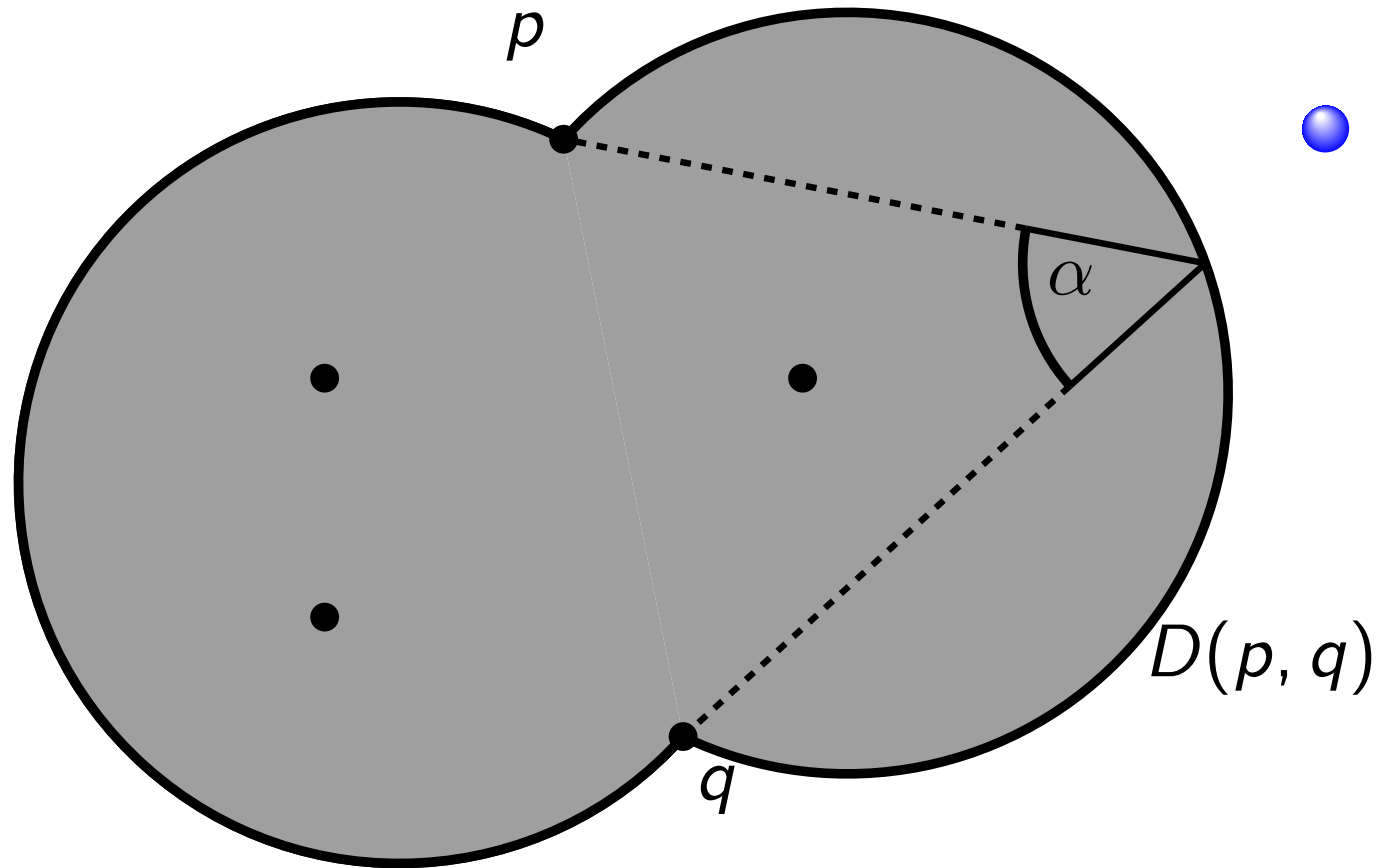
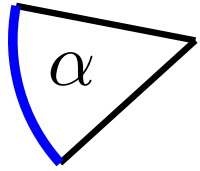
# The Radial Leader Model with Flexible Center Position



- find disk that respects **minimum angle  $\alpha$**

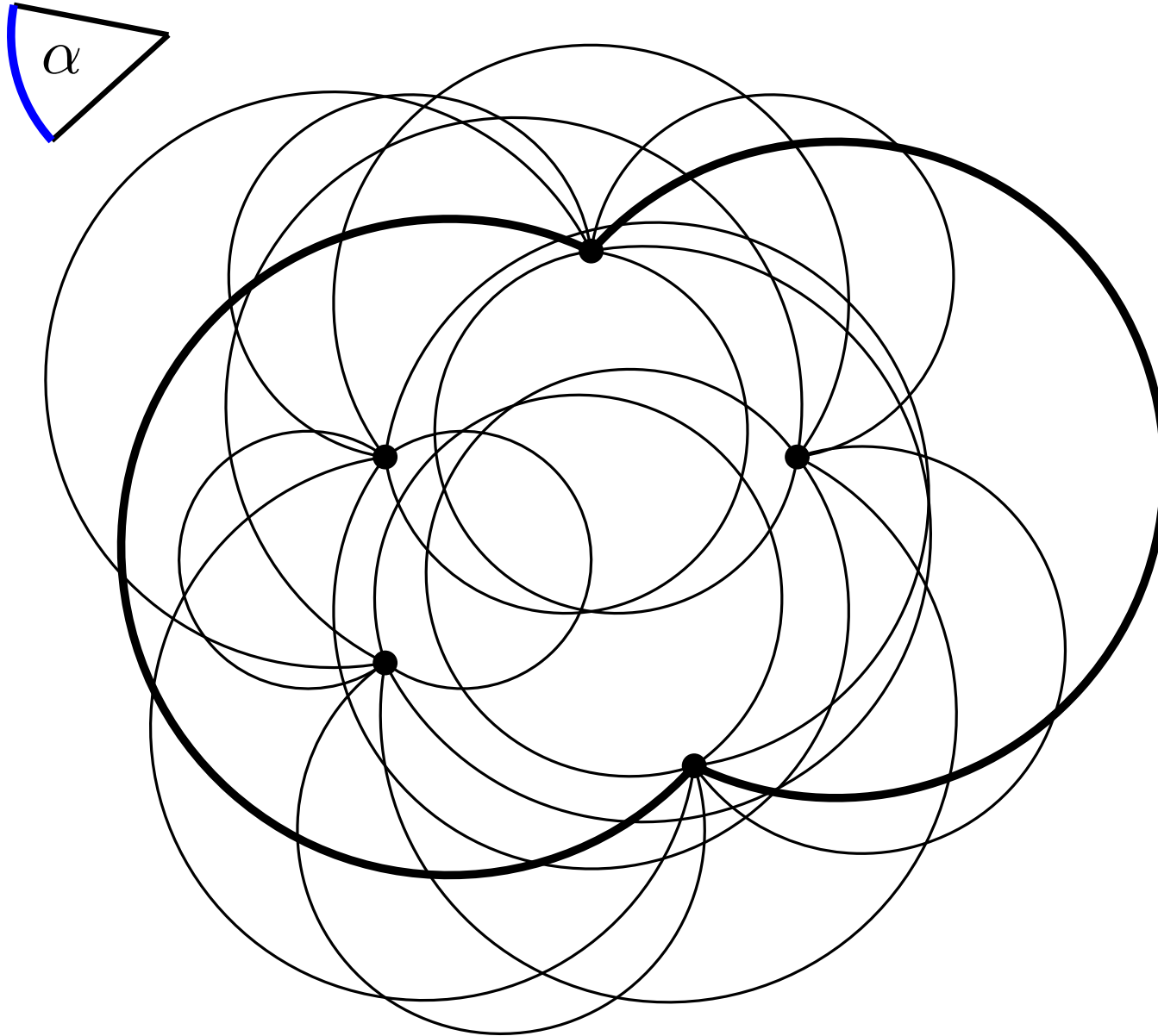


# The Radial Leader Model with Flexible Center Position



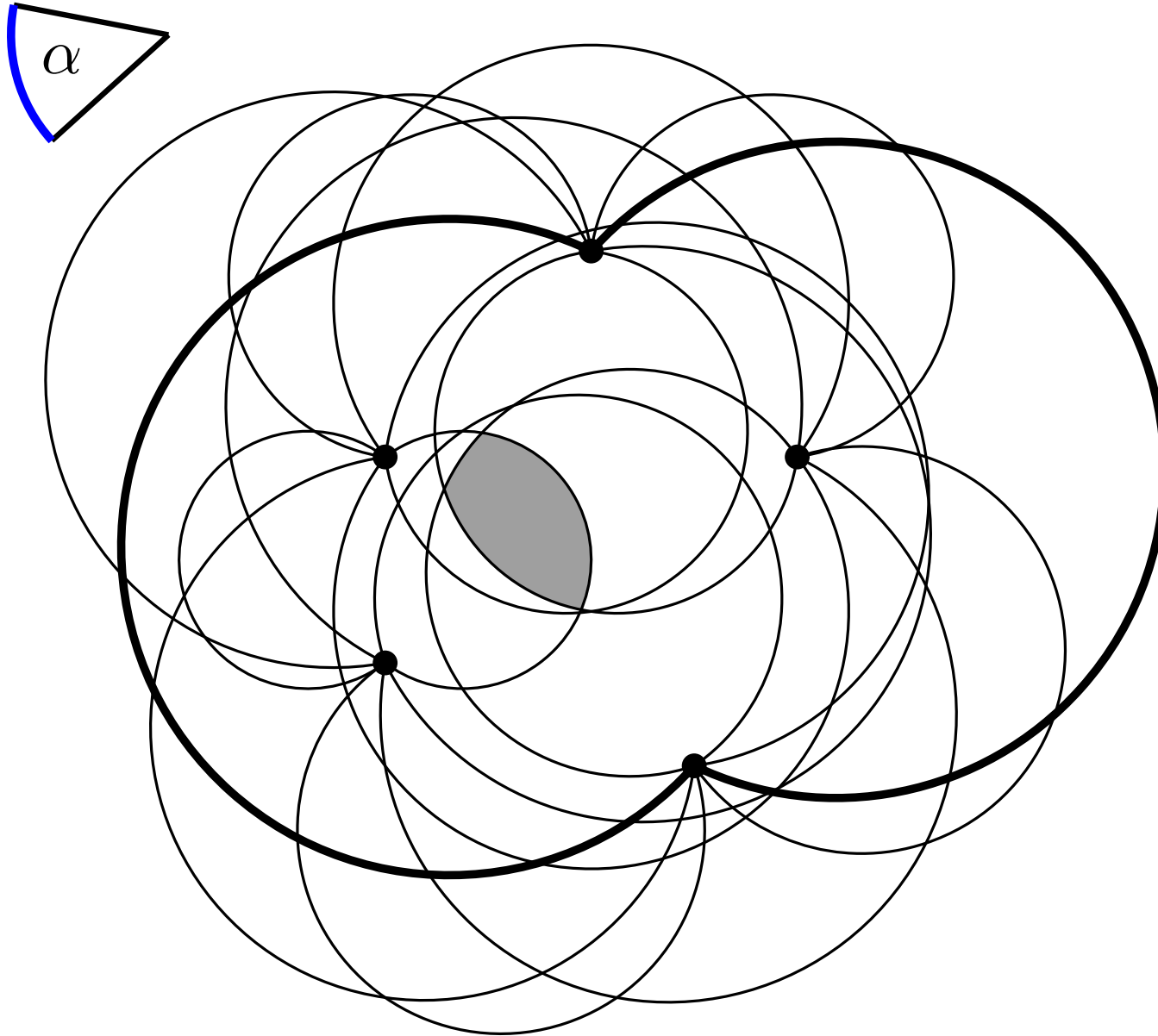
- find disk that respects **minimum angle  $\alpha$**
- consider double disk  $D(p, q)$  of minimum angle  $\alpha$  formed with  $p$  and  $q$

# The Radial Leader Model with Flexible Center Position



- find disk that respects minimum angle  $\alpha$
- consider double disk  $D(p, q)$  of minimum angle  $\alpha$  formed with  $p$  and  $q$
- build arrangement of all  $D(\cdot, \cdot)$

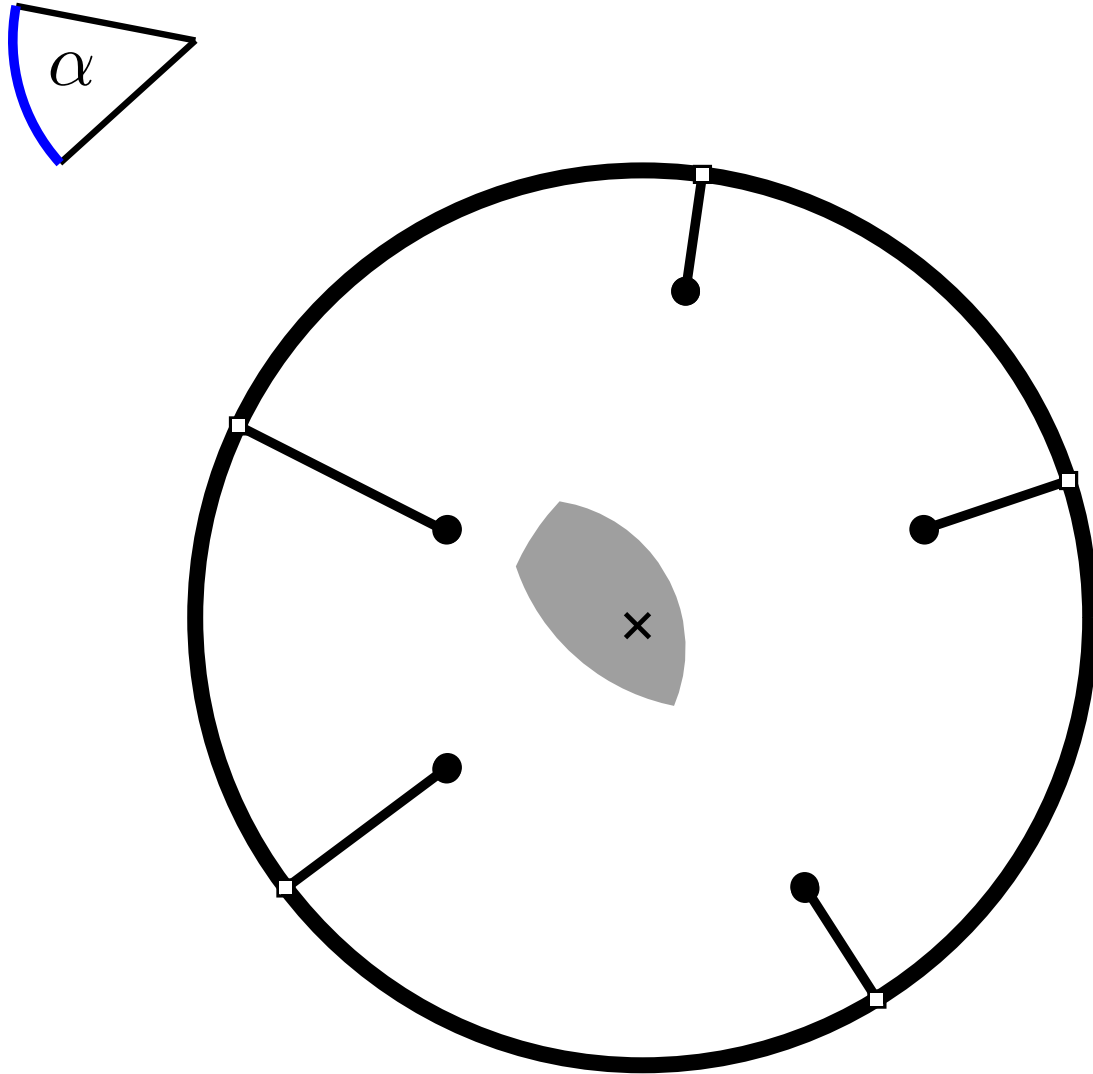
# The Radial Leader Model with Flexible Center Position



- find disk that respects **minimum angle  $\alpha$**
- consider double disk  $D(p, q)$  of minimum angle  $\alpha$  formed with  $p$  and  $q$
- build arrangement of all  $D(\cdot, \cdot)$
- check for intersection (cell of depth  $\binom{n}{2}$ )



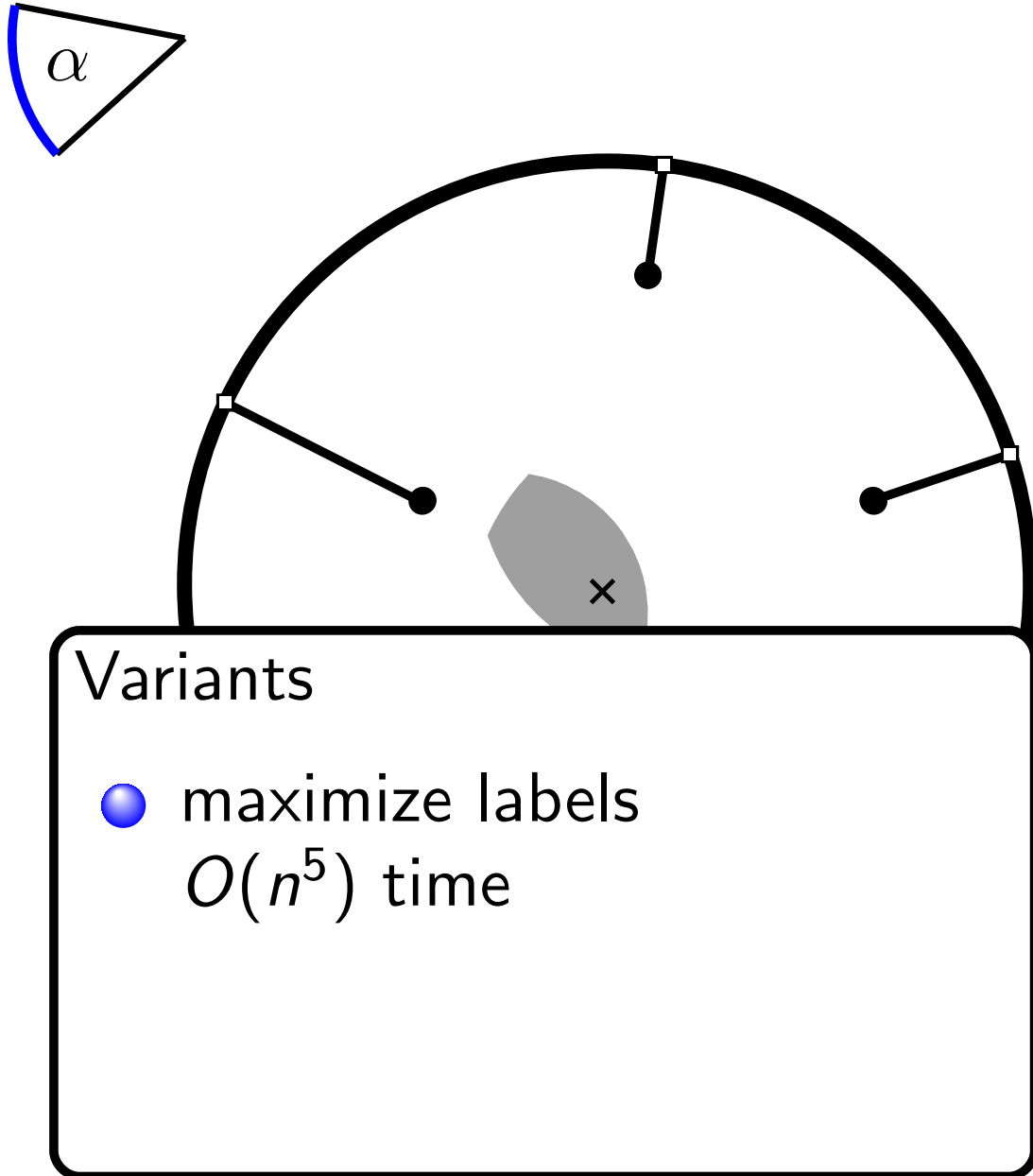
# The Radial Leader Model with Flexible Center Position



- find disk that respects **minimum angle  $\alpha$**
- consider double disk  $D(p, q)$  of minimum angle  $\alpha$  formed with  $p$  and  $q$
- build arrangement of all  $D(\cdot, \cdot)$
- check for intersection (cell of depth  $\binom{n}{2}$ )
- choose center in intersection

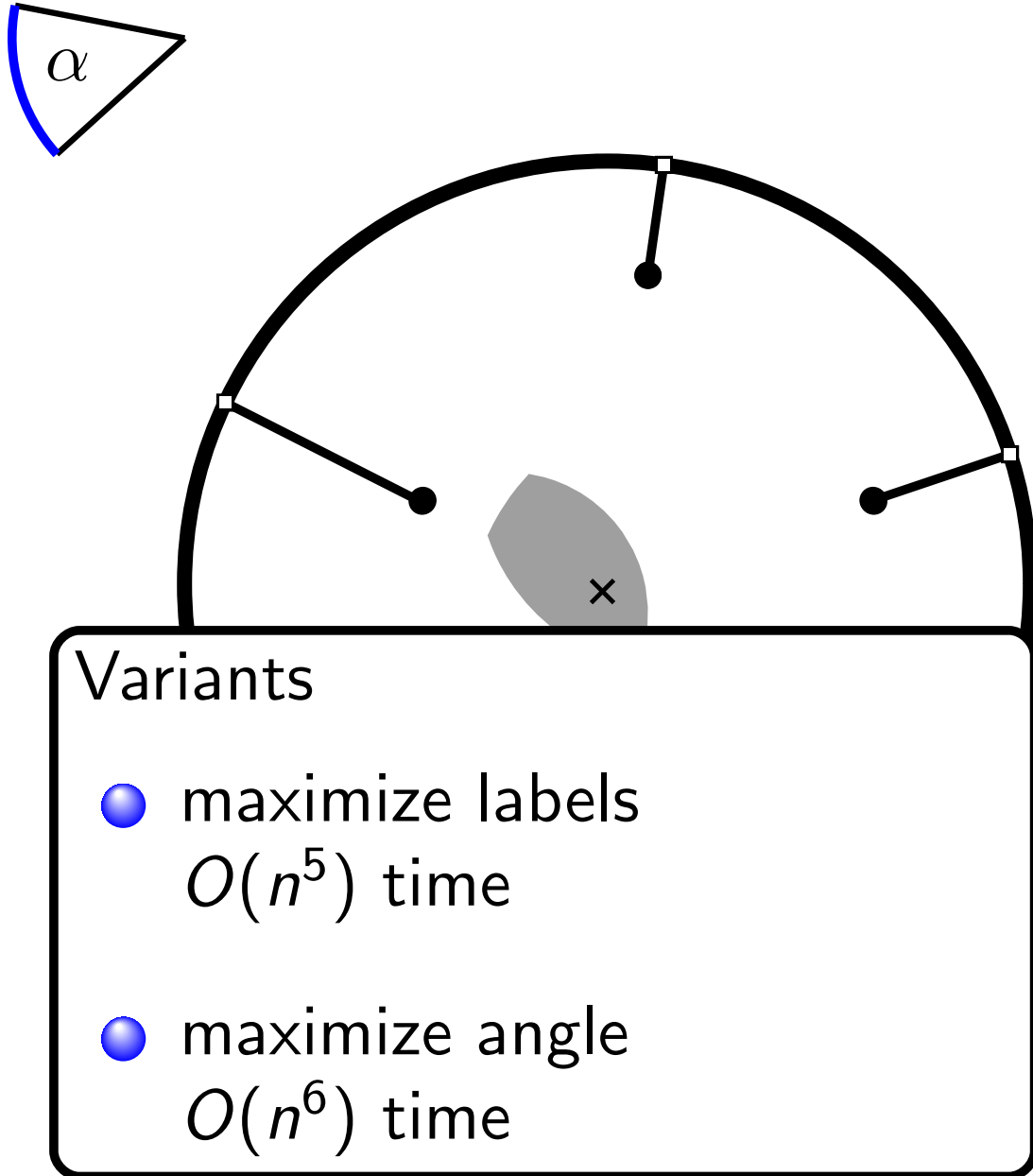
$O(n^4 \log n)$  time

# The Radial Leader Model with Flexible Center Position



- find disk that respects minimum angle  $\alpha$
- consider double disk  $D(p, q)$  of minimum angle  $\alpha$  formed with  $p$  and  $q$
- build arrangement of all  $D(\cdot, \cdot)$
- check for intersection (cell of depth  $\binom{n}{2}$ )
- choose center in intersection  
 $O(n^4 \log n)$  time

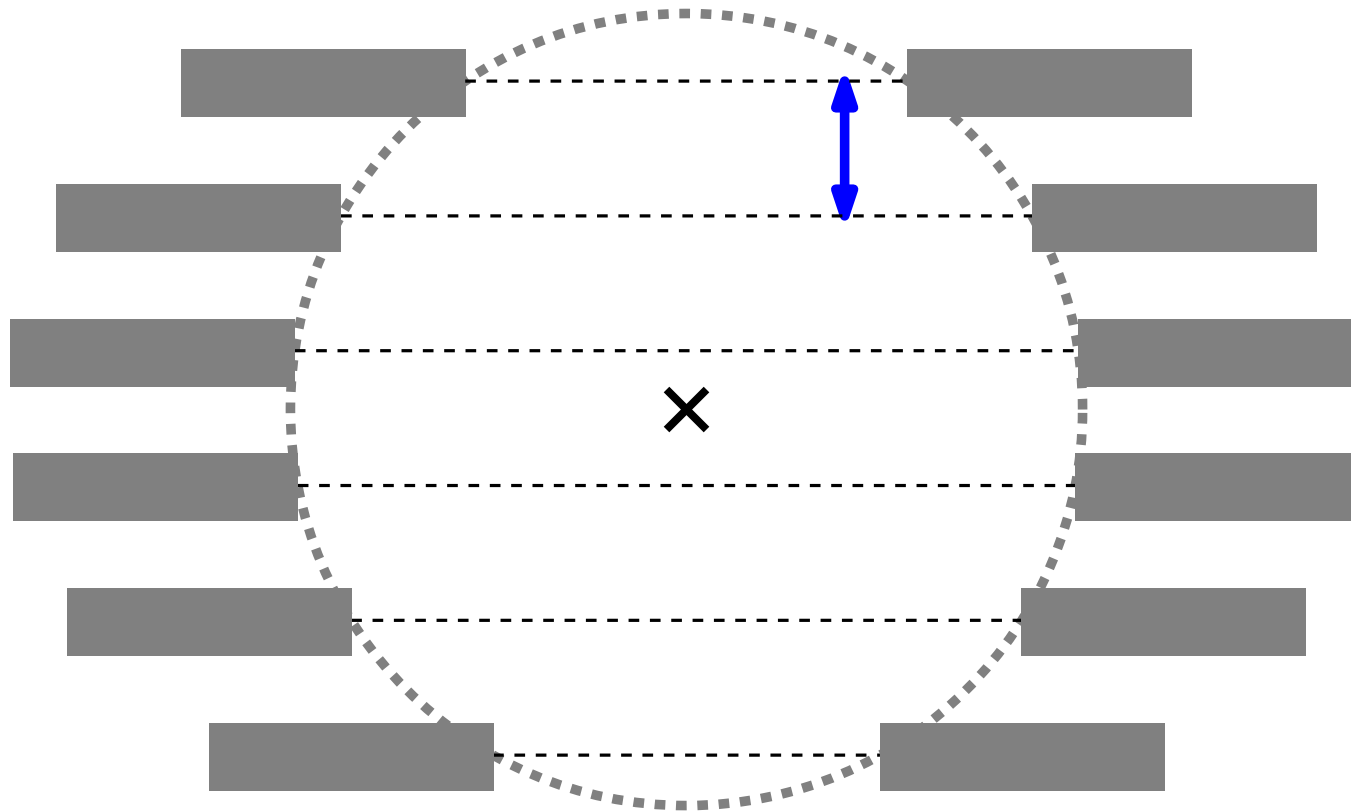
# The Radial Leader Model with Flexible Center Position



- find disk that respects minimum angle  $\alpha$
- consider double disk  $D(p, q)$  of minimum angle  $\alpha$  formed with  $p$  and  $q$
- build arrangement of all  $D(\cdot, \cdot)$
- check for intersection (cell of depth  $\binom{n}{2}$ )
- choose center in intersection  
 $O(n^4 \log n)$  time

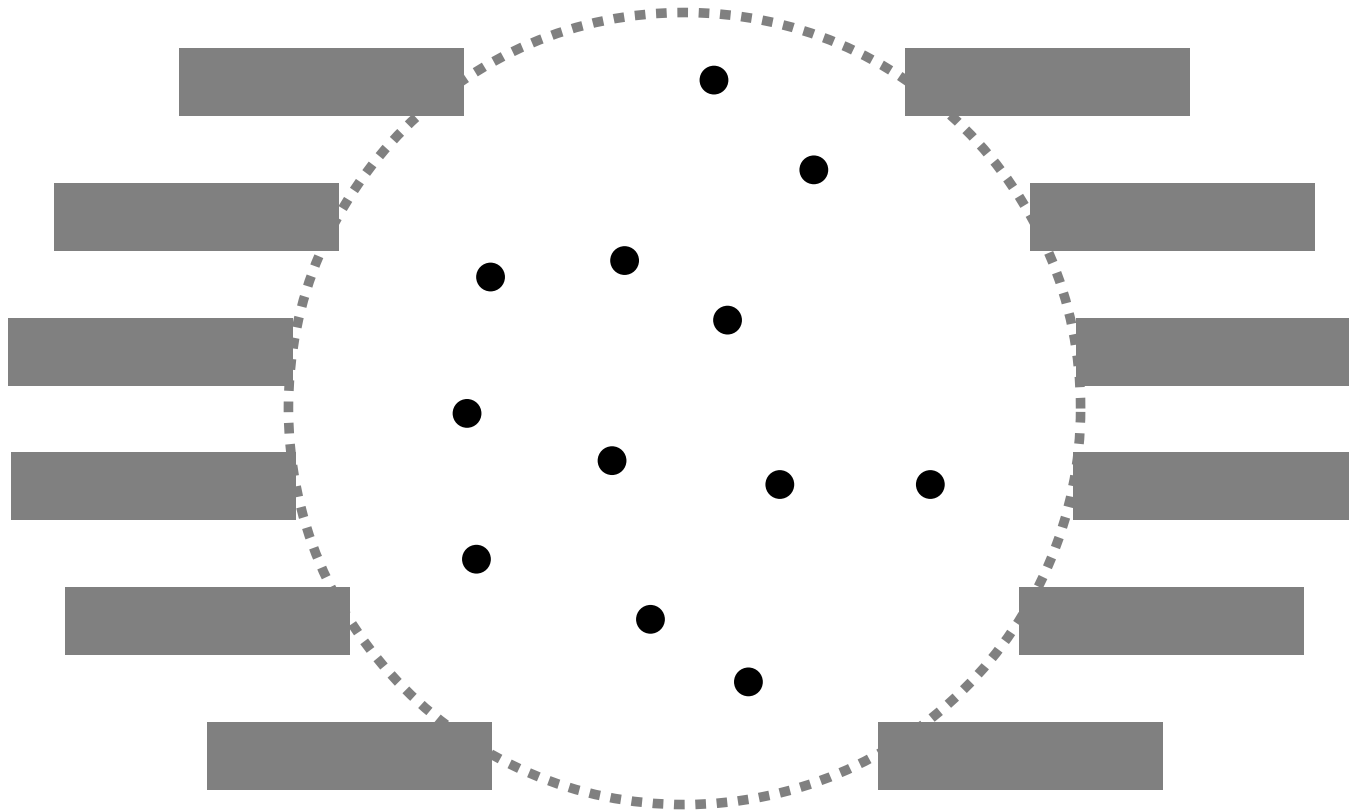
# The free leader model

- labels vertically distributed with unit distances

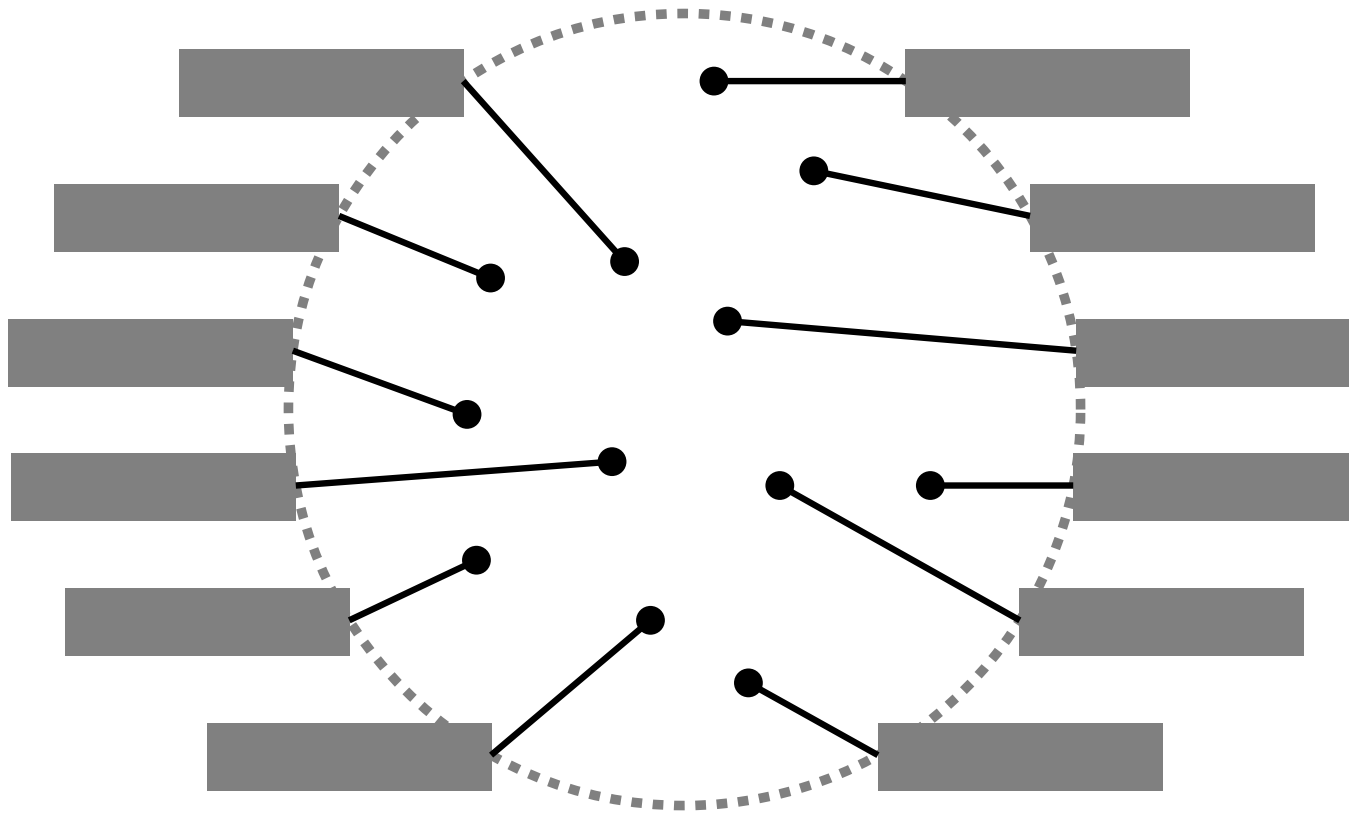


# The free leader model

- labels vertically distributed with unit distances
- compute non-crossing leaders

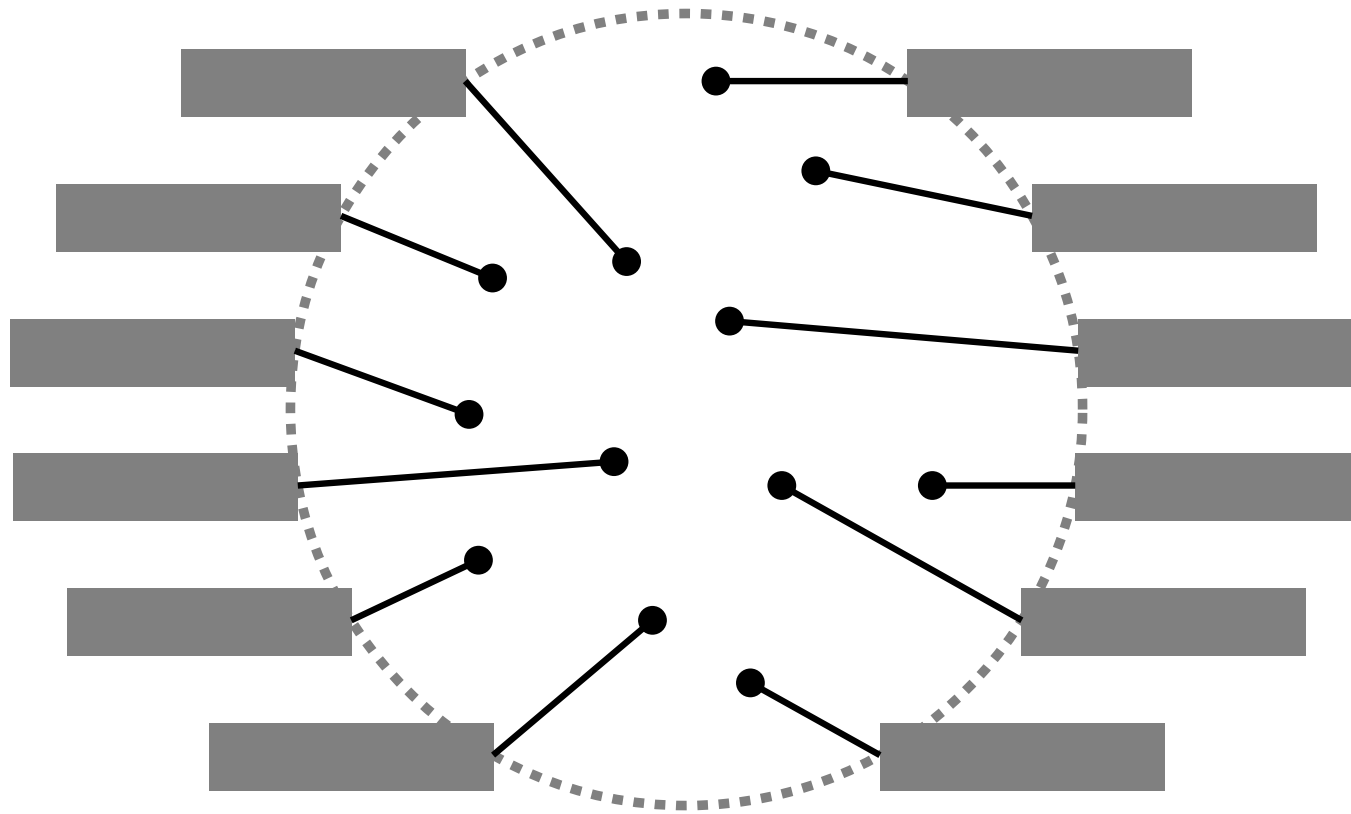


# The free leader model



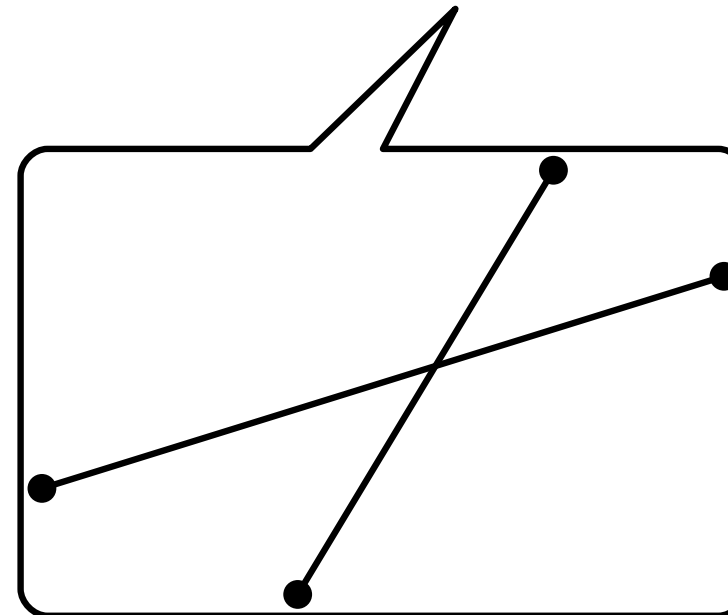
- labels vertically distributed with unit distances
- compute non-crossing leaders
- minimize total leader length: weighted bipartite matching [Bekos et al., 2007]

# The free leader model

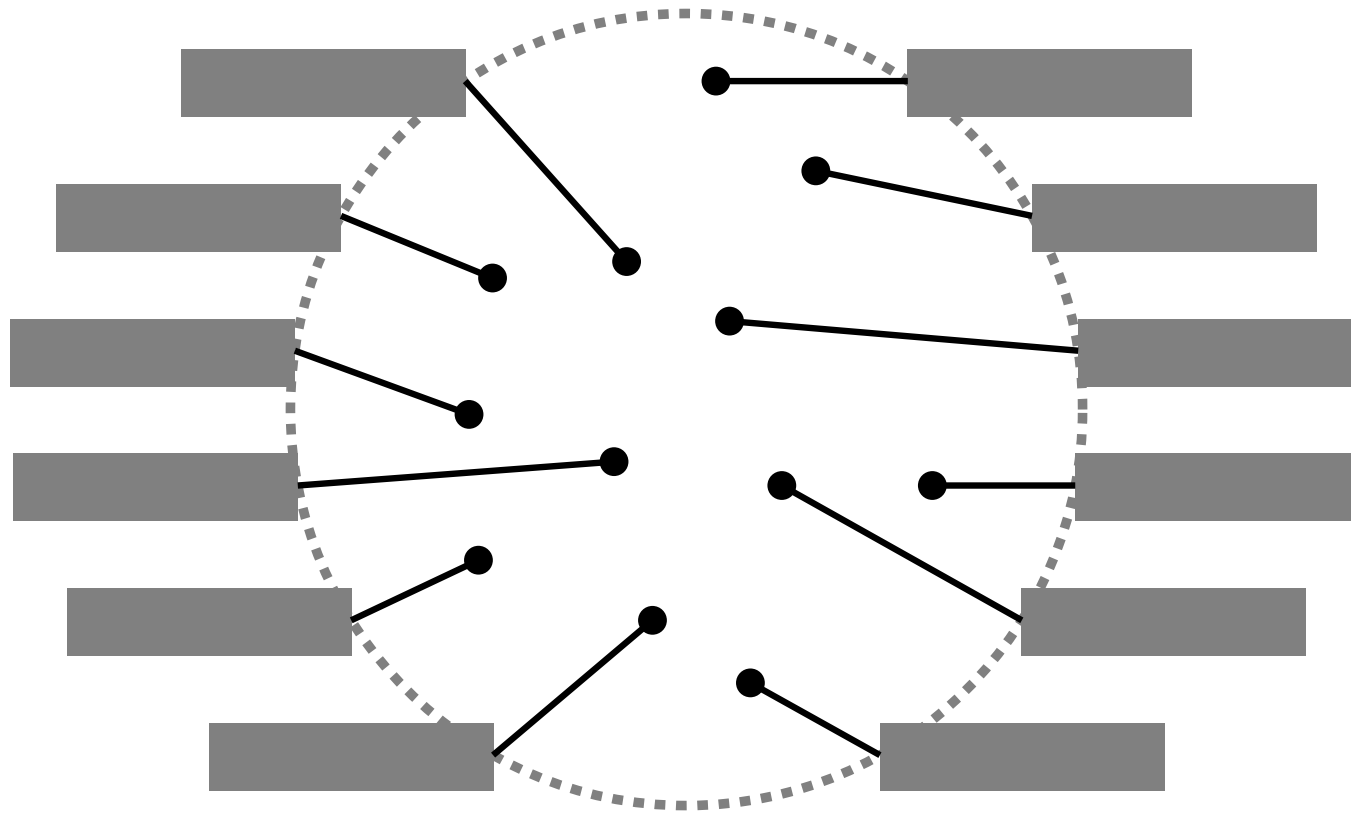


- labels vertically distributed with unit distances
- compute non-crossing leaders
- minimize total leader length: weighted bipartite matching

no crossings

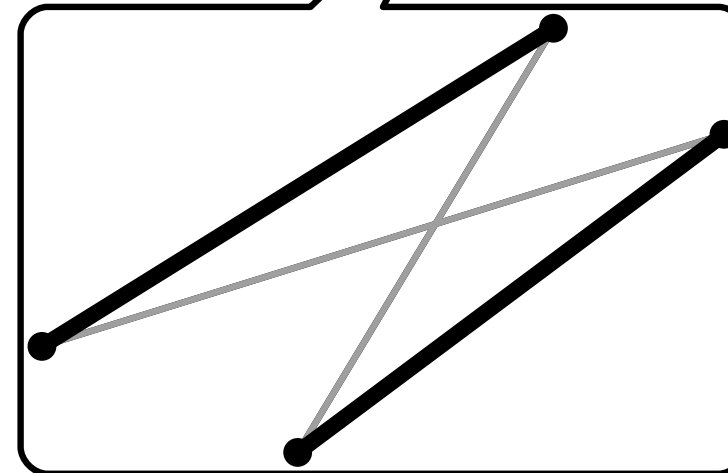


# The free leader model



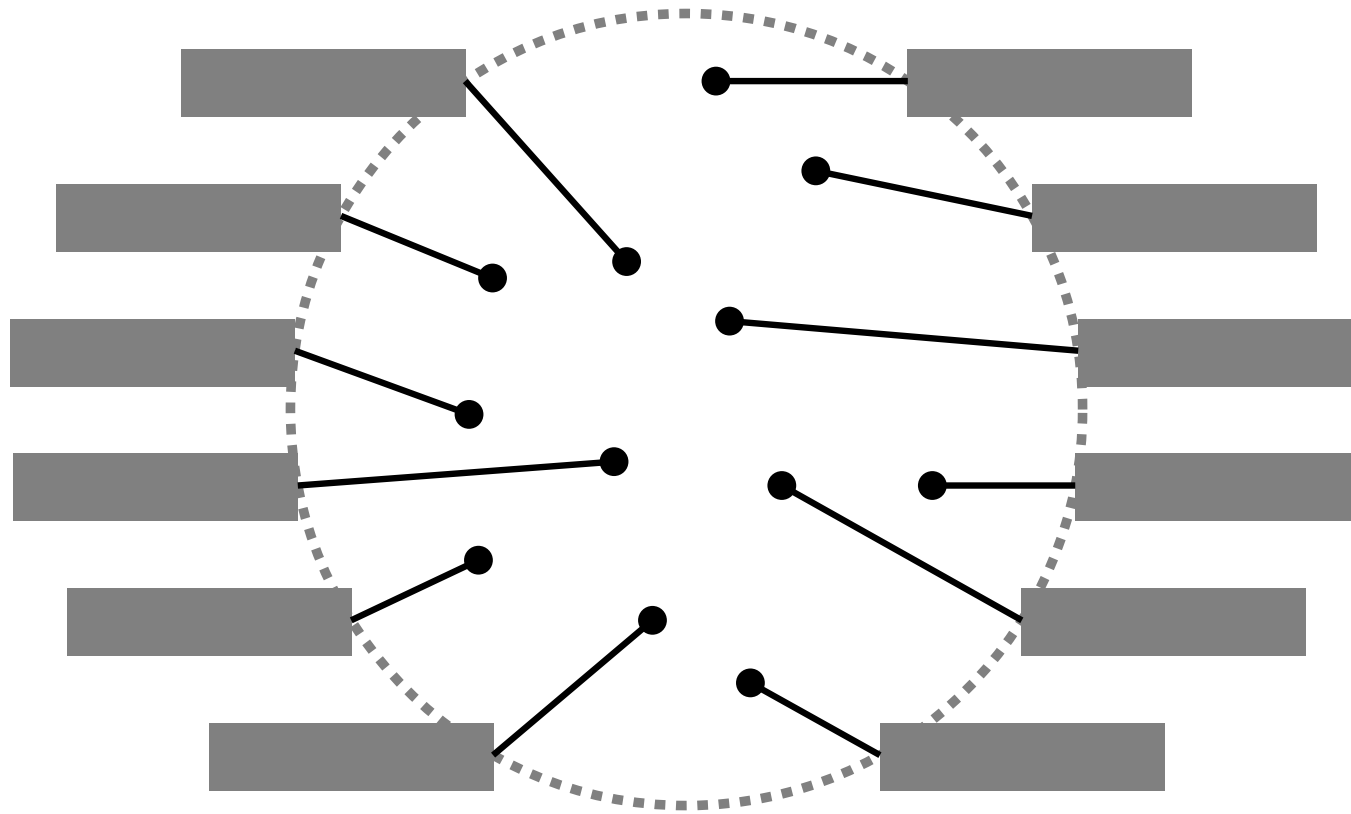
- labels vertically distributed with unit distances
- compute non-crossing leaders
- minimize total leader length: weighted bipartite matching

no crossings





# The free leader model



- labels vertically distributed with unit distances
- compute non-crossing leaders
- minimize total leader length: weighted bipartite matching

no crossings

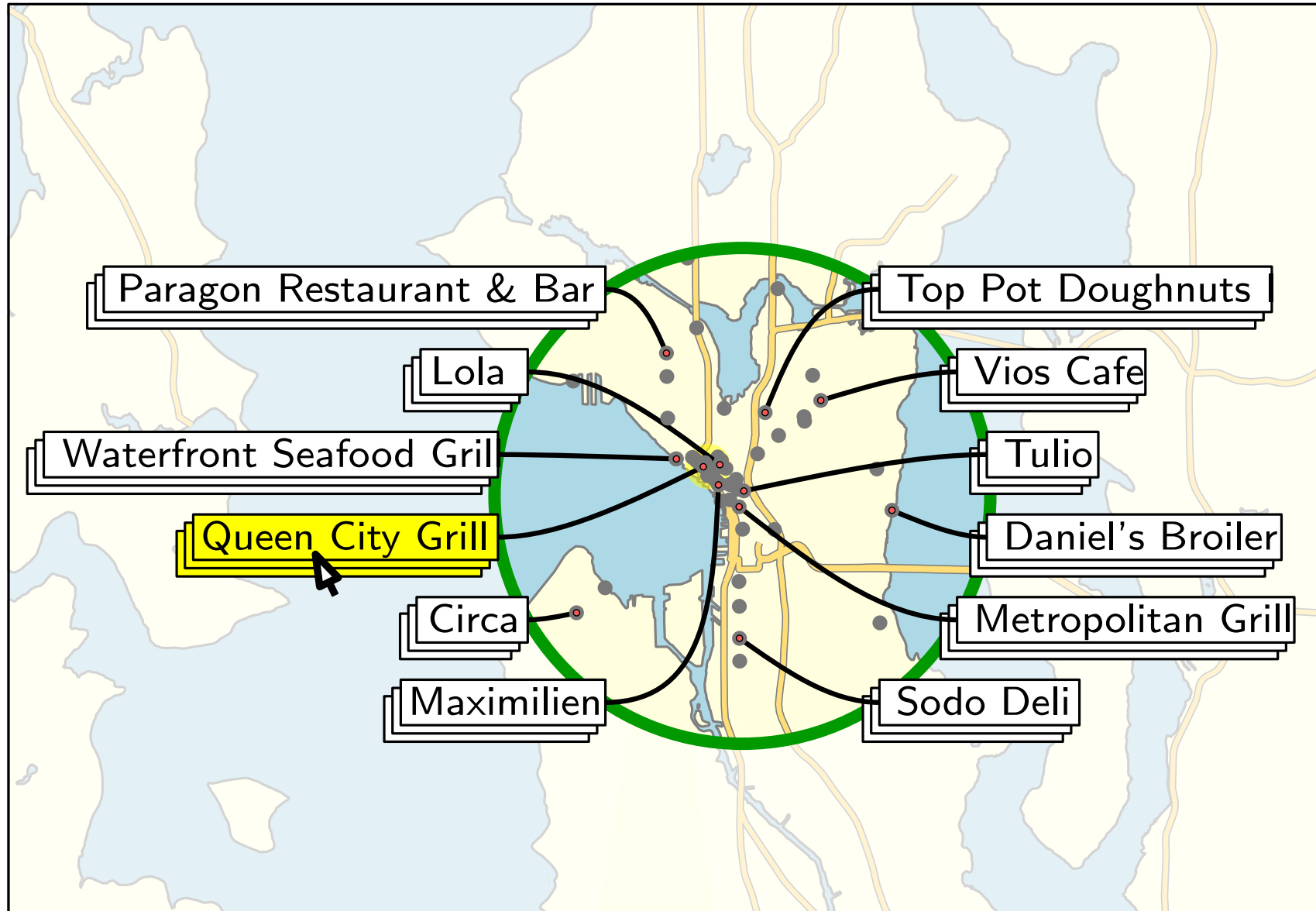


fast



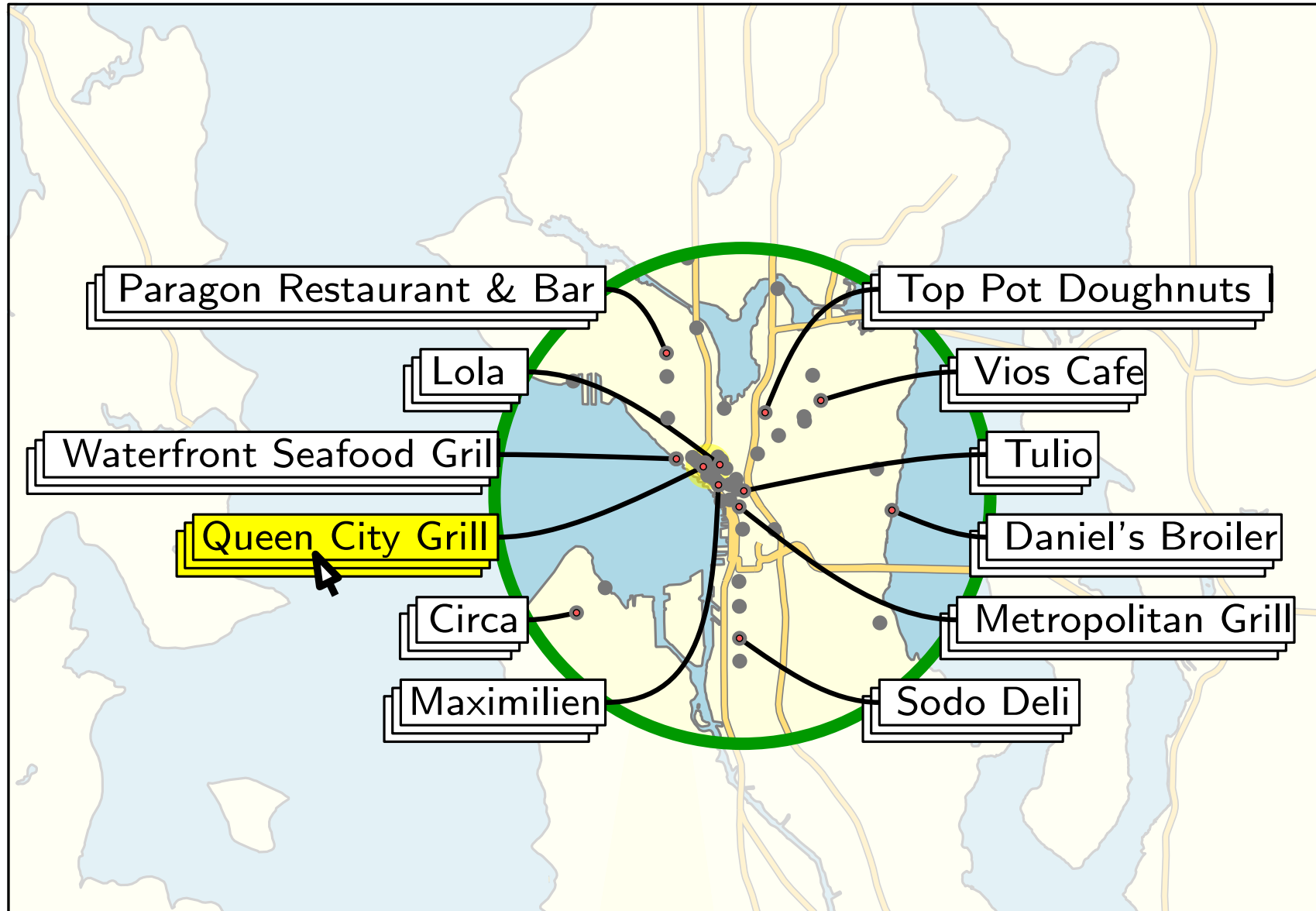
$$O(n^{2+\varepsilon})$$

# Selecting labeled sites



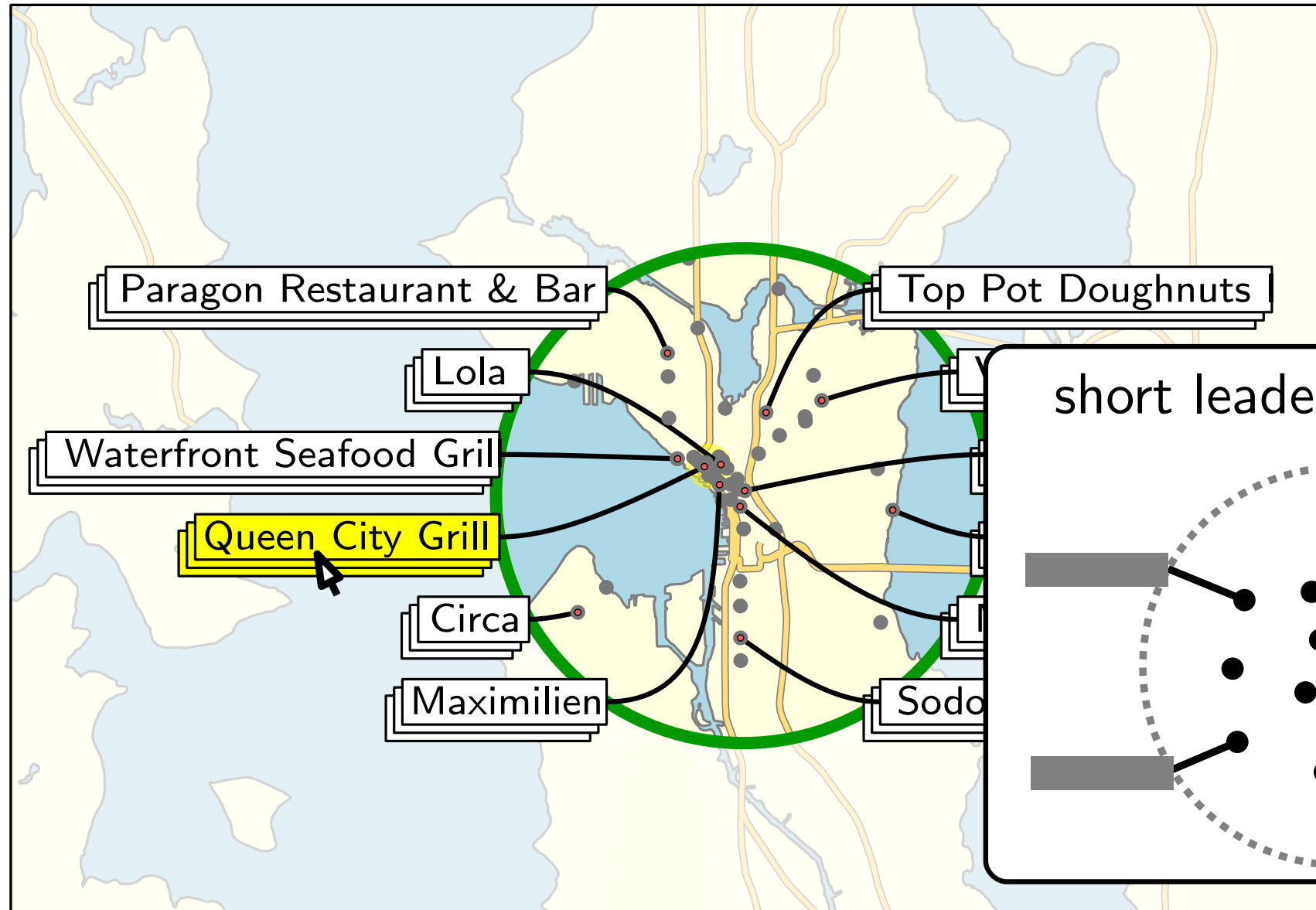
- not all sites can be labeled

# Selecting labeled sites

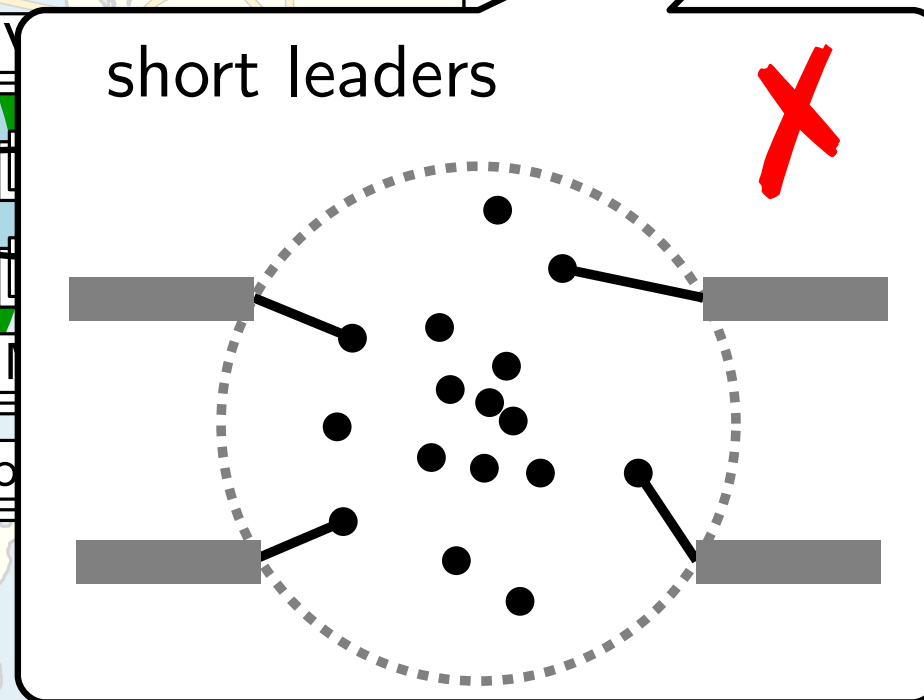


- not all sites can be labeled
- label good subset

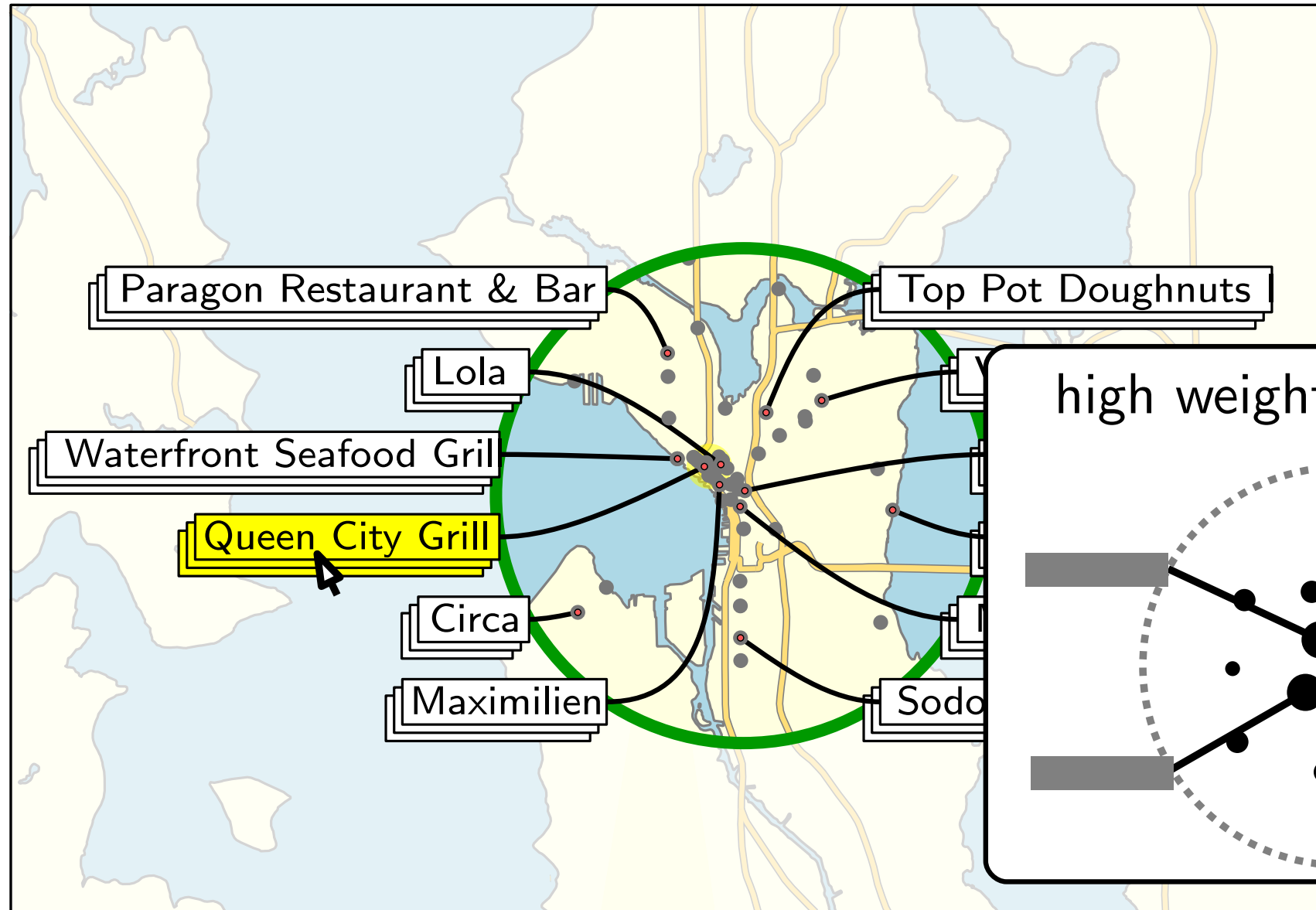
# Selecting labeled sites



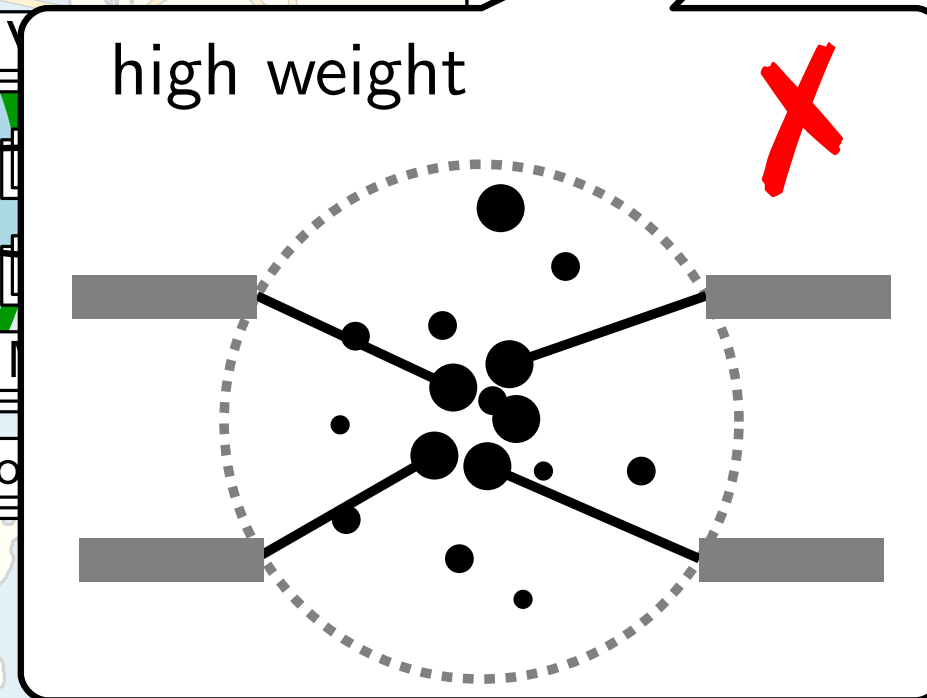
- not all sites can be labeled
- label good subset



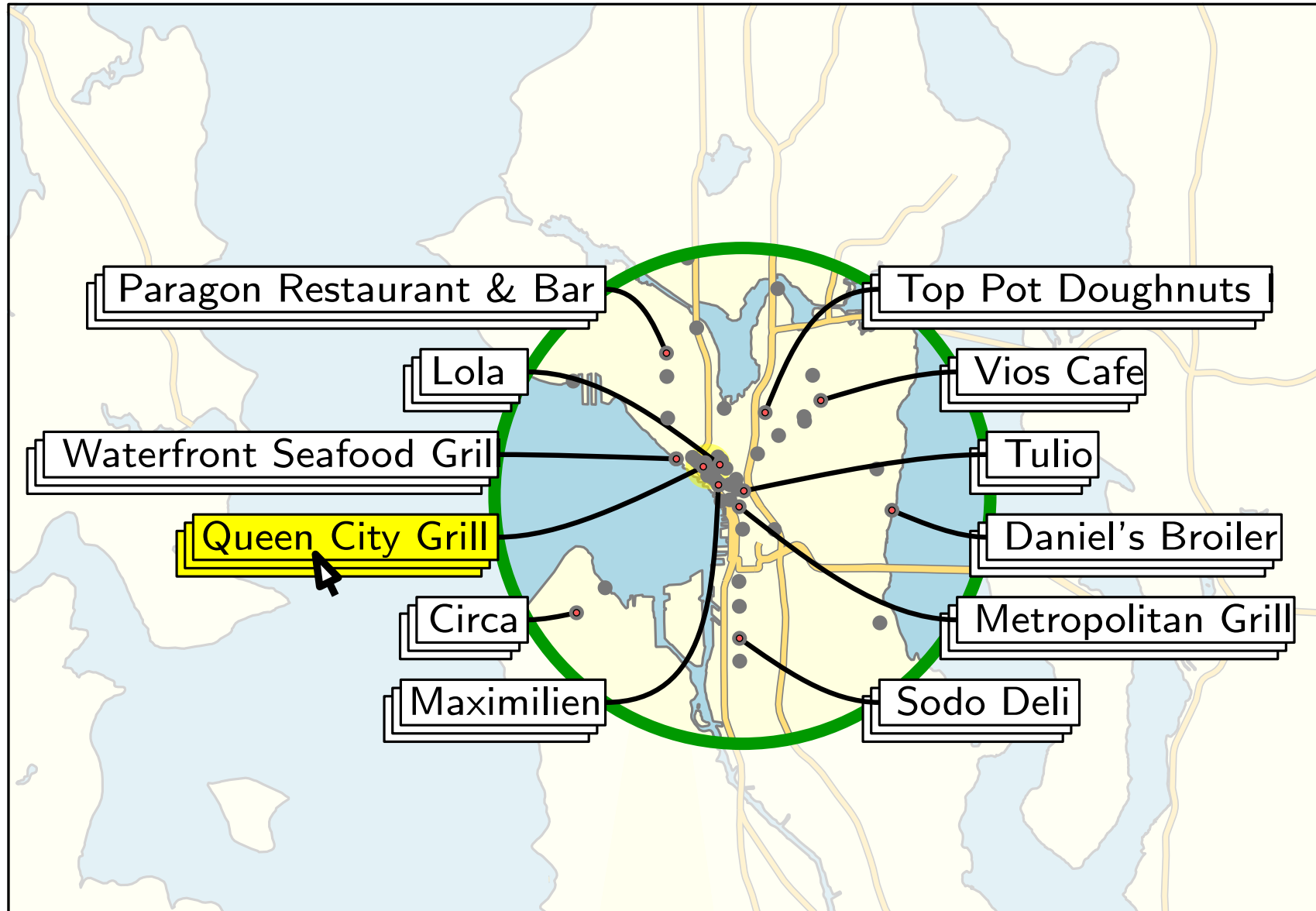
# Selecting labeled sites



- not all sites can be labeled
- label good subset

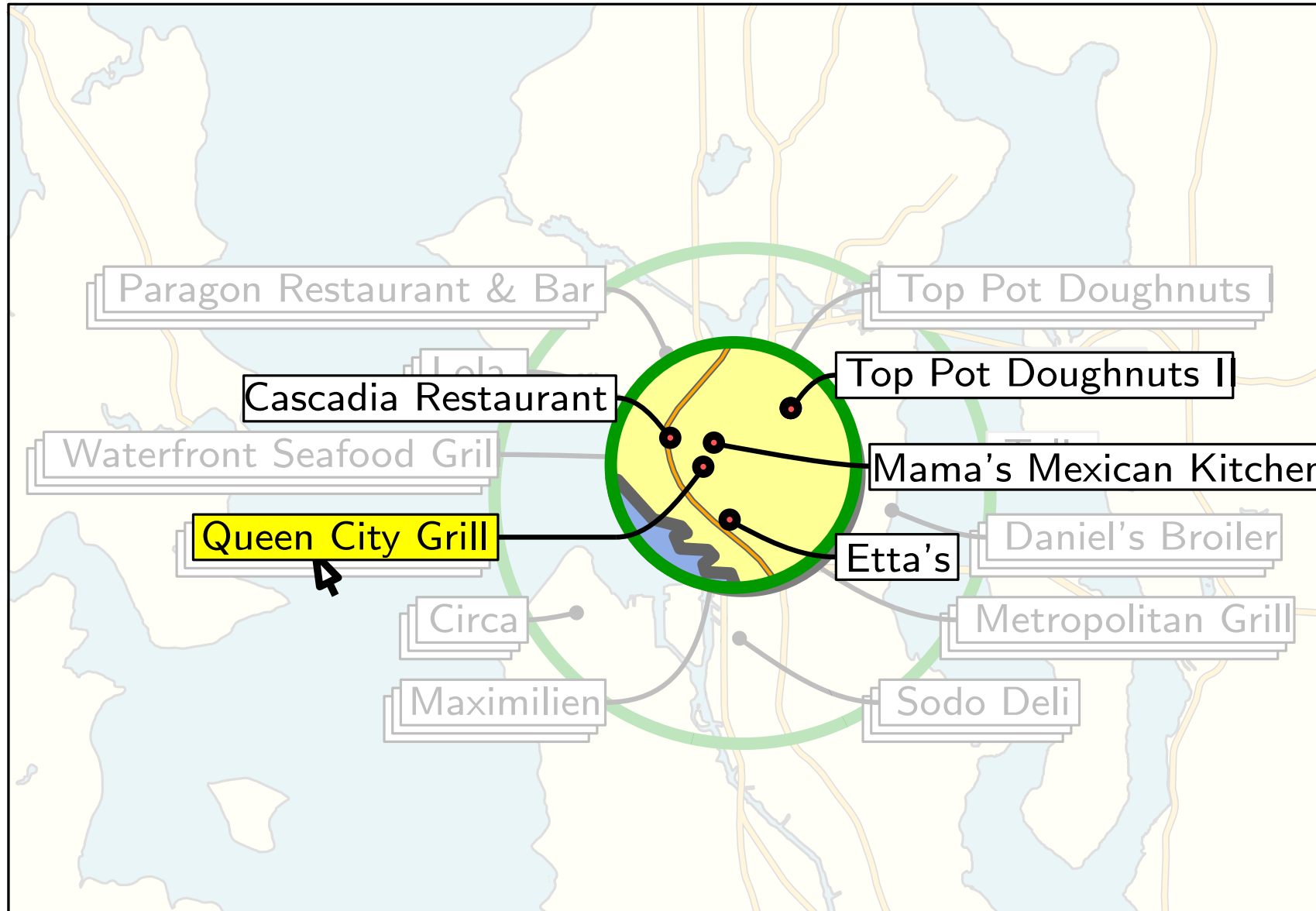


# Selecting labeled sites



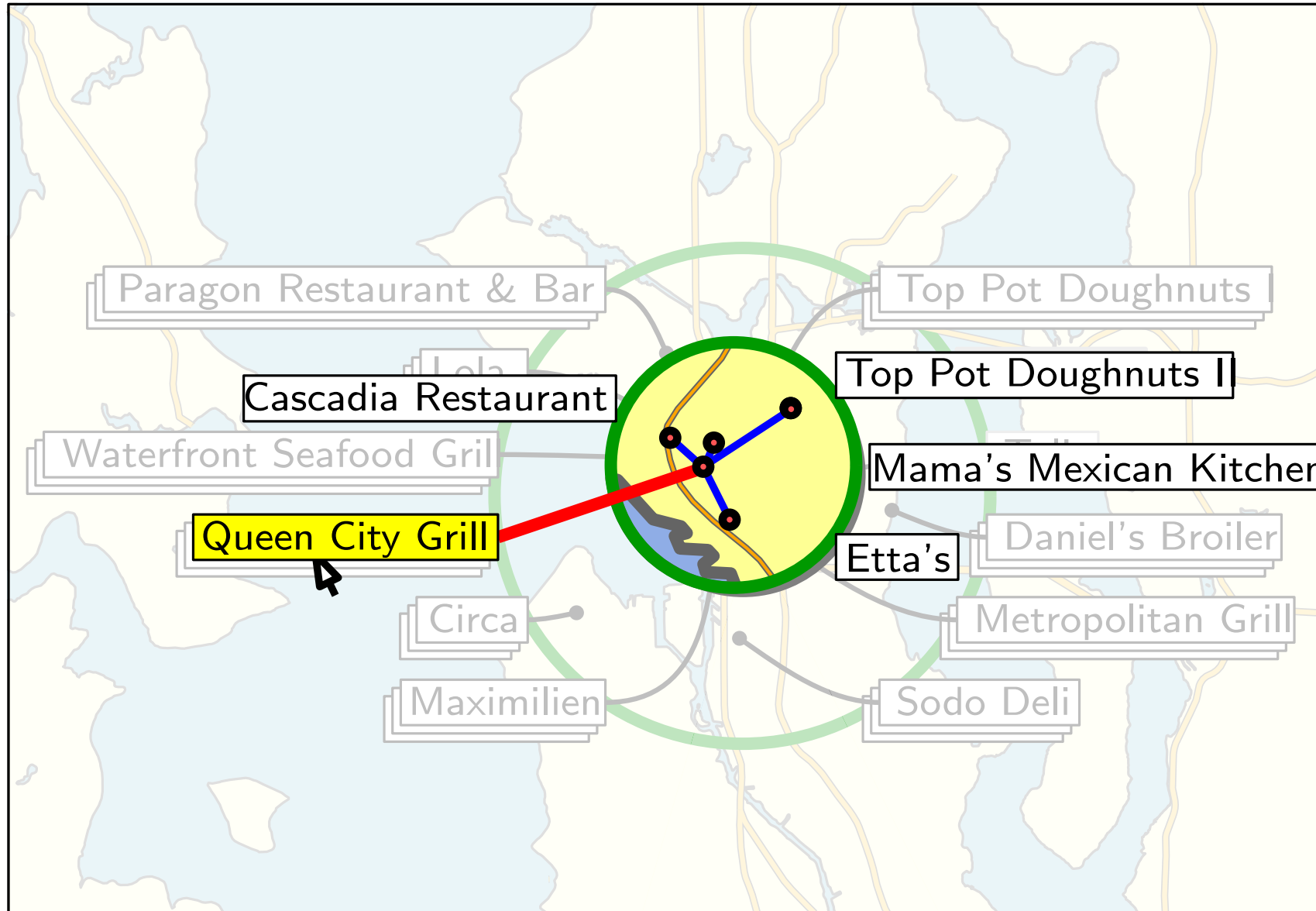
- not all sites can be labeled
- label good subset
  - nice distribution
  - represent all sites

# Clustered Labeling



- 1 labeled site
- $k$  unlabeled sites

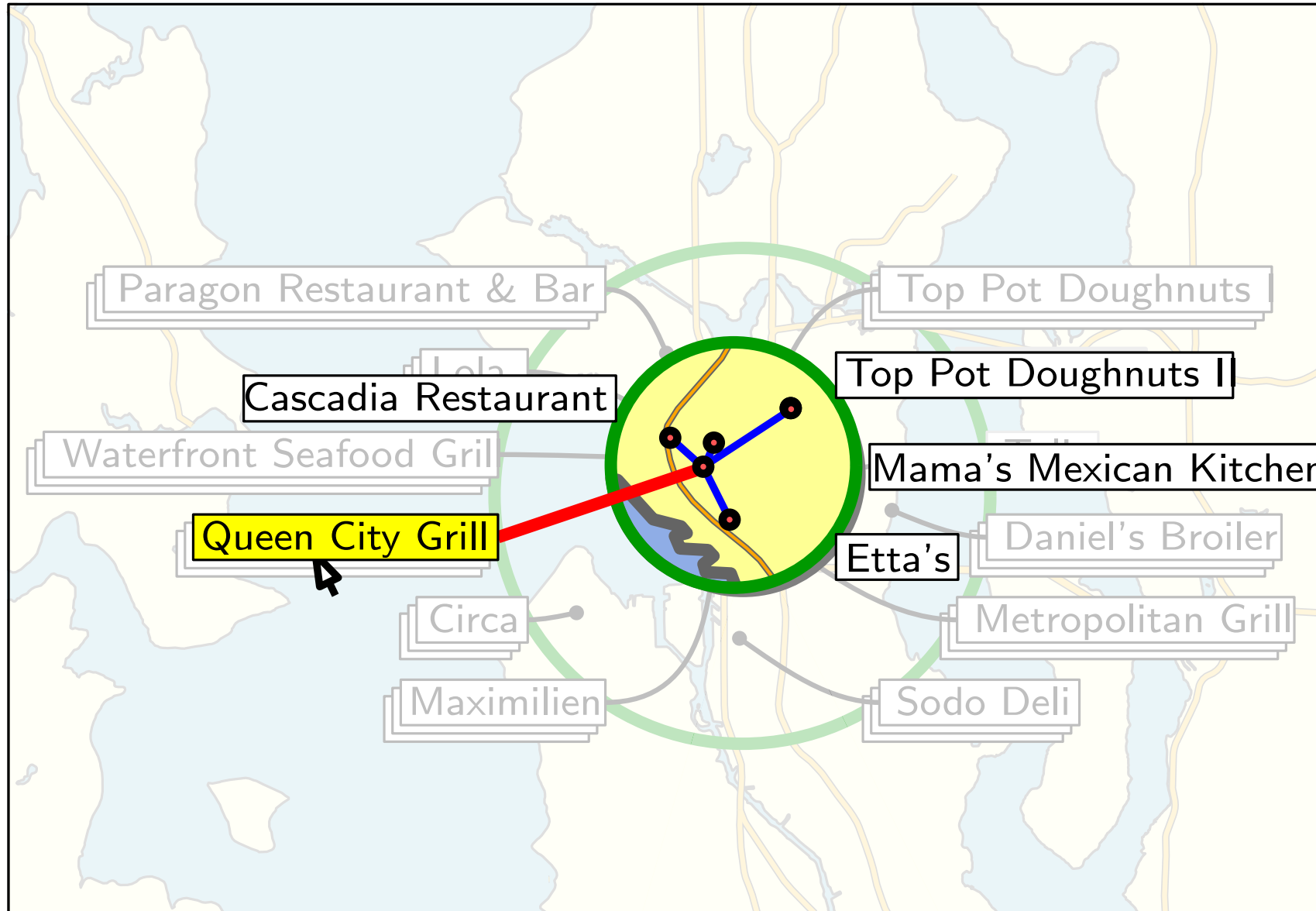
# Clustered Labeling



- 1 labeled site  
→  $k$  unlabeled sites
- minimize leader length + distance to attached sites

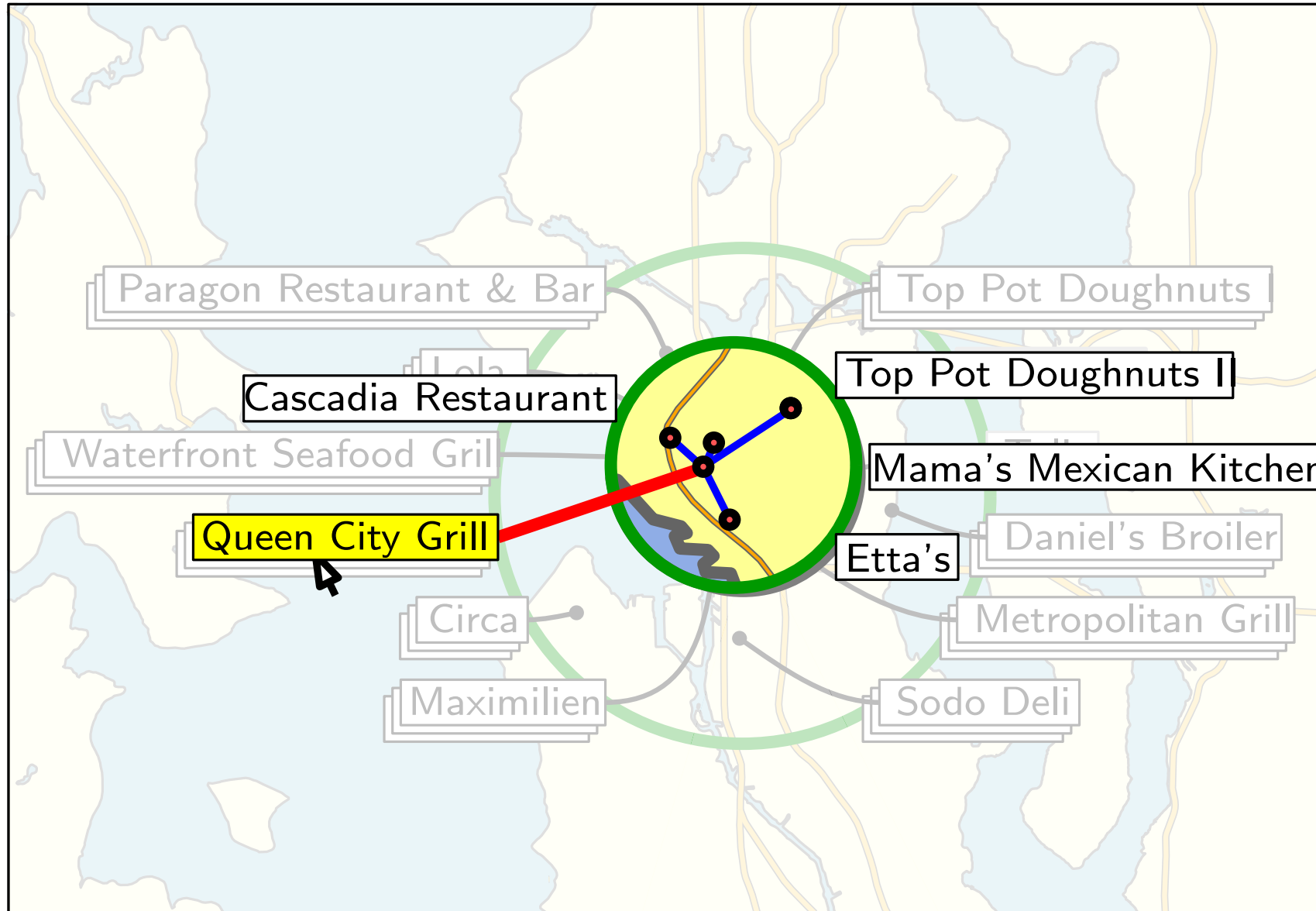


# Clustered Labeling



- 1 labeled site  
→  $k$  unlabeled sites
- minimize leader length + distance to attached sites
- Facility Location model:  
solved by ILP

# Clustered Labeling

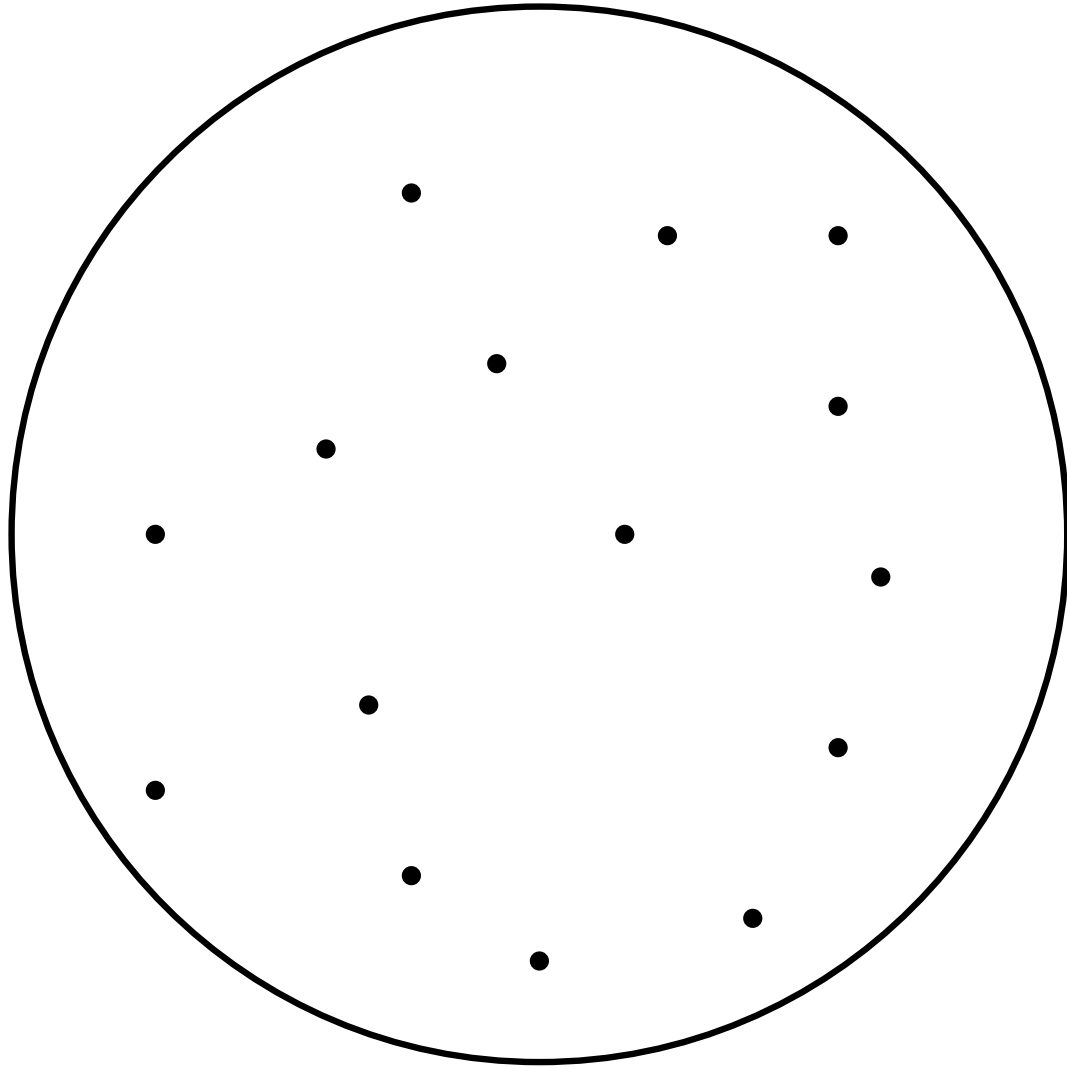


- 1 labeled site  
→  $k$  unlabeled sites
  - minimize leader length + distance to attached sites
  - Facility Location model:  
solved by ILP
- 95 sites, 20 labels:  
124s

# A Heuristic for Clustered Labeling

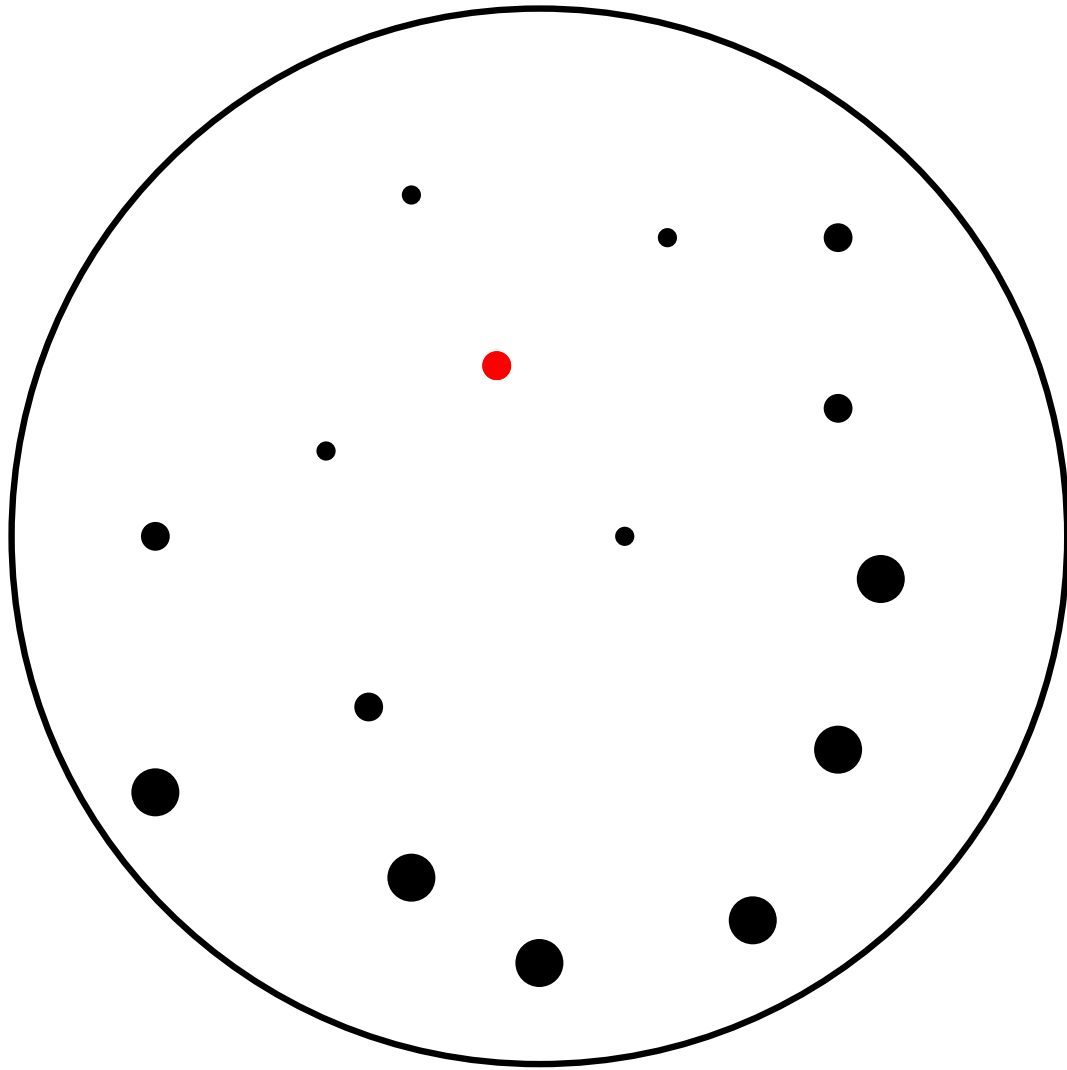
- Randomized initialization heuristic for  $k$ -median/ $k$ -means [Arthur and Vassilvitski, 2007]

# A Heuristic for Clustered Labeling



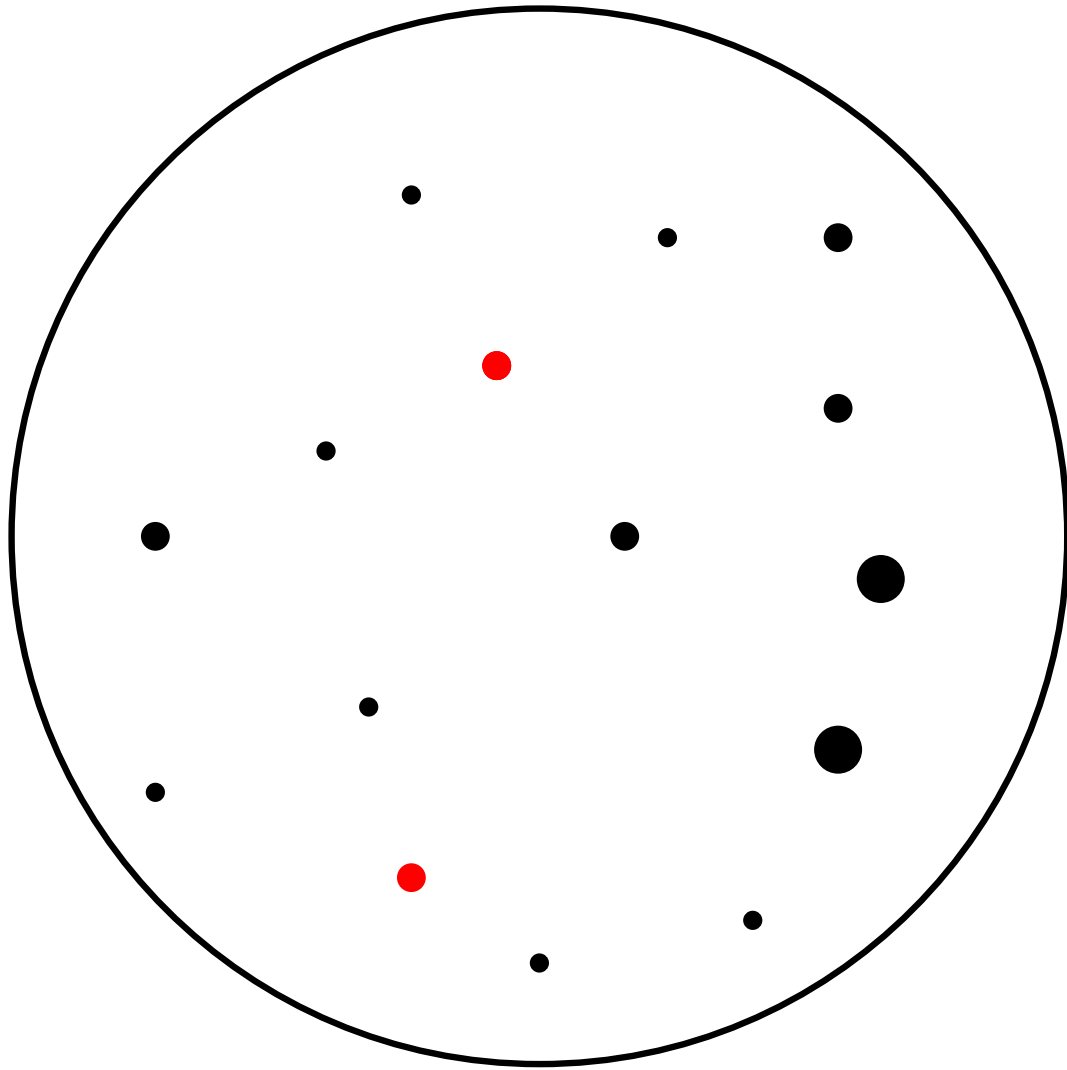
- Randomized initialization heuristic for  $k$ -median/ $k$ -means [Arthur and Vassilvitski, 2007]

# A Heuristic for Clustered Labeling



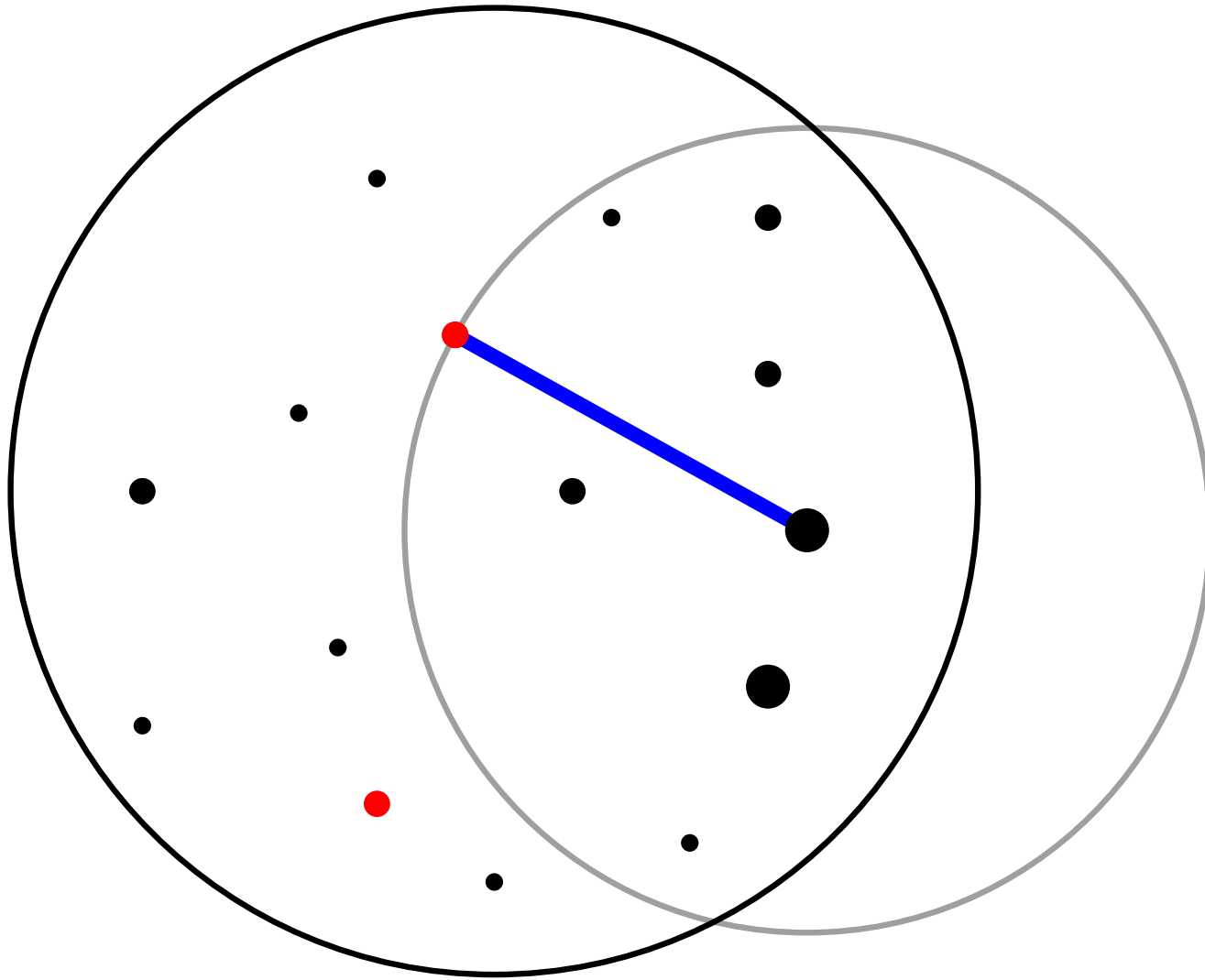
- Randomized initialization heuristic for  $k$ -median/ $k$ -means [Arthur and Vassilvitski, 2007]

# A Heuristic for Clustered Labeling



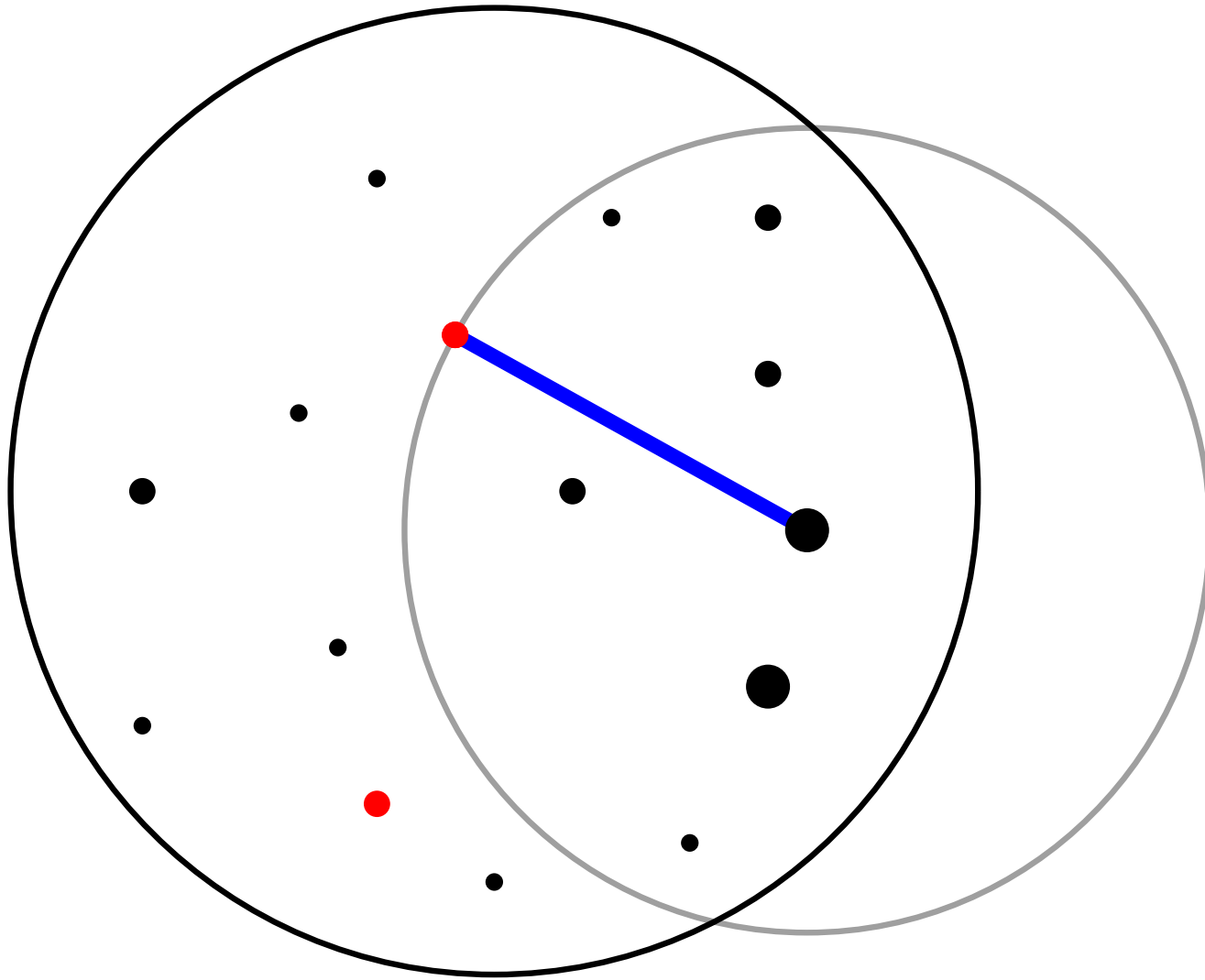
- Randomized initialization heuristic for  $k$ -median/ $k$ -means [Arthur and Vassilvitski, 2007]

# A Heuristic for Clustered Labeling



- Randomized initialization heuristic for  $k$ -median/ $k$ -means [Arthur and Vassilvitski, 2007]
- probability  $\approx \textit{distance}^d$

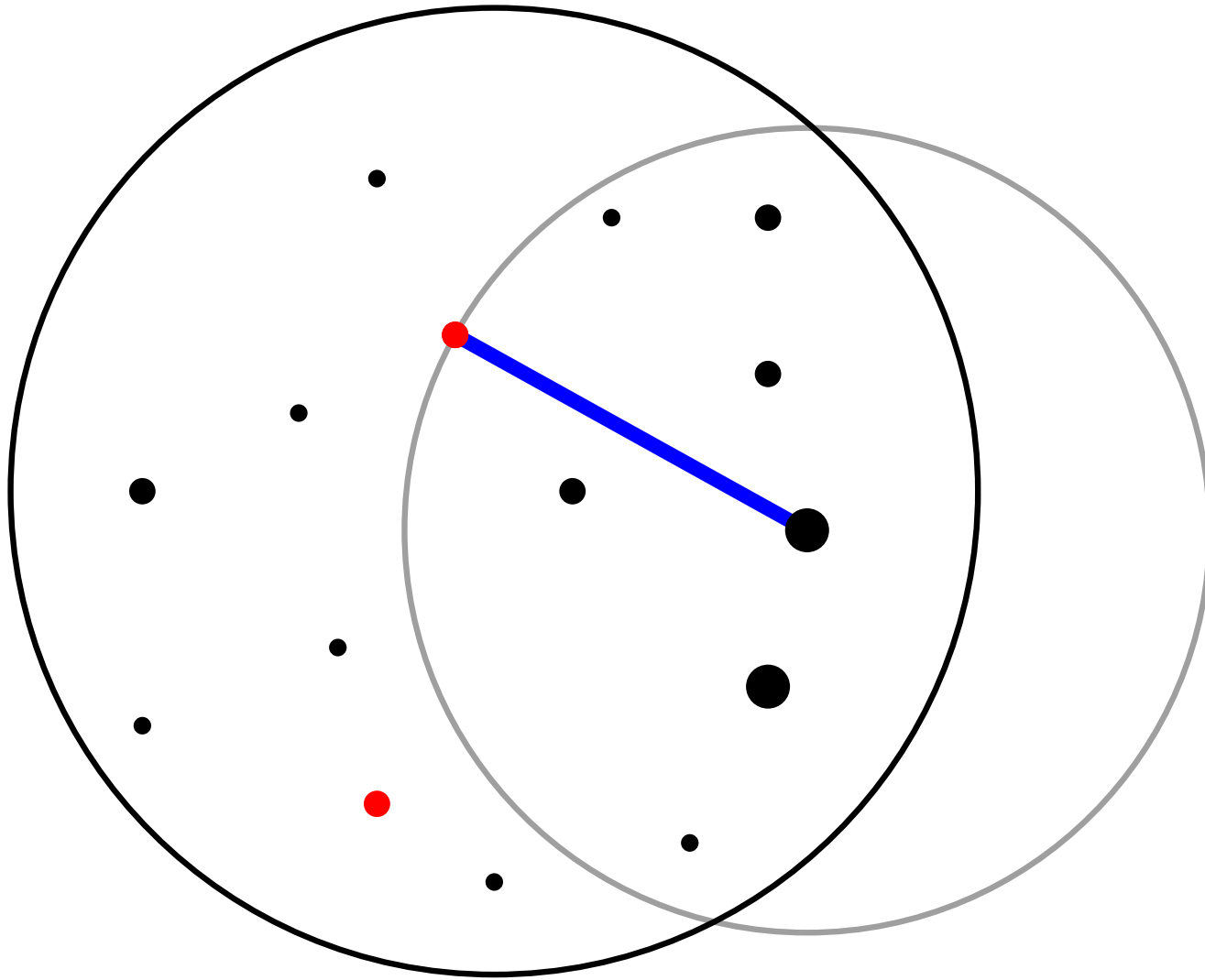
# A Heuristic for Clustered Labeling



- Randomized initialization heuristic for  $k$ -median/ $k$ -means [Arthur and Vassilvitski, 2007]
- probability  $\approx \textit{distance}^d$
- Clustering: assign to closest labeled site

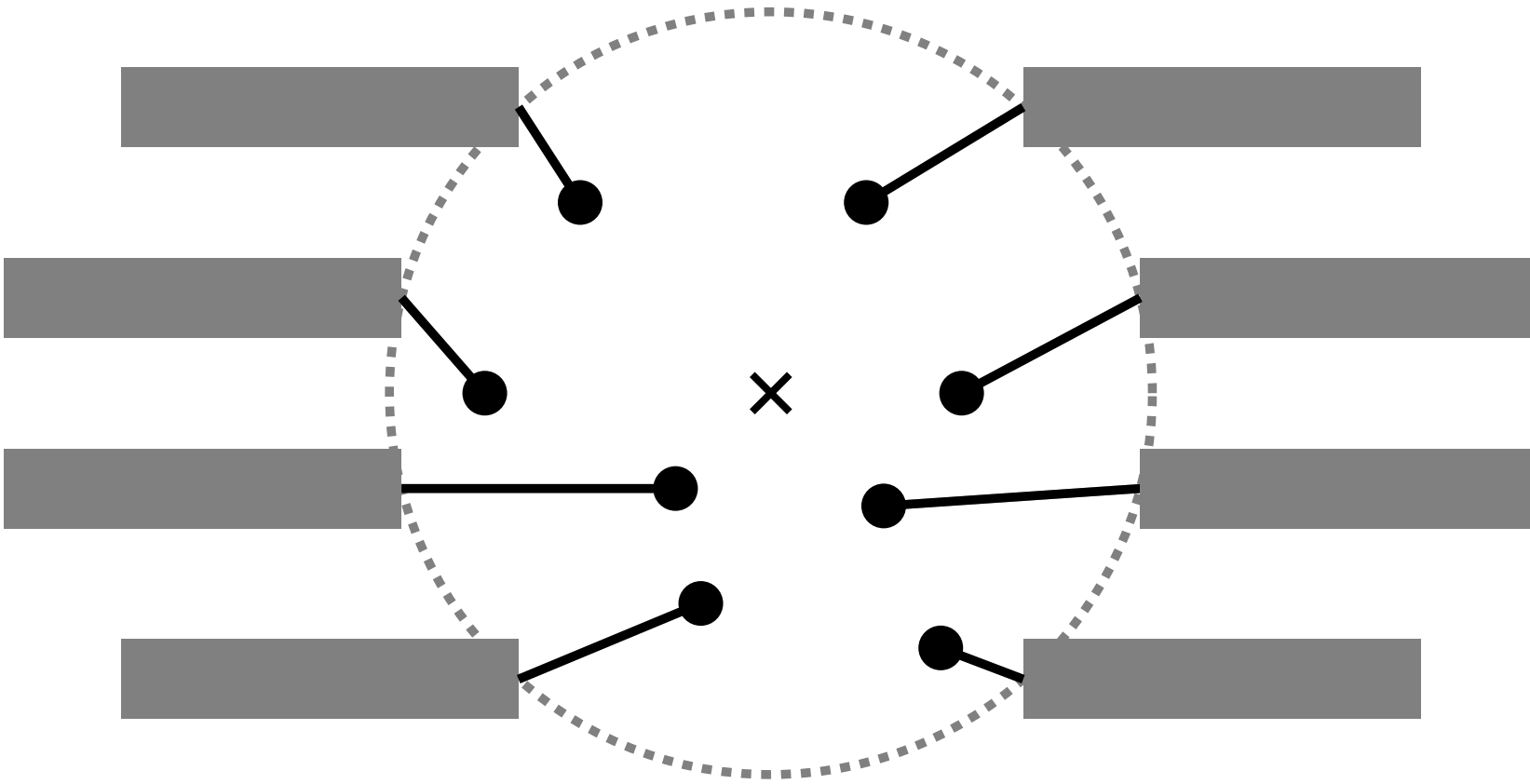


# A Heuristic for Clustered Labeling



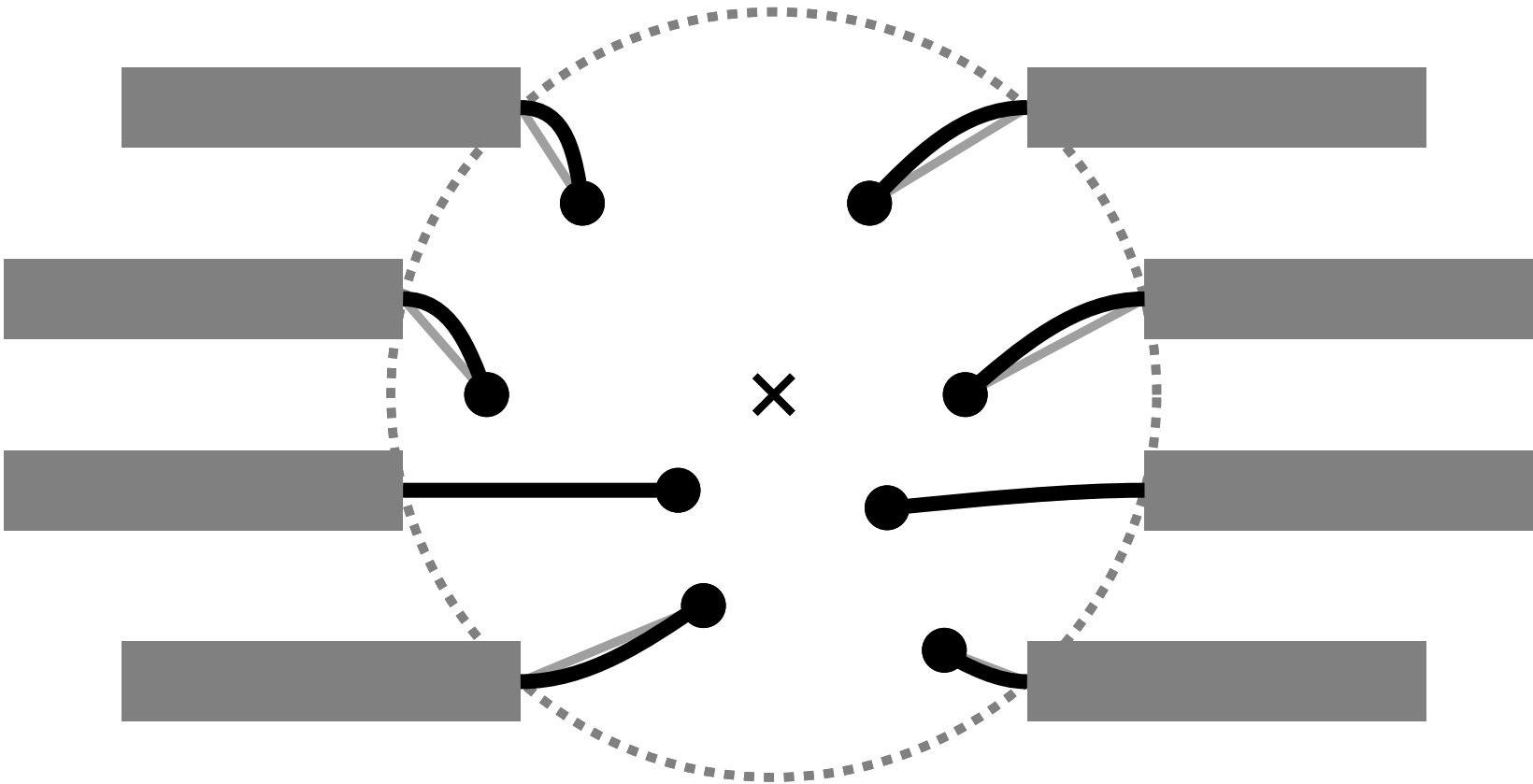
- Randomized initialization heuristic for  $k$ -median/ $k$ -means [Arthur and Vassilvitski, 2007]
- probability  $\approx \textit{distance}^d$
- Clustering: assign to closest labeled site
  - much better than uniform random selection
  - fast

# Bézier Curves as Leaders



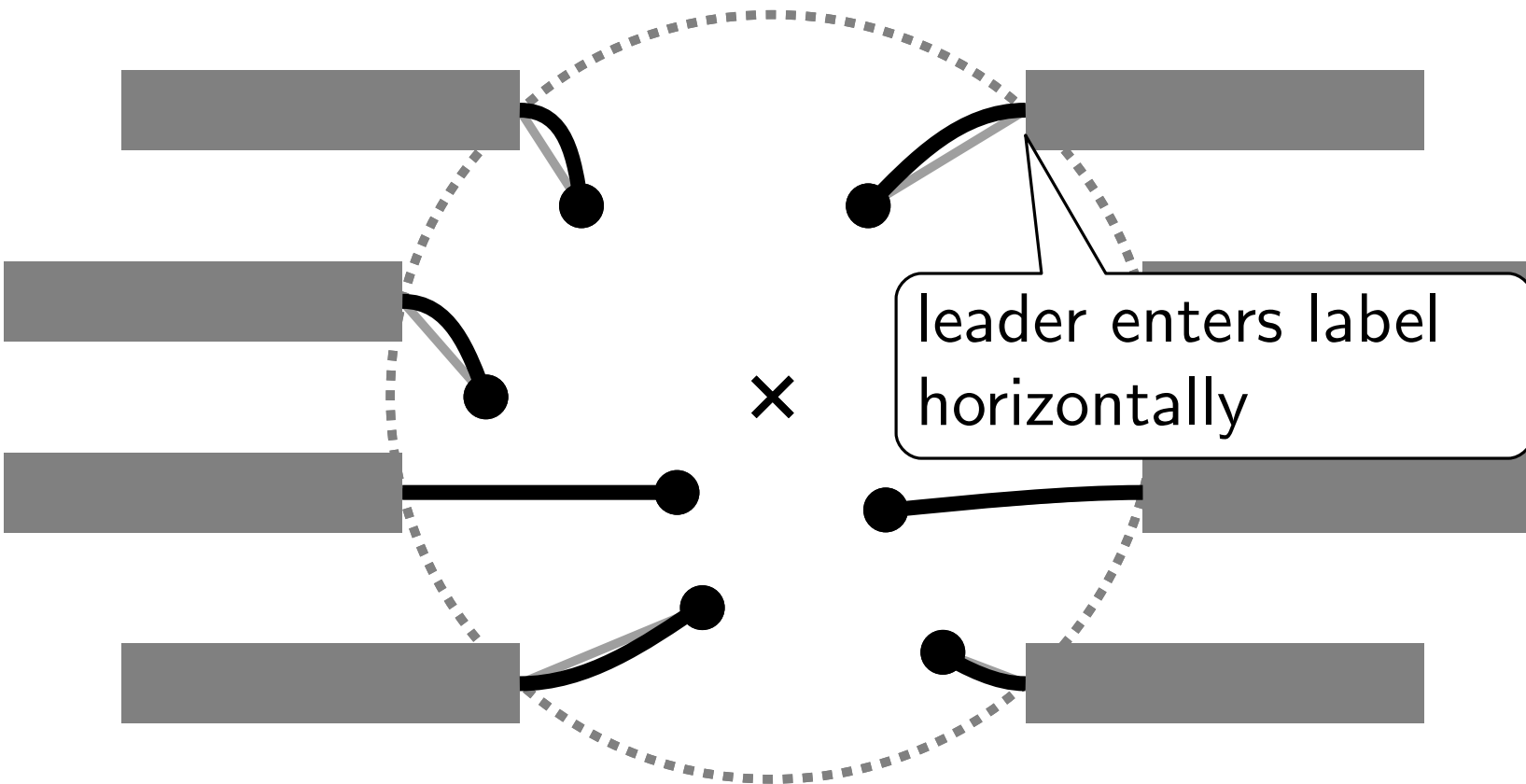
# Bézier Curves as Leaders

● post-processing:

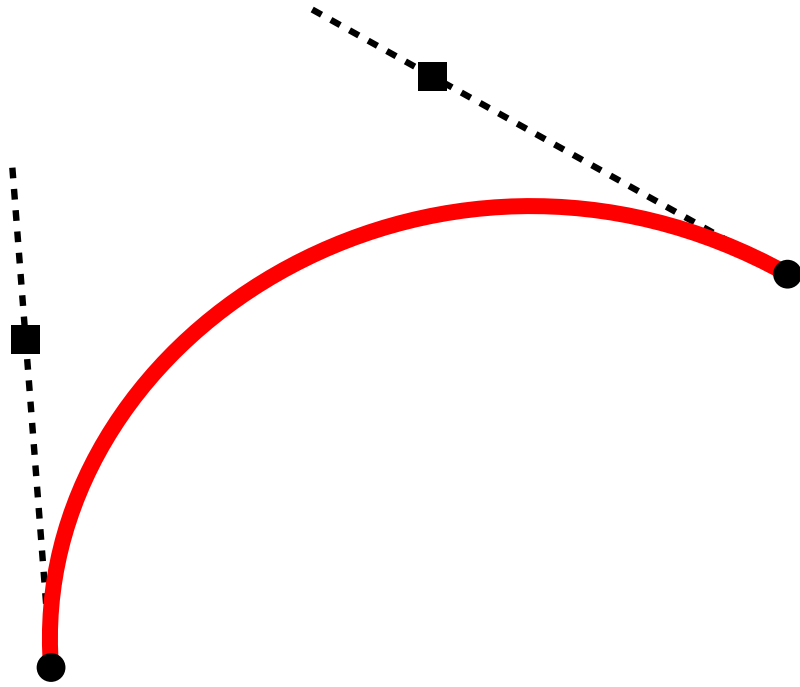


# Bézier Curves as Leaders

● post-processing:

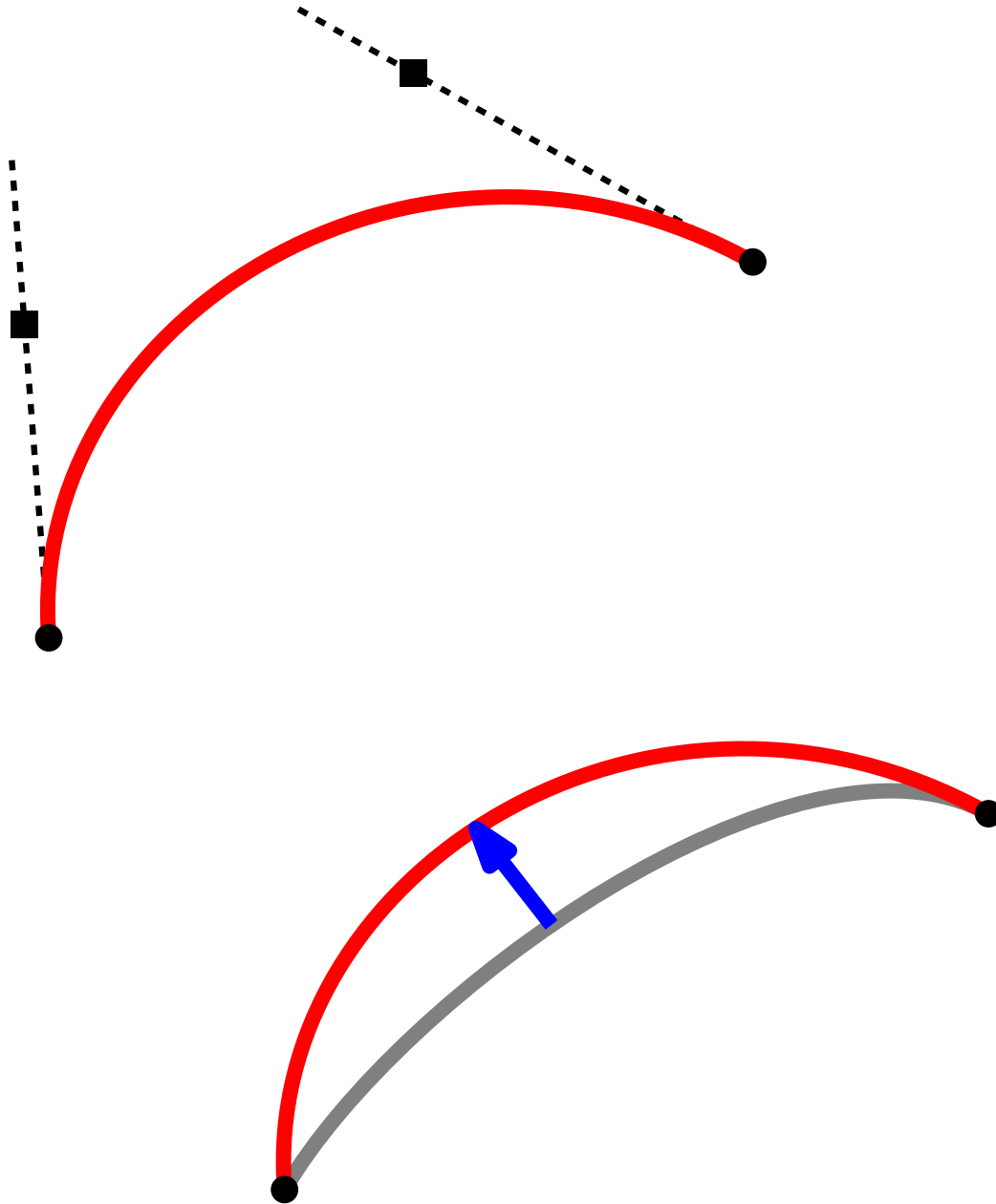


# Bézier Curves as Leaders



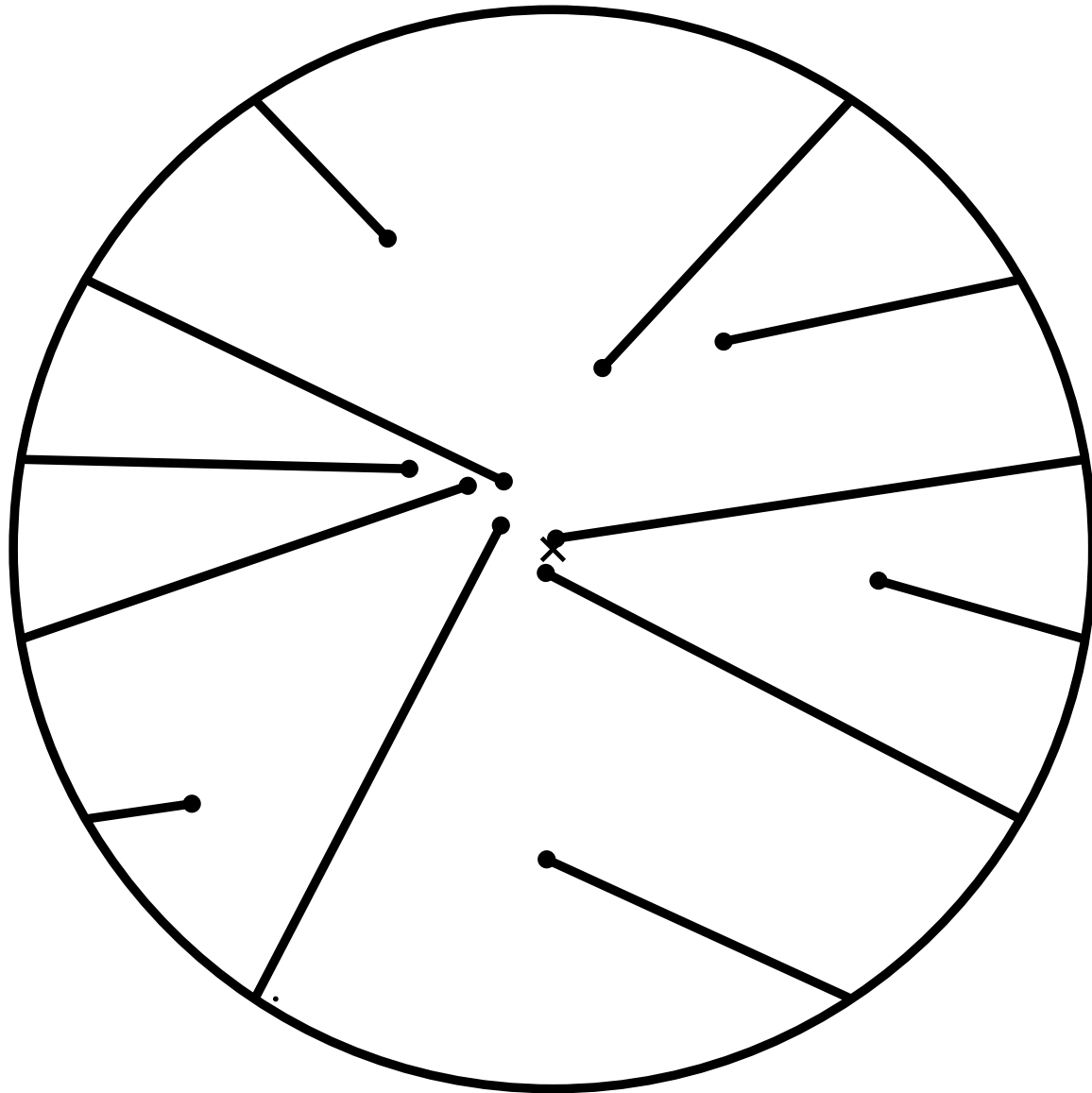
- post-processing:
  - (cubic) Bézier curves

# Bézier Curves as Leaders



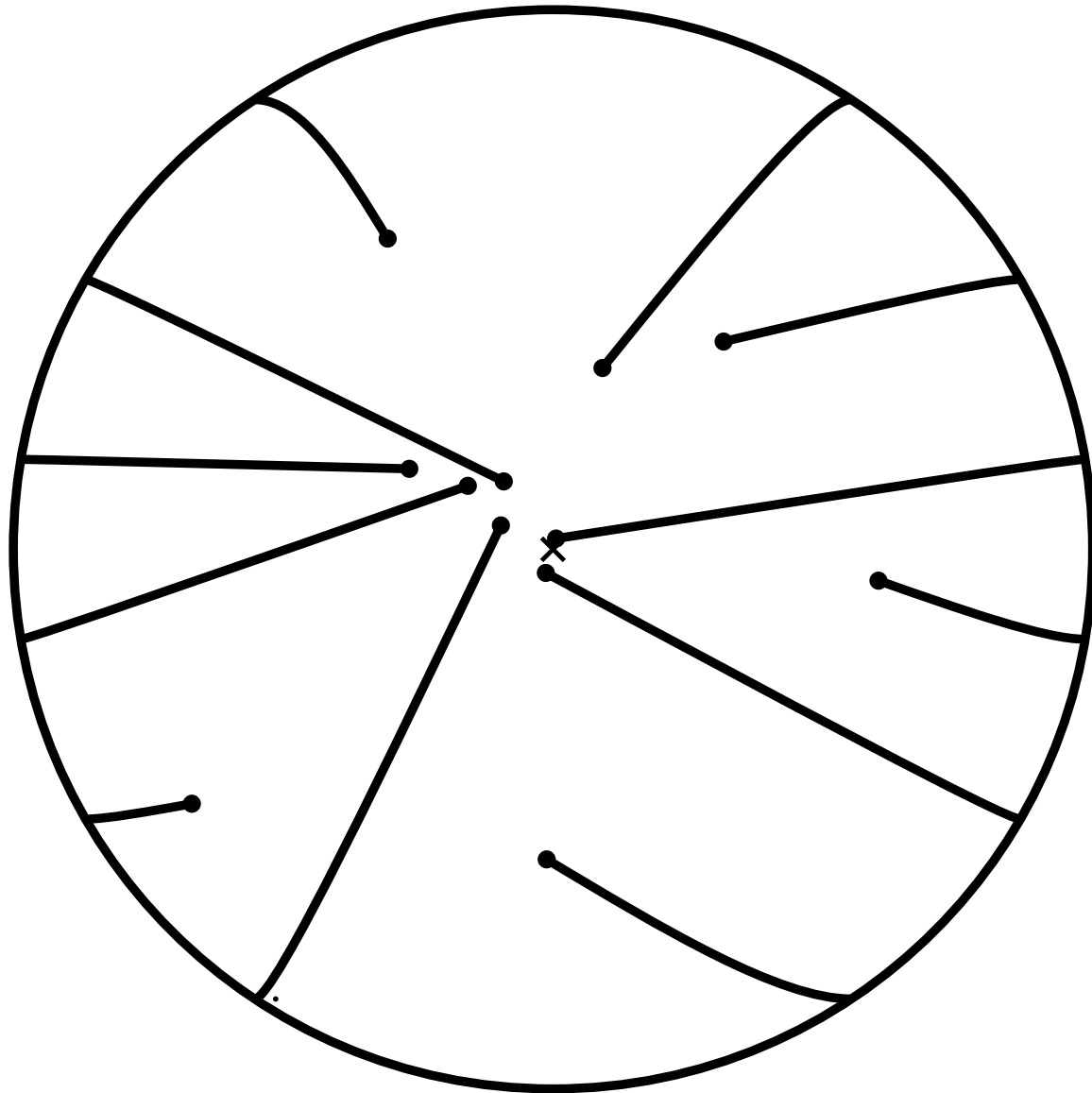
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach

# Bézier Curves as Leaders



- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

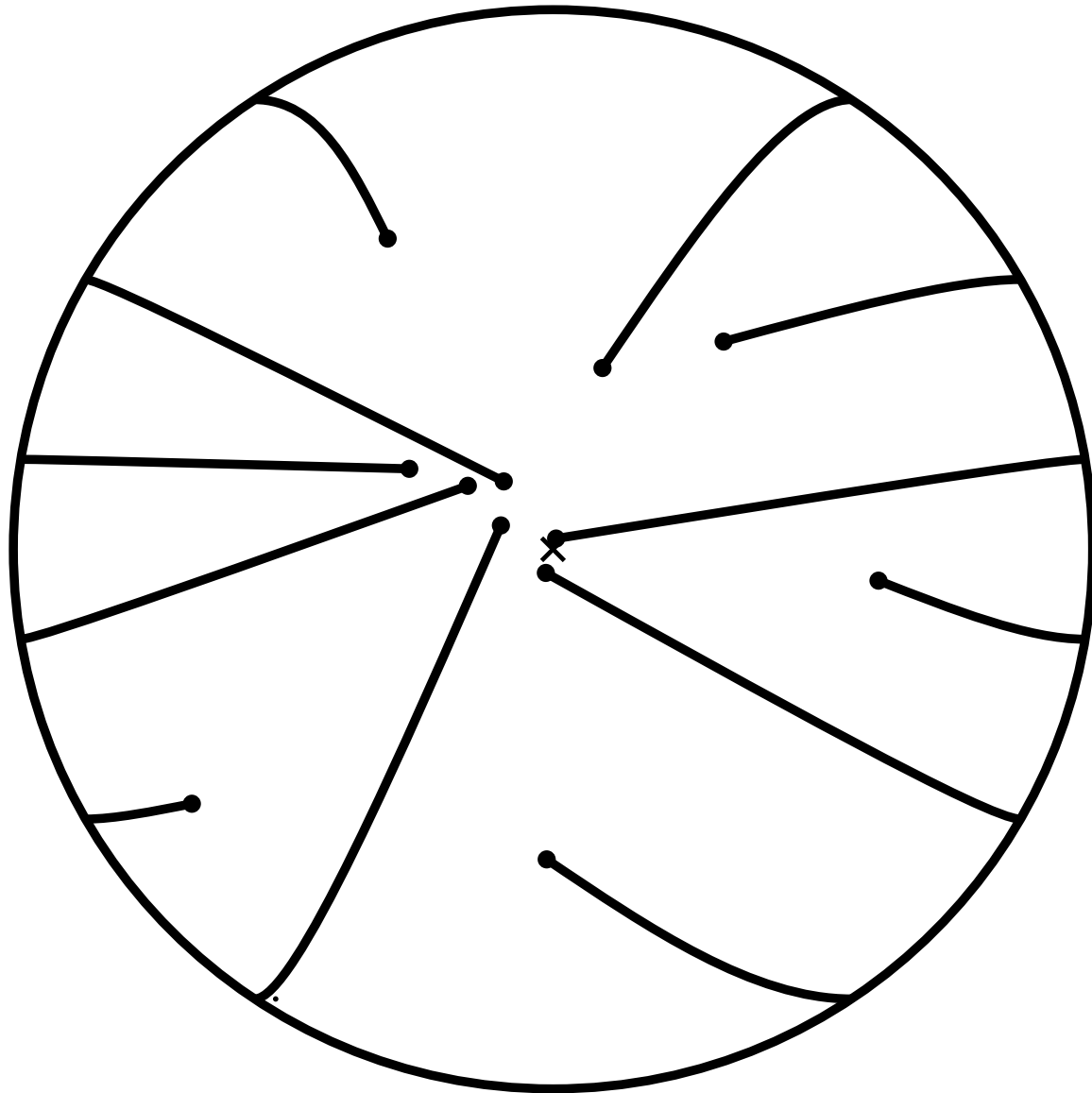
# Bézier Curves as Leaders



- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

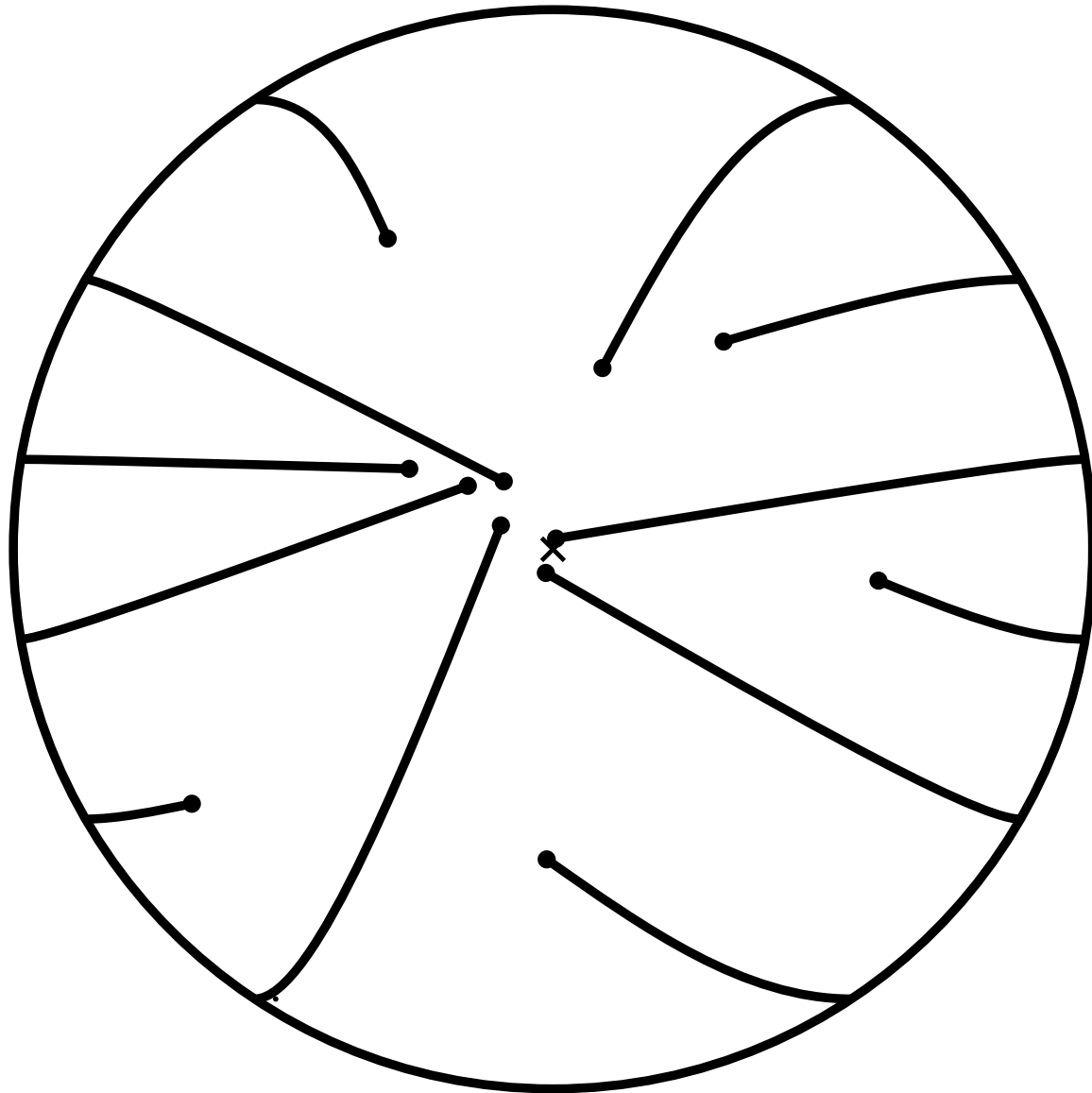


# Bézier Curves as Leaders



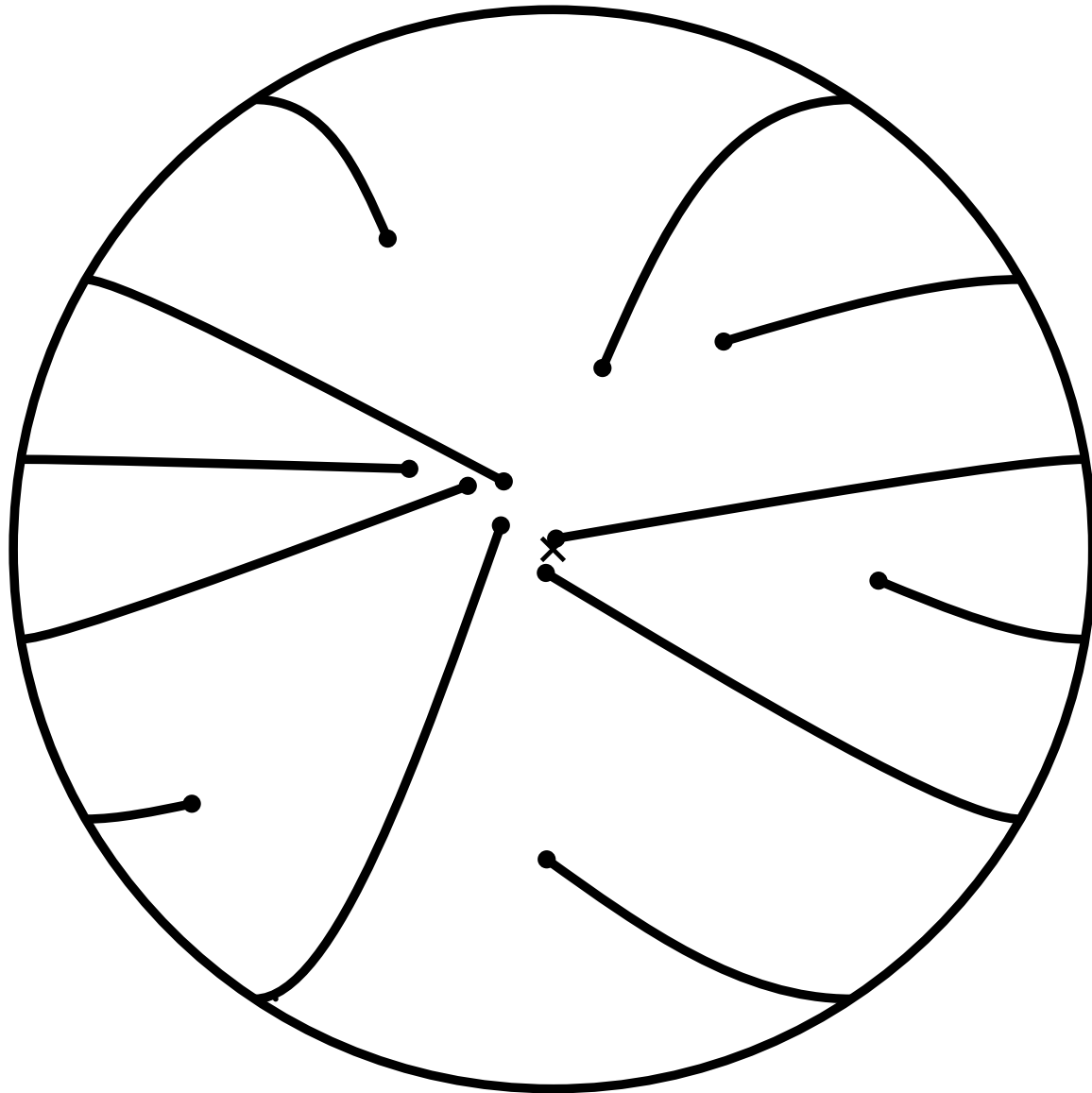
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



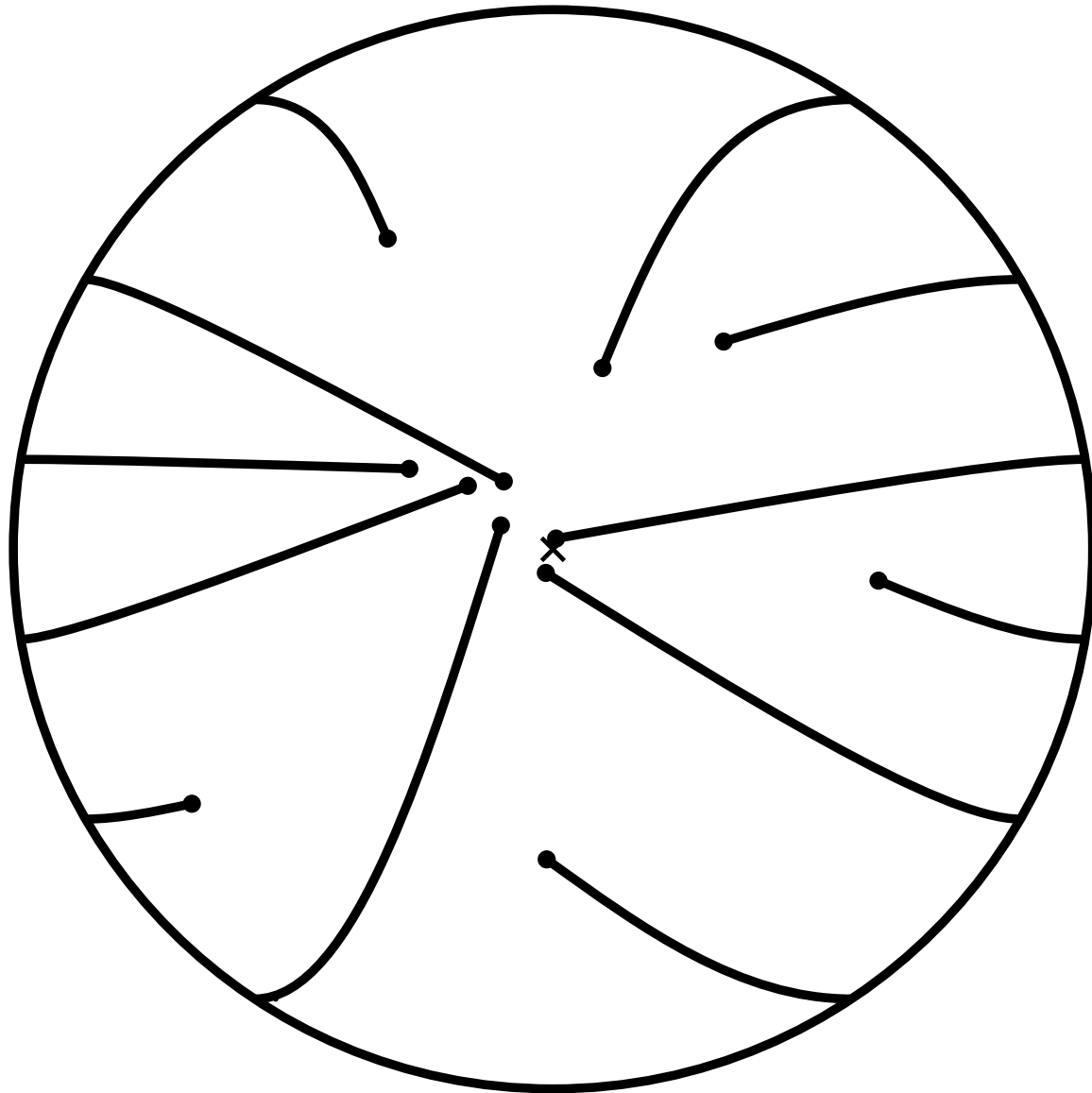
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



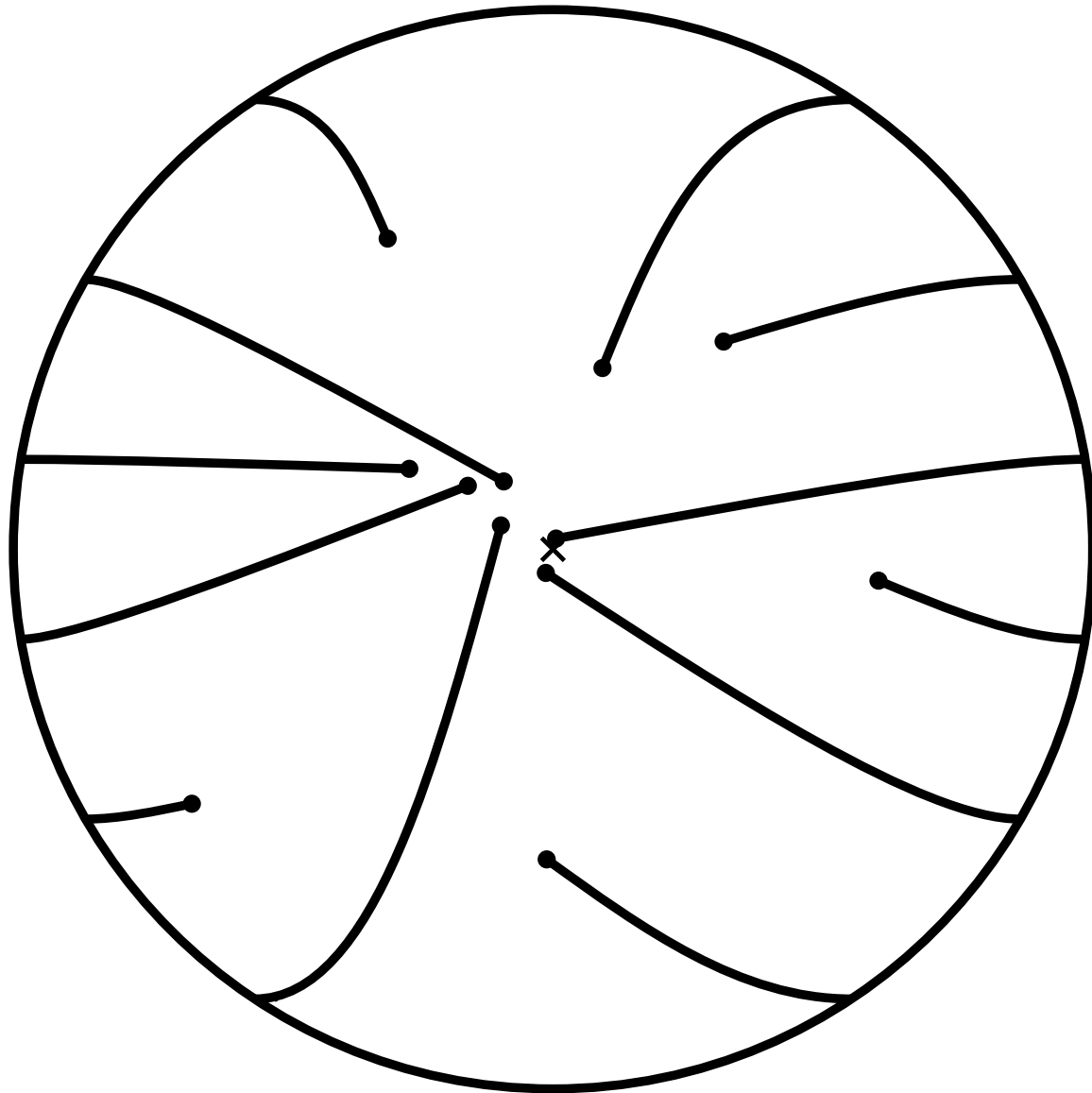
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



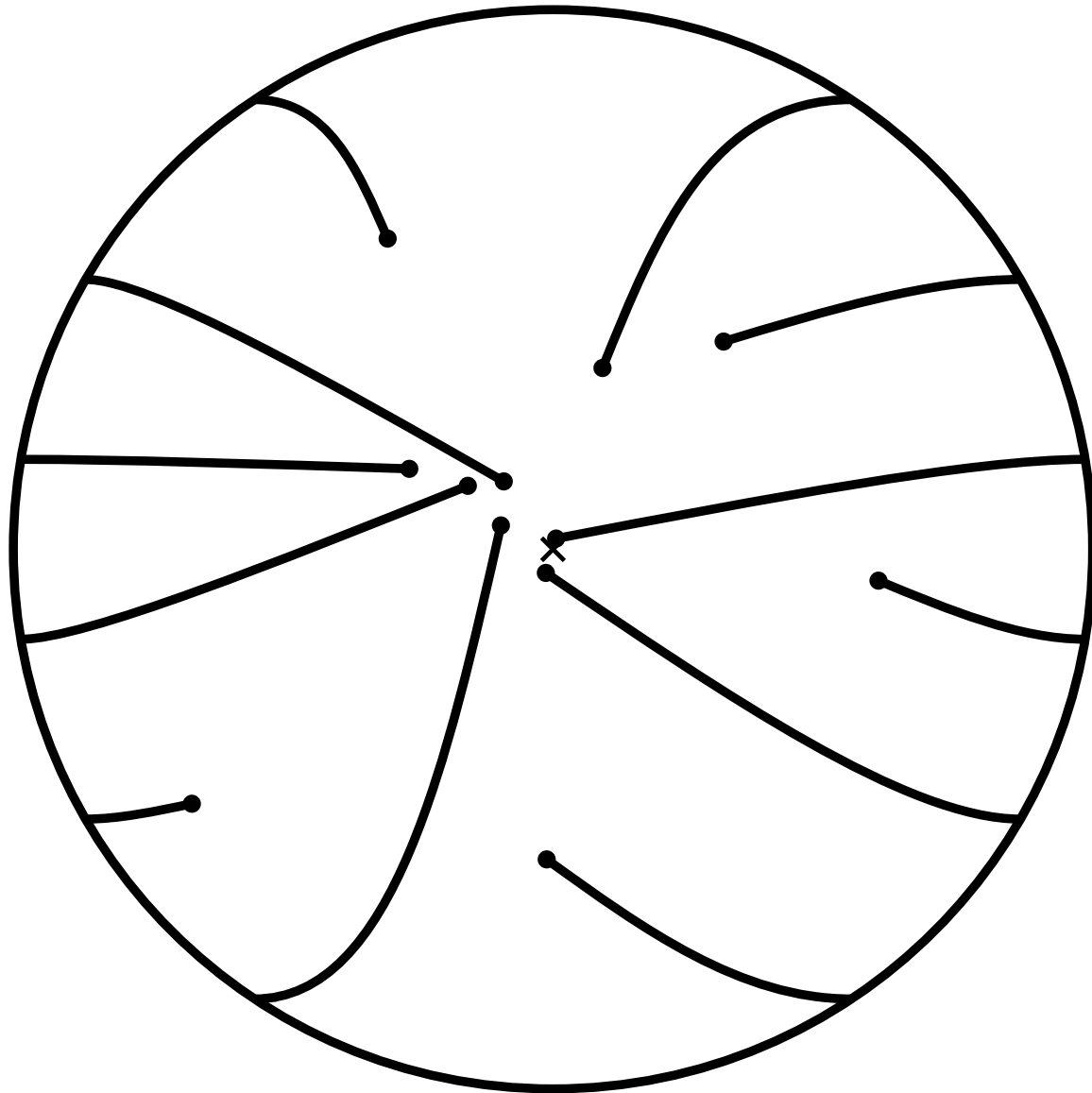
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



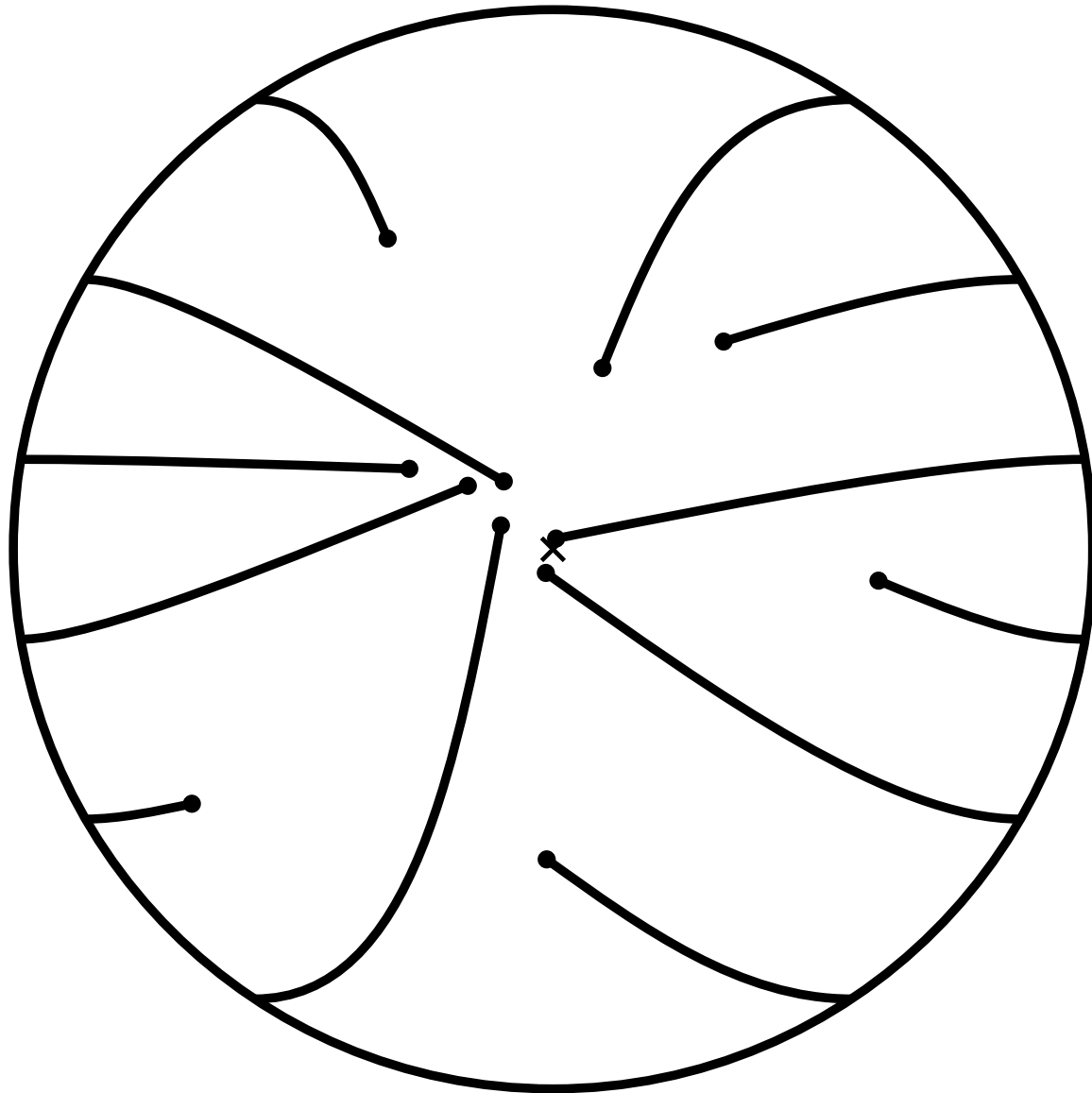
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



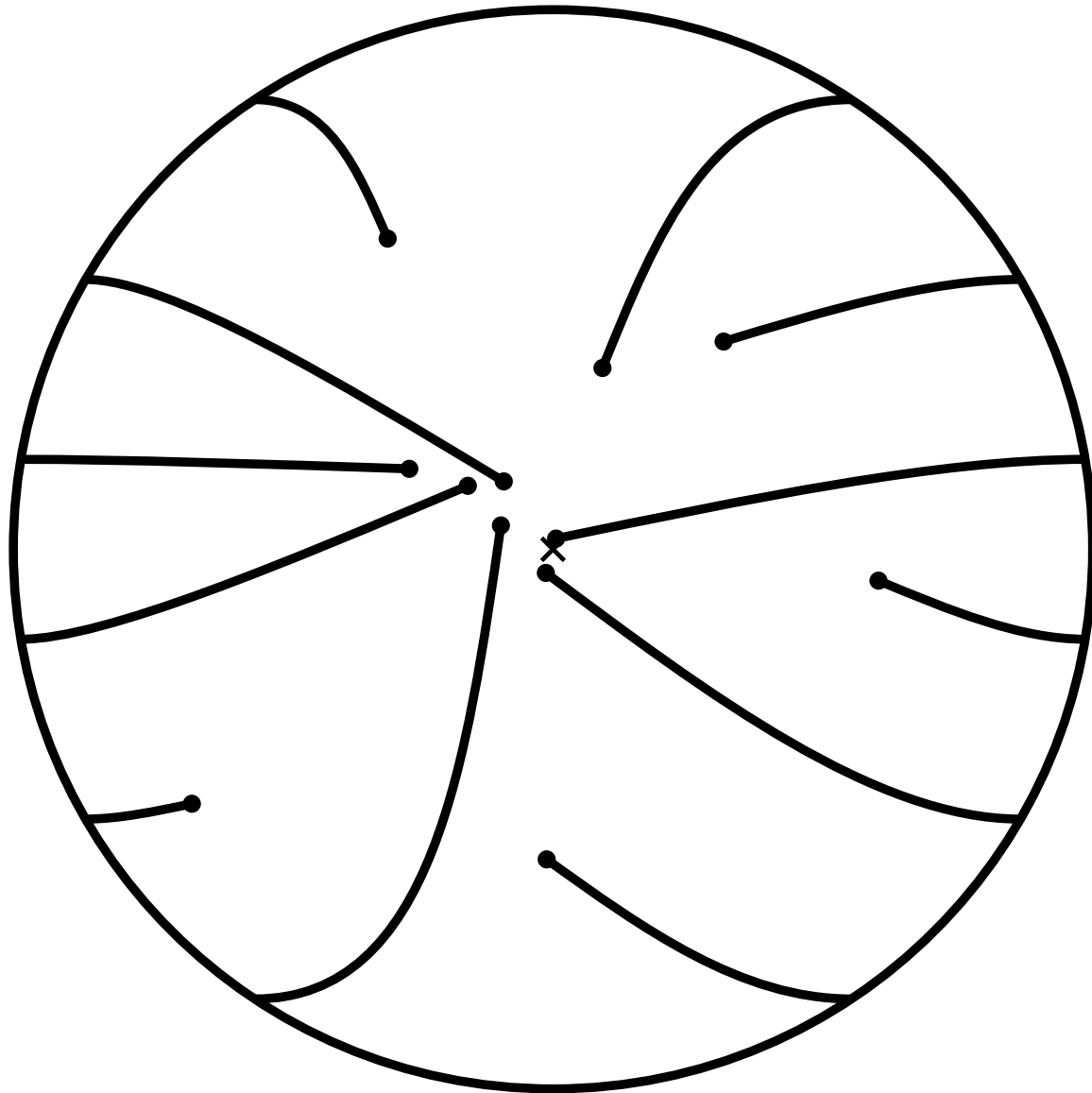
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

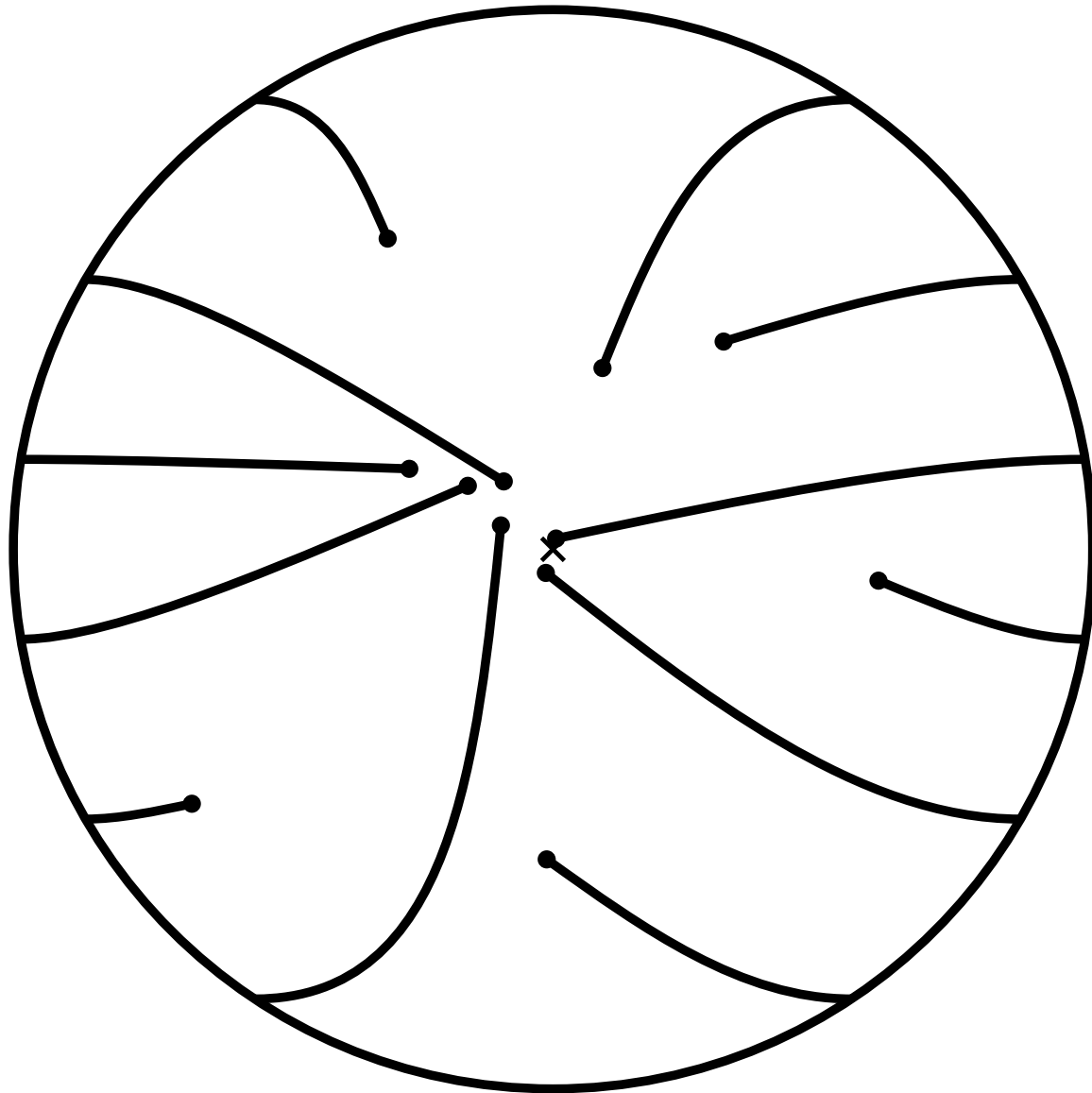
# Bézier Curves as Leaders



- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

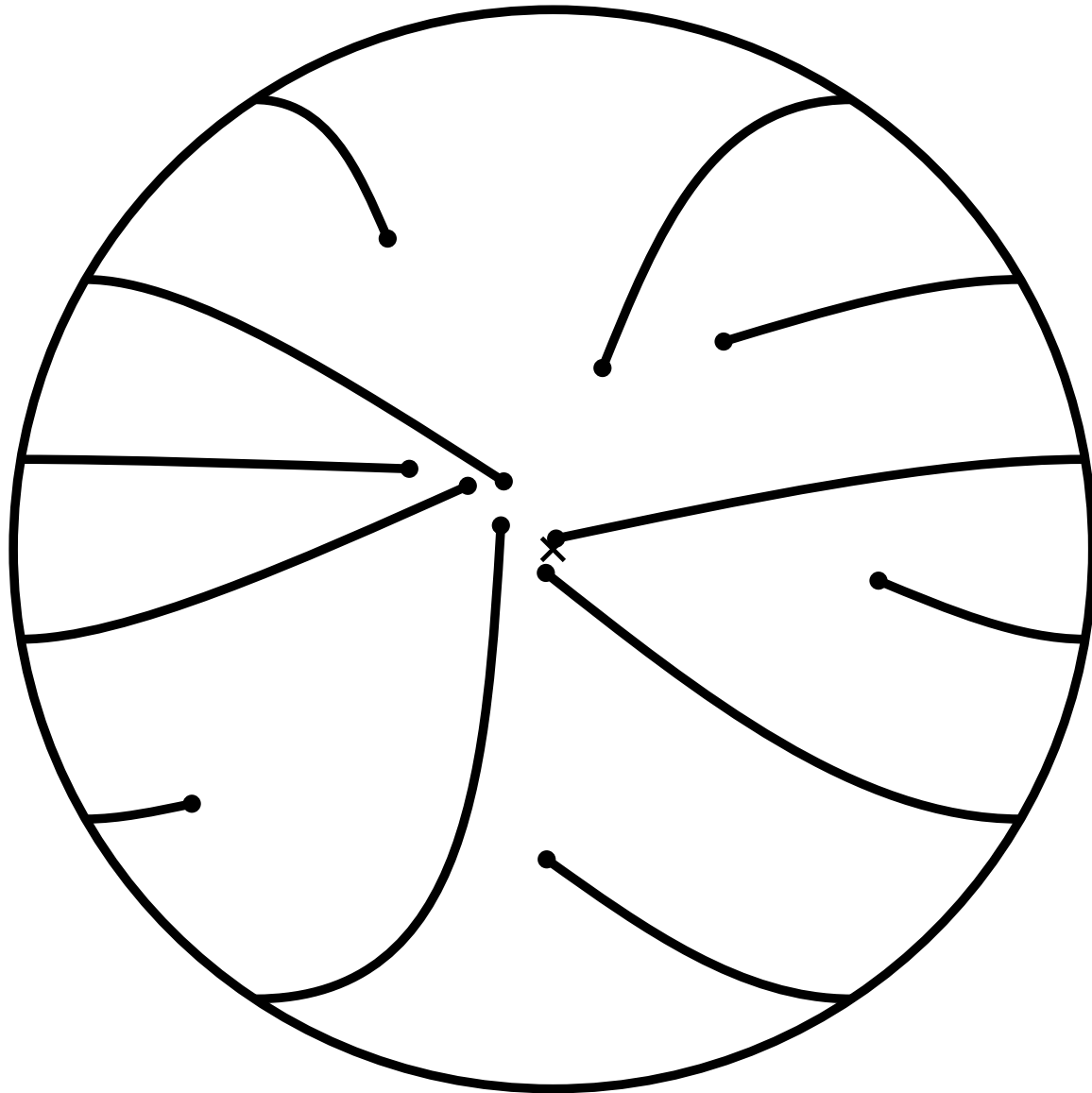


# Bézier Curves as Leaders



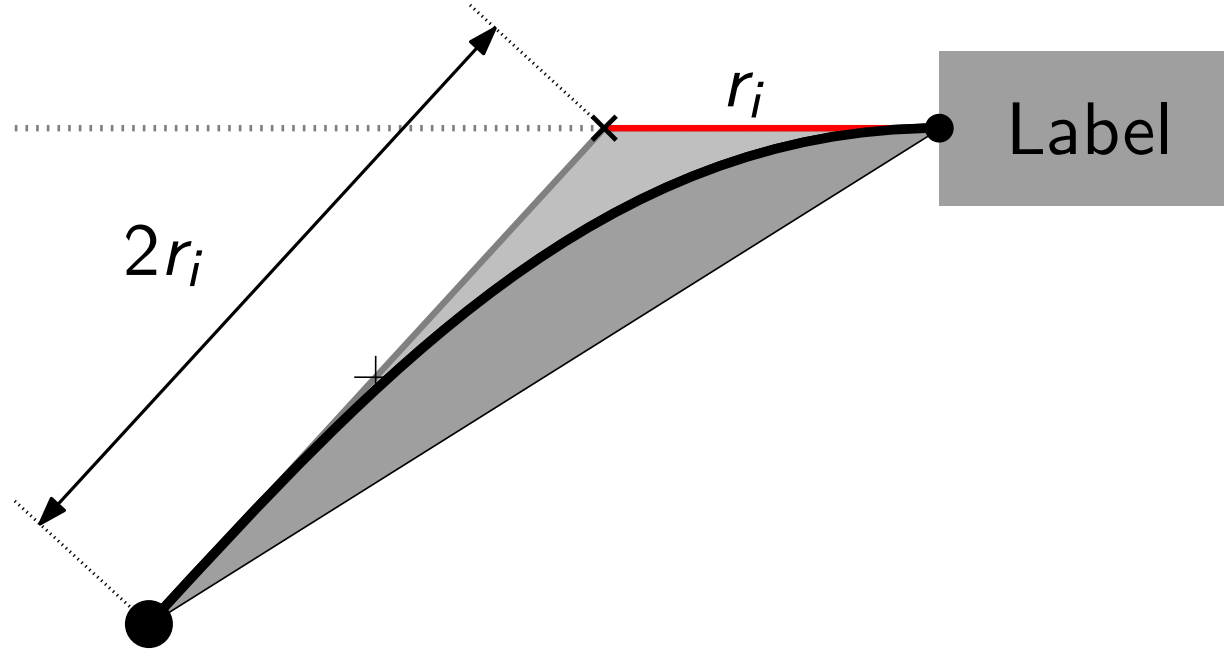
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



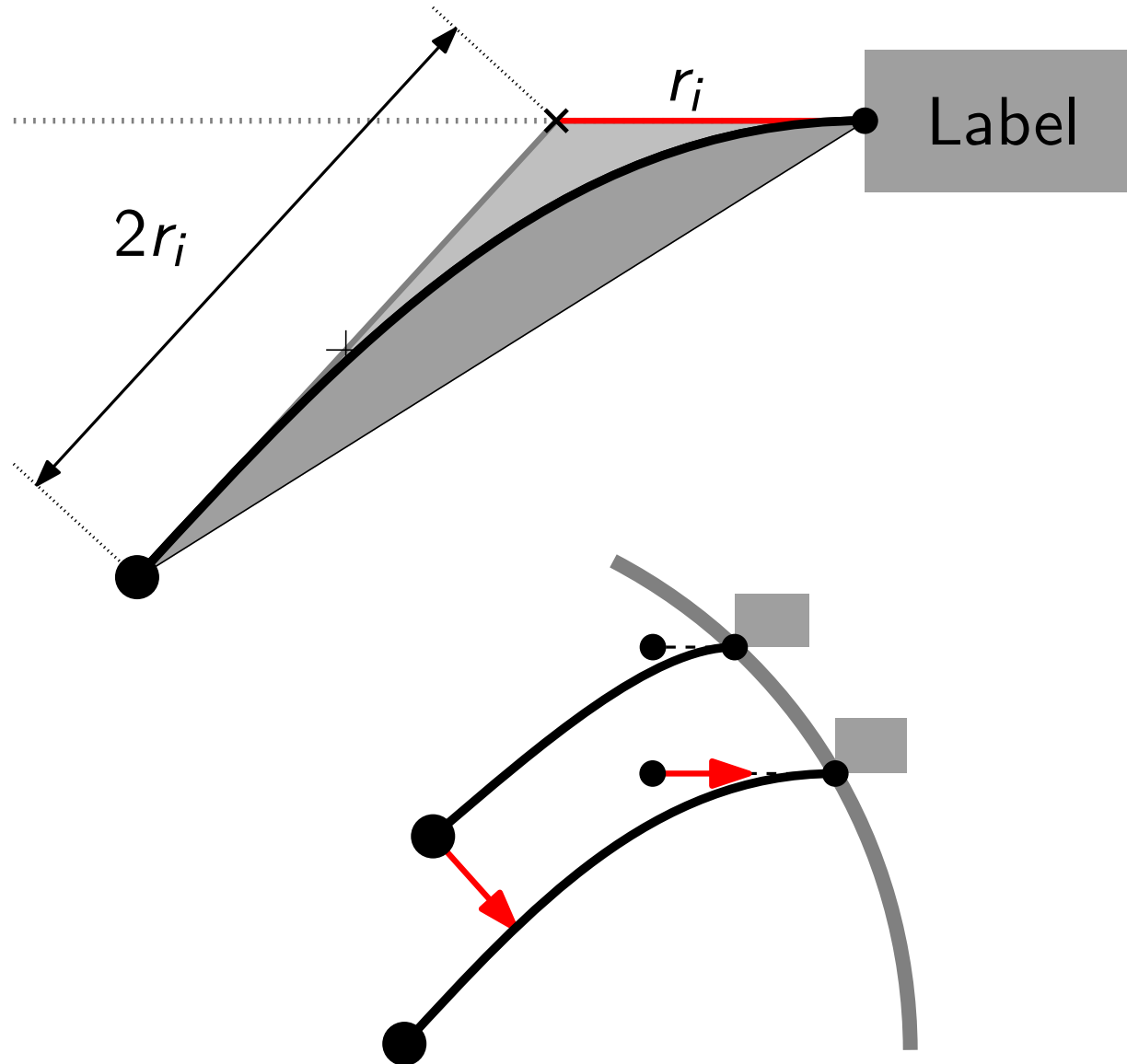
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)

# Bézier Curves as Leaders



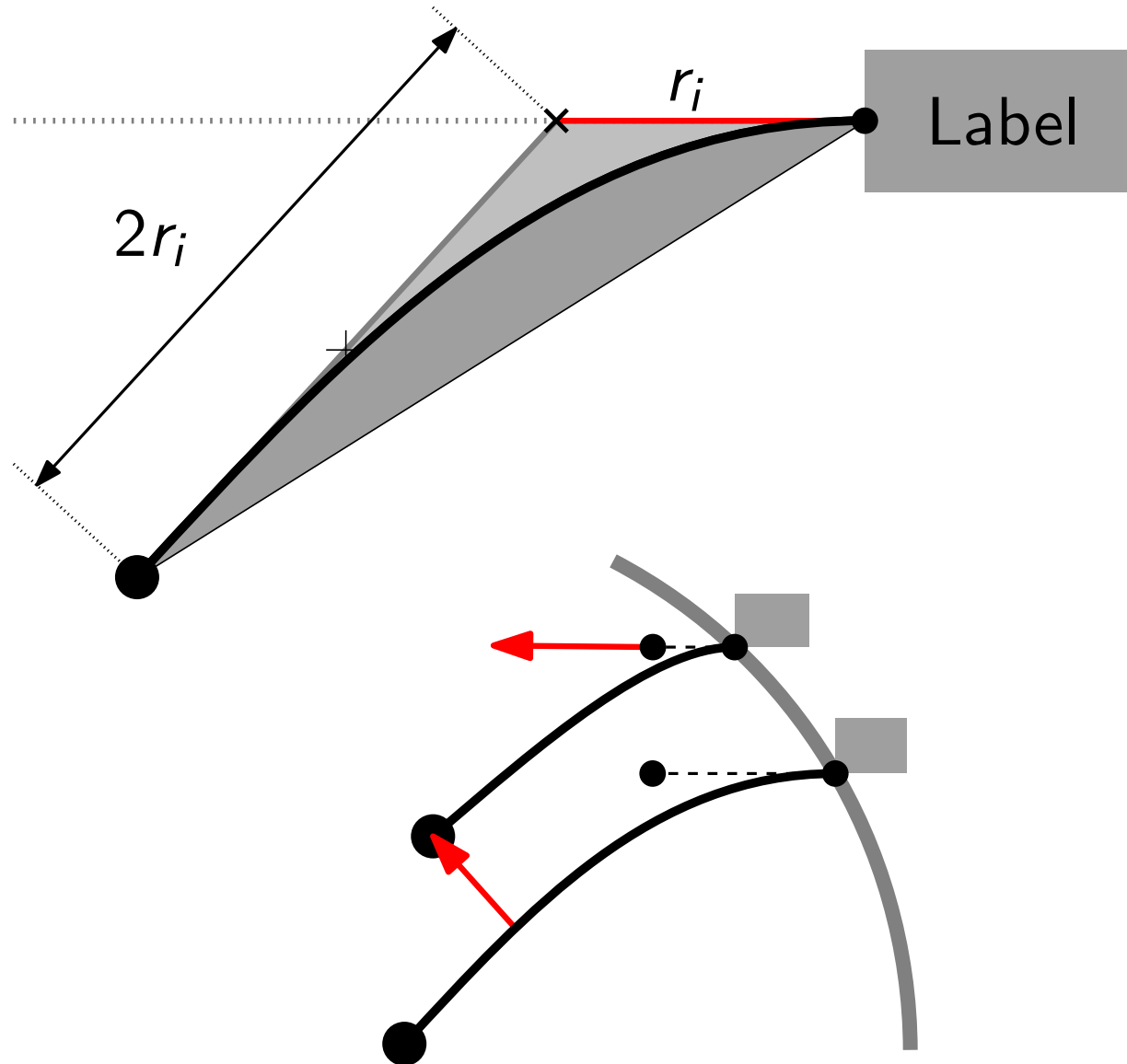
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)
  - move towards desired shape

# Bézier Curves as Leaders



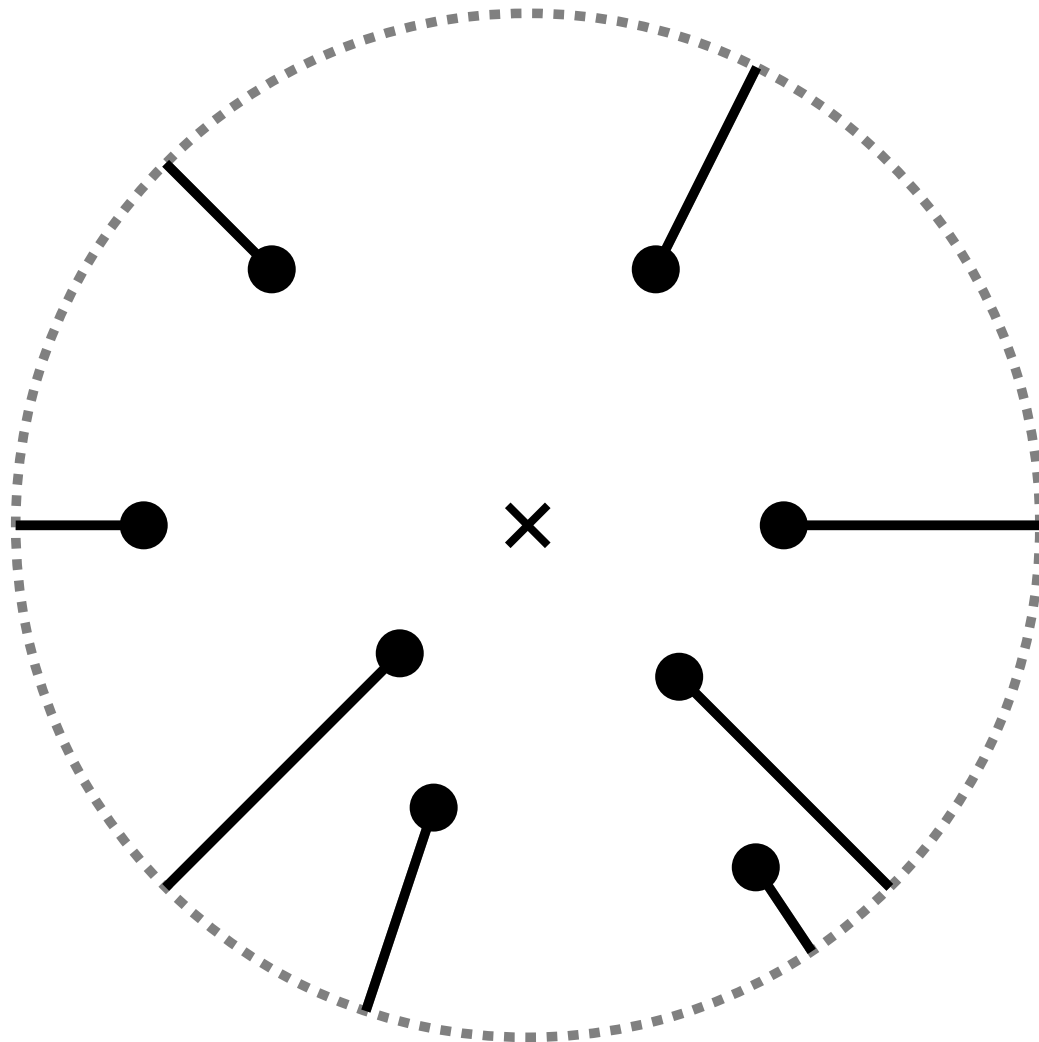
- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)
  - move towards desired shape
  - avoid other leaders

# Bézier Curves as Leaders

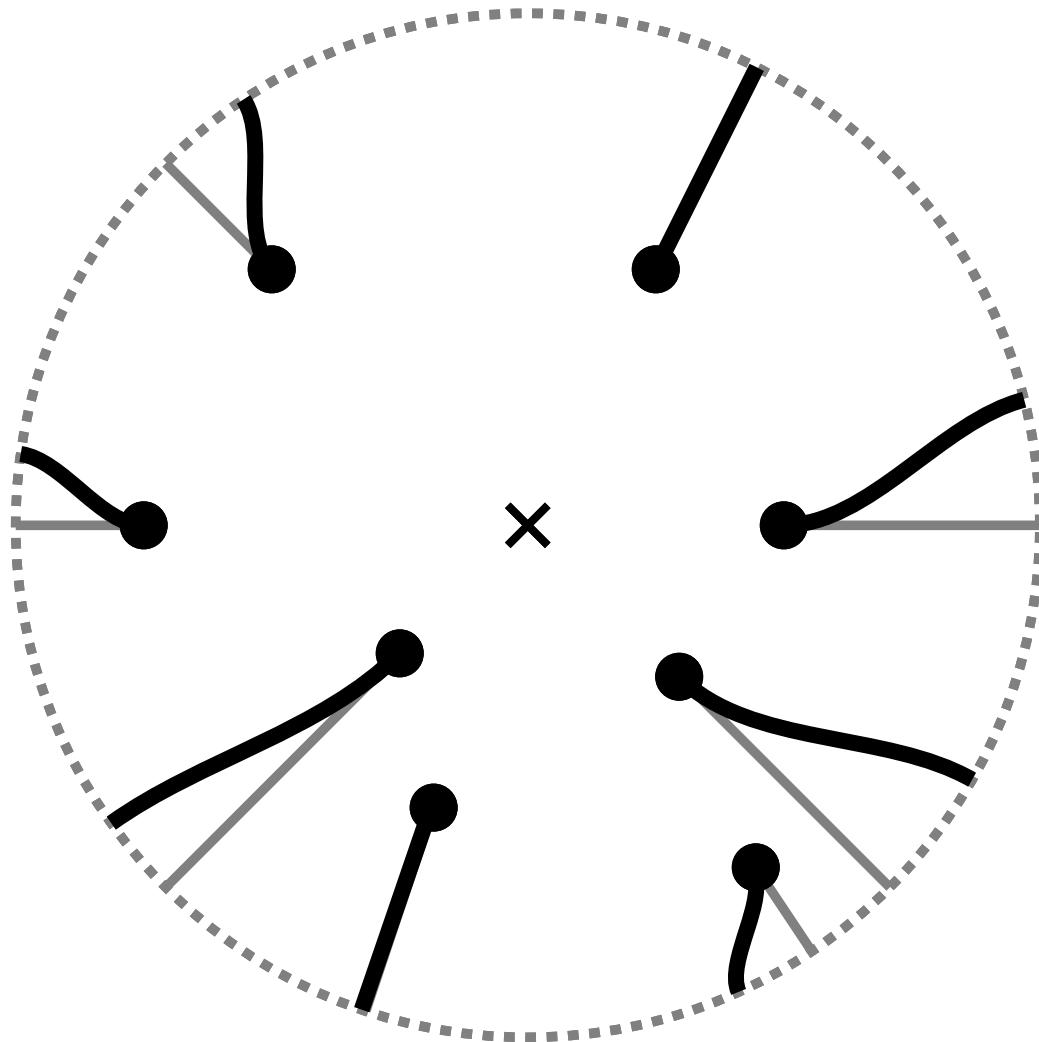


- post-processing:
  - (cubic) Bézier curves
  - force-directed approach
- gradually improve drawing according to desired changes (*forces*)
  - move towards desired shape
  - avoid other leaders

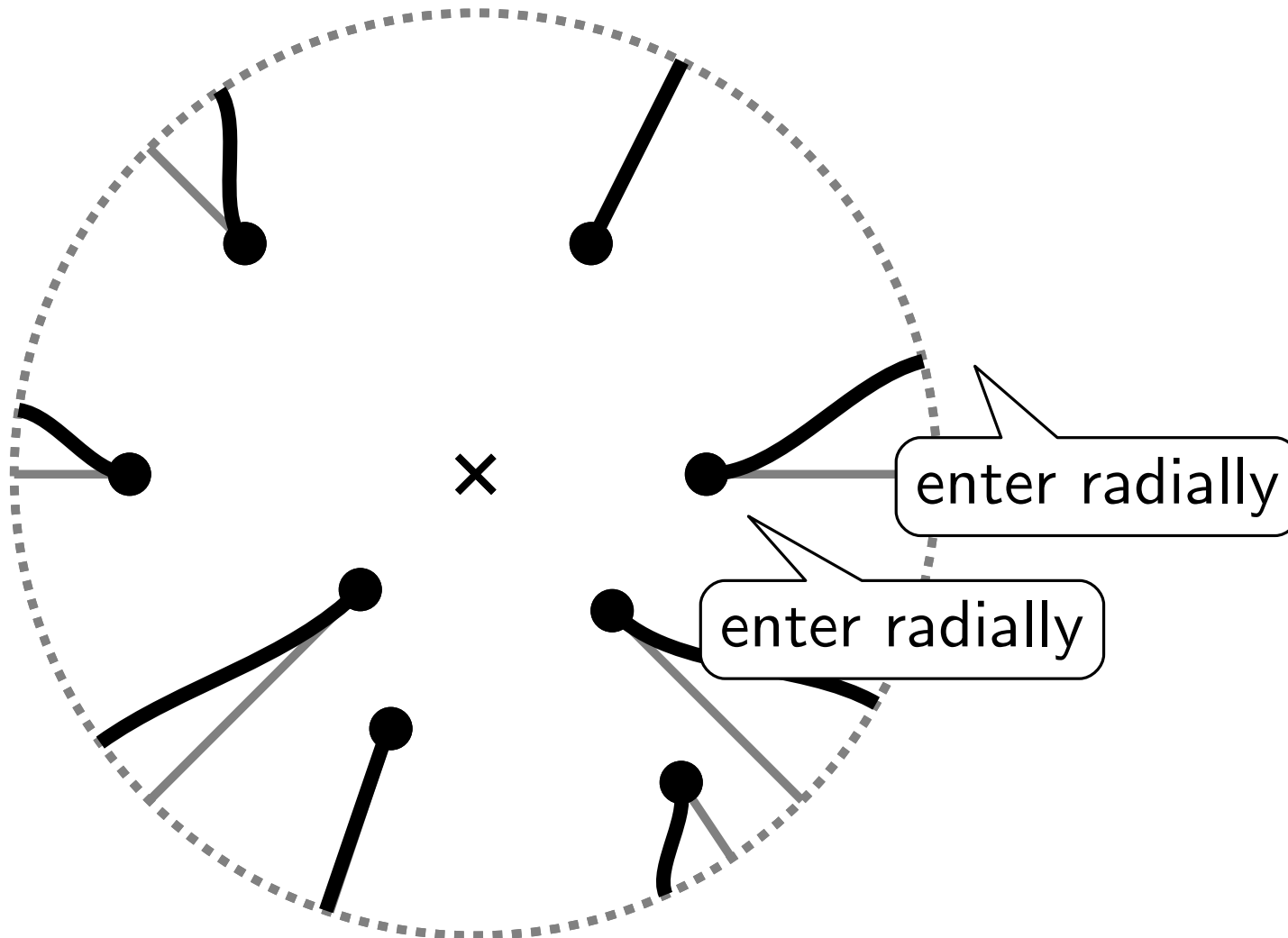
# Curvy Leaders in the Radial Model



# Curvy Leaders in the Radial Model

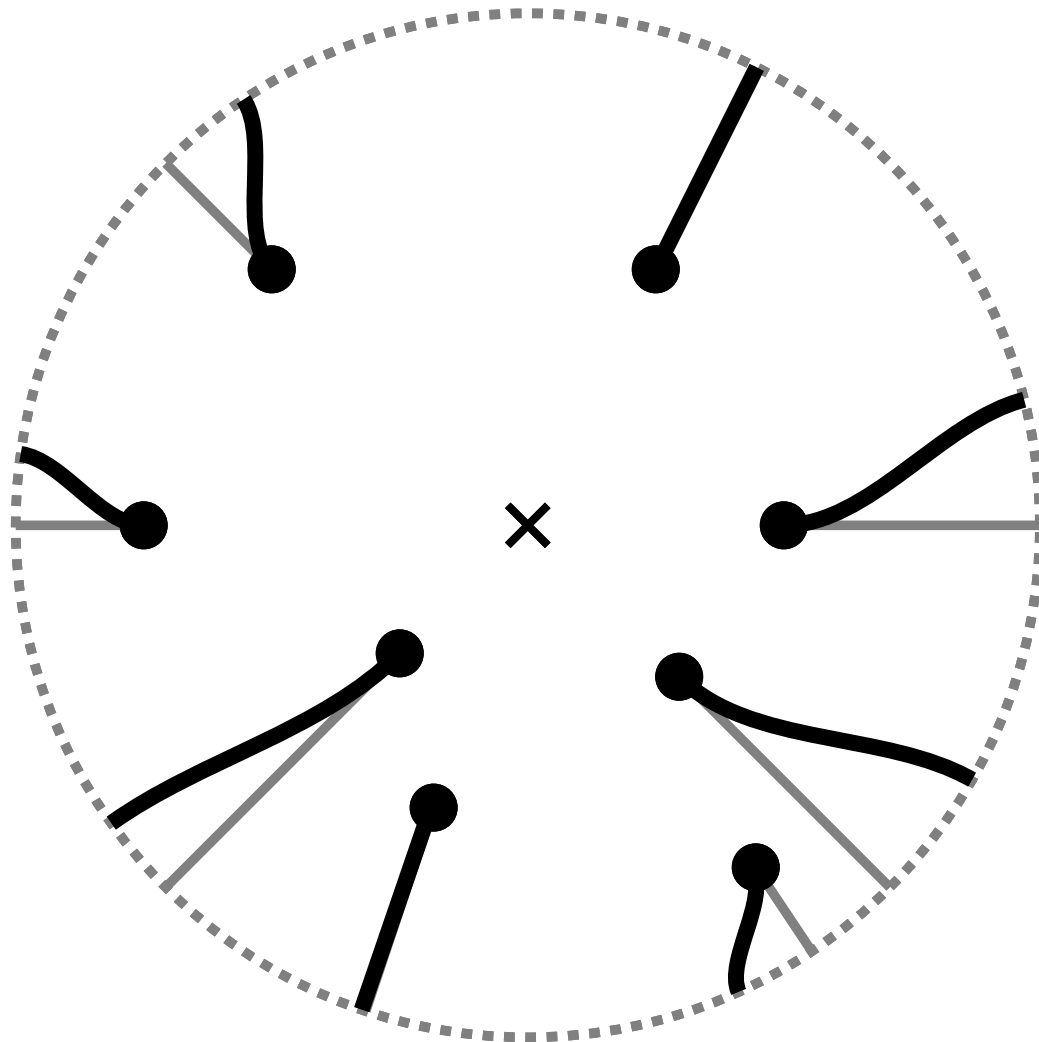


# Curvy Leaders in the Radial Model





# Curvy Leaders in the Radial Model



- move label positions on boundary
- improve angle

# Conclusion and Open Problems

- Free leader model preferred for smaller numbers of labeled sites
- Radial model for many short labels

# Conclusion and Open Problems

- Free leader model preferred for smaller numbers of labeled sites
- Radial model for many short labels
- Faster algorithms for finding a good center in the radial leader model?

# Conclusion and Open Problems

- Free leader model preferred for smaller numbers of labeled sites
- Radial model for many short labels
- Faster algorithms for finding a good center in the radial leader model?
- Make interactive methods more stable during mouse movement.

# Conclusion and Open Problems

- Free leader model preferred for smaller numbers of labeled sites
- Radial model for many short labels
- Faster algorithms for finding a good center in the radial leader model?
- Make interactive methods more stable during mouse movement.

Idea: Weights changing over time

