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Computing the cut distance of two curves

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Visiting Würzburg

Schloss Veitshöchheim

Mainbrücke, Marienburg,

Residenz, Dom...

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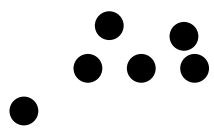

Mainbrücke, Marienburg,

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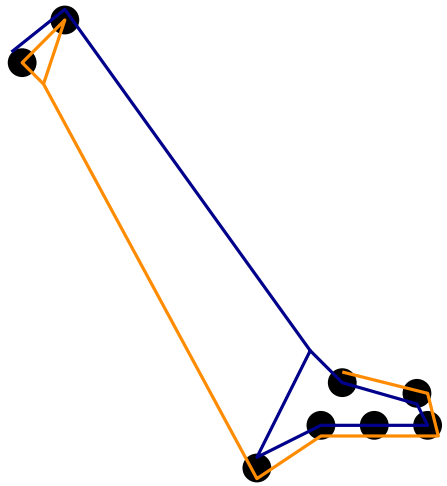
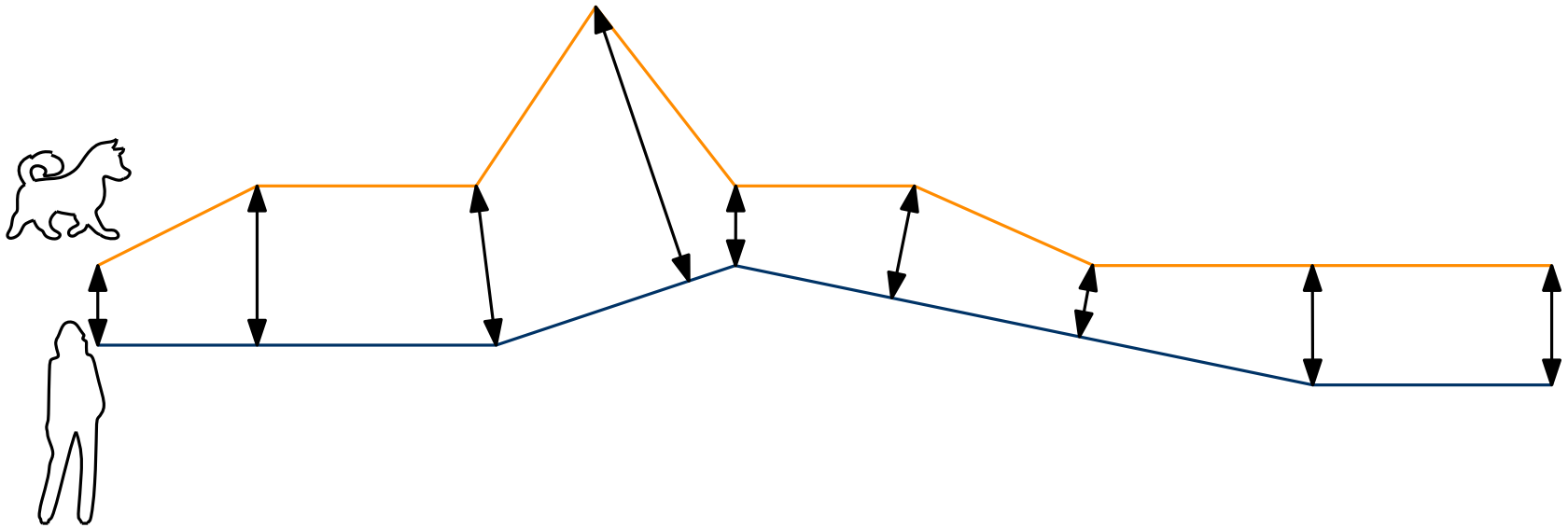


Mainbrücke, Marienburg,

Residenz, Dom...

The Fréchet distance



$$\delta_F(P, Q) = \inf_{\tau,\sigma} \max_{t\in[0,1]} \|P(\tau(t)) - Q(\sigma(t))\|$$

τ, σ range over all orientation-preserving homeomorphisms

The cut distance

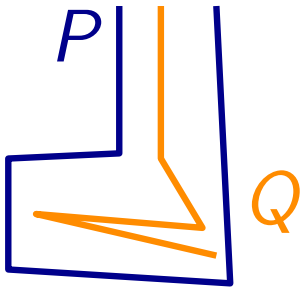
$$\delta_{\text{cut}}(k, P, Q) = \inf_{\tau,\sigma} \max_{t\in[0,1]} \|P(\sigma(t)) - Q(\tau(t))\|$$

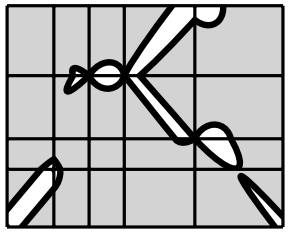
where τ, σ range over all piecewise continuous, surjective functions with $k' < k$ jump discontinuities, each.

The cut distance

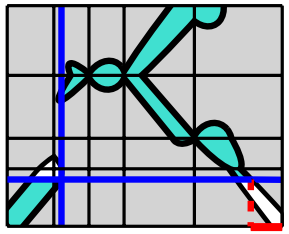
$$\delta_{\text{cut}}(k, P, Q) = \inf_{\tau,\sigma} \max_{t\in[0,1]} \|P(\sigma(t)) - Q(\tau(t))\|$$

where τ, σ range over all piecewise continuous, surjective functions with $k' < k$ jump discontinuities, each.

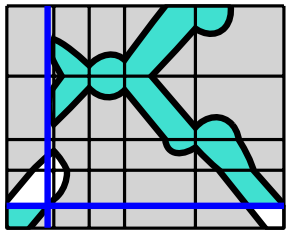




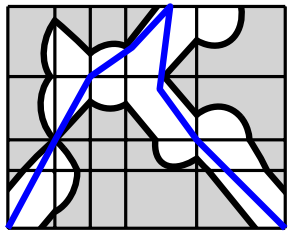
$\varepsilon_1 \geq \delta_{\text{H}}(P, Q)$



$\varepsilon_2 < \delta_{\text{cut}}(2, P, Q)$



$\varepsilon_3 \geq \delta_{\text{cut}}(2, P, Q)$

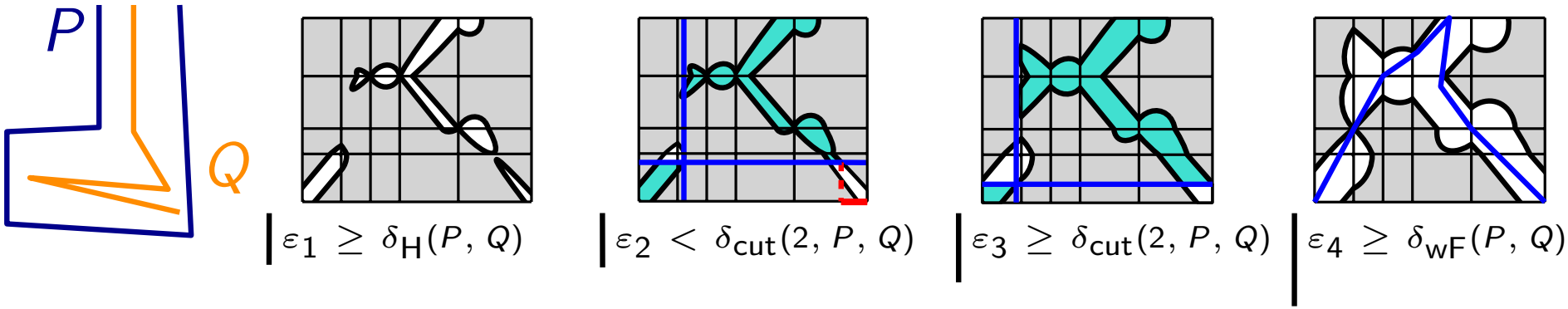


$\varepsilon_4 \geq \delta_{\text{wF}}(P, Q)$

The cut distance

$$\delta_{\text{cut}}(k, P, Q) = \inf_{\tau,\sigma} \max_{t\in[0,1]} \|P(\sigma(t)) - Q(\tau(t))\|$$

where τ, σ range over all piecewise continuous, surjective functions with $k' < k$ jump discontinuities, each.

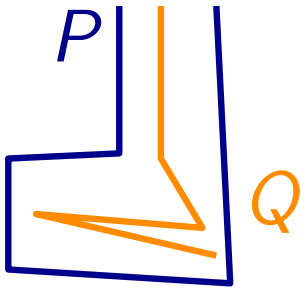


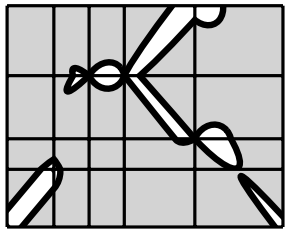
$$F_\varepsilon(P, Q) = \{(t_1, t_2) : \|P(t_1) - Q(t_2)\| \leq \varepsilon\}$$

The cut distance

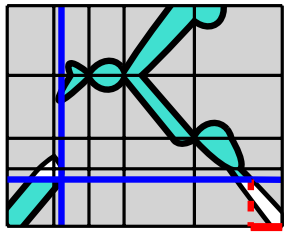
$$\delta_{\mathrm{cut}}(k, P, Q) = \inf_{\tau,\sigma} \max_{t\in[0,1]} \|P(\sigma(t)) - Q(\tau(t))\|$$

where τ, σ range over all piecewise continuous, surjective functions with $k' < k$ jump discontinuities, each.

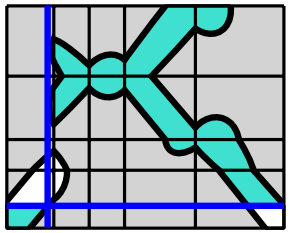




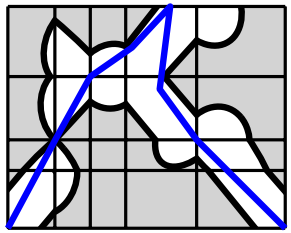
$\varepsilon_1 \geq \delta_{\mathrm{H}}(P, Q)$



$\varepsilon_2 < \delta_{\mathrm{cut}}(2, P, Q)$



$\varepsilon_3 \geq \delta_{\mathrm{cut}}(2, P, Q)$



$\varepsilon_4 \geq \delta_{\mathrm{wF}}(P, Q)$

Observe that $\delta_{\mathrm{H}} \leq \delta_{\mathrm{cut}} \leq \delta_{\mathrm{wF}} \leq \delta_{\mathrm{F}}$.

Placing cut lines

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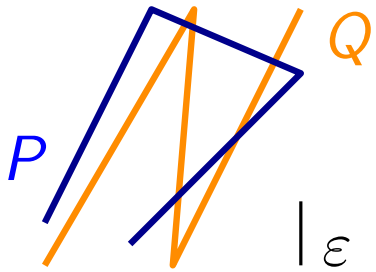

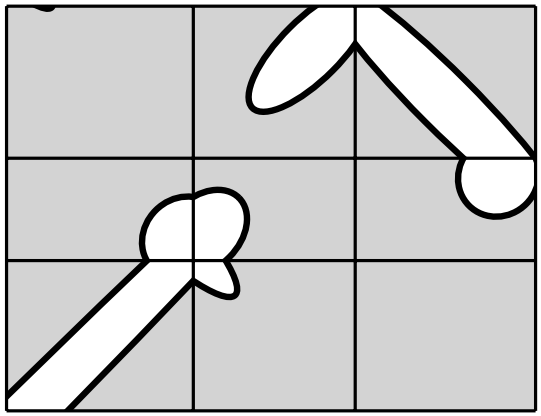

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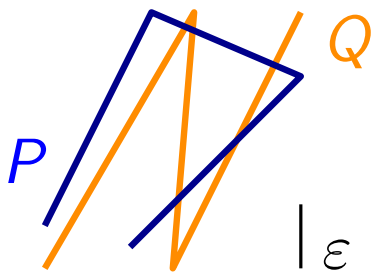
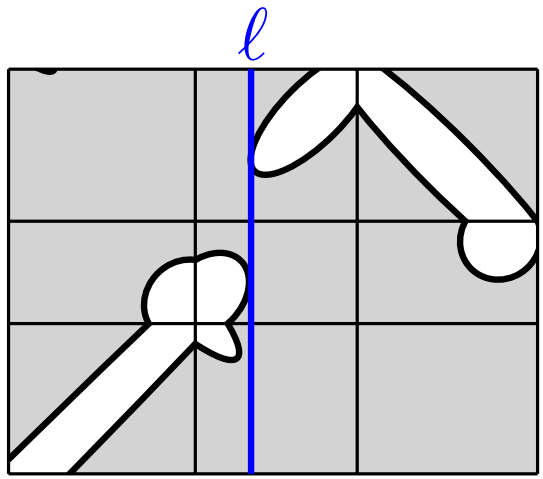
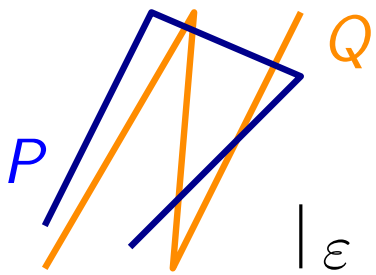
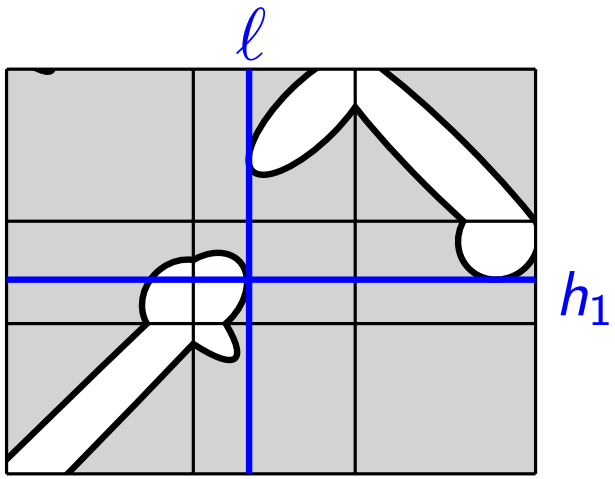




Placing cut lines





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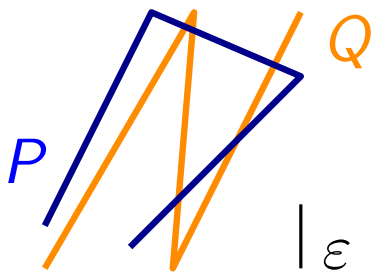
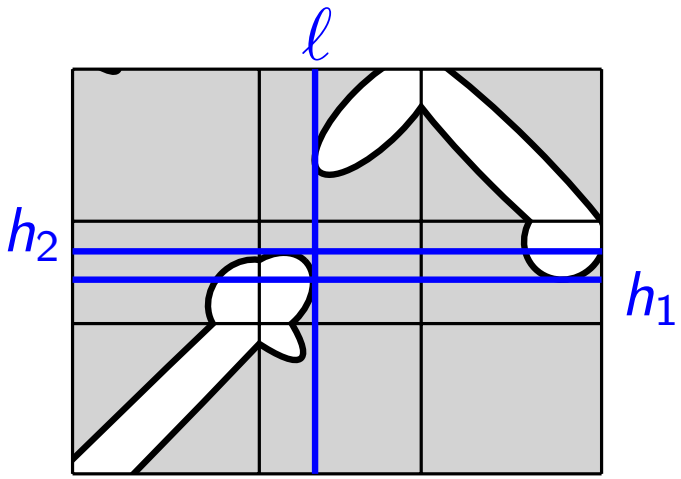
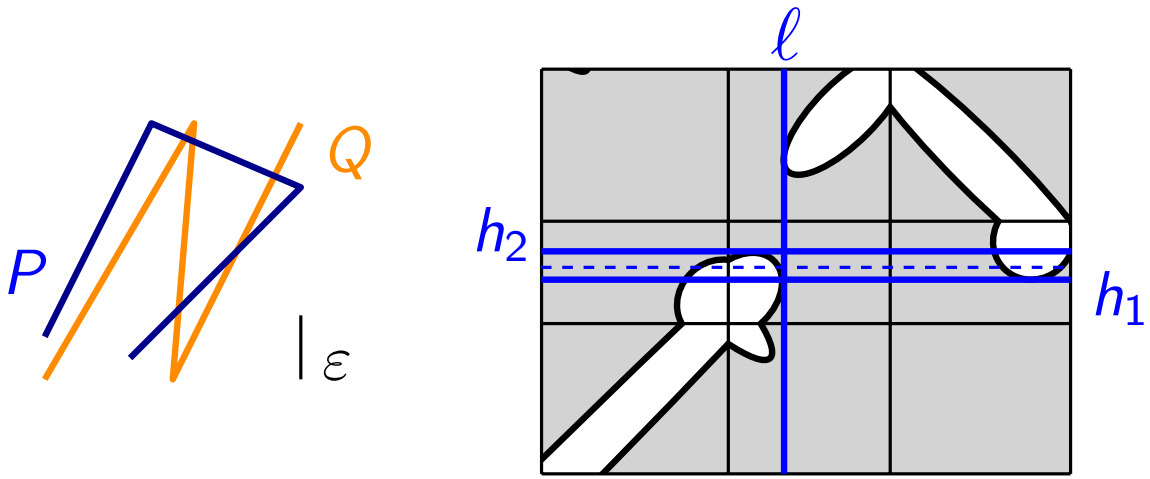


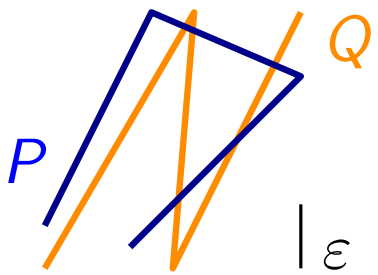


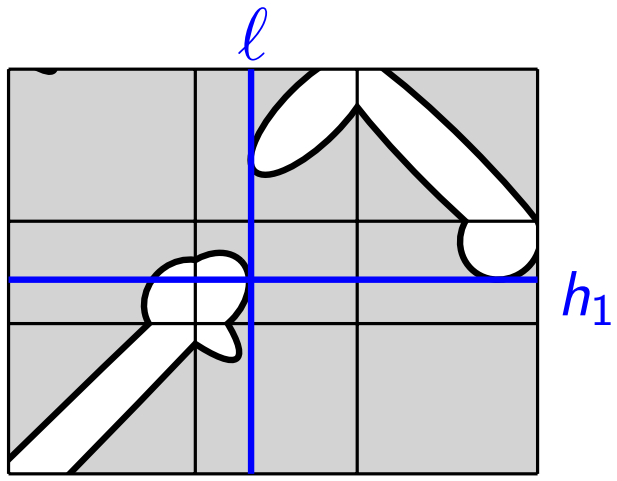
Placing cut lines



Problem: In general, there are infinitely many possible placements for cut lines.

Placing cut lines





For $k = 2$, it is possible to move valid cut lines such that one of them coincides with an extremal point of a component's boundary.

Algorithm for $k = 2$

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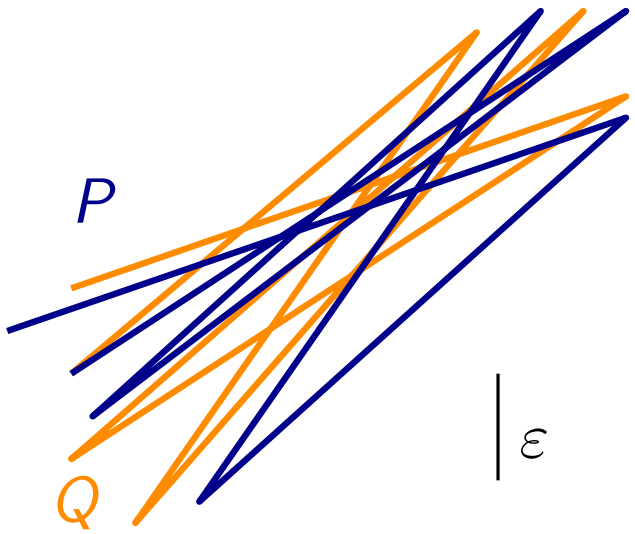
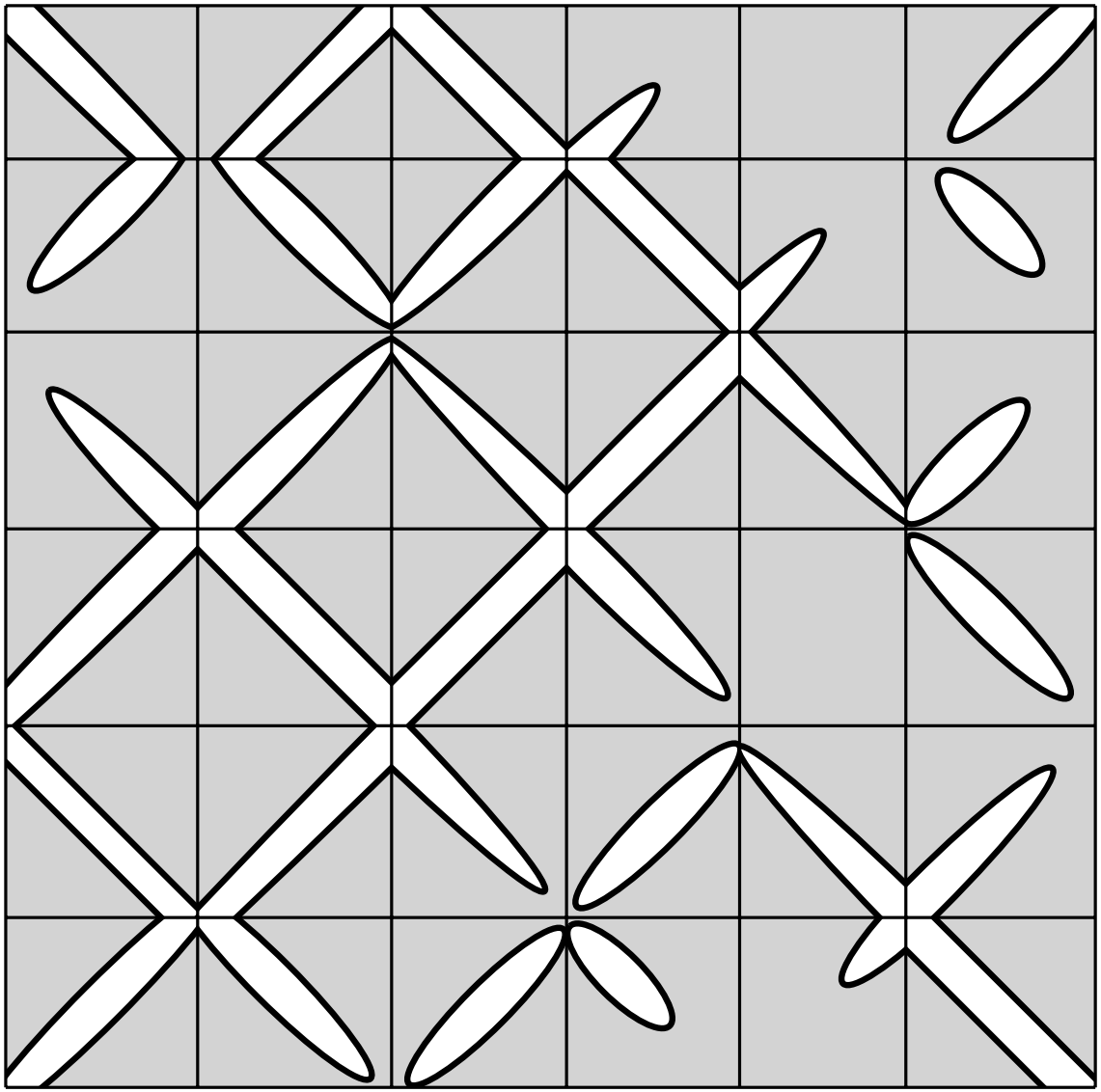
Input: polygonal curves P, Q and $\varepsilon > 0$

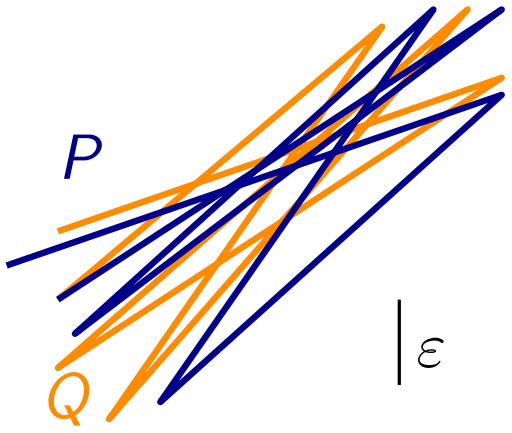


Algorithm for $k = 2$

RUB



1. Compute FSD



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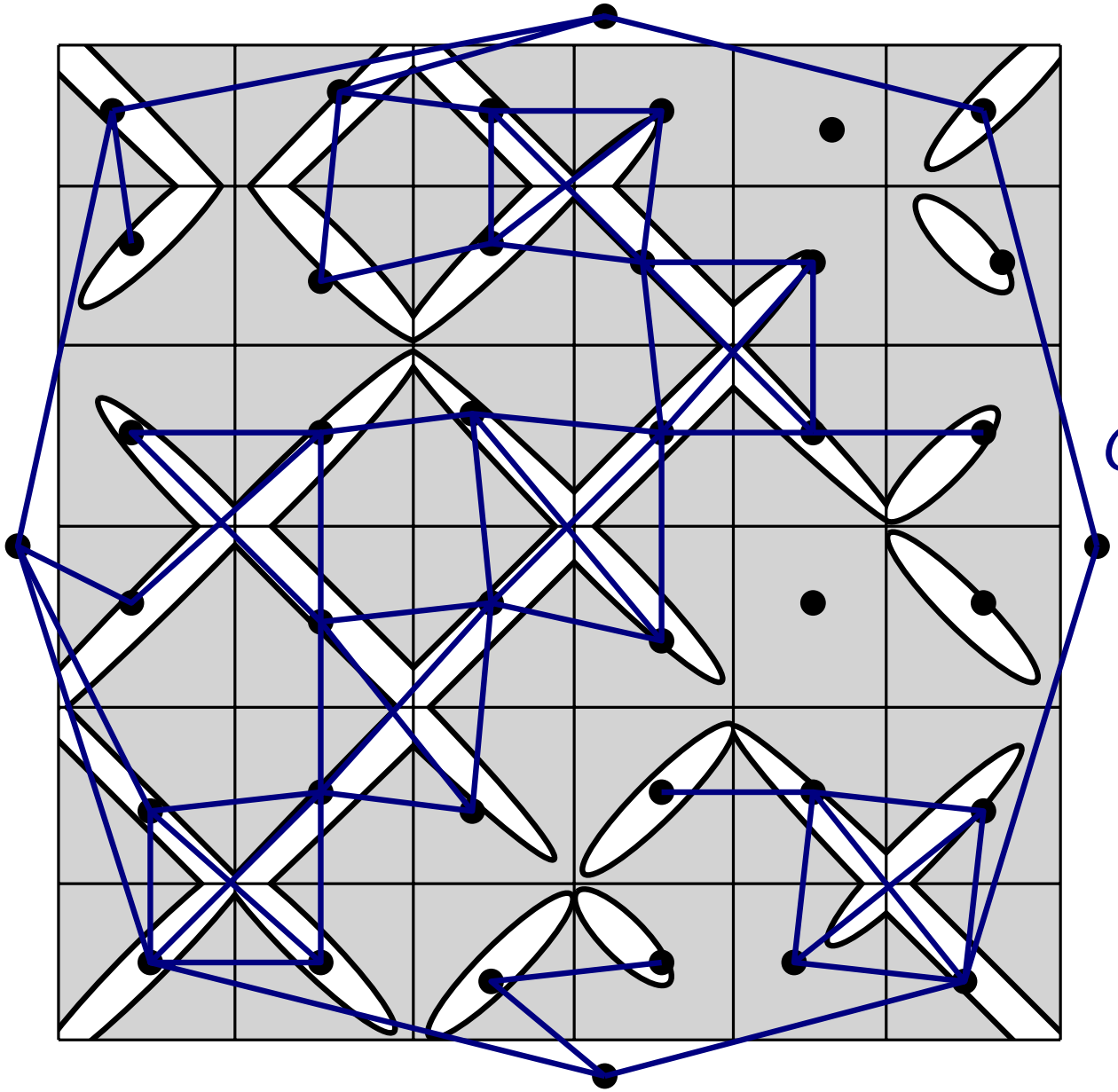

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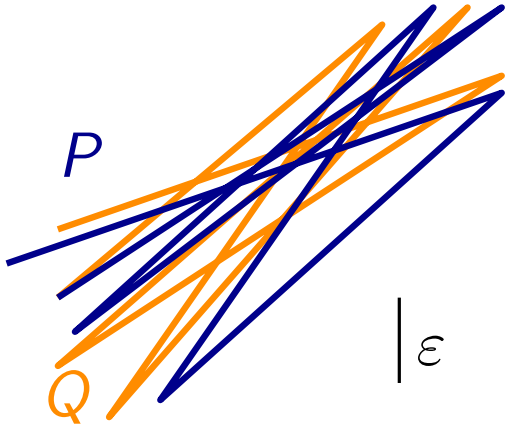
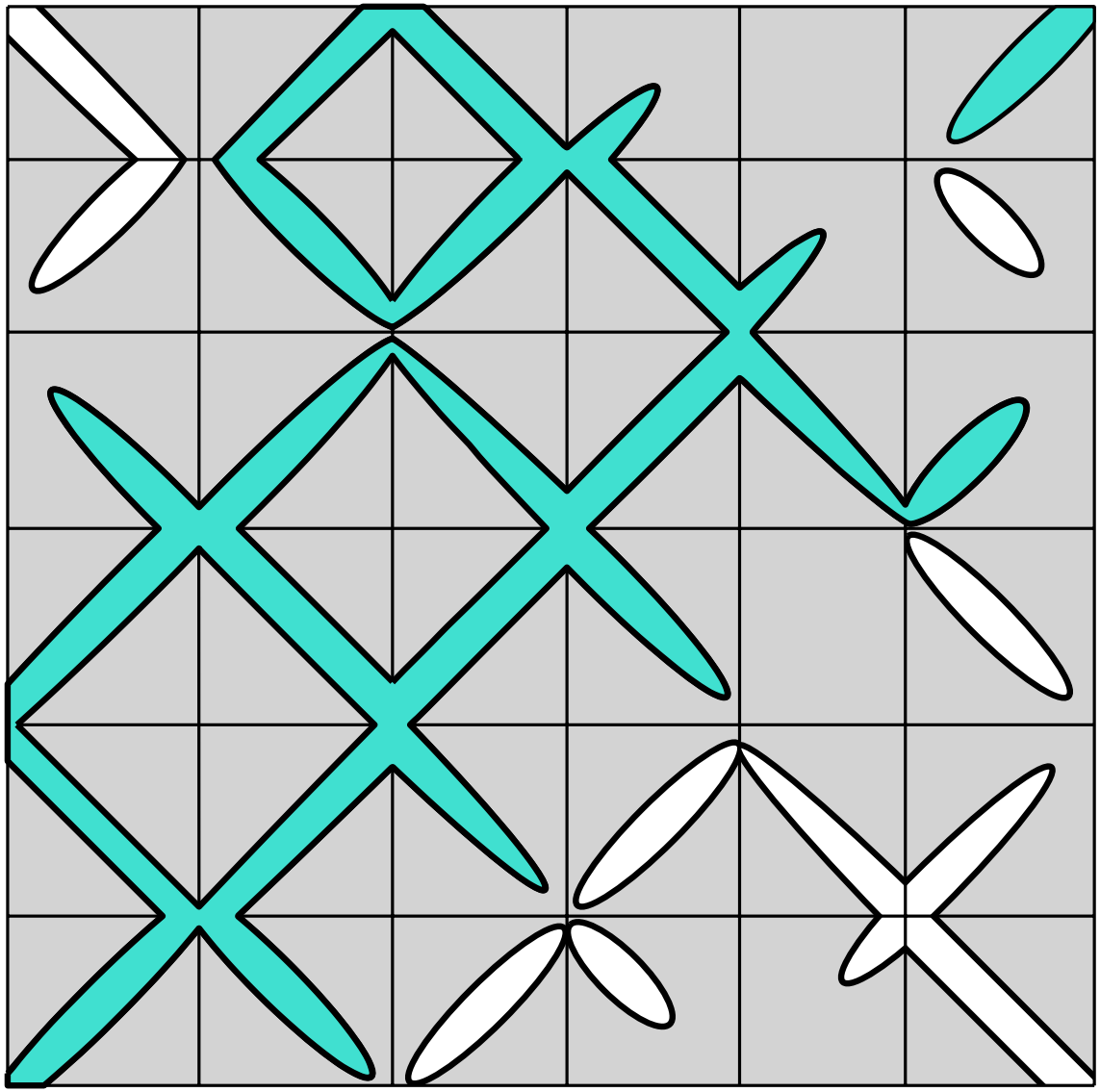
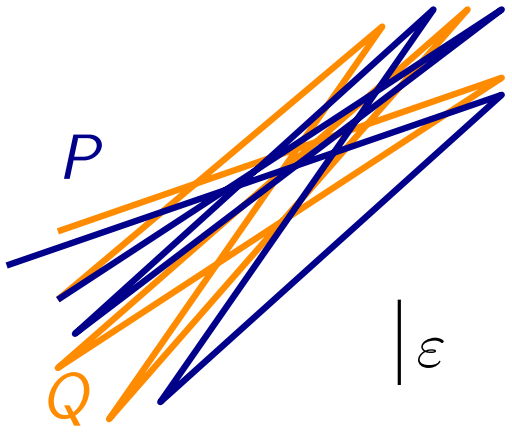


Algorithm for $k = 2$

RUB



1. Compute FSD
2. Identify *candidates* a_i, b_j



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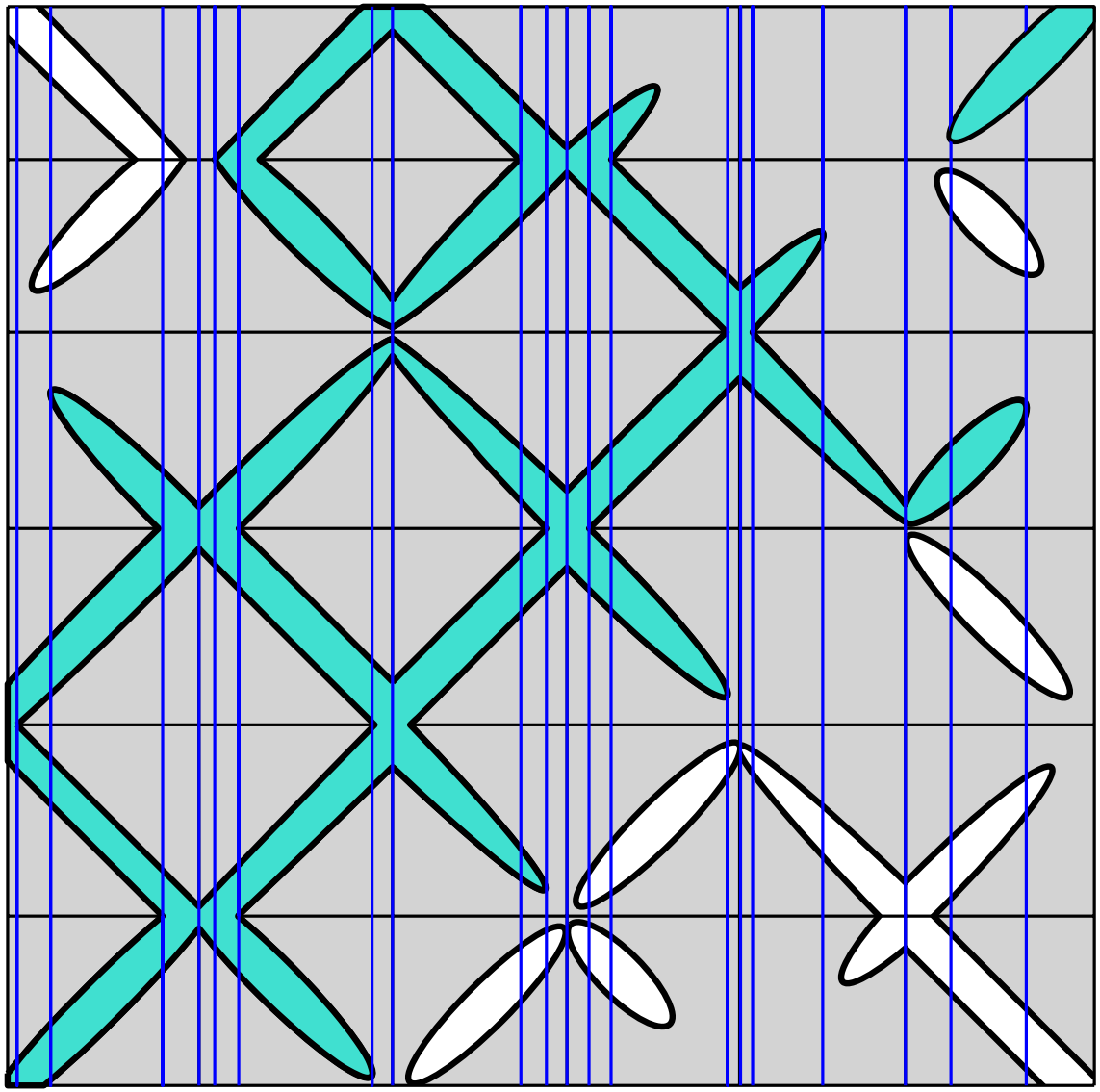
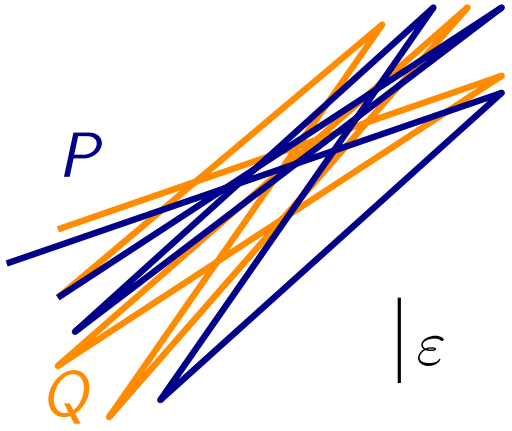


1. Compute FSD
2. Identify *candidates* a_i, b_j
3. Compute cut lines ℓ at extremal points



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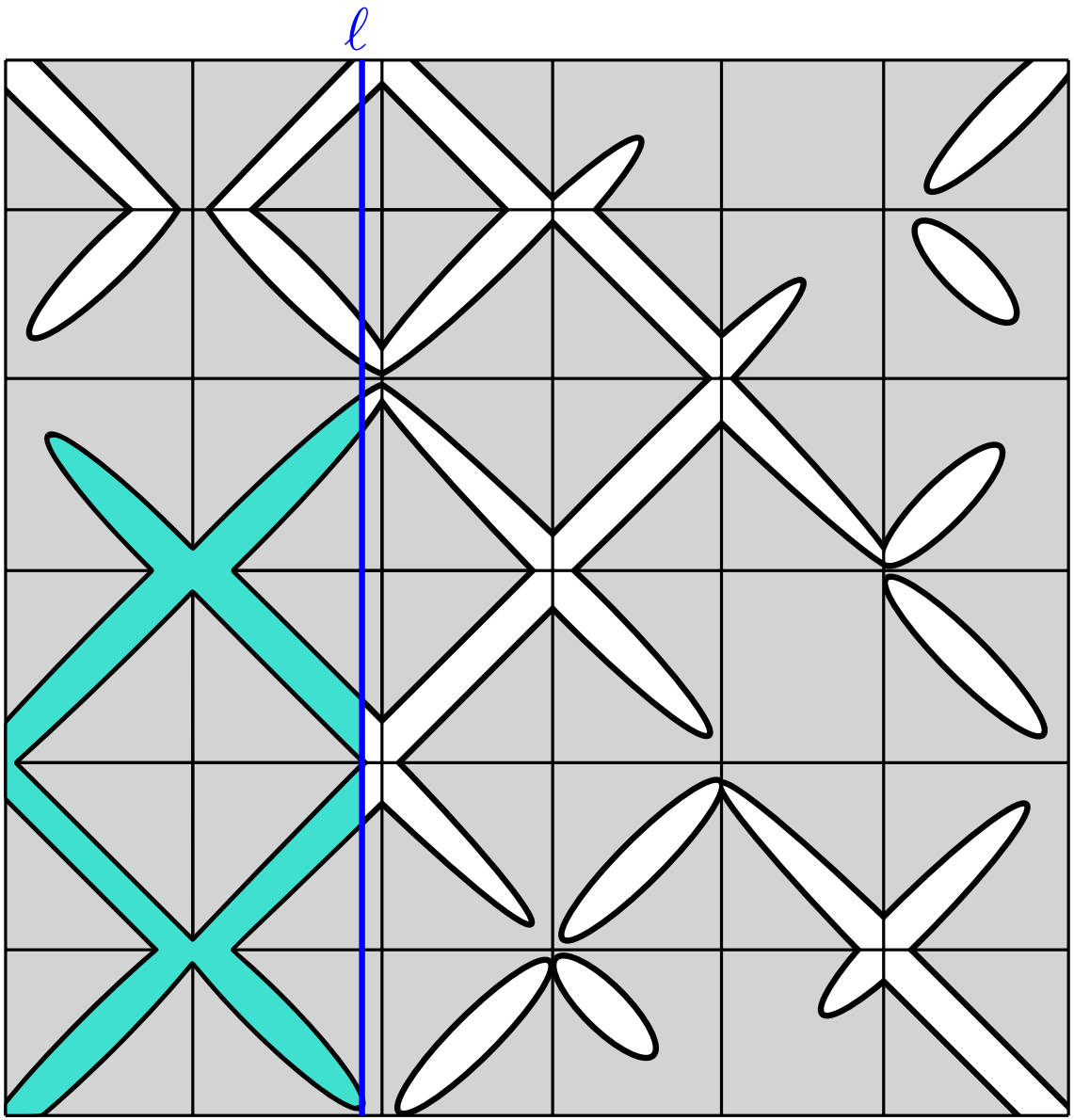

1. Compute FSD
2. Identify *candidates* a_i, b_j
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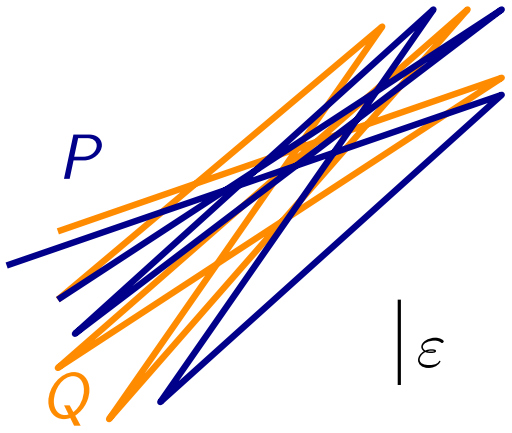
4. Identify a_i^ℓ, b_j^ℓ



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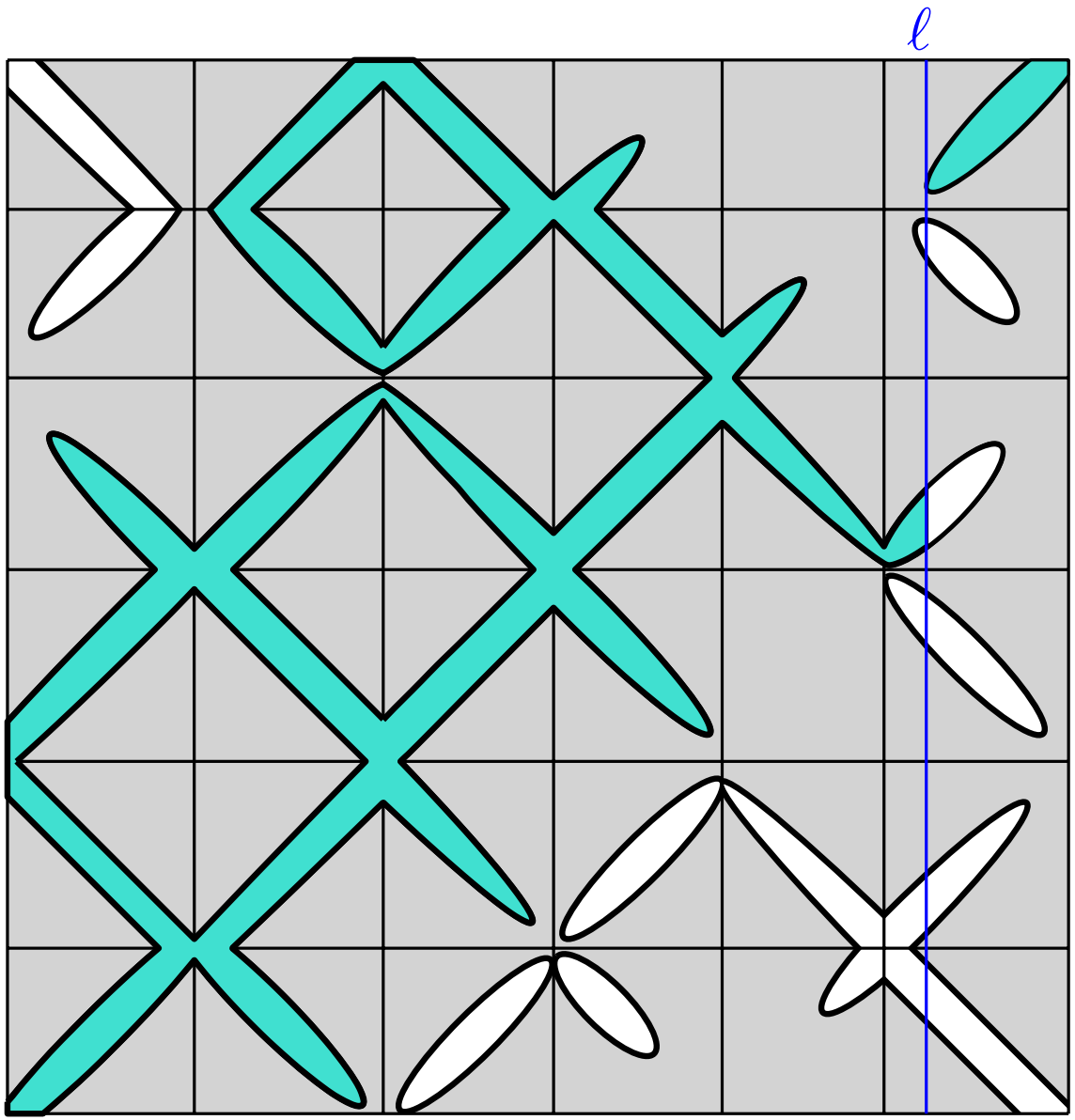

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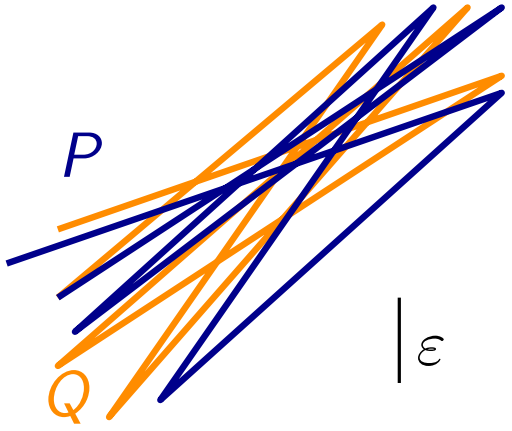

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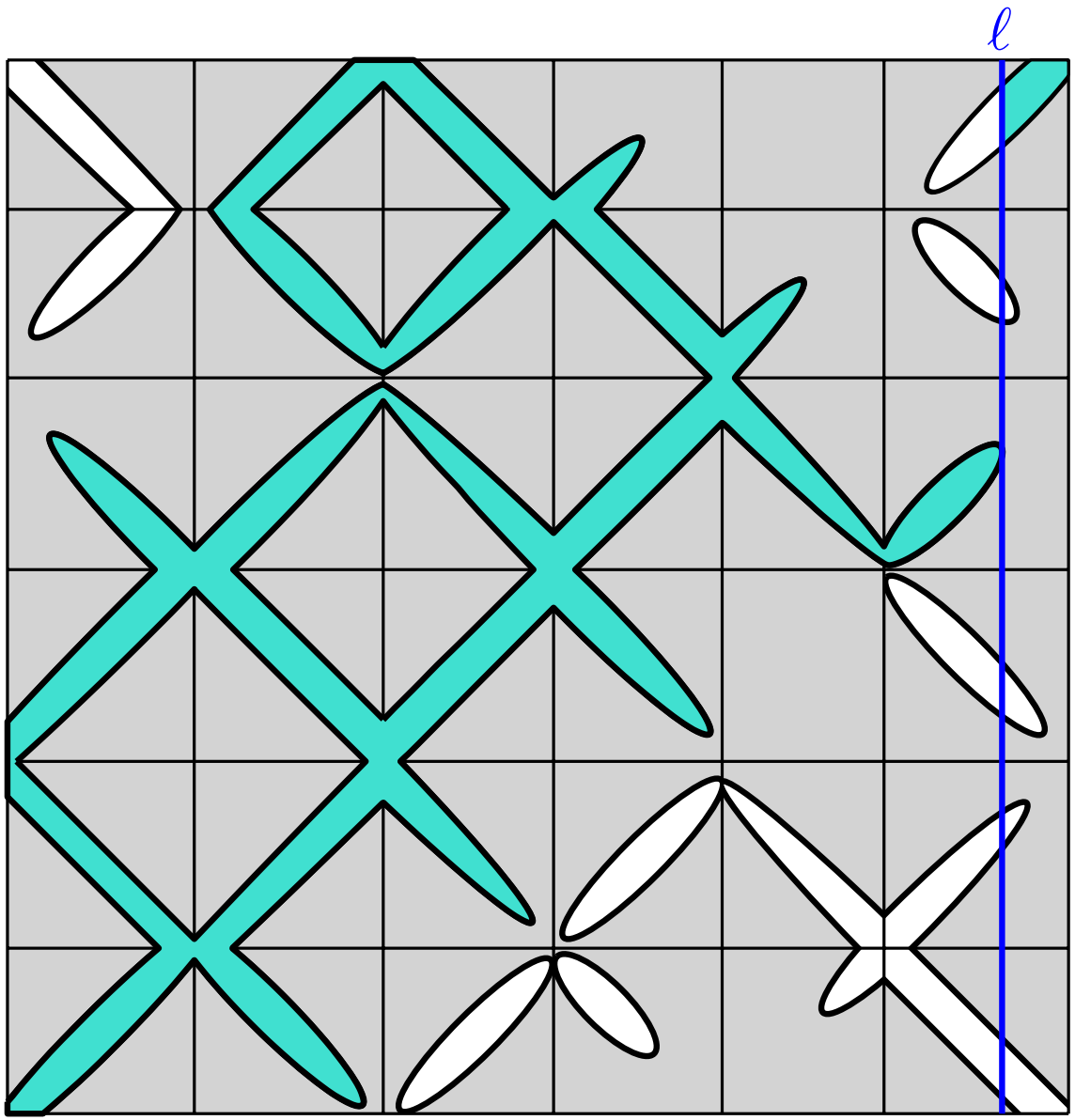

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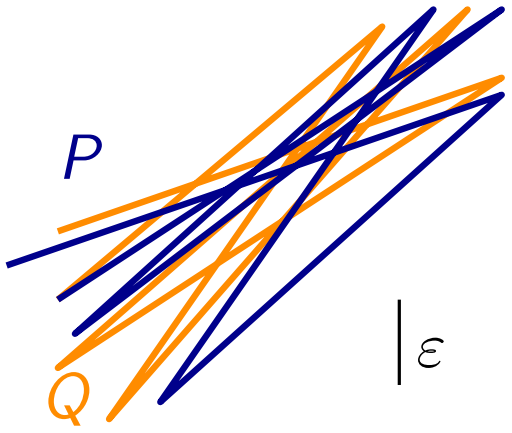

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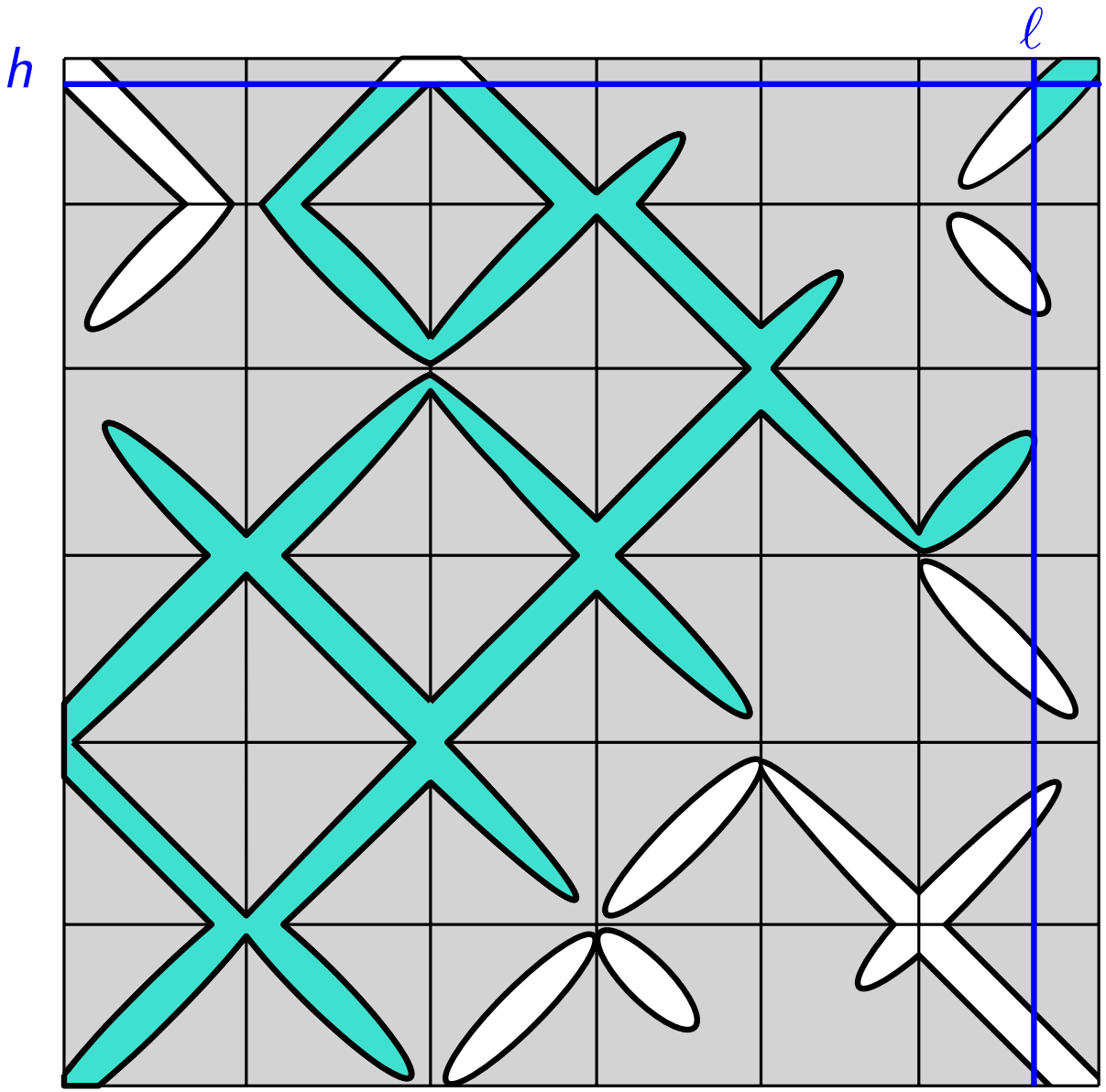
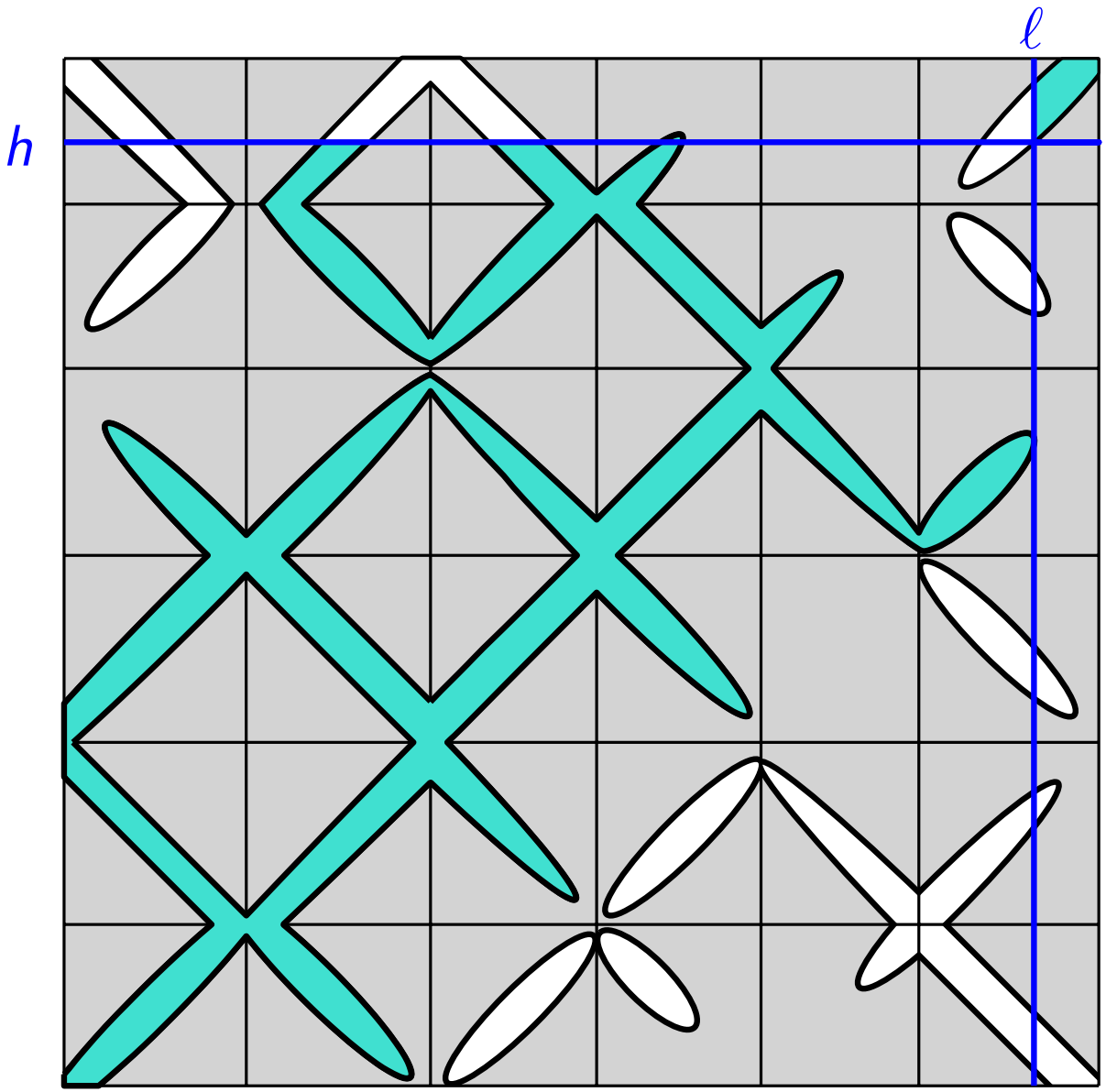


1. Compute FSD
2. Identify *candidates* a_i, b_j
3. Compute cut lines ℓ at extremal points
4. Identify a_i^ℓ, b_j^ℓ
5. Determine horizontal cut h

Algorithm for $k = 2$



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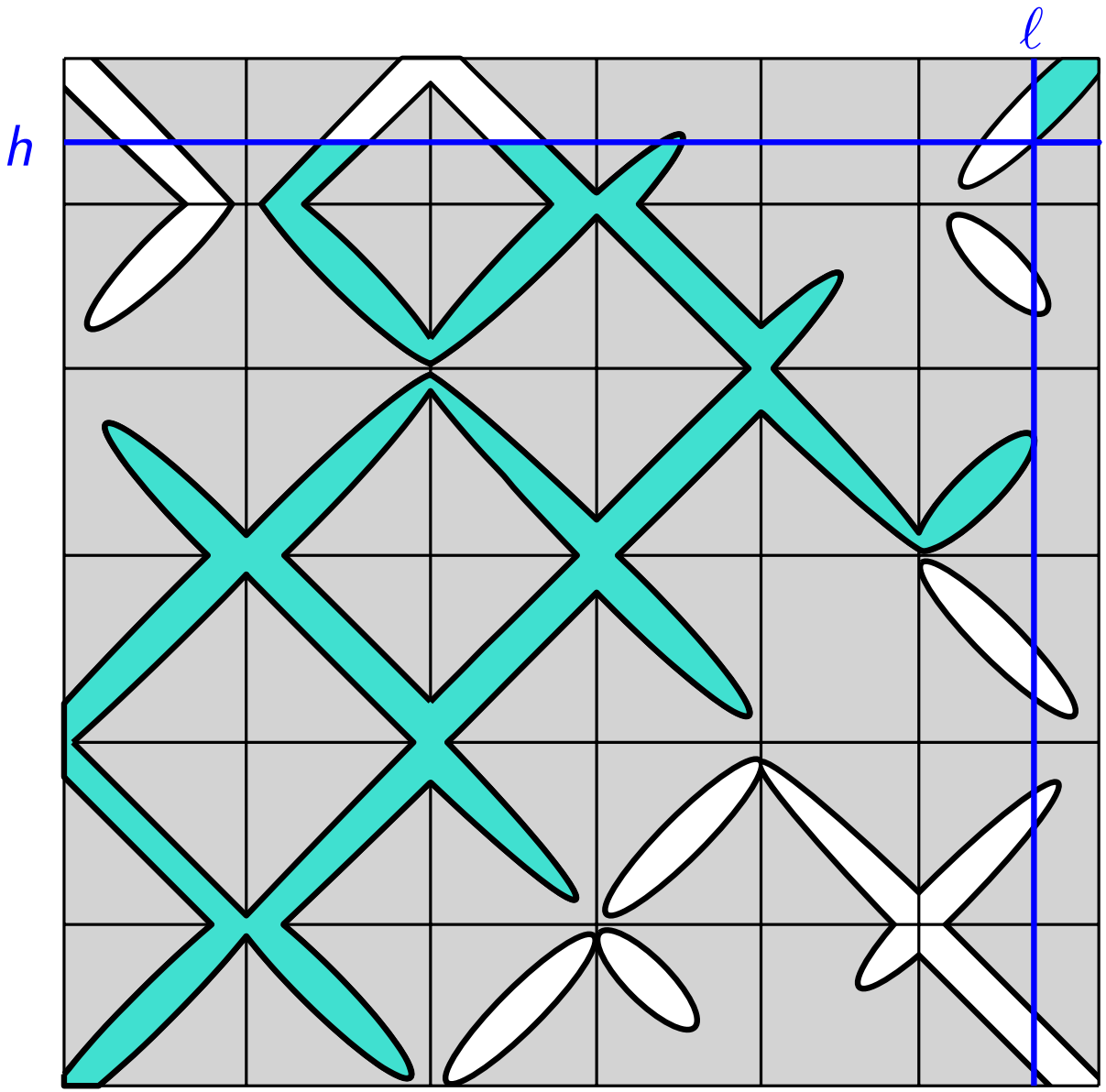

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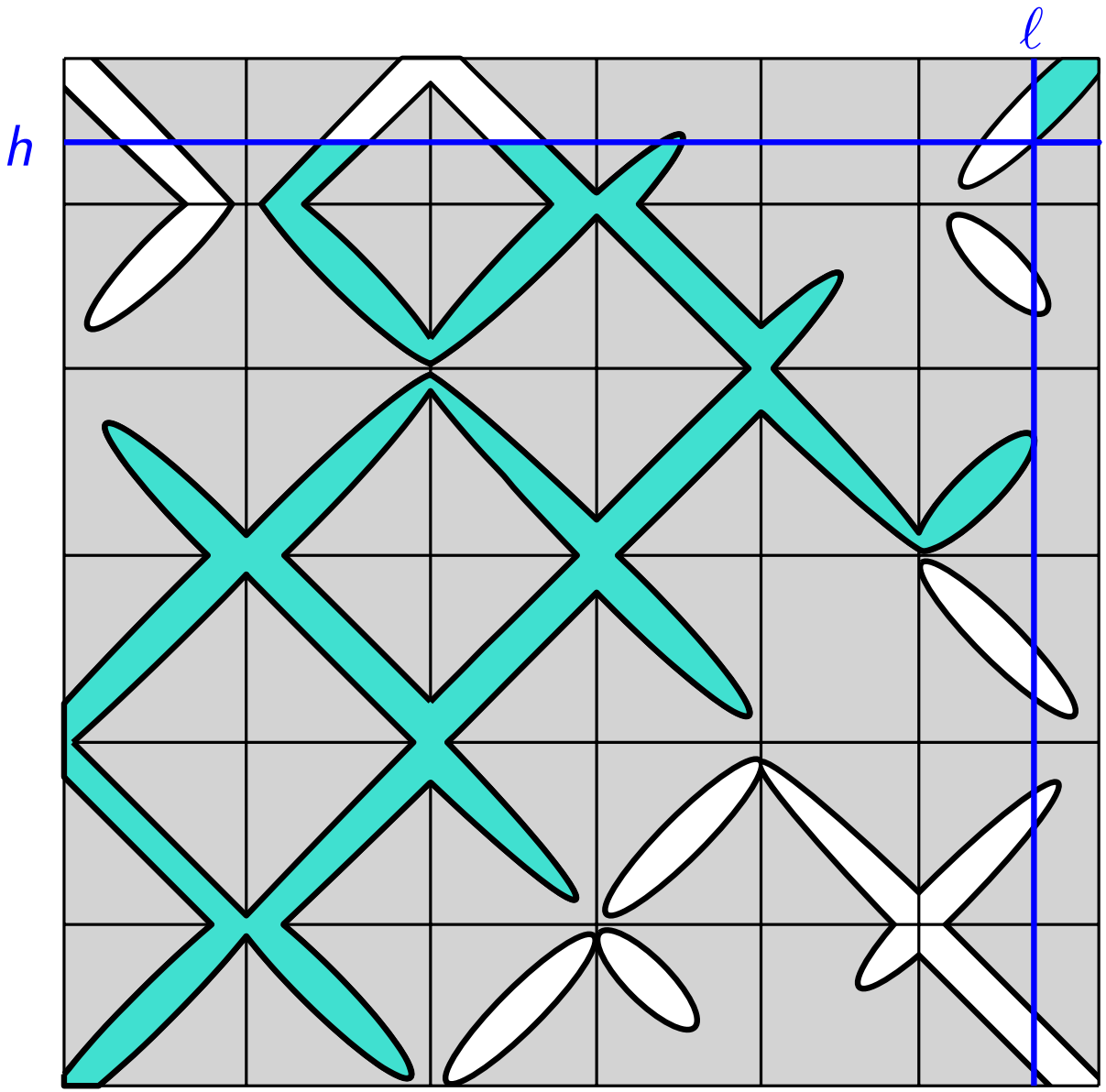
4. Identify a_i^ℓ, b_j^ℓ
5. Determine horizontal cut h

(6. Repeat for h at extremal points)

Algorithm for $k = 2$



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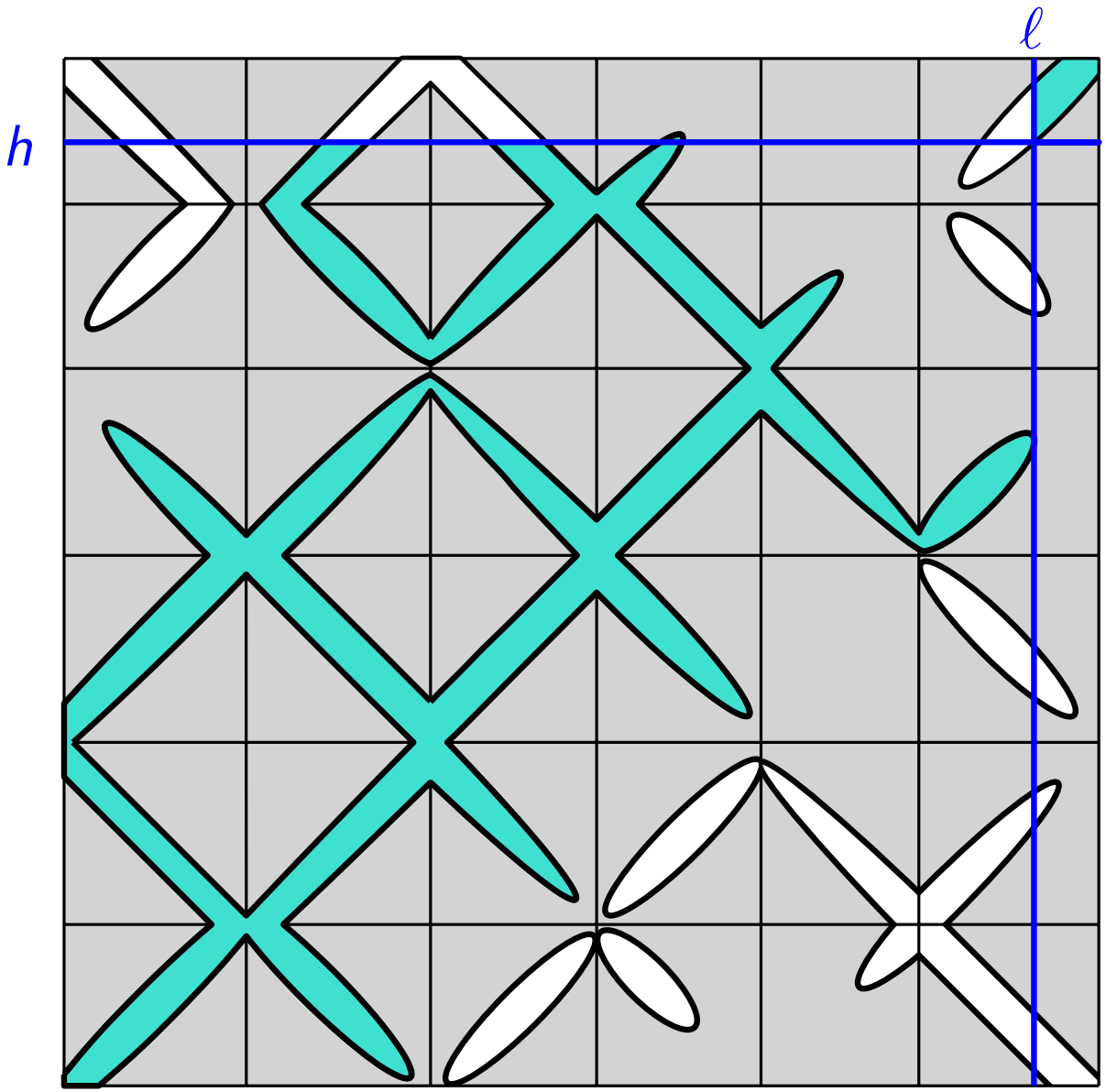
(7. Repeat for opposite quadrants if necessary)

Algorithm for $k = 2$



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(6. Repeat for h at extremal points)

(7. Repeat for opposite quadrants if necessary)

8. Return cut positions

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Overview on the cut distance

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Overview on the cut distance

- Deciding the cut distance is NP-hard in general

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- Optimizing k is APX-hard

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- $\mathcal{O}(n^2 \log n)$ time algorithm for $k = 2$

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Overview on the cut distance

- Deciding the cut distance is NP-hard in general
- Optimizing k is APX-hard
- $\mathcal{O}(n^2 \log n)$ time algorithm for $k = 2$
- Conjecture: already NP-hard for $k = 3$

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Overview on the cut distance

- Deciding the cut distance is NP-hard in general
- Optimizing k is APX-hard
- $\mathcal{O}(n^2 \log n)$ time algorithm for $k = 2$
- Conjecture: already NP-hard for $k = 3$

Thank you!