t-spanners for Transmission Graphs Using the Path-Greedy Algorithm

Stav Ashur and Paz Carmi

EuroCG'20 Würzburg

Overview

Outline

A t-Spanner for a Directed Graph

Let $G = (V, E)$ be a directed graph.

A t-Spanner for Directed Graphs

A t-spanner G' for G is a sparse subgraph $G' \subseteq G$, s.t. for any two vertices $p, q \in G$, there is a directed path from p to q in G' of length at most t times the length of the path from p to q in G.

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Formally:

$$\forall p, q \in V : \quad |\pi_{G'}(p, q)| \leq t \cdot |\pi_G(p, q)|$$

Where $\pi_G(p, q)$ is the shortest directed path from p to q in the graph G, and $|\pi_G(p, q)|$ is its length.

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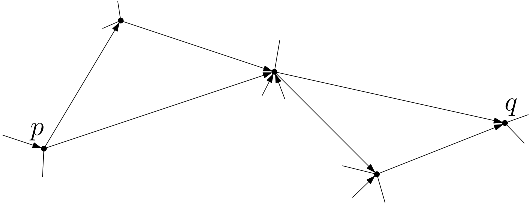

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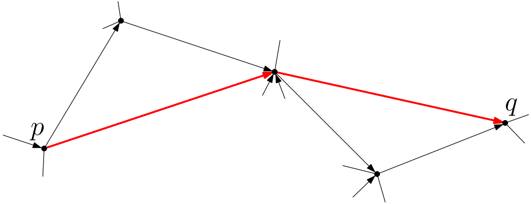

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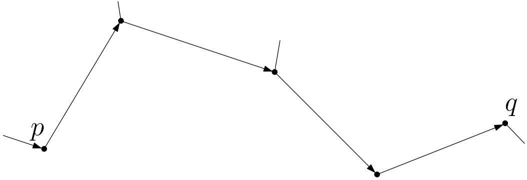

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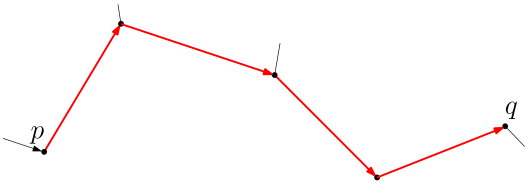

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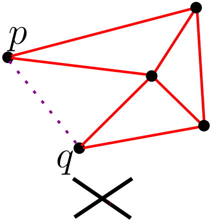
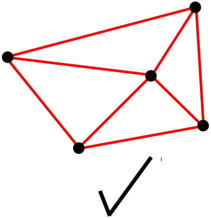
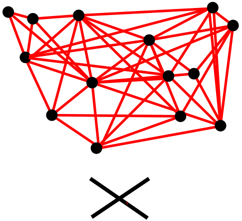
- Small stretch factor (spanning ratio)

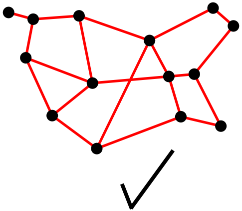




Desired Properties

- Small stretch factor (spanning ratio)
- Small number of edges (linear is preferable)





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- Small stretch factor (spanning ratio)
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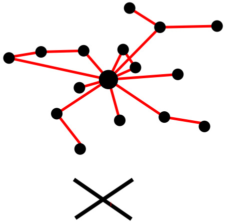

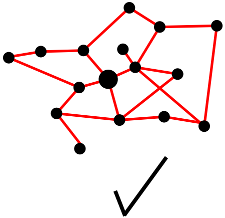

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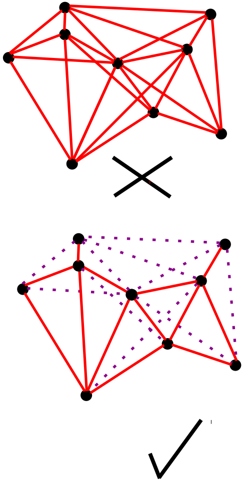
- Weight



Desired Properties

- Small stretch factor (spanning ratio)
- Small number of edges (linear is preferable)
- Bounded degree
- Weight
- Easy construction

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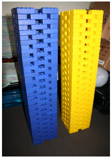

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Outline

Path-Greedy Algorithm

Path-Greedy, $O(n^3 \log n)$

I. Althöfer, G. Das, D. Dobkin, D. Joseph, J. Soares, (1993)

Input: Given a graph $G = (P, E)$, where $P \subset \mathbb{R}^d$, E are edges with Euclidean weights, and a real number $t > 1$.

Output: The Path-Greedy t-spanner $G' = (P, E')$ for G.

$E \leftarrow E$ sorted in non-decreasing order of length
$E' := \emptyset$
$G' := (P, E')$
ForEach $(u, v) \in E$ (in sorted order)
  If $\pi_{G'}(u, v) > t \cdot |uv|$
    $E' := E' \cup \{(u, v)\}$
Return: $G = (P, E')$

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- The Path-Greedy spanner has the best properties
  - $O(n)$ edges
  - Bounded degree
  - Total weight $O(wt(MST(G)))$
  - Very simple
- Its main weakness is its time complexity

≈Greedy

Approximate Greedy, $O(n \log^2 n)$

G. Das and G. Narasimhan, (1997)

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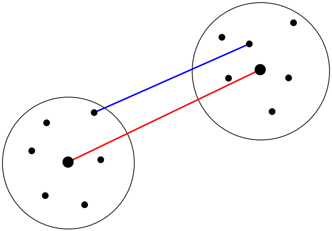
- Approximating Dijkstra's algorithm by querying a cluster graph



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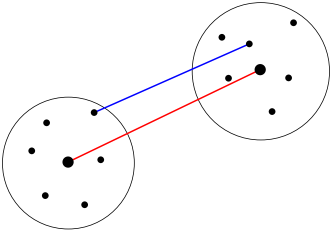
- Calculating a t-spanner in $O(n \log^2 n)$



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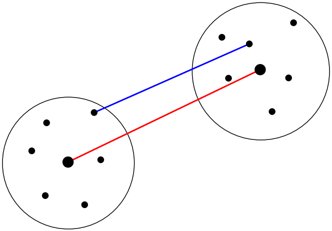
- Calculating a t-spanner in $O(n \log^2 n)$
- Theoretically has good properties as the Path-Greedy spanner



Path Greedy superiority

Experimental Study of Geometric t-Spanners

M. Farshi, & J. Gudmundsson, (2009)

Algorithm	Edges	Degree	$\frac{\textbf{Weight}}{wt(MST)}$
Path-Greedy	36K	17	11
θ-Graph	370K	144	327
$\approx$-Greedy	852K	403	
WSPD spanner	11,119K	5,192	70,470

Table: Results for 8000 random uniformly distributed points with $t = 1.1$

Path-Greedy Improvements

Fast Path-Greedy, $\tilde{O}(n^2)$

P. Bose, P. Carmi, M. Farshi, A. Maheshwari, M. Smid, (2010)

δ-Greedy, $\tilde{O}(n^2)$

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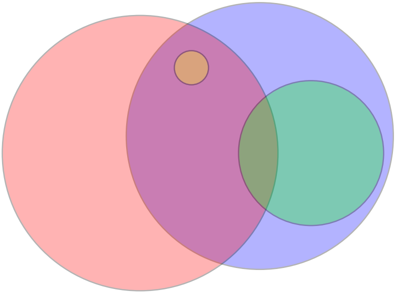
- Maintain cones in directions where t-spanning paths are already guaranteed
- Simpler than Fast-Greedy

Outline

1. Introduction
   - Notations and Definitions
   - Path-Greedy Spanner
2. Transmission Graphs
   - Definitions
   - Results
3. Computing a t-Spanner for Transmission Graphs
   - Algorithm
   - Analysis

Transmission Graphs

$D = \{d_1, ..., d_n\}$ a set of disks in $\mathbb{R}^d$



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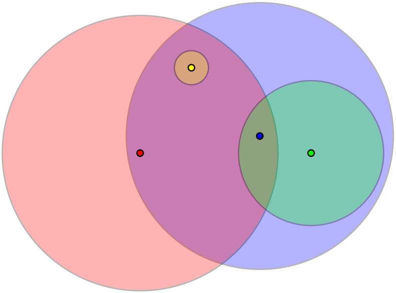

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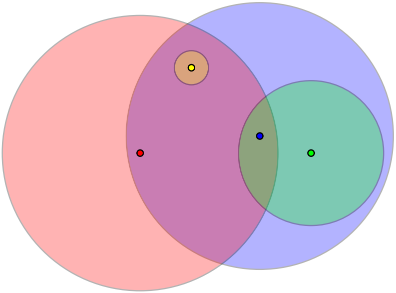

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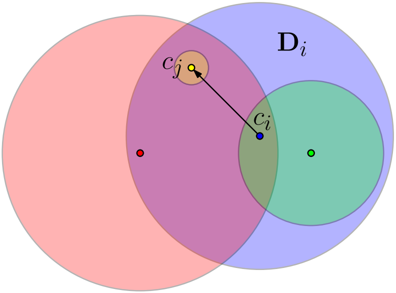

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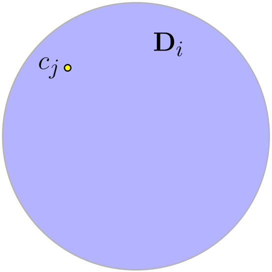

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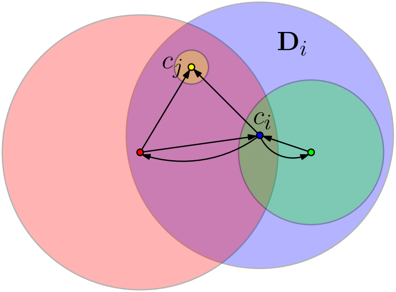

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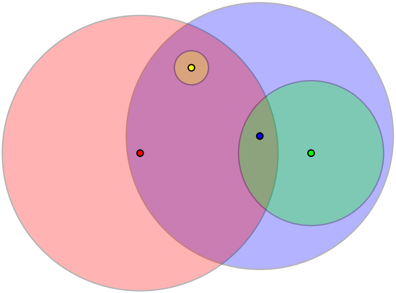

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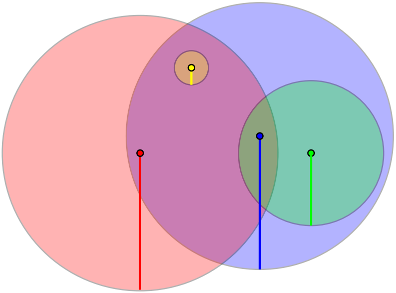

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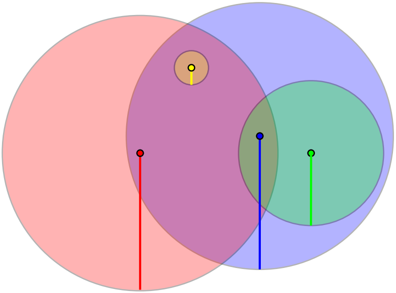

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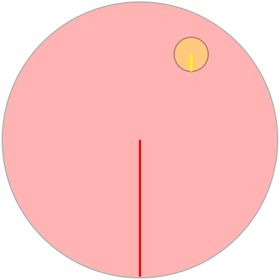
$$r_{max} = max\{r_i\}, r_{min} = min\{r_i\}, \Psi = \frac{r_{max}}{r_{min}}$$



Outline

Transmission graph Spanners

All results are of a t-spanner for any $t > 1$.

$O(n \cdot \Psi)$ edges, $O(m \log n)$ time

D. Peleg, L. Roditty, (2010)

$O(n)$ edges, $O(n \log n + n \log \Psi)$ or $O(n \log^5 n)$ time

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No previous result of a t-spanner with bounded weight.

Our results

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- Weight bounded by a function of n and Ψ
- Disadvantage - runtime, can be reduced by using the δ-Greedy

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Algorithm

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$1 < t \in \mathbb{R}$

Algorithm for computing a t-spanner for the disk graph G of D:

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Claim 1

G' is a t-spanner of G

Outline

Bounded In-Degree

Claim 2

The in-degree of every vertex in G is bounded by a constant that depends on t.

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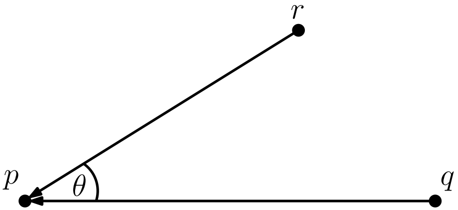

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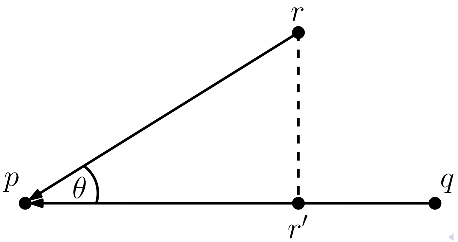

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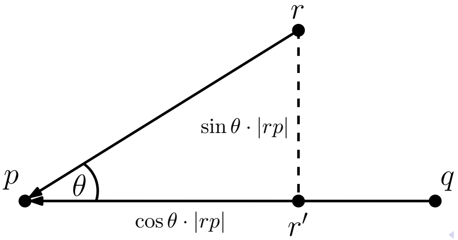

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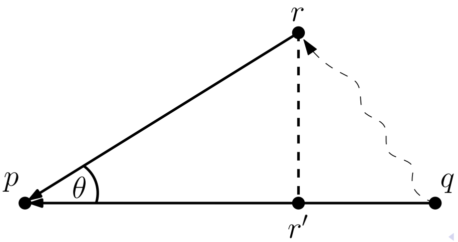

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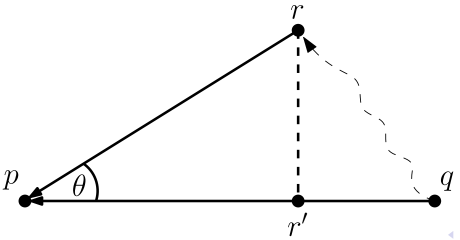

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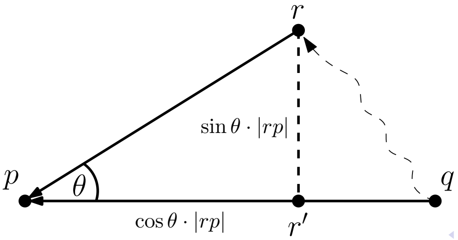

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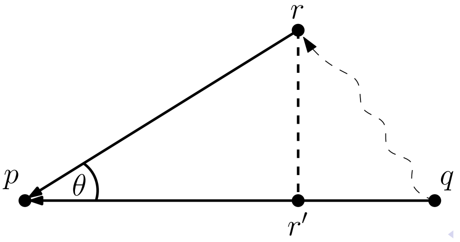

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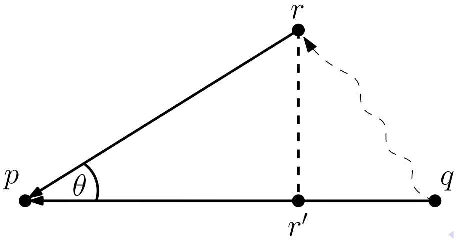

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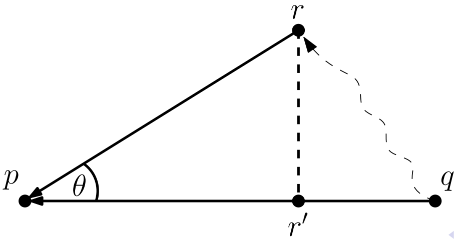

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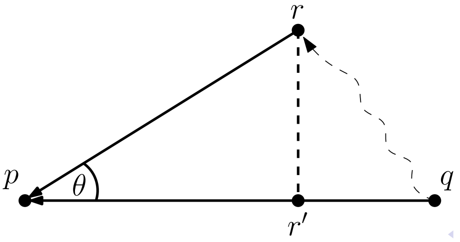

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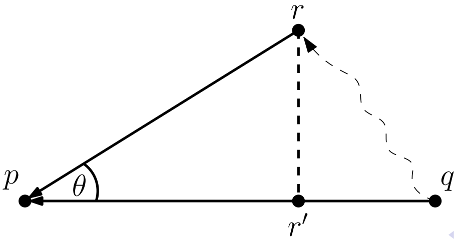

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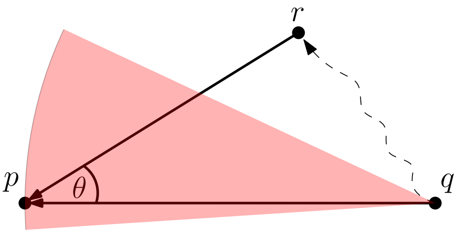

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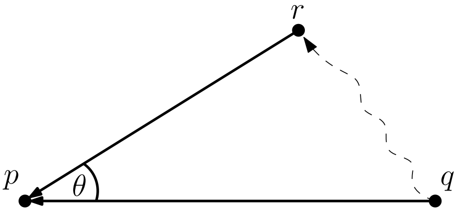

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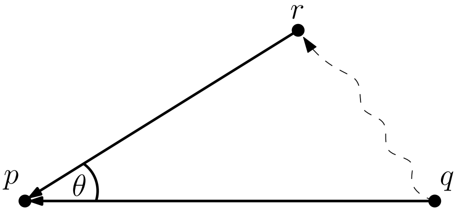

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The weight of G', denoted by $wt(G')$, is bounded by

$$O((1+\Psi)\cdot \log n \cdot wt(MST(G))),$$

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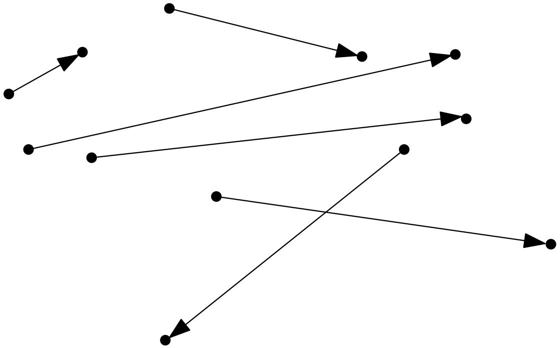

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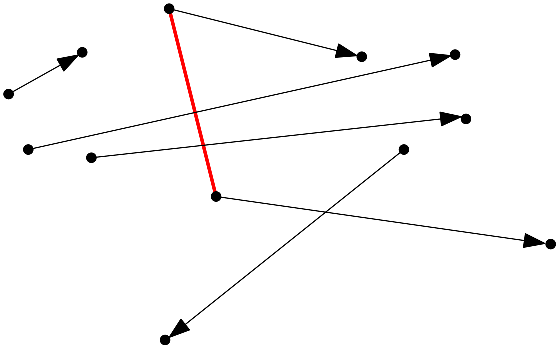

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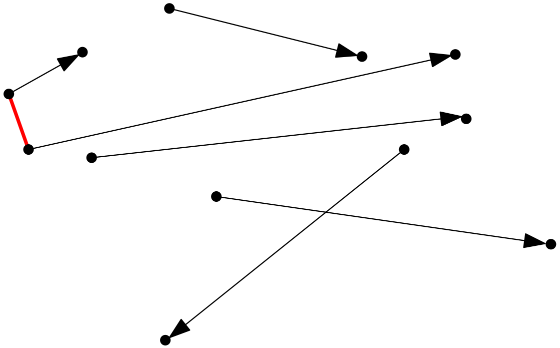

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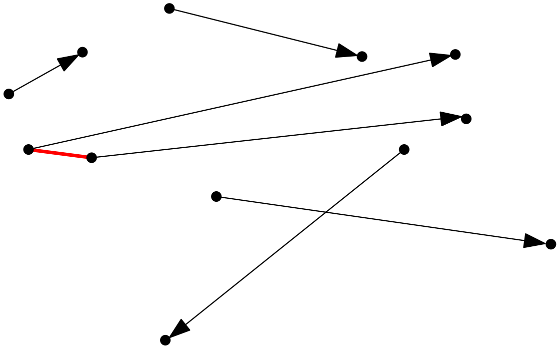

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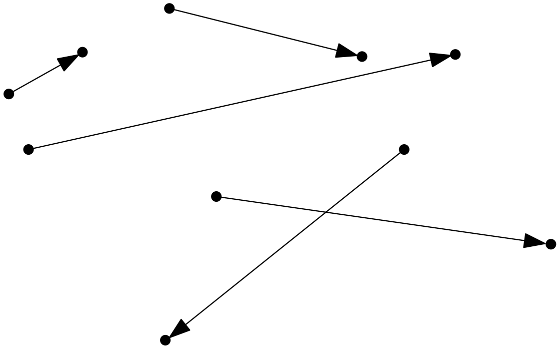

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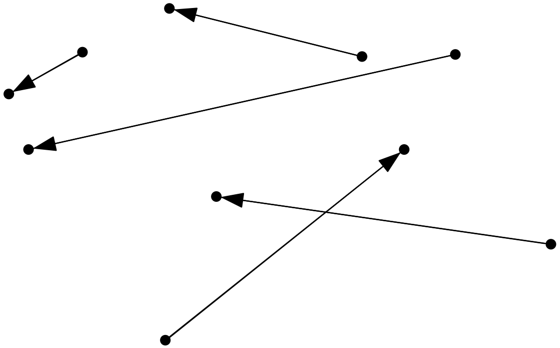

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Lemma [G. Narasimhan, M.Smid 2007]

A set of directed edges E that admit the w-gap property for $w \in \mathbb{R}^+$, then the total weight of E is less than

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We divide $E(G')$ into a constant number of sets that admit the $\frac{1}{\Psi}$-gap property.

Conclusion

1. A simple t-spanner for transmission-graphs
2. Bounded in-degree
3. Bounded weight
4. $O(n^2 \log n)$ runtime

Thank You