Technische Universität Braunschweig

Probing a Set of Trajectories to Maximize Captured Movement

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Motivation


Emeryville
Piedmont
Oakland
San Francisco
Alameda
Daly City
Broadmoor
Brisbane
South San Francisco
San Bruno
Pacifica
Millbrae
Burlingame

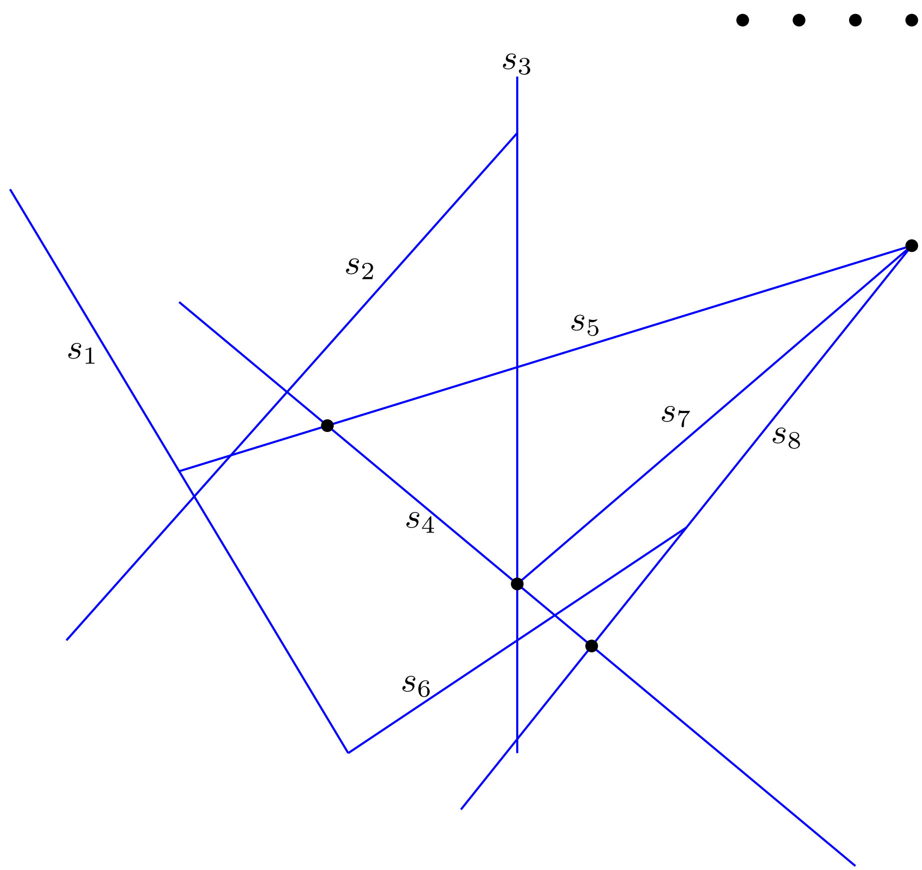
Trajectory Capturing Problem

Given:

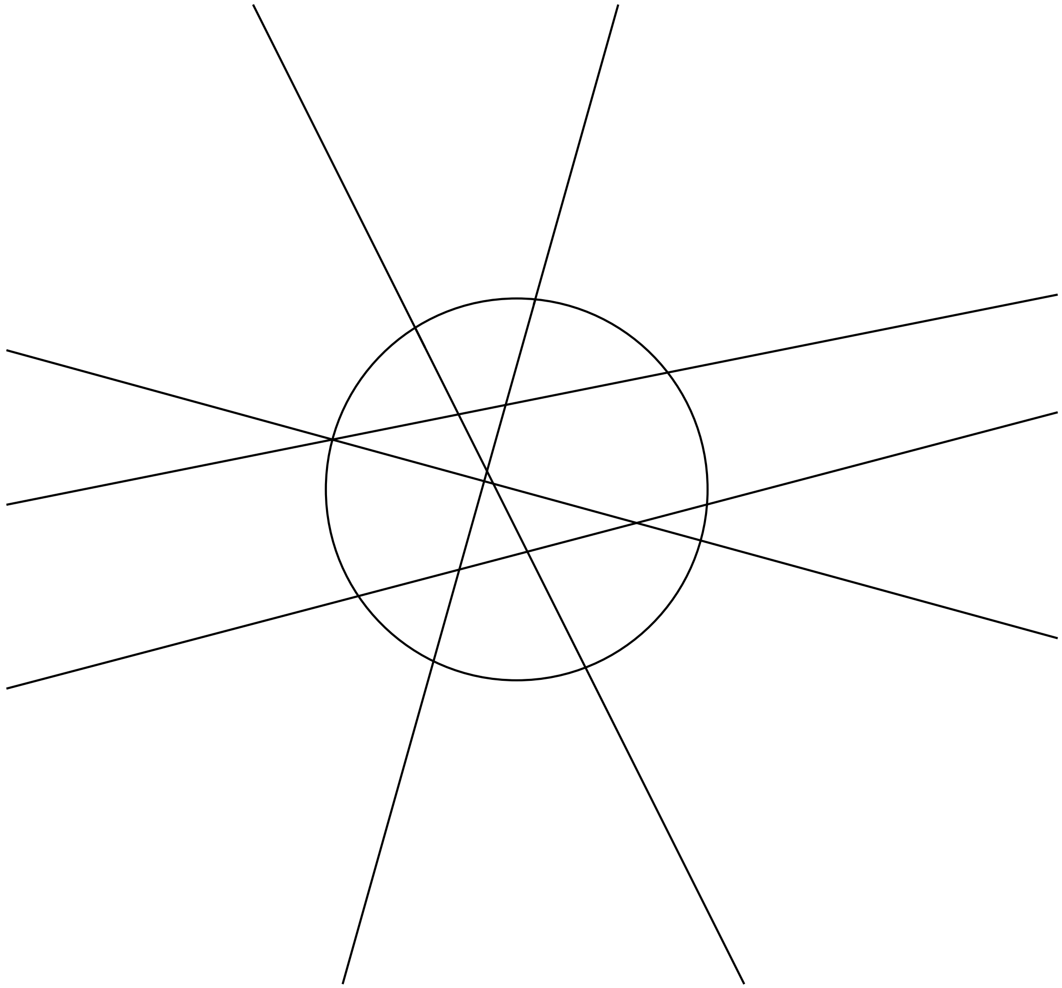
- Set $\mathcal{T}$ of trajectories
- $k \geq 2$ portals/pins

Wanted:

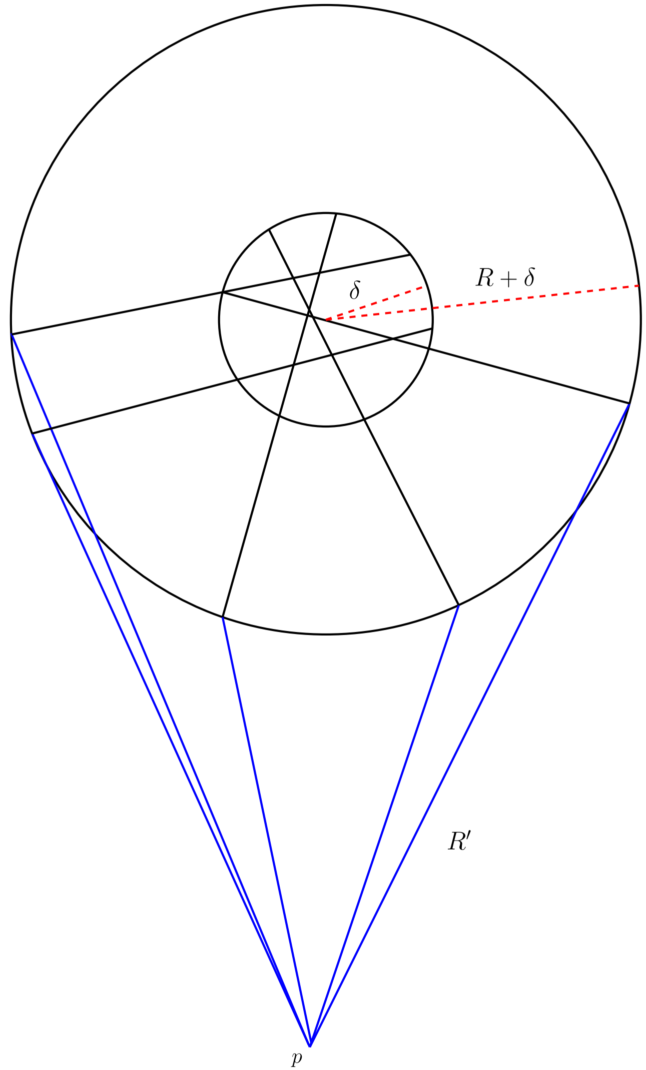
- Place k portals
- maximize the total length
- Measured between two portals



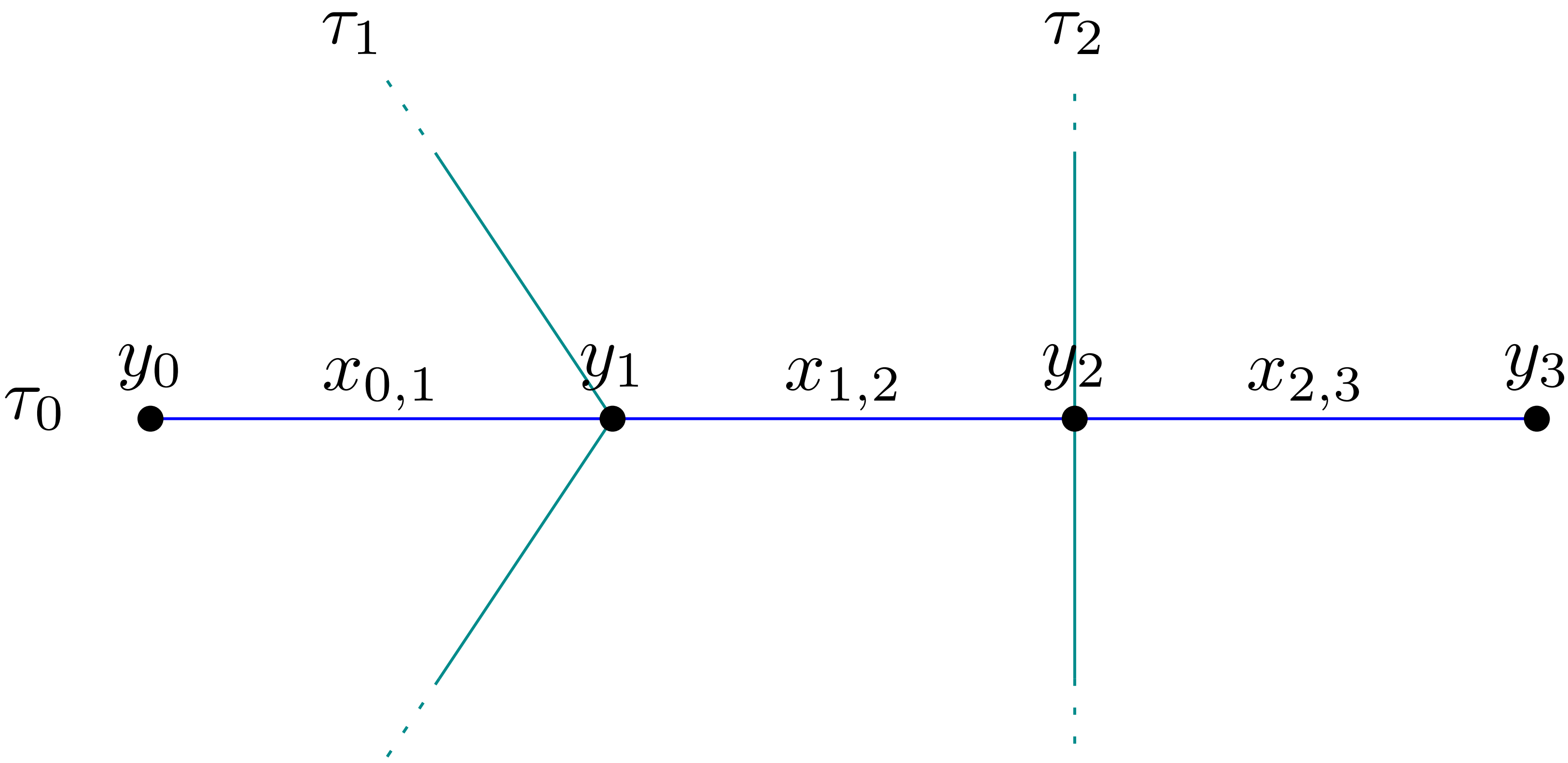
Hardness proof: Geometric graphs



Hardness proof: Geometric graphs



Integer Programming Preliminaries

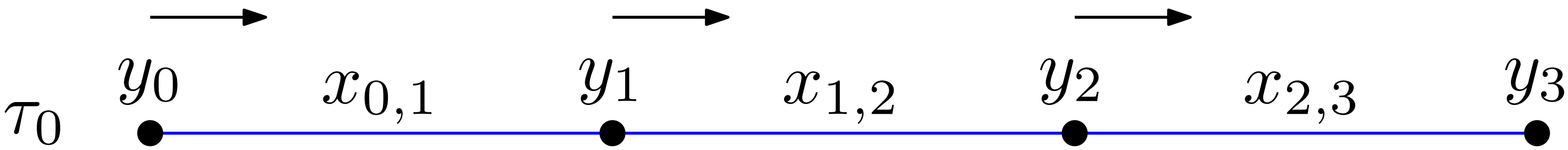


Integer Programming

$$\max \sum_{\tau \in \mathcal{T}, e \in E(\tau)} f(e) x_{\tau,e}$$

$$\sum_{v \in V} y_v \leq k \qquad \text{(Constraint 1)}$$

Integer Programming



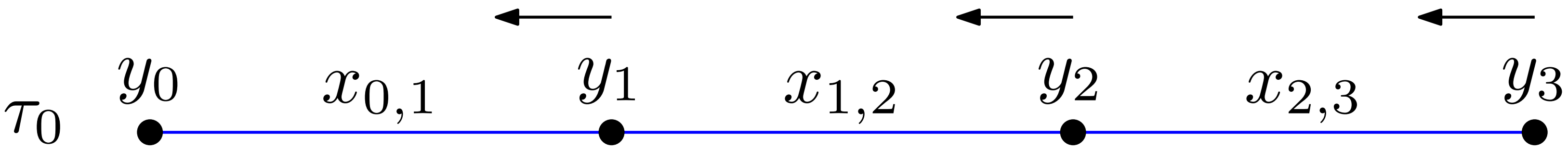
$$y_0 \geq x_{0,1} \qquad x_{0,1} + y_1 \geq x_{1,2} \qquad x_{1,2} + y_2 \geq x_{2,3}$$

Generalization:

$$\forall \tau = (v_0, \ldots, v_l) \in \mathcal{T} :$$

$$\forall i \in 0, \ldots, l-1 : \quad \begin{cases} x_{\tau, v_i v_{i+1}} \leq y_i & \text{if } i = 0, \\ x_{\tau, v_i v_{i+1}} \leq y_i + x_{\tau, v_{i-1} v_i} & \text{else} \end{cases}$$

Integer Programming



$$x_{0,1} \leq y_1 + x_{1,2} \qquad x_{1,2} \leq y_2 + x_{2,3} \qquad x_{2,3} \leq y_3$$

Generalization:

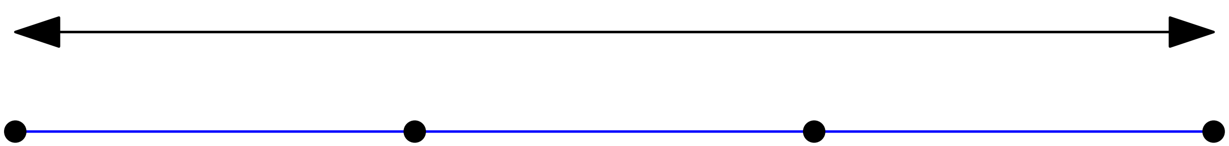
$$\forall \tau = (v_0, \ldots, v_l) \in \mathcal{T} :$$

$$\forall i \in 1, \ldots, l : \qquad \begin{cases} x_{\tau, v_{i-1} v_i} & \leq & y_i & \text{if } i = l, \\ x_{\tau, v_{i-1} v_i} & \leq & y_i + x_{\tau, v_i v_{i+1}} & \text{else} \end{cases}$$

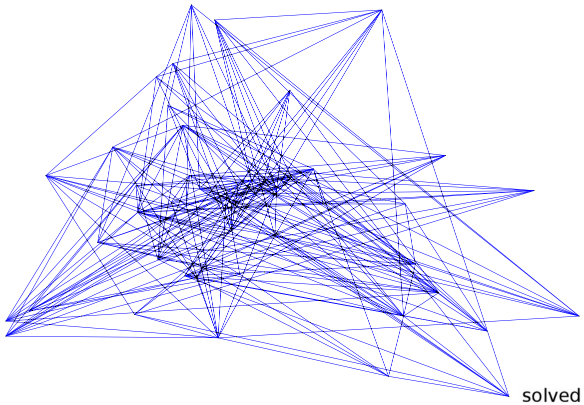
$$\max \sum_{\tau\in\mathcal{T}, e\in E(\tau)} f(e)x_{\tau,e}$$

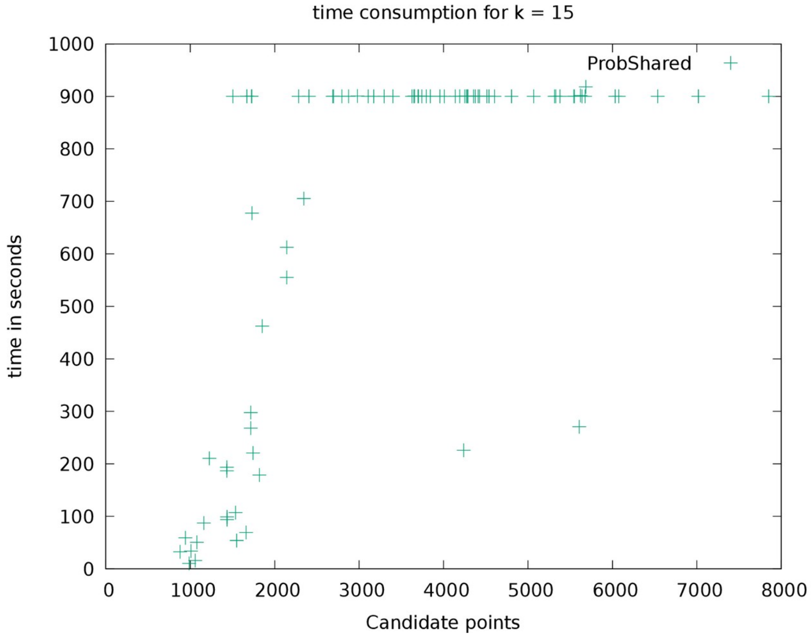
$$\sum_{v\in V} y_v \leq k \qquad \text{(Constraint 1)}$$

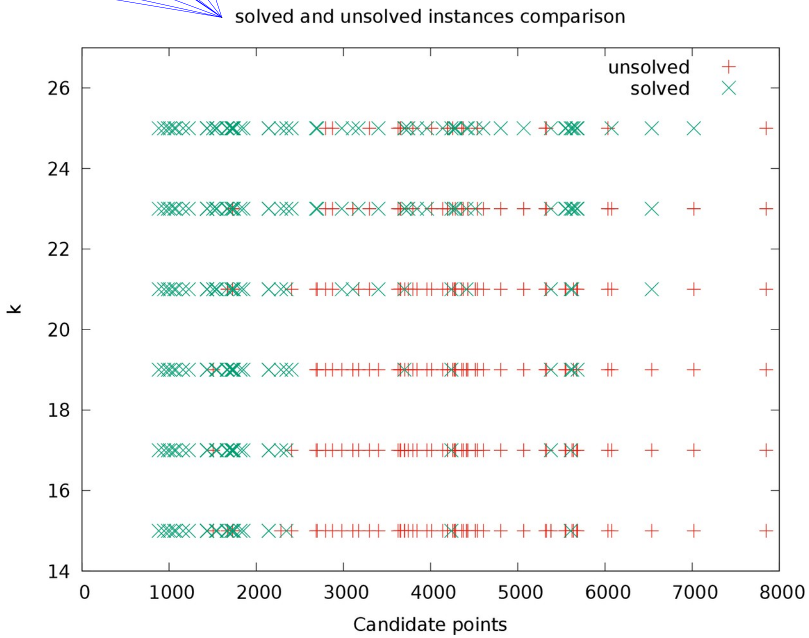
$$\begin{array}{lllll}
\forall \tau = (v_0,\dots,v_l)\in\mathcal{T}: & & & & \\
\quad \forall i\in 0,\dots,l-1: & \left\{\begin{array}{lll} x_{\tau,v_iv_{i+1}} & \leq & y_i \\ x_{\tau,v_iv_{i+1}} & \leq & y_i + x_{\tau,v_{i-1}v_i} \end{array}\right. & \begin{array}{l} \text{if } i=0, \\ \text{else} \end{array} & \text{(Constraint 2)} \\
\quad \forall i\in 1,\dots,l: & \left\{\begin{array}{lll} x_{\tau,v_{i-1}v_i} & \leq & y_i \\ x_{\tau,v_{i-1}v_i} & \leq & y_i + x_{\tau,v_iv_{i+1}} \end{array}\right. & \begin{array}{l} \text{if } i=l, \\ \text{else} \end{array} & \text{(Constraint 3)} \\
\forall v\in V, \tau\in e\in\tau: & x_{\tau,e}, y_v \in \{0,1\} & & &
\end{array}$$



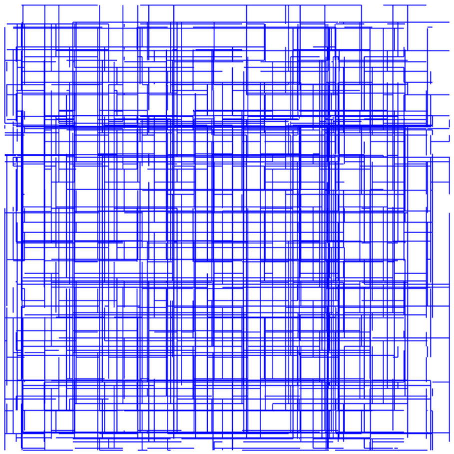
Evaluation: Segments between points

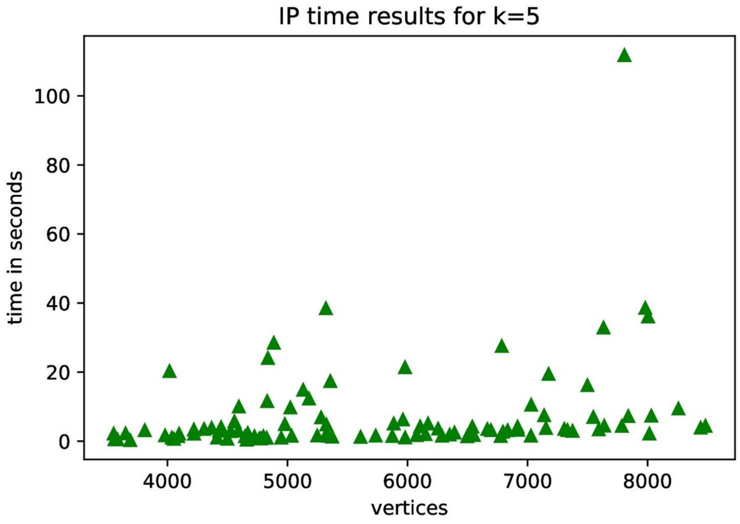


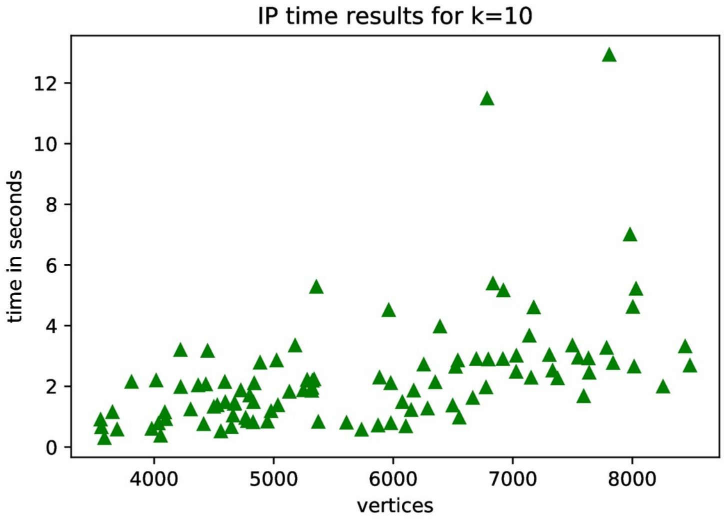




Evaluation: Non-overlapping axis-aligned





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Orinda
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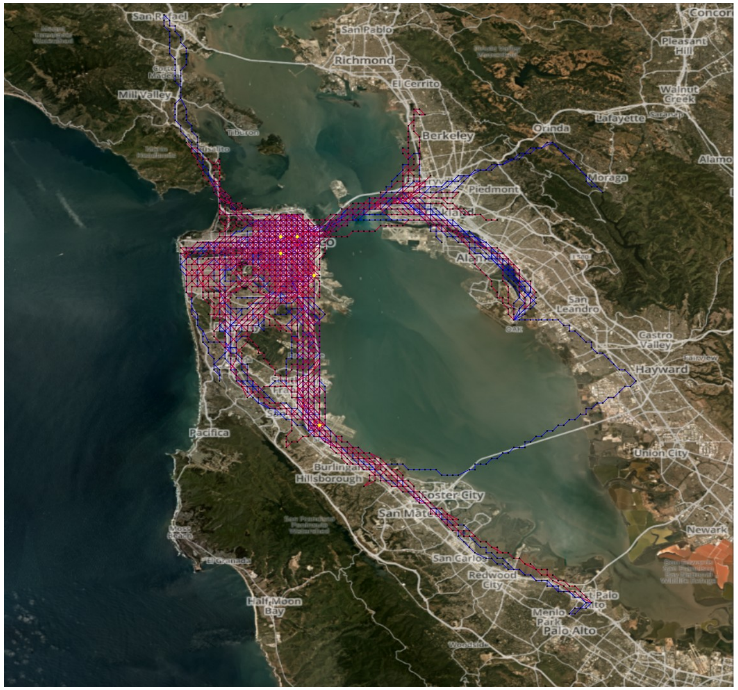
Piedmont
Alamo
Moraga
Alameda
San Leandro
Castro Valley
Daly City
Hayward
Pacifica
Union City
Burlingame
Hillsborough
Foster City
San Mateo
Newark
San Carlos
Redwood City
Half Moon Bay
East Palo Alto
Menlo Park
Palo Alto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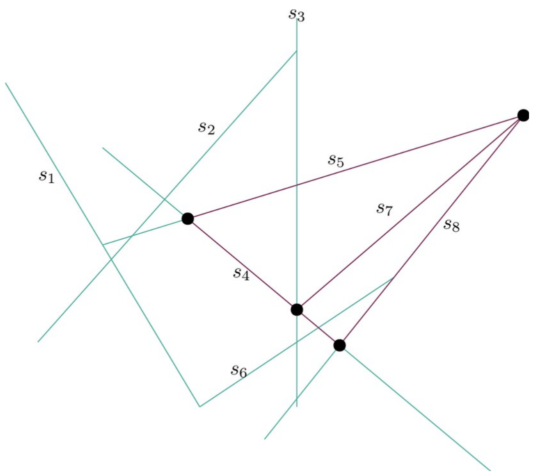
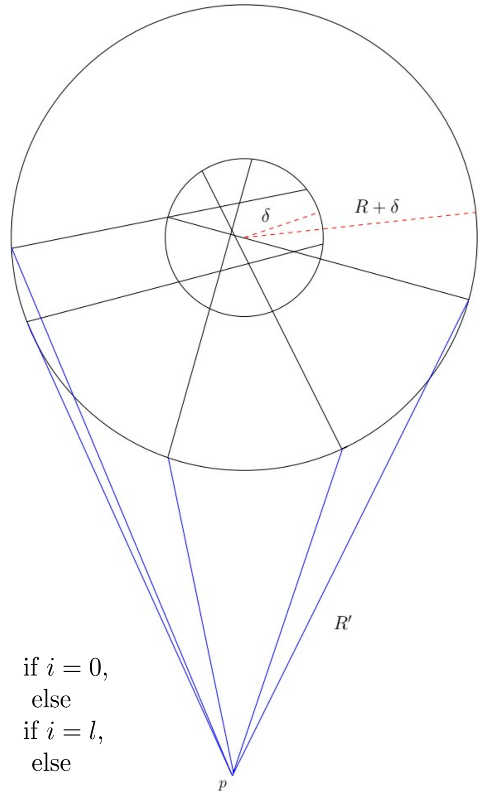
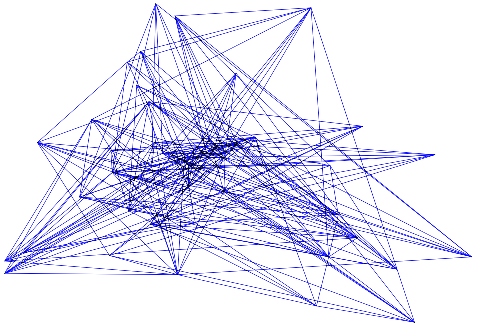
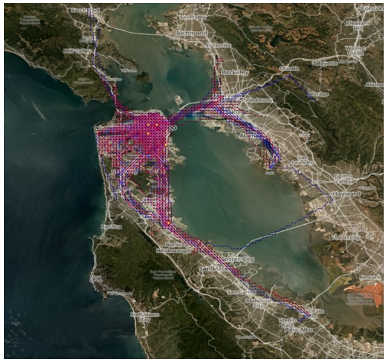
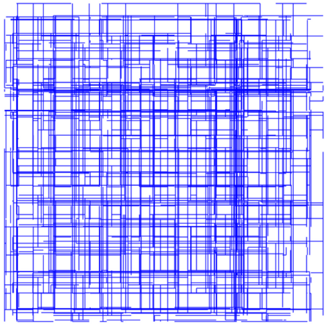
Conclusion











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$$\sum_{v \in V} y_v \leq k$$

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