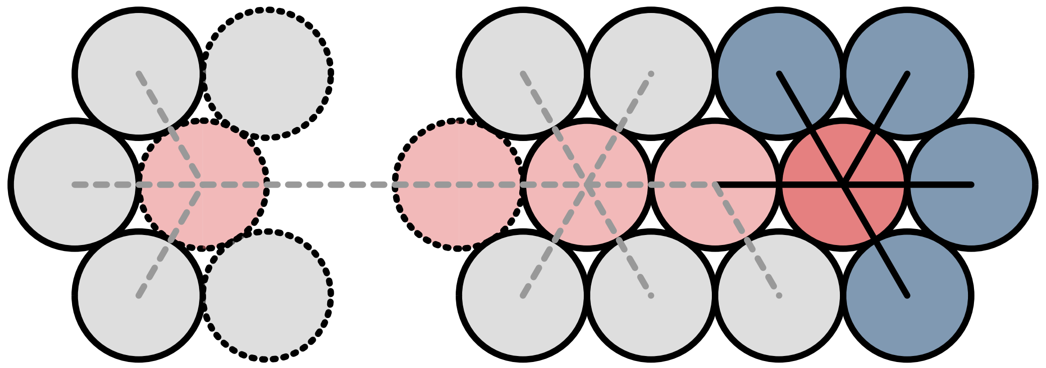
Weak Unit Disk Contact Representations for Graphs **without Embedding**

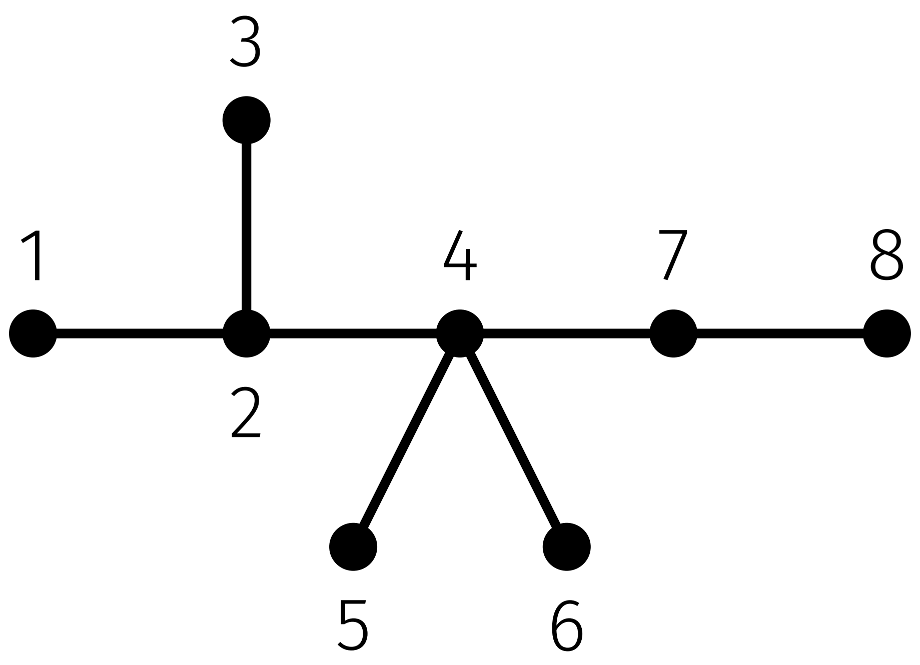
Jonas Cleve, Freie Universität Berlin



EuroCG 2020 – March 16–18 – Würzburg, Germany

Unit Disk Contact Representations

Unit disk contact representation (UDCR) of $G = (V, E)$:



graph G

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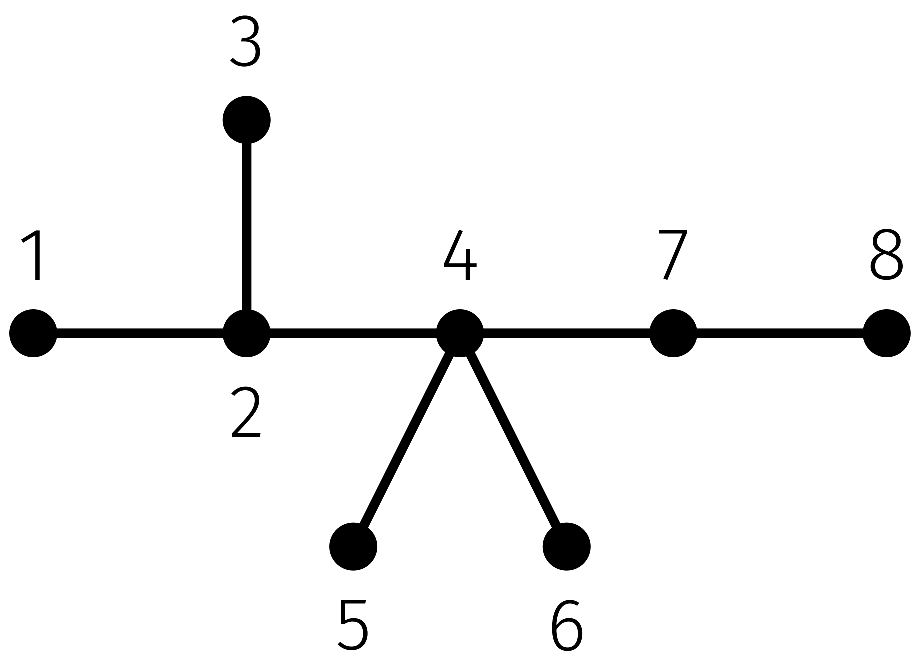

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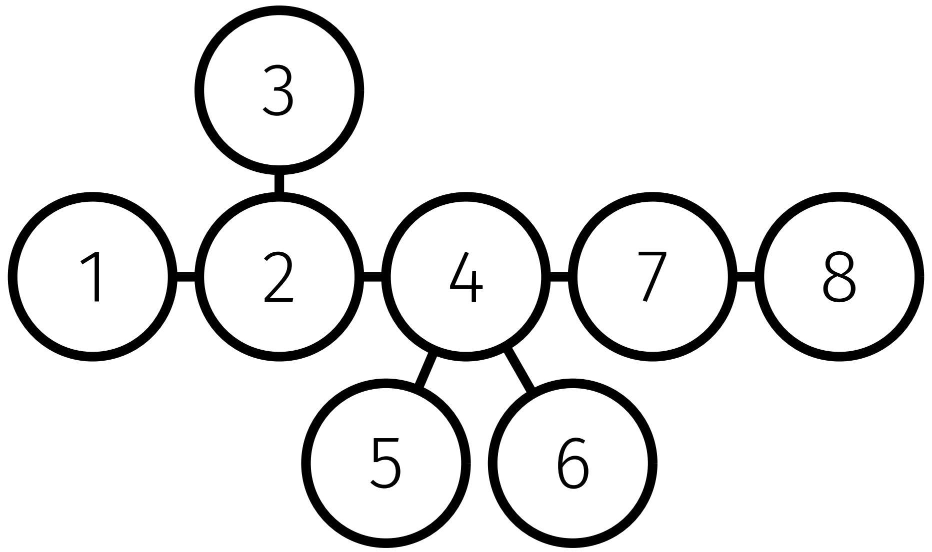

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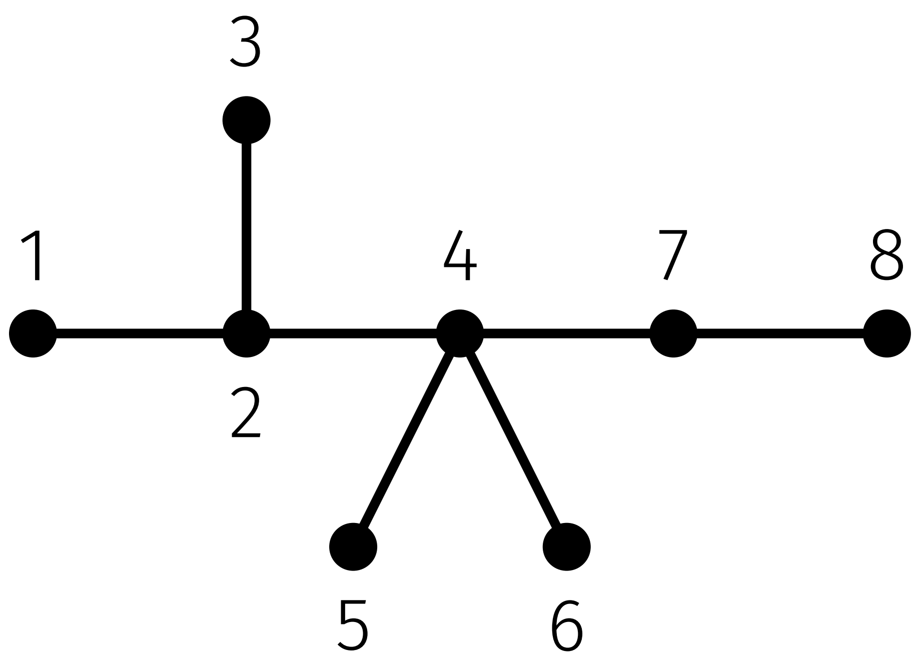

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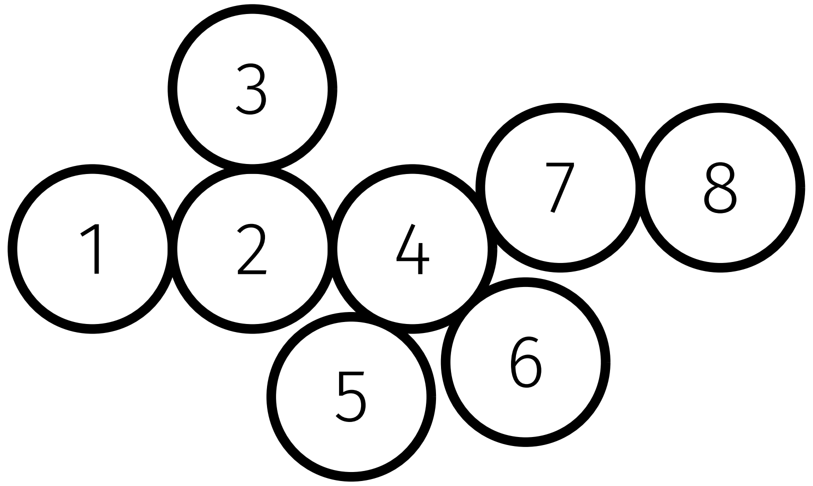

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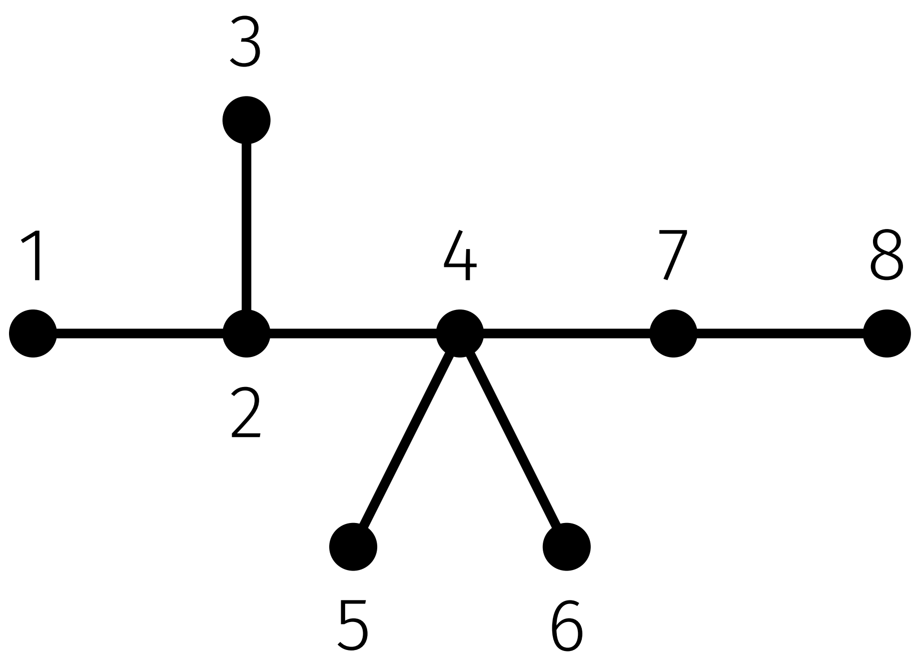

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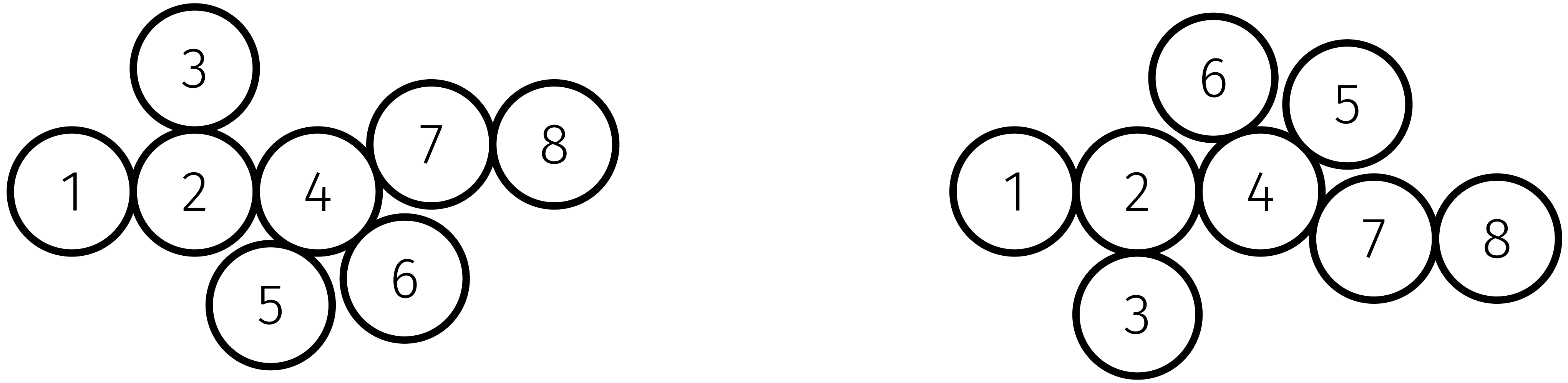

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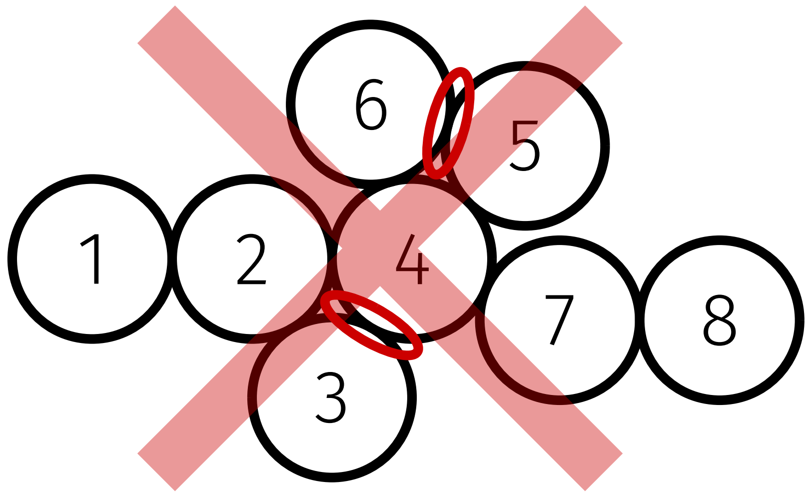

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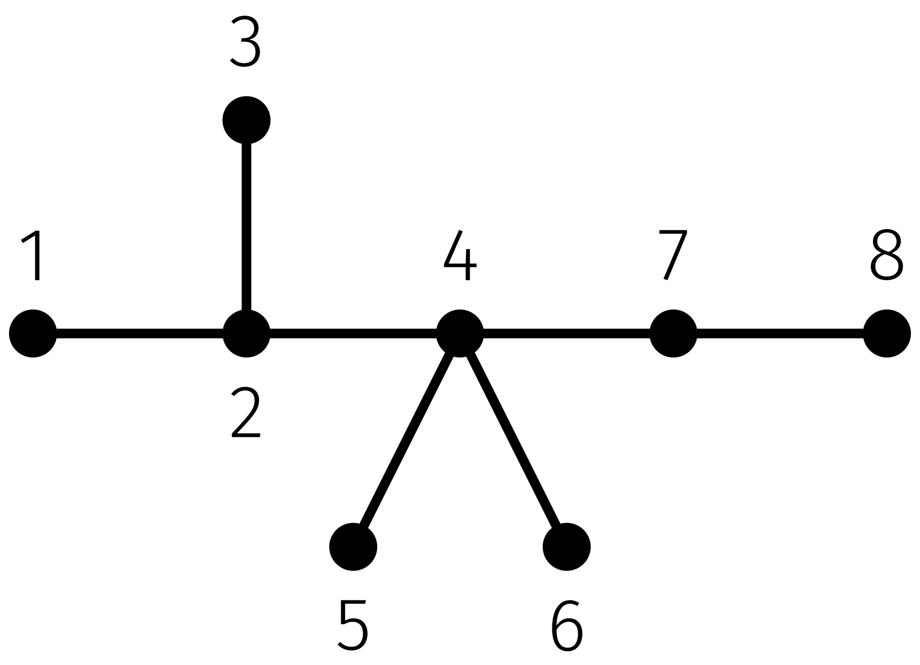

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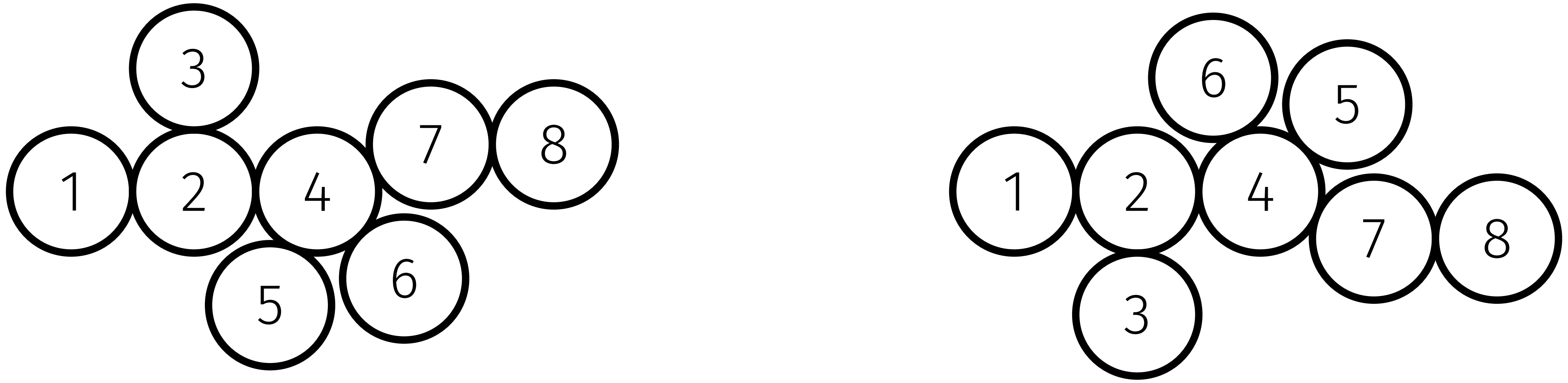

UDCRs of G

Unit Disk Contact Representations

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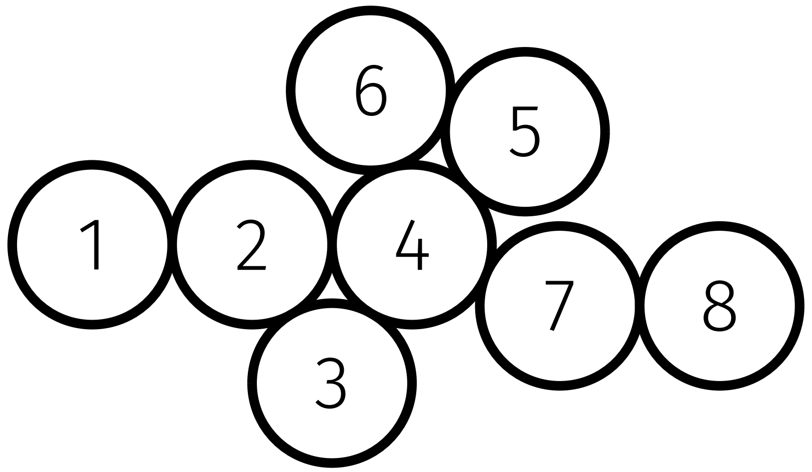

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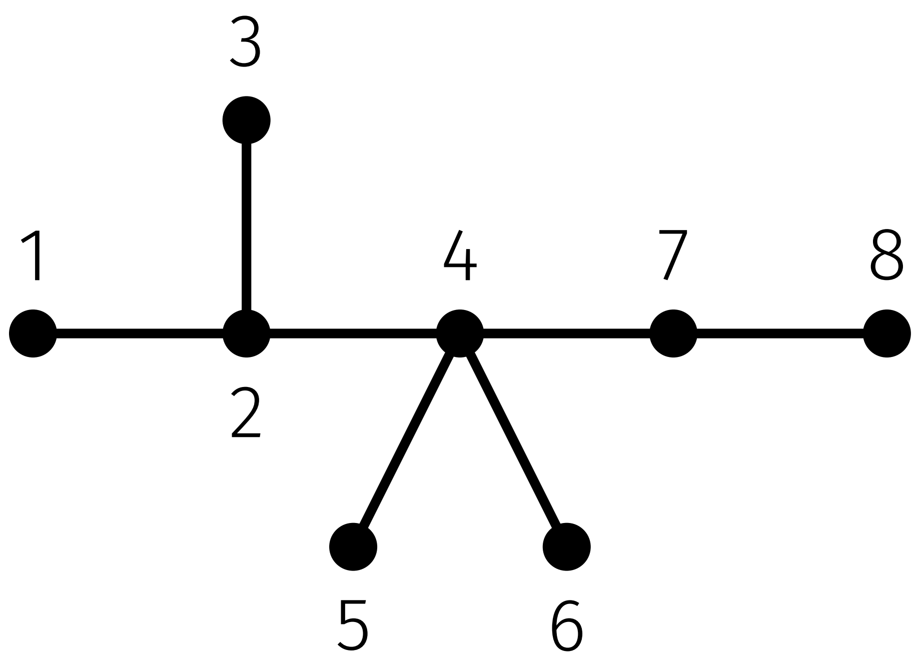

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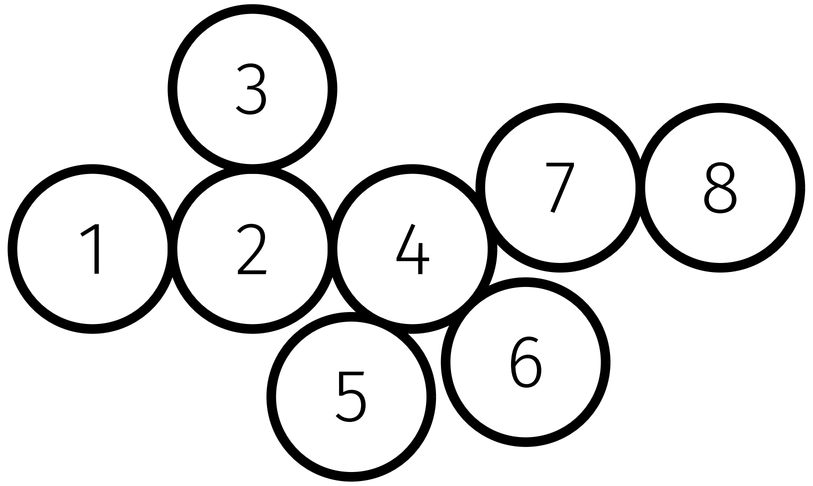

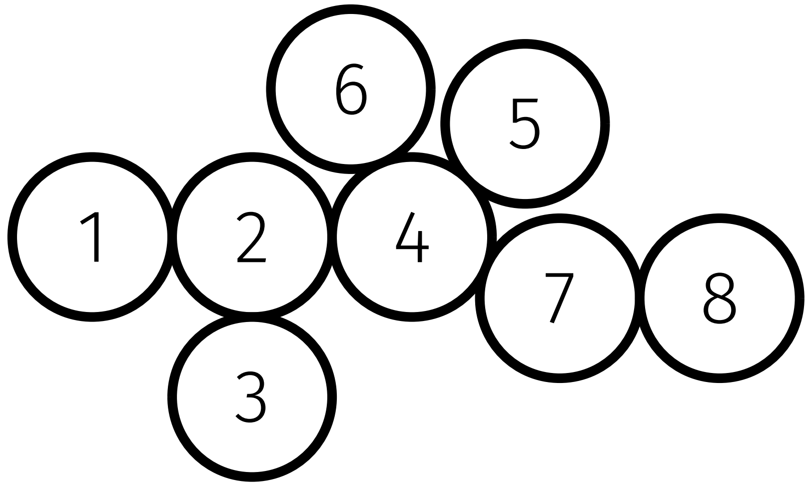

WUDCRs of G

Results

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Existing Work: NP-hardness and construction algorithms.

	star	caterpillar	tree
Weak UDCR		Cleve; 2020	Cleve; 2020
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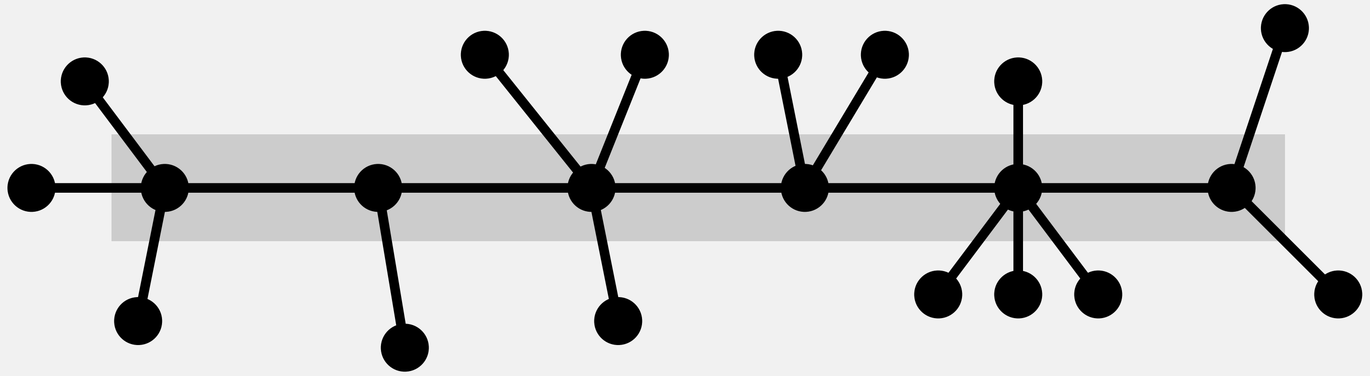
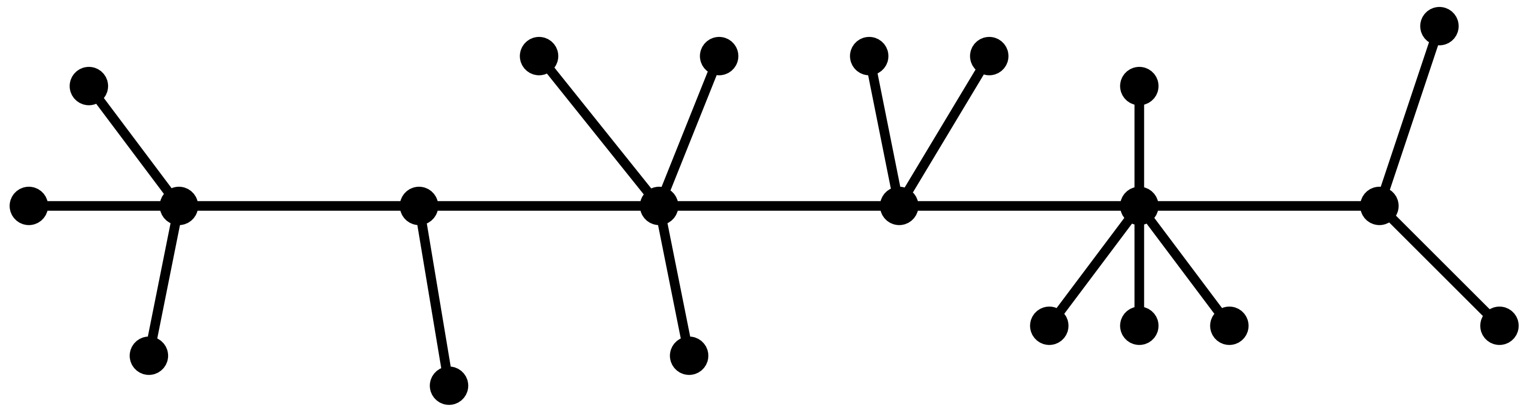
Reminder:
- A **caterpillar** is a tree
- **backbone** path plus leaves

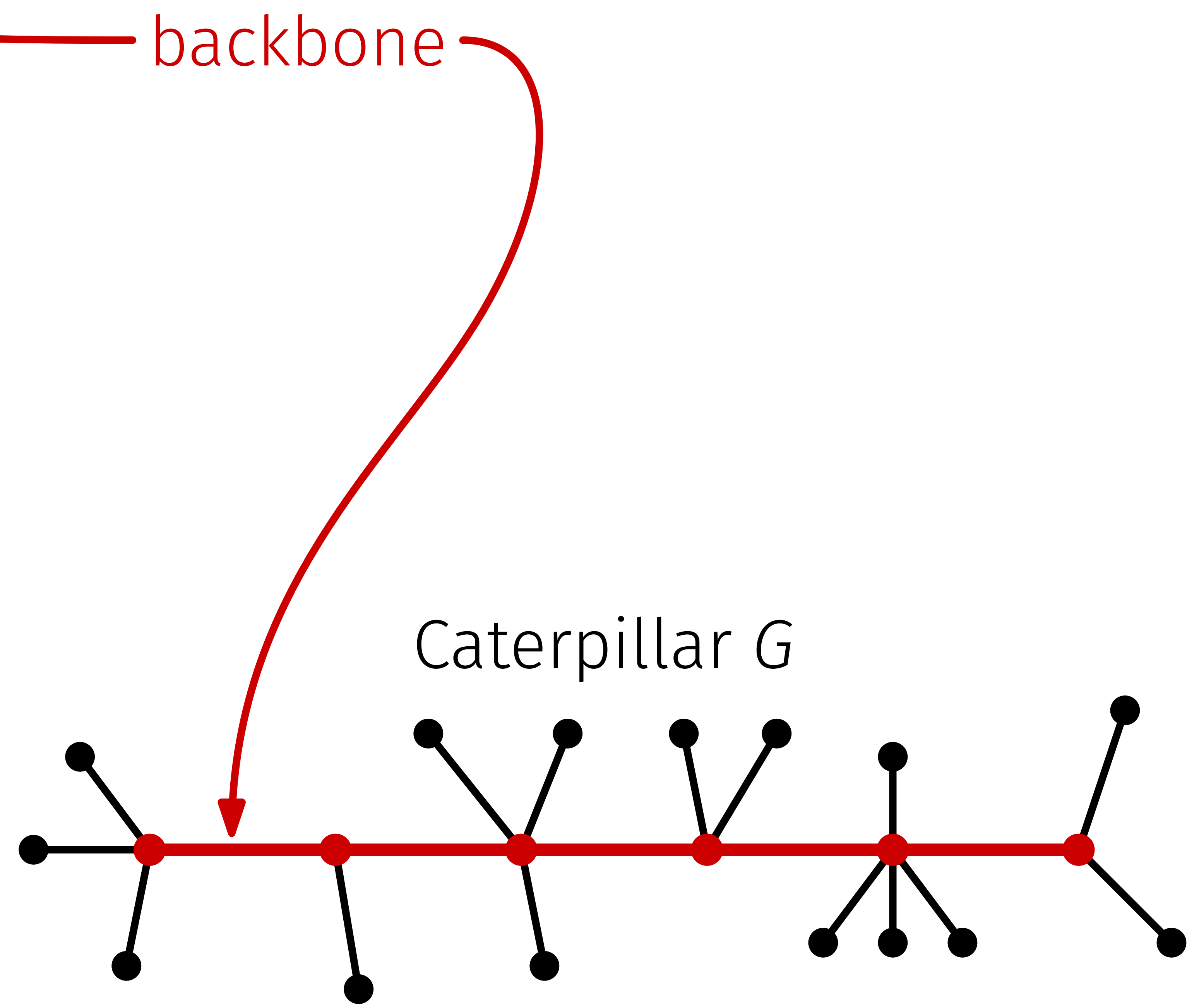


Algorithm:

Caterpillar G



Algorithm: Find longest path $v_0, v_1, \ldots, v_k, v_{k+1}$



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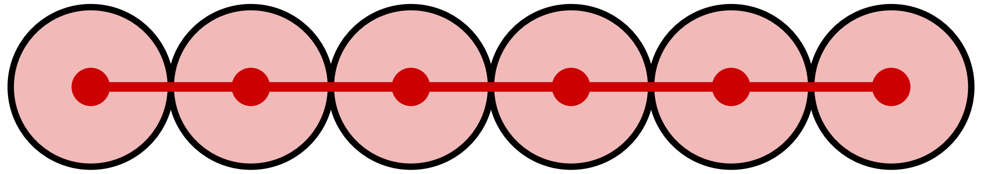
Place backbone disks horizontally



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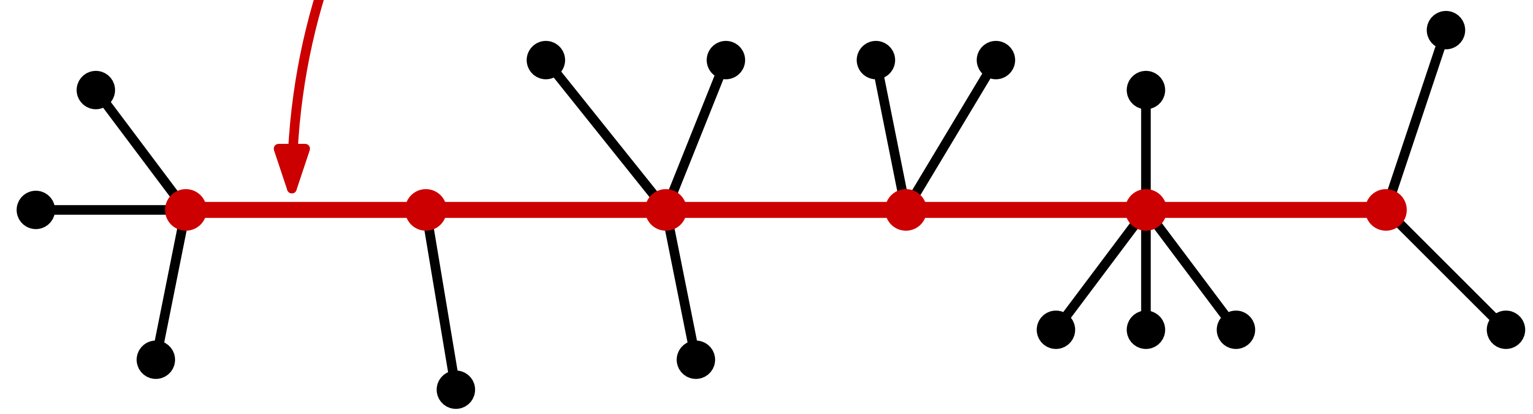

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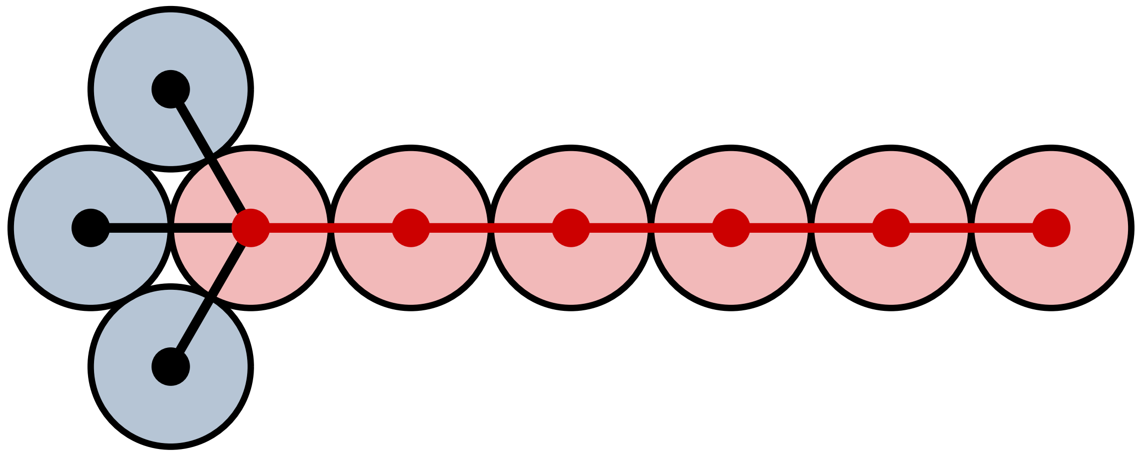
Place backbone disks horizontally

Place leaf disks at leftmost position



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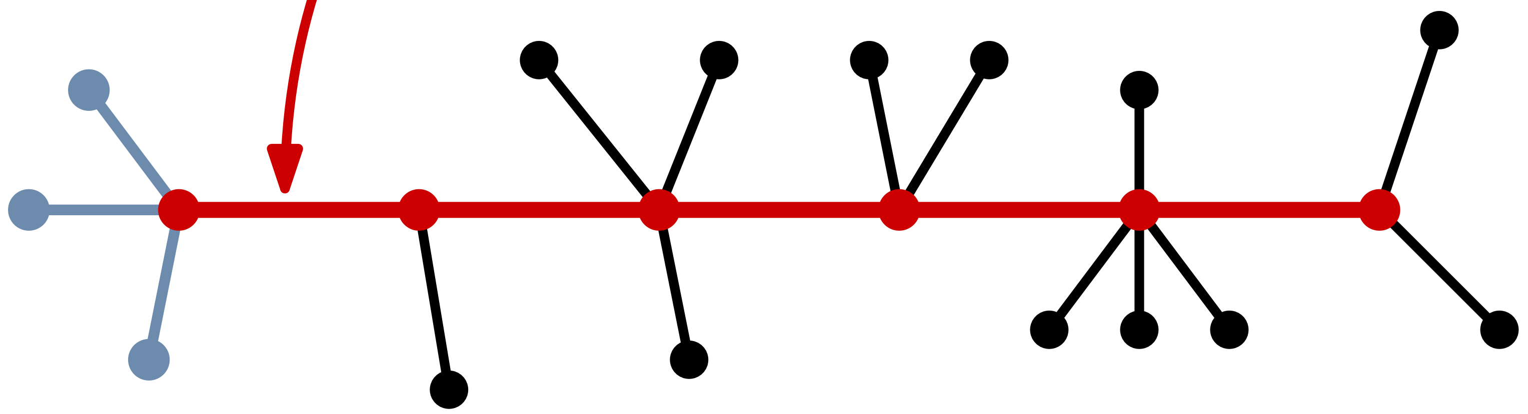

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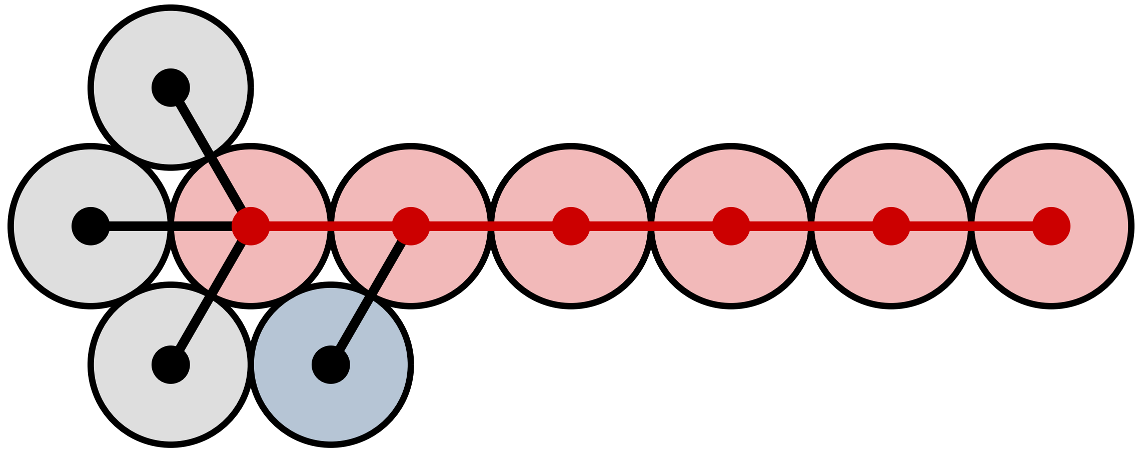

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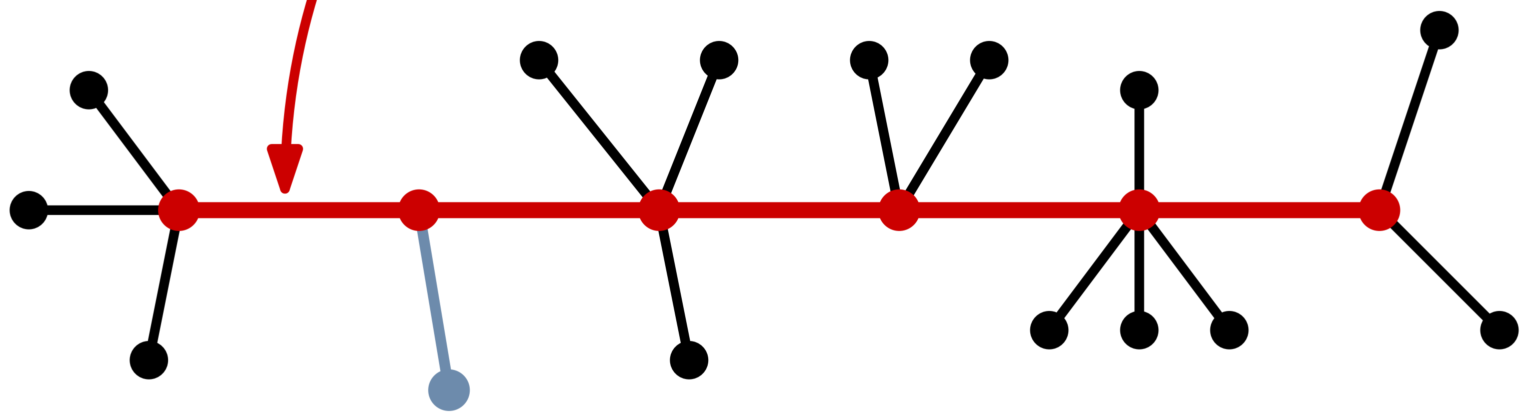

Recognizing Caterpillars in Linear Time

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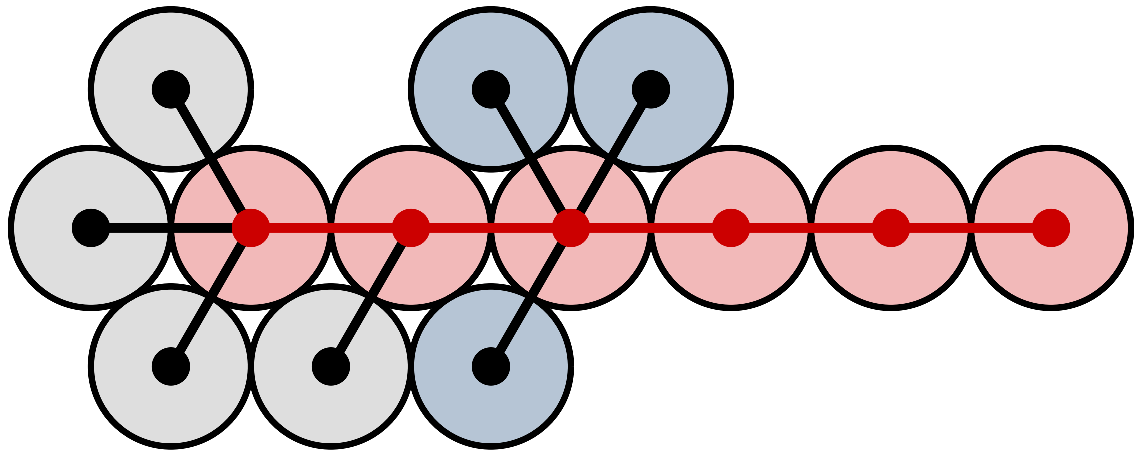

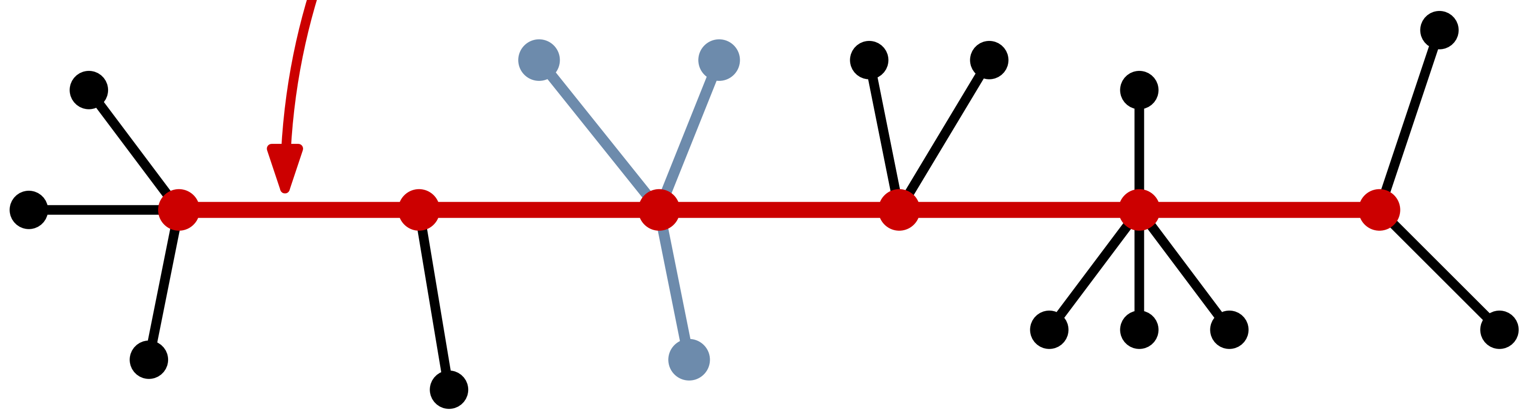

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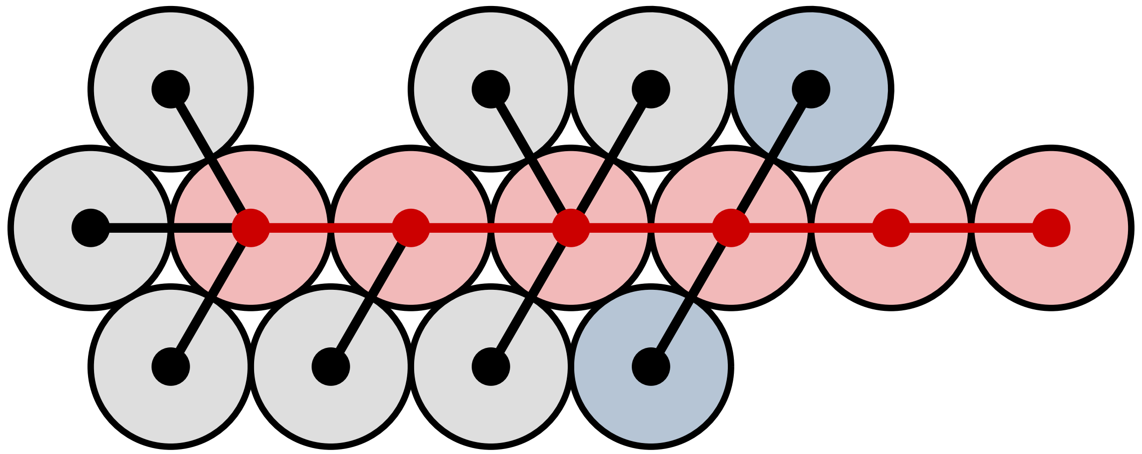

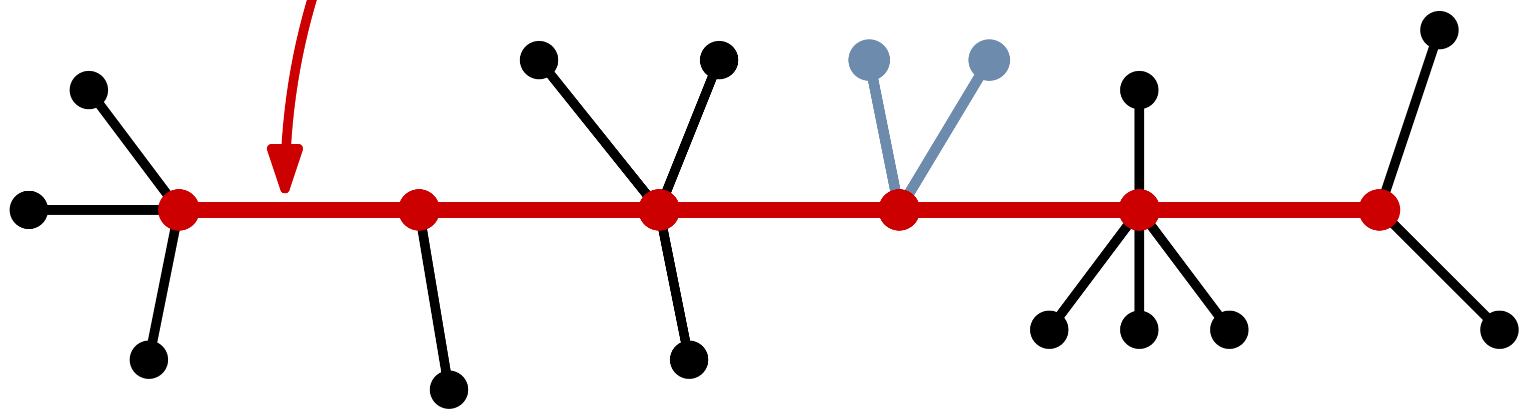

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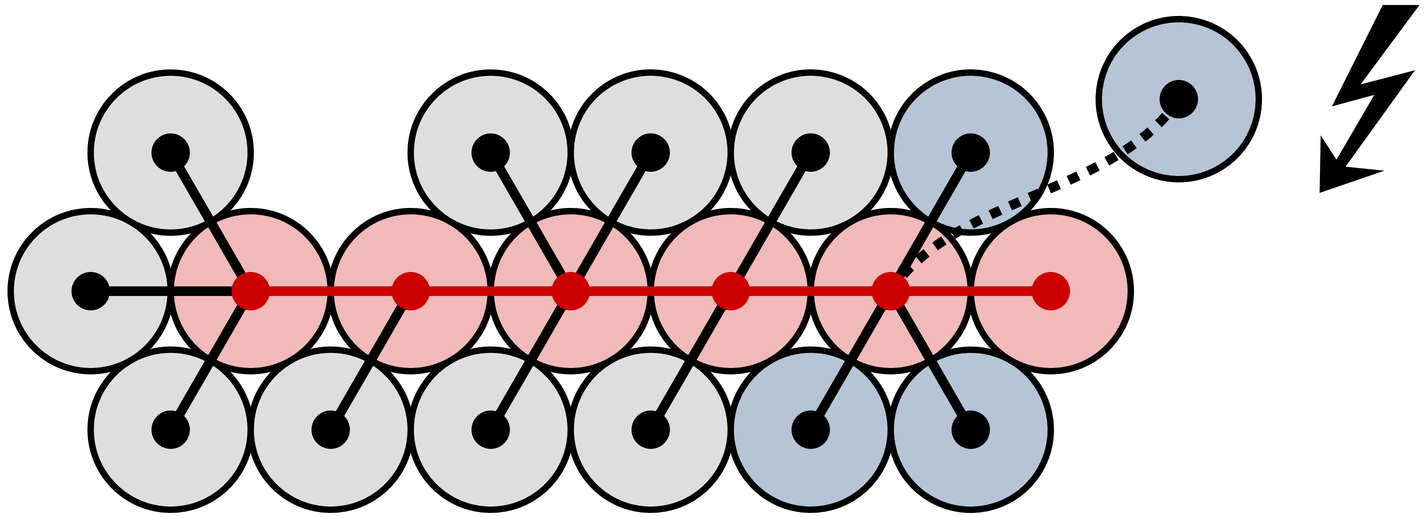

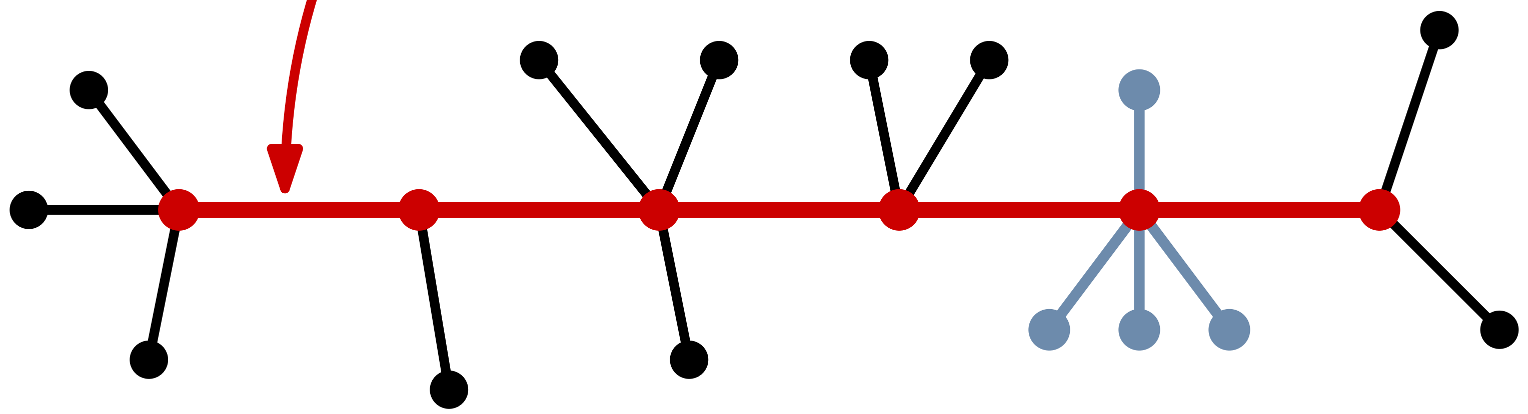

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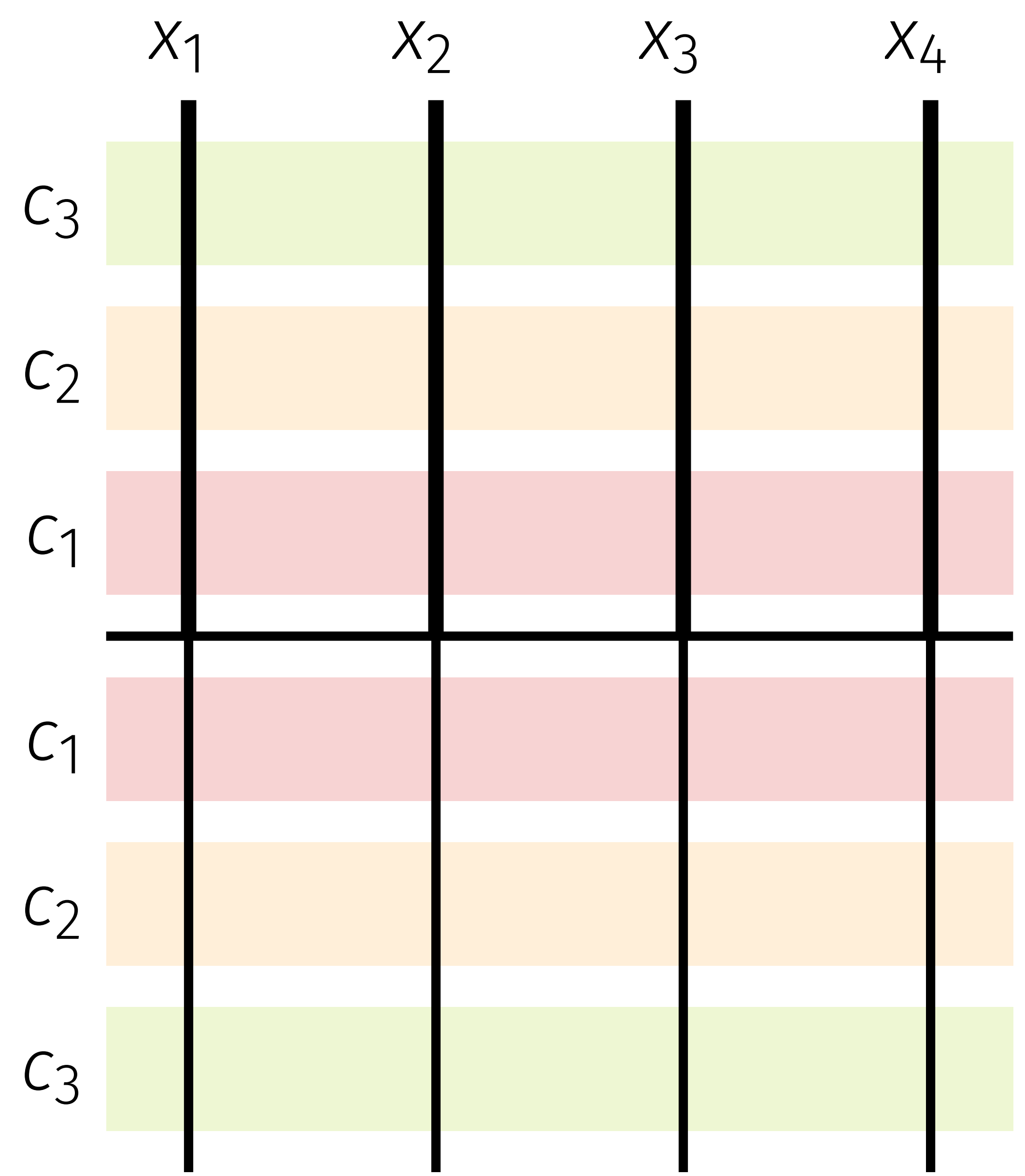

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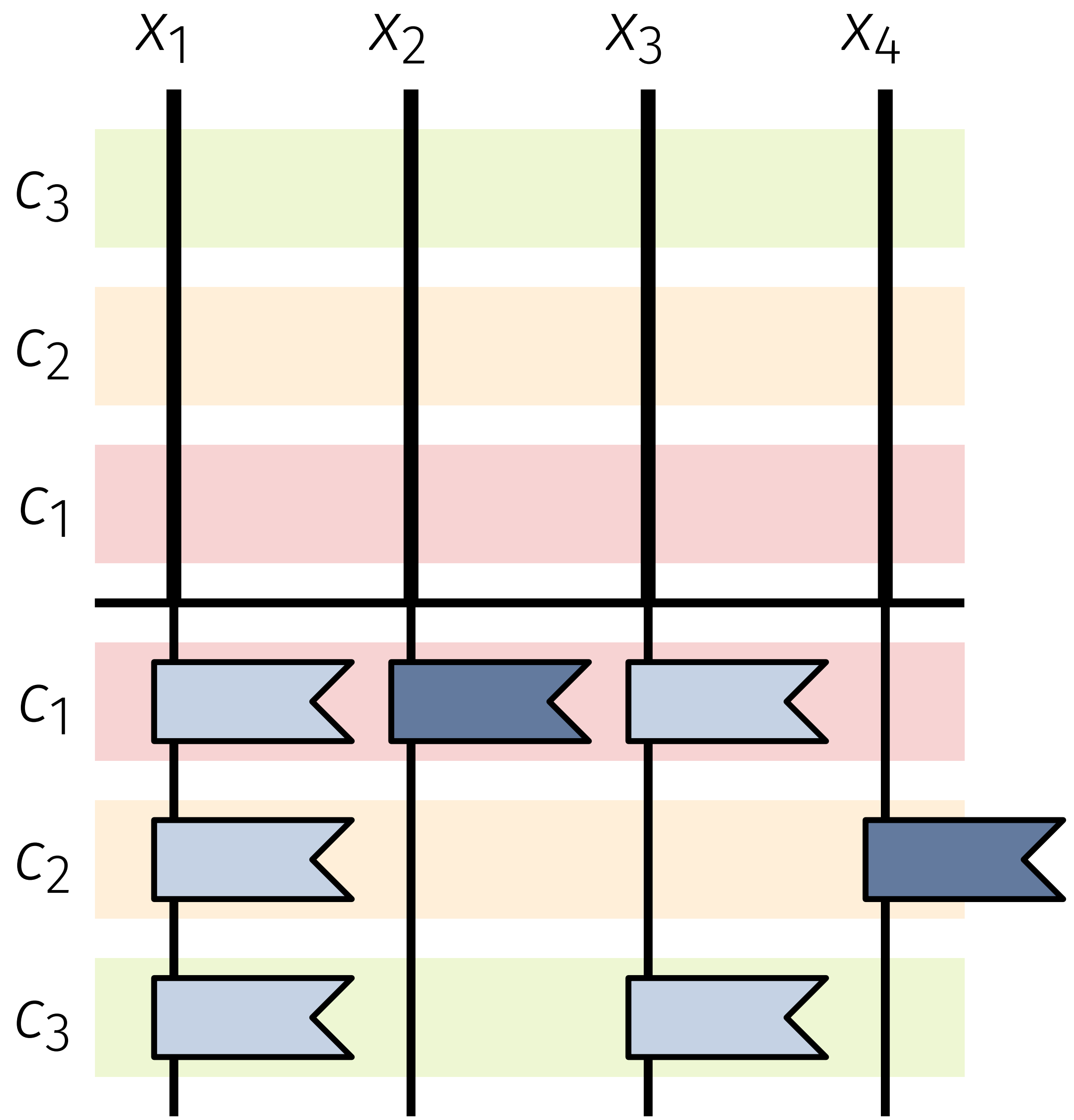

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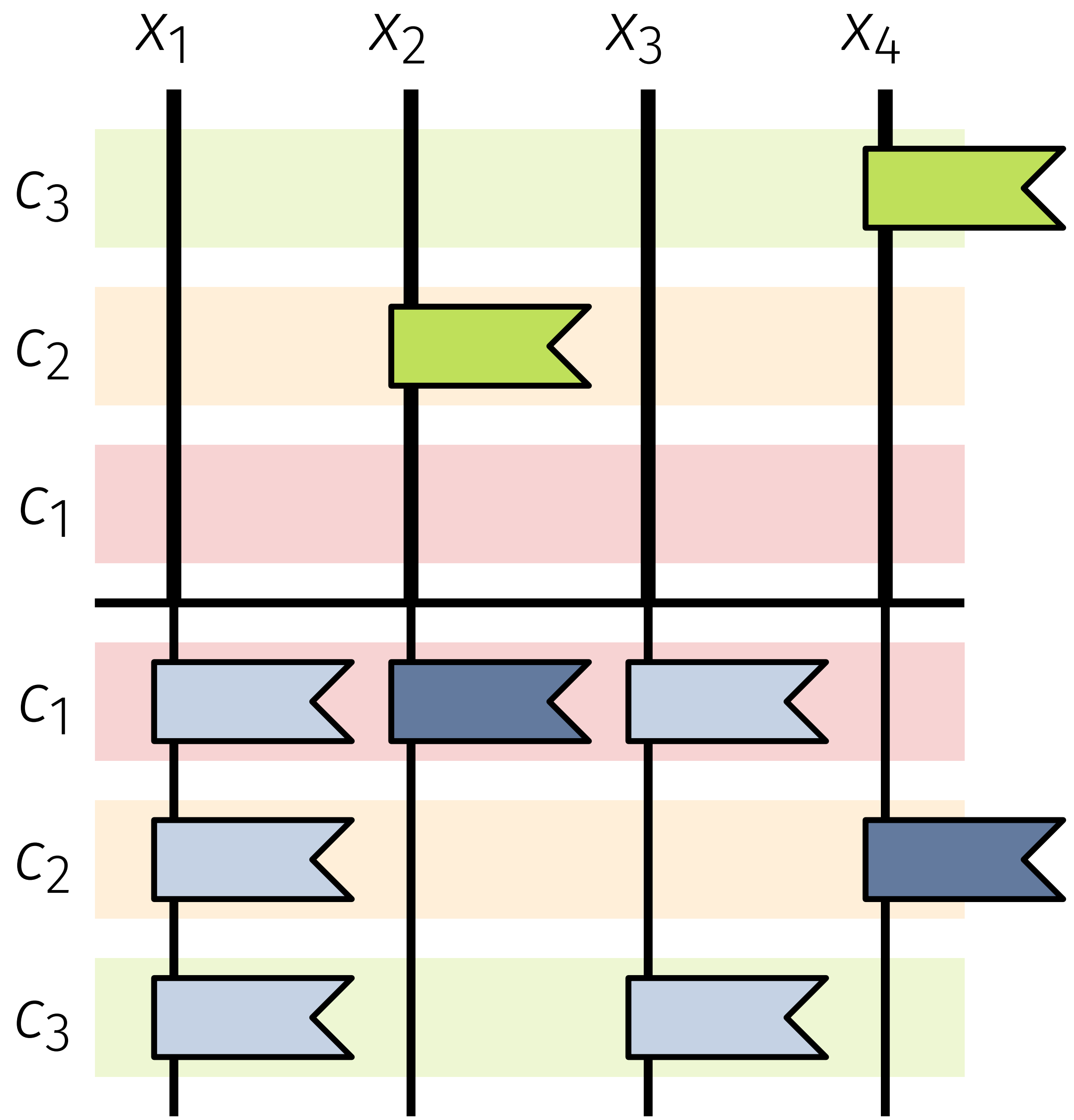

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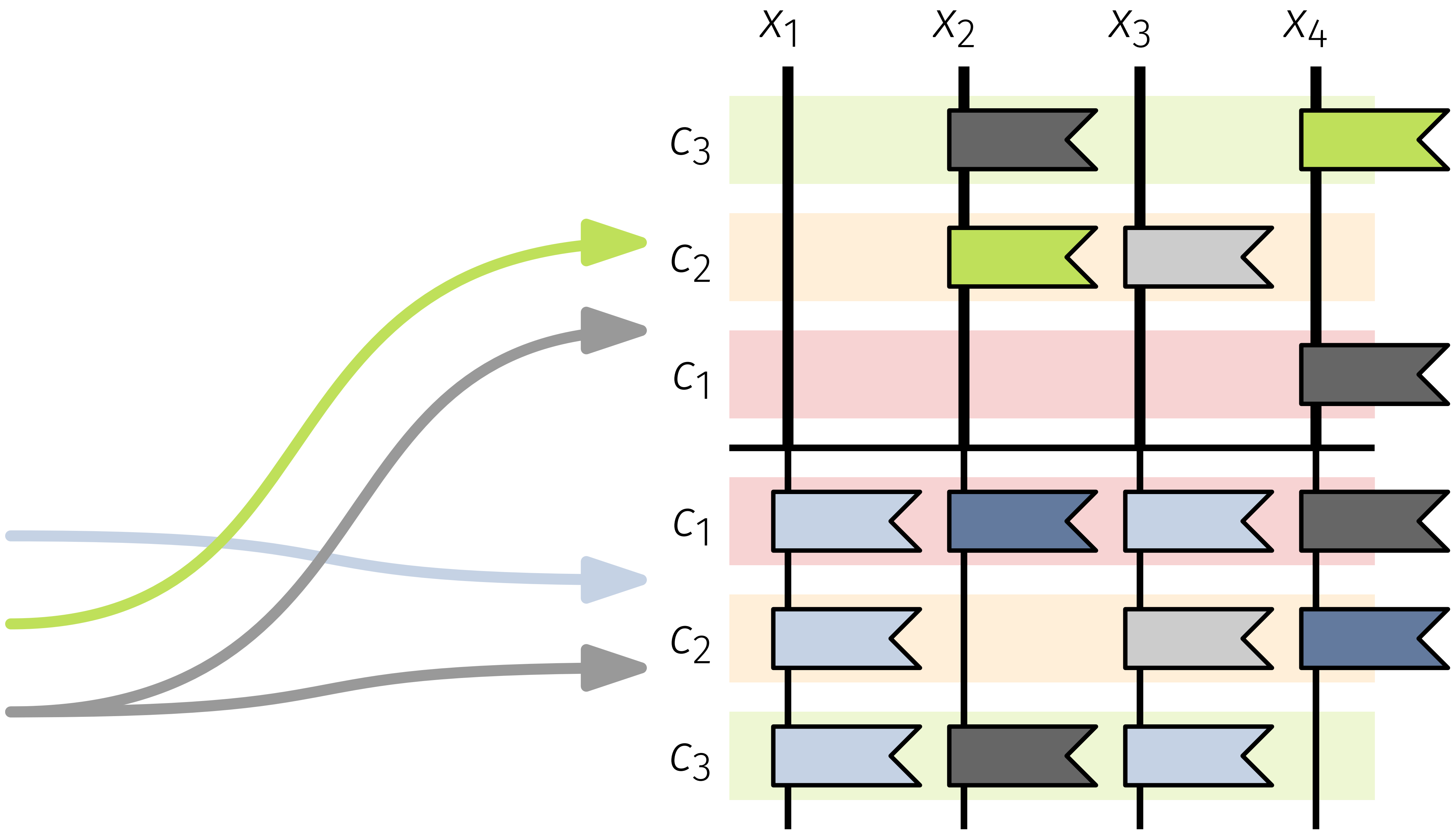

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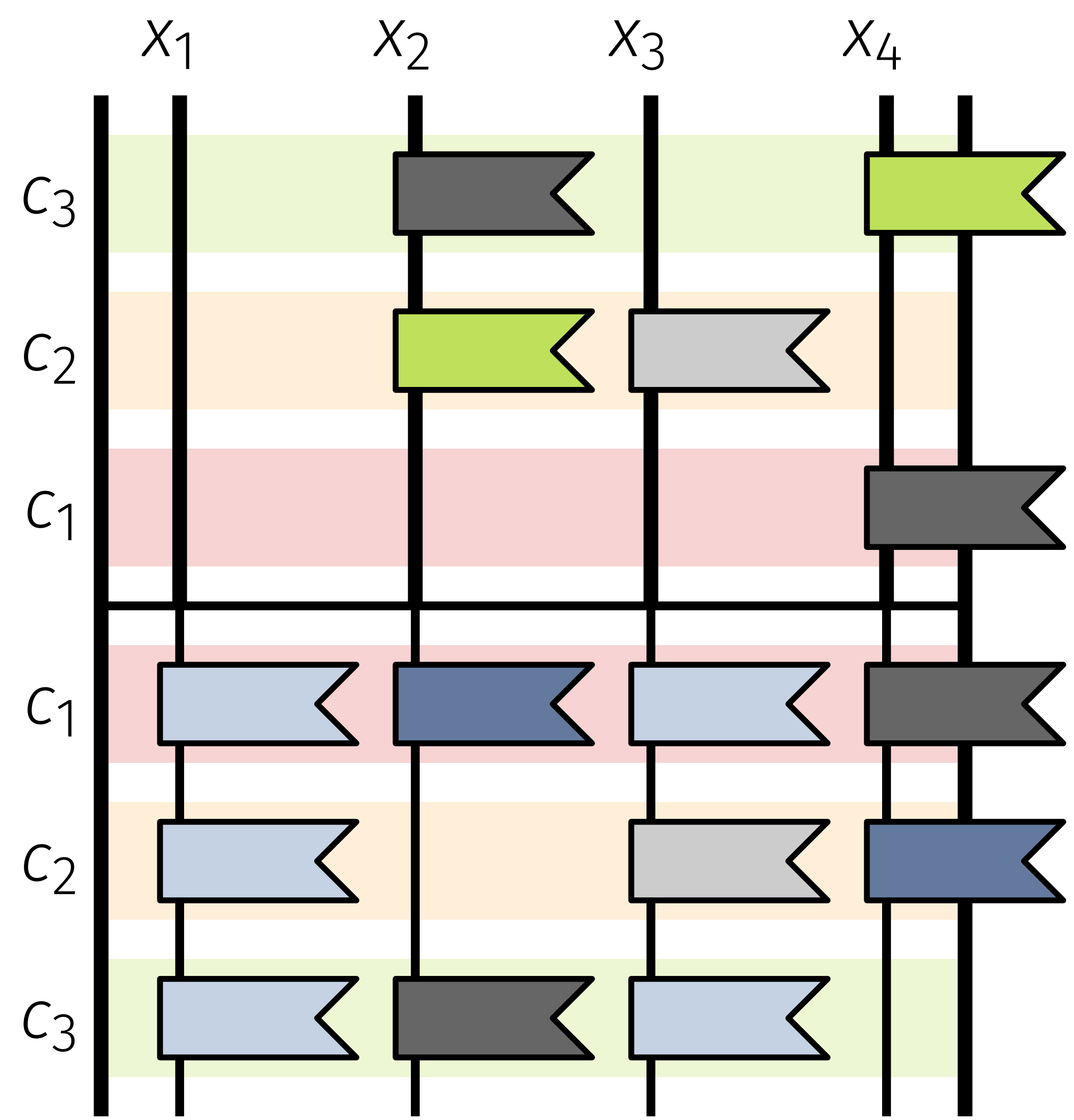

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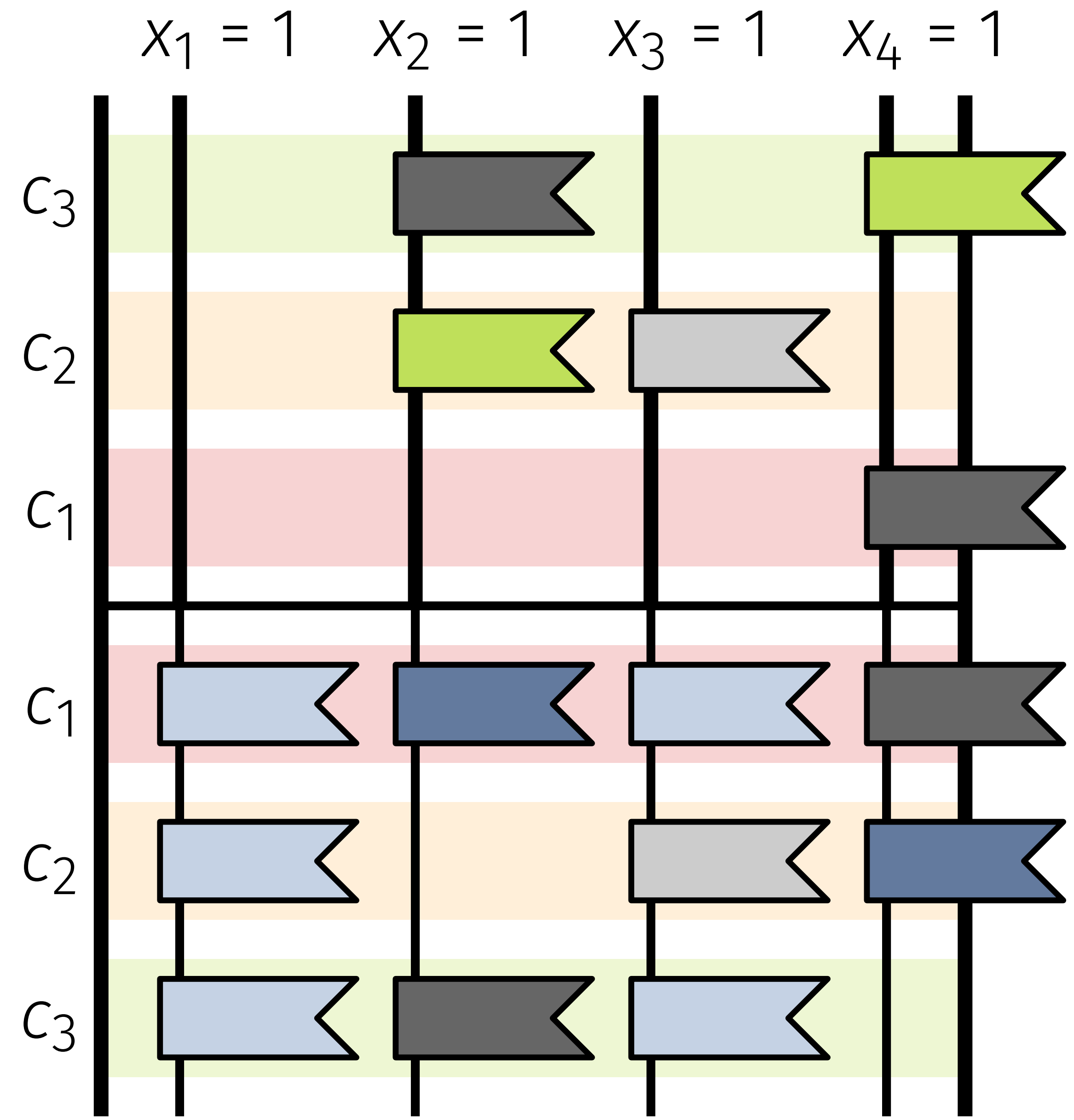

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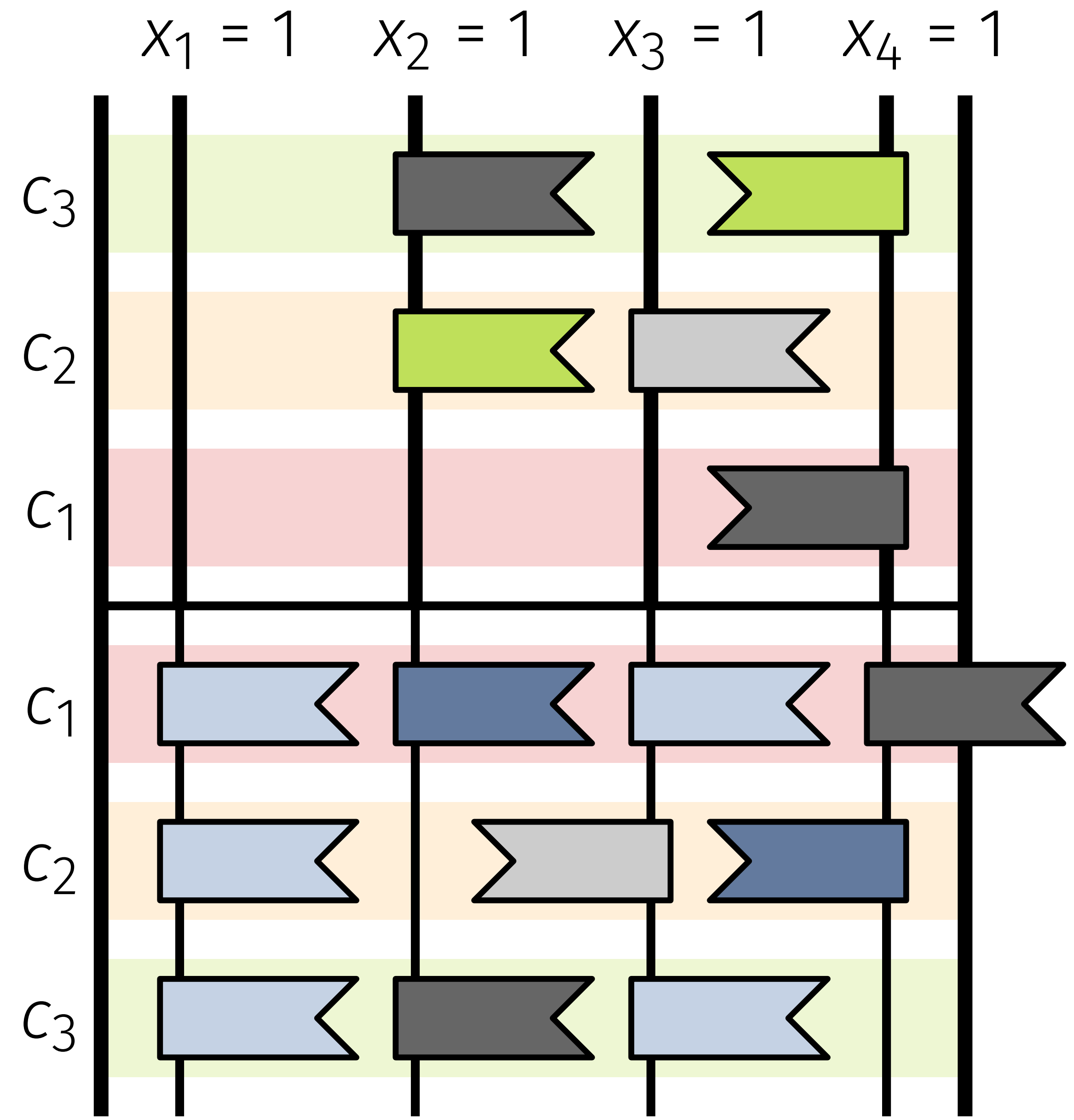

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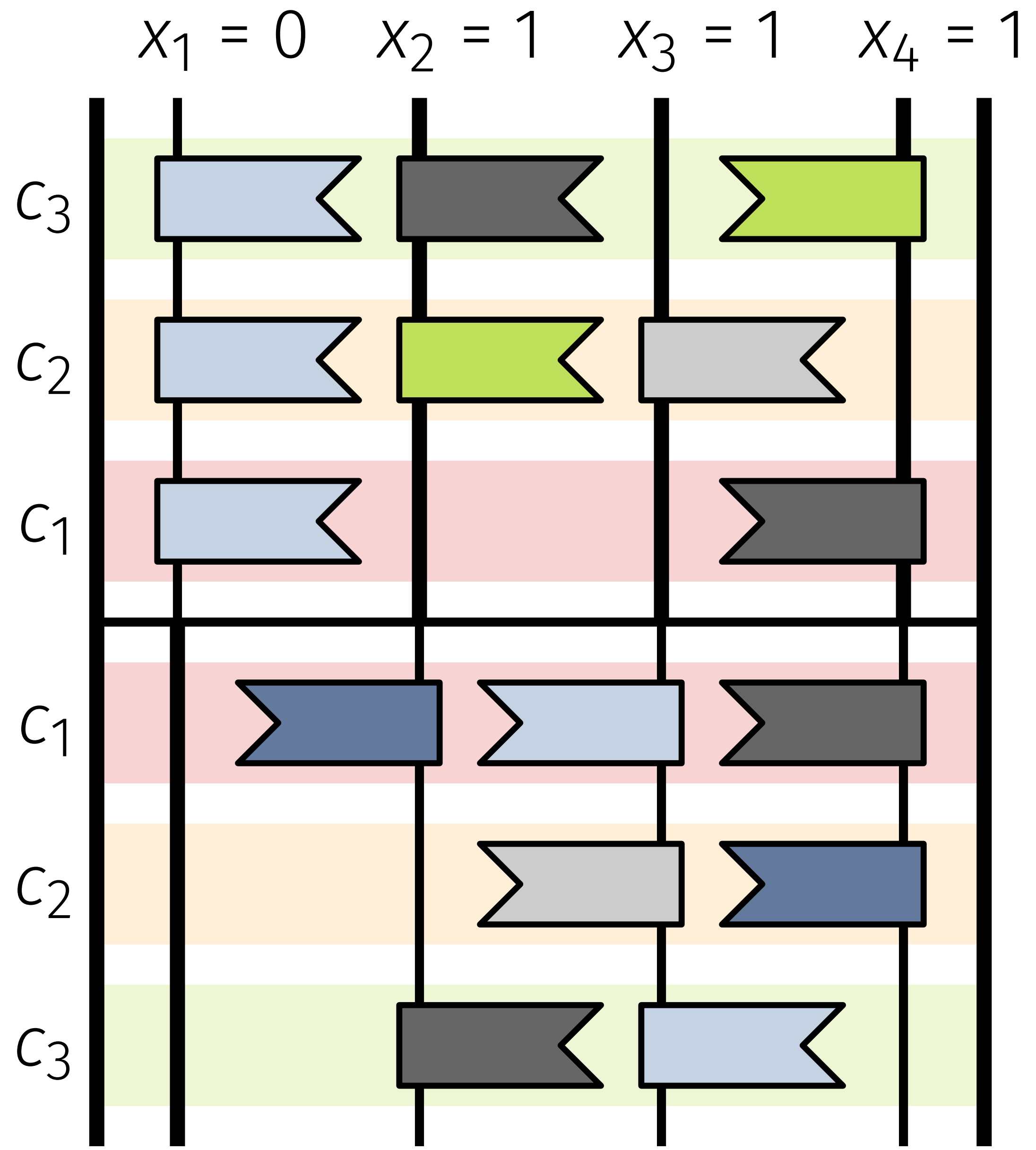

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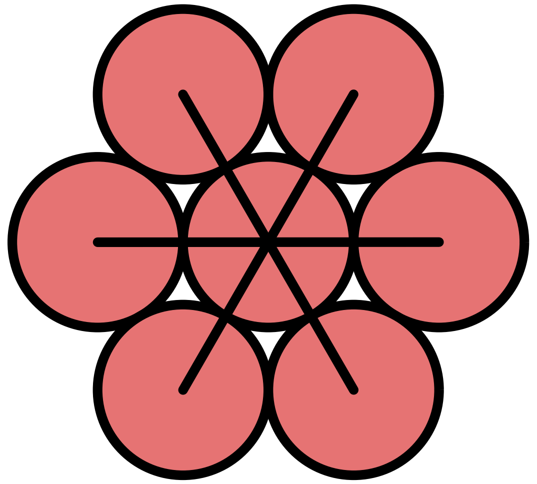
Modeling Line Segments



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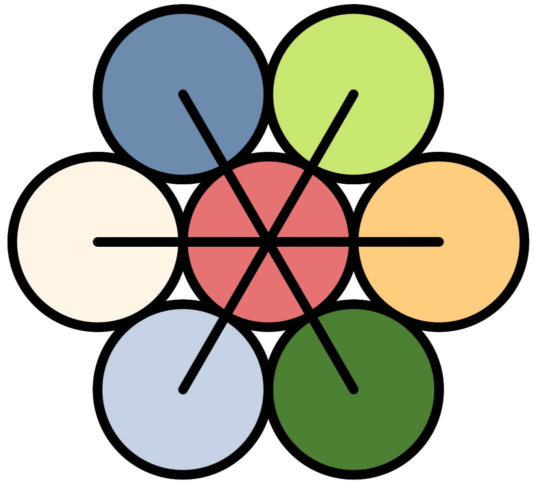

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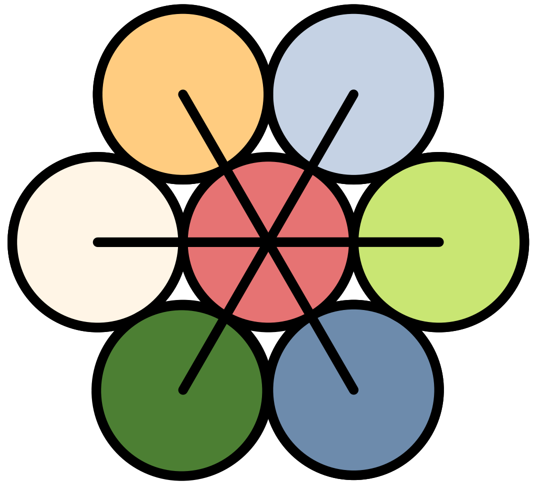

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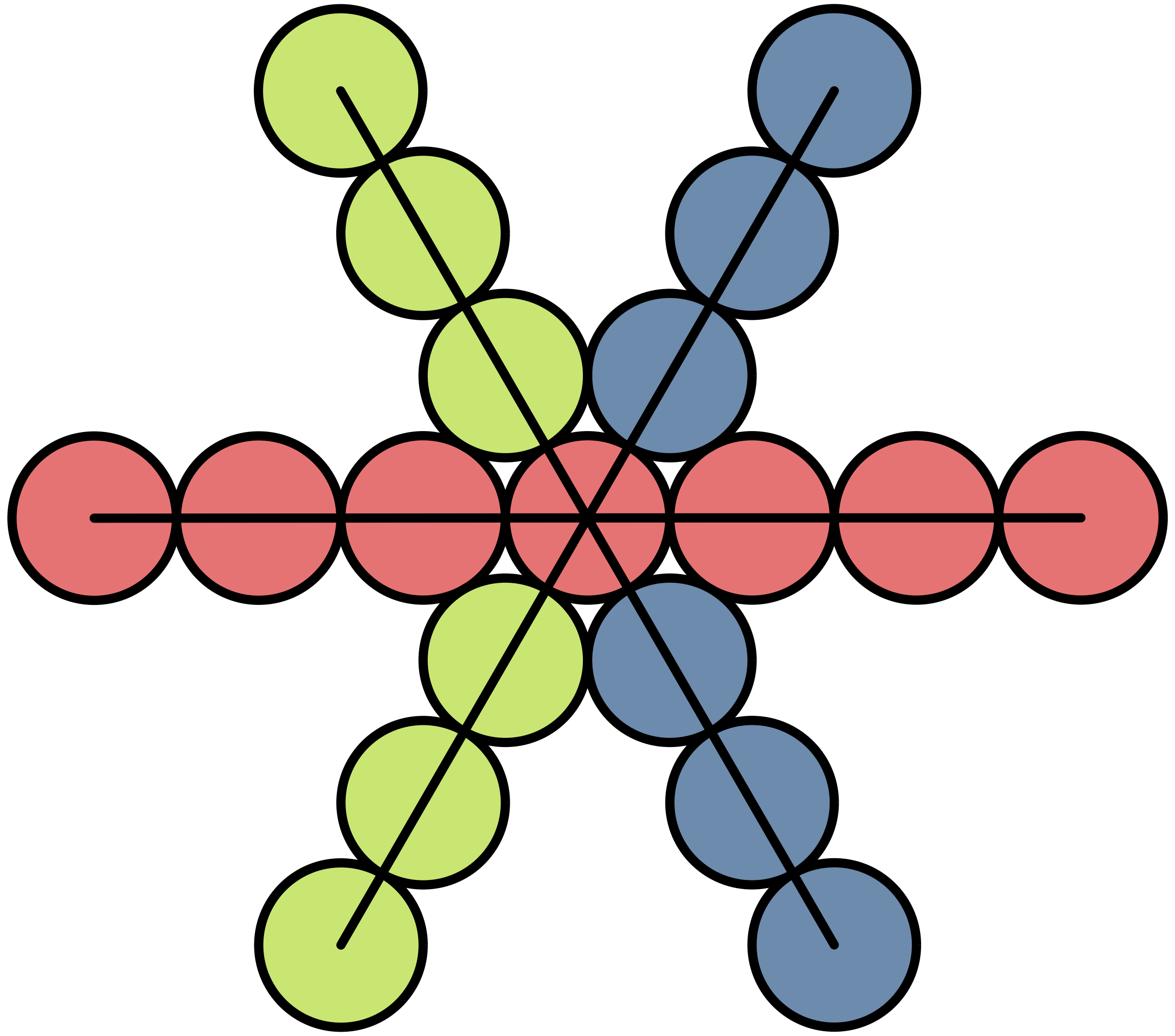

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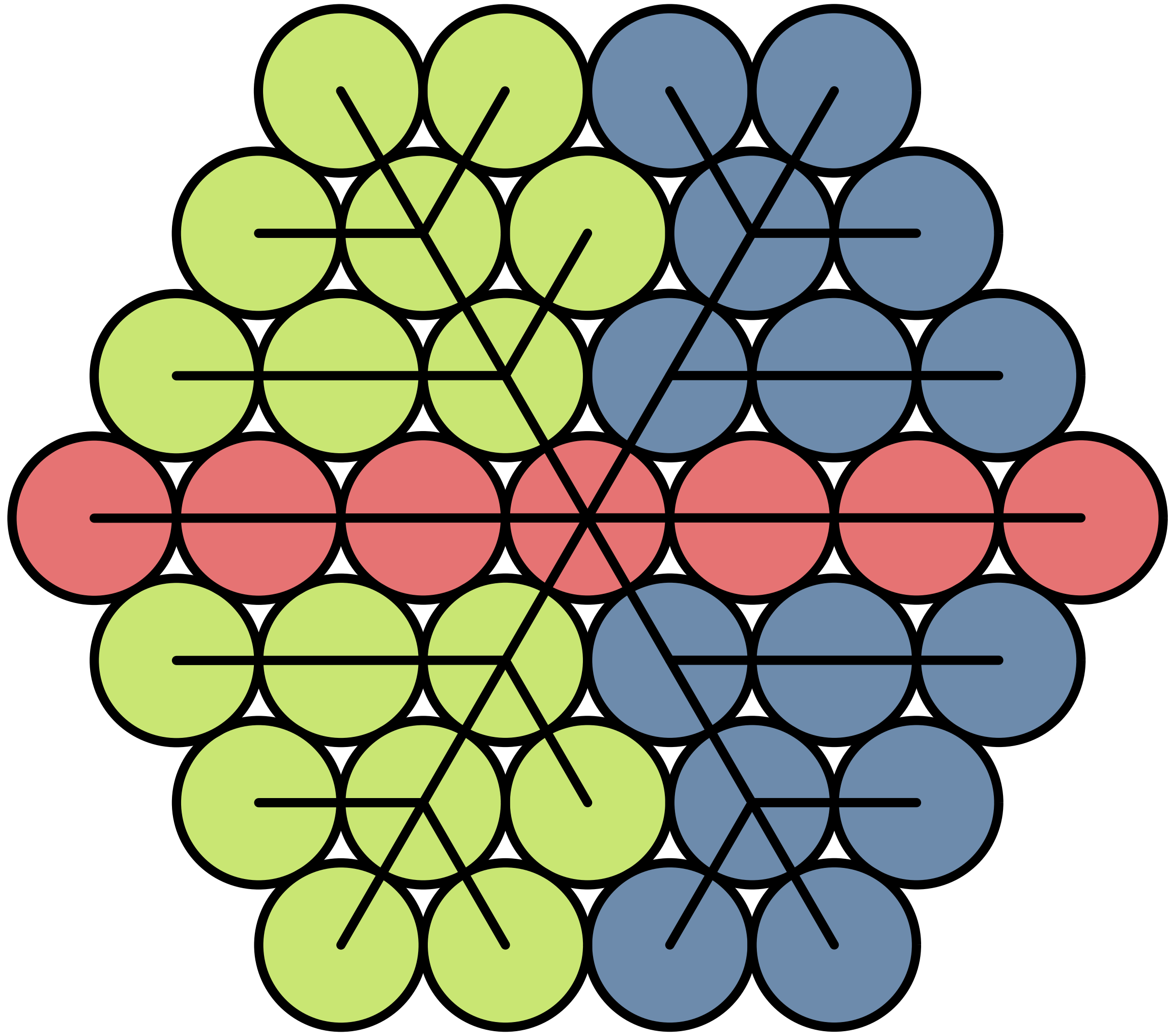

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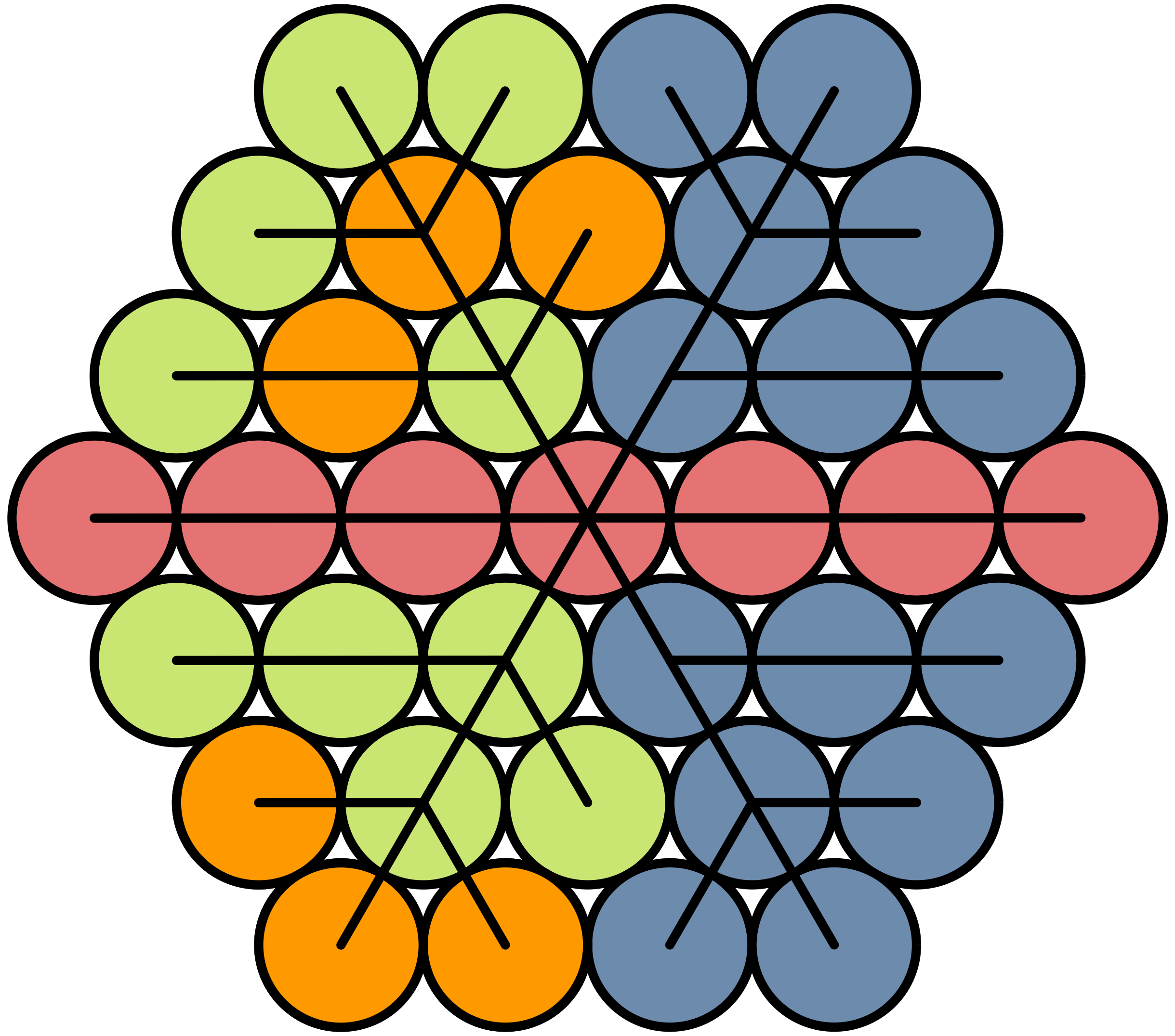

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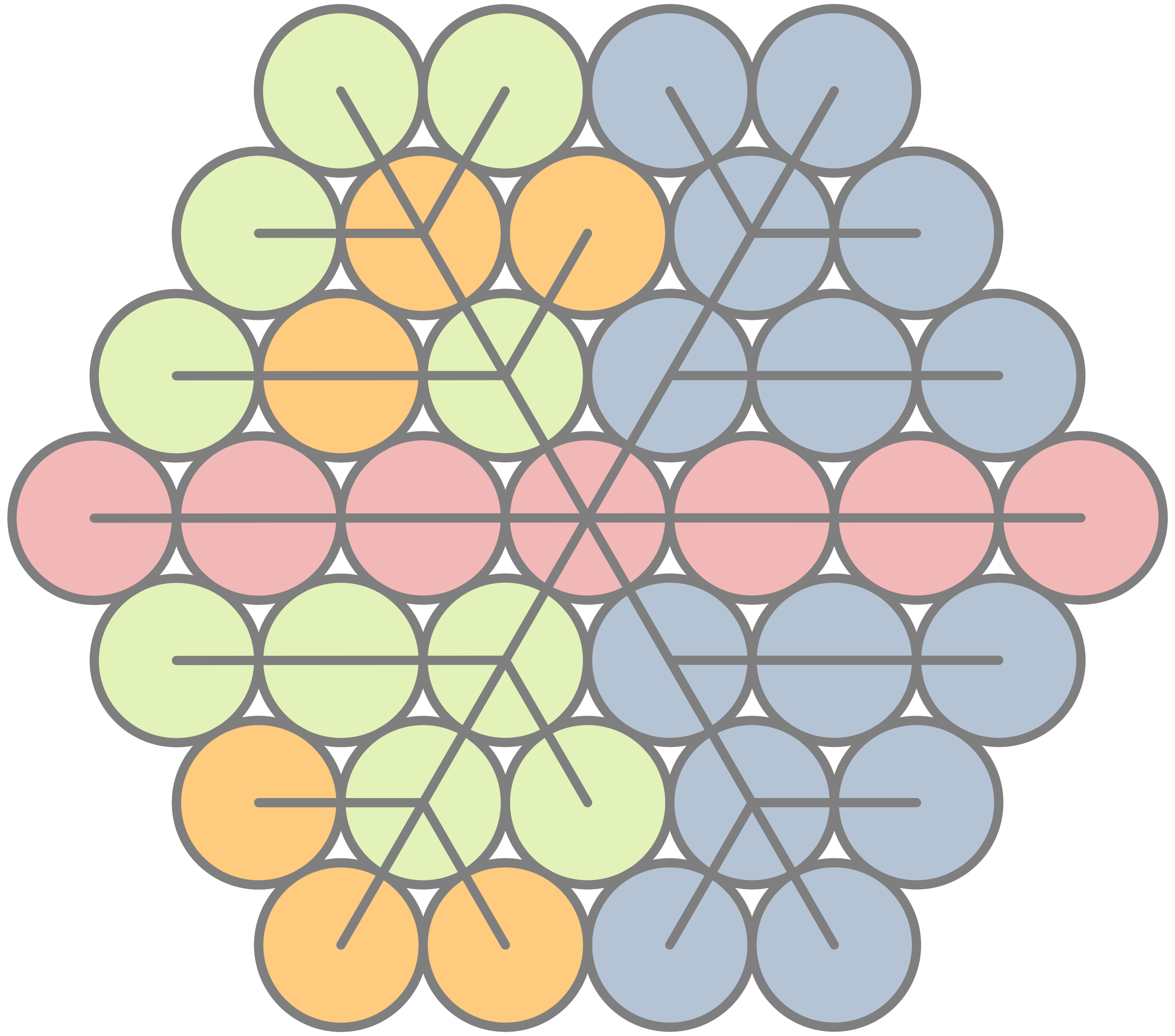
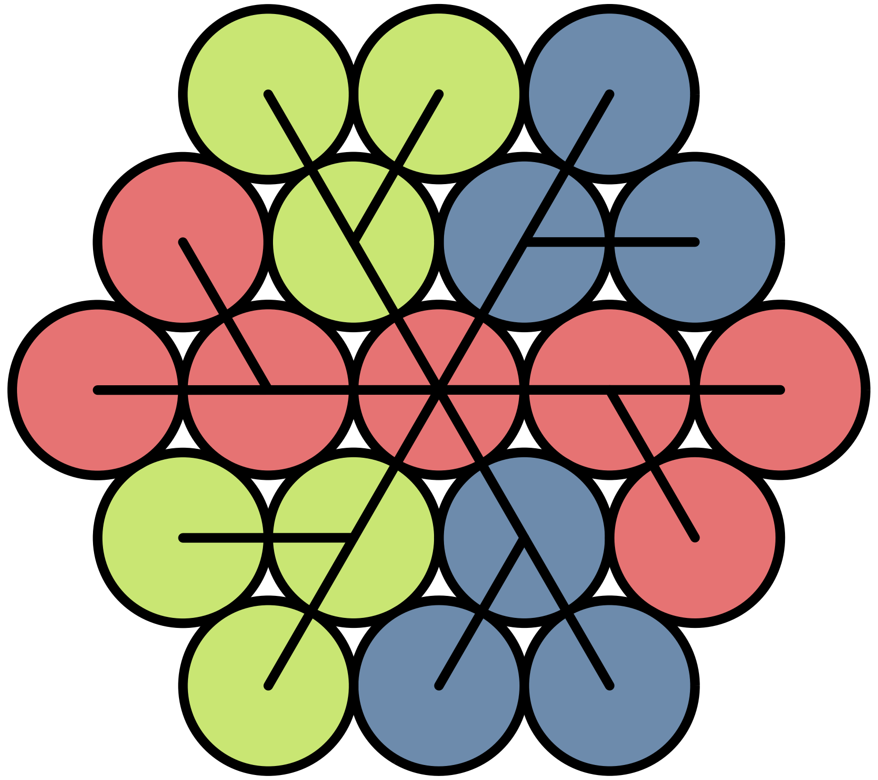
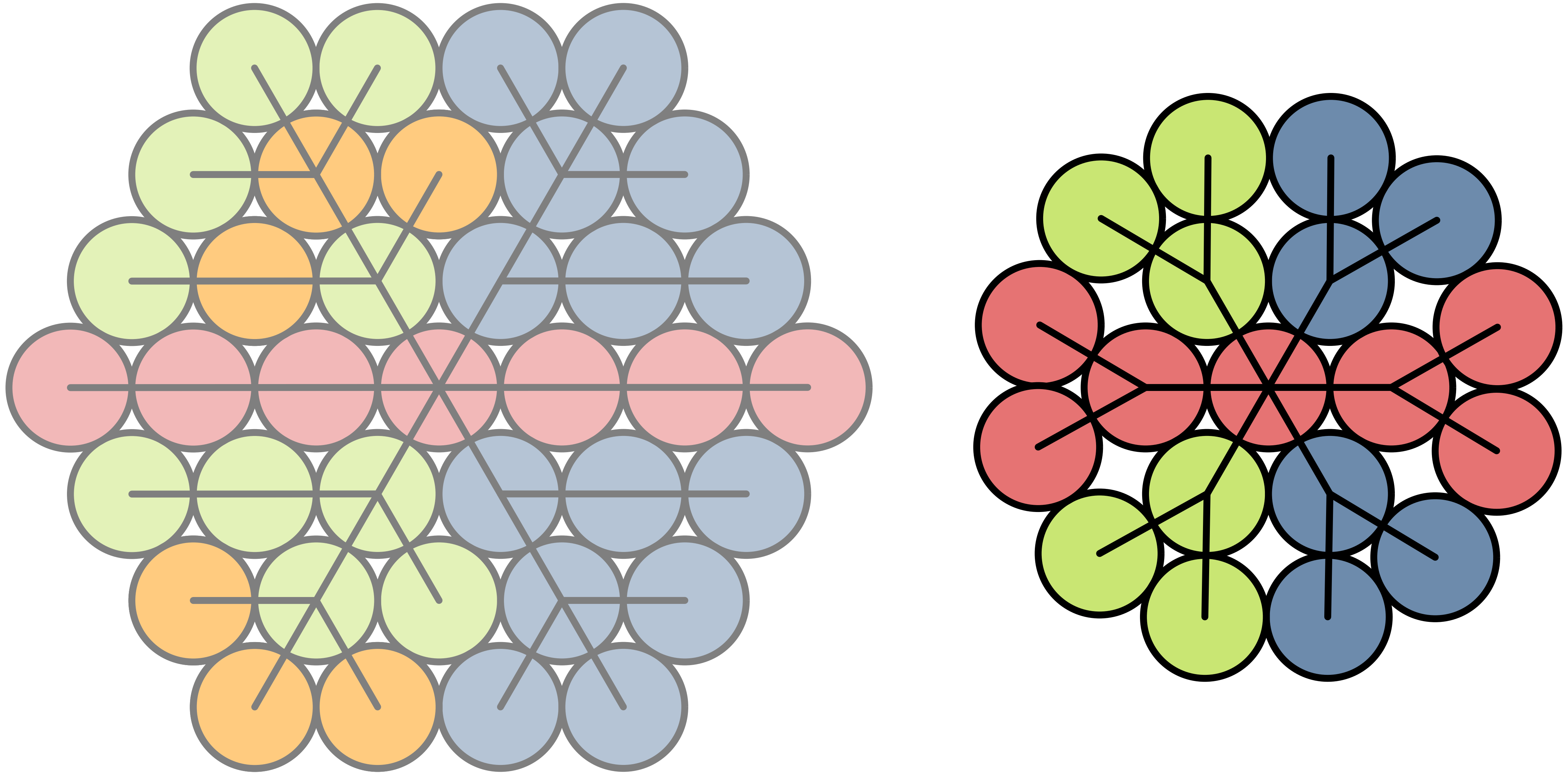




Modeling Line Segments



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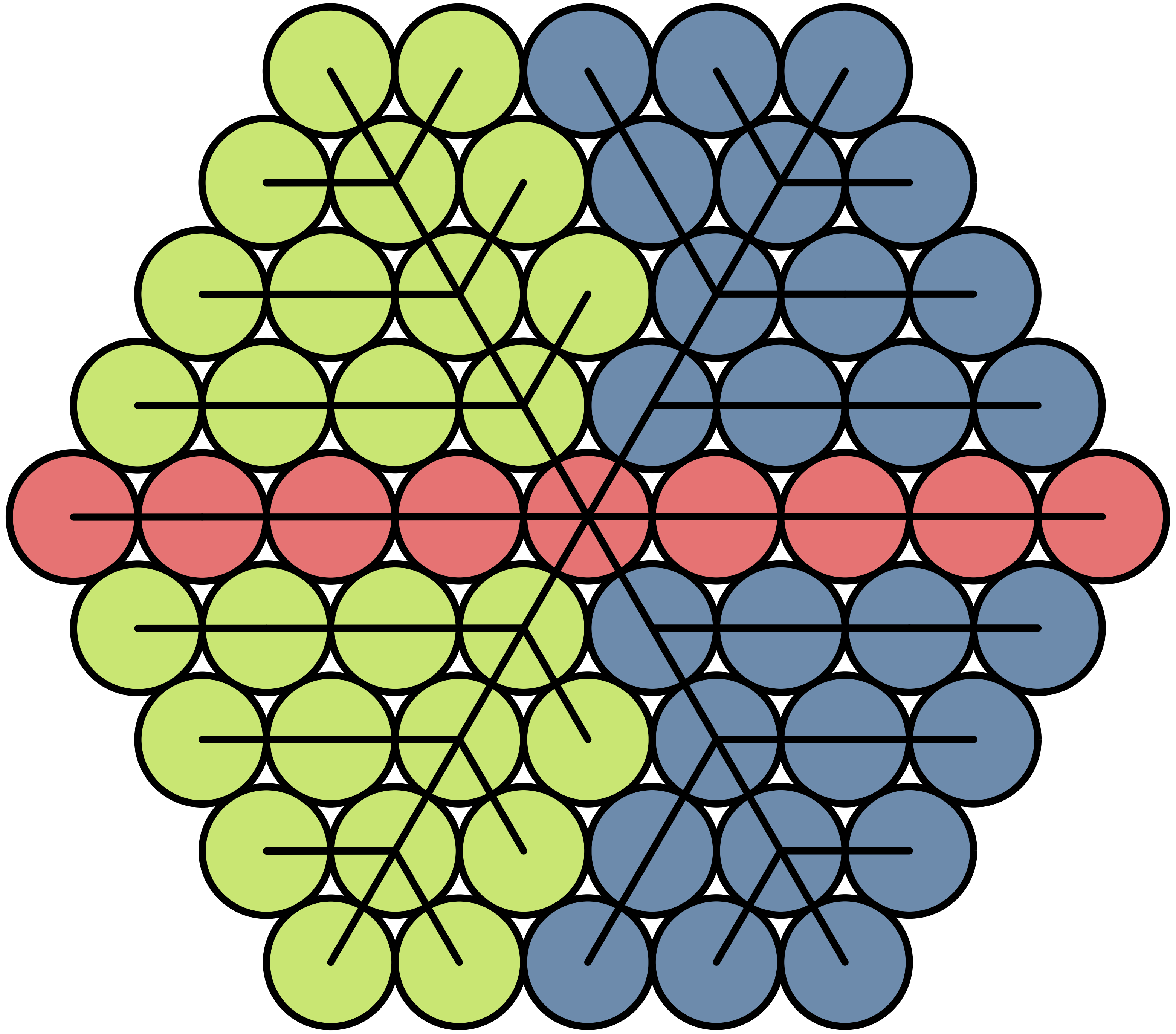

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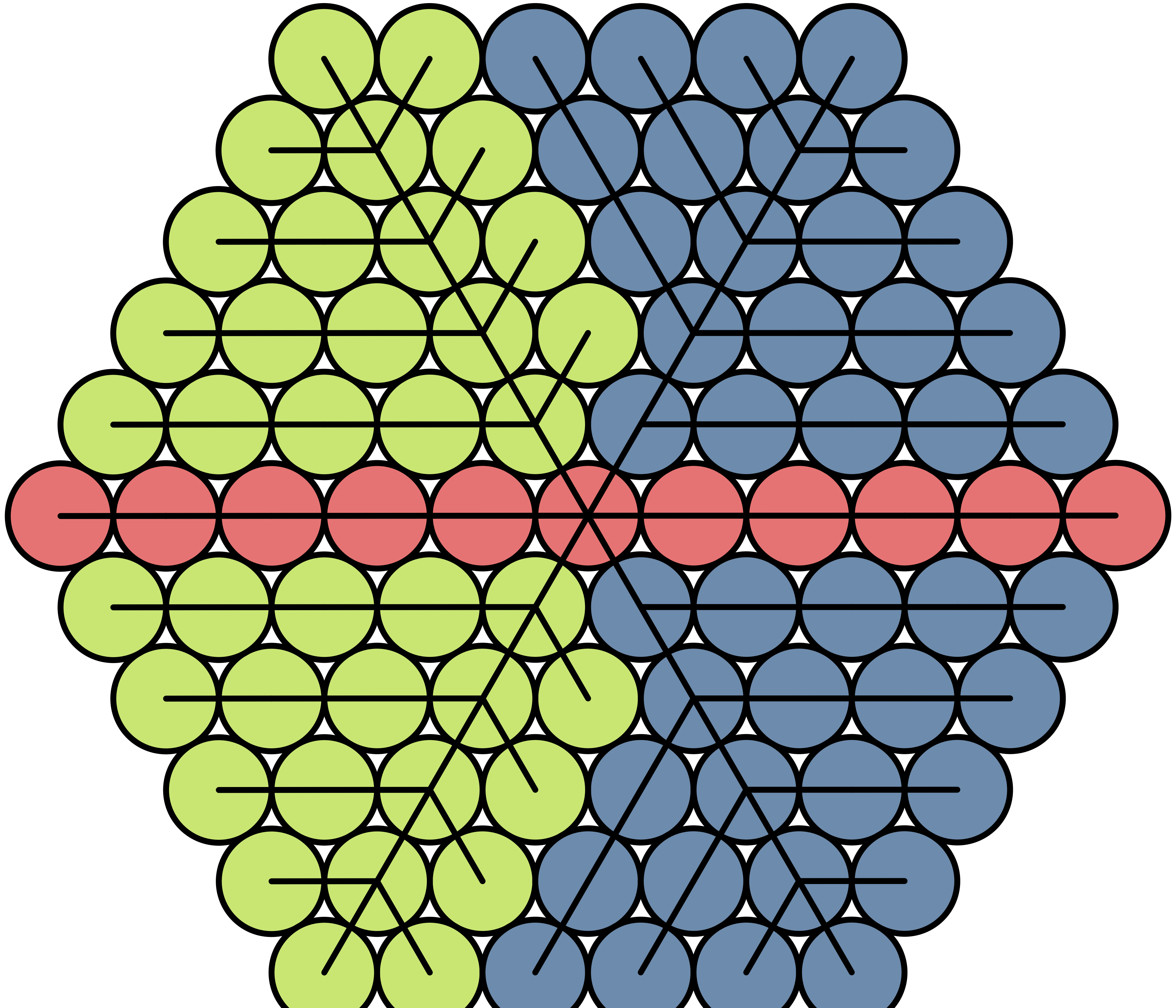

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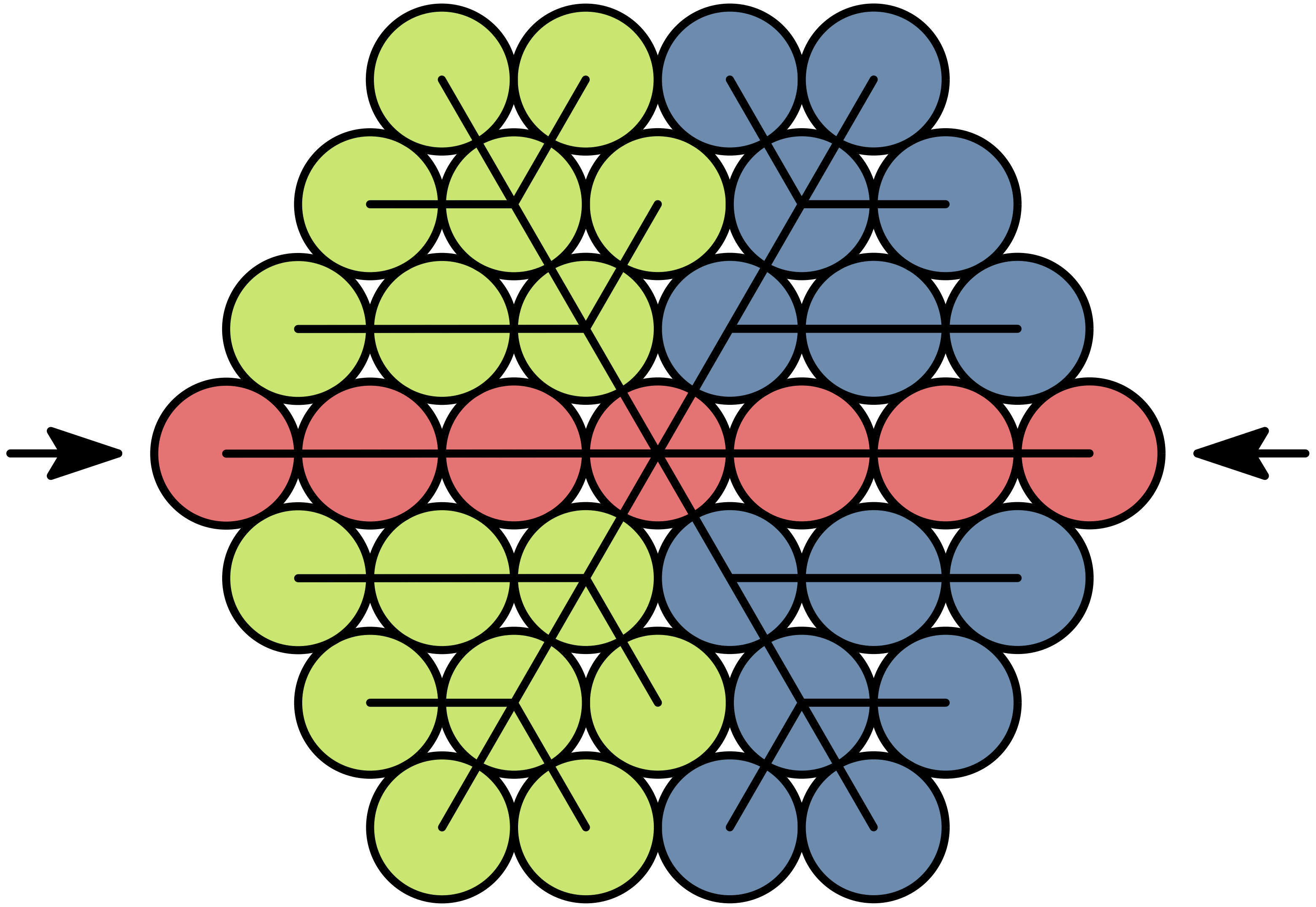

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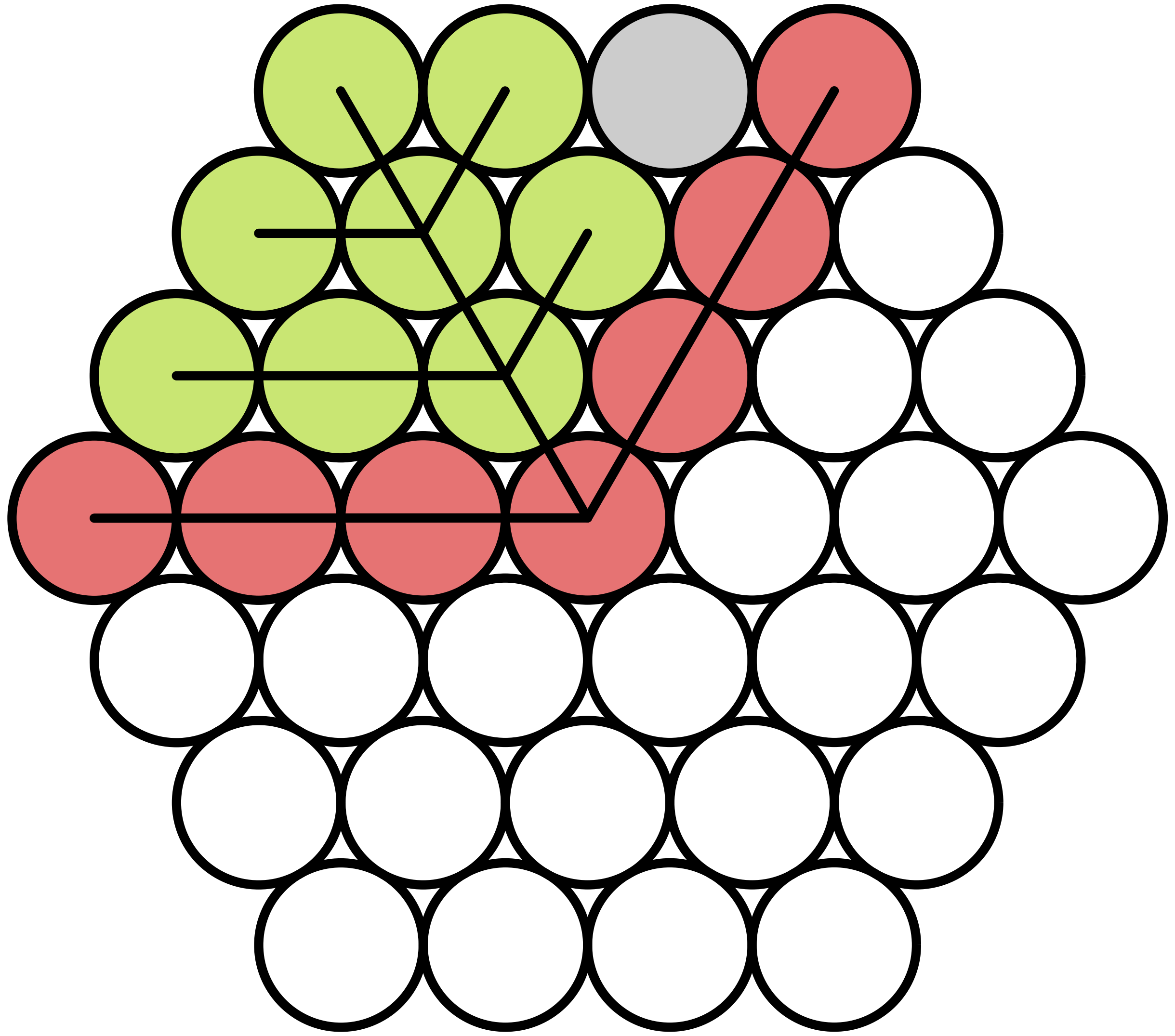

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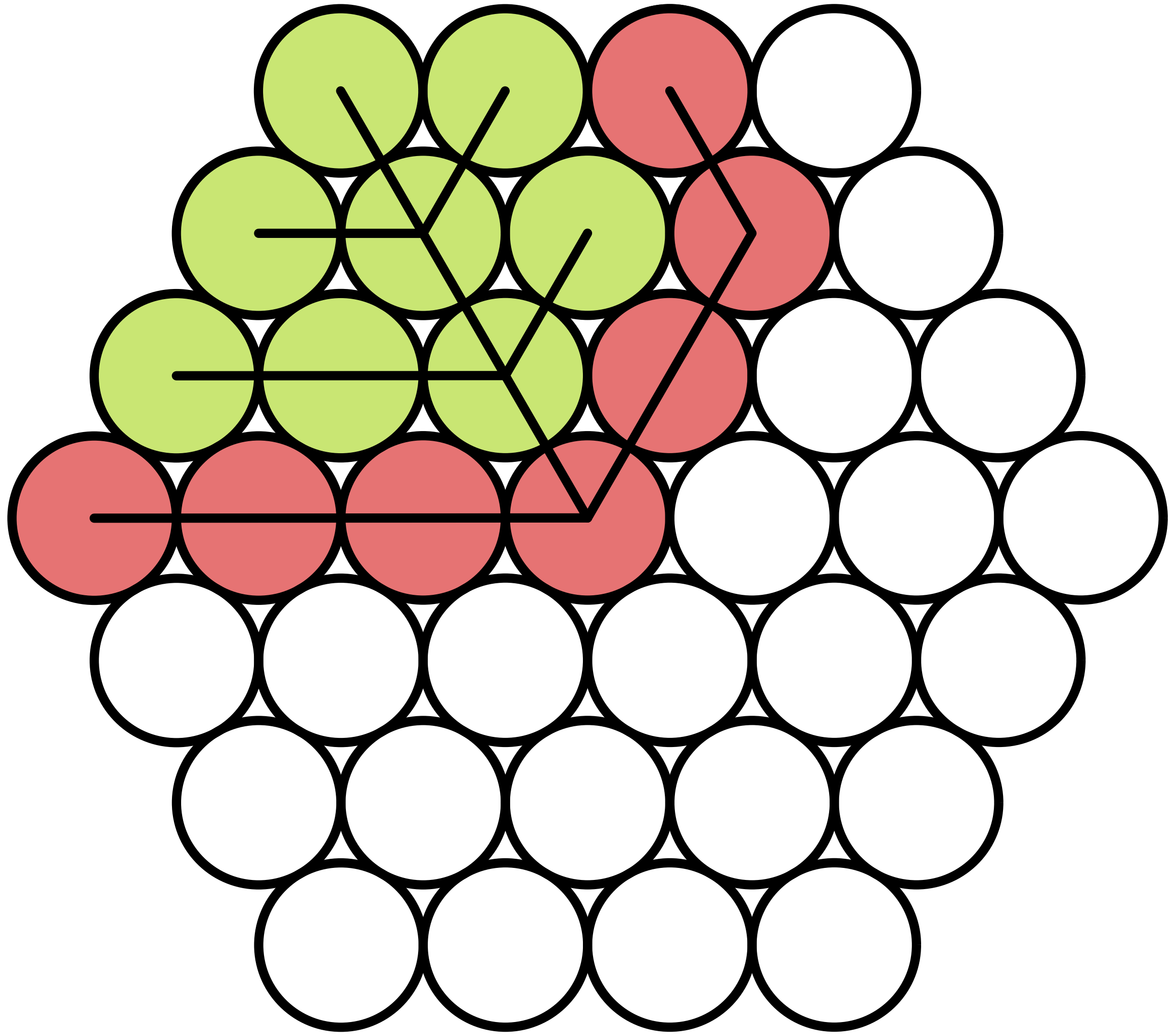

Modeling Line Segments

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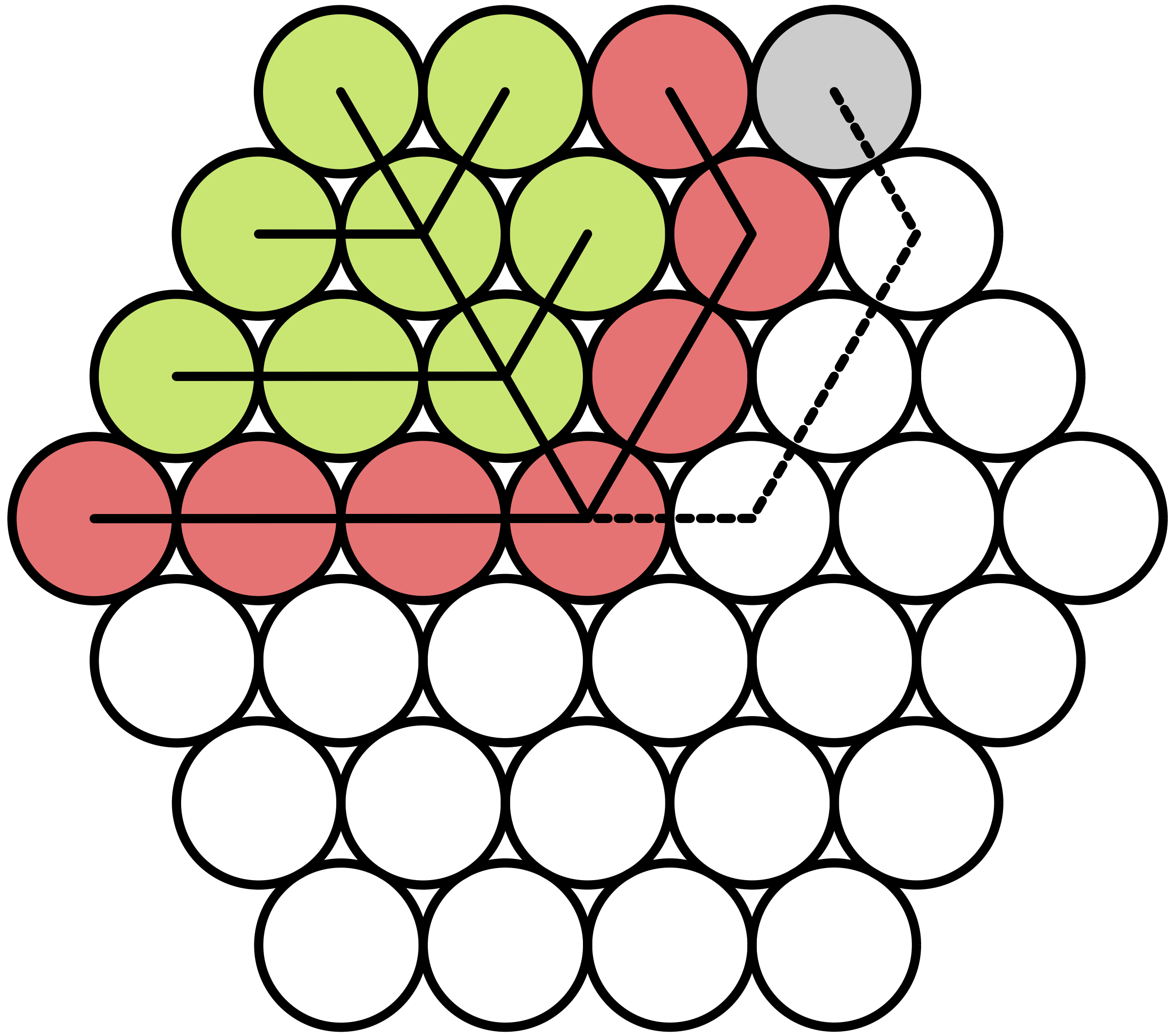

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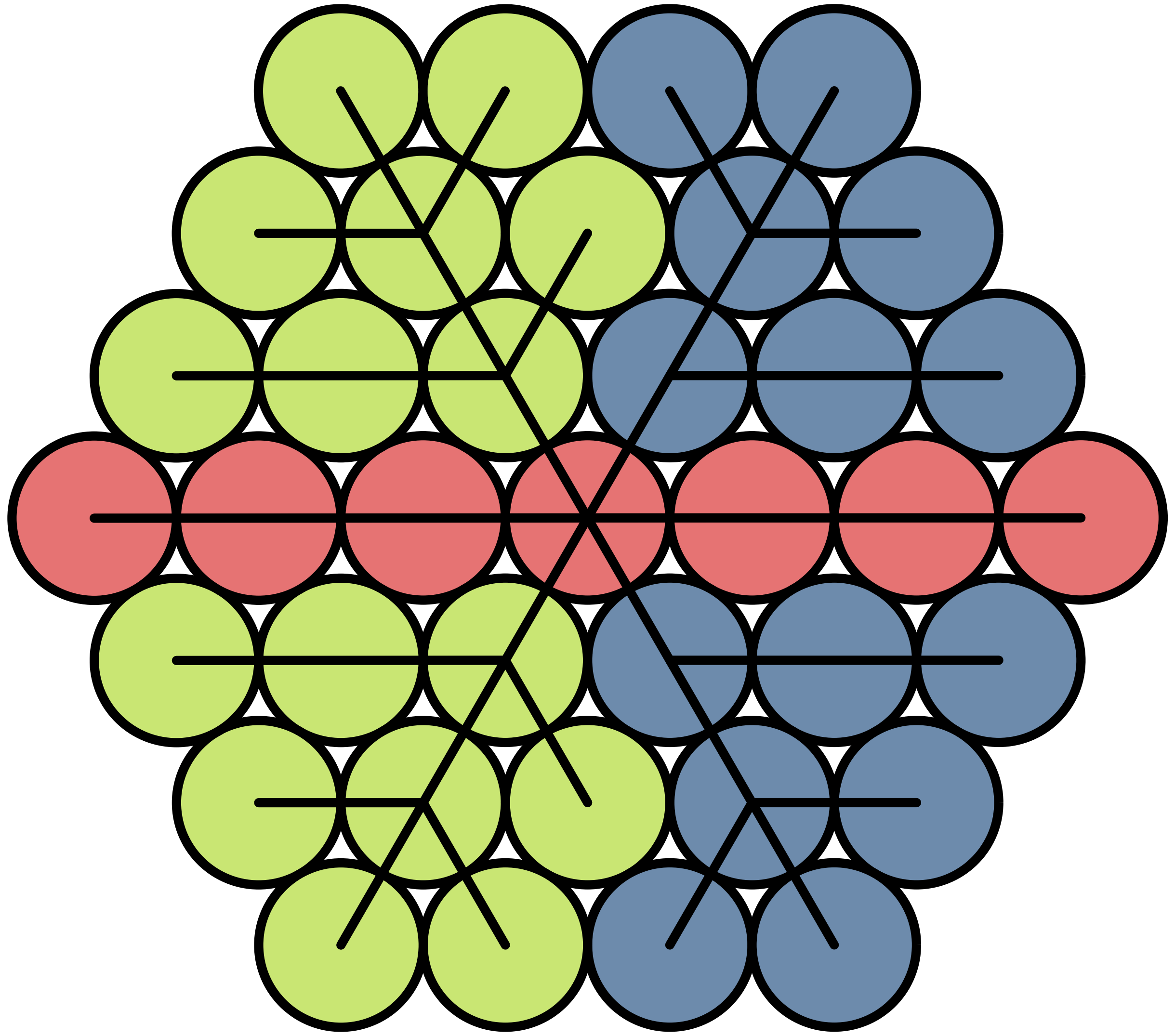

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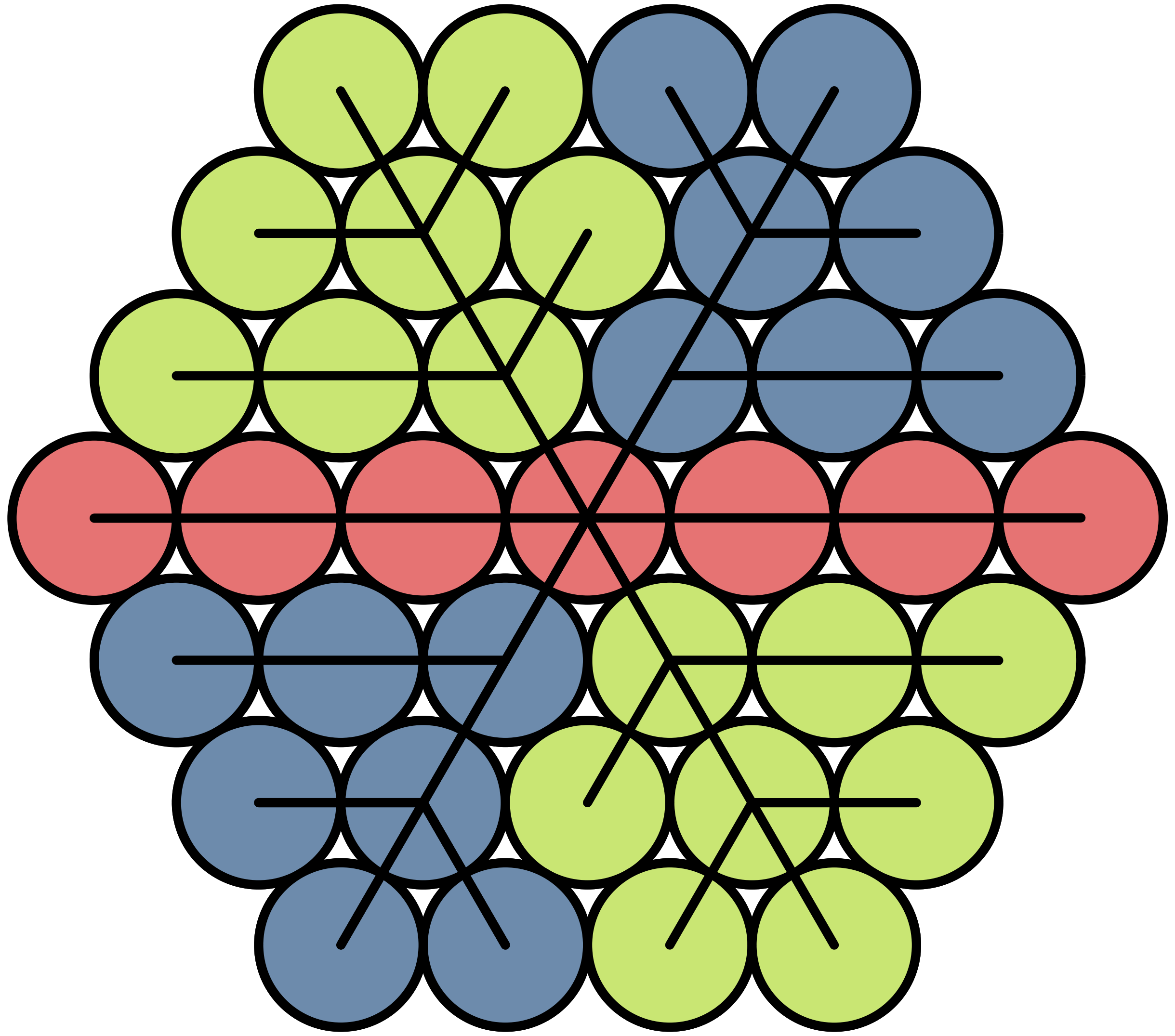
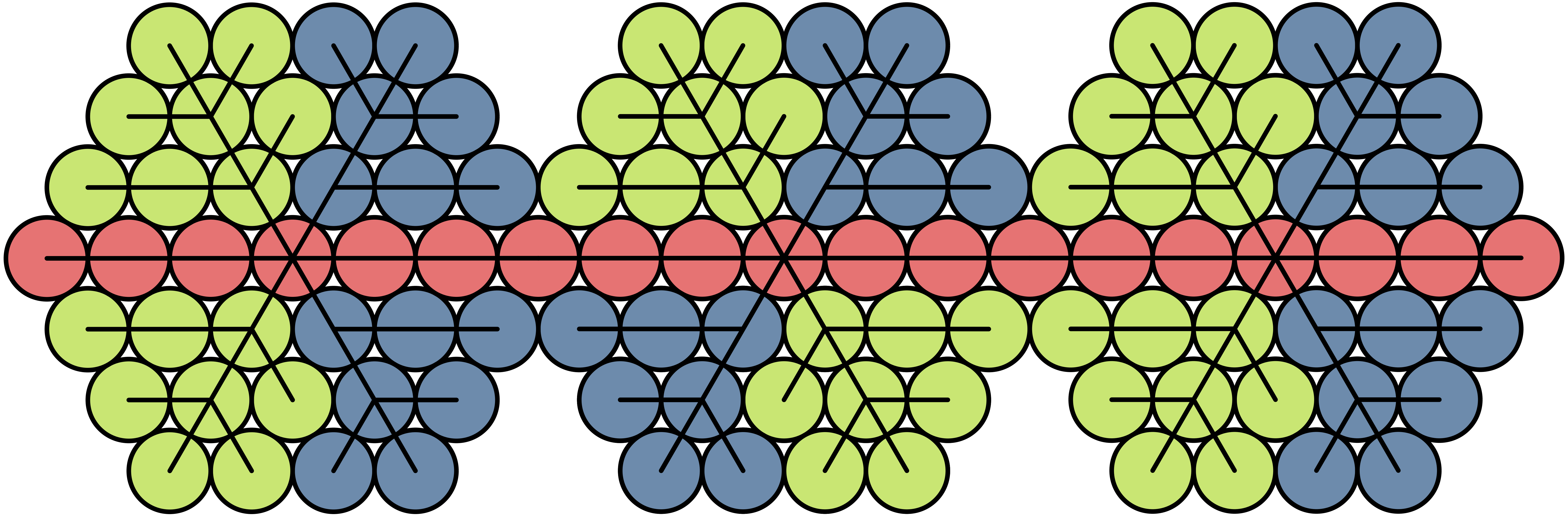
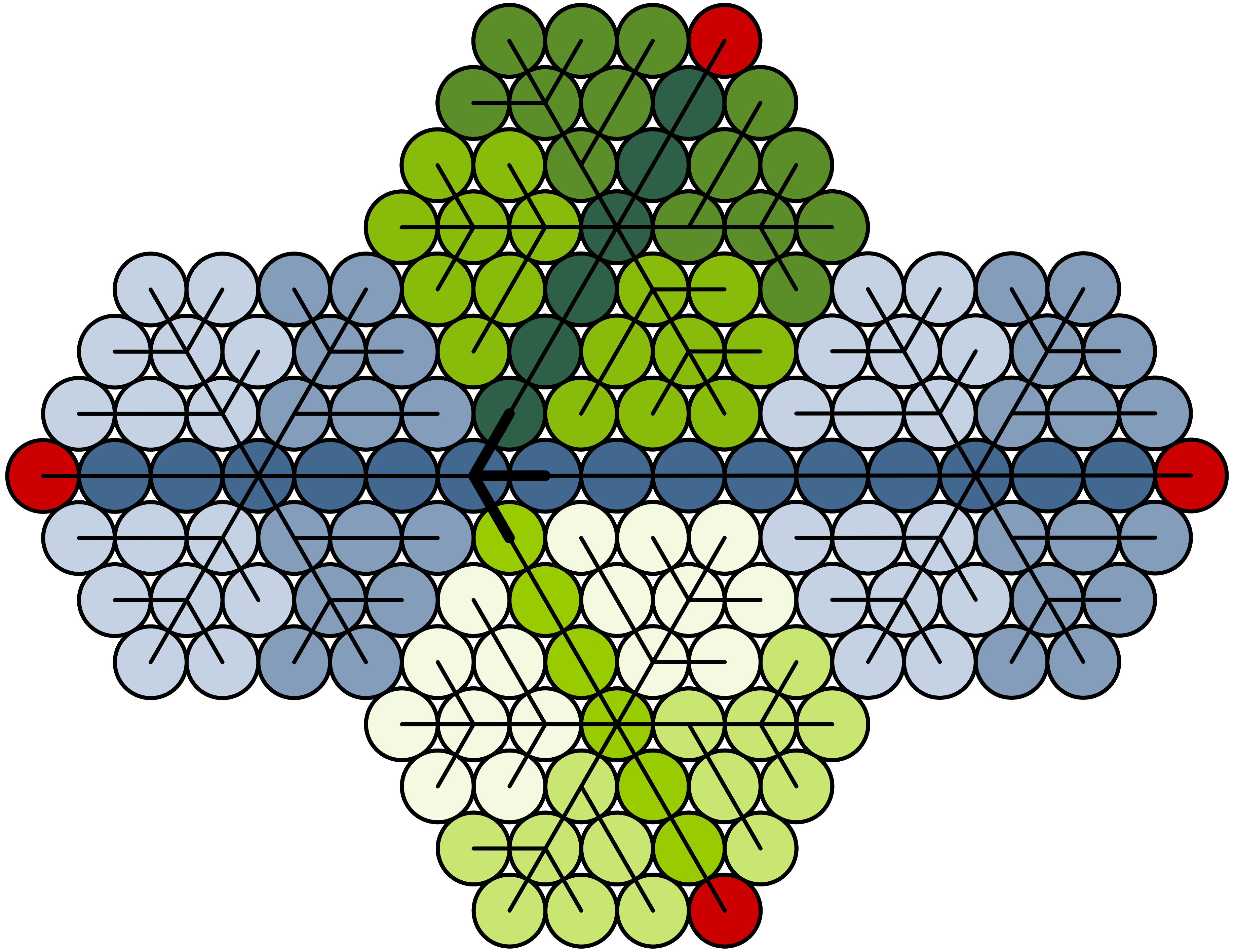


Modeling Line Segments



A Branching Gadget



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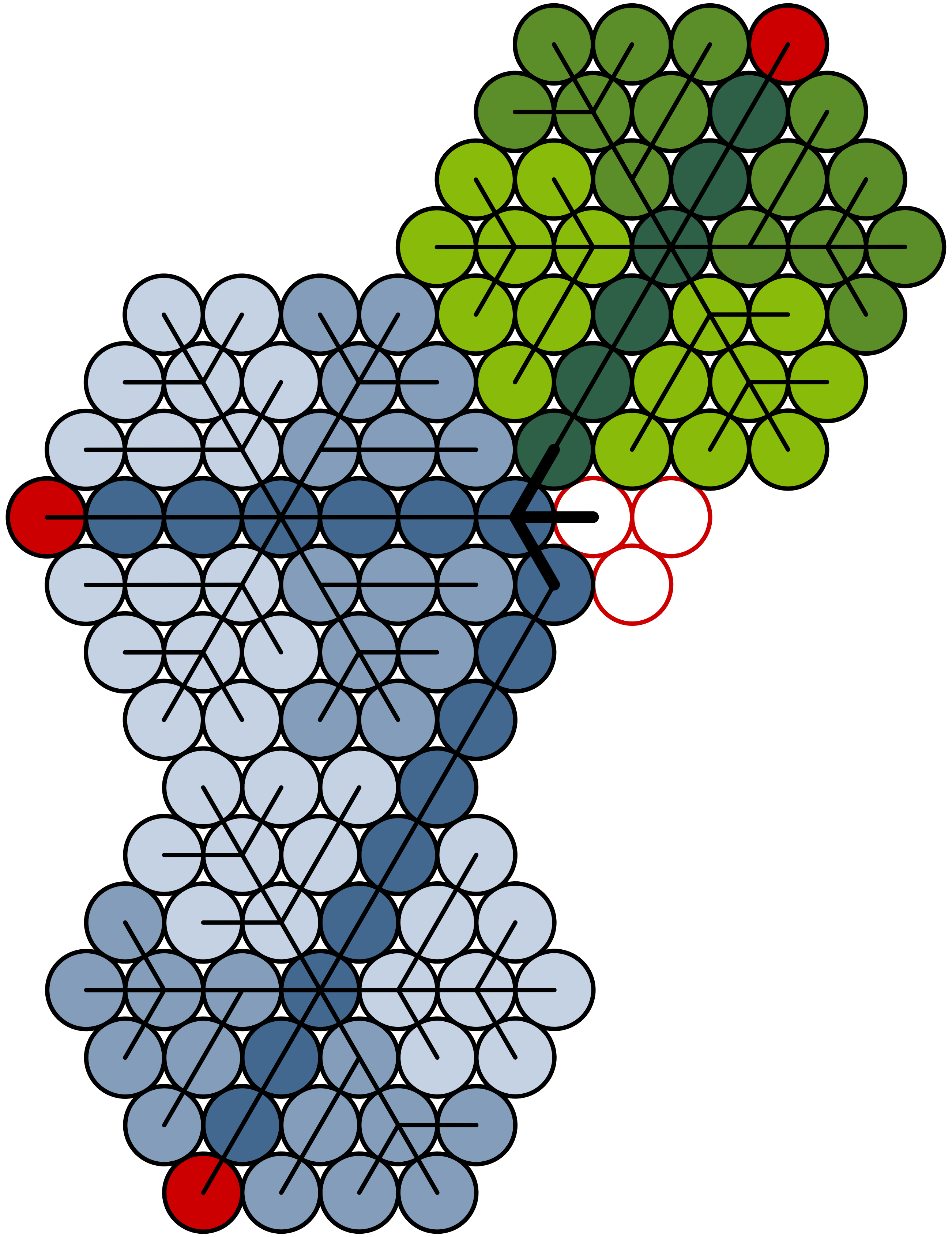

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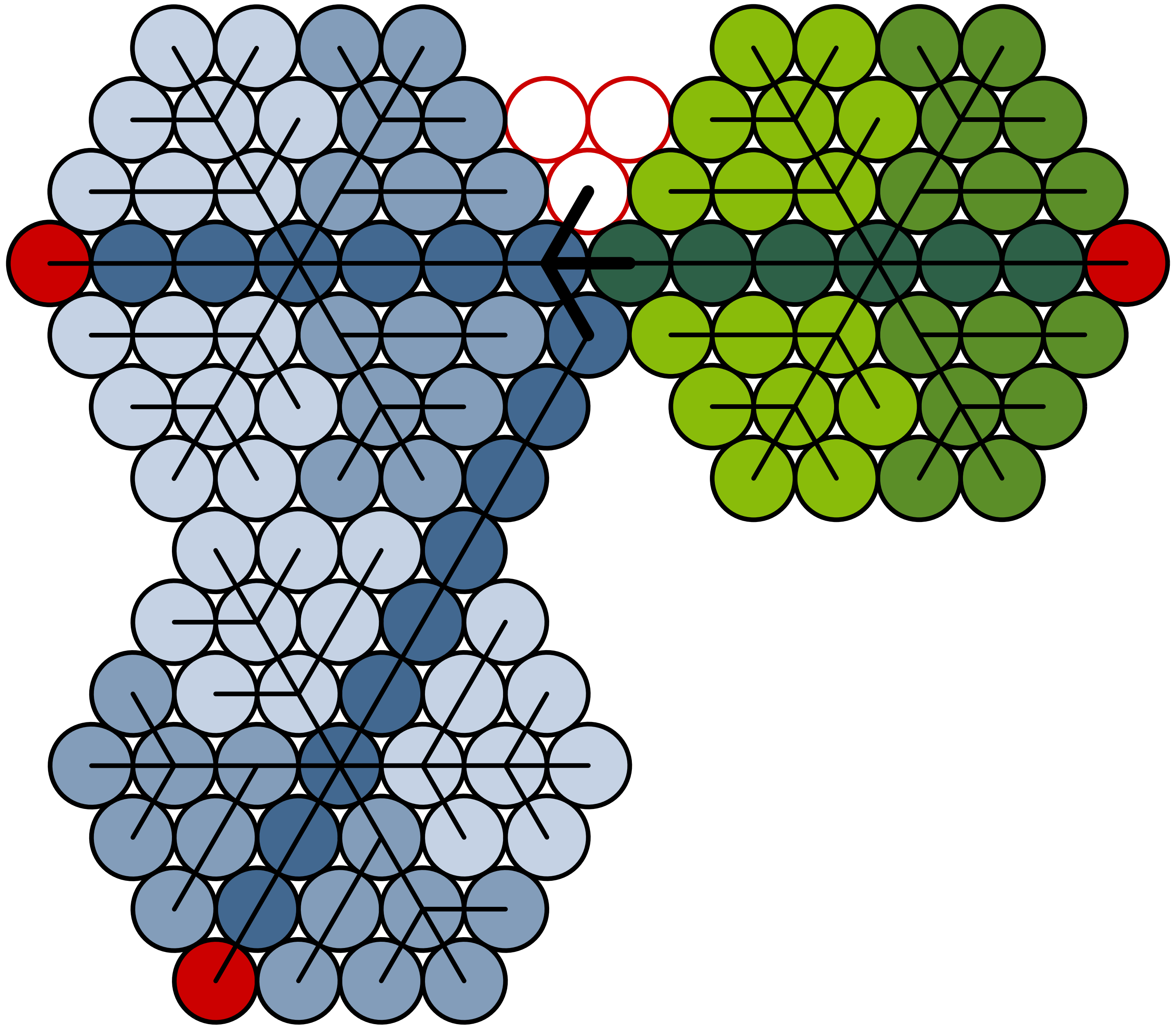

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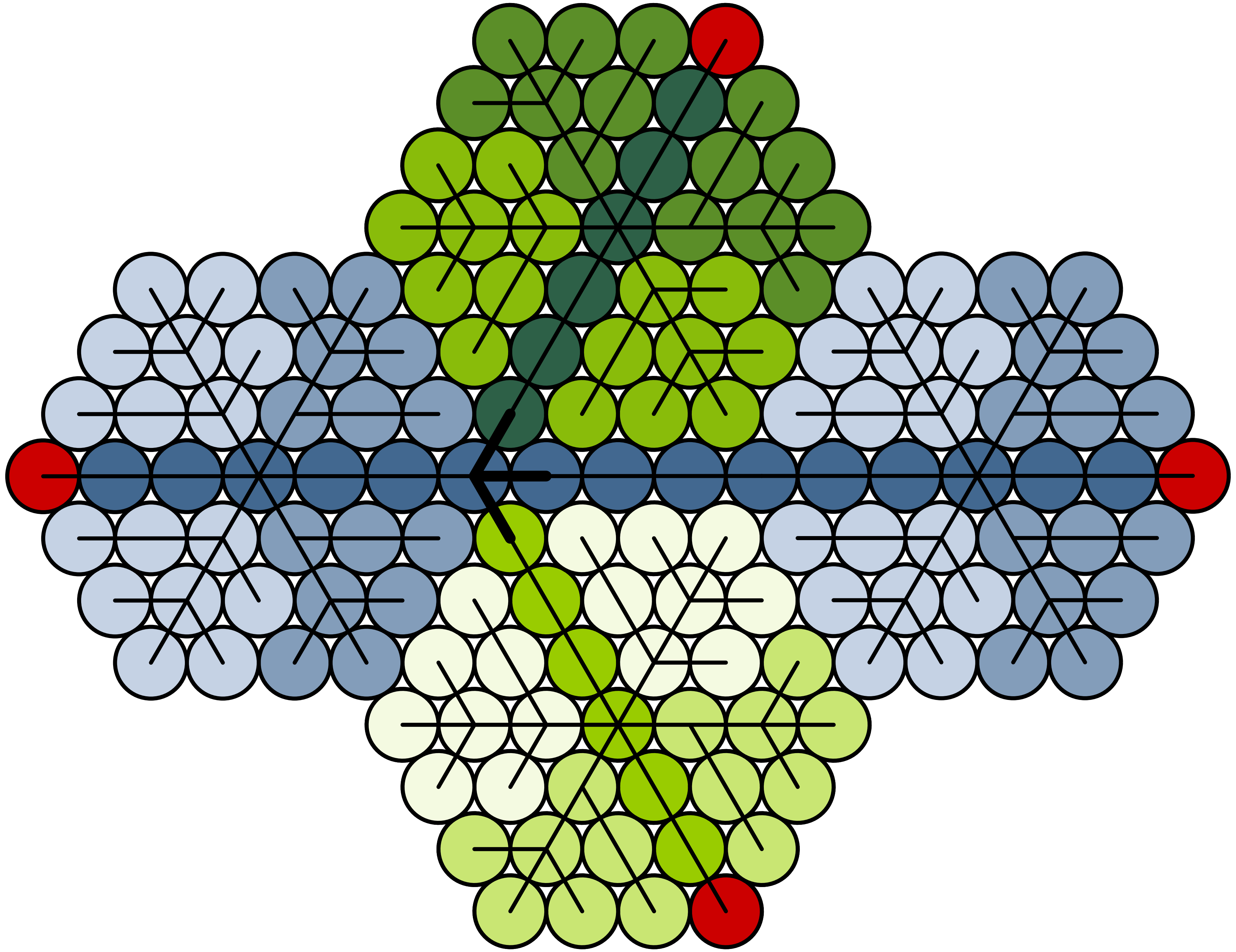

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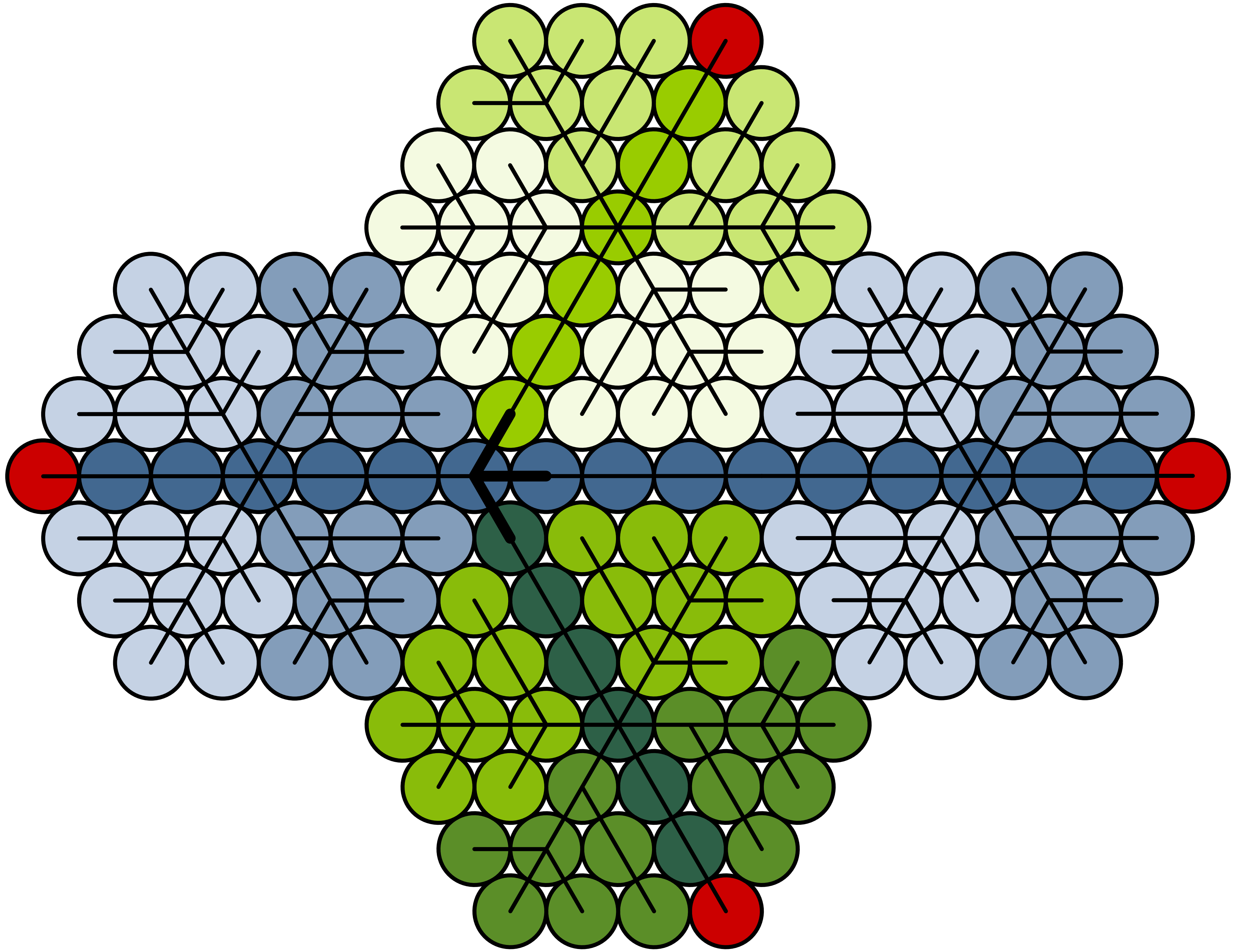

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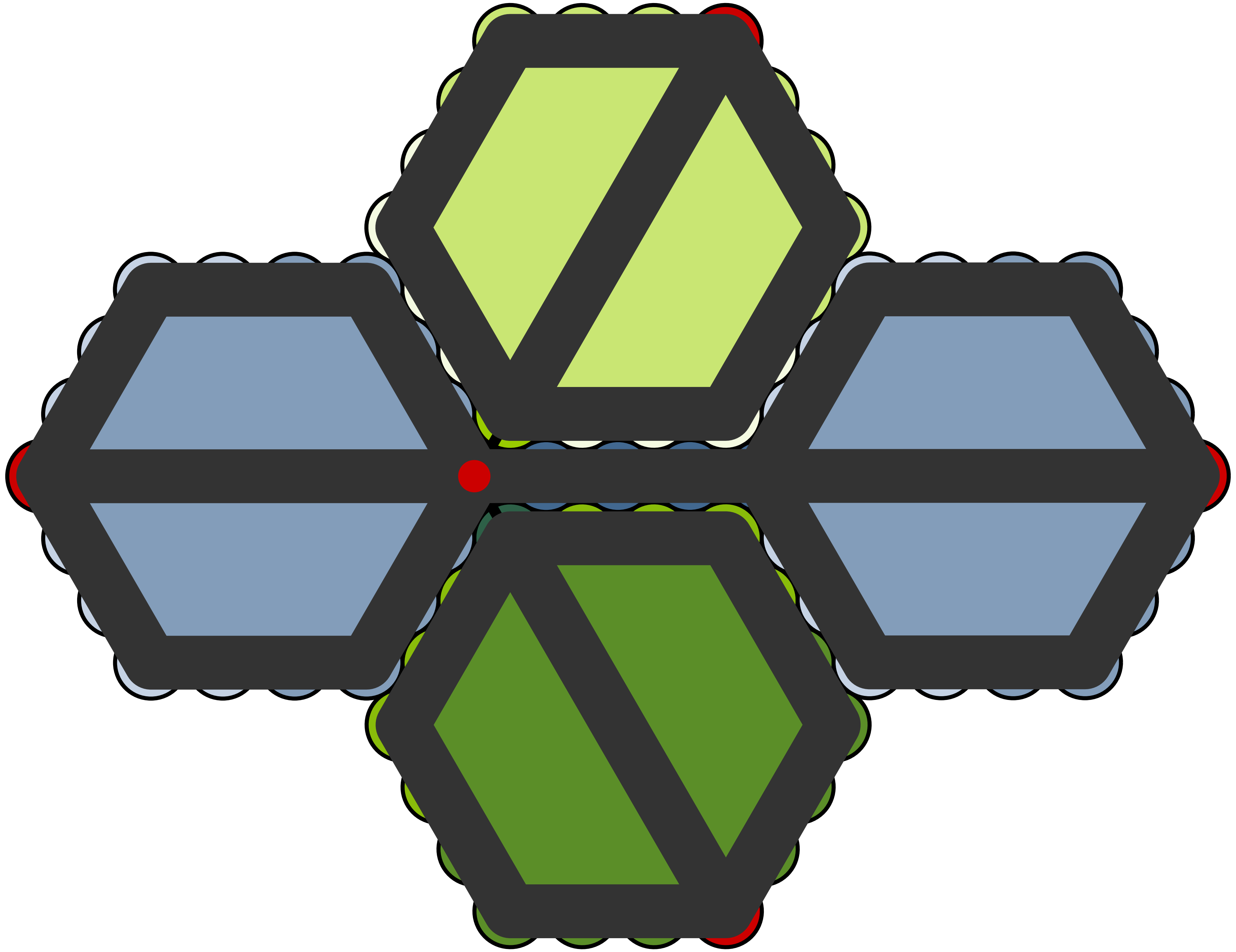
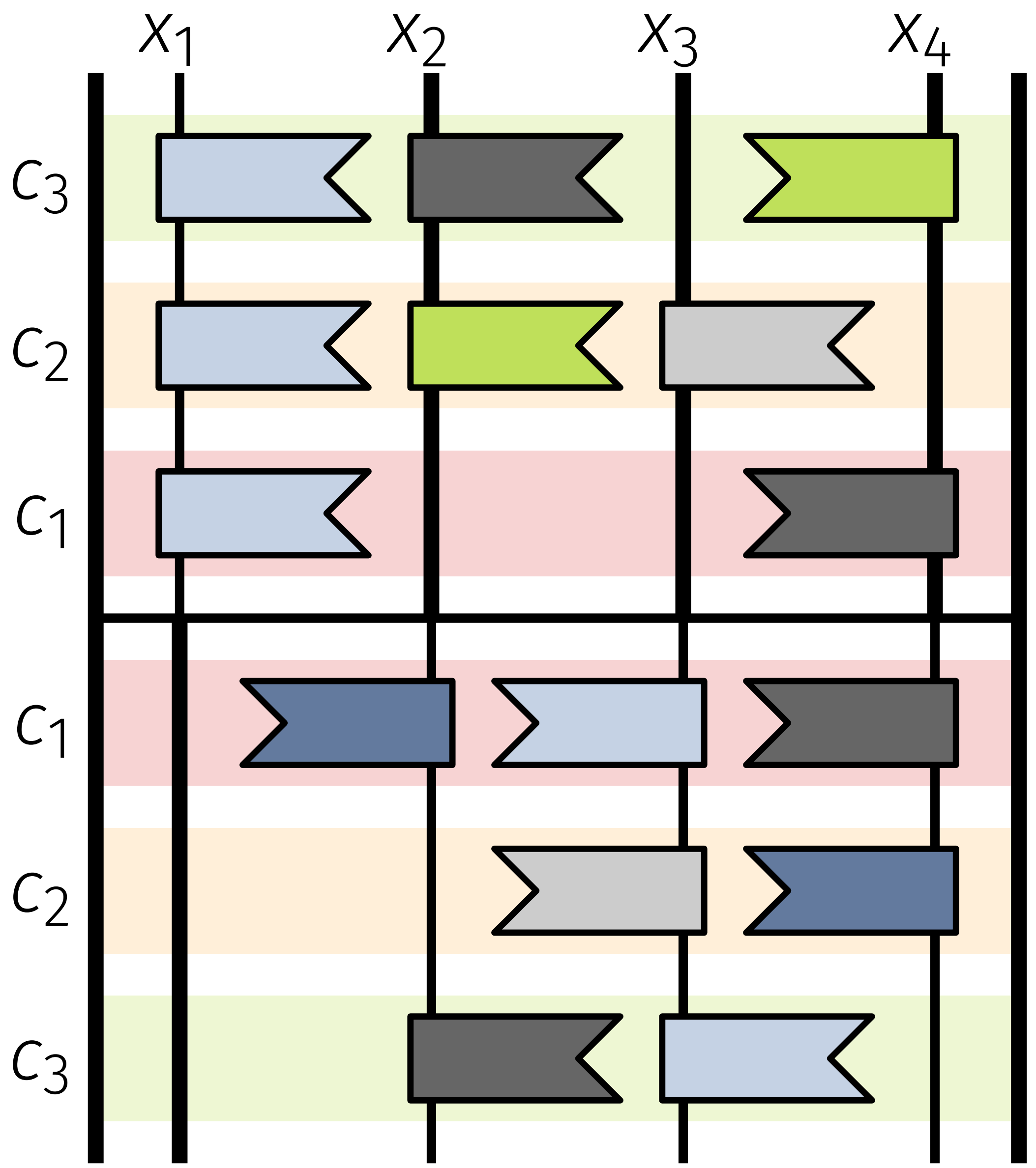
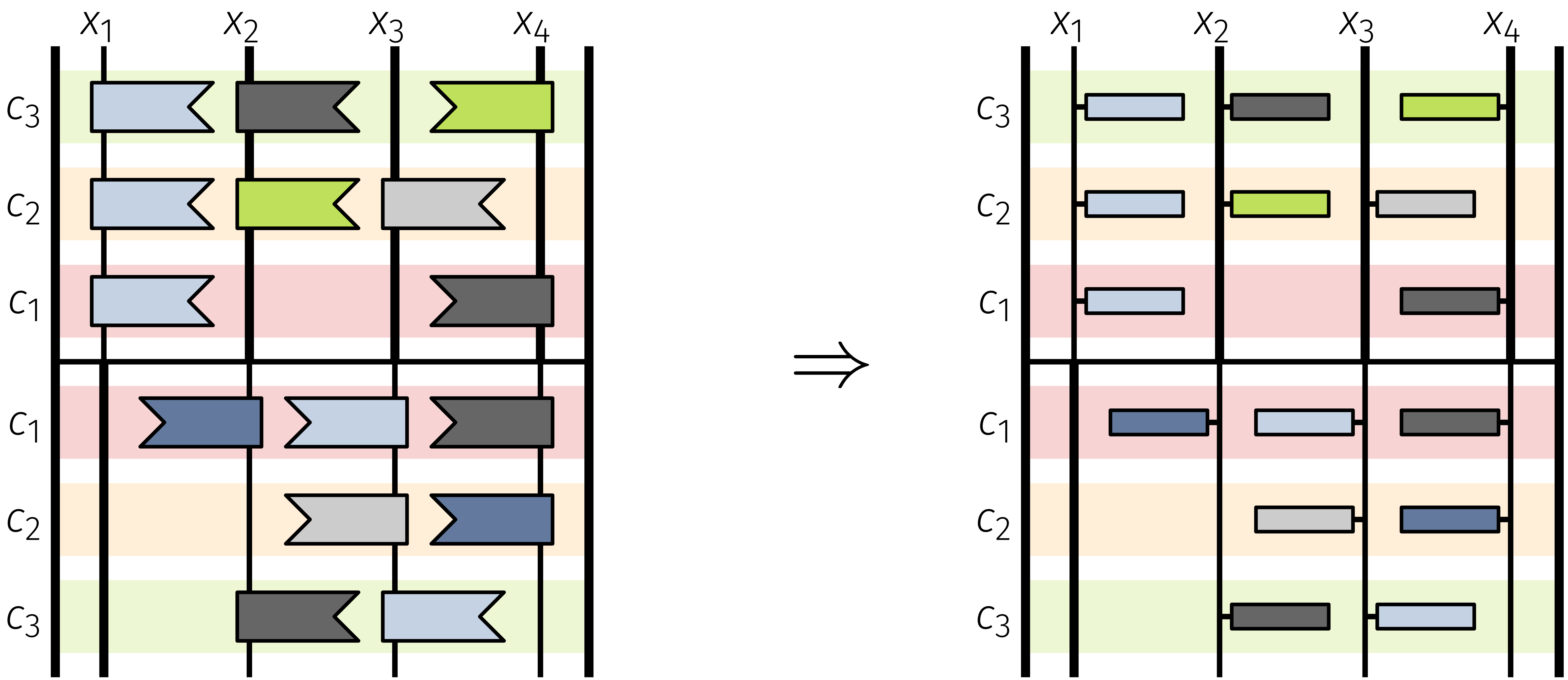


A Branching Gadget

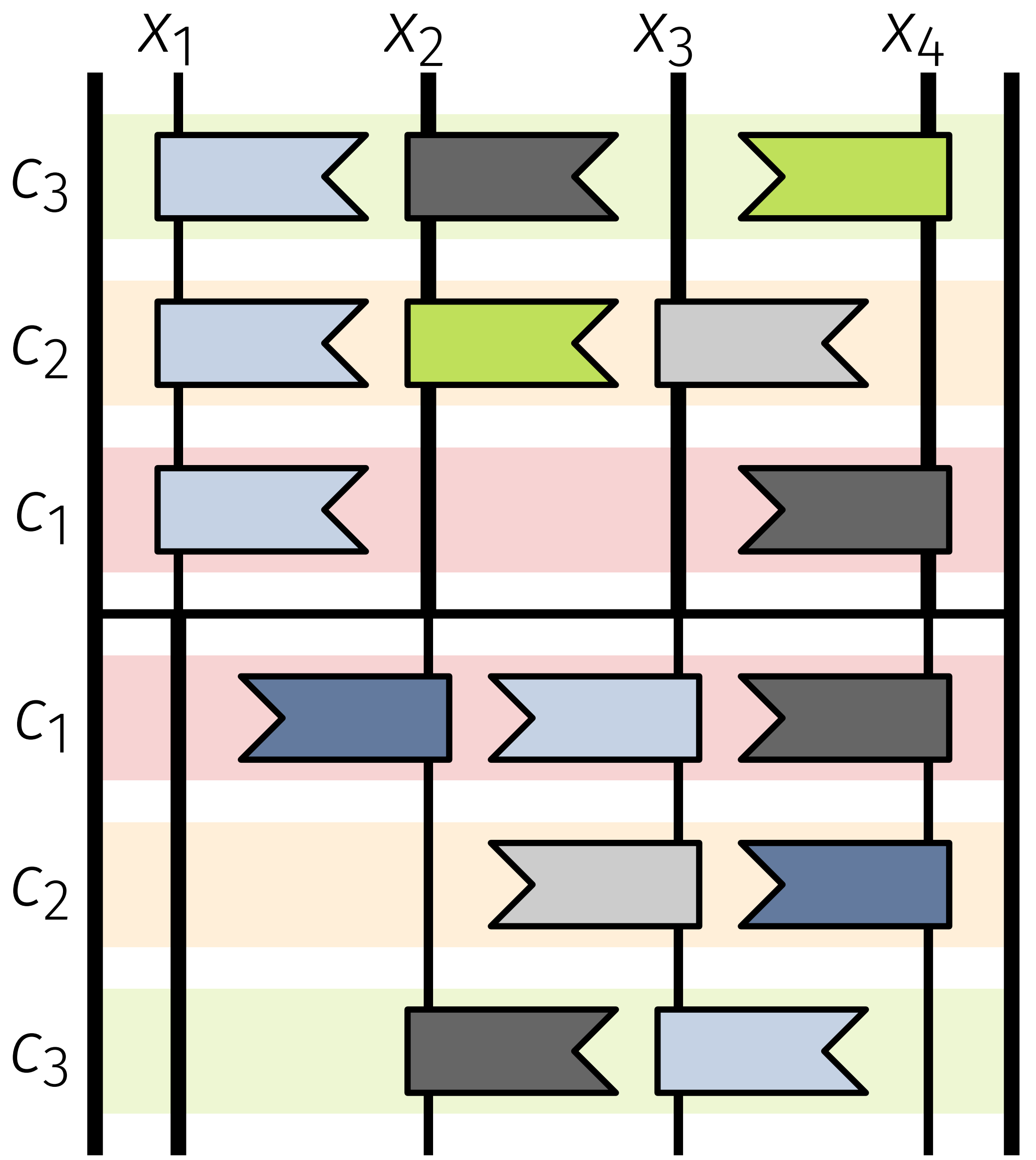
60° Logic Engine



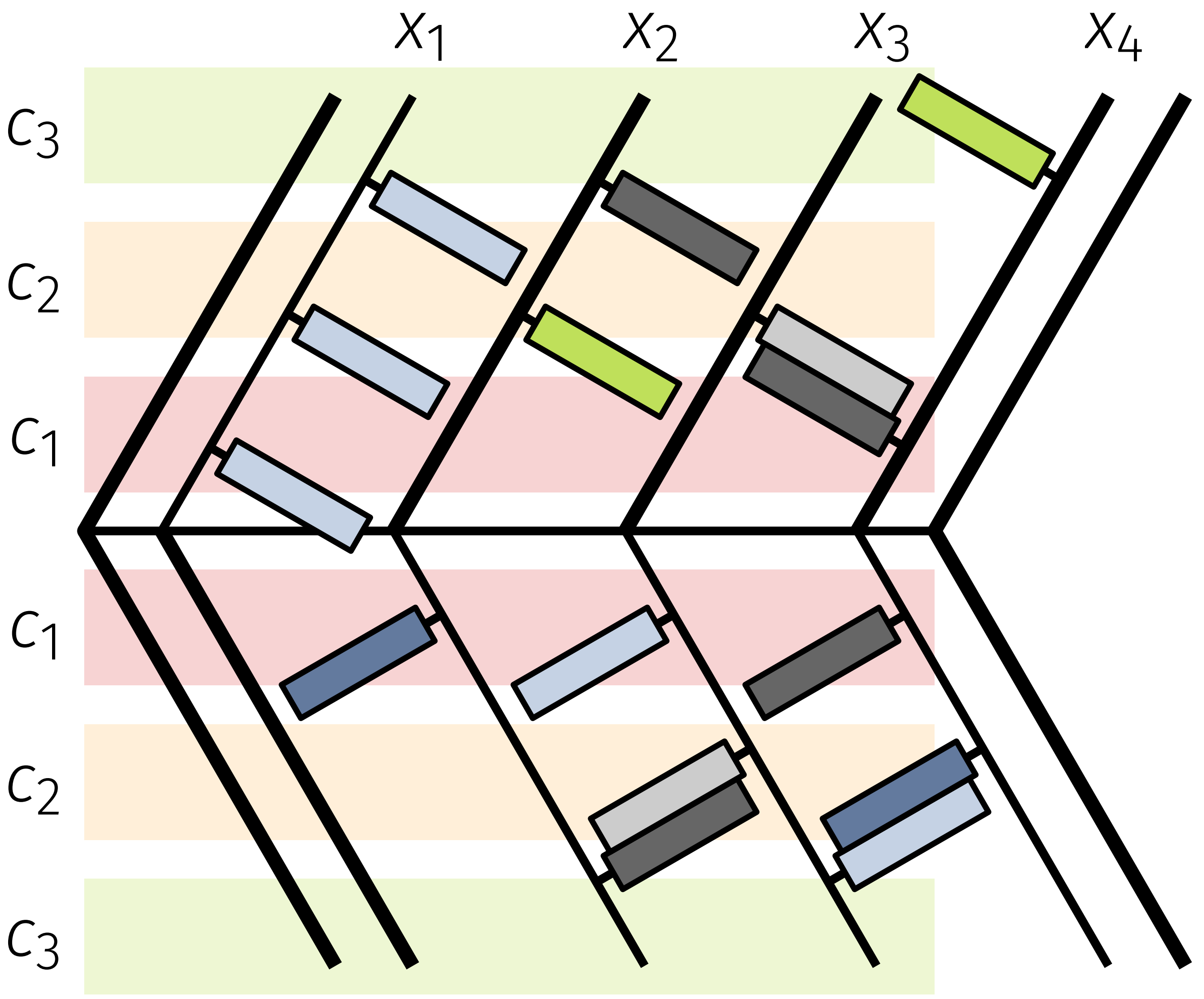
60° Logic Engine



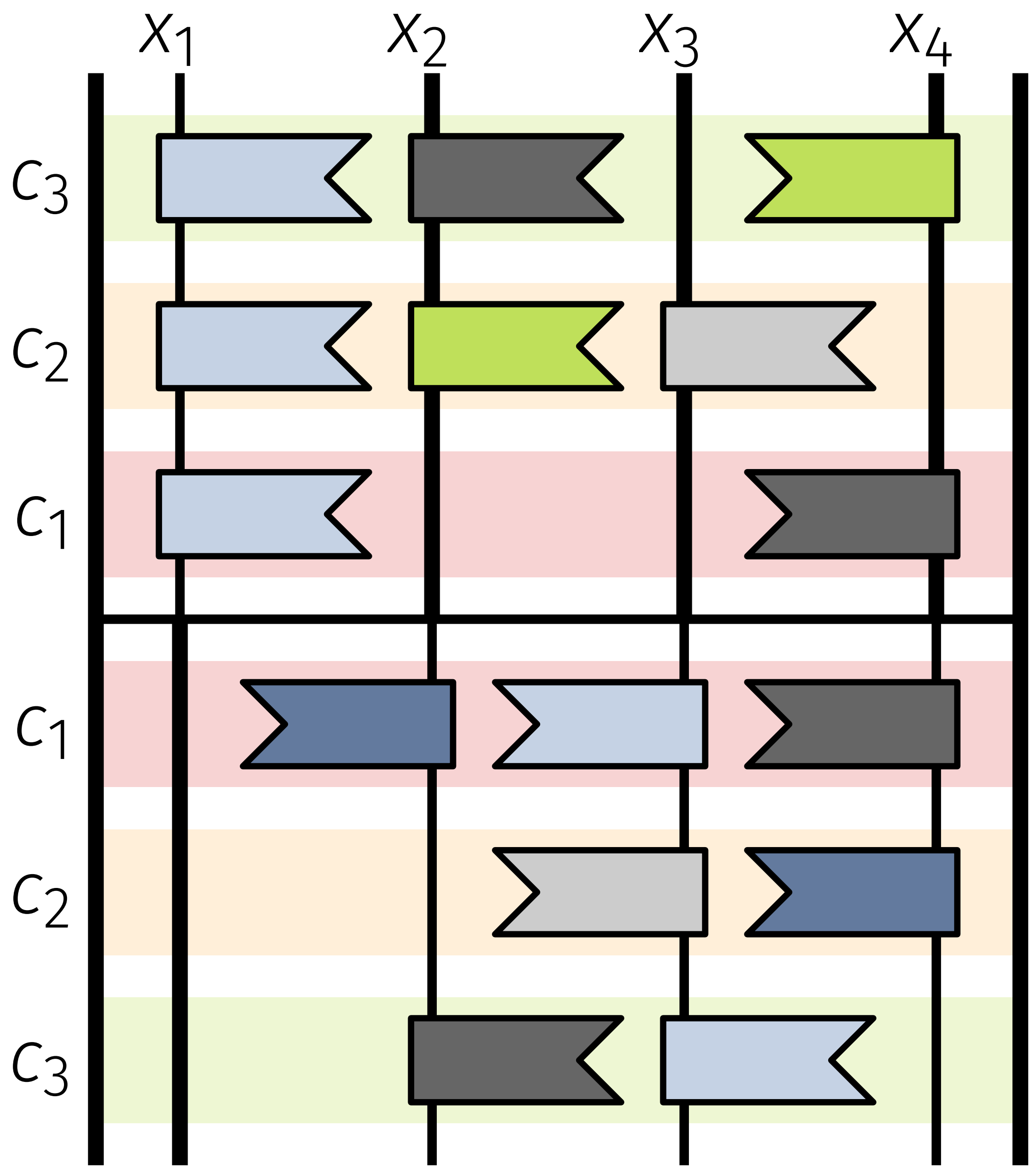
60° Logic Engine



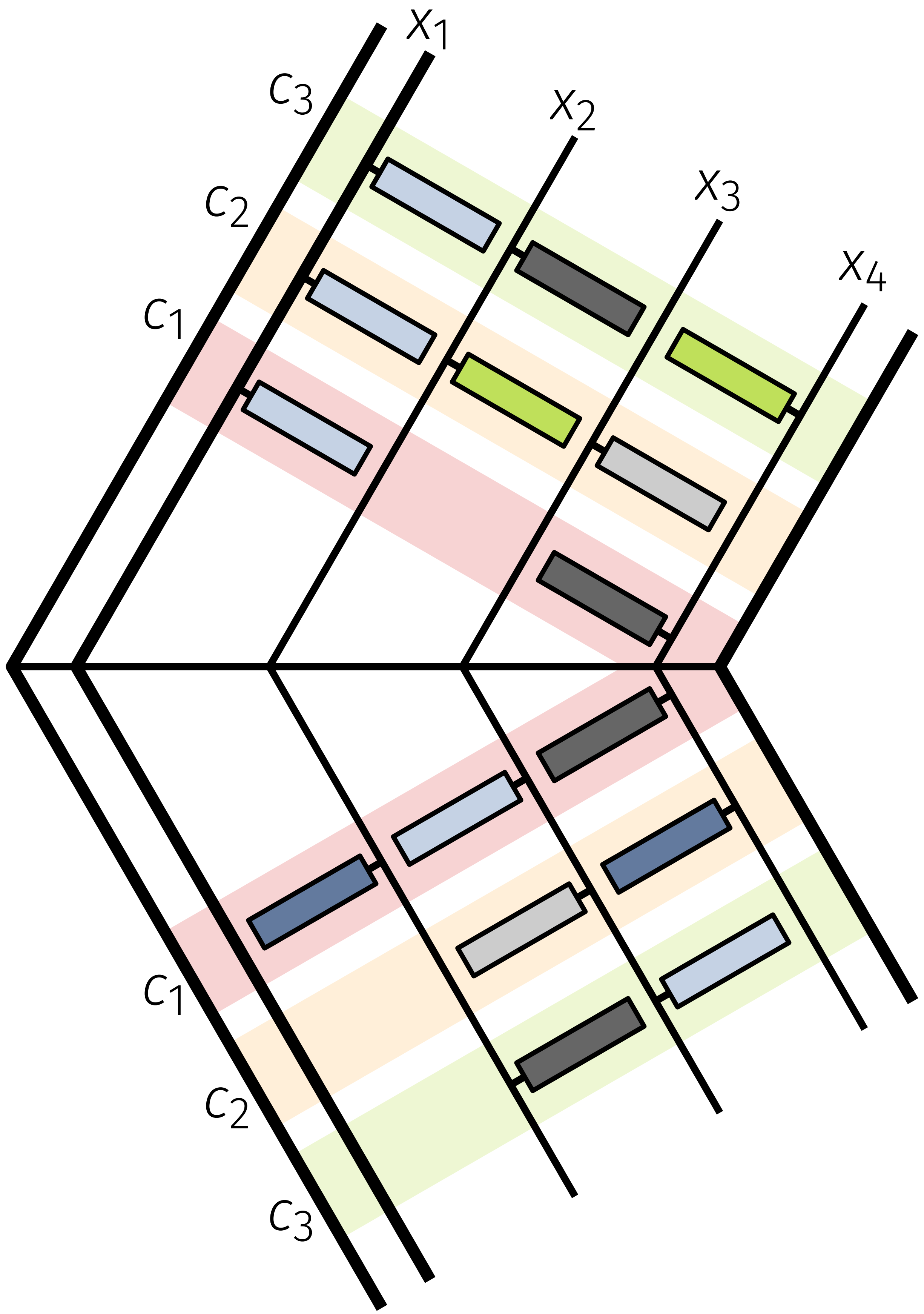
$\Rightarrow$



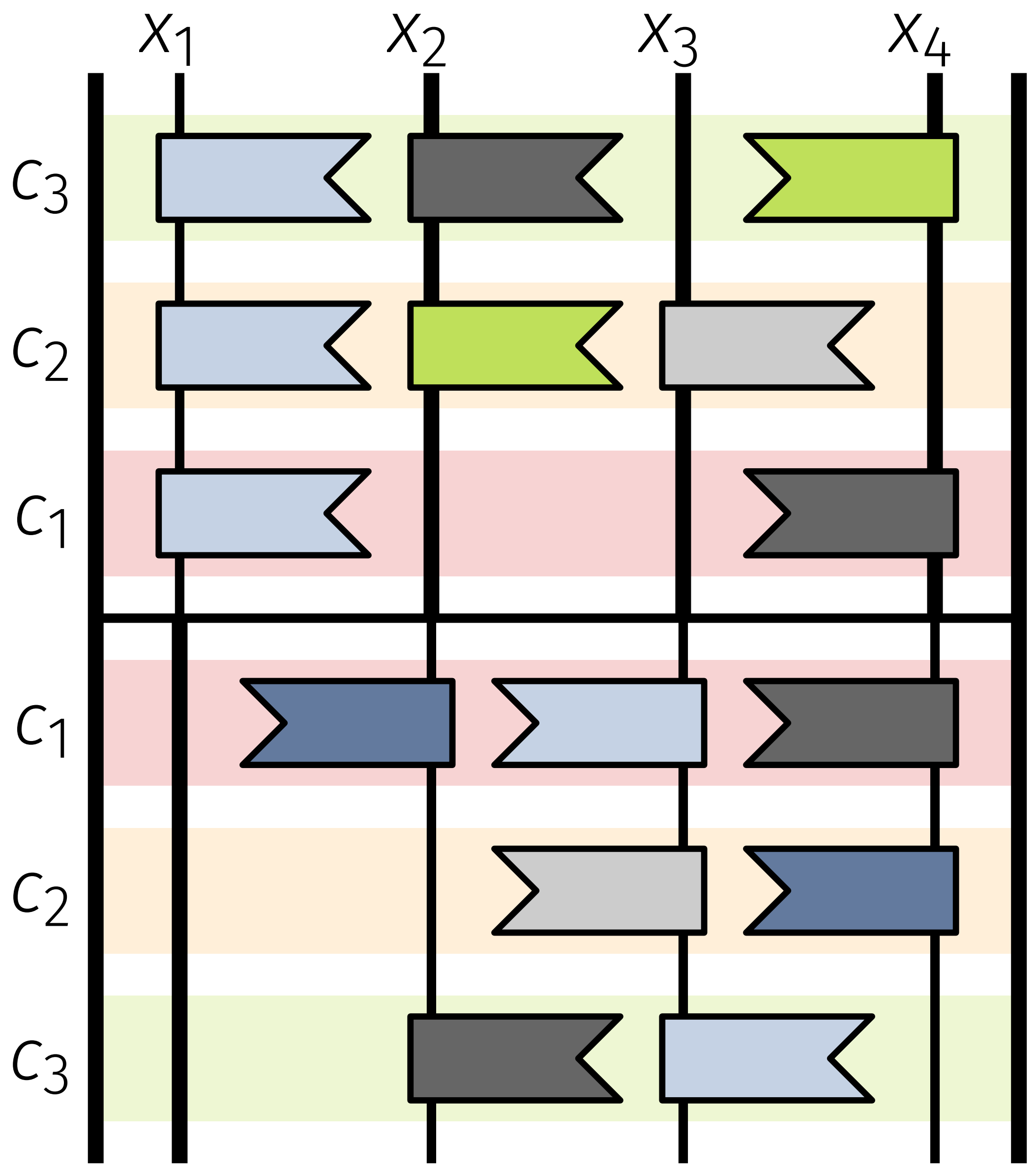
60° Logic Engine



$\Rightarrow$



60° Logic Engine



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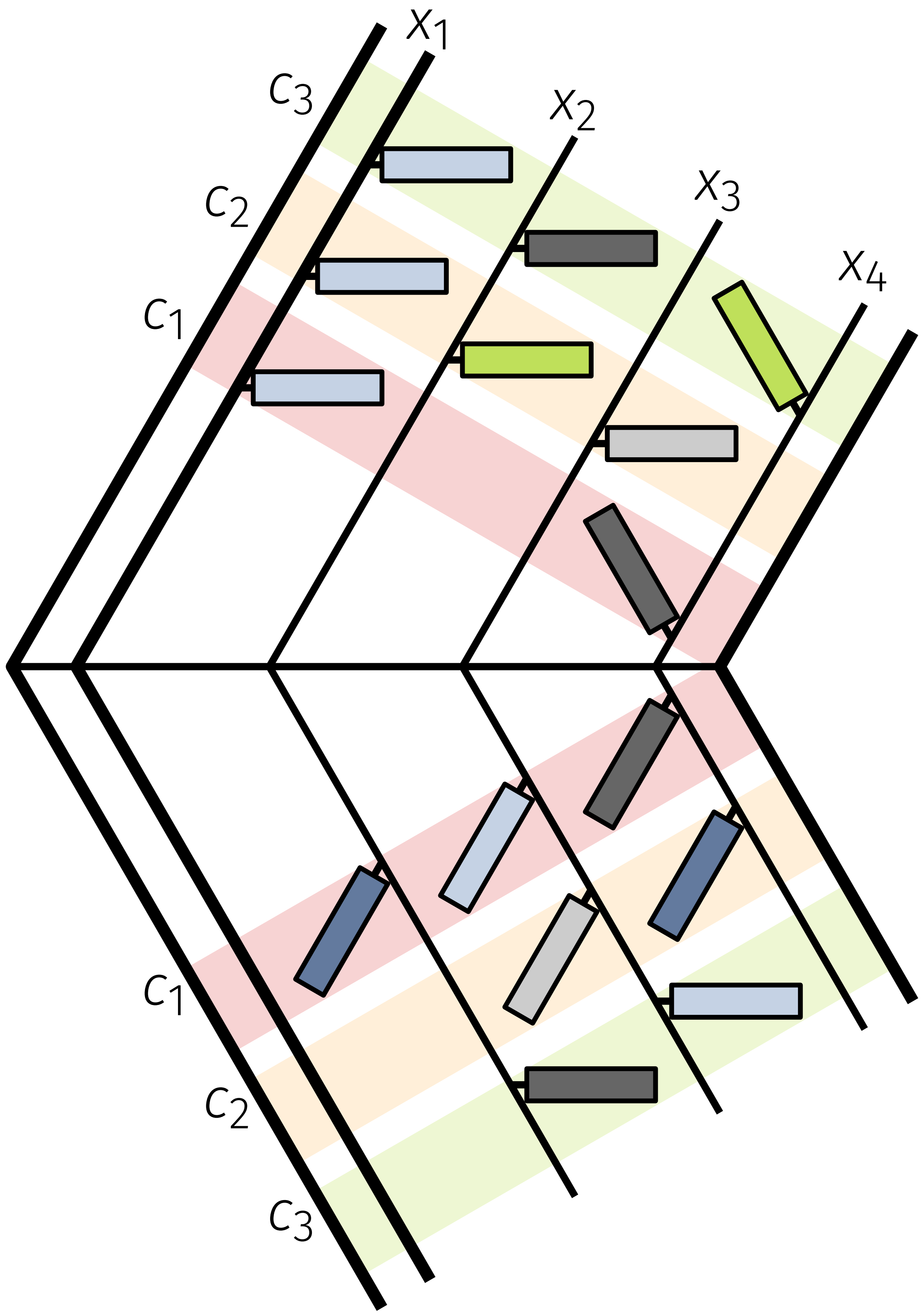
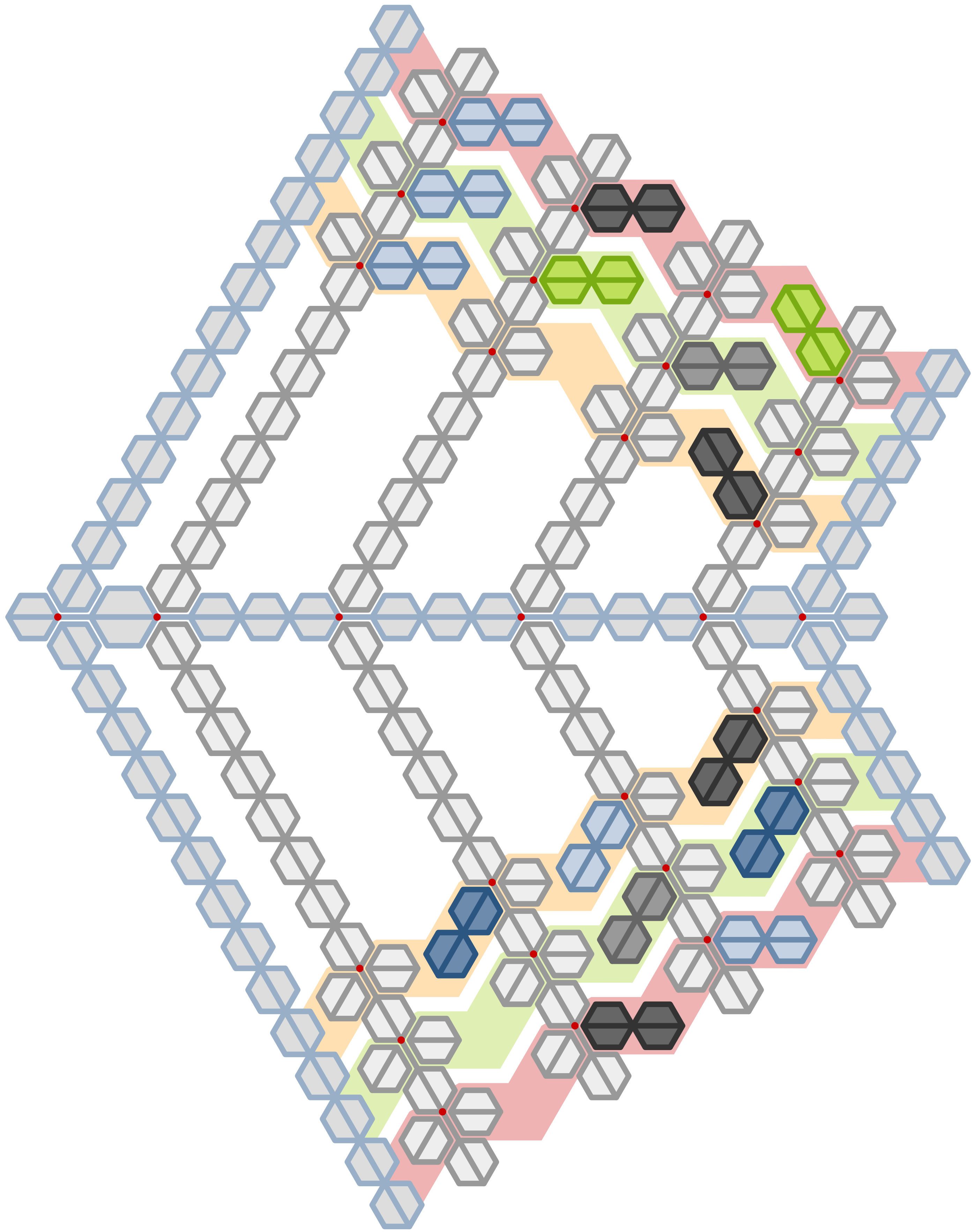
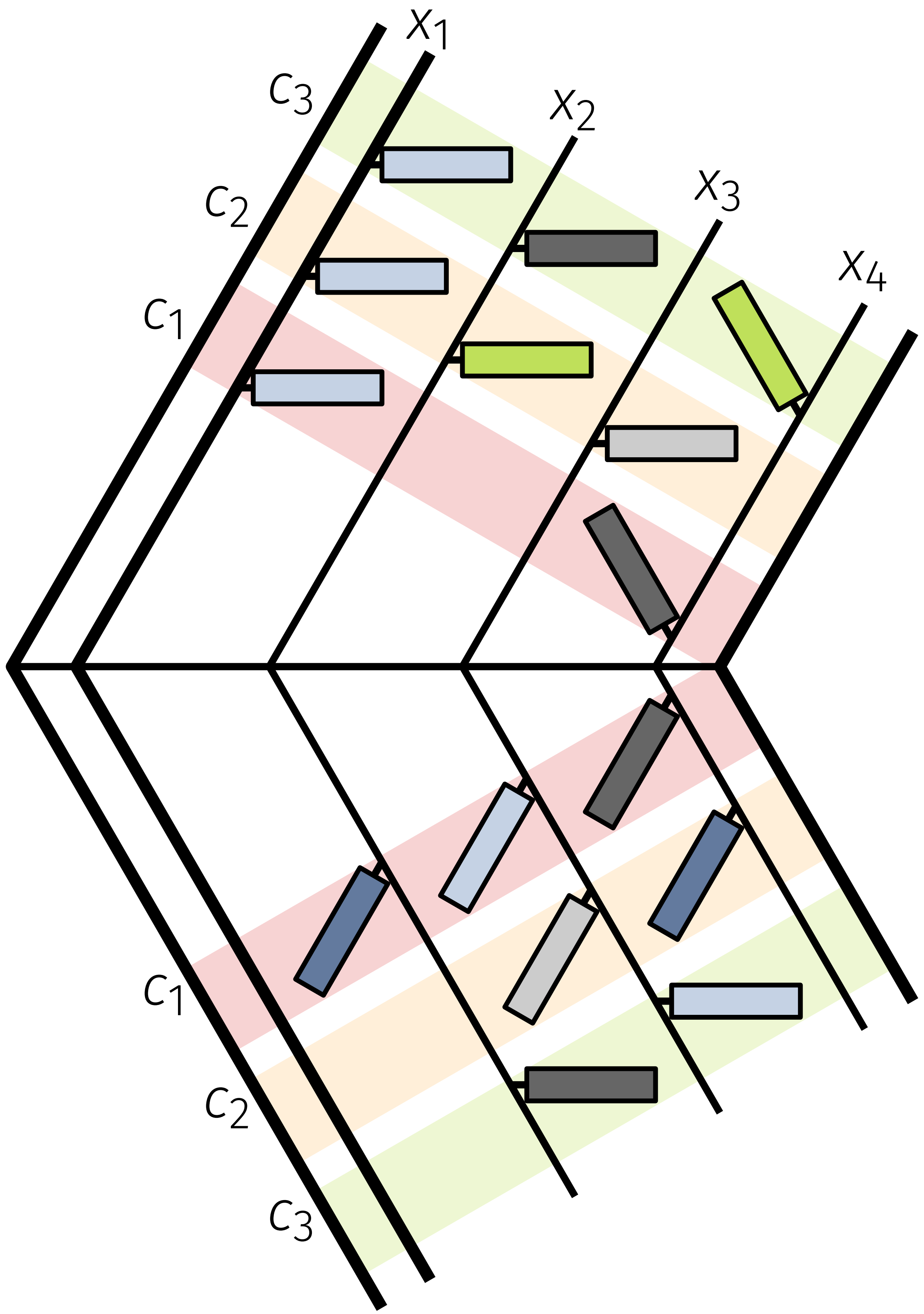


60° Logic Engine



$\Leftarrow$



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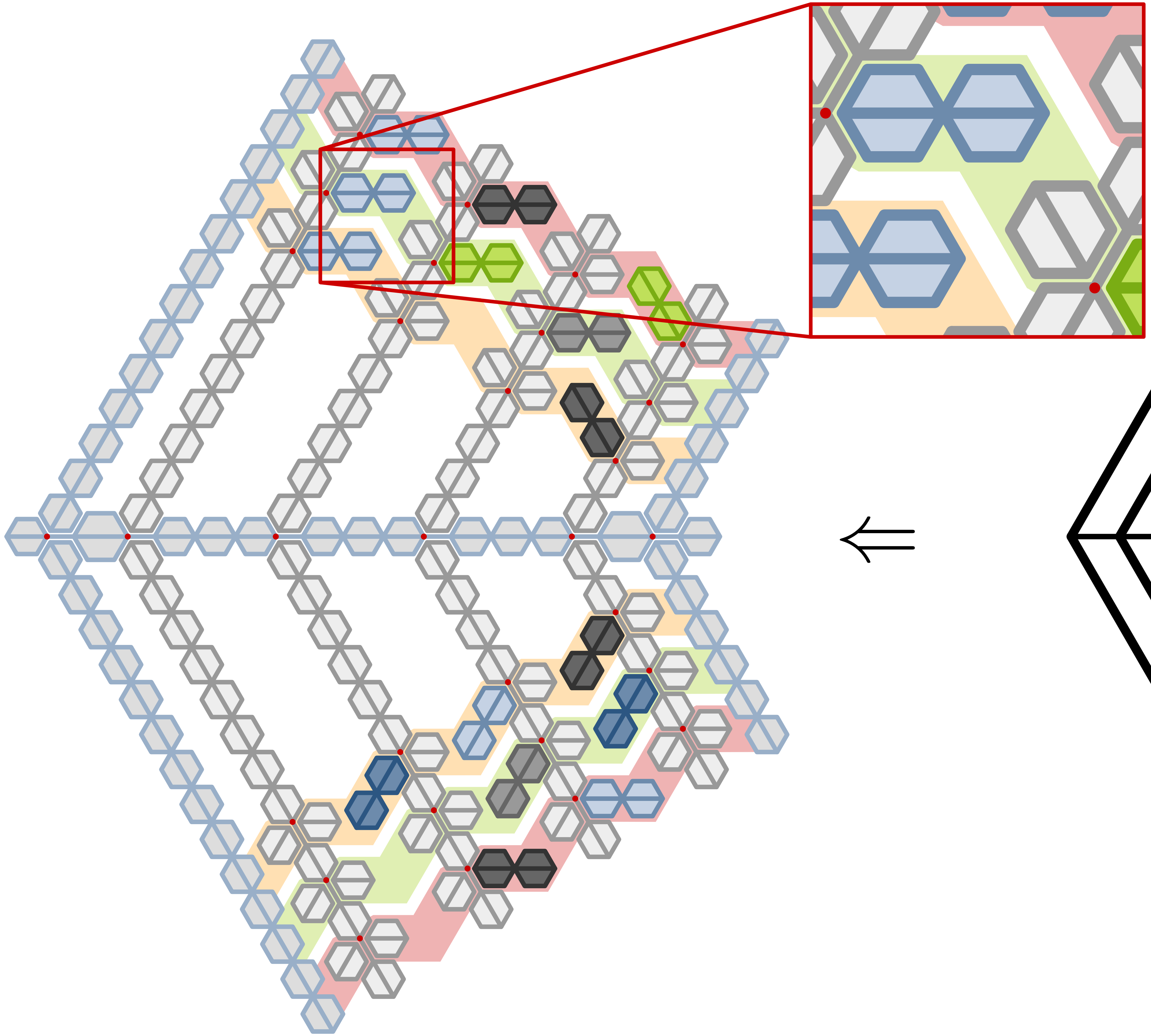

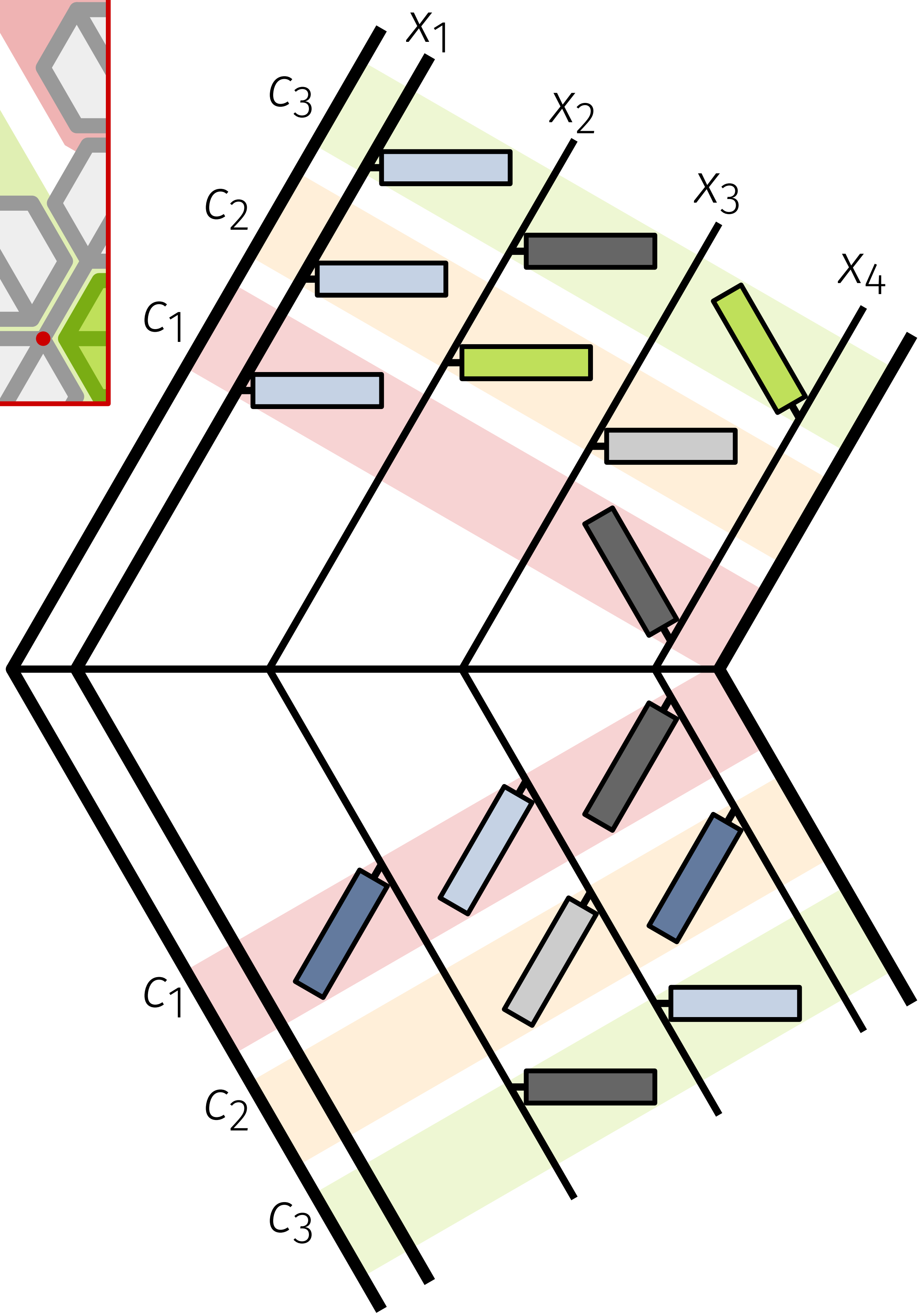

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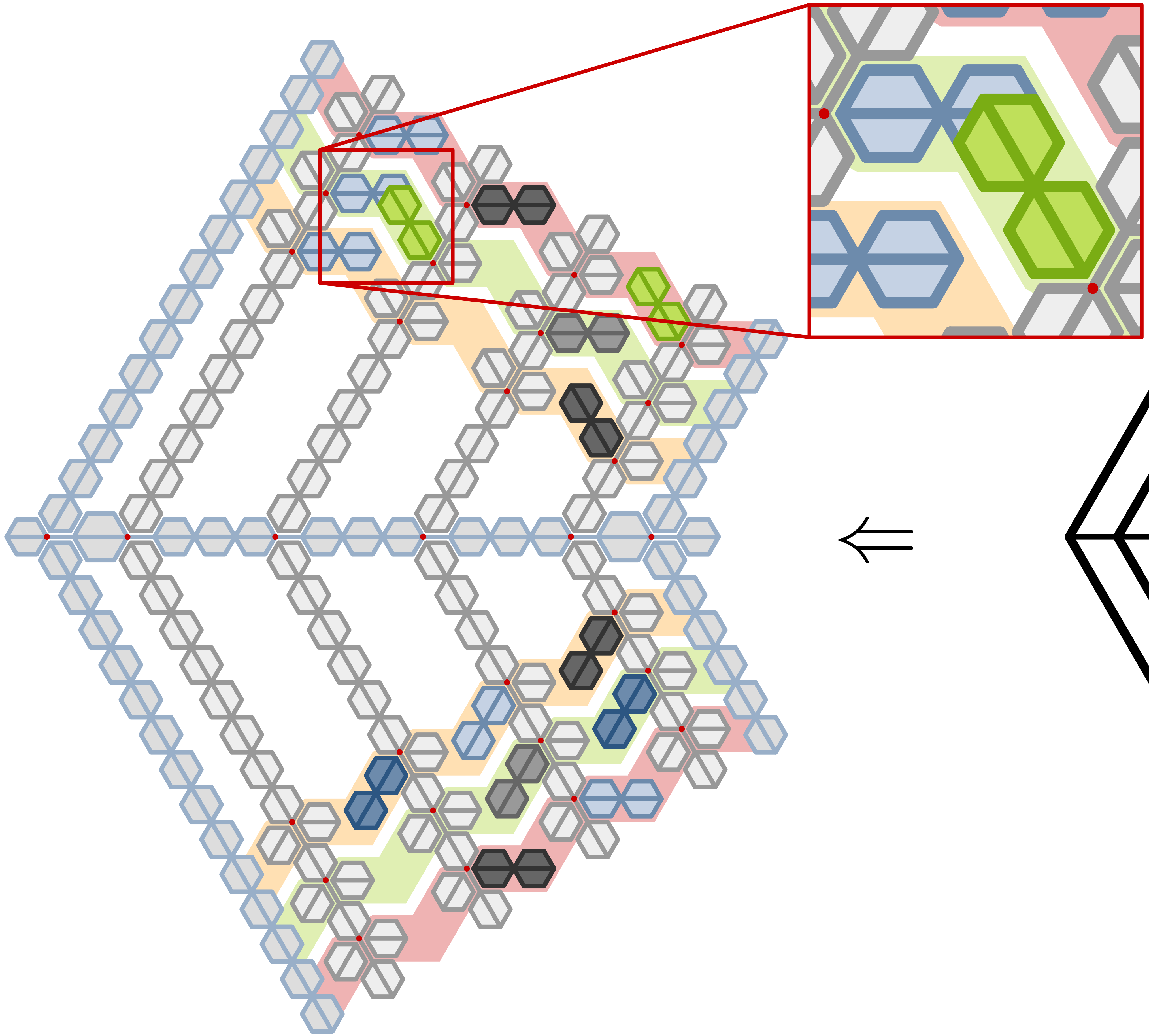

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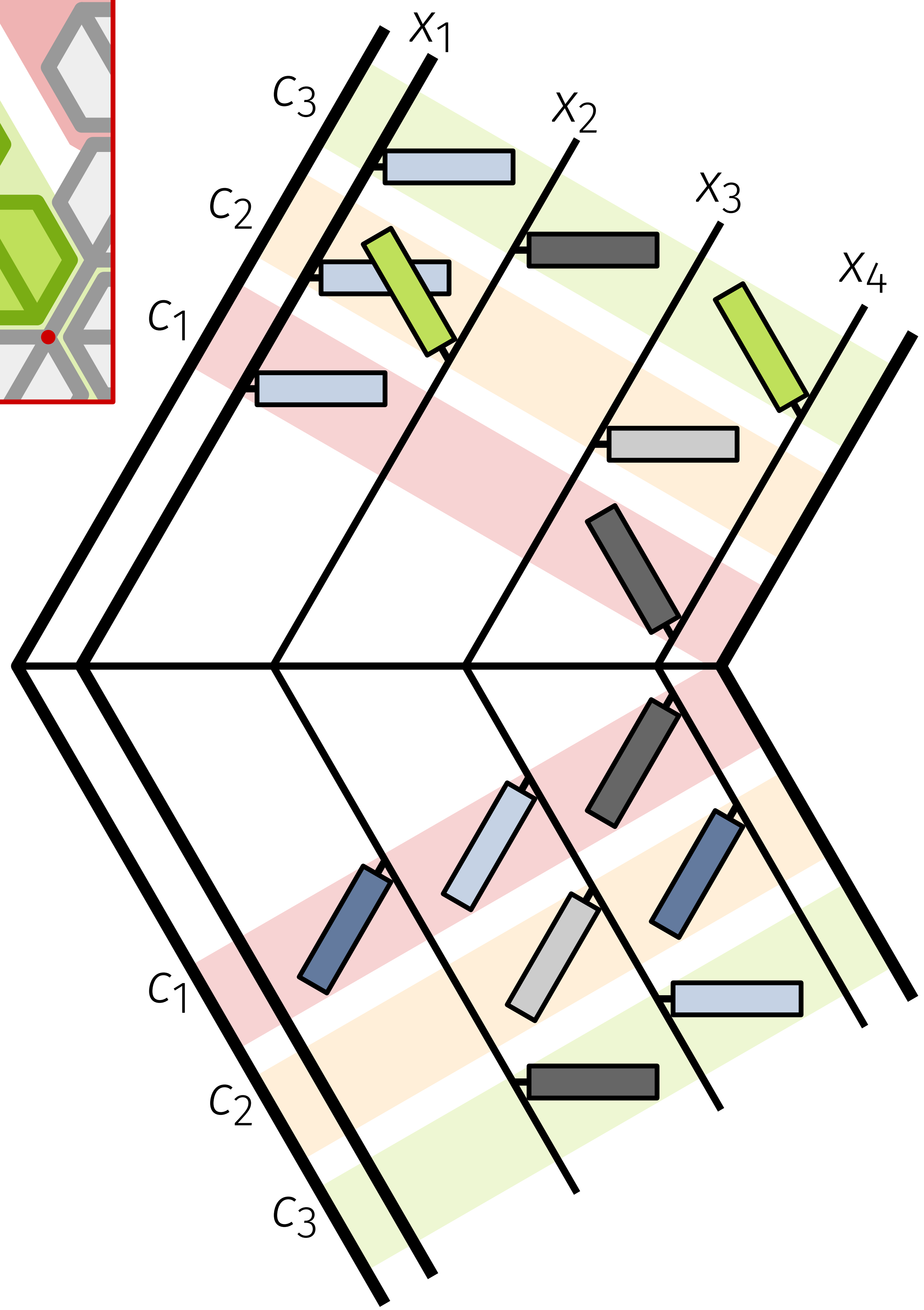
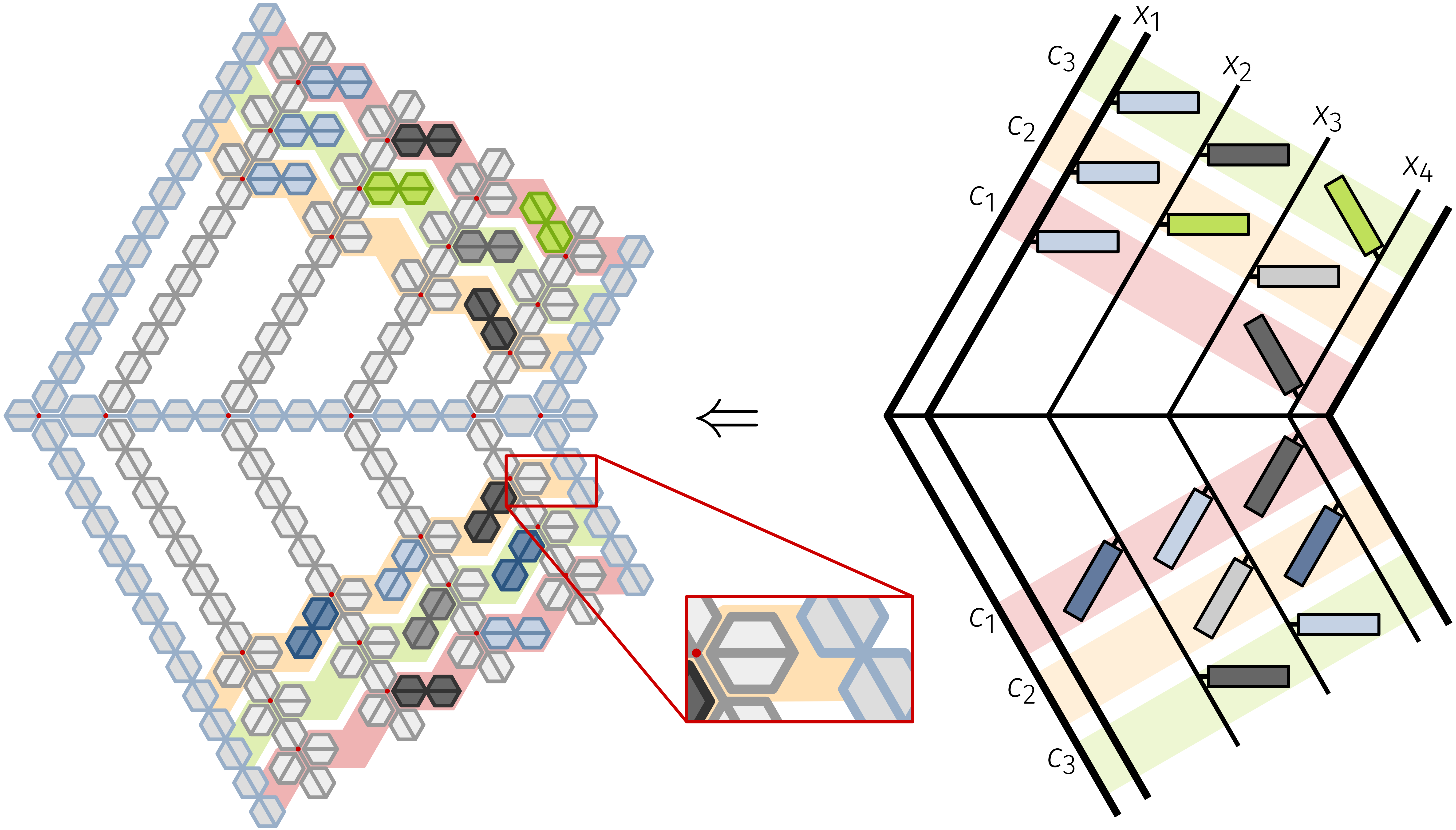
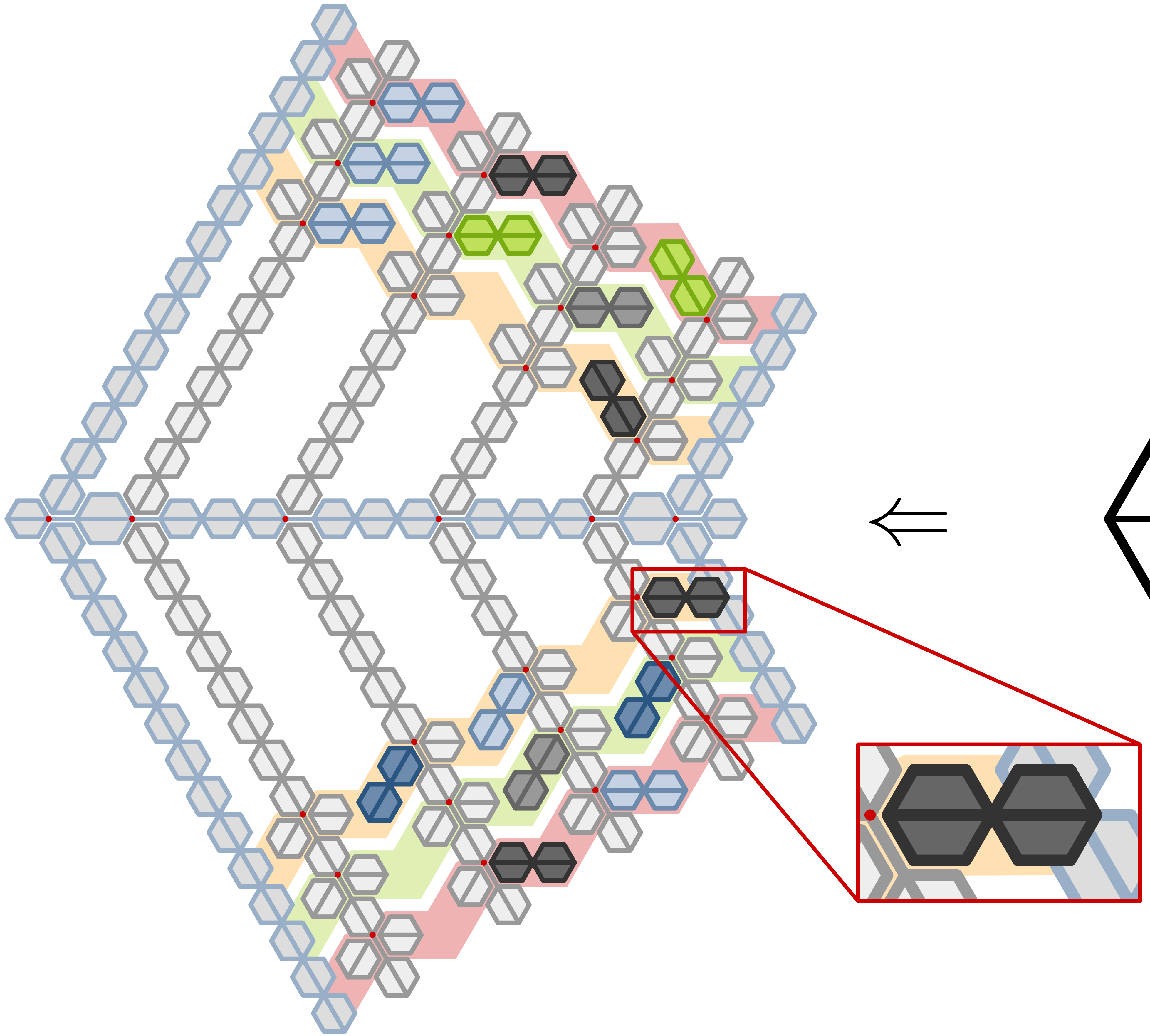


60° Logic Engine



60° Logic Engine



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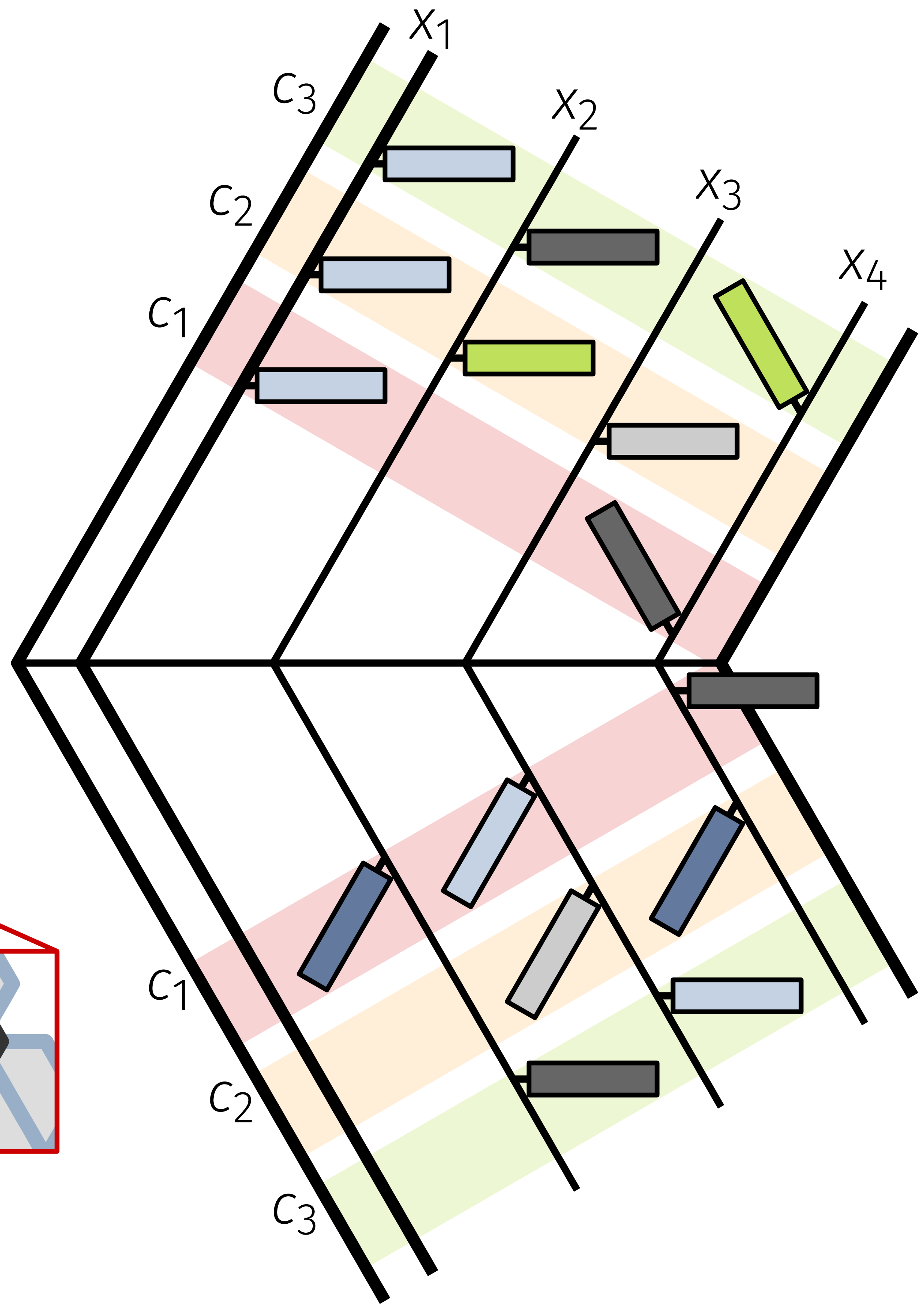

Conclusion

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	star	caterpillar	tree
Weak UDCR		Cleve; 2020	Cleve; 2020
Weak Emb. UDCR		Chiu, Cleve, Nöllenburg; 2019	

Conclusion

	star	caterpillar	tree
Weak UDCR		Cleve; 2020	Cleve; 2020
Weak Emb. UDCR		Chiu, Cleve, Nöllenburg; 2019	

- Recognizing *weak* UDCR for caterpillars takes linear time
- Recognizing *weak* UDCR for trees is NP-hard

Conclusion

	star	caterpillar	tree
Weak UDCR		Cleve; 2020	Cleve; 2020
Weak Emb. UDCR		Chiu, Cleve, Nöllenburg; 2019	

- Recognizing *weak* UDCR for caterpillars takes linear time
- Recognizing *weak* UDCR for trees is NP-hard

NP-completeness?

Conclusion

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	star	caterpillar	lobster	…	tree
Weak UDCR		Cleve; 2020	?		Cleve; 2020
Weak Emb. UDCR		Chiu, Cleve, Nöllenburg; 2019			

- Recognizing *weak* UDCR for caterpillars takes linear time
- Recognizing *weak* UDCR for trees is NP-hard NP-completeness?
- Trees in which all vertices have distance at most k to a central path?
- Lobster graphs (k = 2)?